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ORIGINAL PAPER

Study a bearing-only moving ground target tracking problem using single seismic sensor

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Abstract

The problem of tracking moving ground targets using seismic sensors is considered in this paper. Noisy seismic data induced from a moving ground vehicle is detected and collected by a single, fixed, and passive three-component seismic sensor. Two Bayesian suboptimal estimator, namely the Extended Kalman filter (EKF) and the Unscented Kalman filter (UKF), and the optimal Monte Carlo based particle filter (PF) were used in estimating and tracking the true angular behavior of the target. The comparison between these estimators showed that they have almost the same accuracy in estimating the mean value of the noisy target azimuth. In terms of filter consistency, EKF and PF with a number of particles (NP = 5000) are superior to the UKF estimator.

Keywords Ground vehicle tracking · Extended Kalman filter (EKF) · Unscented Kalman filter (UKF) · Particle filter (PF) · Bearing-only tracking (BOT)

Introduction

Estimation is the process of inferring the value of a quantity (scalar or vector) of interest from inaccurate and uncertain noisy measurements (observations). On the other hand, tracking is the estimation of the state of a moving object based on remote measurements (Bar-Shalom et al. 2004). This is done using one or more sensors at fixed locations or on moving platforms. Tracking might be considered as a special case of estimation.

The state-space approach is the most convenient for handling tracking processes. In this approach, the target's position, velocity, and any other information that might be necessary to describe its kinematic characteristics are contained in a target's state vector. The measurement vector contains the noisy measurements that are related to the state vector. At least two models are required in order to study the behavior of a tracking system or any other dynamic system. The system model describes the evolution of target's state vector with time when a new measurement is received, and the measurement model relates the noisy measurements to the target's state. In almost all real applications, these two models are available in probabilistic formulation. The probabilistic state-space formulation and the requirement for estimating target's state on receipt of new measurements are ideally suited for the Bayesian approach (Rice 2014).

When estimating the state of a dynamic process, such as tracking a moving target, the optimal estimate of the posterior probability density function (pdf) of the state is that one which is based on all the available information, including the set of all received measurements.

If the estimate is required to be updated when receiving a new measurement, a recursive filter based on the Bayesian approach is the convenient solution. The recursive nature of this filter means that there is no need to store all received measurements in a memory of growing size. Reprocessing all the old data when receiving new measurement is also not needed.

The mechanism for updating knowledge about the target state in the light of extra information from new data is usually based on using Bayes theorem. In the Bayesian approach; as a new measurement is received, it is used together with prior

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knowledge about the pdf of the physical phenomena and the measuring devices, in order to sequentially produce statistically minimizes- error estimates of the desired posterior pdf of the target's dynamics parameters.

Related work

Bearings-only tracking (BOT) is an important branch of tracking systems. The basic problem with BOT tracking systems is to estimate the state vector of a moving target, which includes the target's kinematics (position, velocity, acceleration) using only a sequence of noise-corrupted angular measurements provided by one or more passive sensors (observers) which measure only the angle of the target with respect to the positions of the sensors. In the last decade, bearing-only tracking has become a common tracking technique in many important applications. Typical examples are submarine tracking using only noisy measurements provided from one or more passive sonar sensors, aircraft surveillance using radars in the passive mode, and the two-dimensional tracking of moving ground vehicles using passive seismic or acoustic sensors.

In the case of using a single sensor (observer), the problem is difficult because an infinite number of targets moving at different ranges with different velocities can generate the same angular behavior. In such a case, the target range is unobservable, and hence the target state is not fully observable. The necessary observability condition using a single observer can be satisfied only with using a movable observer (Reshma et al. 2013; Mušicki 2009).

The conventional Kalman filter (KF) developed by R.E. Kalman in 1960 (Kalman 1960) is an optimal recursive Bayesian estimator in the minimum mean square error (MMSE) sense for a tracking problem in the environment of linear dynamic and measurement models with additive Gaussian noises. These restrictions make KF unsuitable for almost all practical applications, including the nonlinear bearing-only tracking applications. Many suboptimal methods have been developed for such applications. One of these methods is the most commonly used Extended Kalman Filter (EKF) (Welch and Bishop 2006; Aidala 1979) which linearizes nonlinearities around the predicted target position. However, EKF is known to lack robustness and can diverge if the degree of nonlinearity of the system is high or if the filter is poorly initialized. The Unscented Kalman filters (UKF) (Julier and Uhlmann 2004; Julier et al. 2000) sample and propagate the distribution probability density function at sigma points and propagate these samples through the system nonlinearity to an EKF for prediction and updating. The second class is a numerical Monte Carlo method such as particle filters (PF) (Arulampalam et al. 2002, 2004), which samples nonlinear distributions by a set of random hypothesized samples with their associated weights and calculates the posterior estimate at every time step k as the expected value of these samples and their weights. The above methods are presented in detail in the "Review study of different filters" section.

All of the abovementioned approaches use a single filter to estimate the target's state. In (Mušicki 2009), an approach for single-observer and bearing-only passive tracking is presented. Gaussian mixture measurement presentation, together with a track splitting algorithm, allows space-time integration of the target position uncertainty with a simple algorithm. The bearings-only measurements are incorporated into track as they arrive using a dynamic bank of linear Kalman filters. In (Peach 1995), a multiple-hypotheses approach of the target range estimation is considered. A bank of independent and parallel-operating filters is proposed. This bank of filters is known as the Range-parameterized extended Kalman filter (RP-EKF). An initial estimate of the minimum and the maximum distances between the sensor and the target is assumed. This distance range is divided into a number of subintervals. Each of these subintervals represents one of the hypotheses regarding the true range of the target. Each hypothesis is treated by one of the independent filters. The final estimate is a combination of the estimations of all the filters.

All the abovementioned tracking approaches of the target's dynamical model are defined in the Cartesian coordinate frame, while the measurement model is nonlinear and defined in the polar frame. Another class of trackers uses the modified polar coordinate (MPC-EKF) (Aidala and Hammel 1983; Jawahar and Koteswara Rao 2016). The MPC state vector is nonlinear while the measurement model is linear. The advantage of such trackers is that the unobservable and observable components of the target's state vector are decoupled. Such decoupling prevents covariance matrix ill-conditioning, which is the primary cause of filter instability. The use of this coordinate basis improves the stability and robustness of an EKF-based tracking filter.

This paper is organized as follows. The "Review study of different filters" section gives a review study of the basics of the three filters under study: EKF, PF, and UKF. The "Practical work" section describes the work done in the field including the preparation of the equipment used in collecting the seismic data and the procedure of collecting the data. Results and a comparison of the three filters are presented in the "Comparison results" section. The concluding remarks are given in the "Conclusions" section (5).

Review study of different filters

Extended Kalman filter (EKF)

The conventional Kalman filter (KF) is an optimal recursive Bayesian estimator in the minimum mean square error (MMSE) sense for a tracking problem in the environment of linear dynamic and measurement models with additive Gaussian noises. Basically, the linear Kalman filter consists of two stages:

- Prediction stage uses the dynamic model to predict the target's state and the state error covariance matrix forward from one measurement time to the next.
- Updating (correction) stage uses the current measurement to update (correct) the predicted target state and its error covariance matrix.

Several improved versions of the conventional KF have been developed to alleviate the problem of tracking in nonlinear environment. The EKF is probably the most widely used filter as a suboptimal estimator in tracking problems in the environment of slightly nonlinear dynamic or/and measurement Gaussian distributed models. EKF may get unstable and even may diverge if the degree of nonlinearity of the system is high or if the filter is poorly initialized. In BOT tracking applications, the target dynamic model is commonly defined in a 2D linear Cartesian frame, while the measurement model is defined as a nonlinear model. In such applications, the linear model of the target dynamics is described as (Welch and Bishop 2006):

$$\boldsymbol{X}(k) = \boldsymbol{F}.\boldsymbol{X}(k-1) + \boldsymbol{\Gamma}.\boldsymbol{w}(k) \tag{1}$$

where $X(k) \equiv [x(k)\ \ y(k)\ \ \dot{x}(k)\ \ \dot{y}(k)]^T$ is the state vector of the target at time step k, *including the target's position* $(x(k), y(k))$ and its velocity $(\dot{x}(k),\ \dot{y}(k))$, and the time-invariant matrix $\boldsymbol{F}$ which controls the linear transition of the target state from the previous time step $\boldsymbol{X}(k-1)$ to the current step $X(k)$.

$$F = \begin{bmatrix} 1 & 0 & T & 0 \\ 0 & 1 & 0 & T \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \ and\ \mathrm{r} = \begin{bmatrix} \frac{T^2}{2} & 0 \\ 0 & \frac{T^2}{2} \\ T & 0 \\ 0 & T \end{bmatrix} \tag{2}$$

where T is the sampling period, and $\boldsymbol{w}(k)$ is a (2×1) process noise vector (in acceleration units) which accounts for deviation of the actual target's motion from the assumed dynamic model.

The nonlinear measurement model that relates the measurement vector $\boldsymbol{z}\ (k)$ to the state vector is described by (Welch and Bishop 2006):

$$\boldsymbol{z}(k) = tan^{-1}(\mathrm{x}(k)/\mathrm{y}(k)) \ \ + \boldsymbol{n}(k) \tag{3}$$

where $\boldsymbol{n}(k)$ denotes the additive Gaussian measurement noise in radians. It is assumed that both $\boldsymbol{w}(\mathrm{k})$ and $\boldsymbol{n}(k)$ are white, temporally uncorrelated, and zero-mean Gaussian random variables.

In EKF filters, the nonlinearity in the measurement model is linearized about the predicted target state by applying Taylor's series and using the first-order term while all higher terms are ignored. More details about EKF can be found in (Welch and Bishop 2006).

Particle filters (PF)

Particle filter (PF) method is a Monte Carlo (MC) technique for the solution of the state estimation problems. Particle filters are considered to be a generalized approach in tracking applications. These filters are superior to other tracking filters in tracking nonlinear models (e.g., maneuvering targets) with non-Gaussian errors.

PF represents the distribution of the state vector by a set of NP statistically independent samples (particles). At time step k, each particle consists of a hypothesized state vector $x^i(k)$ and an associated weight $w^i(k)$; $i = 1 : NP$.

As the number of the particles becomes very large, the estimate obtained from the MC-based PF approaches the optimal Bayesian estimate.

However, it is impossible to draw particles directly from the unknown posterior probability density function. The key idea of the PF algorithms is to make use of a well-known probability density function which is called the importance (or proposed) density function, as a prior estimate, to generate the initial statistically independent NP particles $x^i\ (k = 1),\ \ i = 1 : NP$. In other words, the statistics of the initial set of particles are not known in most of the tracking applications. One has to use an initial set of particles with a proposed probability density function. This proposed function is known as the importance (or proposed) density function. As each measurement is received, the particle filter updates recursively each particle state and the weight associated with it.

The estimate of the expected value of the state vector $x(k)$ at step k is obtained by (Welch and Bishop 2006):

$$\mathrm{E}[\mathrm{x}(\mathrm{k})] \approx \sum_{\mathrm{i}=1}^{\mathrm{NP}} \mathrm{x}^{\mathrm{i}}(\mathrm{k}).\mathrm{w}^{\mathrm{i}}(\mathrm{k}) \tag{4}$$

Unfortunately, after few recursions, the weights of most of the particles become too weak to make any effective contribution in the estimation process. This results in wasting most of the computation time and effort in updating useless particles. This problem is known as the degeneracy phenomenon. The direct solution to this problem is to increase the number of particles. This solution will put a heavy burden of the computational resources. Another solution is to initialize the filter by using well-selected importance density.

Particle resampling is the most commonly used technique to avoid the degeneracy phenomenon. The resampling process probabilistically replicates particles with large weights and discards particles with small weights. However, using the resampling technique may lead to another problem, which is

Table 1 Comparison study between the different filters

	Linear/nonlinear	Errors nature	Analytical/Monte Carlo	Optimality
EKF	Simple nonlinearity	Only additive Gaussian	Analytical	Suboptimal
UKF	Nonlinear	Only additive Gaussian	MC + EKF	Suboptimal
PF	Nonlinear	Gaussian/non-Gaussian	MC	Optimal with infinite number of particles

known as sample impoverishment. This problem occurs when many particles are repeated and the diversity of the particles is lost. The particle filters have another drawback; it is based on Monte Carlo which is very demanding in the computational resources. This drawback may make unsuitable for complicated real-time applications. However, resampling can be performed either at any time step or only if the weights of a statistically defined number N_{eff} of particles become lower than a certain weight threshold. The number N_{eff} is calculated as (Aidala 1979)

$$\mathrm{N_{eff}} = \frac{\mathrm{NP}}{1+\mathrm{Var}(\mathrm{w^i(k)})} \tag{5}$$

The unscented Kalman filter UKF

The unscented Kalman filter (UKF) is one of the many approaches that have been developed to generalize the extended Kalman filter to nonlinear systems (Julier and Uhlmann 2004; Julier et al. 2000). It is based on the unscented transformation (UT) (Julier 2002). The unscented transformation is a method for calculating the statistics of a random variable which undergoes a nonlinear transformation.

In UKF filter, the normally distributed N-dimensional state vector is represented by (2N + 1) sigma points. Each of these points represents a hypothesized state vector. These points are carefully and deterministically chosen such that when propagated through any true nonlinearity they capture the posterior mean and covariance accurately up to the third order. This gives an advantage of UKF over EKF in capturing the higher order moments caused by the nonlinear transform, as discussed in (Julier and Uhlmann 2004). Also, UKF does not need to calculate the Jacobean matrices, so the estimation procedure is in general easier and less error-prone. Table 1 summarizes the differences between the three filters

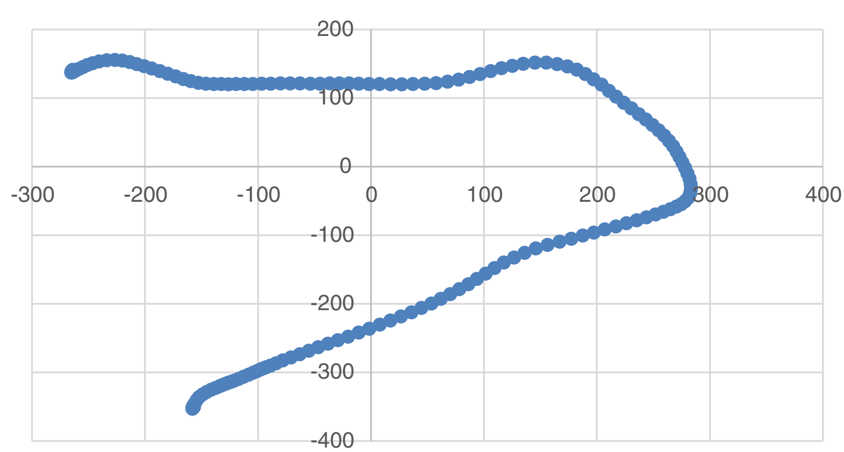
Practical work

Generally, seismic waves can be classified into body waves which travel at a higher speed through the deep interior of the Earth and propagate in three dimensions, and surface waves which travel near the surface of the Earth and propagate in two dimensions. Ground targets (such as cars or trucks) moving over ground generate a succession of impacts generating horizontal soil disturbances. These soil disturbances propagate away from the source as induced horizontal seismic waves. For this reason, in tracking ground targets, the wave of importance for us is the Rayleigh surface wave.

This study used a single three-component seismic sensor. The vertical component detects and provides noisy measurement of the vertical component velocity of the ground particles in the sensor location; the other two components detect and provide noisy measurements of the horizontal components of these velocities.

The relation between the output of each component expressed in volts and its input ground velocity expressed in m/s is defined as the sensitivity of this component. The sensitivity of the used sensor is 750 V·s/m. We are

Fig. 1 Target route



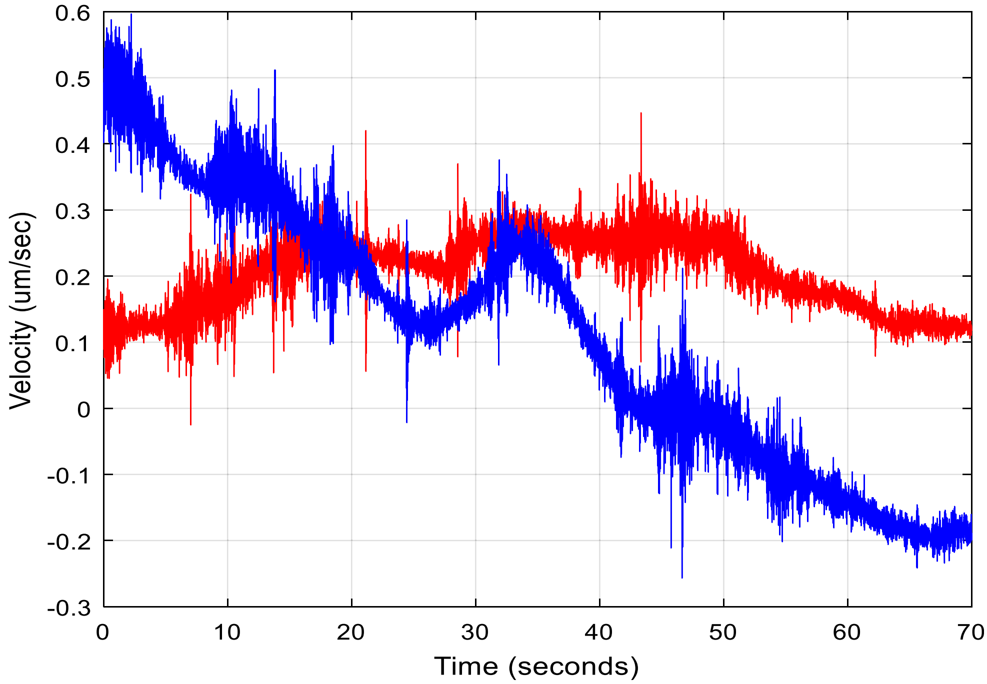


Fig. 2 Measured velocity of ground particles in EW (red) and in NS (blue) directions in μm/s

interested only in the two horizontal East-West and the North-South components.

The analog output signal from the sensor is digitized and stored in a seismic digitizer at a sampling rate of 500 s/s and a resolution of 24 bits. The sensitivity of the digitizer defined as the number of digital bits generated at the digitizer output at a change of one volt at its input (the output of the seismic sensor). The total system sensitivity defined in bits·s/m is the product of the sensor and digitizer sensitivities.

Since the expected seismic wave magnitudes from a ground moving target are much weaker than that expected from Earthquakes or chemical explosions, we have configured the digitizer parameters to provide, with the sensor, a total system sensitivity of 60 bits·s/nm, which is 60 times larger than that used in the national seismic network and large enough to sense the weak magnitudes of the seismic waves generated by the ground target motion. To avoid the effects of wind and the changes in atmospheric temperature and

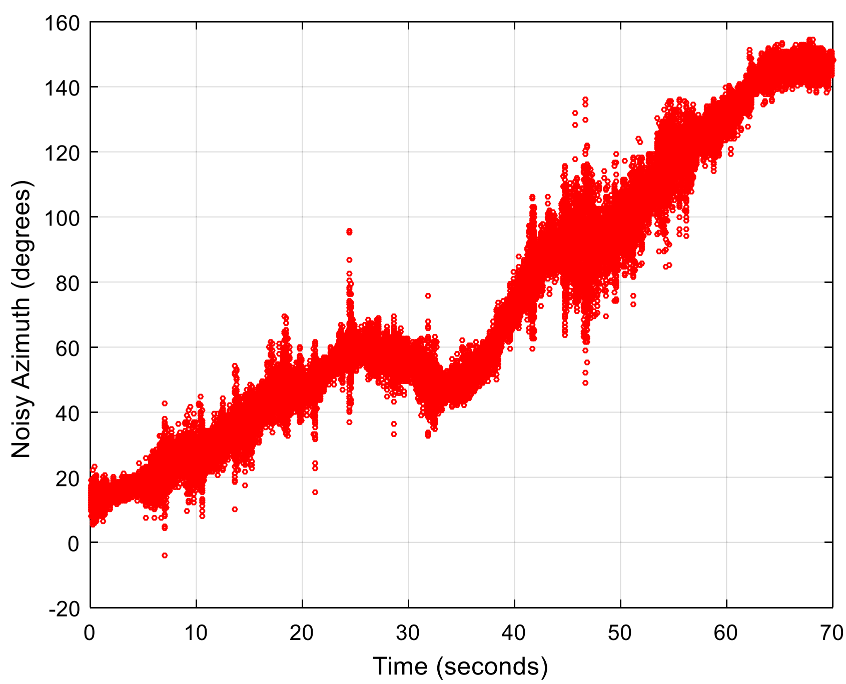


Fig. 3 Noisy azimuth angles

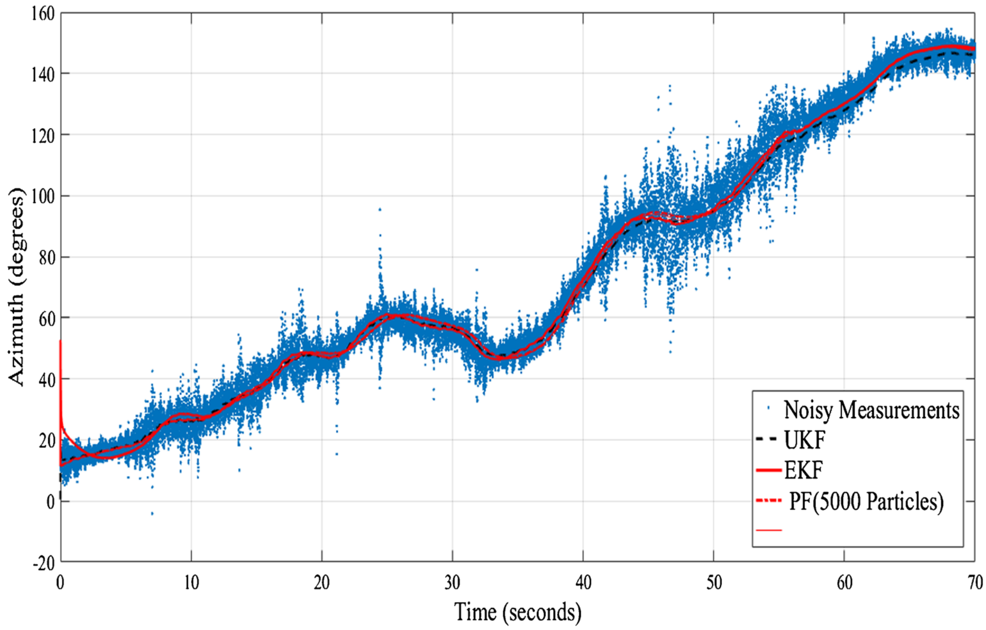


Fig. 4 Mean values of the estimated azimuth angle

pressure, the system was installed at a depth of about 1 m under the ground surface.

Figure 1 shows the target route that we used in this study. It consists of an east-west almost straight 100-m route followed by a first turn at the point (155,152) to the south-east direction and then to a second sharper turn at the point (280,− 45) to the south-west direction. The sensor is located at the point (0, 0). The moving target was a heavy water tank.

Figure 2 shows the noisy measurements of the horizontal velocities of ground particles in μm/s extracted from the sensor. Unfortunately, the data collected from the two horizontal components of the sensor does not provide any information about the target's range or velocity. The only information that could be obtained from these noisy measurements is the changes in the target azimuth while moving along the route. This is the first step in the data processing. For every time sample (k) of the received measurements, the noisy azimuth angle $\theta(k)$ is calculated as (Mušicki 2009):

$$\theta(k) = \tan^{-1}(ew(k)/ns(k)) \ \ , for\ k = 1, 2, ..., M \tag{6}$$

where $ew(k)$ and $ns(k)$ are the measured particle velocities in East-West and North-South directions respectively, while M is

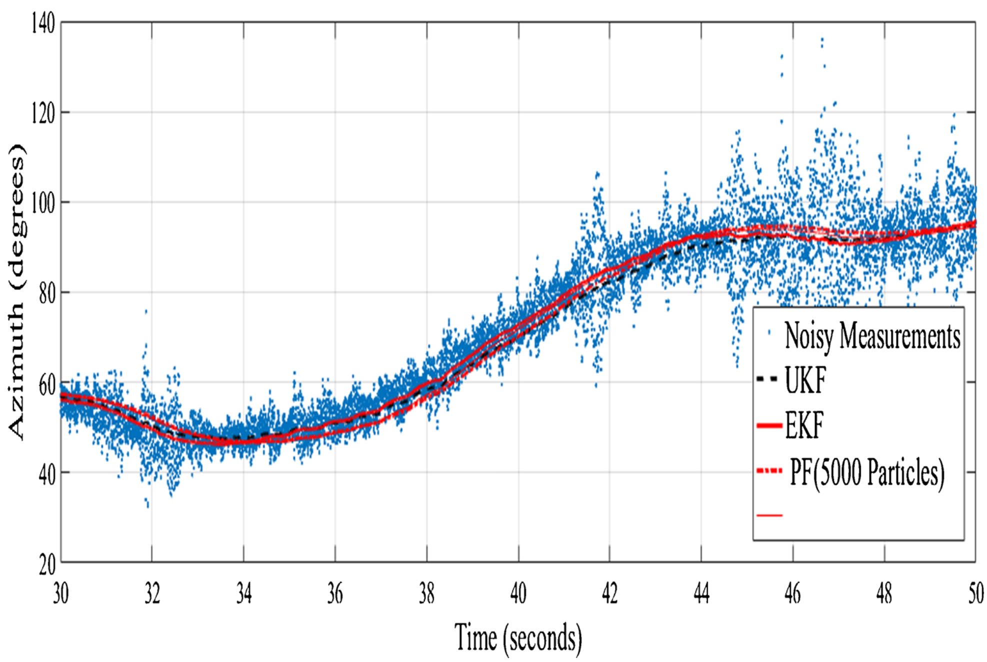


Fig. 5 Zoomed interval of the estimated azimuth angle

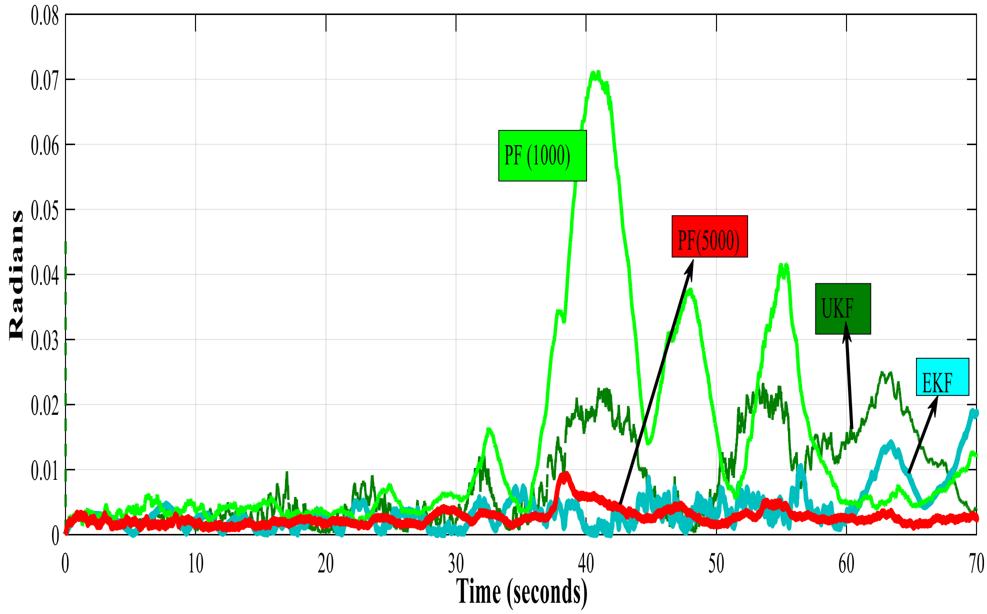


Fig. 6 RMS about the mean values of the estimated azimuth

the number of measured noisy samples. Figure 3 shows the calculated noisy azimuth angles (degrees) of the target.

Comparison results

Filter initialization is a necessary computational step when performing state estimate on dynamical systems. In the initial formulation of any estimating filter it is assumed that the initial value of the state has a known mean value and covariance matrix. If no such data is available, the estimate will have a transient in the initial phase of the estimation process. Poor initialization may lead to filter instability or even a complete filter divergence. In our study, we used a commonly used probabilistic initialization algorithm taken from (Julier et al. 2000). In this initialization algorithm, each of the initial range (r_ini), initial speed (s_ini), and course (c_ini) of the tracked target are assumed to be a random variable with a Gaussian distribution defined as (Julier et al. 2000):

$$\mathrm{r\_ini} = \overline{r} + N(0, \sigma_r), \mathrm{s\_ini} = \overline{s} + N(0, \sigma_s), \mathrm{c\_ini} = \overline{c} + N(0, \sigma_c) \quad (7)$$

where the mean values $\overline{r}$, $\overline{s}$, and $\overline{c}$ are assumed according to a prior knowledge of the field under surveillance, while σ_r, σ_s, and σ_c are the root mean square (RMS) of these distributions. The initial estimate of the covariance matrix is derived from the assumed distributions after the first azimuth measurement is received.

Table 2 Execution time (s)

Filter	Execution time (s)
EKF	1.53
UKF	8.41
PF(NP = 5000)	940

Performance evaluation metric parameters

As a metric parameter, the mean values of the estimates are calculated to compare between the accuracy of the filters, while the RMS values are used to compare between the consistencies of the filters.

Each of the three filters is initialized with the same parameters: r_ini = 200 m, s_ini = 20 km/h, and c_ini = pi/2 radians. A unified root mean square (RMS) of 0.5 is assumed for each of the three random variables. The RMS value of the measurement errors $\boldsymbol{n}(k)$ was assumed to be 0.5 radians, while the RMS of the uncertainty $\boldsymbol{w}(k)$ of the target trajectory was assumed to be 0.5 m/s/s. The particle filter formulation was considered twice, once with a number of particles NP = 1000 particles and again with NP = 5000. Initialization of the particle filter was carried out by sampling NP times from the distribution used to initialize both EKF and UKF filters.

After initializing each of the three filters, the calculated noisy azimuth angles are applied sequentially to an algorithm consists of a number M = 35,000 of iterations. The ith iteration gives an estimate of the target true azimuth at the sampling time $(i.\ \partial t)$, where $\delta t = 0.002$ s is the sampling period. After completing the total M iterations, we get an estimate of the target true azimuth angels for a period of 70 s.

In the comparison between the performances of the three filters, the performance of each filter is evaluated by running the total M iterations a number of n_run = 20 runs. In each run, the filter is initialized by a different realization of the initializing scheme. Thus, we obtain 20 different estimates at every sampling time $(i \cdot \partial t)$.

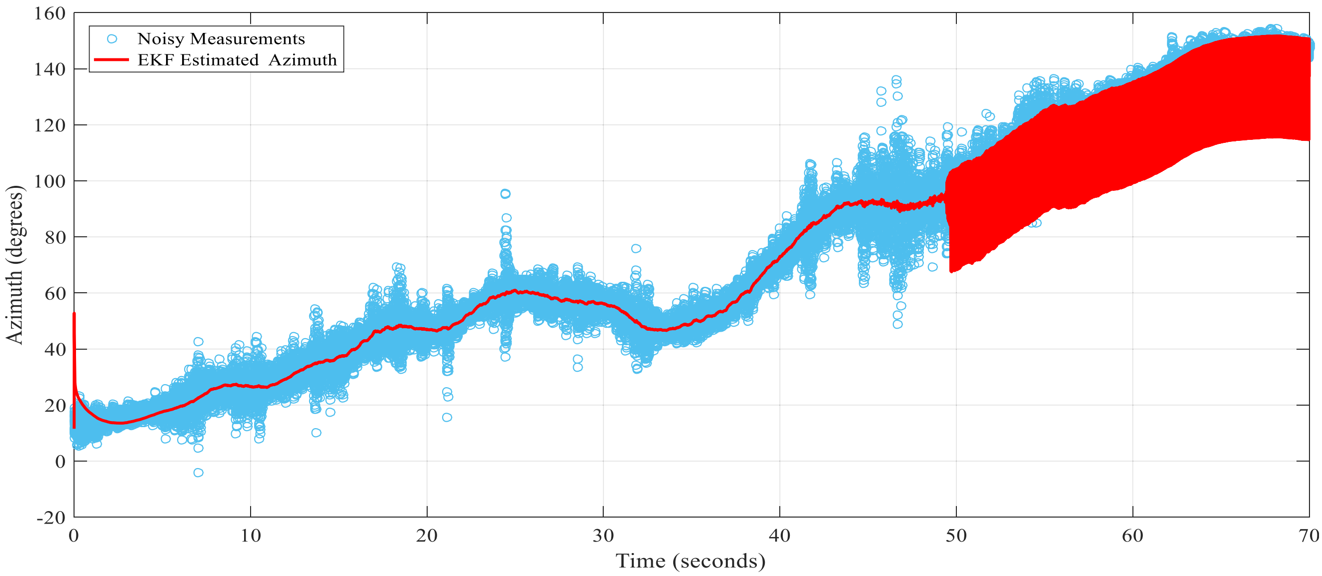


Fig. 7 Divergence of EKF

The mean values $\left(\overline{\theta}\,(i\cdot\partial t)\right)$ and the RMS values $(\theta_{rms}(i\cdot\partial t))$ about the means at each sampling time $(i\cdot\partial t)$ are computed as follows in equations (8) and (9) and taken as measures of the filter accuracy and consistency.

$$\overline{\theta}(i.\partial t) = \frac{1}{n\_run}\sum\nolimits_1^{n\_run}\theta(i.\partial t), \quad ..i = 1,2,\ldots ....M \tag{8}$$

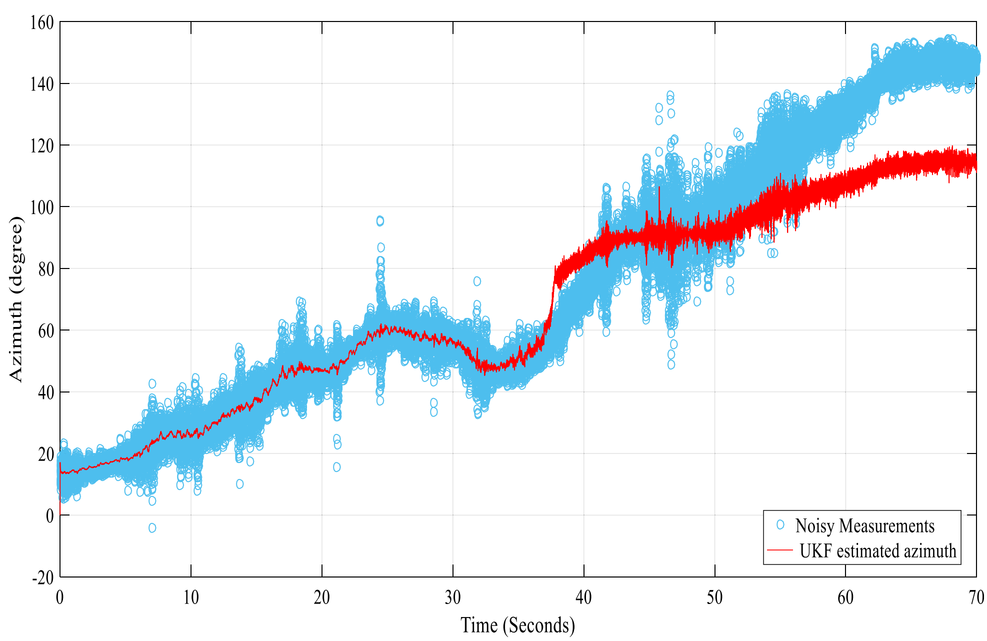
$$\theta_{rms}(i.\partial t) = \sqrt{\frac{1}{n\_run}\sum\nolimits_1^{n\_run}\left(\theta(i.\partial t)-\overline{\theta}(i.\partial t)\right)^2} \quad ..i = 1,2,..M \tag{9}$$

Result analysis and discussion

The estimated mean values of the target azimuth obtained from the three filters are shown on Fig. 4. The time segment between 30 and 50 s is zoomed and illustrated in Fig. 5. These figures show that the estimated mean value of the three filters appears to be almost the same. This means that the three filters have almost the same accuracy in the mean sense.

The RMS values obtained from the three filters are shown on Fig. 6. Both EKF and PF (NP = 5000) provided almost the same RMS along the target track, while both

Fig. 8 Divergence of UKF



UKF and PF (NP = 1000) showed an RMS peak at the second turn after 40 s from the start. This means that EKF and PF (NP = 5000) filters are more consistent than UKF and PF (NP = 1000) filters. The peak in the RMS of PF (NP = 1000) is significantly larger than that of the other filters, which means that this filter formulation is not a good choice. The peak in the RMS value of the particle PF (NP = 1000) is more than three times larger than that of UKF. It needed a larger number of particles to obtain similar performance to the other filters, which reflects passively on its computational load.

In terms of computational load, Table 2 shows the time needed for every one of the three filters to execute one run. The absolute value of the execution time needed for every filter was given directly by Matalab-R2016a.

In order to test the sensitivity of each filter to the initialization parameters, we modified the initial range to be 100 m instead of 200 m while keeping all the other initialization parameters without change. Upon repeating the 20 computational runs again, we found that EKF has lost the target five times, as shown on Fig. 7, with a loss probability of 25%, while UKF has lost the target three times, as shown on Fig. 8, with a loss probability of 15%.

Conclusions

This paper compared between the performances of three estimating filters, namely EKF, UKF, and PF when tracking the azimuth of a real ground target, using a single seismic sensor. The comparison showed that if the filters were properly initialized, the three filters have similar performances in estimating the mean value of the target azimuth. In terms of filter consistency, EKF and PF (NP = 5000) showed the same RMS levels. PF (NP = 1000) showed significantly higher RMS value than the other filters, meaning that it is less consistent than the others. Regarding the computational time, EKF is superior to the other filters. The PF filter (NP = 5000) is most demanding of computational time. This is the cost of its improved performance.

Regarding the sensitivity of the filters to the initialization parameters, EKF showed a larger probability in losing the target than UKF if the filters were poorly initialized.

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