KSME

Springer

Journal of Mechanical Science and Technology 35 (12) 2021

Original Article
DOI 10.1007/s12206-021-1131-x

Keywords:
· Adaptive kriging model
· Electromagnetic performance
· Integrated structural-electromagnetic analysis
· Reflector antenna
· Reliability-based design optimization

Correspondence to:
Naigang Hu
nghu@xidian.edu.cn

Citation:
Hu, N., Xu, W. (2021). Reliability-based design of reflector antennas with integrated structural-electromagnetic analysis using adaptive kriging modeling. Journal of Mechanical Science and Technology 35 (12) (2021) 5601~5610.
http://doi.org/10.1007/s12206-021-1131-x

Received September 7th, 2020
Revised June 10th, 2021
Accepted September 7th, 2021

† Recommended by Editor Hyung Wook Park

Reliability-based design of reflector antennas with integrated structural-electromagnetic analysis using adaptive kriging modeling

Naigang Hu[1,2] and Wanye Xu[1,2]

[1]Key Laboratory of Electronic Equipment Structure Design of Ministry of Education, Xidian University, Xi'an 710071, China, [2]Shaanxi Key Laboratory of Space Solar Power Station System, Xidian University, Xi'an 710071, China

Abstract Reflector antennas have been widely used in various applications, such as telecommunication and radio astronomy. However, they suffer from electromagnetic (EM) performance degradation caused by structural uncertainties. In this study, a reliability-based design optimization (RBDO) method for reflector antennas that involves EM performance with adaptive kriging modeling is developed for the first time to the authors' knowledge. An integrated structural-EM analysis procedure for antennas with material property uncertainties is introduced. Then, an adaptive kriging-based reliability analysis method is developed to alleviate the heavy computational costs. To improve kriging modeling efficiency, a priori knowledge of antennas is utilized to reduce the traditional initial sampling region. Finally, the RBDO problem is formulated as a double-loop process with a reliability index constraint. Simulation results show that the proposed method can significantly improve structural reliability with considerably less computational cost.

1. Introduction

Reflector antennas are among the best solutions for the requirements of cost-effective, high-gain, and high-performance antenna systems [1]; and they have been widely used in remote sensing, telecommunication, radio astronomy, and other applications [2]. Nevertheless, an antenna's electromagnetic (EM) performance is invariably affected by structural uncertainties in practical engineering, such as manufacturing, material property, geometric, and loading condition errors. These factors must be considered in the antenna design procedure [3] to achieve reliable EM performance.

Early studies focused on the effects of structural errors on the reflector surface's root-mean-square (RMS) accuracy [4, 5]. Radiation performance, i.e., gain loss, can be estimated with the RMS value by using the well-known Ruze formula [6]. Nevertheless, this method overlooks other aspects of EM performance, such as side-lobe level and cross-polarization [7], which can be important in particular applications. Therefore, such antenna designs suffer from essentially pure structural design problems with RMS accuracy as the design objective [8, 9].

Some researchers [10-12] investigated the effects of random surface errors on EM performance [6]. The characteristics of the average radiation pattern with the influence of random surface errors were provided [13]. In their study, errors were assumed to have a Gaussian distribution with zero mean and specific standard deviation. Thus, the task of structural designers is to find solutions that satisfy surface RMS requirement. In addition to probabilistic models that can quantify uncertain errors, interval models [14] are also utilized to estimate the variation interval of the average radiation pattern [15, 16]. Nevertheless, all these previous studies are still unable to guide the structural design of reflector antennas efficiently [17] because only average radiation performance with respect to the amplitude and distribution of random surface errors is considered.

An integrated structural-EM concept is developed [18, 19] to construct a direct relationship between antenna actual errors (not just hypothetical surface errors) and EM performance. Moreover, the mean value and variance of far-field directivity with respect to structural random dimensional errors are developed [7]. Then, an optimization framework is introduced with the average boresight directivity (the mean value) as the design objective when the cable lengths of the mesh reflector exhibit imperfections. Compared with traditional structural designs, this method can yield design candidates with better EM performance [17]. Nevertheless, the variation characteristics of the optimal design performance are unclear. Therefore, antenna EM performance reliability, which can provide clear and explicit demands to structural design, is difficult to measure. To the authors' knowledge, however, the reliability of an antenna structure that considers EM performance has not yet been investigated.

For reflector antennas, EM performance, such as antenna gain and side-lobe level, is significantly affected by reflector surface distortion. The shape variation of the surface changes the phase of EM waves and then severely influences the aforementioned antenna performance, particularly when the working frequency is high. For example, for a reflector antenna working at 8 GHz, a 10 % deviation of Young's modulus will lead to an additional gain loss of 0.2 dB, causing approximately 1 % shortening of the signal receiving distance, which may reach several or tens of kilometers. This condition may not be acceptable in certain circumstances. Therefore, the uncertainties of Young's modulus are considered in the current study.

By extending the integrated structural-EM concept from the deterministic design into the reliability-based design, a new reliability-based design method for reflector antennas with Young's modulus uncertainties is developed. This method is the major contribution of this work. The design problem is formulated as a structural weight minimization problem with a reliability index constraint. An adaptive kriging model is utilized to approximate the implicit relationship between uncertain material parameters, e.g., Young's modulus and antenna gain, to relieve the computational burden. The optimal design candidates can be obtained with acceptable reliability, which is of considerable significance in antenna structural design.

The remainder of this paper is organized as follows. Sec. 2 presents the proposed multidisciplinary reliability analysis method for reflector antennas that considers EM performance. The formulation of the reliability-based design optimization (RBDO) is provided in Sec. 3. The framework is described in Sec. 4 with different antenna design problems. Conclusions are drawn in Sec. 5.

2. Reliability analysis of antenna structure that considers EM performance

Reliability analysis/assessment is extremely important in RBDO methodology [20]. Although studies on structural RBDO are abundant [21-23], the application of RBDO to antenna multidisciplinary design is highy limited. In this section, an efficient reliability analysis procedure for antenna EM performance with respect to uncertain material parameters, i.e., Young's modulus, is developed.

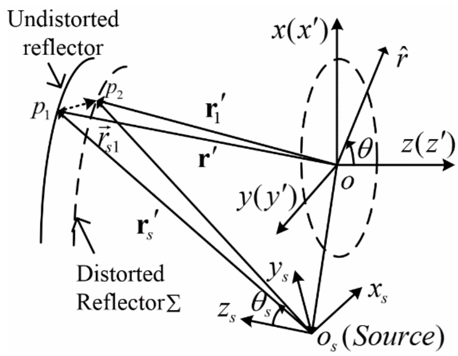

Fig. 1. Reflector antenna illuminated by an arbitrarily located source.

2.1 Integrated structural-EM antenna analysis procedure

The physical optics (PO) method is adopted in this work to calculate the far-field pattern of reflector antennas; this method achieves good accuracy in the main beam and the first few side-lobe regions [24]. Before implementing PO to obtain the shape (i.e., the node coordinates) of the distorted reflector, finite element analysis (FEA) should be conducted with each realization of uncertain material parameters. The formulation of the method is as follows [25]:

$$\mathbf{E}(\theta,\phi) = -jk\eta \frac{e^{-jkr}}{4\pi r}\left(\hat{\hat{I}} - \hat{r}\hat{r}\right)\iint \mathbf{J}(\mathbf{r}')\cdot e^{jk\hat{r}\cdot\mathbf{r}'}d\Sigma \tag{1}$$

where $\mathbf{E}$ is the electric field intensity, and θ, ϕ is the spherical coordinates. $j=\sqrt{-1}$, $k = 2\pi/\lambda$ (λ is the wavelength), and $\eta = 120\pi$. r is the distance from the observation point to the coordination system's origin, and $\hat{r}$ is the corresponding unit vector. $\hat{\hat{I}}$ is a unit dyadic, and $\hat{r}\hat{r}$ is the dyadic of $\hat{r}$. $\mathbf{J}(\mathbf{r}') = 2\hat{n}\times\mathbf{H}^i$ is the induced current; $\hat{n}$ is the outward unit normal vector; and $\mathbf{J}$ and $\mathbf{H}^i$ are the induced current and magnetic field, respectively. Vector $\mathbf{r}'$ locates the integration point, and Σ is the distorted reflector surface. The details of the parameters are provided in Fig. 1.

Antenna gain is a measure of antenna efficiency and directivity [1]. Gain and directivity are coincident in the current study because efficiency is assumed as 100 %. Then, antenna gain (far-field directivity) is expressed as

$$G(\theta,\phi) = \frac{4\pi r^2}{\eta P_{rad}}\mathbf{E}(\theta,\phi)\mathbf{E}^*(\theta,\phi)\ . \tag{2}$$

Notably, the directivity and gain mentioned below imply the values in the maximum radiation intensity direction, which elicit the most concern among designers. In practical engineering, antenna gains are always smaller than the ideal gain (when the reflector has no deformation, i.e., the entire structure is a rigid body), and the ideal gain becomes a constant value once feed characteristics and reflector size are fixed. Therefore, gain loss can be used in antenna design as follows:

$$\Delta G = G_{\text{ideal}} - G . \tag{3}$$

2.2 Reliability analysis with EM performance

A variety of methods have been developed to perform reliability analysis [26-29] in the past decades. The reliability index approach (RIA) [30], which is one of the outstanding representatives, is adopted in the current study. RIA transforms the probabilistic constraint in RBDO into a reliability index restriction, and index β can be computed by solving the equivalent constraint optimization as follows:

$$\begin{aligned} &\text{Min} \quad \beta = \|\mathbf{U}\| \\ &\text{s.t.} \quad \Delta G(\mathbf{U}) - \Delta \mathrm{G}^{\mathrm{U}} = 0 \end{aligned} \tag{4}$$

where $\mathbf{U}$ is the uncertain parameter vector (Young's modulus) in standard normal space, and ΔG^{U} is the prescribed upper bound of gain loss. The index is defined as the shortest distance between the origin to the limit state surface defined by $\Delta G(\mathbf{U}) - \Delta \mathrm{G}^{\mathrm{U}} = 0$. The corresponding point in the limit state surface is called the most probable point (MPP). Probabilistic reliability can be transformed into

$$Prop(\Delta G(\mathbf{U}) \le \Delta G^{\mathrm{U}}) = -\Phi(\beta) \tag{5}$$

where Φ is the standard normal cumulative distribution function. In Eqs. (4) and (5), the RBDO problem can be formulated as a nonlinear optimization problem with an index constraint, as described in Sec. 3.

2.3 Adaptive kriging model approximation scheme

The computation of gain loss constraint values in Eq. (4), including FEA and EM analysis, is typically implicit and expensive to evaluate. Nevertheless, reliability analysis requires high accuracy, and thus, it typically entails a relatively large amount of function calls. This heavy computational burden severely limits the practical implementation of RBDO in antenna design. To alleviate this problem, surrogate model-based methods have elicited attention in improving the efficiency of RBDO problems by replacing the original limit state functions with surrogate models [31-33]. In the current work, the widely used kriging model [34] is adopted to approximate the relationship between gain loss and material Young's modulus.

The function $\Delta G(\mathbf{U})$ in standard normal space is approximated using the adaptive kriging model to compute the reliability index. The entire calculation process of the kriging model is implemented through MATLAB's DACE toolbox [35] in the current study.

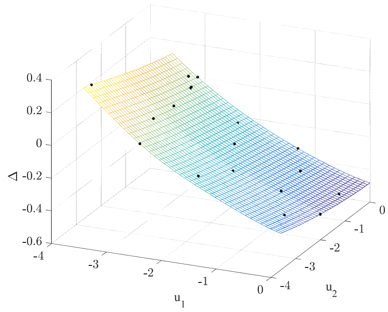

Fig. 2. Mesh plot of the constraint function ΔG in standard normal space.

2.3.1 Determination of sampling region size with prior knowledge

A large initial sampling region is typically required in a reliability analysis [36] to construct the surrogate model if prior knowledge regarding the design problem is scant. Under such condition, a scheme [37] is provided to determine the size of the sampling region on the basis of the maximum target index:

$$r = (1.2 + 0.3nc)\beta_{\text{tar}} \tag{6}$$

where r is the radius of the local adaptive region, which is a circle (2D) or a super sphere in *n* dimensions ($n > 3$). β_{tar} is the target reliability index. $nc \in [0, 1]$ is the nonlinearity coefficient, and it maintains a constant value of 0.2 during the entire design process in the current work. For additional details regarding nc, readers can refer to Ref. [38].

In reflector antennas, the relationship between gain loss and material Young's modulus is nonlinear but monotonic, as shown in Fig. 2. From the viewpoint of mechanics, structural stiffness increases as material Young's modulus increases. The surface distortion induced by the structural load can then be alleviated, and gain loss decreases accordingly. In the current study, prior knowledge, i.e., monotonic characteristics, is used to adjust the initial sampling region to a certain subspace that is described as

$$\begin{cases} \left\{\mathbf{U} \middle| \|\mathbf{U}\| \le r^2, \ \mathrm{u}_i \ge 0, \ \mathrm{i} = 1, 2, \ldots, \mathrm{N}\right\} & \text{if } \ \Delta G(\mathbf{0}) \ge \Delta G^{\mathrm{U}} \\ \left\{\mathbf{U} \middle| \|\mathbf{U}\| \le r^2, \ \mathrm{u}_i \ge 0, \ \mathrm{i} = 1, 2, \ldots, \mathrm{N}\right\} & \text{if } \ \Delta G(\mathbf{0}) \le \Delta G^{\mathrm{U}} \end{cases} . \tag{7}$$

As shown in Eq. (7) and Fig. 3, the original sampling region, i.e., the super sphere, can be reduced into its $1/2^N$. The MPP is certainly located in the new sampling region due to the monotonic characteristics. Approximation accuracy can be apparently improved if the same number of samplings is used. This condition is important in antenna RBDO problems.

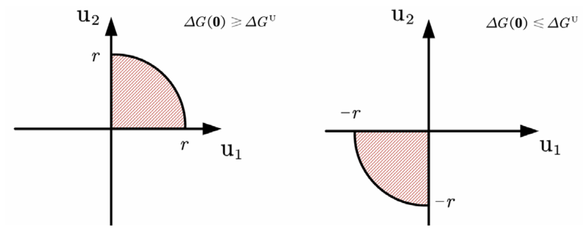

Fig. 3. Sampling region for different designs.

2.3.2 Importance boundary sampling (IBS) scheme

When the limit state function ($\Delta G(\mathbf{U})=\Delta G^{\mathrm{U}}$) is approximated, the samples should be located near the constraint boundary and the MPP to improve efficiency [19]. In the current work, the IBS method [39] is adopted to generate an adaptive kriging model. The IBS criterion is given as follows:

$$\mathrm{IBS}=\begin{cases}\sum_{j=1}^{N}\varphi\left(\dfrac{\hat{g}_j(\mathbf{U})}{\sigma_{g_j}(\mathbf{U})}\right)\cdot D(\mathbf{U})\cdot \mathrm{e}^{\tau \mathrm{I}(\mathbf{U})} & \text{if } \hat{g}_j(\mathbf{U})\ge 0,\ \forall j\\ 0 & \text{otherwise}\end{cases} \tag{8}$$

where

$$\mathrm{I}(\mathbf{U})=\frac{\max\left\{\left\|\mathbf{U}_{\mathrm{i}}-\mathbf{U}_{\mathrm{MPP}}\right\|\right\}_{\mathrm{i}=1}^{\mathrm{m}}-\left\|\mathbf{U}-\mathbf{U}_{\mathrm{MPP}}\right\|}{\max\left\{\left\|\mathbf{U}_{\mathrm{i}}-\mathbf{U}_{\mathrm{MPP}}\right\|\right\}_{\mathrm{i}=1}^{\mathrm{m}}-\min\left\{\left\|\mathbf{U}_{\mathrm{i}}-\mathbf{U}_{\mathrm{MPP}}\right\|\right\}_{\mathrm{i}=1}^{\mathrm{m}}}. \tag{9}$$

The closer point $\mathbf{U}$ is from current $\mathbf{U}_{\mathrm{MPP}}$, the higher the importance coefficient $\mathrm{I}(\mathbf{U})$ will be, and the range of $\mathrm{I}(\mathbf{U})$ is evidently [0, 1]. An exponential expression $\mathrm{e}^{\tau \mathrm{I}(\mathbf{U})}$ is used to identify the critical parts of the current limit state boundaries, and τ is a constant and set as 20 in the current study. For additional details, readers can refer to Ref. [39].

1) The procedure of the proposed reliability antenna analysis is illustrated in Fig. 4, and the relevant explanations are as follows.

2) The current study adopts the double-loop optimization scheme; hence, the current design variables $\mathbf{X}$ should be provided before each inner reliability analysis process.

The initial samples are generated using the Latin hypercube method within the region described in Sec. 2.3.2. The number of samples in the current work is set as 15 to ensure relatively good accuracy that is higher than the required value $(n+1)(n+2)/2$ [40]. Then, the constraint values are evaluated by applying a multidisciplinary analysis procedure.

3) The kriging model is constructed using the aforementioned samples and the corresponding responses.

4) Reliability analysis is performed with the constructed kriging model in the k th iteration.

5) If the design converges, then the entire process is stopped. The convergence criterion is

$$\left|\beta_{\mathrm{k}}-\beta_{\mathrm{k-1}}\right|/\left|\beta_{\mathrm{k-1}}\right|\le\varepsilon, \tag{10}$$

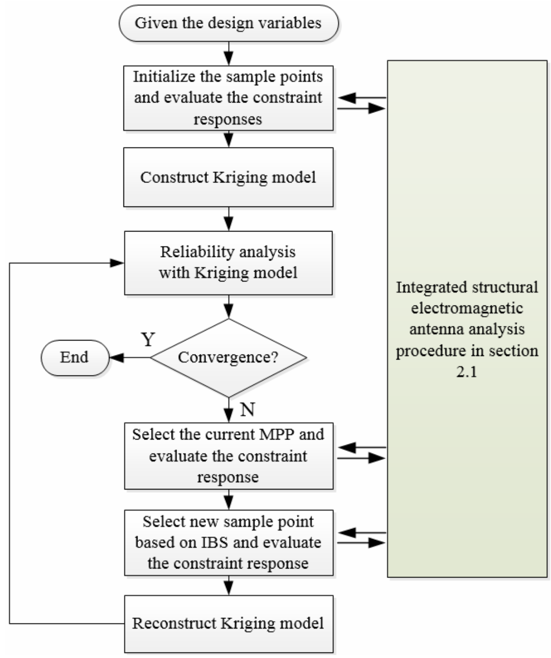

Fig. 4. Flowchart of the reliability analysis with the adaptive kriging model.

with $\varepsilon=0.001$ as the prescribed tolerance.

6) Otherwise, the current MPP and the point based on the IBS criterion are selected in the sample set. Then, the constraint values are separately evaluated.

7) Return to step 4.

3. Integrated structural-EM RBDO

The formulation of RBDO is given in Eq. (11), wherein the failure probability constraint of EM performance (gain loss) is in the form of a reliability index.

$$\begin{aligned}&\text{Find} && \mathbf{X}=(x_1,x_2,\dots,x_n)^{\mathrm{T}}\\ &\text{Min} && W(\mathbf{X})\\ &\text{s.t.} && \beta(\Delta G(\mathbf{X},\mathbf{P})\le\Delta G^{\mathrm{U}})\le\beta_{\mathrm{tar}}\\ &&& \mathbf{X}^{\mathrm{L}}\le\mathbf{X}\le\mathbf{X}^{\mathrm{U}}\end{aligned} \tag{11}$$

where the design variable $\mathbf{X}$ is the cross-section radius of the backup structure (BUS) beams. The objective is to minimize structural weight, and the formula is given as

$$W(\mathbf{X})=\sum_{i=1}^{n}\left[\rho\pi x_i^2\left(\sum_{q=1}^{\mathrm{NUM}_i}l_q\right)\right]+\mathrm{W}_0 \tag{12}$$

where ρ is the material density of BUS, i.e., the density of steel. NUM_i is the number of beams in the i th group, and W_0 is the weight of the structural components not involved in the optimization, e.g., reflector panels. Structural uncertain parameters $\mathbf{P}=(p_{\mathrm{b}},p_{\mathrm{p}})^{\mathrm{T}}$ include material properties, i.e., the Young's moduli of the BUS (steel) and reflector panels (alumi-

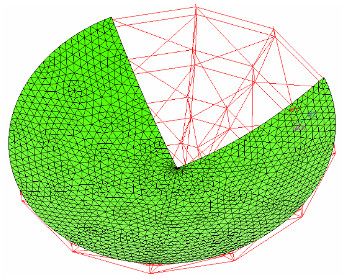

Fig. 5. Reflector antenna with a diameter of 8 m.

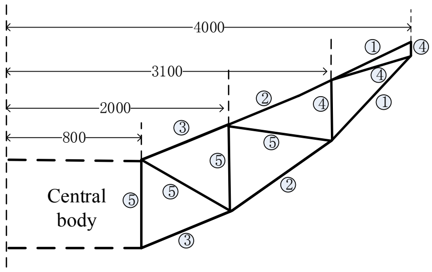

Fig. 6. BUS and design variables.

num) in the original space. The reliability index β is obtained using the method described in Sec. 2.

4. Numerical examples

The structural design problem of a reflector antenna with a diameter of 8 m is investigated to validate the proposed method. As shown in Fig. 5, the parabolic reflector antenna structure consists of 12 radial beams and 4 rings. Approximately 1/4 of the reflector panels are hidden in the figure to show the antenna BUS. Fig. 6 depicts one of the twelve radial beams, the design variables, and the five categories of the BUS beams involved in the design problem, which are numbered in the figure. The working frequency of the antenna is 8 GHz, and the structural load is the antenna's self-weight. The upper bound of gain loss ΔG^U is set as 1.2 dB. These examples are tested in MATLAB with the optimizer "fmincon."

4.1 Reliability analysis of reflector antennas by using different methods

The IBS method adopted in the current work was developed by Chen et al. [39]. In the present study, we integrate the scheme that reduces the initial sampling region size via the monotonic characteristics into this method. In this section, the reliability analysis process with the proposed adaptive kriging model, the constraint boundary sampling (CBS)-based adaptive kriging model [41], and the real model are implemented separately to demonstrate the advantages of the proposed method described in Sec. 2.3. Notably, the design vector is set as $\mathbf{X} = (10, 10, 10, 10, 10)^T$.

The results obtained using the real model are assumed as accurate. As indicated in Table 1, the proposed model and the

Table 1. Reliability analysis results of the three methods.

	Proposed kriging model	CBS-based kriging method	Real model
β	2.7760433	2.7760626	2.7759838
Number of function calls	13	31	54

Table 2. Material properties in the FEA model.

	Mean of Young's modulus (GPa)	Density (kg/m^3)	Poisson's ratio
BUS (steel)	210	7800	0.3
Reflector (aluminum)	70	2700	0.3

Table 3. Design parameters for antenna RBDO problems.

		σ_s (GPa)	σ_a (GPa)	β_{tar}
Deterministic	Case 0	/	/	/
Different target indices	Case 1	20	5	2.5
	Case 2	20	5	2.8
	Case 3	20	5	3.0
Different uncertainty levels	Case 4	10	2.5	3.0
	Case 5	10	5	3.0
	Case 6	20	10	3.0
	Case 7	25	10	3.0

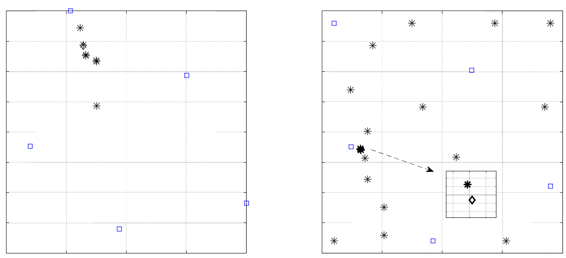

Fig. 7. Sample points for the two methods in the reliability analysis.

CBS-based model can obtain relatively accurate results. However, the computational effort of the proposed model is considerably less than those of the others. Fig. 7 shows the sample points during the reliability analysis process. In the figure, "□" represents the initial sample point; "*" is the sequentially added point during the adaptive kriging modeling; and "◇" is the optimal point, i.e., the most probable point. The region of the proposed model (left figure) is reduced to 1/4 of the region of the CBS-based model (right figure) by using the scheme described in Sec. 2.3.1. Moreover, the added points are located more "wisely," i.e., the points are all around the final optimal point. For the CBS-based model, however, a large number of the added points do not contribute to the convergence of the reli-

Table 4. Summary of the optimization results with different target indices.

		r_1 (mm)	r_2 (mm)	r_3 (mm)	r_4 (mm)	r_5 (mm)	W (kg)	β	Number of function calls
Case 0		0.8808	1.7482	1.0916	6.4347	0.8709	645.3301	0	–
Case 1	Analytical	2.1289	4.5640	4.1854	12.8234	3.2630	722.6431	2.4993	32284
	Proposed	2.1539	4.6002	4.1884	12.7714	3.3290	722.7115	2.4999	7357 (22.79 %)
Case 2	Analytical	2.9986	6.6874	5.7663	15.4696	5.7183	793.1233	2.8000	19152
	Proposed	3.0150	6.7293	5.8199	15.4718	5.5450	793.1008	2.7999	9254 (48.32 %)
Case 3	Analytical	4.8207	9.7345	8.6072	18.8655	9.7037	942.9357	3.0000	33033
	Proposed	5.0476	10.5185	8.5367	18.4334	9.0966	943.5916	2.9998	4900 (14.83 %)

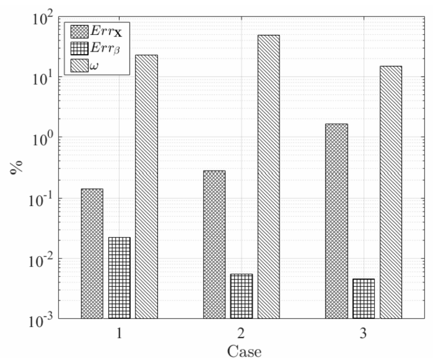

Fig. 8. Comparison of the optimal results with different target indices.

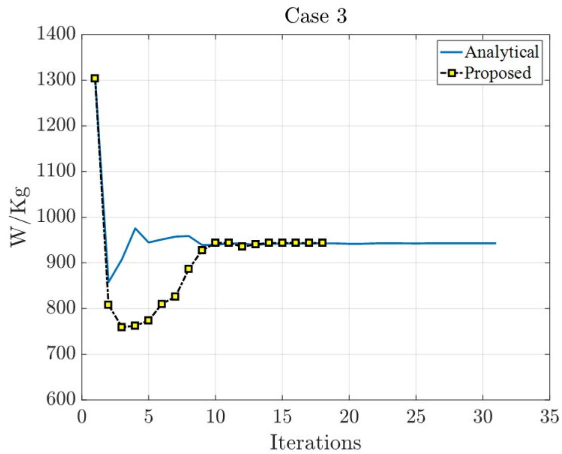

Fig. 9. Iteration history of case 3 with different methods.

ability analysis process. This phenomenon can explain the differences found in Table 1.

The structure's material parameters are listed in Table 2. In this work, the two Young's moduli are assumed to follow the Gaussian distribution with mean values. The deterministic design (case 0) and seven RBDO with different target reliability indices β_{tar} (cases 1-3) and uncertainty levels (cases 4-7), i.e., the standard deviations of the moduli, are implemented. To demonstrate the efficiency of the proposed method, for cases 1-7, the optimization that requires the actual performance functions (real model), i.e., without the kriging method, is also implemented and labeled as the analytical method. That is, this method requires FEA and the MATLAB-based PO method in every evaluation of antenna gain loss. The optimal results obtained using this method are assumed as accurate. This assumption can be used to justify the proposed method. The only difference is that the proposed method utilizes the adaptive kriging model in reliability analysis, whereas the analytical method does not. The standard variances of the moduli and target indices for the design cases are listed in Table 3.

4.2 RBDO of reflector antennas with different target indices

Three indicators are introduced to assess the optimal results of the new method. The first two are the relative errors of the optimal design $\mathrm{Err}_{\mathbf{X}}$ and the reliability index Err_β, which are shown as follows:

$$\mathrm{Err}_{\mathbf{X}} = \left\| \mathbf{X}^*_{\mathrm{prop}} - \mathbf{X}^*_{\mathrm{Ana}} \right\| / \left\| \mathbf{X}^{\mathrm{U}} - \mathbf{X}^{\mathrm{L}} \right\|, \tag{13}$$

$$\mathrm{Err}_\beta = \left| \beta_{\mathrm{prop}} - \beta_{\mathrm{Ana}} \right| / \beta_{\mathrm{Ana}} \tag{14}$$

where $\mathbf{X}^*$ is the optimal design (i.e., $r_1, \dots, r_5$). The two indicators represent the approximation level (accuracy) of the kriging-based method over the analytical method from the viewpoints of design and index, respectively. The third indicator is the ratio ω of the function call number of the new method to that of the analytical method (provided in parentheses in Tables 4 and 5). In contrast with the accuracy indicators, the ω values can be regarded as an efficiency improvement indicator.

The optimal results of cases 0-3 are listed in Table 4. The deterministic design has the lightest weight because it has no reliability requirement, and thus, its index is zero. For cases 1-3, the optimal weight increases with an increase of the target index. For every case, the new method can produce optimal results that are as good as those of the analytical method.

The three indicators are illustrated in Fig. 8, where the two relative errors are generally smaller than 1 %. The values of ω indicate that the new kriging-based method can improve design efficiency by more than 50 %. Fig. 9 shows the iteration history of the two methods for case 3, with the proposed method converging after 18 iterations, while the others need more than 30 iterations.

Table 5. Summary of the optimization results with different uncertainty levels.

		r_1 (mm)	r_2 (mm)	r_3 (mm)	r_4 (mm)	r_5 (mm)	W (kg)	β	Number of function calls
Case 4	Analytical	1.4376	2.6578	2.2498	8.8954	1.5255	666.4966	3.0013	16587
	Proposed	1.4283	2.6372	2.2634	8.8964	1.5435	666.4717	2.9998	3800 (22.91 %)
Case 5	Analytical	1.4011	2.8234	2.4336	9.6786	1.5779	673.1002	2.9999	14031
	Proposed	1.4082	2.7993	2.4623	9.6775	1.5774	673.0997	3.0002	5463 (38.94 %)
Case 6	Analytical	12.0642	16.2507	13.2753	33.8312	13.2142	1551.9426	3.0000	10540
	Proposed	13.1283	15.6660	13.3987	34.3392	11.6800	1551.9785	2.9996	5547 (52.63 %)
Case 7	Analytical	0.1057	19.4837	22.9958	99.2122	15.4002	4992.6485	3.0000	36854
	Proposed	0.1059	19.5144	22.9200	99.2374	15.3623	4992.1501	2.9999	9545 (25.90 %)

4.3 RBDO of reflector antennas with different uncertainty levels

Table 5 provides the optimization results for cases 4-7, and the three indicators are shown in Fig. 10. Under different uncertainty levels, the proposed method can still achieve relatively better results with few function calls. This finding has considerable implications for engineering structure design. The iteration history of case 7 in Fig. 11 also proves the higher efficiency and better convergence performance of the proposed method.

Fig. 9 shows a slight change in function value between the two methods after approximately 15 iterations. By changing the convergence tolerance value, considerably less iteration number may be expected for the analytical method. Nevertheless, the proposed method is still more efficient because it requires significantly less number of actual function calls (FEA and EM analysis) in each iteration (reliability analysis) with the help of surrogate models.

In Fig. 11, the proposed method performs remarkably better than the analytical method. With approximately 20 iterations, the optimization procedure can obtain a near-optimal solution. By contrast, the analytical method requires more than 40 iterations to achieve a similar design. In addition, Tables 3 and 4 show that the optimal design and performance of the two methods are similar, indicating that the proposed method can achieve nearly the same solution with considerably less computational effort.

In Sec. 2.2, first-order RIA is adopted to perform reliability analysis for the antenna design problem. Monte Carlo simulations with actual performance functions are tested here to verify the rationality and accuracy of RIA. The reliability P_{MCS} of the optimal solutions with cases 3-7, i.e., the probability satisfying the gain loss constraint, is illustrated in Fig. 12. All the probability values are larger than 99.73 %, indicating that the FORM assumption is acceptable for the uncertain antenna design problems.

Fig. 13 provides the nominal values of gain loss of the optimal designs obtained using the proposed method. Given that case 0 is the deterministic design, the ΔG_0 value will certainly reach the constraint bound ΔG^{U} after optimization, signifying that the design cannot withstand any system uncertainty or

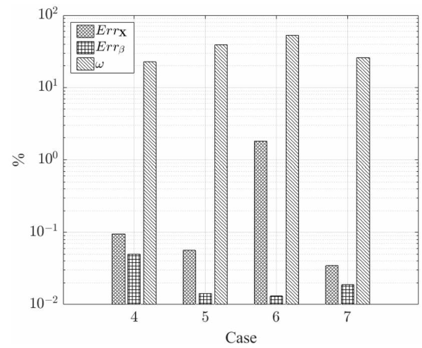

Fig. 10. Comparison of the optimization results with different uncertainty levels.

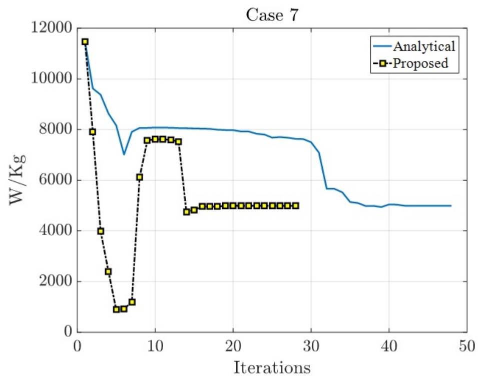

Fig. 11. Iteration history of case 7 by using different methods.

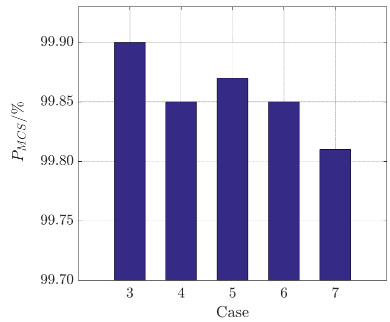

Fig. 12. Reliability testing via MCS for cases 3-7 when the proposed method was used.

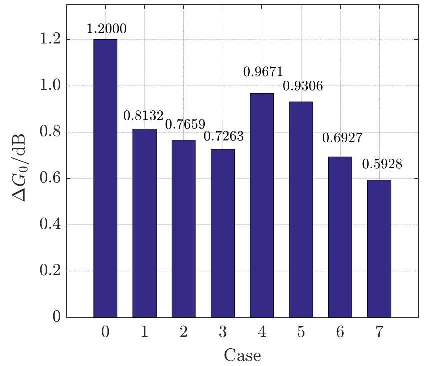

Fig. 13. Nominal values of ΔG for the optimal designs when the proposed method was used.

error. For cases 1-3 with the same uncertainty level, ΔG_0 decreases when the index increases. For cases 4-7, ΔG_0 decreases when the uncertainty level increases with same index.

5. Conclusions

Uncertainties in the antenna structure typically result in the deterioration of EM performance. In the current study, an integrated structural-EM RBDO method is developed to design a reflector antenna, and an adaptive kriging model method is developed to perform inner reliability analysis. Several antenna design examples with various target indices or uncertainty levels are tested to verify the accuracy and efficiency of the proposed method. RBDO can achieve good balance between minimizing objectives and ensuring structural safety by using a probabilistic reliability constraint rather than deterministic constraints.

Several conclusions can be drawn as follows.

1) Material property uncertainties significantly affect the reliability of antenna EM performance, and research on the RBDO of reflector antennas is necessary.

2) The proposed RBDO method can naturally produce reliable solutions with unavoidable structural uncertainties. It exhibits significant superiority over traditional deterministic designs.

3) Compared with the original analytical method, the proposed design method can yield optimal designs with nearly no accuracy loss with an average of 70 % less function calls. Efficiency can be improved further if slightly more accuracy loss is acceptable. The current work demonstrates the bright prospects of surrogate model-based methods for the RBDO of antennas.

Important topics that remain to be investigated in the future are as follows.

1) The effects of material uncertainties on antenna gain are the focus of the current study. Research that considers multi-source uncertain factors and multi-performance remains to be conducted.

2) The double-loop optimization adopted in the current study exhibits high accuracy but low efficiency. Decoupled design techniques [42] can be investigated in future work.

3) For large-dimension reflector antennas, the structure may become more complicated and a sensitivity analysis based on a multidisciplinary analysis procedure may be necessary to identify the critical design variables for further improving efficiency.

Acknowledgments

The work was supported by the National Natural Science Foundation of China (Grant No. 51905402), the Natural Science Basic Research Plan in Shaanxi Province of China (Grant No. 2019JQ-378), the China Postdoctoral Science Foundation (Grant No. 2019M653868XB), and the Fundamental Research Funds for the Central Universities.

References

[1] C. A. Balanis, *Modern Antenna Handbook*, 1st Ed., John Wiley and Sons, Inc., Hoboken (2008).

[2] J. W. M. Baars and H. J. Kärcher, *Radio Telescope Reflectors,* Springer, New York (2018).

[3] N. Chahat, R. E. Hodges, J. Sauder, M. Thomson, E. Peral and Y. Rahmat-Samii, CubeSat deployable ka-band mesh reflector antenna development for earth science missions, *IEEE Transactions on Antennas and Propagation*, 64 (6) (2016) 2083-2093.

[4] J. M. Hedgepeth, Influence of fabrication tolerances on the surface accuracy of large antenna structures, *AIAA Journal*, 20 (5) (1982) 680-686.

[5] A. Meguro, M. Jin and K. Ando, A modular cable-mesh deployable structure for large-scale satellite communication antennas, *Electronics and Communications in Japan Part*, 77 (8) (1994) 90-100.

[6] J. Ruze, Antenna tolerance theory-a review, *Proceedings of the IEEE*, 54 (4) (1966) 633-642.

[7] S. Zhang, J. Du and B. Duan, Integrated structural electromagnetic analysis of mesh reflectors with structural random dimensional errors, *AIAA Journal*, 53 (10) (2015) 2838-2844.

[8] J. Du, C. Wang, H. Bao and L. Wang, Robust shape adjustment with finite element model updating for mesh reflectors, *AIAA Journal*, 55 (4) (2017) 1450-1459.

[9] J. Du, H. Bao and C. Cui, Shape adjustment of cable mesh reflector antennas considering modeling uncertainties, *Acta Astronautica*, 97 (2014) 164-171.

[10] K. Bahadori and Y. Rahmat-Samii, Characterization of effects of periodic and aperiodic surface distortions on membrane reflector antennas, *IEEE Transactions on Antennas and Propagation*, 53 (9) (2005) 2782-2791.

[11] S. Sinton and Y. Rahmat-Samii, Random surface error effects on offset cylindrical reflector antennas, *IEEE Transactions on Antennas and Propagation*, 51 (6) (2003) 1331-1337.

[12] P. Lian, C. Wang, W. Wang and B. Xiang, Electromagnetic performance analysis of reflector antennas with non-uniform

errors along radius, *Journal of Systems Engineering and Electronics*, 27 (5) (2016) 961-967.

[13] Y. Rahmat-Samii, An efficient computational method for characterizing the effects of random surface errors on the average power pattern of reflectors, *IEEE Transactions on Antennas and Propagation*, 31 (1) (1983) 92-98.

[14] J. Wu, X. Han and Y. Tao, Kinematic response of industrial robot with uncertain-but-bounded parameters using interval analysis method, *Journal of Mechanical Science and Technology*, 33 (1) (2019) 333-340.

[15] P. Rocca, L. Manica and A. Massa, Interval-based analysis of pattern distortions in reflector antennas with bump-like surface deformations, *IET Microwaves Antennas and Propagation*, 8 (15) (2014) 1277-1285.

[16] P. Rocca, N. Anselmi and A. Massa, Interval arithmetic for pattern tolerance analysis of parabolic reflectors, *IEEE Transactions on Antennas and Propagation*, 62 (10) (2014) 4952-4960.

[17] S. Zhang and B. Duan, Integrated structural-electromagnetic optimization of cable mesh reflectors considering pattern degradation for random structural errors, *Structural and Multidisciplinary Optimization*, 61 (4) (2020) 1621-1635.

[18] J. S. Liu and L. Hollaway, Integrated structure-electromagnetic optimization of large reflector antenna systems, *Structural Optimization*, 16 (1998) 29-36.

[19] S. L. Padula, H. M. Adelman, M. C. Bailey and R. T. Haftka, Integrated structural electromagnetic shape control of large space antenna reflectors, *AIAA Journal*, 27 (6) (1989) 817-819.

[20] S. Lee, Reliability based design optimization using response surface augmented moment method, *Journal of Mechanical Science and Technology*, 33 (4) (2019) 1751-1759.

[21] Y. Luo, A. Li and Z. Kang, Reliability-based design optimization of adhesive bonded steel-concrete composite beams with probabilistic and non-probabilistic uncertainties, *Engineering Structures*, 33 (7) (2011) 2110-2119.

[22] C. Wang, H. G. Matthies, M. Xu and Y. Li, Hybrid reliability analysis and optimization for spacecraft structural system with random and fuzzy parameters, *Aerospace Science and Technology*, 77 (2018) 353-361.

[23] D. Zhang, S. Liu, J. Wu, Y. Wu and J. Liu, An active learning hybrid reliability method for positioning accuracy of industrial robots, *Journal of Mechanical Science and Technology*, 34 (8) (2020) 3363-3372.

[24] S. X. Zhang, B. Y. Duan, H. Bao and P. Y. Lian, Sensitivity analysis of reflector antennas and its application on shaped geo-truss unfurlable antennas, *IEEE Transactions on Antennas and Propagation*, 61 (11) (2013) 5402-5407.

[25] Y. Rahmat-Samii, A comparison between GO/aperture-field and physical-optics methods of offset reflectors, *IEEE Transactions on Antennas and Propagation*, 32 (3) (1984) 301-306.

[26] Z. Meng, D. Zhang, G. Li and B. Yu, An importance learning method for non-probabilistic reliability analysis and optimization, *Structural and Multidisciplinary Optimization*, 59 (4) (2019) 1255-1271.

[27] C. Jiang, W. Zhang, B. Wang and X. Han, Structural reliability analysis using a copula-function-based evidence theory model, *Computers and Structures*, 143 (2014) 19-31.

[28] Y. Shi, Z. Lu, S. Chen and L. Xu, A reliability analysis method based on analytical expressions of the first four moments of the surrogate model of the performance function, *Mechanical Systems and Signal Processing*, 111 (2018) 47-67.

[29] J. Zheng, Z. Luo, C. Jiang, B. Ni and J. Wu, Non-probabilistic reliability-based topology optimization with multidimensional parallelepiped convex model, *Structural and Multidisciplinary Optimization*, 57 (6) (2018) 2205-2221.

[30] W. Li and Y. Li, An effective optimization procedure based on structural reliability, *Computers and Structures*, 52 (5) (1994) 1061-1067.

[31] M. Moustapha and B. Sudret, Surrogate-assisted reliability-based design optimization: a survey and a unified modular framework, *Structural and Multidisciplinary Optimization*, 60 (5) (2019) 2157-2176.

[32] R. Jin, X. Du and W. Chen, The use of metamodeling techniques for optimization under uncertainty, *Structural and Multidisciplinary Optimization*, 25 (2) (2003) 99-116.

[33] L. Zhang, Z. Lu and P. Wang, Efficient structural reliability analysis method based on advanced kriging model, *Applied Mathematical Modelling*, 39 (2) (2015) 781-793.

[34] L. Hong, H. Li, K. Peng and H. Xiao, A novel kriging based active learning method for structural reliability analysis, *Journal of Mechanical Science and Technology*, 34 (4) (2020) 1545-1556.

[35] DACE, *A Matlab kriging Toolbox*, Technical University of Denmark (2002).

[36] B. Gaspar, A. P. Teixeira and C. G. Soares, Adaptive surrogate model with active refinement combining kriging and a trust region method, *Reliability Engineering and System Safety*, 165 (2017) 277-291.

[37] X. Li, H. Qiu, Z. Chen, L. Gao and X. Shao, A local kriging approximation method using MPP for reliability-based design optimization, *Computers and Structures*, 162 (2016) 102-115.

[38] Z. Chen, H. Qiu, L. Gao, X. Li and P. Li, A local adaptive sampling method for reliability-based design optimization using kriging model, *Structural and Multidisciplinary Optimization*, 49 (3) (2014) 401-416.

[39] Z. Chen et al., An important boundary sampling method for reliability-based design optimization using kriging model, *Structural and Multidisciplinary Optimization*, 52 (1) (2015) 55-70.

[40] B. J. Bichon, M. S. Eldred, L. P. Swiler, S. Mahadevan and J. M. McFarland, Efficient global reliability analysis for nonlinear implicit performance functions, *AIAA Journal*, 46 (10) (2008) 2459-2468.

[41] T. H. Lee and J. J. Jung, A sampling technique enhancing accuracy and efficiency of metamodel-based RBDO: constraint boundary sampling, *Computers and Structures*, 86 (13-14) (2008) 1463-1476.

[42] J. Lim, B. Lee and I. Lee, Sequential optimization and reliability assessment based on dimension reduction method for accurate and efficient reliability-based design optimization, *Journal of Mechanical Science and Technology*, 29 (4) (2015) 1349-1354.

Naigang Hu was born in Shaanxi, China in 1991. He obtained his B.S. and Ph.D. from Xidian University, Xi'an, China in 2011 and 2018, respectively. He is currently a Lecturer at the School of Electromechanical Engineering, Xidian University. His major research interests include antenna electromagnetic analysis, antenna structural design, integrated structural–electromagnetic design in reflector antennas, and design methods under uncertainties.

Wanye Xu was born in Henan, China in 1989. He obtained his B.S. and Ph.D. from Xidian University, Xi'an, China in 2010 and 2015, respectively. He is currently a Lecturer at the School of Electromechanical Engineering, Xidian University. His major research interests include electromechanical integrated design of large ground-based radomes, structural-electrical-thermal integrated analysis and design of airborne radomes, and tolerance analysis and robust design of radomes.