Multidim Syst Sign Process (2014) 25:759–773
DOI 10.1007/s11045-013-0229-4

A novel range alignment method for ISAR based on linear T/R array model

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Received: 19 June 2012 / Revised: 2 February 2013 / Accepted: 19 February 2013 /
Published online: 8 March 2013

Abstract In inverse synthetic aperture radar (ISAR) imaging, translation compensation should be done before range-Doppler imaging process, and range alignment is the first step for translation compensation. In order to remove the limitation of integer range bin and align the echoes precisely for ISAR range alignment, combining with the advantage of array signal processing at fractional unit delay compensation, we propose a novel range alignment method based on linear transmitting/receiving (T/R) array. Firstly the ISAR imaging system is modeled as a linear T/R array. Then based on the snapshot imaging model of linear T/R array, range alignment is accomplished by wave path difference compensation which is transformed into the phase difference compensation in frequency domain between adjacent array elements. The phase difference compensation consists of integer range bin alignment and decimal time delay compensation which is implemented by the phase rotation's estimation and compensation in frequency domain. Finally, the results of simulation data and real radar data are provided to demonstrate the effectiveness of the proposed method.

Keywords Inverse synthetic aperture radar (ISAR) · Linear transmitting/receiving array · Range alignment · Imaging

1 Introduction

Inverse synthetic aperture radar (ISAR) imaging of non-cooperative target in the air, space, or on the sea is very important for high resolution target image generation and automatic target identification (Xing et al. 2009). In past decades, the research of ISAR has mainly focused

This work was supported by Industry-academic Joint Technological Innovations Fund Project of Jiangsu Province (BY2012187), NSFC (61101082), Natural Science Foundation of Jiangsu Province (BK2011562), Natural Science Foundation of Jiangsu Province (BK2010376) and Graduate Innovation Program of Jiangsu Province (CXZZ11_0043).

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on high-resolution imaging, translation compensation methods, target recognition (Kim et al. 2000; Delisle and Wu 1994; Gunaratne and Bruton 2011; Zhu et al. 2009), etc.

Inverse synthetic aperture radar (ISAR) utilizes the range-Doppler principle to obtain the desired image of a revolving target (Mensa 1982). Before the Fourier transform, translation compensation should be done to eliminate the effects of the translation between the radar and the target in range. The translation compensation consists of range alignment and phase adjustment (Su and Yuan 2011). Range alignment is the first step for translation compensation, and phase adjustment is the second one.

Many methods have been proposed to accomplish range alignment. Typical methods include the maximum-correlation method (Delisle and Wu 1994; Mensa 1982), the minimum entropy method (Zhu et al. 2009), the peak method (Mensa 1982), the frequency domain method (Su and Yuan 2011), and so on. Most of these methods are based on the similarity of the echoes' envelops, so their accuracy is limited to integer range bin. Even if two echoes are aligned by using these methods, the alignment error still exists, up to half a range bin.

The linear T/R antenna array can receive the echoes from different directions of the same target (Delisle and Wu 1994). To align the response echoes of different linear T/R array elements, the time delay caused by wave path difference can be compensated in frequency domain by introducing a phase rotation, and fractional time delay can be made to obtain more accurate wave path difference compensation (Hachabiboglu et al. 2007; Foo and Kashyap 2003). The core problem for wideband pulse array is the realization of variable delay circuits to compensate the channel propagation delay accurately. Now the problem can be solved by combined the fractional sample unit delay filters with delay lines (Foo and Kashyap 2003; Wang et al. 2006). The fractional delay filters (FDF) can be implemented by many methods, such as spline-based interpolation (Viola and Walker 2005), Lagrange interpolation (Wang et al. 2006) and so on. A widely used and proved effective FDF method is Lagrange interpolation filters (Wang et al. 2006; Dooley and Nandi 1999).

In this paper, combining the advantage of array signal processing in fractional sample unit delay, a novel range alignment method for ISAR range alignment based on the equivalent linear T/R array model is proposed. Firstly, the ISAR data acquisition process is modeled as a uniform linear T/R antenna array. Then considering the equivalent relationship between the linear T/R array and ISAR echo data structure, the echo data of ISAR is rearranged as the array received data. Subsequently, the range alignment method based on the linear T/R array is given. Between adjacent array elements, the wave path difference compensation corresponding to range alignment can be implemented by integer range bin alignment and decimal time delay compensation, which can be done by the estimation and compensation of the phase rotation in frequency domain. The estimation of the phase slope is based on the least-square linear fitting method, and the compensation of phase rotation is based on the circular shifting characteristic of discrete Fourier transform (DFT). At last, after the range alignment is accomplished, the phase adjustment can be done, and the range-Doppler imaging will finally be obtained. The theoretical analysis and the results of simulation data and real radar data show that, with the same phase adjustment method and range-Doppler imaging, the quality of range alignment based on linear T/R array is better than regular range alignment methods. Particularly, to verify the effectiveness of the proposed method on decimal time delay compensation, the Lagrange interpolation FDF method is also compared.

2 Range alignment model based on linear T/R array

According to Skolnik (2002) and Zhu et al. (2010), the ISAR radar is supposed to work in a "move-stop-move" mode during the data acquisition process. That is to say, the target is considered to be stationary in range domain during the transmitting and receiving of a repetition pulse. The ISAR imaging model is shown in Fig. 1a.

The structure of ISAR echoes data matrix is shown in Fig. 2, in which N is the repetition pulse number in azimuth direction, M is the sampling number in range direction. The echoes data is an $N \times M$ matrix. The shadow parts denote the distribution area of echoes scatterer points of target, and the black dots represent the positions of a same scatterer point in different echoes.

For ISAR imaging model shown in Fig. 1a, supposing T_p is the pulse repetition time(PRT), then the accumulating time for T/R chirp pulse is $N \cdot T_p$ and the horizontal movement distance of the target is $d_a = T_p \cdot v$.

For linear T/R array, the time delay caused by the wave path difference between adjacent elements can be compensated in frequency domain by estimating the phase rotation, and fractional time unit delay can be made to obtain more accurate time delay compensation (Delisle and Wu 1994; Hachabiboglu et al. 2007). Inspired by this, the data acquisition process of ISAR in Fig. 1a is modeled as a single snapshot of the linear T/R array in Fig. 1b. The n_{th} $(n = 1, 2, \ldots, N)$ T/R repetition pulse sampling data for ISAR is corresponding to the singe snap-shot space sampling of the n_{th} T/R array element in Fig. 1b. Based on the equivalent relationship shown in Fig. 1, the matrix of echoes data shown in Fig. 2 can be rearranged as the matrix of received array data. Thus, the range alignment for ISAR can be implemented by path difference compensation between adjacent array elements in the equivalent linear T/R array.

For the model shown in Fig. 1b, the envelope delay between adjacent array elements is

$$\tau_d = \Delta d / c \tag{1}$$

where c is the wave velocity, and Δd is the wave path difference which has to be estimated and compensated between adjacent array elements. Δd is mainly determined by d_a and the

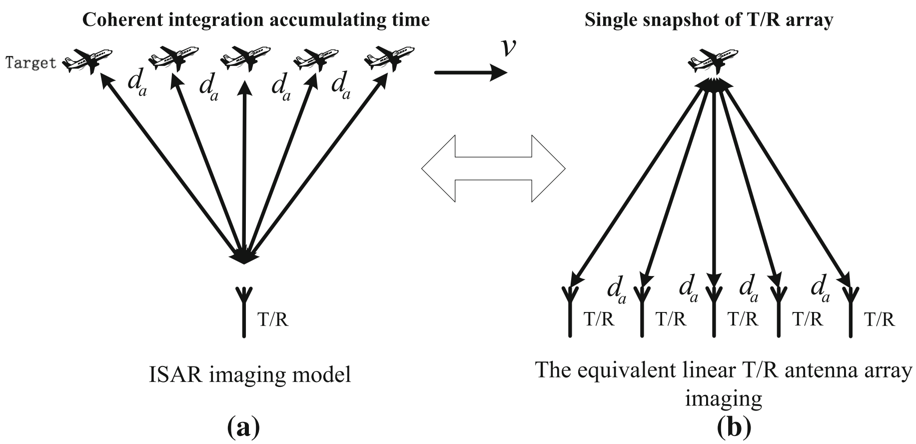


Fig. 1 Equivalent relationship between ISAR imaging and the linear T/R antenna array imaging, d_a is the linear extent of target position between two transmitting/receiving (T/R) moments, and v is the target's velocity

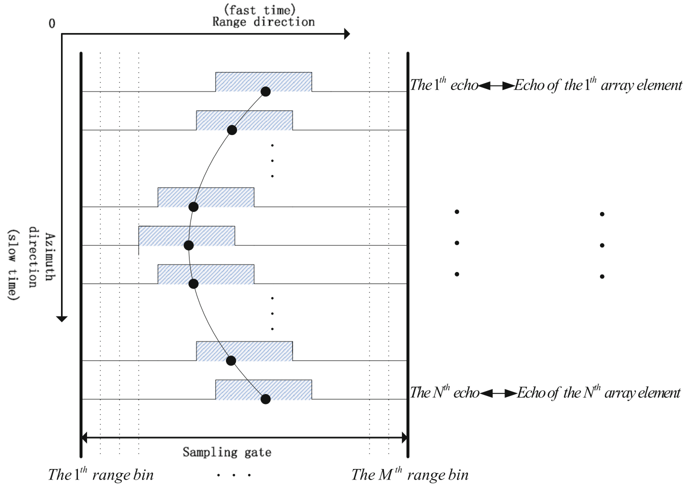
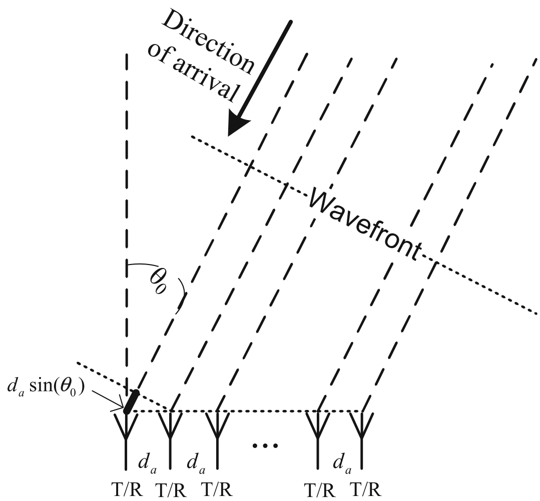


Fig. 2 The structure of ISAR echoes data matrix

Fig. 3 The linear T/R array model in far field assumption



direction-of-arrival (DOA) of the returned signal. In the condition of far field assumption, the receive signal of the array elements is plane wave, which means the DOAs of the reflected signal from the same target for all array elements are parallel (Ward 1995; Liu et al. 2013). The linear T/R array model in far field assumption is shown in Fig. 3.

For the model shown in Fig. 3, the wave path difference between adjacent array elements Δd can be expressed as $\Delta d = d_a \sin(\theta_0)$. If the parameters in the model shown in Fig. 1b don't satisfy the far field assumption, the expression of Δd should be more complex. In our application, the envelope delay between adjacent array elements τ_d expressed in (1) is the parameter that we want to estimate from echoes data. The expression of Δd is not the research emphasis of this paper, and τ_d or Δd in (1) are taken to the discussion as an entirety in the follows.

3 Range alignment method based on linear T/R array model

Supposing $fr_1(t)$ and $fr_2(t)$ are the echoes of two adjacent array elements, the relationship between $fr_1(t)$ and $fr_2(t)$ can be expressed as

$$fr_1(t) \cong \sigma_{1,2} fr_2(t - \tau_d) \tag{2}$$

where t is the fast time in range direction, $\sigma_{1,2}$ is an overall reflectivity function between the adjacent array elements, and $\sigma_{1,2}$ is independent with t and τ_d.

Based on the rearranged array echoes in Sect. 2, to avoid phase-wrapping between adjacent array elements in frequency domain, integer range bin alignment should be done firstly. To break through the limitation of integer range bin of most traditional methods, the decimal time delay, which is corresponding to the decimal range bin that has to be aligned, should also be compensated. Thus, the proposed range alignment method based on linear T/R array model consists of integer range bin alignment and decimal time delay compensation. Then Δd is divided to the integer range bins Δr and the decimal range bin $c \cdot \tau_r$, therefore, (1) can be re-expressed as

$$\tau_d = \Delta r / c + \tau_r \tag{3}$$

In order to avoid phase wind-up between different array element echoes, the integer range bin alignment should be accomplished first so that different array element echoes from the same scatterer can be aligned at the same range bin. Then a compensation factor is needed to make the decimal time delay compensation in time domain, which can be done by phase rotation compensation in frequency domain. The flowchart of ISAR imaging based on the proposed range alignment method is shown in Fig. 4. We will illustrate the two steps of range alignment in the following context.

Because of the time-displacement characteristic of Fourier transform, the time delay in time domain is corresponding to the phase rotation in frequency domain. Therefore decimal time delay compensation for range alignment can be made in frequency domain. We take Fourier transform on both sides of (2), then

$$\mathscr{F}(fr_1) = \sigma_{1,2} \mathscr{F}(fr_2) \exp(-j2\pi f(\Delta r/c + \tau_r)) \tag{4}$$

where f is the center frequency.

Then the phase difference corresponding to the wave path difference can be obtained

$$\Delta\phi_{1,2} = 2\pi f \Delta r / c + 2\pi f \tau_r \tag{5}$$

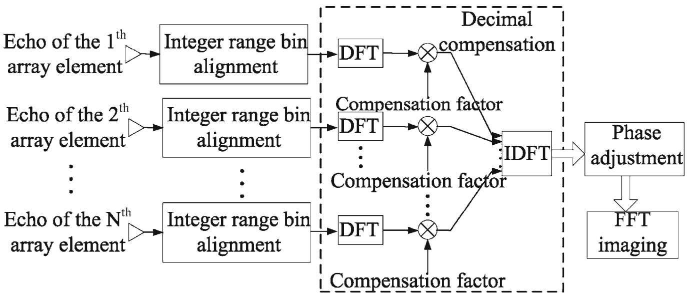


Fig. 4 The flowchart of ISAR imaging based on the proposed method

In (5), $\Delta\phi_1 = 2\pi f \Delta r/c$ is the phase rotation that can be compensated in integer range bin alignment, and $\Delta\phi_2 = 2\pi f \tau_r$ denotes the phase rotation of the decimal time delay in frequency domain that has to be compensated.

Integer range bin alignment can be accomplished by many methods introduced in Sect. 1. In our range alignment method, the key problems focus on the estimation and compensation of the phase rotation caused by the decimal time delay. Thus, a simple and efficient integer range bin alignment method named maximum-correlation (Delisle and Wu 1994; Mensa 1982) is adopted here. The maximum-correlation method calculates the correlation value of adjacent repetition pulse sampling data which is corresponding to space sampling data of adjacent array elements in our method, and finds the maximum correlation value to locate the integer range bins that have to be shifted.

After the integer range bins are aligned, let $x_1(m); m = 1, 2, \ldots, M$ and $x_2(m); m = 1, 2, \ldots, M$ be the discrete sampling of the echoes of the two adjacent array elements. According to (2) and (5), to be aligned with the sampling echo $x_2(m)$, $x_1(m)$ has to make a decimal time delay by introducing a phase rotation in frequency domain

$$x_1(m - \tau_r) = \frac{1}{M} \sum_{k=-M/2}^{k=M/2-1} \left\{ \left[\sum_{m'=0}^{M-1} x_1(m') \exp\left(-j\frac{2\pi}{M}km'\right) \right] \exp\left(-j\frac{2\pi}{M}k\tau_r\right) \right\} \times \exp\left(j\frac{2\pi}{M}km\right) \tag{6}$$

Combining with the circular shifting characteristic of discrete fourier transform (DFT), (6) can also be expressed as

$$x_1(m - \tau_r) = IDFT[X(k)] \tag{7}$$

where

$$X(k) = \begin{cases} DFT\left[x_1(m)\right] \exp\left(-j\frac{2\pi}{M}k\tau_r\right), & 0 \le k \le \frac{M}{2} - 1 \\ DFT\left[x_1(m)\right] \exp\left[-j\frac{2\pi}{M}(k - M)\tau_r\right], & \frac{M}{2} \le k \le M - 1 \end{cases} \tag{8}$$

Hence, the phase rotation that has to be compensated in frequency domain is

$$\Delta\phi_2(k) = \begin{cases} \frac{2\pi}{M}k\tau_r, & 0 \le k \le \frac{M}{2} - 1 \\ \frac{2\pi}{M}(k - M)\tau_r, & \frac{M}{2} \le k \le M - 1 \end{cases} \tag{9}$$

After the phase rotation between the two adjacent array elements is compensated in frequency domain and the IDFT operation is taken, the decimal time delay compensation is accomplished and the range alignment based on linear T/R array model is finished. According to (9), the $\frac{2\pi}{M}\tau_r$ is the phase slope of $\Delta\phi_2(k)$. Thus, in order to make the decimal time delay compensation, the remaining problem is how to extract the phase slope between the two adjacent array elements in frequency domain.

4 Phase slope extraction

Considering the structure of $\Delta\phi_2(k)$, two stages of least-square linear fitting are needed to estimate the phase rotation $\Delta\phi_2(k)$. Because the phase slope τ_r is same for the two stages, the observation values of $\Delta\phi_2(k)$ in the stage of $0 \le k \le \frac{M}{2} - 1$ is used to estimate the phase slope. According to (9), the least-square approximation of $\Delta\phi_2(k)$ can be written as

$$\Delta\tilde{\phi_2}(k) = b \times k, \quad 0 \le k \le \frac{M}{2} - 1 \tag{10}$$

Based on the minimum mean-square error (MMSE) criterion, b can be estimated as

$$b = \frac{K\sum_{k=1}^{K} k\Delta\varphi(k) - \sum_{k=1}^{K} k \sum_{k=1}^{K} \Delta\varphi(k)}{K\sum_{k=1}^{K} k^2 \left(\sum_{k=1}^{K} \Delta\varphi(k)\right)^2} \tag{11}$$

where $K = \frac{M}{2} - 1$, and $\Delta\varphi(k)$ is the observation value of $\Delta\phi_2(k)$ in the stage of $0 \le k \le \frac{M}{2} - 1$.

The cross-power spectrum of the adjacent array elements' echo is used to get the observation value of phase rotation $\Delta\varphi(k)$ (Mensa 1982; Jiang and Chen 2007). Let $s_1(k)$ and $s_2(k)$ be the observation value of the two adjacent array elements after the integer range bins.

$$\begin{aligned} s_1(k) &= x_1(k) + n_1(k) \\ s_2(k) &= x_1(k - \tau_r) + n_2(k) \end{aligned} \tag{12}$$

$n_1(k)$ and $n_2(k)$ are mutually independent zero-mean Gaussian white noise, and they are all independent with $x_1(k)$.

Suppose the Fourier transform of $x_1(k)$, $s_1(k)$, and $s_2(k)$ are $X_1(k)$, $S_1(k)$, and $S_2(k)$. The cross-power spectrum of $s_1(k)$ and $s_2(k)$ can be expressed as

$$\begin{aligned} &S_1(m)S_2^*(m) \\ &\quad = |X_1(k)|^2 \exp(j\Delta\varphi(k)) + X_1(k)N_2^*(k) + X_1^*(k)N_1(k)\exp(j\Delta\varphi(k)) + N_1(k)N_2^*(k) \end{aligned} \tag{13}$$

where * is the complex conjugation operator, $N_1(k)$ and $N_2(k)$ are the Fourier transform of $n_1(k)$ and $n_2(k)$.

$$W(k) = X_1(k)N_2^*(k) + X_1^*(k)N_1(k)\exp(j\Delta\varphi(k)) + N_1(k)N_2^*(k) \tag{14}$$

Because $n_1(k)$ and $n_2(k)$ are independent with $x_1(k)$, $W(k)$ is the noise parts of the cross-power spectrum of $s_1(k)$ and $s_2(k)$. $|X_1(k)|^2$ in (13) can be substituted by the module value of the cross-power spectrum $\left|S_1(m)S_2^*(m)\right|$ with high signal to noise ratio (SNR). Then based on (13) and (11), the phase slope can be extracted.

Considering (9) and (11), if the envelop delay to be estimated is longer than 1 range bin, the phase rotation may be wrapped by 2π, which needs to be unwrapped. In the proposed linear T/R array method, the integer range bin alignment is accomplished first. Thus the phase-unwrapping process can be avoided during the estimation and compensation of the phase rotation caused by the decimal time delay.

With the extracted phase slope, combining (7) and (8), the phase rotation can be compensated in frequency domain, and after the IDFT operation is taken, the range alignment is finished. Then the translation compensation can be accomplished by phase adjustment. Finally, range-Doppler ISAR imaging can be obtained.

5 Experiments

5.1 Simulation test

To verify the given method in this paper, simulation results are given and analyzed in this section. The simulation parameters are listed in Table 1.

Table 1 Simulation parameters

System parameters	Numerical value
Signal bandwidth	400 MHz
Signal pulse width	10 μs
Signal carrier frequency	10 GHz
Wavelength	0.03 m
Pulse repetition frequency	10,000 Hz
Sampling frequency	450 MHz
Velocity of target	300 m/s
Target initial coordinate	(0, 600, 2,500)

Supposing that the ISAR imaging radar transmits linear frequency modulated (LFM) signal with the identical bandwidth and center frequency. The LFM signal can be expressed as

$$s(t) = rect\left(\frac{t}{T}\right)\exp\left(j2\pi f_c t + j\pi K_p t^2\right) \tag{15}$$

where $rect\left(\frac{t}{T}\right) = \begin{cases} 1 \; |t| \le T/2 \\ 0 \; |t| > T/2 \end{cases}$, f_c is the center frequency, T is the pulse width, and $K_p = B/T$ is the frequency modulation rate.

An airplane model containing twelve scatter points is used for imaging simulation. A set of simulation data, 256 repetition pulse echoes with 512 range bins each, is taken to evaluate our method. We perform the range alignment by the maximum-correlation method, the minimum entropy method and the proposed linear T/R array method. Specially, to verify the performance of the proposed method on decimal time delay compensation, the Lagrange interpolation FDF method (See "Appendix") is also compared. The Lagrange interpolation FDF method has satisfied performance if the order of FDF is larger than 10 (Wang et al. 2006; Laakso et al. 1996). The 11th order FDF is utilized in the Lagrange interpolation FDF method for the decimal time delay compensation in this paper. Combining (9) and (10), the decimal sample unit delay $\tau_r = \frac{M}{2\pi}b$ is used to generate the Lagrange interpolation FDF.

To better demonstrate the performance of range alignment, the phase adjustments are all realized by the phase gradient autofocus (PGA) algorithm (Munoz-Ferreras and Perez-Martinez 2009). The final imaging results are obtained by range-Doppler process, which are given in Figs. 5, 6, 7 and 8.

With decimal time delay compensation, the linear T/R array method and the Lagrange interpolation FDF method have more accurate range alignment than the other two integer range alignment methods. Range alignment has to be made before phase adjustment and range-Doppler imaging. Better range alignment should results in better ISAR image. To measure the focus quality of an image, the entropy of an image is defined (Mensa 1982; Jeong et al. 2008) as

$$E = -\sum_{n=0}^{N-1}\sum_{m=0}^{M-1} |D(n,m)|^2 \ln |D(n,m)|^2 \tag{16}$$

where $D(n, m)$ is complex image data. n and m are the sequence numbers in azimuth and range. The better the focus quality of ISAR image is, the smaller the entropy is.

Fig. 5 ISAR image based on the maximum-correlation range alignment method

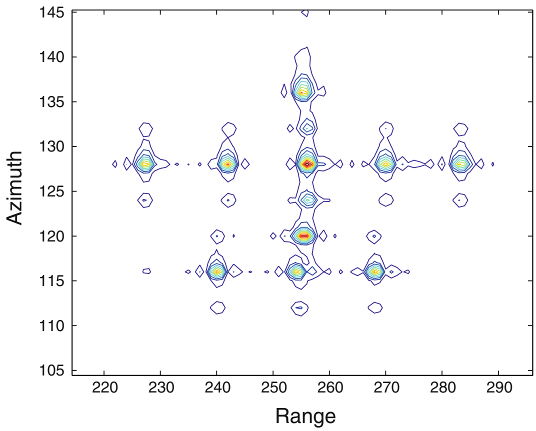


Fig. 6 ISAR image based on the minimum entropy range alignment method

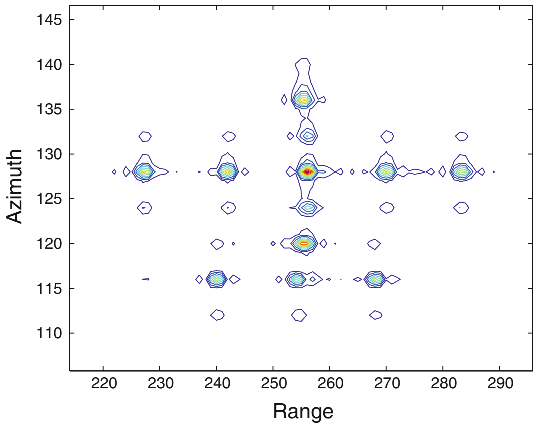
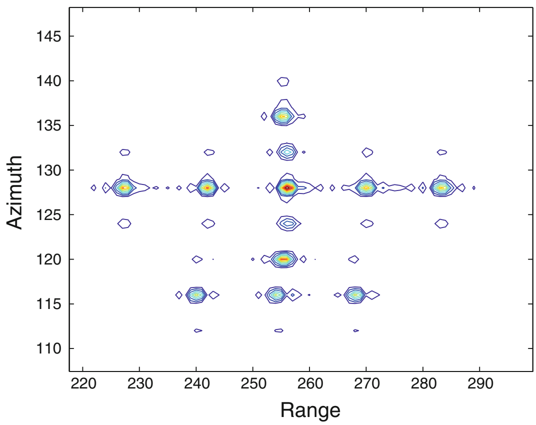


Fig. 7 ISAR image based on the Lagrange interpolation FDF method



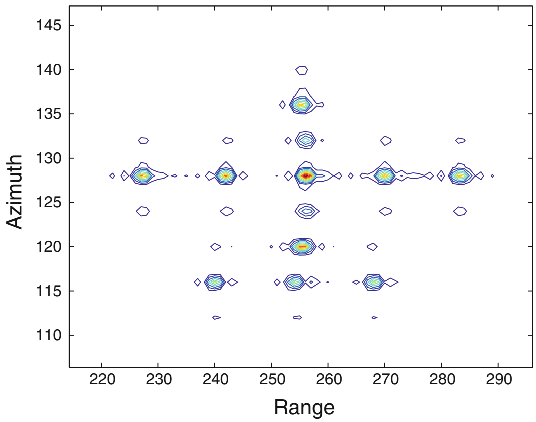


Fig. 8 ISAR image based on the linear T/R array range alignment method

The entropies of the images in Figs. 5, 6, 7 and 8 can be calculated by (16), the entropies of the images based on the maximum-correlation method, the minimum entropy method, the Lagrange interpolation FDF method and the linear T/R array method are -5.3388×10^{13}, -5.5749×10^{13}, -5.94118×10^{13} and -5.94130×10^{13}. Obviously, the linear T/R array method results in the smallest entropy. The focus quality of the ISAR image based on the proposed method is slightly better than the Lagrange interpolation FDF method.

With decimal time delay compensation, the linear T/R array method and the Lagrange interpolation FDF method have different computational loads. In order to analyze the computational loads of the proposed method and the Lagrange interpolation FDF method, an echo pulse with M range bins in the echo data is taken as an example. The computational loads analysis mainly focuses on the decimal time delay compensation process. After the phase slope extraction (the echo data has finish the FFT operation), for the proposed linear T/R array method, the decimal time delay compensation process contains one M-point complex multiplications (Mc) for phase rotation compensation and one M-point IFFT operation to translate the data to time domain. The M-point IFFT computation needs $\frac{M}{2}\log_2 M$ times of Mc and $M\log_2 M$ times of complex additions (Ac). Thus, the decimal time delay compensation process of the proposed method requires $M + \frac{M}{2}\log_2 M$ times of Mc and $M\log_2 M$ times of Ac. For the Lagrange interpolation FDF method, it is assumed that the FDF is L^{th} order (the length of the FDF is $L + 1$). The decimal time delay compensation process needs $M \times (L + 1)$ times of Mc and $M \times L$ times of Ac. The computational load is mainly determined by Mc operations rather than Ac operations. In this section, $M = 512$. Thus, if the order of the Lagrange interpolation FDF is higher than 4, the proposed linear T/R array method has more computational efficiency than the Lagrange interpolation FDF method.

In summary, with the same phase adjustment method and range-Doppler imaging process, the linear T/R array method removes the limitation of integer range bin alignment, has better range alignment performance, better computational efficiency and results in better focus quality of ISAR image than the other two methods.

5.2 Real data test

The proposed method can also be tested with real ISAR data. The real data was collected from an in-flight aircraft Yak-42 using static ISAR on the ground. 256 echo pulses and 256 range bins are processed to form ISAR image. Likewise, to better evaluate the performance of range alignment, the phase adjustments are all realized by the PGA algorithm, and the final imaging results are obtained by range-Doppler process. Two sets of 256 echo pulses are used here, and the results are shown in Figs. 9, 10, 11 and 12.

Entropy defined in the manuscript can be used to measure the focus quality of the imaging results. The entropies of the ISAR images in Figs. 9, 10, 11 and 12 are shown in Table 2.

Obviously, with decimal time delay compensation, the entropy of the ISAR image based on the proposed linear T/R array range alignment method and the Lagrange interpolation FDF method are smaller than the other two methods for each echo set. Between the two decimal time delay compensation methods, the proposed linear T/R array method also results in a better focus quality of ISAR image than the Lagrange interpolation FDF method. Based on the computational loads analysis in Sect. 5.1, M is 256 in this section, if the order of the Lagrange interpolation FDF is higher than 4, the proposed linear T/R array method has more computational efficiency than the Lagrange interpolation FDF method. Thus, due to the most accurate range alignment, the focus quality of the proposed linear T/R array range alignment method is superior to the other three methods.

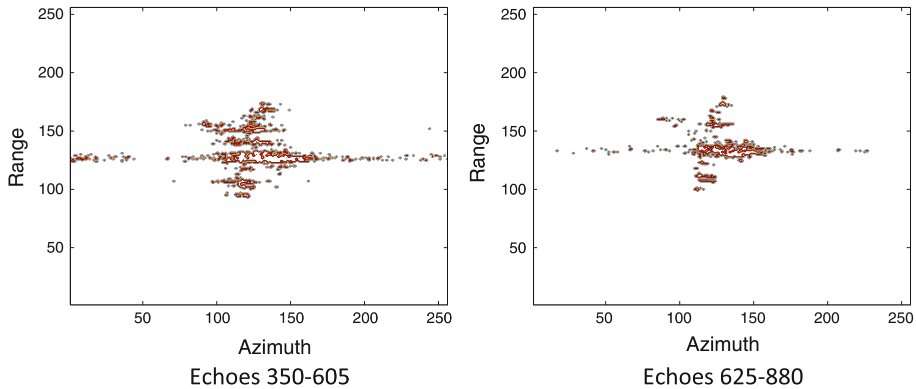


Fig. 9 Images based on the maximum-correlation method

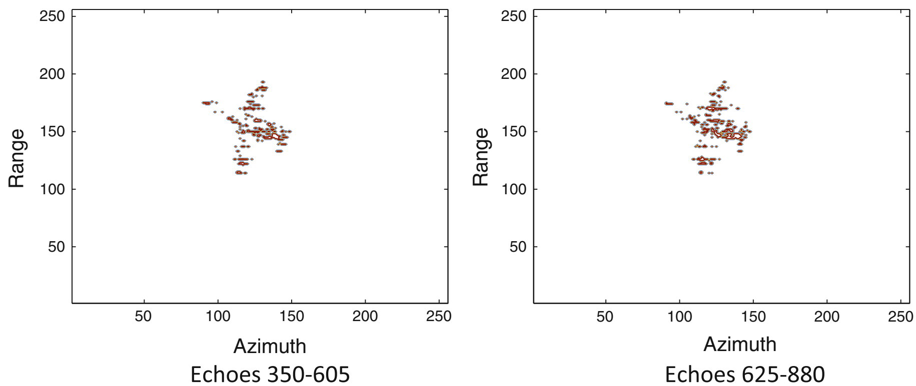


Fig. 10 Images based on the minimum entropy method

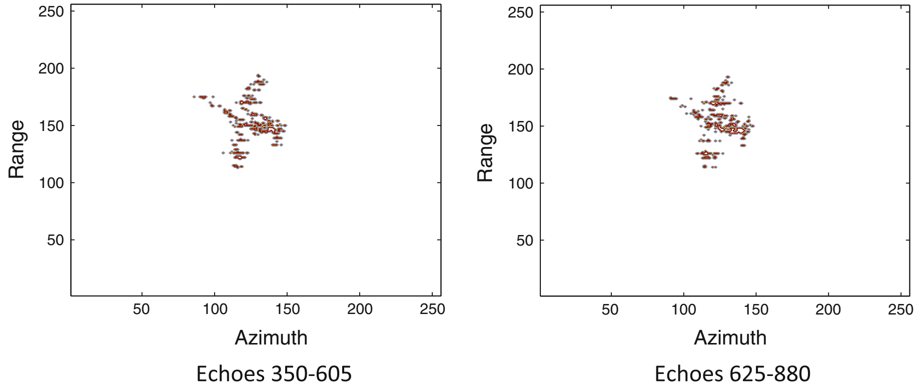


Fig. 11 Images based on the Lagrange interpolation FDF method

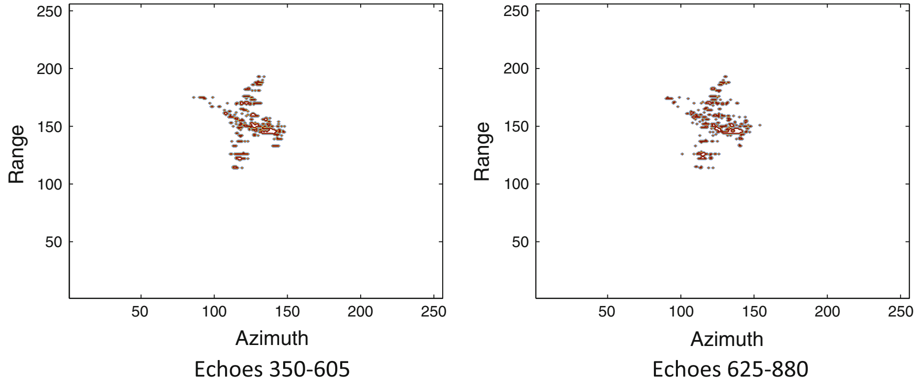


Fig. 12 Images based on the linear T/R array range alignment method

Table 2 Entropies of ISAR images

Echo sets	The maximum-correlation method	The minimum entropy method	The Lagrange interpolation FDF method	The linear T/R array method
Echoes 350–605	-1.2568×10^{14}	-1.2886×10^{14}	-1.8710×10^{14}	-1.9523×10^{14}
Echoes 625–880	-1.1903×10^{14}	-1.2012×10^{14}	-1.2023×10^{14}	-1.2064×10^{14}

6 Conclusion

In this paper, a novel range alignment method combining with the linear T/R array model is analyzed and verified. In the range alignment method based on the linear T/R array model, after integer range bin alignment, the decimal time delay compensation for range alignment is realized in the frequency domain by estimating the phase rotation between the two adjacent array elements. Compared with other methods, the linear T/R array method results in better performance in focus quality of ISAR images, and the limitation of integer rang bins alignment for traditional methods is removed.

7 Appendix: The Lagrange interpolation FDF method

Take two discretely sampled signals that have to be aligned as an example, the signals are denoted here as:

$$x(k) = s(k) + \xi_1(k)$$
$$y(k) = s(k - D) + \xi_2(k) \quad (17)$$

where $s(k)$ is the source signal, $\xi_1(k)$ and $\xi_2(k)$ are independent stationary zero-mean white Gaussian noises, and k (k = 1, 2, …, M) is the discrete range bins.

D is the time delay after sampling period normalization. D is a noninteger number that can be split into the integer and fractional part. In this paper, the integer range bin alignment is accomplished first. Thus, the D in (17) is supposed to be the decimal time delay that has to be aligned. The Lagrange interpolation fractional delay filter (FDF) is a popular method to solve the decimal time delay compensation problem (Viola and Walker 2005; Dooley and Nandi 1999; Laakso et al. 1996).

The coefficients of the Lagrange interpolation fractional delay filters can be obtained in an explicit form as

$$h_D(n) = \prod_{\substack{i=0 \\ i \neq n}}^{N} \frac{D - i}{n - i}, \quad n = 0, 1, 2, \ldots, N \quad (18)$$

where N is the order of the FDF. Thus, the length of the FDF is N+1.

The Lagrange interpolation FDF method is implemented as

$$\begin{aligned} &When\ N\ is\ odd, \\ &x(k - D) = \sum_{n=0}^{N} h_D(n) x\left(\frac{N+1}{2} + k - n\right). \\ &When\ N\ is\ even, \\ &x(k - D) = \sum_{n=0}^{N} h_D(n) x\left(\frac{N}{2} + k - n\right). \end{aligned} \quad (19)$$

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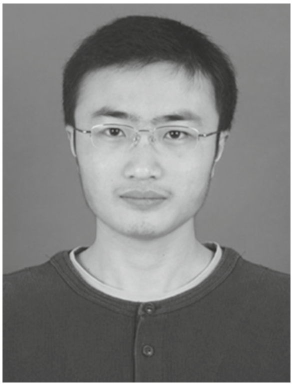

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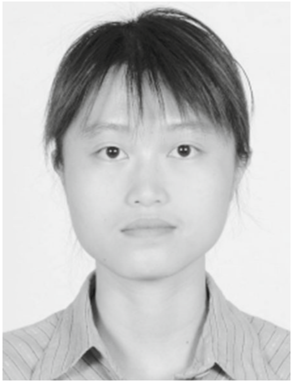

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