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ORIGINAL PAPER

# An optimized transmultiplexer using combinational window functions

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**Abstract** This paper proposes an efficient approach for the design of $M$-channel maximally decimated near-perfect reconstruction (NPR) type transmultiplexer. Cosine modulation is used to design the synthesis and analysis sections of the transmultiplexer. The prototype filter is designed by using high sidelobe falloff rate (SLFOR) combinational window functions. A bisection-type optimization algorithm has been applied to minimize the interference parameters like inter-channel interference (ICI) and inter-symbol interference (ISI). The proposed algorithm has certain advantages than earlier reported work. The algorithm is of generalized nature, independent of the window function used in the design of the prototype filter. Second, it is fast and computationally efficient as only a single parameter is used as a variable which provides almost uniform interference level in all subchannels. Design examples are included for comparison with earlier reported work. Very small values of ICI and ISI have been obtained by using variable and combinational window functions.

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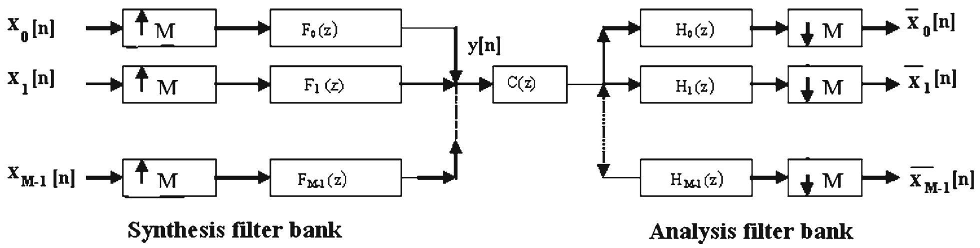
## 1 Introduction

Orthogonal frequency division multiplexing (OFDM) and discrete multitone transmission (DMT) are the widely used technologies in multicarrier communication [1]. Both technologies employ the inverse discrete Fourier transform (IDFT) and discrete Fourier transform (DFT) for the modulation and demodulation of the signals. Due to multipath fading over wireless communication, the consecutive OFDM symbols overlap at the receiver and gives rise to ISI [2]. In order to minimize the ISI, the OFDM system makes use of the guard band, which results in loss of spectral efficiency [3]. In addition, since the DFT-based modulation filters have sidelobes of the order of $-13$ dB, the significant spectral overlap between the sub-carriers causes ICI [4]. Many authors have worked on this issue and suggested filterbank-based multicarrier transmission systems (FB-MCTs), such as the overlapped DMT [4], filtered multitone transmission (FMT) [5] and discrete wavelet multitone transmission (DWMT) [6]. These FB-MCTs systems use filters of greater length than the rectangular filters of DMT systems and typically yield results with improvement in sidelobes attenuation, lower levels of ICI, ISI and greater robustness to narrowband interference [7]. Moreover, there are other efficient techniques for the implementation of FB-MCTs such as complex-modulated transmultiplexers (CXMTs), cosine-modulated transmultiplexers (CMTs) and sine-modulated transmultiplexers (SMTs) [8]. In these systems, all the filters are designed in an efficient manner by modulation of a single prototype filter. In the real communication world, where the transmission channel itself introduces considerable distortion, it is better to go

**Fig. 1** *M*-Channel maximally decimated transmultiplexer



for NPR side [9]. In NPR systems, the constraints and complexity of perfect reconstruction (PR) is relaxed by allowing a small amount of interferences. The NPR system is beneficial since it provides better stopband performance with less complexity [10]. Therefore, this paper focuses on the design of NPR-type CMTs systems.

In a previously reported series of publications, Martin et al. [9,11,12] have designed optimized transmultiplexers using fixed window functions of the Blackman window family. The reported optimization algorithm has some constraints on the variation of the cutoff frequency of the prototype filter [9]. Also in [12], more than one parameter is taken as adjustable parameters during the optimization process.

In this proposed work, a NPR-type CMTs is designed using variable and combinational window functions. The main contribution of this paper can be summarized as below:

1. The fixed window functions have certain limitations in selecting input parameters. Therefore, popular variable window functions with high sidelobe falloff rate (SLFOR) are used for the design of prototype filters [13–16].
2. A new iterative unconstrained optimization technique is used for optimizing the interference parameters.

This paper is organized as follows: The NPR CMTs with their performance parameters are described in Sect. 2. Section 3 gives closed form expressions of the window functions in tabular form. The expression for cosine modulation of prototype filter is also included in this section. The optimization algorithm is described in Sect. 4. Section 5 provides some design examples, and a comparative discussion has been made with earlier reported work. Finally, in Sect. 6, some concluding remarks with utility of designed CMTs in the field of communication are elaborated.

## 2 Transmultiplexer system

The *M*-channel maximally decimated transmultiplexer is shown in Fig. 1. Basically, it is a TDM-FDM-TDM converter [17]. It consists of synthesis block at transmitter end and precedes the analysis block at receiver end. At the transmitter end, *M*-input signals are first interpolated by the factor of *M* and synthesized into one composite signal using synthesis filterbank $F_k(z)$ for $k = 0, 1, \ldots, M-1$. Conversely, at the receiver end, the composite signal is split out into *M*-output signals with the help of the analysis filterbank $H_k(z)$ and then decimated by a factor of *M*.

The $z$-transform of the output at particular $l$th subchannel is given by (1), in terms of $z$-transform of the *M*-input signals [9]:

$$\hat{X}_l(z) = \sum_{k=0}^{M-1} \left[ \frac{1}{M} \sum_{m=0}^{M-1} H_l\left(z^{1/M} W^m\right) C\left(z^{1/M} W^m\right) F_k\left(z^{1/M} W^m\right) \right] X_k(z) \tag{1}$$

where $W^m = \mathrm{e}^{-j2\pi m/M}$ and $C(z)$ represents the transmission characteristics of the transmission channel.

Equation (1) can also be expressed in transfer function form as:

$$\hat{X}_l(z) = \sum_{k=0}^{M-1} T_{lk}(z)\, X_k(z) \tag{2}$$

where $T_{lk}(z)$ is the transfer function between the output of the $l$th subchannel and the input of the $k$th subchannel, which is defined as:

$$T_{lk}(z) = \frac{1}{M} \sum_{m=0}^{M-1} H_l(z^{1/M} W^m) C(z^{1/M} W^m) F_k(z^{1/M} W^m) \tag{3}$$

In case of PR-type transmultiplexer with the ideal channel, $T_{lk}(z)$ should be zero for $l \neq k$ and one for $l = k$, *i.e*., the total error and interference are vanished.

### 2.1 Performance measuring parameters

The output at the receiver end is not a perfect replica of the input signal. These signals have distorted due to mixing of undesired interference during the processing and transmission. The amplitude of these interference signals is considered as performance measure of the designed system. The following are the performance measure parameters in the transmultiplexer system.

**Table 1** The window functions

| S. No. | Name | Window equation | SLFOR |
|---|---|---|---|
| 1 | Kaiser [16] | $w(n)=\frac{I_o\left[\beta\sqrt{1-\left(1-\frac{2n}{N-1}\right)^2}\right]}{I_o[\beta]},\quad 0\le n\le(N-1)$<br>where $I_o[.]$ is the modified zeroth-order Bessel function, and $\beta$ is window shape parameter | −6 dB/octave |
| 2 | PC6 [13–15] | $w_{PC6}(n)=\begin{cases}\gamma_6\,[l_6(n)]+(1-\gamma_6)\,[d_6(n)], & \lvert n\rvert\le\frac{N}{2}\\ 0, & \lvert n\rvert>\frac{N}{2}\end{cases}$<br>where $0\le\gamma_6\le 3.7$<br>$l_6(n)=\begin{cases}1-24\left\lvert\frac{n}{N}\right\rvert^2\left(1-2\left\lvert\frac{n}{N}\right\rvert\right), & \lvert n\rvert<\frac{N}{4}\\ 2\left(1-2\left\lvert\frac{n}{N}\right\rvert\right)^3, & \frac{N}{4}\le\lvert n\rvert\le\frac{N}{2}\end{cases}$<br>$d_6(n)=\cos^6\left(\frac{n\pi}{N}\right),\quad \lvert n\rvert\le\frac{N}{2}$ | −24 to −42 dB/octave |
| 3 | PC4 [13,15] | $w_{PC4}(n)=\begin{cases}\gamma_4\,[l_4(n)]+(1-\gamma_4)\,[d_4(n)], & \lvert n\rvert\le\frac{N}{2}\\ 0, & \lvert n\rvert>\frac{N}{2}\end{cases}$<br>where $0\le\gamma_4\le 8.235$<br>$l_4(n)=\frac{1}{\pi}\left\lvert\sin\left(\frac{2\pi n}{N}\right)\right\rvert+\left(1-2\left\lvert\frac{n}{N}\right\rvert\right)\cos\left(\frac{2\pi n}{N}\right),\ \lvert n\rvert\le\frac{N}{2}$<br>$d_4(n)=\cos^4\left(\frac{n\pi}{N}\right),\ \lvert n\rvert\le\frac{N}{2}$ | −24 to −30 dB/octave |

**Table 2** Performance comparison with earlier reported works

| | Window function | $N$ | $\omega_c$ | $\Delta_s$ | ICI (dB) | ISI (dB) |
|---|---|---|---|---|---|---|
| Reported work Martin et al. [11] | Modi. Blackman | 48 | – | −50 | −49.81 | −46.06 |
| Proposed method | PC6 | 48 | $0.15\pi$ | −50 | −100.17 | −72.24 |
| | PC4 | 48 | $0.08\pi$ | −50 | −99.97 | −72.23 |
| | Kaiser | 48 | $0.12\pi$ | −50 | −108.58 | −72.24 |
| Reported work Martin et al. [23] | Blackman | 64 | – | ≈ −70 | −90.81 | −50.77 |
| | Kaiser | 64 | – | −100 | −92.22 | −44.03 |
| Proposed method | Kaiser | 64 | $0.11\pi$ | −100 | −92.43 | −72.22 |

The ICI is the leakage of signal from the remaining $(M-1)$ subchannels to given particular subchannel. This occurs due to interference of the filters in their stopband. The ICI in the $l$th subchannel can be conveniently measured as [9]:

$$E_{\mathrm{ICI}}(l)=\frac{1}{\pi}\int_0^{\pi}\left(\sum_{k=0,k\neq l}^{M-1}\lvert T_{lk}(\mathrm{e}^{j\omega})\rvert^2\right)\mathrm{d}\omega \tag{4}$$

where $T_{lk}(\mathrm{e}^{j\omega})$ can be obtained from (3). On the other hand, the ISI is caused by the interference of other symbols in the same subchannel. This occurs due to the non-ideal nature of the filters within their passband and the fact that channel frequency response is not uniform for all subchannels. The ISI in the $l$th subchannel can be measured as [9]:

$$E_{\mathrm{ISI}}(l)=\frac{1}{\pi}\int_0^{\pi}\left(\lvert T_{ll}(\mathrm{e}^{j\omega})-1\rvert^2\right)\mathrm{d}\omega \tag{5}$$

where $T_{ll}$ can be obtained from (3).

## 3 Prototype filter design

Up to now, several approaches have been developed by the researchers for the design of prototype filters for CMTs [18, 19]. The windowing approach for the design of linear phase $N$th-order FIR prototype filter has certain inherent advantages [16]. Martin et al. [12] has designed CMTs using cosine window functions, which are fixed window functions and

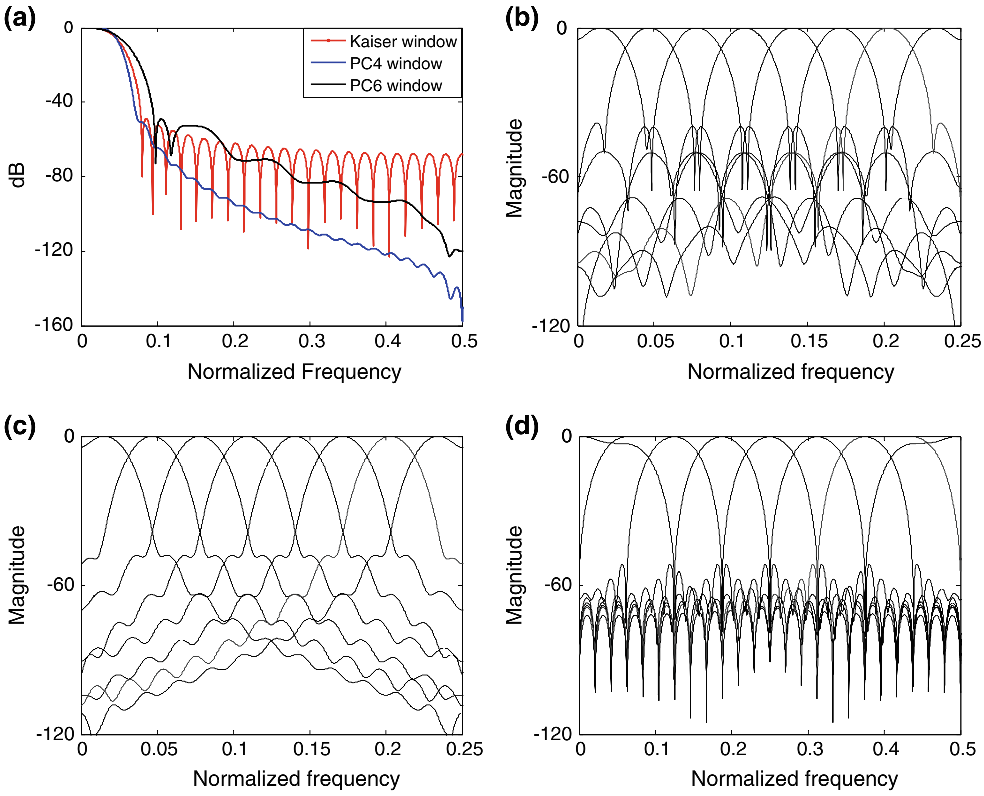


**Fig. 2** **a** Magnitude responses of prototype filter for $\Delta_s = -50\,\text{dB}$; 8-channel CMTs using, **b** PC6 window, **c** PC4 window, **d** Kaiser window function

have certain well-known limitations. Therefore, in this work, variable window functions with high SLFOR are used to design the prototype filter.

The impulse response coefficients of a causal *N*th-order linear phase FIR filter $p[n]$ are given as:

$$p[n] = w[n]h[n] \tag{6}$$

where $h[n]$ is the impulse response of a causal ideal lowpass filter given by:

$$h[n] = \frac{\sin\left(\omega_c\left(n - 0.5N\right)\right)}{\pi\left(n - 0.5N\right)} \tag{7}$$

where $\omega_c$ is the cutoff frequency of the causal ideal lowpass filter, and $w[n]$ is the window function satisfying $w[N-n] = w[n]$. The window functions under consideration are Kaiser window (variable) [16], Parzen-$\cos^6(n\pi/N)$(PC6) and Papoulis-$\cos^4(n\pi/N)$ (PC4) combinational variable window functions. These windows are designed by combining a data window and a lag window in a linear manner [13–15] and provide high SLFOR with better far end attenuation [13]. These characteristics can provide better suppression of the interference parameters than other window functions. The closed form expression for these window functions is given in Table 1.

The order ($N$) of the filter designed by using above-mentioned window functions can be estimated by the following formula:

$$N = \left\lceil \frac{D}{\Delta\omega} \right\rceil + 1 \tag{8}$$

where $D$ is the window width parameter, and $\Delta\omega$ is the normalized transition width $=(\omega_s - \omega_p)/2\pi$. $\lceil x \rceil$ represents the smallest integer greater than or equal to $x$. A filter designed by the use of a window is specified by three parameters—cutoff frequency ($\omega_c$), filter order ($N$) and window width parameter ($D$) for the desired stopband attenuation ($\Delta_s$) and appropriately chosen transition bandwidth ($\Delta\omega$).

### 3.1 Cosine modulation

Cosine modulation is one of the efficient techniques to design the synthesis and analysis sections of a transmultiplexer with minimum computational cost. In CMTs, all the filters of synthesis and analysis sections are obtained by cosine

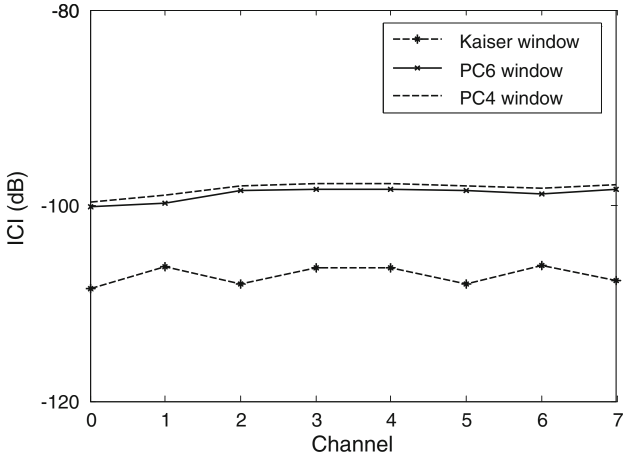


**Fig. 3** Subchannel versus ICI plot for 8-channel CMTs

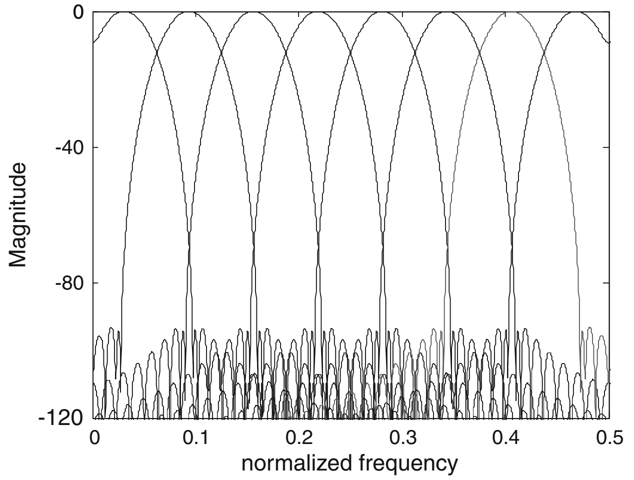


**Fig. 4** Eight-channel CMTs using Kaiser window for $\Delta_s = -100\,\text{dB}$

modulation of a linear phase lowpass prototype FIR filter [16].

$$\left.\begin{aligned} f_r(n) &= 2p(n)\cos\left[(2r+1)\frac{\pi}{2M}\left(n-\frac{N}{2}\right)-(-1)^r\frac{\pi}{4}\right]\\ h_r(n) &= 2p(n)\cos\left[(2r+1)\frac{\pi}{2M}\left(n-\frac{N}{2}\right)+(-1)^r\frac{\pi}{4}\right]\end{aligned}\right\}$$

$$\text{for}\quad \begin{array}{l} 0 \le r \le (M-1) \quad \text{and} \\ 0 \le n \le N \end{array} \tag{9}$$

where $f_r(n)$ and $h_r(n)$ are the filters of synthesis and analysis section, respectively.

**Table 3** The comparative performance for stopband attenuation $(\Delta_s) = -35$ and $-56\,\text{dB}$

| Window function | $\Delta_s$ | $N$ | ICI (dB) | ISI (dB) |
|---|---|---|---|---|
| PC6 | | 46 | −109.29 | −72.22 |
| PC4 | −35 | 42 | −103.61 | −72.23 |
| Kaiser | | 32 | −95.61 | −72.24 |
| PC6 | | 82 | −102.05 | −72.24 |
| PC4 | −56 | 102 | −97.99 | −72.23 |
| Kaiser | | 54 | −92.59 | −72.22 |

**Table 4** The comparative performance for filter order ($N$) = 46 and 92

| Window function | $N$ | $\Delta_s$ | ICI (dB) | ISI (dB) |
|---|---|---|---|---|
| PC6 | | −35 | −109.09 | −72.22 |
| PC4 | 46 | −36 | −106.75 | −72.23 |
| Kaiser | | −48 | −107.83 | −72.24 |
| PC6 | | −59 | −98.90 | −72.24 |
| PC4 | 92 | −51 | −102.67 | −72.23 |
| Kaiser | | −90 | −98.78 | −72.22 |

## 4 Optimization technique

In NPR system, the PR conditions are certainly relaxed by allowing a small amount of error and interferences. These interferences can be minimized by applying suitable optimization techniques. Much work has been done in this field [14,18–20]. Initially, Johnston [20] developed a nonlinear optimization technique for the design of filterbanks. Later on, many prominent authors such as Creusere et al. [18] and Lin et al. [19] and others [14,21] have simplified it and developed single-variable linear optimization techniques. In these algorithms, a single-variable parameter is varying as per search direction and calculates the corresponding values of the objective function. Different authors used different objective function and obtained the minimum possible value of reconstruction error in a filterbank. Since transmultiplexer is dual of filterbank [17], therefore similar optimization techniques can be applied to transmultiplexer with different objective functions [22]. In this proposed work, a single-variable optimization technique is used with ICI given in (4) as an objective function, i.e., $[g(\omega_c) = |E_{\text{ICI}}(l)|]$. The cutoff frequency ($\omega_c$) of the prototype filter is selected as a variable parameter. The proposed optimization technique is independent of the window function. Initially, the supplied input parameters, i.e., sampling rate, passband and stopband frequencies, number of bands, passband ripple and stopband attenuation are specified. Based on these inputs, the filter order and initial value of cutoff frequency will be calculated, then the window coefficients of the selected window function are determined and they remain fixed. The filter coefficients of the prototype lowpass filter and filters of synthesis and analysis section

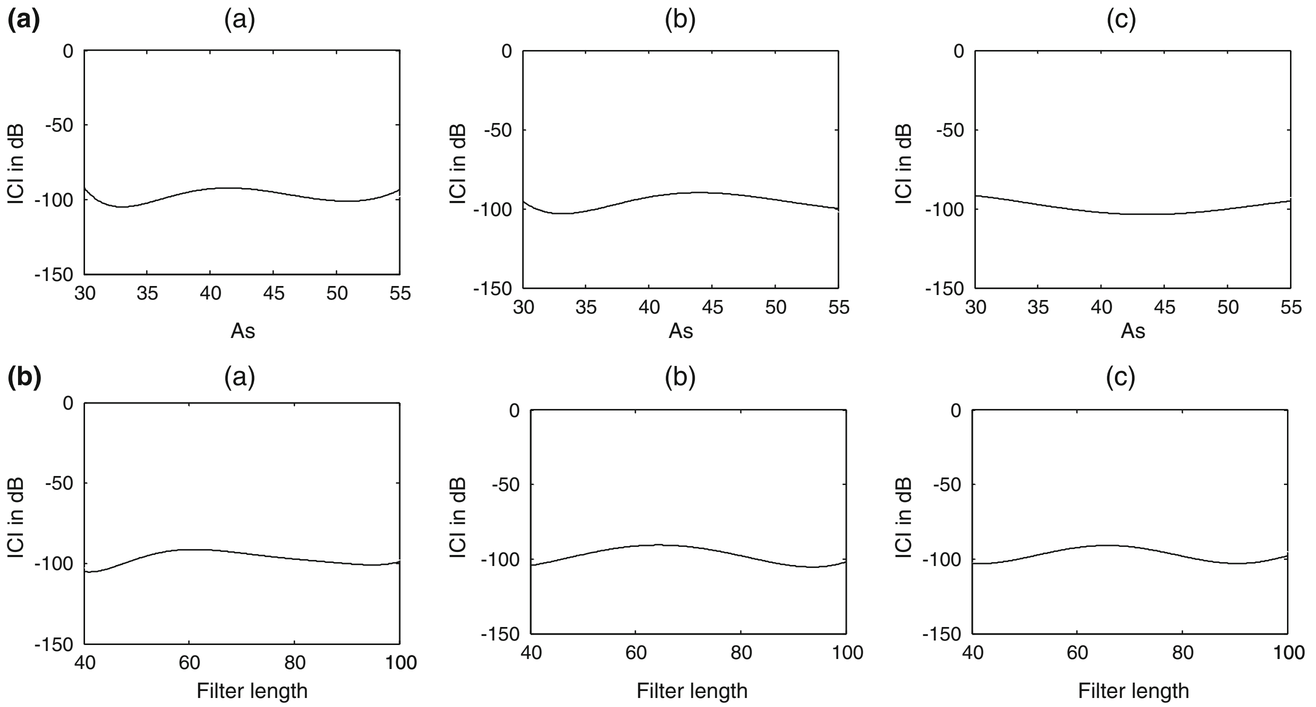


**Fig. 5** **a** Plot of stopband attenuation $\Delta_s$ versus ICI for (a) PC4 window (b) PC6 window (c) Kaiser window. **b** Plot of filter length versus ICI for (a) PC4 window (b) PC6 window (c) Kaiser window

are determined using cosine modulation. The ICI in a particular subchannel for current value of cutoff frequency is calculated. The absolute value of ICI is selected as an objective function. In the optimization routine cutoff frequency of the prototype filter is continuously iterates as per the search direction and calculate the corresponding value of these parameters. The algorithm terminates the loop as it attains the optimum value of the objective function. The flow chart of the optimization algorithm is given in "Appendix". It is implemented on MATLAB 6.5 version on Pentium IV Processor.

## 5 Design examples

The evaluation of the proposed optimization technique is done with the help of the examples. The first two examples compare the performance with the earlier reported work [11,23]. The third one shows the optimal performance and the comparison with widely used Kaiser window function.

In the first example, the input parameters such as stopband attenuation and filter order are taken like in reported work [11], and correspondingly, the initial value of the cutoff frequency is calculated for the chosen values of passband and stopband frequencies. These input parameters are fixed before calling the optimization algorithm. Only one parameter, i.e., cutoff frequency, is variable during optimization and attains the minimum value of the objective function. The obtained values of different parameters are given in Table 2. The frequency responses of prototype filter for different window functions are shown in Fig. 2a. The magnitude responses of 8-channel transmultiplexer are shown in Fig. 2b–d.

Since transmultiplexers multiplex the multiple data coming from different sources. Therefore, it is necessary to analyze the ICI level in each subchannel, which is shown in Fig. 3.

In the second example, the Kaiser window function is selected to show the superiority of the proposed algorithm with the reported publication [23]. The same input parameters, i.e., stopband attenuation ($-100$ dB) and filter order (64), are taken in this example. PC6 and PC4 window functions cannot be used as they are limited up to $-68.69$ and $-61.08$ dB, respectively. The obtained values of ICI and ISI are given in Table 2. The magnitude response of 8-channel transmultiplexer using Kaiser window is shown in Fig. 4.

In another case study, Tables 3 and 4 show the comparative performance between the combinational window functions, i.e., PC6, PC4 and the popular Kaiser window function.

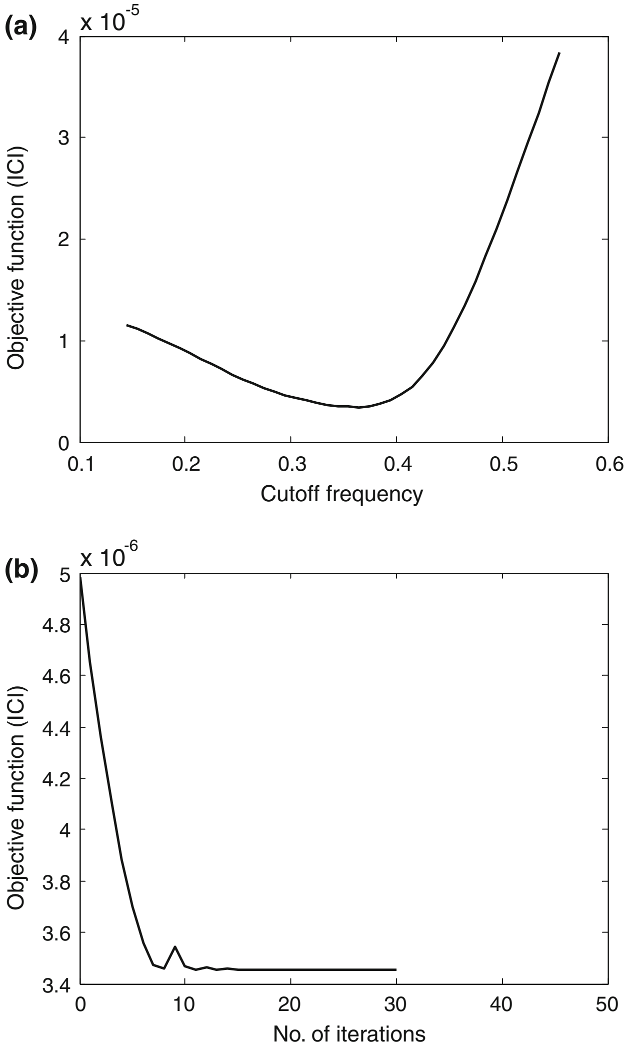


**Fig. 6** **a** Plot between cutoff frequency ($\omega_c$) and objective function (ICI). **b** No. of iterations versus objective function (ICI) using PC6 window function for $\Delta_s = 35$ dB, $N = 46$ and $M = 8$

It is quite important to analyze the output of a transmultiplexer against variation in the input quantities. This factor decides the range of application of the system. Therefore, Fig. 5a and b show the variation of ICI with respect to stopband attenuation and filter length, respectively.

### 5.1 Discussion

In the last half decade, Martin et al. [9,11,12,23] have done excellent work in this field. They used cosine-based fixed window functions for the design of the prototype filter of transmultiplexers. A specific four-variable optimization technique is used in their design, which optimizes the weight terms of the window function. As per [12], this technique requires a good starting point for obtaining the optimum results. However, in the proposed work, a single-variable generalized optimization technique is applied. Variable combinational window functions with high SLFOR are used for the design of prototype filter. These window functions provide more flexibility and better performance than fixed window functions. As it is clear from the Fig. 2a that these window functions have better far end attenuation than widely used Kaiser window function. This property provides better suppression of ICI when compared to Kaiser window function. It is evident from the results of the quoted examples that the obtained values of interferences parameters are much better than earlier reported work. It is clear from Fig. 3 that the used optimization technique provides almost the same interference level in all subchannels of the transmultiplexers. Therefore, there is no constraint of selecting a particular subchannel for a given particular application. It is also clear from Fig. 5 that the interference level is almost constant of $\approx -100$ dB, in the range of stopband attenuation varying from $-30$ to $-55$ dB and filter length from 35 to 100.

The Fig. 6a shows that the proposed optimization technique is of convex nature. The convexity of the objective function, as numerically validated in Fig. 6a, guarantees that the optimization algorithm terminates at all, not only that local minima are also global. The algorithm is converging within few iterations as shown in Fig. 6b. These results conclude that the designed transmultiplexer is suitable for a wide range of stopband attenuation as well as filter length.

## 6 Conclusions

A simple and an efficient design for CMTs is presented. The proposed algorithm is a single-variable bisection-type optimization technique. It provides very small and stable interference levels against variation in the input parameters, i.e., stopband attenuation and filter order. Also, the interference level in all subchannels is almost equal. Therefore, there is no burden of selecting a particular subchannel for a given particular application. This wide range stability makes it useful for a variety of applications of audio and video fields.

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## Appendix

See Fig. 7.

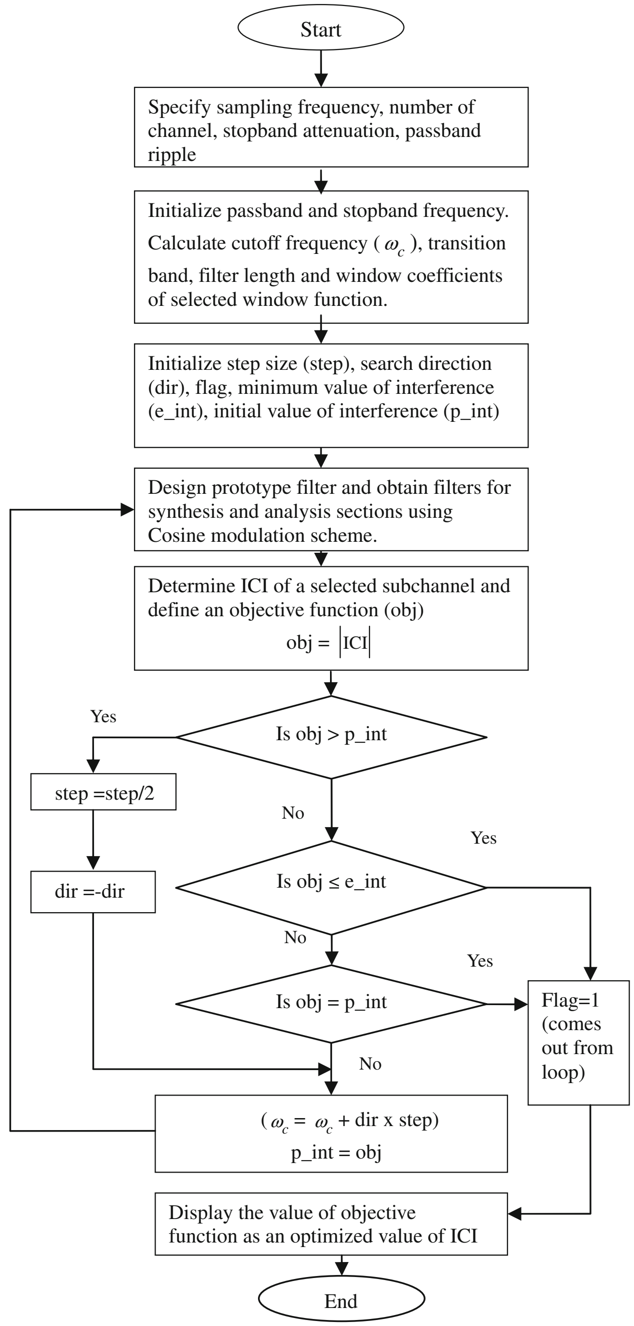

**Fig. 7** Flow-chart of optimization

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