Wireless Pers Commun (2015) 84:3057–3074
DOI 10.1007/s11277-015-2782-7

Spectrum Efficiency Evaluation of SM MC-CDM with ZF USIC Under Different Adaptive Transmission Techniques

Lakshmi S. Pai[1] · **Vidhyacharan Bhaskar**[2,3]

Published online: 29 May 2015

Abstract In this paper, closed form expressions for capacities per unit bandwidth for Spatially Multiplexed Multicarrier-Code Division Multiplexing with Zero Forcing Unified Successive Interference Cancellation detection are derived for optimal power and rate adaptation, optimal rate adaptation with constant transmit power, channel inversion with fixed rate, and truncated channel inversion adaptation policies, taking into account the effect of perfect channel estimation at the receiver. Optimal power and rate adaptation policy provides the highest capacity over other adaptation policies. Truncated channel inversion policy suffers a large capacity penalty relative to the optimal power adaptation policy. However, the capacity penalty for the truncated channel inversion policy is lower compared to channel inversion with fixed rate policy. Furthermore, we derive analytical results for (1) capacity statistics, (2) Complementary Cumulative Distribution Function, and (3) Probability Density Function.

Keywords Spatially multiplexed multicarrier code division multiplexing · Unified successive interference cancellation · Optimal power and rate adaptation · Optimal rate with constant transmit power · Channel inversion with fixed rate · Truncated channel inversion

✉ Vidhyacharan Bhaskar
meetvidhyacharan@yahoo.com; charanvb@sfsu.edu; vidhyacharan.bhaskar@mail.npu.edu; vcharan@gmail.com

Lakshmi S. Pai
lakshmi.s.pai@gmail.com

[1] Department of Electronics and Communication Engineering, Matha College of Technology, Ernakulam 683511, Kerala, India

[2] Department of Electrical and Computer Engineering, San Francisco State University, 1600 Holloway Avenue, San Francisco, CA 94132, USA

[3] Department of Electrical Engineering, Northwestern Polytechnic University, 47650 Westinghouse Dr., Frémont, CA 94539, USA

1 Introduction

The common feature of next generation wireless technologies is the convergence of multimedia services such as speech, audio, video, image and data. This indicates that future wireless communications for high-quality multimedia and internet-based applications require high throughput and reliability under limited power and bandwidth resources. Consequently, emphasis is placed on system design goals and on the need for tradeoffs among basic system parameters such as Signal-to-Noise Ratio (SNR), probability of error, and bandwidth expenditure [1].

Numerous researchers have worked on the study of channel capacity over fading channels. Alouini and Goldsmith investigated the theoretical spectral efficiency limits of adaptive modulation in [2]. In [2], the capacity of a single user flat fading channel with perfect channel measurement information at the transmitter and receiver is derived for various adaptive transmission policies. In [3], the general theory developed in [2] is applied to obtain closed form expressions for the capacity of Rayleigh fading channels under different adaptive transmission and diversity combining techniques.

Mobile radio links are subjected to severe multipath fading due to the combination of randomly delayed, reflected, scattered, and diffracted signal components [4]. Capacity analysis of multipath fading channels becomes an important and fundamental issue in the design and study of new generation of wireless communication systems due to scarce radio spectrum available and to the rapidly growing demand for wireless services. Accordingly, there have been many papers dealing with the channel capacity of Rayleigh, Nakagami, Rician, and Generalized Gamma fading channels.

In [5], the capacity of Nakagami Multipath Fading (NMF) channels with an average power constraint for various adaptation policies has been studied. Closed-form solutions for NMF channel capacity (with and without diversity) for each adaptation strategy are compared with the additive white Gaussian noise channel capacity. Karmani and Sivarajan [6] found bounds and approximations for the capacity of mobile cellular communication networks based on CDMA. The authors have developed efficient analytic techniques for capacity calculations of CDMA cellular networks. In [7], the authors presented a study of capacity evaluation of various multiuser MIMO schemes in cellular environments. This study provides vital information for applying multiuser MIMO schemes in multicell environments. In [8], the ergodic capacity of spatially multiplexed multi-carrier code division multiple access (MC CDMA) system with unified successive interference cancellation (USIC) is discussed.

In [9], the channel capacity per unit bandwidth for different adaptation policies over Generalized Rayleigh fading channels has been computed. Closed-form expressions for the spectral efficiencies for the three adaptation policies were derived for the single antenna reception and Maximal Ratio Combining (MRC) diversity reception cases. In [10], closed-form expressions for the single user capacity of Selection Combining Diversity (SCD) system were derived, taking into account the effect of imperfect channel estimation at the receiver. Recently, there has been some work dealing with the channel capacity of different fading channels employing different adaptive schemes such as [11] and the references therein. Despite the ability of the generalized gamma distribution to characterize so many different fading/shadowing channel models, only recently it has been applied to the field of digital communications over fading channels [12]. In this paper, we extend the work of other researchers by analyzing the spectral efficiency of SM MC-CDM with zero forcing (ZF) USIC under different adaptive transmission techniques.

The paper is organised as follows: In Sect. 2, we derive closed-form expressions for capacities per unit bandwidth of SM MC-CDM with ZF USIC under different adaptation schemes, such as optimal adaptation policy, constant power policy, and channel inversion policy respectively. The capacity statistics, CDF and PDF are derived in Sect. 3. Section 4 presents numerical results of the comparison of capacities per unit bandwidth for the various adaptation policies. The main outcomes of the paper are summarized in Sect. 5.

2 Capacity Adaptation Policies

Let us consider a SM MC-CDM system with ZF USIC employed at the receiver. We assume perfect channel knowledge at the receiver. The distribution of the received SNR of SM MC-CDM with ZF USIC is approximated to be the inverse gamma distribution [8]. The PDF of received SNR is given by

$$f_\gamma(\gamma) = \left(\frac{\beta}{\zeta}\right)^{\alpha} \gamma^{\alpha-1} \exp\left(\frac{-\beta\gamma}{\zeta}\right) \frac{1}{\Gamma(\alpha)}; \quad \gamma \geq 0, \tag{1}$$

where α is the shape parameter, β is the scale parameter and $\zeta = \frac{E_s}{M\sigma_n^2}$, M denotes the number of transmit antennas, E_s is the signal energy and denominator represents the noise energy.

Adapting certain parameters of the transmitted signal to the channel fading leads to better utilization of the channel capacity. The main idea behind adaptive method is real-time balancing of the link budget through adaptive variation of the transmitted power level, symbol rate, constellation size, coding rate/scheme, or any combination of these parameters. Thus, without sacrificing BER, these schemes provide a much higher average spectral efficiency by taking advantage of the time-varying nature of the wireless channel. This section investigates the three adaptation policies: optimal simultaneous power and rate adaptation, constant power with optimal rate adaptation, and channel inversion with fixed rate.

2.1 Optimal Simultaneous Power and Rate Adaptation Policy

Since we have assumed that the variation in SNR is sent back to the transmitter through an error free feedback path, it allows the source to adapt both its power and rate to the actual channel conditions. Given an average transmit power constraint, the channel capacity of a fading channel with received CNR distribution and optimal power and rate adaptation $\left(\langle C\rangle_{\mathrm{opra}}(\mathrm{b/s})\right)$ is given in [3] as

$$\frac{\langle C\rangle_{\mathrm{opra}}}{B} = \int_{\gamma_0}^{\infty} \log_2\left(\frac{\gamma}{\gamma_0}\right) f_\gamma(\gamma) d\gamma, \tag{2}$$

where B is the channel bandwidth and γ_0 is the optimal cut off CNR level below which data transmission is suspended. This optimal cut off must satisfy the condition [3].

$$\int_{\gamma_0}^{\infty} \left(\frac{1}{\gamma_0} - \frac{1}{\gamma}\right) f_\gamma(\gamma) d\gamma = 1. \tag{3}$$

To achieve the capacity in (2), the channel fading level must be tracked at both the receiver and the transmitter, and the transmitter has to adapt its power and rate accordingly, allocating high power levels and rates for good channel conditions (γ large), and lower power levels and rates for unfavourable channel conditions (γ small). Since no data is sent when $\gamma < \gamma_0$, the optimal policy suffers a probability of outage, P_{out}, equal to the probability of no transmission given by [14]

$$P_{\text{out}} = P[\gamma \leq \gamma_0] = \int_0^{\gamma_0} f_\gamma(\gamma) d\gamma, \tag{4}$$

and substituting $f_\gamma(\gamma)$ from (1) into the integral of (3), it is found that γ_0 must satisfy

$$\frac{\Gamma_{\text{CIGF}}(\alpha, \gamma_0)}{\gamma_0} - \Gamma_{\text{CIGF}}(\alpha - 1, \gamma_0) = \Gamma(\alpha), \tag{5}$$

where $\Gamma_{\text{CIGF}}(\alpha, \gamma_0) = \int_{\gamma_0}^{\infty} t^{\alpha-1} e^{-t} dt$ is the Complementary Incomplete Gamma Function (CIGF) (section 8.35, Page 890 of [15]). Substituting $x = \gamma_0$ in (5) and define

$$g(x) = \frac{\Gamma_{\text{CIGF}}(\alpha, x)}{x} - \Gamma_{\text{CIGF}}(\alpha - 1, x) - \Gamma(\alpha). \tag{6}$$

Note that $\frac{dg(x)}{dx} = -\frac{1}{x^2}\Gamma_{\text{CIGF}}(\alpha, x) \quad < 0 \forall x > 0$. Moreover, from (6) $\lim_{x \to 0^+} g(x) = +\infty$ and $\lim_{x \to +\infty} g(x) = -\Gamma(\alpha) < 0$. Thus, it is concluded that there is a unique positive γ_0 for which $g(\gamma_0) = 0$, which satisfies (5). Numerical results from MATLAB show that $\gamma_0 \in$ [0, 1]. An approximation for the distribution of post processing SNR of SM MC-CDM with ZF USIC is shown in (1), and substituting (1) into (2), we have

$$\frac{\langle C \rangle_{\text{opra}}}{B} = \int_{\gamma_0}^{\infty} \log_2\left(\frac{\gamma}{\gamma_0}\right) \left(\frac{\beta}{\zeta}\right)^{\alpha} \gamma^{\alpha-1} \exp\left(-\frac{\beta}{\zeta}\gamma\right) \frac{1}{\Gamma(\alpha)} d\gamma. \tag{7a}$$

Making change of variables in the above integral, $\frac{\langle C \rangle_{\text{opra}}}{B}$ is obtained as

$$\frac{\langle C \rangle_{\text{opra}}}{B} = \frac{1}{\ln 2} \sum_{k=0}^{\alpha-1} \frac{\Gamma_{\text{CIGF}}\left(k, \frac{\beta\gamma_0}{\varsigma}\right)}{k!}. \tag{7b}$$

Equation (7b) can also be written as [9]

$$\frac{\langle C \rangle_{\text{opra}}}{B} = \frac{1}{\ln 2} \left(E_1\left(\frac{\beta\gamma_0}{\varsigma}\right) + \sum_{k=1}^{\alpha-1} \frac{\Gamma_{\text{CIGF}}\left(k, \frac{\beta\gamma_0}{\varsigma}\right)}{k!} \right), \tag{8}$$

where $E_1(.)$ is the exponential integral of the first order function defined as $E_1(x) = \int_1^{\infty} \frac{e^{-xt}}{t} dt$ [13].

Asymptotic approximation of capacity as a result of OPRA policy can be obtained by letting $\gamma \to \infty$ using the series expansion of exponential integral of the first order given by [3]

$$E_1(x) = -E - \ln(x) - \sum_{i=1}^{\infty} \frac{(-x)^i}{i.i!}, \tag{9}$$

where E = 0.5772156659 is the Euler-Mascheroni constant. Then, the capacity per unit bandwidth (in bits/s/Hz) using OPRA policy is approximated asymptotically as

$$\frac{\langle C \rangle_{\text{opra}}}{B} = \frac{1}{\ln 2}\left(\left(-E - \ln\left(\frac{\beta\gamma_0}{\varsigma}\right) + \left(\frac{\beta\gamma_0}{\varsigma}\right)\right) + \sum_{k=1}^{\alpha-1} \frac{\Gamma_{\text{CIGF}}\left(\frac{k, \beta\gamma_0}{\varsigma}\right)}{k!}\right). \tag{10}$$

The capacity per unit bandwidth expression of OPRA can be upper bounded by applying Jensen's inequality to (2) as follows

$$\frac{\langle C \rangle_{\text{opra}}^{\text{UB}}}{B} = \ln(E[\gamma]) = \ln\left(\int_0^{\infty} \gamma f(\gamma) d\gamma\right) = \ln\left(\frac{\Gamma(\alpha+1)\varsigma}{\beta\Gamma(\alpha)}\right), \tag{11}$$

where E[γ] is the expectation operator. The outage probability associated with this policy is obtained by substituting (1) into (4) and is given by

$$\begin{aligned} P_{\text{out}} &= \frac{1}{\Gamma(\alpha)}\left(\frac{\beta}{\zeta}\right)^{\alpha} \int_0^{\gamma_0} \gamma^{\alpha-1} \exp\left(-\frac{\beta\gamma}{\varsigma}\right) d\gamma, \\ &= \frac{1}{\Gamma(\alpha)} \Gamma_{\text{IGF}}\left(\frac{\gamma_0\beta}{\varsigma}, \alpha\right), \end{aligned} \tag{12}$$

where $\Gamma_{\text{IGF}}(x, \alpha) = \int_0^x t^{\alpha-1} e^{-t} dt$ is the Incomplete Gamma Function (IGF) (section 8.36, Page 890 of [15]).

2.2 Optimal Rate Adaptation with Constant Transmit Power Policy

Applying optimal rate adaptation policy to channel fading with constant transmit power, the channel capacity per unit bandwidth becomes

$$\frac{\langle C \rangle_{\text{ora}}}{B} = \int_0^{\infty} \log_2(1+\gamma) f_{\gamma}(\gamma) d\gamma, \tag{13}$$

and the expression obtained for ZF USIC is obtained by substituting (1) into (13), and is given as

$$\frac{\langle C \rangle_{\text{ora}}}{B} = \frac{1}{\Gamma(\alpha)}\left(\frac{\beta}{\zeta}\right)^{\alpha} \int_0^{\infty} \log_2(1+\gamma)\gamma^{\alpha-1} \exp\left(-\frac{\beta\gamma}{\zeta}\right) d\gamma. \tag{13a}$$

Let us define

$$I_{\alpha}\left(\frac{\beta}{\zeta}\right) = \int_0^{\infty} t^{\alpha-1} \ln(1+t) e^{-\frac{\beta}{\zeta}t} dt.$$

Making change of variables in the integral of (13a), we have

$$\frac{\langle C\rangle_{\text{ora}}}{B} = \frac{1}{\Gamma(\alpha)\ln 2}\left(\frac{\beta}{\varsigma}\right)^{\alpha} I_{\alpha}\left(\frac{\beta}{\varsigma}\right). \tag{14}$$

The evaluation of $I_{\alpha}\left(\frac{\beta}{\varsigma}\right)$, where α is a positive integer, is derived in Appendix B of [3]. Using the result of [3], (14) is rewritten as

$$\frac{\langle C\rangle_{\text{ora}}}{B} = \log_2(e)e^{\frac{\beta}{\varsigma}}\sum_{k=0}^{\alpha-1}\left(\frac{\beta}{\varsigma}\right)^{k}\Gamma_{\text{CIGF}}\left(-k,\frac{\beta}{\varsigma}\right), \tag{15}$$

which can also be expressed in the form of Poisson distribution as

$$\frac{\langle C\rangle_{\text{ora}}}{B} = \log_2(e)P_{\alpha}\left(\frac{\beta}{\varsigma}\right)E_1\left(\frac{\beta}{\varsigma}\right) + \sum_{k=1}^{\alpha-1}\frac{P_k\left(\frac{\beta}{\varsigma}\right)P_{\alpha-k}\left(-\frac{\beta}{\varsigma}\right)}{k}, \tag{16}$$

where $P_k(.)$ is given as $P_k(\mu) = e^{-\mu}\sum_{j=0}^{k-1}\frac{\mu^j}{j!}$ [11]. In fact, (13) represents the capacity of the fading channel without transmitter feedback (i.e., channel fade level known at the receiver only). The capacity per unit bandwidth, $\frac{\langle C\rangle_{\text{ora}}}{B}$ can be upper bounded using Jensen's inequality to (13) as follows:

$$\frac{\langle C\rangle_{\text{ora}}^{UB}}{B} = \ln(1+E[\gamma]) = \ln\left(1+\frac{\Gamma(\alpha+1)\varsigma}{\beta\Gamma(\alpha)}\right). \tag{17}$$

Capacity per unit bandwidth by adopting ORA policy can be approximated in the high SNR region as [10]

$$\log_2(1+\gamma) \approx \log_2(\gamma). \tag{18}$$

By using this approximation, (13) can be rewritten as

$$\frac{\langle C\rangle_{\text{ora}}}{B} = \int_0^{\infty}\log_2(\gamma)f_{\gamma}(\gamma)d\gamma. \tag{19}$$

Substituting (1) into (19), capacity approximation in high SNR region can be obtained as

$$\begin{aligned}\frac{\langle C\rangle_{\text{ora}}}{B} &= \frac{1}{\Gamma(\alpha)}\left(\frac{\beta}{\zeta}\right)^{\alpha}\int_0^{\infty}\log_2(\gamma)\gamma^{\alpha-1}\exp\left(-\frac{\beta\gamma}{\zeta}\right)d\gamma \\ &= \frac{\Gamma(\alpha)\left[\psi(\alpha)-\ln\left(\frac{\beta}{\varsigma}\right)\right]}{\Gamma(\alpha)\ln(2)},\end{aligned} \tag{20}$$

where $\psi(\alpha)$ is the Euler-Psi function [15].

In low SNR region I, capacity can be approximated as [10]

$$\log_2(1+\gamma) \approx \sqrt{\gamma}. \tag{21}$$

Rewriting the capacity per unit bandwidth expression for ORA policy in low SNR region I in (13), we have

$$\begin{aligned}\frac{\langle C\rangle_{\text{ora}}^{\text{LSRI}}}{B} &\approx \frac{1}{\Gamma(\alpha)}\left(\frac{\beta}{\zeta}\right)^{\alpha}\int_{0}^{\infty}\sqrt{\gamma}\,\gamma^{\alpha-1}\exp\left(-\frac{\beta}{\zeta}\gamma\right)d\gamma \\ &= \frac{\varsigma\Gamma(\alpha+0.5)}{\beta\Gamma(\alpha)}.\end{aligned} \tag{22}$$

In a similar way, capacity in low SNR region II can be approximated by using the expression [10]

$$\log_2(1+\gamma) \approx \frac{1}{\ln(2)}\left(\gamma - \frac{1}{2}\gamma^2\right). \tag{23}$$

By substituting (23) into (13), the capacity per unit bandwidth expression for low SNR region II using ORA policy, we have

$$\begin{aligned}\frac{\langle C\rangle_{\text{ora}}^{\text{LSRII}}}{B} &\approx \frac{1}{\Gamma(\alpha)}\left(\frac{\beta}{\zeta}\right)^{\alpha}\int_{0}^{\infty}\frac{1}{\ln(2)}\left(\gamma-\frac{1}{2}\gamma^2\right)\gamma^{\alpha-1}\exp\left(-\frac{\beta}{\zeta}\gamma\right)d\gamma \\ &= \frac{\varsigma\Gamma(\alpha+1)}{\beta\Gamma(\alpha)\ln 2} - \frac{\varsigma^2\Gamma(\alpha+2)}{2\beta^2\Gamma(\alpha)\ln 2}.\end{aligned} \tag{24}$$

2.3 Channel Inversion with Fixed Rate Policy

The channel capacity when the transmitter adapts its power to maintain a constant carrier-to-noise ratio (CNR) at the receiver (i.e., inverts the channel fading) was also investigated in [3]. This technique uses fixed-rate modulation and a fixed code design, since the channel after channel inversion appears as a time-invariant AWGN channel. As a result, channel inversion with fixed rate is the least complex technique to implement assuming good channel estimates are available at the transmitter and receiver. Channel inversion is an adaptive transmission technique whereby the transmitter uses the channel information fed back by the receiver in order to invert the channel fading. The channel capacity with this technique is derived from the capacity of an AWGN channel, and is given in [3] (Eq. (46) of [3]) as

$$\frac{\langle C\rangle_{\text{cifr}}}{B} = \log_2\left[1 + \frac{1}{\int_0^\infty \frac{f_\gamma(\gamma)}{\gamma}d\gamma}\right], \tag{25}$$

and the expression obtained for ZF USIC is given as

$$\begin{aligned}\frac{\langle C\rangle_{\text{cifr}}}{B} &= \log_2\left[1 + \frac{1}{\frac{1}{\Gamma(\alpha)}\left(\frac{\beta}{\zeta}\right)^{\alpha}\int_0^\infty \frac{1}{\gamma}\gamma^{\alpha-1}\exp\left(-\frac{\beta}{\zeta}\gamma\right)d\gamma}\right] \\ &= \log_2\left[1 + \frac{\Gamma(\alpha)}{\left(\frac{\beta}{\varsigma}\right)\Gamma(\alpha-1)}\right]; \quad \alpha > 1.\end{aligned} \tag{26}$$

Channel inversion with fixed rate suffers a large capacity penalty relative to the other techniques since a large amount of the transmitted power is required to compensate for the deep channel fades. A better approach is to use a modified inversion policy which inverts the channel fading only above a fixed cut-off fade depth. The capacity per unit bandwidth with TIFR policy is given as Eq. (47) of [3]

$$\frac{\langle C\rangle_{\mathrm{tifr}}}{B} = \log_2\left[1 + \frac{1}{\int_0^{\infty} \frac{f_\gamma(\gamma)}{\gamma} d\gamma}\right](1 - P_{\mathrm{out}}), \tag{27}$$

and is found to be

$$\frac{\langle C\rangle_{\mathrm{tifr}}}{B} = \log_2\left[1 + \frac{\Gamma(\alpha)}{\frac{\beta}{\varsigma}\Gamma\left(\alpha - 1, \frac{\beta\gamma_0}{\varsigma}\right)}\right]\left[1 - \frac{\Gamma\left(\frac{\beta\gamma_0}{\varsigma}, \alpha\right)}{\Gamma(\alpha)}\right]; \quad \alpha > 1. \tag{28}$$

The cut-off level can be selected to achieve a specified outage probability or, alternatively, to maximize (27).

3 Capacity Statistics

In this section, we focus on deriving exact analytical expressions for capacity statistics of SM MC-CDM with ZF USIC, assuming perfect channel knowledge at the receiver and no channel knowledge at the transmitter with average input-power constraint. Performance analysis of MC-CDMA systems under Nakagami-Hoyt fading is discussed briefly in a short paper in [16]. The mathematical analysis using probability theory related to computation of CDF, CCDF and PDF of capacity are discussed in the following subsection.

3.1 Complementary Cumulative Distribution Function (CCDF)

The CDF of capacity C is defined as [10]

$$F_C(C) = \mathrm{Prob}\,(\overline{C} \le C) = \int_0^{2^{C-1}} f_\gamma(\gamma) d\gamma. \tag{29}$$

Averaging over the distributions of received instantaneous SNR, results in the expression for CDF of capacity as

$$\begin{aligned} F_C(C) &= \frac{1}{\Gamma(\alpha)}\left(\frac{\beta}{\zeta}\right)^{\alpha} \int_0^{2^{C-1}} \gamma^{\alpha-1} \exp\left(-\frac{\beta}{\zeta}\gamma\right) d\gamma \\ &= \frac{1}{\Gamma(\alpha)} \Gamma_{\mathrm{IGF}}\left(\frac{\beta}{\varsigma}(2^C - 1), \alpha\right). \end{aligned} \tag{30}$$

The CCDF can be obtained as [10]

$$\overline{F_C}(C) = 1 - F_C(C) = 1 - \frac{1}{\Gamma(\alpha)}\Gamma_{IGF}\left(\frac{\beta}{\varsigma}(2^C - 1), \alpha\right). \tag{31}$$

3.2 Probability Density Function (PDF)

The PDF of C is defined as the derivative of $F_C(C)$ with respect to C. Taking the derivative of $F_C(C)$ in (30) results in

$$f_C(c) = C\left(\frac{\beta}{\varsigma}\right)^{\alpha} 2^{C-1}\left(2^C - 1\right)^{\alpha-1}\exp\left(-\frac{\beta(2^C - 1)}{\varsigma}\right). \tag{32}$$

4 Numerical Results

In this section, we provide some numerical results that illustrate the mathematical derivation of the channel capacity per unit bandwidth as a function of received SNR in dB for different adaptation policies.

Figure 1 shows the calculated channel capacity per unit bandwidth (spectrum efficiency) as a function of SNR for various adaptation policies. These curves are obtained using closed form expressions (8), (16), (26) and (28). The optimal cut-off SNR, γ_0, of the adaptation policies, OPRA and TIFR, is obtained using MATLAB from (6) as $\gamma_0 = 0.516788$ for $\alpha = 3$. From Fig. 1, it is clear that the OPRA policy yields a significant increase in capacity as compared to ORA and CIFR policies. The spectral efficiency curve obtained using TIFR policy lies in between the curves obtained for OPRA policy and ORA policy. As the shape parameter increases, there is an improvement in the spectral efficiencies calculated for all adaptation policies. The shape parameter indicates the fading severity or scintillation index. This indeed varies from deep fade to light fade. As α increases, the fluctuations in the signal diminishes and this is in turn improves the system performance.

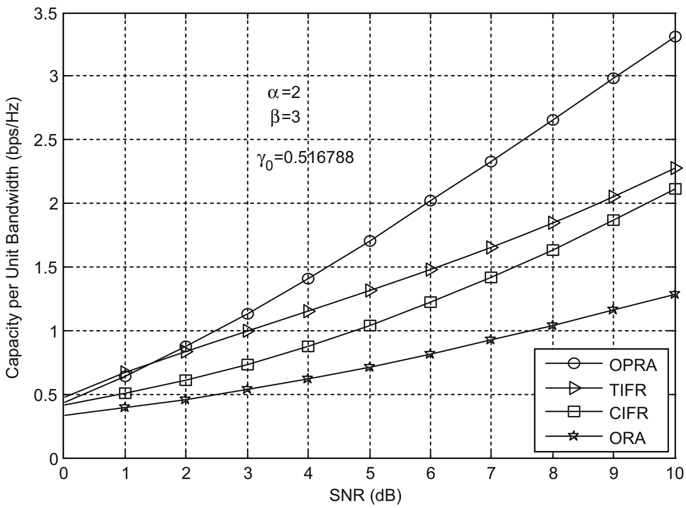


Fig. 1 Capacity per unit bandwidth for different adaptation policies

Figure 2a shows a decrease in capacity for the OPRA policy with increase in β. The scale parameter, β, relates to the inverse of average fading power. When β increases the average fading power decreases, and correspondingly, the capacity also decreases. In Fig. 2b and Fig. 3, the outage probability is plotted versus average SNR. The outage probability is high when α is low as in Fig. 2b and low when β is low as in Fig. 3. Outage probability is associated with OPRA policy since this policy adapts power and rate only if

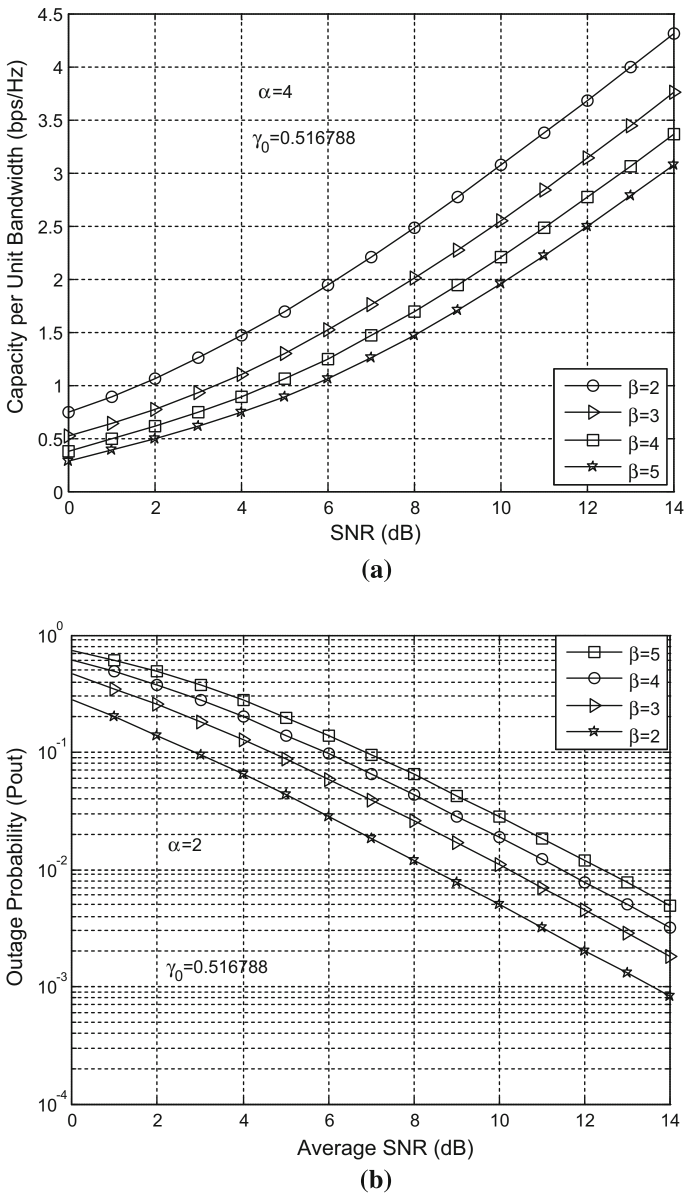


Fig. 2 **a** Capacity per unit bandwidth for OPRA policy for varying β, **b** Outage probability of OPRA policy for varying β

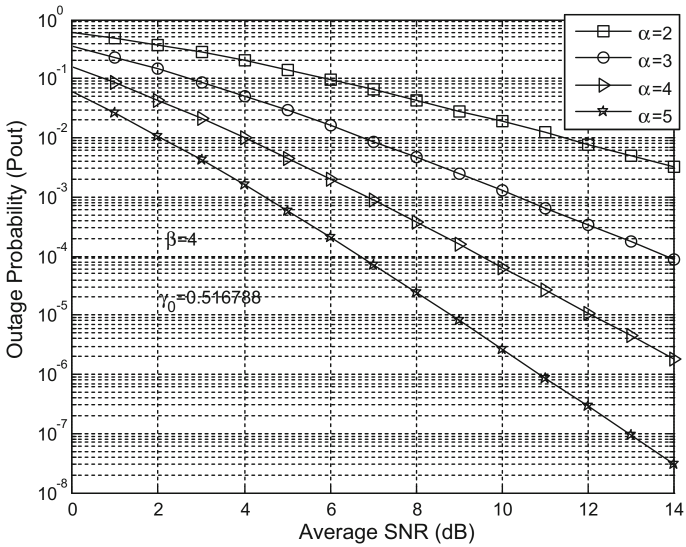


Fig. 3 Outage probability of OPRA policy for varying α

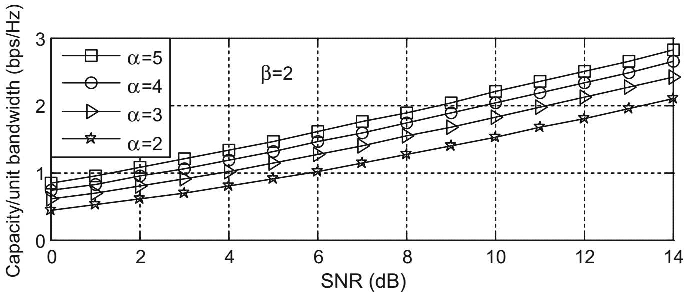
the instantaneous SNR of the channel is above the optimal cut-off SNR. The optimal cut-off SNR is obtained numerically using MATLAB and is found to be 0.516788 in Figs. 1, 2a, b and 3.

Figure 4a and b show the capacity per unit bandwidth obtained using ORA policy for (a) varying α and constant β ($\beta = 2$), and (b) varying β and constant α ($\alpha = 4$) cases. As expected, the capacity increases with increase in SNR. Figure 5a and b show the capacity per unit bandwidth obtained using CIFR policy for (a) varying α and constant β ($\beta = 2$), and (b) varying β and constant α ($\alpha = 2$) cases. As expected, the capacity increases with increase in SNR.

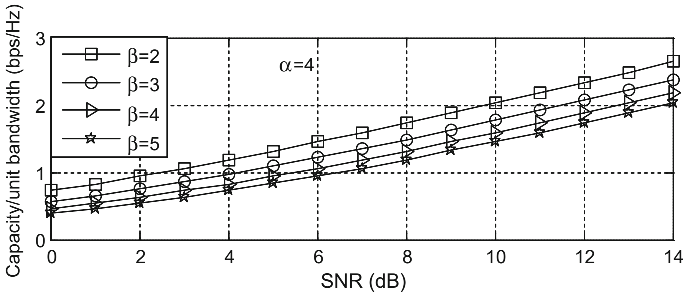
In Fig. 6a and b, increase in β for a given α reduces the capacity of the channel, as expected. This is because an increase in β decreases the average fading power of the channel which is responsible for the decrease in capacity. The CIFR policy is impractical during severe fading conditions and a modified version known as TIFR policy is developed. In the TIFR scheme, transmission is suspended if the received SNR falls below a threshold. As expected, Figs. 6a and b show that the TIFR scheme provides a better capacity compared to the CIFR scheme.

Figure 7 shows the asymptotic approximation of OPRA policy. Figure 7 clearly depicts the comparison between the theoretical result and asymptotic approximation associated with OPRA policy. This is plotted using closed form expressions, (8) and (10), respectively. The optimal cut-off SNR is found numerically using MATLAB and is 0.516788. This value of γ_0 is used to plot expression (8). Asymptote of a curve is a line such that the distance between the curve and the line approaches zero at infinity. Figure 7 also shows the tightness between the theoretical result and asymptotic approximation at high SNRs.

Figure 8a and b show the upper bound of OPRA policy for varying values of $\alpha$and β. Figure 8a and b clearly depict that the theoretical results are well within the upper bound



(a)



(b)

Fig. 4 **a** Capacity per unit bandwidth for ORA policy for varying α and constant β, **b** Capacity per unit bandwidth for ORA policy for varying β and constant α

curve. Capacity of this system is able to achieve a maximum upper bound shown by Figs. 8a and b. The closed form expression in (11) is used to get this result.

Figure 9a and b show the upper bound of ORA policy with varying α and varyingβ, and the closed form expression given in (17) is used to plot the result. This result also agrees with the previous explanation of OPRA. But, if upper bounds of OPRA and ORA are compared, then it is shown that the OPRA policy performs much better than the ORA policy.

Figure 10a and b show capacity approximations of ORA policy in low SNR region I and low SNR region II, respectively. Figure 10a and b are obtained using the closed form expressions, (22) and (24), respectively. From Fig. 10a, it is observed that (21) will be satisfied only for certain range of SNR (-30 to -5 dB). For SNR above this range, the approximation is not valid. From Fig. 10b, it is observed that (23) will be satisfied if the range of SNR is below -15 dB. For values of SNR greater than this, (23) will not be satisfied.

Figure 11a depicts the PDF curves for different values of α, with $\beta = 4$, at SNR = 15 dB. The closed form expression (32) is used to plot the result. Figure 11a shows that the capacity distribution has a Gaussian-like shape. It also shows that the relative frequency of occurrence of capacity increases as the value of shape parameter increases. The PDF is a continuous function that describes the relative likelihood for capacity to occur at a given point. The PDF of capacity to fall within a particular region is given by the integral of PDF of capacity over the region. Figure 11b shows the CDF curves

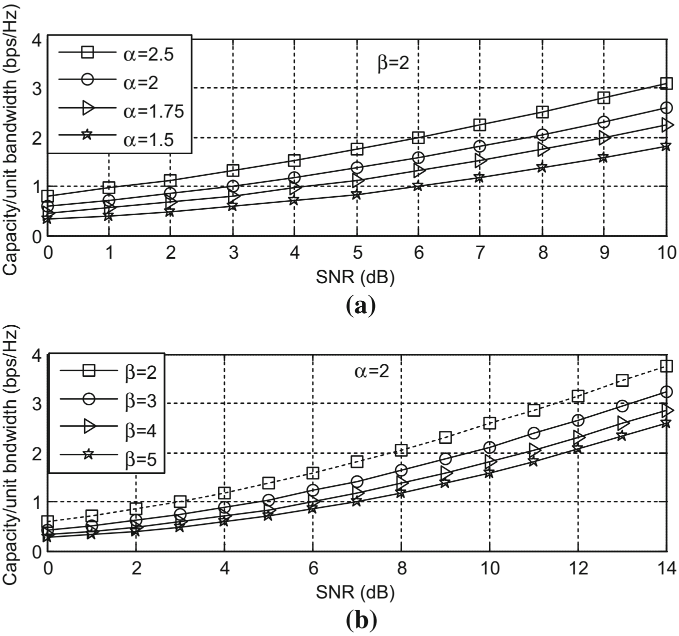


Fig. 5 **a** Capacity per unit bandwidth for CIFR policy for varying α and constant β, **b** Capacity per unit bandwidth for CIFR policy for varying β and constant α

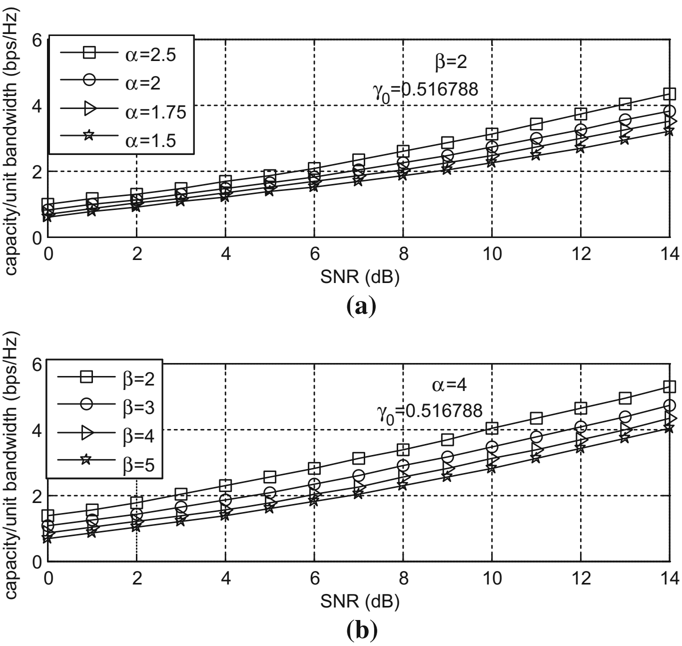


Fig. 6 **a** Capacity per unit bandwidth for TIFR policy for varying α and constant β, **b** Capacity per unit bandwidth for TIFR policy for varying β and constant α

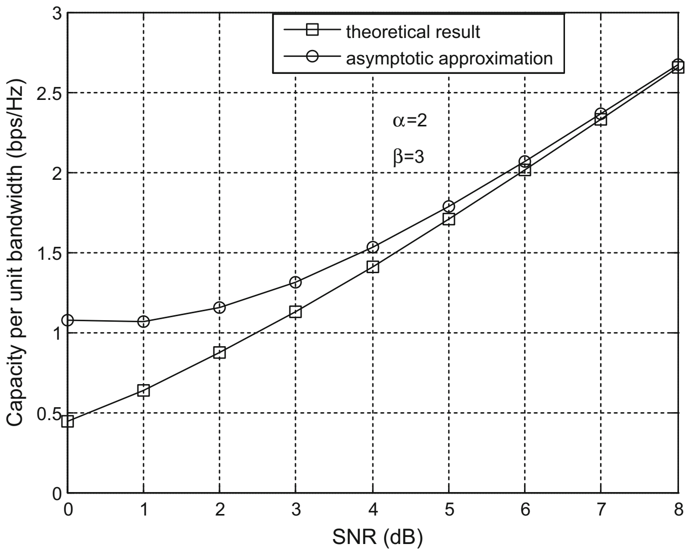


Fig. 7 Asymptotic approximation of OPRA policy for $\alpha = 2$ and $\beta = 3$

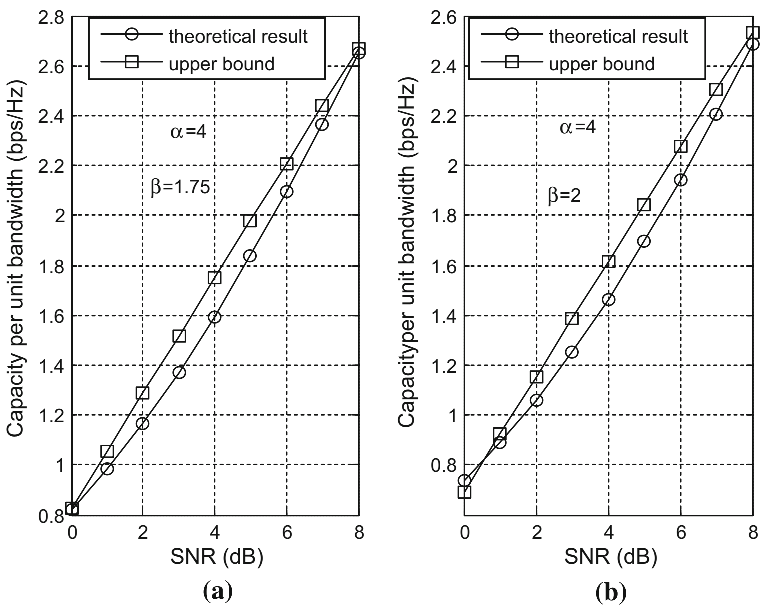


Fig. 8 **a** Comparison of upper bound and theoretical result of OPRA policy for $\alpha = 4$, $\beta = 1.75$, **b** Comparison of upper bound and theoretical result of OPRA policy with $\alpha = 4$, $\beta = 2$

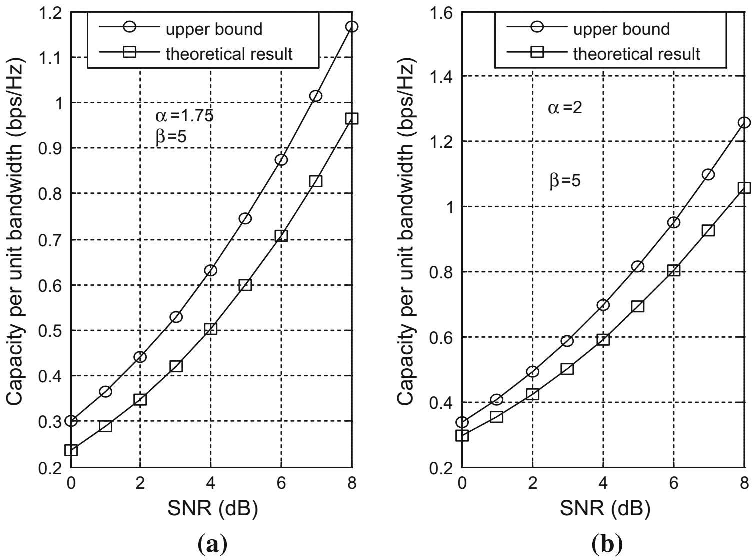


Fig. 9 **a** Comparison of upper bound and theoretical result of ORA policy for $\alpha = 1.75$, $\beta = 5$, **b** Comparison of upper bound and theoretical result of ORA policy for $\alpha = 2$, $\beta = 5$

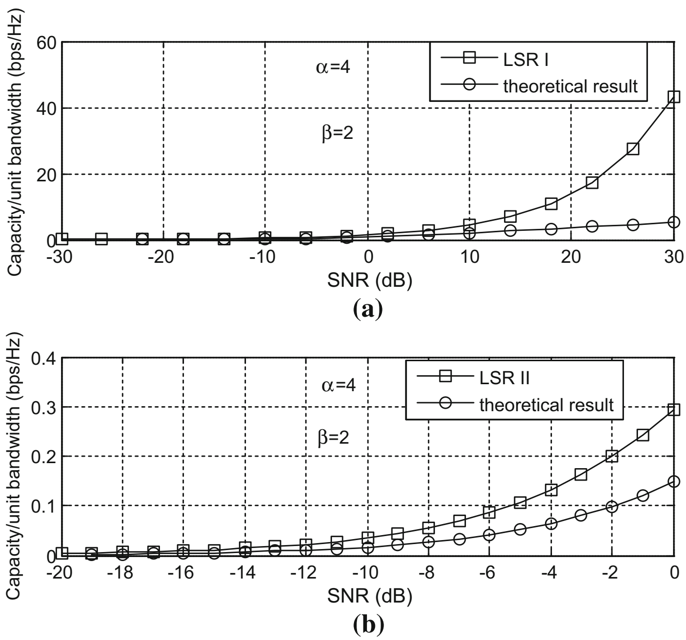


Fig. 10 **a** Capacity approximations of ORA policy between LSR I and theoretical result, **b** Capacity approximations of ORA policy between LSR II and theoretical result

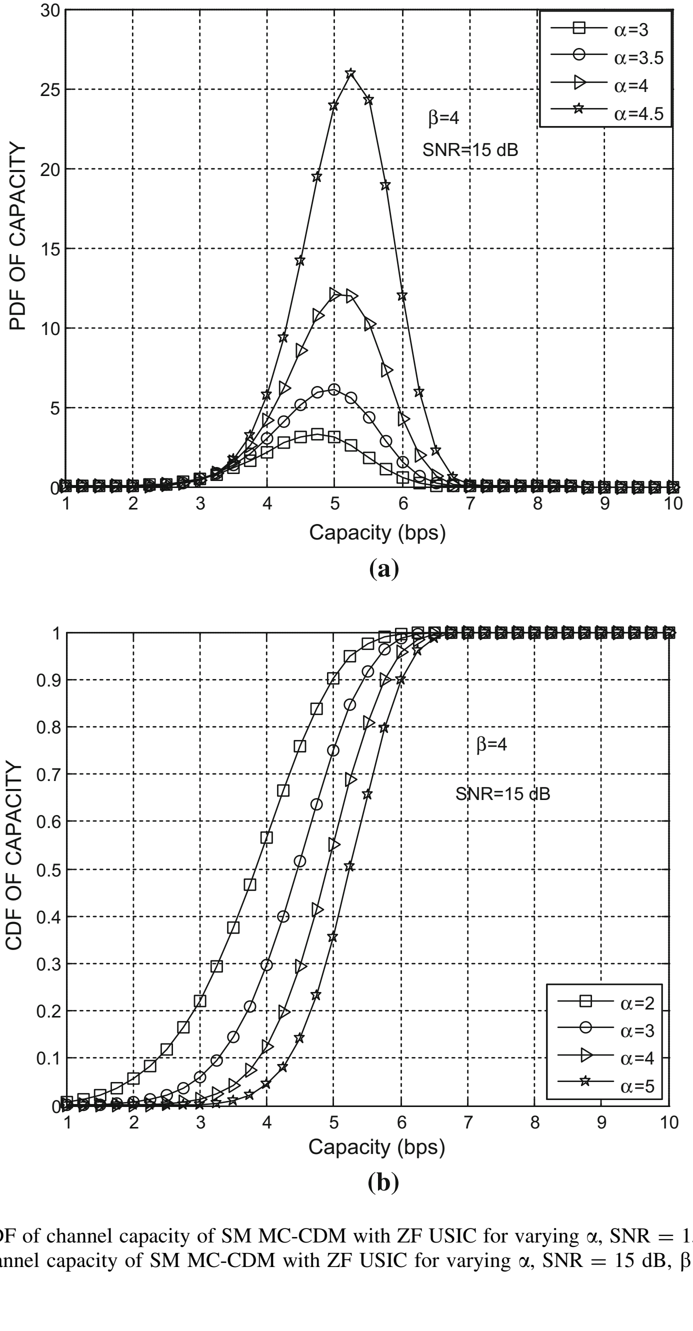


Fig. 11 **a** PDF of channel capacity of SM MC-CDM with ZF USIC for varying α, SNR = 15 dB, β = 4, **b** CDF of channel capacity of SM MC-CDM with ZF USIC for varying α, SNR = 15 dB, β = 4

for different values of α and β. The distribution function describes the probability that the capacity random variable,$\overline{C}$, with a given probability distribution will be found at a value less than or equal to C.

5 Conclusions

The spectrum efficiency or channel capacity per unit bandwidth of SM MC CDM with ZF USIC for different adaptation policies including their approximations and upper bounds are investigated in this paper. Numerical results show that OPRA policy outperforms ORA policy. Optimal power and rate adaptation policy provides the highest capacity over the other adaptation policies. Truncated channel inversion policy suffers a large capacity penalty because of higher probability of outage relative to the optimal power adaptation policy and constant transmit power policy. Furthermore, we have provided analytical results for the PDF and CDF of capacity.

References

1. Sklar, B. (1988). *Digital communications—Fundamentals and applications*. Englewood Cliffs: PTR Prentice Hall.
2. Goldsmith, A., & Varaiya, P. (1997). Capacity of fading channels with channel side information. *IEEE Transactions on Information Theory, 43*(6), 1986–1992.
3. Alouini, M. S., & Goldsmith, A. (1999). Capacity of Rayleigh fading channels under different adaptive transmission and diversity combining techniques. *IEEE Transactions on Vehicular Technology, 48*(4), 1165–1181.
4. Jakes, W. (1994). *Microwave mobile communications* (2nd ed.). Piscataway, NJ: IEEE Press.
5. Alouini, M. S. and Goldsmith, A., (1997) "Capacity of Nakagami multipath fading channels," 47th *IEEE vehicular technology conference*, *1*, pp. 358–362. Phoenix, AZ, USA.
6. Karmani, G., and Sivarajan, K., (2001) "Capacity evaluation for CDMA cellular systems," *Proceedings of IEEE international symposium on computer and communication societies*, *1*, pp. 601–610. Anchorage, Alaska, USA.
7. Joung, J., Kim, E., Lim, S., and Lee, Y. (2006). "Capacity evaluation of various multiuser MIMO schemes in downlink cellular environments," *International symposium on personal, indoor and mobile radio communications (PIMRC 06)*, (pp. 1–5). Helsinki, Finland.
8. Eslami, M., & Krzymien, W. (2008). Performance of spatially multiplexed MC-CDM with zero-forcing unified successive interference cancellation detection. *IET Communications, 3*(8), 1410–1419.
9. Bhaskar, V. (2007). Spectral efficiency evaluation for MRC diversity schemes over generalized Rician fading channels. *International Journal of Wireless Information Networks, 14*(3), 209–223.
10. Fawaz, S., Salam, A., & Arun, K. G. (2009). Spectral efficiency evaluation for selection combining diversity scheme over slow fading. *IET Communications, 3*(9), 1443–1451.
11. Bhaskar, V. (2007). Capacity evaluation for equal gain diversity schemes over generalized Rayleigh fading channels. *International Journal of Electronics and Communications, 3*(14), 235–240.
12. Bithas, P. S., Sagias, N. C., & Mathiopoulos, P. T. (2007). GSC diversity receivers over generalized-gamma fading channels. *IEEE Communication Letters, 11*(12), 964–966.
13. Kang, M., & Alouini, M. S. (2006). Capacity of MIMO Rician channels. *IEEE Transactions on Wireless Communications, 5*(1), 112–122.
14. Duman, T. M., & Ghrayeb, A. (2007). *Coding for MIMO communication systems*. Chichester, England: Wiley.
15. Gradshteyn, I. S., & Ryzhik, I. M. (2000). *Table of integrals, series and products* (7th ed.). Jamestown Road, London: Academic Press.
16. Bhaskar, V., & Pai, L. S. (2012). Performance analysis of MC-CDMA systems under Nakagami Hoyt fading. *Wireless Personal Communications Journal*. doi:10.1007/s11277-012-0669-4

Lakshmi S. Pai received her B.E. degree in Electronics and Communication from Manipal Institute of Technology, Manipal University, Karnataka, India in 2003. She completed her M. Tech in Communication Systems from S.R.M University, Chennai in 2011. Her research interests include the broad areas of Wireless communications, Error Control Coding, and MIMO. Currently, she is working as an Assistant Professor in the Department of Electronics and Communication Engineering at Matha College of Technology, Ernakulam, Kerala, India. She has published two Refereed International Journals.

Dr. Vidhyacharan Bhaskar received the B.Sc. degree in Mathematics from the University of Madras, Chennai, India in 1992, M.E. degree in Electrical & Communication Engineering from the Indian Institute of Science, Bangalore in 1997, and the M.S.E. and Ph.D. degrees in Electrical Engineering from the University of Alabama in Huntsville in 2001 and 2002, respectively. During 2002–2003, he was a Post Doctoral fellow with the Communications research group at the University of Toronto, Canada, where he worked on the applications of space-time coding for wireless communication systems. During 2003–2006, he was an Associate Professor in the Department of Information Systems and Telecommunications at the University of Technology of Troyes, France. From Jan. 2007 to May 2014, he was a Full Professor in the Department of Electronics and Communication Engineering at S.R.M. University, Kattankulathur, India. Currently, he is an Adjunct Professor in the (1) Department of Electrical and Computer Engineering at San Francisco State University, and the (2) Department of Electrical Engineering at Northwestern Polytechnic University, California. His research interests include MIMO wireless communications, signal processing, error control coding and queuing theory. He has published around 94 Refereed Journal papers, presented around 70 Conference papers in various International Conferences over the past 12 years, published a book on “Adaptive Rate Coding for A-CDMA Systems” in Jan 2011, a book on “Higher-Order s-to-z mapping functions for digital filters,” in March 2013, and has also co-authored a book on MATLAB in 1998. He is on the Board of Reviewers of Refereed Journal articles published by IEEE, Springer, EURASIP, and Elsevier. His name was nominated for inclusion in the 2011 Edition of Who’s Who in the World. In Jan. 2012, he received the award for the “Best Academic Researcher” and in Dec. 2013, he received the “Best Professor” award from the Association of Scientists, Developers and Faculty, Pondicherry, India. In Sep. 2012, he received the Prof. SVC Aiya Memorial Award for outstanding contributions in teaching and doing research on Wireless Communications, Error control coding, Signal processing, and Queuing theory. In Dec. 2013, he also received a National Award for Outstanding Academic from the India Society of Technical Education. He is a Senior member of the IEEE (SM-IEEE) and is a member of IET (M-IET, UK). He is a Fellow of Institute of Electronics and Telecommunication Engineers (F-IETE), and a Fellow of Institute of Engineers (F-IE), Kolkata, India. He is also a Life member of the Indian Society of Technical Education (LM-ISTE) and a member of the Indian Science Congress (M-ISC).