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ORIGINAL ARTICLE

Almost periodic solutions for fuzzy cellular neural networks with multi-proportional delays

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Abstract In this paper, a class of fuzzy cellular neural networks with multi-proportional delays is investigated. By applying contraction mapping fixed point theorem and differential inequality techniques, some sufficient conditions are established for the existence and global attractivity of a unique almost periodic solution for the model, which improve and supplement existing ones. Moreover, a numerical example is given to illustrate the feasibility and application of the obtained results.

Keywords Fuzzy cellular neural networks · Almost periodic solution · Existence · Global attractivity · Multi-proportional delay

Mathematics Subject Classification 34C25 · 34K13 · 34K25

1 Introduction

As is well known, both in biological and man-made neural networks, delays are inevitable, due to various reasons. For instance, time delays can be caused by the finite switching speed of amplifier circuits in neural networks [21]. Therefore, fuzzy cellular neural networks (FCNNs) with delays have been extensively applied in psychophysics, speech, perception, robotics, adaptive pattern recognition, vision, and image processing (see [1, 9, 23, 31]). When FCNNs model is used to describe the biological dynamics with periodically varying environment, the coefficients and delays in the model are usually periodically time-varying, and there have been extensive results on the problem of the existence and stability of periodic solutions of FCNNs with time-varying delays in the literature. We refer the reader to [2, 6, 17, 19, 25, 26] and the references cited therein.

On the other hand, time delays involving in cellular neural networks (CNNs) may be proportional in theory, that is to say, the proportional delay function $\tau(t) = t - qt$ is a monotonically increasing function with the increase of time $t > 0$, where q is a constant and satisfies $0 < q < 1$. In fact, the proportional delay is one of the many objective-existent delay types such as the proportional delay usually is required in web quality of service routing decision, which is because it is convenient to control the networks running time according to the network allowed delays [5, 30, 32–35]. Moreover, the systems with proportional delays have many interesting applications, for example, collection of current by the pantograph of an electric locomotive [18], electrodynamics [7], nonlinear dynamics [3, 20], and probability theory on algebraic structures [4].

Here, it is worth noting that, if we consider the effects of the environmental factors, almost periodicity is sometimes more realistic and more general than periodicity, and thus, people have paid much attention to the study of existence and stability of almost periodic solutions and pseudo almost periodic solutions for CNNs with time-varying delays and distributed delays because of its successful applications in variety of areas such as signal processing, pattern recognition, chemical processes, nuclear reactors, biological systems, static image processing, associative memories,

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optimization problems and so on (see [10–15, 22, 27–29] and the references cited therein). However, to the best of our knowledge, there is no result on problem of almost periodic solutions for FCNNs with proportional delays.

Motivated by the above discussions, the main purpose of this paper is to establish some sufficient conditions on the existence and global attractivity of almost periodic solutions for the following FCNNs with multi-proportional delays:

$$\begin{cases} \dot{x}_i(t) = -c_i(t)x_i(t) + \sum_{j=1}^{n} a_{ij}(t)f_j(x_j(t)) + \sum_{j=1}^{n} b_{ij}(t)u_j(t) + I_i(t) \\ \qquad + \bigwedge_{j=1}^{n} \alpha_{ij}(t)g_j(x_j(q_{ij}t)) + \bigvee_{j=1}^{n} \beta_{ij}(t)g_j(x_j(q_{ij}t)) \\ \qquad + \bigwedge_{j=1}^{n} T_{ij}(t)u_j(t) + \bigvee_{j=1}^{n} H_{ij}(t)u_j(t), \ t \geq t_0 > 0, \\ x_i(s) = \varphi_i(s), \ s \in [\rho_i t_0, \ t_0], \ i \in J = \{1, 2, \ldots, n\}, \end{cases} \tag{1.1}$$

where $\alpha_{ij}(t), \beta_{ij}(t), T_{ij}(t)$ and $H_{ij}(t)$ are the elements of the fuzzy feedback MIN template, fuzzy feedback MAX template, fuzzy feedforward MIN template and fuzzy feedforward MAX template, respectively; $a_{ij}(t)$ and $b_{ij}(t)$ are the elements of feedback template and feedforward template; $\bigwedge, \bigvee$ denote the fuzzy AND and fuzzy OR operation, respectively; $x_i(t), u_i(t)$ and $I_i(t)$ denote the state, input and bias of the ith neuron, respectively; $c_i(t)$ represents the rates with which the i-th neuron will reset its potential to the resting state in isolation when disconnected from the networks and external inputs; $f_i(\cdot)$ and $g_i(\cdot)$ denote the nonlinear activation functions; $q_{ij}, i, j \in J$ are proportional delay factors and satisfy $0 < q_{ij} \leq 1$, and $q_{ij}t = t - (1 - q_{ij})t$, in which $\tau_{ij}(t) = (1 - q_{ij})t$ is the transmission delay function, and $(1 - q_{ij})t \to \infty$ as $q_{ij} \neq 1, t \to \infty$; $\varphi_i(s)$ denotes the initial value of $x_i(s)$ at $s \in [\rho_i t_0, \ t_0]$, $\rho_i = \min_{1 \leq j \leq n} \{q_{ij}\}$, and $\varphi_i \in C([\rho_i t_0, \ t_0], \mathbb{R})$. When coefficients and activation functions in (1.1) are continuous, it can be shown by the method-of-steps given in Hale and Verduyn Lunel [8] that the solution of (1.1) exists and is unique.

The remaining of this paper is organized as follows. In Sect. 2, we give some basic definitions and lemmas, which play an important role in Sect. 3 to establish the existence of almost periodic solutions of (1.1). Here we also study the global attractivity of almost periodic solutions. The paper concludes with an example to illustrate the effectiveness of the obtained results by numerical simulation.

2 Preliminaries

In this section, we shall first recall some basic definitions, lemmas which are used in what follows.

For convenience, we denote by $\mathbb{R}^n(\mathbb{R} = \mathbb{R}^1)$ the set of all $n-$dimensional real vectors (real numbers). For any $\{x_i\} = (x_1, \ x_2, \ldots, x_n) \in \mathbb{R}^n$, we let $|x|$ denote the absolute-value vector given by $|x| = \{|x_i|\}$, and define $\|x\| = \max_{i \in J} |x_i|$. A matrix or vector $A \geq 0$ means that all entries of A are greater than or equal to zero. $A > 0$ can be defined similarly. For matrices or vectors A_1 and A_2, $A_1 \geq A_2$ (resp. $A_1 > A_2$) means that $A_1 - A_2 \geq 0$ (resp. $A_1 - A_2 > 0$). $C(\mathbb{R}, \mathbb{R}^n)$ denotes the set of continuous functions from $\mathbb{R}$ to $\mathbb{R}^n$.

Definition 2.1 (*see* [6]) Let $u(t) \in C(\mathbb{R}, \mathbb{R}^n)$. $u(t)$ is said to be almost periodic on $\mathbb{R}$ if, for any $\varepsilon > 0$, the set $T(u, \varepsilon) = \{\delta : \|u(t + \delta) - u(t)\| < \varepsilon \text{ for all } t \in \mathbb{R}\}$ is relatively dense, *i.e.*, for any $\varepsilon > 0$, it is possible to find a real number $l = l(\varepsilon) > 0$ with the property that, for any interval with length $l(\varepsilon)$, there exists a number $\delta = \delta(\varepsilon)$ in this interval such that $\|u(t + \delta) - u(t)\| < \varepsilon$, for all $t \in \mathbb{R}$.

We denote by $AP(\mathbb{R}, \mathbb{R}^n)$ the set of the almost periodic functions from $\mathbb{R}$ to $\mathbb{R}^n$. Then $(AP(\mathbb{R}, \mathbb{R}^n), \| \cdot \|_\infty)$ is a Banach space, where $\| \cdot \|_\infty$ denotes the supremum norm $\|f\|_\infty := \sup_{t \in \mathbb{R}} \|f(t)\|$ (see [6]). For $h \in C(\mathbb{R}, \mathbb{R})$, let h^+ and h^- be defined as

$$h^+ = \sup_{t \in \mathbb{R}} |h(t)|, \quad h^- = \inf_{t \in \mathbb{R}} |h(t)|.$$

It will be assumed that $c_i, a_{ij}, b_{ij}, \tau_{ij}, \alpha_{ij}, \beta_{ij}, H_{ij}, T_{ij}, I_i, u_i : \mathbb{R} \to \mathbb{R}$ are almost periodic functions, and $i, j \in J$. We also make the following assumptions which will be used later.

(H_0) For each $i \in J$,

$$M[c_i] = \lim_{T \to +\infty} \frac{1}{T} \int_t^{t+T} c_i(s)ds > 0,$$

and there exist a bounded and continuous function $\tilde{c}_i : \mathbb{R} \to (0, \ +\infty)$ and a positive constant K_i such that

$$e^{-\int_s^t c_i(u)du} \leq K_i e^{-\int_s^t \tilde{c}_i(u)du} \text{ for all } t, s \in \mathbb{R} \text{ and } t - s \geq 0.$$

(H_1) For each $j \in J$, there exist nonnegative constants L_j^f and L_j^g such that

$$|f_j(u) - f_j(v)| \leq L_j^f |u - v|, |g_j(u) - g_j(v)| \leq L_j^g |u - v|, \text{ for all } u, \ v \in \mathbb{R}.$$

(H_2) There exist positive constants $\xi_1, \xi_2, \ldots, \xi_n$ and α_i such that

$$\sup_{t \in \mathbb{R}} \left\{ -\tilde{c}_i(t) + K_i \left[\xi_i^{-1} \sum_{j=1}^{n} |a_{ij}(t)| L_j^f \xi_j + \xi_i^{-1} \sum_{j=1}^{n} (|\alpha_{ij}(t)| + |\beta_{ij}(t)|) L_j^g \xi_j \right] \right\} < -\alpha_i, \ i \in J.$$

Lemma 2.1 (see [24]). Let $x_j, \bar{x}_j, \theta_{ij}, \kappa_{ij} \in \mathbb{R}$, $h_j : \mathbb{R} \to \mathbb{R}$ be continuous functions, and $i, j \in J$, then we have

$$\left|\bigwedge_{j=1}^{n} \theta_{ij} h_j(x_j) - \bigwedge_{j=1}^{n} \theta_{ij} h_j(\bar{x}_j)\right| \le \sum_{j=1}^{n} |\theta_{ij}||h_j(x_j) - h_j(\bar{x}_j)|,$$

and

$$\left|\bigvee_{j=1}^{n} \kappa_{ij} h_j(x_j) - \bigvee_{j=1}^{n} \kappa_{ij} h_j(\bar{x}_j)\right| \le \sum_{j=1}^{n} |\kappa_{ij}||h_j(x_j) - h_j(\bar{x}_j)|.$$

Lemma 2.2 Let $x(t) \in AP(\mathbb{R}, \mathbb{R})$, and $q \in \mathbb{R}$ be a constant. Then, $x(qt) \in AP(\mathbb{R}, \mathbb{R})$.

Proof We only consider the case of $q > 0$ since other situations can be dealt with by the analogous approach. For convenience, denote $x(qt)$ by $y(t)$. By the almost periodicity of $x(t)$, one can see that for any $\varepsilon > 0$, there exists $l = l(\varepsilon) > 0$, for any interval with length l, there exists a number τ in this interval such that

$$|x(t + \tau) - x(t)| < \varepsilon, \quad t \in \mathbb{R}, \tag{2.1}$$

For the ε given above, choose $\frac{1}{q} l > 0$, then $[a, a + \frac{1}{q} l] (a \in \mathbb{R})$ is an arbitrary interval with length $\frac{1}{q} l > 0$. Then there exists a $\tau \in [qa, qa + l]$, such that (2.1) holds. Clearly, $\frac{1}{q}\tau \in [a, a + \frac{1}{q} l]$, we deduce from (2.1) that

$$\begin{aligned} |y(t + \frac{1}{q}\tau) - y(t)| &= |x(q(t + \frac{1}{q}\tau)) - x(qt)| \\ &= |x(qt + \tau) - x(qt)| < \varepsilon, \quad t \in \mathbb{R}. \end{aligned}$$

This proves Lemma 2.2. □

Lemma 2.3 For $i, j \in J$, let $x_j, \alpha_{ij}, \beta_{ij} \in AP(\mathbb{R})$, $q_{ij} \in \mathbb{R}$ and (H_1) hold, then

$$\bigwedge_{j=1}^{n} \alpha_{ij}(t) g_j(x_j(q_{ij}t)), \bigvee_{j=1}^{n} \beta_{ij}(t) g_j(x_j(q_{ij}t)) \in AP(\mathbb{R}, \mathbb{R}),\ i \in J.$$

Proof It follows from (H_1) that $g_j (j \in J)$ is uniformly continuous on $\mathbb{R}$. By Lemma 2.2 and [6, pp. 5, Theorem 1.9], we have

$$g_j(x_j(q_{ij}t)) \in AP(\mathbb{R}, \mathbb{R}), \quad i, j \in J.$$

Let $M^g = \max_{i,j \in J}\{\sup_{t \in \mathbb{R}} |g_j(x_j(q_{ij}t))|\}, M^\alpha = \max_{i,j \in J}\{\sup_{t \in \mathbb{R}} |\alpha_{ij}(t)|\}$. For any $\varepsilon > 0$, from [6, pp. 19, Corollary 2.3] and the definition of the uniformly almost periodic family, it is possible to find a real number $l = l(\varepsilon) > 0$ with the property that, for any interval with length l, there exists a number $\delta = \delta(\varepsilon)$ in this interval such that

$$|\alpha_{ij}(t + \delta) - \alpha_{ij}(t)| < \frac{\varepsilon}{2n(M^g + M^\alpha)}, \quad i, j \in J, \tag{2.2}$$

and

$$|g_j(x_j(q_{ij}(t + \delta))) - g_j(x_j(q_{ij}t))| < \frac{\varepsilon}{2n(M^g + M^\alpha)}, \quad i, j \in J. \tag{2.3}$$

With the help of (2.2), (2.3) and Lemma 2.1, we get

$$\begin{aligned} &\left|\bigwedge_{j=1}^{n} \alpha_{ij}(t+\delta) g_j(x_j(q_{ij}(t+\delta))) - \bigwedge_{j=1}^{n} \alpha_{ij}(t) g_j(x_j(q_{ij}t))\right| \\ &\le \left|\bigwedge_{j=1}^{n} \alpha_{ij}(t+\delta) g_j(x_j(q_{ij}(t+\delta))) - \bigwedge_{j=1}^{n} \alpha_{ij}(t) g_j(x_j(q_{ij}(t+\delta)))\right| \\ &\quad + \left|\bigwedge_{j=1}^{n} \alpha_{ij}(t) g_j(x_j(q_{ij}(t+\delta))) - \bigwedge_{j=1}^{n} \alpha_{ij}(t) g_j(x_j(q_{ij}t))\right| \\ &\le \sum_{j=1}^{n} |g_j(x_j(q_{ij}(t+\delta)))||\alpha_{ij}(t+\delta) - \alpha_{ij}(t)| \\ &\quad + \sum_{j=1}^{n} |\alpha_{ij}(t)||g_j(x_j(q_{ij}(t+\delta))) - g_j(x_j(q_{ij}t))| \\ &\le M^g \sum_{j=1}^{n} |\alpha_{ij}(t+\delta) - \alpha_{ij}(t)| + M^\alpha \sum_{j=1}^{n} |g_j(x_j(q_{ij}(t+\delta))) \\ &\quad - g_j(x_j(q_{ij}t))| \\ &< M^g \sum_{j=1}^{n} \frac{\varepsilon}{2n(M^g + M^\alpha)} + M^\alpha \sum_{j=1}^{n} \frac{\varepsilon}{2n(M^g + M^\alpha)} \\ &< \varepsilon, \end{aligned}$$

which implies

$$\bigwedge_{j=1}^{n} \alpha_{ij}(t) g_j(x_j(q_{ij}t)) \in AP(\mathbb{R}, \mathbb{R}),\ i \in J.$$

Similarly, we have

$$\bigvee_{j=1}^{n} \beta_{ij}(t) g_j(x_j(q_{ij}t)) \in AP(\mathbb{R}, \mathbb{R}),\ i \in J.$$

This completes the proof. □

Remark 2.1 Note that

$$\begin{aligned} &\left|\bigwedge_{j=1}^{n} T_{ij}(t+\delta) u_j(t+\delta) - \bigwedge_{j=1}^{n} T_{ij}(t) u_j(t)\right| \\ &\le \sum_{j=1}^{n} |T_{ij}(t+\delta)||u_j(t+\delta) - u_j(t)| + \sum_{j=1}^{n} |u_j(t)||T_{ij}(t+\delta) - T_{ij}(t)| \end{aligned}$$

and

$$\begin{aligned} &\left|\bigvee_{j=1}^{n} H_{ij}(t+\delta) u_j(t+\delta) - \bigvee_{j=1}^{n} H_{ij}(t) u_j(t)\right| \\ &\le \sum_{j=1}^{n} |H_{ij}(t+\delta)||u_j(t+\delta) - u_j(t)| \\ &\quad + \sum_{j=1}^{n} |u_j(t)||H_{ij}(t+\delta) - H_{ij}(t)|, \quad i \in J. \end{aligned}$$

Using a similar way to that in Lemma 2.3, one can show

$$\bigwedge_{j=1}^{n} T_{ij}(t)u_j(t), \ \bigvee_{j=1}^{n} H_{ij}(t)u_j(t) \in AP(\mathbb{R}, \mathbb{R}), \ i \in J.$$

3 Main results

In this section, we establish sufficient conditions on the existence and global attractivity of almost periodic solutions of (1.1).

Theorem 3.1 Let (H_0), (H_1) and (H_2) hold. Then, there exists a unique almost periodic solution of system (1.1).

Proof Set $\bar{x}_i(t) = \xi_i^{-1}x_i(t), \quad i \in J$, then we can transform (1.1) into the following system

$$\begin{aligned}\bar{x}_i'(t) = & -c_i(t)\bar{x}_i(t) \\ & + \xi_i^{-1}\sum_{j=1}^{n} a_{ij}(t)f_j(\xi_j\bar{x}_j(t)) + \xi_i^{-1}\sum_{j=1}^{n} b_{ij}(t)u_j(t) + \xi_i^{-1}I_i(t) \\ & + \xi_i^{-1}\bigwedge_{j=1}^{n} \alpha_{ij}(t)g_j(\xi_j\bar{x}_j(q_{ij}t)) + \xi_i^{-1}\bigvee_{j=1}^{n} \beta_{ij}(t)g_j(\xi_j\bar{x}_j(q_{ij}t)) \\ & + \xi_i^{-1}\bigwedge_{j=1}^{n} T_{ij}(t)u_j(t) + \xi_i^{-1}\bigvee_{j=1}^{n} H_{ij}(t)u_j(t), \quad i \in J.\end{aligned}$$

Let $\varphi \in AP(\mathbb{R}, \mathbb{R}^n)$, it follows from Lemma 2.3 and Remark 2.1 that

$$\begin{aligned}& \xi_i^{-1}\sum_{j=1}^{n} a_{ij}(t)f_j(\xi_j\varphi_j(t)) + \xi_i^{-1}\sum_{j=1}^{n} b_{ij}(t)u_j(t) + \xi_i^{-1}I_i(t) \\ & + \xi_i^{-1}\bigwedge_{j=1}^{n} \alpha_{ij}(t)g_j(\xi_j\varphi_j(q_{ij}t)) + \xi_i^{-1}\bigvee_{j=1}^{n} \beta_{ij}(t)g_j(\xi_j\varphi_j(q_{ij}t)) \\ & + \xi_i^{-1}\bigwedge_{j=1}^{n} T_{ij}(t)u_j(t) + \xi_i^{-1}\bigvee_{j=1}^{n} H_{ij}(t)u_j(t) \in AP(\mathbb{R}, \mathbb{R}), \ i \in J.\end{aligned} \tag{3.1}$$

Then, notice that $M[c_i(t)] > 0, \ i \in J$, in view of (3.1), it follows from Lemma 2.1 in [22] that the nonlinear almost periodic differential equations,

$$\begin{aligned}\bar{x}_i'(t) = & - c_i(t)\bar{x}_i(t) + \xi_i^{-1}\sum_{j=1}^{n} a_{ij}(t)f_j(\xi_j\varphi_j(t)) \\ & + \xi_i^{-1}\sum_{j=1}^{n} b_{ij}(t)u_j(t) + \xi_i^{-1}I_i(t) \\ & + \xi_i^{-1}\bigwedge_{j=1}^{n} \alpha_{ij}(t)g_j(\xi_j\varphi_j(q_{ij}t)) + \xi_i^{-1}\bigvee_{j=1}^{n} \beta_{ij}(t)g_j(\xi_j\varphi_j(q_{ij}t)) \\ & + \xi_i^{-1}\bigwedge_{j=1}^{n} T_{ij}(t)u_j(t) + \xi_i^{-1}\bigvee_{j=1}^{n} H_{ij}(t)u_j(t), \ i \in J,\end{aligned} \tag{3.2}$$

has exactly one almost periodic solution:

$$\begin{aligned}x^{\varphi}(t) = \{x_i^{\varphi}(t)\} = & \Bigg\{\int_{-\infty}^{t} e^{-\int_s^t c_i(u)du}\Bigg[\xi_i^{-1}\sum_{j=1}^{n} a_{ij}(s)f_j(\xi_j\varphi_j(s)) \\ & + \xi_i^{-1}\sum_{j=1}^{n} b_{ij}(s)u_j(s) + \xi_i^{-1}I_i(s) \\ & + \xi_i^{-1}\bigwedge_{j=1}^{n} \alpha_{ij}(s)g_j(\xi_j\varphi_j(q_{ij}s)) + \xi_i^{-1}\bigvee_{j=1}^{n} \beta_{ij}(s)g_j(\xi_j\varphi_j(q_{ij}s)) \\ & + \xi_i^{-1}\bigwedge_{j=1}^{n} T_{ij}(s)u_j(s) + \xi_i^{-1}\bigvee_{j=1}^{n} H_{ij}(s)u_j(s)\Bigg]ds\Bigg\}.\end{aligned} \tag{3.3}$$

Now, we define a mapping $T : AP(\mathbb{R}, \mathbb{R}^n) \to AP(\mathbb{R}, \mathbb{R}^n)$ by setting

$$(T\varphi)(t) = x^{\varphi}(t), \quad \forall \varphi \in AP(\mathbb{R}, \mathbb{R}^n).$$

We next prove that the mapping T is a contraction mapping of $AP(\mathbb{R}, \mathbb{R}^n)$. In fact, in view of (3.3), (H_0), (H_1) and (H_2), for $\varphi, \psi \in AP(\mathbb{R}, \mathbb{R}^n)$, we have

$$\begin{aligned}& |(T\varphi)(t) - (T\psi)(t)| \\ & = \{|((T\varphi)(t) - (T\psi)(t))_i|\} \\ & = \Bigg\{\Bigg|\int_{-\infty}^{t} e^{-\int_s^t c_i(u)du}\Bigg[\xi_i^{-1}\sum_{j=1}^{n} a_{ij}(s)(f_j(\xi_j\varphi_j(s)) - f_j(\xi_j\psi_j(s))) \\ & \quad + \xi_i^{-1}\left(\bigwedge_{j=1}^{n} \alpha_{ij}(s)g_j(\xi_j\varphi_j(q_{ij}s)) - \bigwedge_{j=1}^{n} \alpha_{ij}(s)g_j(\xi_j\psi_j(q_{ij}s))\right) \\ & \quad + \xi_i^{-1}\left(\bigvee_{j=1}^{n} \beta_{ij}(s)g_j(\xi_j\varphi_j(q_{ij}s)) - \bigvee_{j=1}^{n} \beta_{ij}(s)g_j(\xi_j\psi_j(q_{ij}s))\right)\Bigg]ds\Bigg|\Bigg\} \\ & \le \Bigg\{K_i\int_{-\infty}^{t} e^{-\int_s^t \tilde{c}_i(u)du}\Bigg[\xi_i^{-1}\sum_{j=1}^{n} |a_{ij}(s)|L_j^f\xi_j|\varphi_j(s) - \psi_j(s)| \\ & \quad + \xi_i^{-1}\sum_{j=1}^{n}(|\alpha_{ij}(s)| + |\beta_{ij}(s)|)L_j^g\xi_j|\varphi_j(q_{ij}s) - \psi_j(q_{ij}s)|\Bigg]ds\Bigg\} \\ & \le \Bigg\{K_i\int_{-\infty}^{t} e^{-\int_s^t \tilde{c}_i(u)du}\Bigg[\xi_i^{-1}\sum_{j=1}^{n} |a_{ij}(s)|L_j^f\xi_j \\ & \quad + \xi_i^{-1}\sum_{j=1}^{n}(|\alpha_{ij}(s)| + |\beta_{ij}(s)|)L_j^g\xi_j\Bigg]ds\|\varphi(t) - \psi(t)\|_\infty\Bigg\} \\ & \le \left\{\int_{-\infty}^{t} e^{-\int_s^t \tilde{c}_i(u)du}[\tilde{c}_i(s) - \alpha_i]ds\|\varphi(t) - \psi(t)\|_\infty\right\} \\ & \le \left\{\int_{-\infty}^{t} e^{-\int_s^t \tilde{c}_i(u)du}(1 - \frac{\alpha_i}{\tilde{c}_i^+})\tilde{c}_i(s)ds\|\varphi(t) - \psi(t)\|_\infty\right\} \\ & \le \left\{(1 - \frac{\alpha_i}{\tilde{c}_i^+})\|\varphi(t) - \psi(t)\|_\infty\right\},\end{aligned}$$

and

$$\|(T\varphi)(t) - (T\psi)(t)\|_\infty \le \max_{i\in J}\{1 - \frac{\alpha_i}{\tilde{c}_i^+}\}\|\varphi(t) - \psi(t)\|_\infty,$$

which implies that the mapping $T : AP(\mathbb{R}, \mathbb{R}^n) \longrightarrow AP(\mathbb{R}, \mathbb{R}^n)$ is a contraction mapping. Therefore, the mapping T possesses a unique fixed point

$$x^{**} = (x_1^{**}(t), x_2^{**}(t), \ldots, x_n^{**}(t)) \in AP(\mathbb{R}, \mathbb{R}^n),\ Tx^{**} = x^{**},$$

and x^{**} satisfies (3.2). So (1.1) has a unique continuously differentiable almost periodic solution $x^* = (\xi_1 x_1^{**}(t), \xi_2 x_2^{**}(t), \ldots, \xi_n x_n^{**}(t))$. The proof of Theorem 3.1 is now completed. □

In what follows, we investigate the attractivity of the solutions for (1.1).

Theorem 3.2 Under the assumptions of Theorem 3.1, system (1.1) has a unique almost periodic solution $x^*(t)$, and there exist two positive constants Λ and σ, which are independent of solutions of (1.1), such that for arbitrary solution $x(t)$ of (1.1) associated with initial value $\varphi(t) = \{\varphi_i(t)\}$, the following inequality holds

$$\|x(t) - x^*(t)\| \le \Lambda \frac{\max_{i\in J}\{\sup_{s\in[\rho_i t_0,\, t_0]} |\varphi_i(s) - x_i^*(s)|\}}{(1+t)^\sigma} \quad \text{for all} \quad t \ge t_0.$$

Proof Obviously, by Theorem 3.1, (1.1) has a unique almost periodic solution $x^*(t) = \{x_i^*(t)\}$. Suppose that $x(t) = \{x_i(t)\}$ is an arbitrary solution of (1.1) associated with initial value $\varphi(t) = \{\varphi_i(t)\}$. We denote $z_i(t) = x_i(t) - x_i^*(t), t \ge \rho_i t_0, i \in J$, and $\|z\|_\xi = \max_{i\in J}\{\sup_{s\in[\rho_i t_0,\, t_0]} |\varphi_i(s) - x_i^*(s)|\}$. Then

$$\begin{aligned} z_i'(t) = & -c_i(t) z_i(t) + \sum_{j=1}^n a_{ij}(t)[f_j(x_j(t)) - f_j(x_j^*(t))] \\ & + \left[\bigwedge_{j=1}^n \alpha_{ij}(t) g_j(x_j(q_{ij}t)) - \bigwedge_{j=1}^n \alpha_{ij}(t) g_j(x_j^*(q_{ij}t))\right] \\ & + \left[\bigvee_{j=1}^n \beta_{ij}(t) g_j(x_j(q_{ij}t)) - \bigvee_{j=1}^n \beta_{ij}(t) g_j(x_j^*(q_{ij}t))\right], \quad i \in J. \end{aligned} \tag{3.4}$$

Define a continuous function $\Gamma_i(\omega)$ by setting

$$\begin{aligned} \Gamma_i(\omega) = \sup_{t\ge 0}\Big\{ & \omega\xi_i - \tilde{c}_i(t)\xi_i + \sum_{j=1}^n K_i[|a_{ij}(t)|L_j^f + (|\alpha_{ij}(s)| \\ & + |\beta_{ij}(s)|) L_j^g e^{\omega \ln\frac{1}{q_{ij}}}]\xi_j \Big\}, \end{aligned}$$

where $\omega \in [0, \min_{i\in J} \inf_{t\ge 0} \tilde{c}_i(t)],\ i \in J$. Then, from (H_2) , we have

$$\begin{aligned} \Gamma_i(0) = \sup_{t\ge 0}\Big\{ & -\tilde{c}_i(t)\xi_i + \sum_{j=1}^n K_i[|a_{ij}(t)|L_j^f + (|\alpha_{ij}(s)| \\ & + |\beta_{ij}(s)|) L_j^g]\xi_j \Big\} < 0, \\ & i \in J, \end{aligned}$$

which, together with the continuity of $\Gamma_i(\omega)$ and the facts that

$$\frac{\sigma\xi_i}{1+t} \le \sigma\xi_i,\ \ln\left(\frac{1+t}{1+q_{ij}t}\right) \le \ln\frac{1}{q_{ij}} \quad \text{for all} \quad t \ge 0,$$

implies that we can choose a constant $\sigma \in (0, \min_{i\in J} \inf_{t\ge 0} \tilde{c}_i(t))$ such that $\Gamma_i(\sigma) < 0$, and

$$\begin{aligned} & \sup_{t\ge 0}\Big\{ \frac{\sigma\xi_i}{1+t} - \tilde{c}_i(t)\xi_i + \sum_{j=1}^n K_i[|a_{ij}(t)|L_j^f + (|\alpha_{ij}(s)| \\ & \quad + |\beta_{ij}(s)|) L_j^g e^{\sigma \ln(\frac{1+t}{1+q_{ij}t})}]\xi_j \Big\} \\ & \le \sup_{t\ge 0}\Big\{ \sigma\xi_i - \tilde{c}_i(t)\xi_i + \sum_{j=1}^n K_i[|a_{ij}(t)|L_j^f + (|\alpha_{ij}(s)| \\ & \quad + |\beta_{ij}(s)|) L_j^g e^{\sigma \ln\frac{1}{q_{ij}}}]\xi_j \Big\} \\ & = \Gamma_i(\sigma) < 0. \end{aligned} \tag{3.5}$$

For any $\varepsilon > 0$, consider the functions $V_i(t), i \in J$, defined as follows

$$V_i(t) = M\xi_l^{-1}(\|z\|_\xi + \varepsilon)\xi_i e^{-\sigma \ln\frac{1+t}{1+t_0}},\ M \ge \max\{1, \max_{i\in J} K_i\},\ t \ge 0, \xi_l = \min_{i\in J} \xi_i.$$

Therefore,

$$\begin{aligned} V_\kappa(q_{\kappa j}t) & = M\xi_l^{-1}(\|z\|_\xi + \varepsilon)\xi_\kappa e^{-\sigma \ln\frac{1+q_{\kappa j}t}{1+t_0}} \\ & = M\xi_l^{-1}(\|z\|_\xi + \varepsilon)\xi_\kappa e^{-\sigma \ln\frac{1+t}{1+t_0}} e^{\sigma \ln(\frac{1+t}{1+q_{\kappa j}t})} \\ & \le V_\kappa(t) e^{\sigma \ln\frac{1}{q_{\kappa j}}} \quad \text{for all} \quad t \ge t_0, \kappa, j \in J, \end{aligned} \tag{3.6}$$

and

$$|z_i(t_0)| < \xi_l^{-1}(\|z\|_\xi + \varepsilon)\xi_i \le M\xi_l^{-1}(\|z\|_\xi + \varepsilon)\xi_i = V_i(t_0), \quad i \in J. \tag{3.7}$$

We next claim that

$$|z_i(t)| < V_i(t) \quad \text{for all } t > t_0, \ i \in J. \tag{3.8}$$

Otherwise, there must exist $i \in J$ and $\theta_1 \in (t_0, +\infty)$ such that

$$|z_i(\theta_1)| = V_i(\theta_1) = M\xi_l^{-1}(\|z\|_\xi + \varepsilon)\xi_i e^{-\sigma \ln\frac{1+\theta_1}{1+t_0}}, \tag{3.9}$$

and

$$|z_j(t)| < V_j(t) \quad \text{for all } t \in [t_0, \theta_1], \ j \in J. \tag{3.10}$$

Note that

$$\begin{aligned} z_i'(s) + c_i(s)z_i(s) &= \sum_{j=1}^{n} a_{ij}(s)[f_j(x_j(s)) - f_j(x_j^*(s))] \\ &+ \left[\bigwedge_{j=1}^{n} \alpha_{ij}(s)g_j(x_j(q_{ij}s)) - \bigwedge_{j=1}^{n} \alpha_{ij}(s)g_j(x_j^*(q_{ij}s))\right] \\ &+ \left[\bigvee_{j=1}^{n} \beta_{ij}(s)g_j(x_j(q_{ij}s)) - \bigvee_{j=1}^{n} \beta_{ij}(s)g_j(x_j^*(q_{ij}s))\right], \\ & s \in [t_0, t], t \in [t_0, \theta_1]. \end{aligned} \tag{3.11}$$

Multiplying both sides of (3.11) by $e^{\int_{t_0}^{s} c_i(u)du}$, and integrating it on $[t_0, t]$, we get

$$\begin{aligned} z_i(t) = z_i(t_0)e^{-\int_{t_0}^{t} c_i(u)du} &+ \int_{t_0}^{t} e^{-\int_s^t c_i(u)du}\Bigg\{\sum_{j=1}^{n} a_{ij}(s)[f_j(x_j(s)) - f_j(x_j^*(s))] \\ &+ \left[\bigwedge_{j=1}^{n} \alpha_{ij}(s)g_j(x_j(q_{ij}s)) - \bigwedge_{j=1}^{n} \alpha_{ij}(s)g_j(x_j^*(q_{ij}s))\right] \\ &+ \left[\bigvee_{j=1}^{n} \beta_{ij}(s)g_j(x_j(q_{ij}s)) - \bigvee_{j=1}^{n} \beta_{ij}(s)g_j(x_j^*(q_{ij}s))\right]\Bigg\}ds, \\ & t \in [t_0, \theta_1]. \end{aligned}$$

In view of Lemma 2.1, (3.5), (3.6), (3.7) and (3.10) yield

$$\begin{aligned} |z_i(\theta_1)| = \Bigg| z_i(t_0)e^{-\int_{t_0}^{\theta_1} c_i(u)du} &+ \int_{t_0}^{\theta_1} e^{-\int_s^{\theta_1} c_i(u)du}\Bigg\{\sum_{j=1}^{n} a_{ij}(s)[f_j(x_j(s)) - f_j(x_j^*(s))] \\ &+ \left[\bigwedge_{j=1}^{n} \alpha_{ij}(s)g_j(x_j(q_{ij}s)) - \bigwedge_{j=1}^{n} \alpha_{ij}(s)g_j(x_j^*(q_{ij}s))\right] \\ &+ \left[\bigvee_{j=1}^{n} \beta_{ij}(s)g_j(x_j(q_{ij}s)) - \bigvee_{j=1}^{n} \beta_{ij}(s)g_j(x_j^*(q_{ij}s))\right]\Bigg\}ds\Bigg| \\ < \xi_l^{-1}(\|z\|_\xi + \varepsilon)\xi_i K_i e^{-\int_{t_0}^{\theta_1} \tilde{c}_i(u)du} &+ \int_{t_0}^{\theta_1} e^{-\int_s^{\theta_1} \tilde{c}_i(u)du} K_i \sum_{j=1}^{n}[|a_{ij}(s)|L_j^f|z_j(s)| \\ &+ (|\alpha_{ij}(s)| + |\beta_{ij}(s)|)L_j^g|z_j(q_{ij}s)|]ds \\ \le \xi_l^{-1}(\|z\|_\xi + \varepsilon)\xi_i K_i e^{-\int_{t_0}^{\theta_1} \tilde{c}_i(u)du} &+ \int_{t_0}^{\theta_1} e^{-\int_s^{\theta_1} \tilde{c}_i(u)du} K_i \sum_{j=1}^{n}[|a_{ij}(s)|L_j^f \\ &+ (|\alpha_{ij}(s)| + |\beta_{ij}(s)|)L_j^g e^{\sigma \ln(\frac{1+s}{1+q_{ij}s})}] \\ &\times \xi_l^{-1}M(\|z\|_\xi + \varepsilon)\xi_j e^{-\sigma \ln\frac{1+s}{1+t_0}}ds \\ = M\xi_l^{-1}(\|z\|_\xi + \varepsilon)\xi_i e^{-\sigma \ln\frac{1+\theta_1}{1+t_0}}\Bigg\{\frac{K_i}{M}e^{-\int_{t_0}^{\theta_1}(\tilde{c}_i(u) - \frac{\sigma}{1+u})du} &+ \int_{t_0}^{\theta_1} e^{-\int_s^{\theta_1}(\tilde{c}_i(u) - \frac{\sigma}{1+u})du}\xi_i^{-1}K_i \sum_{j=1}^{n}[|a_{ij}(s)|L_j^f \\ &+ (|\alpha_{ij}(s)| + |\beta_{ij}(s)|)L_j^g e^{\sigma \ln(\frac{1+s}{1+q_{ij}s})}]\xi_j ds\Bigg\} \\ < M\xi_l^{-1}(\|z\|_\xi + \varepsilon)\xi_i e^{-\sigma \ln\frac{1+\theta_1}{1+t_0}}\Bigg\{\frac{K_i}{M}e^{-\int_{t_0}^{\theta_1}(\tilde{c}_i(u) - \frac{\sigma}{1+u})du} &+ \int_{t_0}^{\theta_1} e^{-\int_s^{\theta_1}(\tilde{c}_i(u) - \frac{\sigma}{1+u})du}(\tilde{c}_i(s) - \frac{\sigma}{1+s})ds\Bigg\} \\ = M\xi_l^{-1}(\|z\|_\xi + \varepsilon)\xi_i e^{-\sigma \ln\frac{1+\theta_1}{1+t_0}} &\times \left[1 - (1 - \frac{K_i}{M})e^{-\int_{t_0}^{\theta_1}(\tilde{c}_i(u) - \frac{\sigma}{1+u})du}\right] \\ \le M\xi_l^{-1}(\|z\|_\xi + \varepsilon)\xi_i e^{-\sigma \ln\frac{1+\theta_1}{1+t_0}}, & \end{aligned}$$

which contradicts (3.9). Hence, (3.8) holds. Letting $\varepsilon \longrightarrow 0^+$, we have from (3.8) that

$$\|x(t) - x^*(t)\| = \max_{i\in J} |z_i(t)| \le \Lambda \frac{\max_{i\in J}\{\sup_{s\in[\rho_i t_0,\, t_0]} |\varphi_i(s) - x_i^*(s)|\}}{(1+t)^\sigma} \quad \text{for all } t \ge t_0, \tag{3.12}$$

where $\Lambda = M\xi_l^{-1}\xi^u(1+t_0)^\sigma$ and $\xi^u = \max_{i\in J} \xi_i$. This proves Theorem 3.2.

4 An example

In this section, we present an example to check the validity of the main results obtained in Sect. 3.

Example 4.1 Consider the following FCNNs with multi-proportional delays:

$$\begin{cases} x_1'(t) = & -(1+\frac{3}{2}\sin 30t)x_1(t) + \frac{\sin\sqrt{2}t}{40} f_1(x_1(t)) + \frac{\sin\sqrt{3}t}{40} f_2(x_2(t)) \\ & +(\sin t)e^{-2\sin t} + (\cos 2t)e^{-2\sin t} + \sin 4t \\ & +\frac{1}{320}(\cos t)g_1(x_1(\frac{1}{2}t)) \bigwedge \frac{1}{320}(\cos t)g_2(x_2(\frac{1}{2}t)) \\ & +\frac{1}{320}(\sin t)g_1(x_1(\frac{1}{2}t)) \bigvee \frac{1}{320}(\sin t)g_2(x_2(\frac{1}{2}t)), \\ x_2'(t) = & -(1+\frac{3}{2}\cos 30t)x_2(t) + \frac{\cos\sqrt{2}t}{40} f_1(x_1(t)) + \frac{\cos\sqrt{3}t}{40} f_2(x_2(t)) \\ & +(\sin t)e^{-2\sin t} + (\cos 2t)e^{-2\sin t} + \sin 5t \\ & +\frac{1}{320}(\cos t)g_1(x_1(\frac{1}{2}t)) \bigwedge \frac{1}{320}(\cos t)g_2(x_2(\frac{1}{2}t)) \\ & +\frac{1}{320}(\sin t)g_1(x_1(\frac{1}{2}t)) \bigvee \frac{1}{320}(\sin t)g_2(x_2(\frac{1}{2}t)), \end{cases} \tag{4.1}$$

where $t \ge 1$, $f_1(x) = f_2(x) = \frac{1}{18}\frac{|x+1|-|x-1|}{2}$, $g_1(x) = g_2(x) = \frac{1}{18}x$, $x_i(s) = \varphi_i(s), s \in [\frac{1}{2},\ 1]$, and $\varphi_i \in C([\frac{1}{2},\ 1], \mathbb{R})$, $i,j = 1,2$.

Obviously,

$$c_1(t) = 1 + \frac{3}{2}\sin 30t, \quad c_2(t) = 1 + \frac{3}{2}\cos 30t,$$
$$I_1(t) = \sin 4t, \quad I_2(t) = \sin 5t,$$
$$b_{11}(t) = b_{21}(t) = \sin t, \quad b_{12}(t) = b_{22}(t) = \cos 2t,$$
$$u_1(t) = u_2(t) = e^{-2\sin t},$$

$$H_{ij}(t) = T_{ij}(t) = 0, \quad \tilde{c}_i(t) = 1, \quad e^{-\int_s^t c_i(u)du} \le e^{\frac{1}{10}} e^{-(t-s)},$$
$$i = 1,2\ , t \ge s,$$

$$\xi_i = 1, \quad q_{ij} = \frac{1}{2} \quad L_i^f = L_i^g = \frac{1}{18}, \quad K_i = e^{\frac{1}{10}}, \quad i = 1,2,$$

$$a_{11}(t) = \frac{\sin\sqrt{2}t}{40}, \quad a_{12}(t) = \frac{\sin\sqrt{3}t}{40}, \quad a_{21}(t) = \frac{\cos\sqrt{2}t}{40},$$
$$a_{22}(t) = \frac{\cos\sqrt{3}t}{40},$$

and

$$\alpha_{ij}(t) = \frac{1}{320}(\cos t), \ \ \beta_{ij}(t) = \frac{1}{320}(\sin t), i,j = 1,2,$$

we obtain

$$\sup_{t\ge 0}\{-\tilde{c}_i(t)\xi_i + \sum_{j=1}^n K_i[|a_{ij}(t)|L_j^f + (|\alpha_{ij}(t)| + |\beta_{ij}(t)|)L_j^g]\xi_j\} < -0.6, \ \ i = 1,2, \tag{4.2}$$

which imply that system (4.1) satisfies (H_0), (H_1) and (H_2). Moreover, from (4.2), we can choose $\sigma = 0.01$ such that (3.5) holds. Then, Theorem 3.2 implies that system (1.1) has a unique almost periodic solution $x^*(t)$, and

$$\|x(t) - x^*(t)\|_\infty \le \Lambda \frac{\max_{i\in J}\{\sup_{s\in[\frac{1}{2},\ 1]} |\varphi_i(s) - x_i^*(s)|\}}{(1+t)^{0.01}} \quad \text{for all } t \ge 1,$$

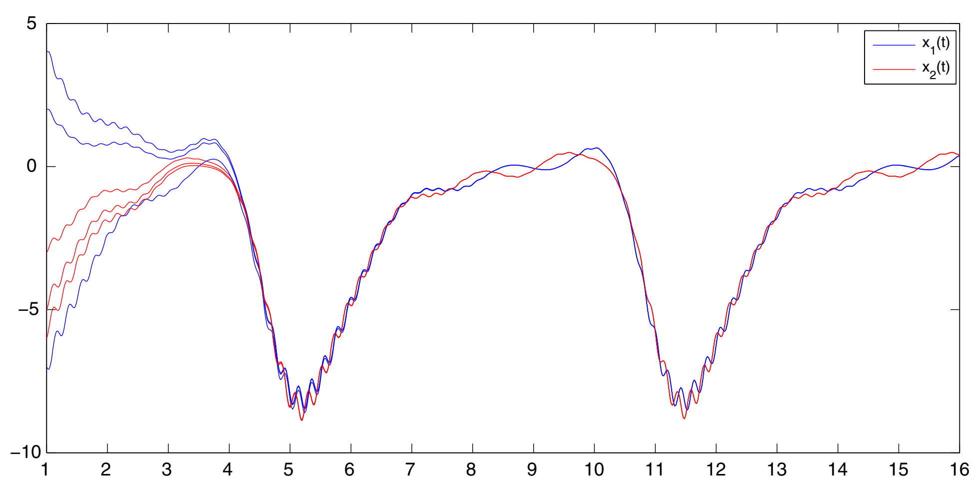

Fig. 1 Numerical solutions $x(t) = (x_1(t), x_2(t))^T$ of system (4.1) for initial values $(2,-3)^T, (-7,-6)^T, (4,-5)^T$, respectively. This implies that the almost periodic solution of system (4.1) is globally attractive

where $\Lambda = 1 + e^{\frac{1}{10}} 2^{0.01}$. The numerical simulations in Fig. 1 strongly support the conclusion.

Remark 4.1 It is worth mentioning that, all scholars in [2, 6, 17, 19, 25, 26] and [10–15, 22, 27–29] have studied the dynamics on CNNs and FCNNs under the fundamental condition that the leakage term coefficient function is not oscillating, i.e., $\inf_{t \in \mathbb{R}} c_i(t) > 0 (i \in J)$. In system (4.1), the time-varying leakage coefficients

$$c_1(t) = 1 + \frac{3}{2} \sin 30t \quad \text{and} \quad c_2(t) = 1 + \frac{3}{2} \cos 30t$$

are oscillating. Thus, all results in the above references cannot be applied to imply that all solutions of (4.1) converge globally to the almost periodic solution. Here, we employ a novel proof to establish some criteria to guarantee the existence and stability of almost periodic solutions for fuzzy cellular neural networks with multi-proportional delays. The method used in this paper provides a possible approach to study the problem on almost periodic solutions of other FCNNs with multi-proportional delays and oscillating leakage term coefficients. These issues are worthy to research in near future works.

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