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Editorial

Symbolic dynamics is a powerful tool in the study of dynamical systems. The purpose of symbolic dynamics is to provide a simplified picture of complicated dynamics, that gives some insight into its complexity. To this end, the state space of the system is partitioned in a finite number of pieces, and the exact trajectories of individual points are traded off by the trajectory relative to that partition. These so-called coarse-grained trajectories turn out to be realisations of a stationary random process with a finite alphabet. In particular, the entropy of a dynamical system can be approximated by the Shannon entropy of any of its symbolic dynamics (the finer the partition, the better the approximation). Today, symbolic dynamics is an independent field of theoretical physics and applied mathematics with applications to such important disciplines as cryptology, time series analysis, and data-compression.

Furthermore, if the state space is endowed with an order relation (like the "smaller-or-equal than" relation between real numbers), then we can assign to each point a so-called ordinal pattern of length L, that is, a permutation that displays the ranking of that point and its $L-1$ successors in its trajectory. The state space then decomposes in sets of points with the same ordinal pattern. A symbolic dynamics based on this type of partition, whose alphabet consists of the $L!$ permutations of length L, is called an ordinal symbolic dynamics. The Shannon entropy of this particular symbolic dynamics is called permutation entropy of order L.

Since Bandt and Pompe introduced permutation entropy in their celebrated paper of 2002, this idea has been intensively discussed. For example, there are interesting results and open questions concerning the relation between the permutation entropy of a measure-preserving transformation and its Kolmogorov-Sinai entropy. On the other hand, the tools of ordinal symbolic dynamics are being used in the analysis of observed time series with remarkable success, e.g., for identifying and discriminating different brain states in epilepsy research and in anesthesiology, for heart rate variability analysis, and for testing independence. The term *permutation complexity* was coined to refer to this group of theoretical ideas and practical tools grown out of the concept of the ordinal pattern.

The scope of this issue of European Physical Journal/Special Topics on Recent Progress in Symbolic Dynamics and Permutation Complexity is three-fold. First, to give the opportunity to researchers working on theoretical and/or practical aspects of symbolic dynamics and permutation complexity to present their latest results to a broad audience in a focused way. Second, to inform firsthand about the new trends and open problems in this area, with special emphasis on the applications. For this reason, the issue begins with a tutorial review where both practitioners and theoreticians will find valuable information and stimuli for their work. And, last but not least, the third scope is to celebrate the ten years of the field!

The tutorial review has been contributed by *M. Riedl*, *A. Müller*, and *N. Wessel* [1] with the scope of providing non-experts in symbolic dynamics with techniques to apply permutation entropy even in a sophisticated manner. After giving a compact definition of permutation entropy, the authors explain the detailed steps when calculating it. In the main part of their paper they discuss how to select the algorithmic parameters appropriately and which artifacts may occur.

This procedure is illustrated with paradigmatic model systems from nonlinear dynamics and stochastic processes.

The rest of the 23 contributions that make up the present issue have been divided in three parts as follows.

- Theoretical aspects I: Concepts, structural results and methods
- Theoretical aspects II: Coupling
- Applications to real world time series

The following short descriptions are partially based on the referee reports.

Part I consists of 7 papers.

I-1) *J.M. Amigó* and *K. Keller* [2] give a short overview on the state of the art concerning theoretical aspects of permutation entropy. The main point of the paper is to review two approaches to permutation entropy, with a focus on its relation to the Kolmogorov-Sinai entropy. Approach I goes back to the original ideas of Bandt and Pompe in 2002 and has also been discussed by Keller and coworkers since 2007. Approach II is due to Amigó and coworkers and was shown by them to be equivalent to the Kolmogorov-Sinai entropy in the ergodic case in 2005 and in a general setting in 2012. The authors formulate open questions that might motivate other scientists to work in this field.

I-2) *J.S. Cánovas*, *A.Guillamón*, and *S. Vera* [3] provide an empirical comparative study of various statistical tests based on permutations with respect to the BDS test, which is a statistic based on correlation integrals. The paper emphasises the convenience of evaluating what test should be used depending on some decisions the researcher can make. According to the main results, the researcher has to take into account the free parameters of the tests and how they affect the empirical size and power of the tests. The paper also hints the relevance of constructing "omnibus" tests, as no one of the considered through out the paper is superior to the rest in all scenarios.

I-3) *M.F. Correia*, *C.C. Ramos*, and *S.M. Vinagre* [4] study the iteration of an operator on an infinite-dimensional space of differentiable interval maps. Starting from an m-modal map, this operator assigns to each differentiable interval map its concatenation with the first one. For topological characterisation of the dynamics of the operator, the authors stress symbolic dynamics. More precisely, they use itineraries and kneading sequences in order to study changes of the number and of the relative location of critical points and values. Finally, the authors present numerical results concerning the number of critical values when starting from a special type of bimodal maps.

I-4) *Y. Hirata* and *K. Aihara* [5] propose a method to obtain a partition that best specifies the locations of points for a time series generated from a stochastic system. An application to real data sets is shown. A particular kind of partition, called "generating", preserves all deterministic dynamical information in the symbolic representation, but such partitions are not obvious beyond one dimension. Existing methods to find them require significant knowledge of the dynamical evolution operator. Indeed for a stochastic complex system a generating partition does not exist. The authors propose an extension of the k-means algorithm for time series where how consecutive points are encoded by the partition is taken into account.

I-5) *M. Matilla-García*, *I. Morales*, and *M. Ruiz* [6] propose a method to test heteroskedasticity using symbolic dynamics. Their idea is to symbolise the modelling errors to find whether there is any structure in the errors or not. For this, the ordinal pattern distribution underlying the time series of estimated squared errors is considered. The advantage of the test obtained in this way is that nearly no assumption on the process behind the time series considered is needed. The new test is discussed with respect to statistical size and power and is compared with other heteroskedasticity tests.

I-6) *P. Pompe* [7] presents a novel method for testing independency between one-dimensional time series on one hand, and for measuring complexity of a single time one-dimensional series on the other hand. For this, a new quantity called LE-statistic based only on the ordinal structure of time series is introduced. It is shown that this quantity is related to the Lyapunov exponent and that it is well suited for the application to short time series. In the paper, the LE-statistic is discussed from a theoretical viewpoint and by means of examples. Moreover, the performance of the LE-statistics in relation both to well-known statistics as Spearman's σ and Kendall's τ and to the permutation entropy is investigated.

I-7) *V.M. Unakafova*, *A.M. Unakafov*, and *K. Keller* [8] study the relation between permutation entropy (as formulated by Bandt and Pompe) and Kolmogorov-Sinai entropy. As stated in a recent paper of them, the equality of both entropies can be traced back to an asymptotic property of the relation between entropies of ordinal patterns and words of ordinal patterns. This paper represents a further step in this difficult and challenging pursuit, consisting of the consideration of mixing maps. This additional hypothesis allows the authors to prove an inequality on the difference of the entropy of ordinal patterns and words of ordinal patterns short of what is needed to conclude the above mentioned equality in the one-dimensional case.

In Part II, 5 papers are presented.

II-1) *W. Bunk*, *J.M. Amigó*, *T. Aschenbrenner*, and *R. Monetti* [9] discuss transcripts, which are permutations obtained from two ordinal patterns by multiplying one of them by the inverse of the other. This internal operation allows compare two ordinal patterns. Transcripts, which were introduced by Monetti et. al. in a former paper, are used to describe coupling between two or more given time series. The authors demonstrate the idea of transcripts, which are represented by permutation matrices in the given paper. Moreover, they illustrate the methods presented by considering data from coupled Roessler systems and electroencephalographic data.

II-2) *T. Haruna*, and *K. Nakajima* [10] present an ordinal-based analysis of ergodic finite-alphabet stationary stochastic processes (SSP). They provide two approaches to the estimation of the excess entropy and transfer entropy of SSPs. The first one is based on the approximation of SSP by hidden Markov models: it is shown that the permutation entropies of the n-th approximation of ergodic SSP converge to the corresponding entropies of the SSP as n tends to infinity. The second approach is related to a different handling with the equality case: the excess entropy and the transfer entropy coincide with their "modified permutation" analogues.

II-3) *D. Kugiumtzis* [11] presents an extensive comparison among several information directionality measures (some of them based on ordinal patterns, some others not) and quantifies their performances to indicate information flow in non–linear multivariate dynamical systems. Such measures are important for studying coupled systems on the base of multivariate time–series obtained from such systems. In particular, the author considers partial versions of the Transfer Entropy, the Symbolic Transfer Entropy, and the Transfer Entropy on Rank Vectors, which were introduced to account for the influence of confounding variables; in other words to account for additional conditions that may influence information flow.

II-4) *R. Monetti*, *J.M. Amigó*, *T. Aschenbrenner*, and *W. Bunk* [12] discuss the coupling complexity index which is an information-theoretic coupling measure based on transcripts (see also II-1) and was introduced in a former paper by the authors. The properties of the index are demonstrated by time series obtained from three delay-coupled autoregressive models. Moreover, the coupling complexity index is applied for differentiating between parts of frontal lobe epilepsy data related to pre-ictal and ictal epoches, respectively. Finally, a coupling complexity rate for symbolic random processes is introduced.

II-5) *K. Nakajima* and *T. Haruna* [13] propose measures called symbolic local transfer entropies in order to investigate information flows in spatiotemporal systems. The measures are tested for coupled map lattice (CML) systems and for the Bak-Sneppen model (BS-model). It is shown that they are able to reveal the characteristic spatiotemporal profile in such systems. For CML systems, the robustness of the measures with respect to noise and their performance compared to conventional methods are discussed. For BS-models, the local information profile is particularly explained by using the measures.

Part III comprises the following 10 contributions.

III-1) *D. Arroyo*, *P. Chamorro*, *J.M. Amigó*, *F.B. Rodríguez*, and *P. Varona* [14] present measures based on ordinal symbolic dynamics and test them for their applicability to the analysis of neurophysiological data. These are permutation entropy, joint permutation entropy, and the relative cardinality of forbidden patterns. In addition to the application to the "raw signal", multi-resolution approximations of the signals are calculated by wavelet techniques and subjected to a permutation entropy analysis, as well. These measures from ordinal symbolic dynamics are compared to established data analysis methods. For this purpose experiments were conducted to obtain time series from the cardiac ganglion and the pyloric central pattern generator network of the stomatogastric nervous system in shore crabs (*Carcinus maenas*). The measured time series are membrane potentials and Ca2+ concentrations. Different typical tasks in neurophysiology serve as illustrations to stress the properties, strengths and weaknesses of the measures.

III-2) *T. Aschenbrenner*, *R. Monetti*, *J.M. Amigó* , and *W. Bunk* [15] examine in their paper the coupling behaviour between different frequency bands of solo violin music using the transcripts between different ordinal symbolic dynamics. The hypothesis is that violins produced by different luthiers are characterised by a unique coupling complexity between the different frequency bands of the respective violins. The coupling complexity is used to quantify the transcripts between different ordinal symbolic dynamics. Support vector machines are used to classify the coupling complexities. Analysing the same piece of music performed by the same violinist using different violins, the authors show that each violin has a unique interaction between its different frequency bands.

III-3) *D. Cysarz*, *A. Porta*, *N. Nicola Montano*, *P. van Leeuwen*, *J. Kurths*, and *N. Wessel* [16] use methods of symbolic dynamics, to analyse the interbeat interval series obtained during graded head-up tilt. The aim of the paper is to examine the usefulness of the methods of symbolic dynamics to reflect the sympathetic and parasympathetic modulations of the autonomic nervous system. In the article, three methods of transformation of the interbeat interval series into symbolic time series are applied. In each method, words of length 3 were considered and grouped according to variations of the symbols. The symbolic patterns analysis reveals that there are strong relations between symbolic indexes and the modulation of the autonomic nervous system. Permutation entropy is another method applied in the paper. It decreases with the increase of tilt angle for all methods of symbolisation, which reflects parasympathetic modulation.

III-4) *G. de Polsi*, *C. Cabeza*, *A.C. Marti*, and *C. Masoller* [17] introduce a novel alphabet for the analysis of coupled oscillators, specifically two identical pendulums coupled by a frictionless sliding platform. The alphabet consists of four letters corresponding to the crossings of the vertical axis, either clock-wise or anticlock-wise, by each pendulum. The authors show that the Shannon entropy of the ensuing symbolic dynamics provides insights into the different motion and synchronisation regimes observed, beyond the usual Lyapunov analysis.

III-5) *J.I. Deza*, *M. Barreiro*, and *C. Masoller* [18] present a method to construct and analyse global climate networks based on the estimation of mutual information of ordinal patterns from gridded surface air temperature time series. They investigate topological changes in the climate networks at time scales ranging from intra-seasonal to inter-annual by introducing relevant lags in the ordinal patterns. Using the area weighted connectivity for interpretations, the authors are able to identify centres of connectivity in extra-tropical regions as well, e.g., the Labrador Sea. They further show that the connections to the El Niño-Southern Oscillation (ENSO) region are highly localised at intra-seasonal time scales, with regions at higher latitudes getting increasingly connected to the ENSO region as the time scale is increased. They contrast their results with the more established method of constructing climate networks using simply the mutual information of the time series. They also investigate the impact of changing versus fixed link density as two distinct approaches to network construction and see the differences in the network obtained by using one or the other. The authors show the impact of changing the bin size in the estimation of mutual information on the results and highlight the critical role of significance testing in constructing networks from time series.

III-6) *G. Graff*, *B. Graff*, *A. Kaczkowska*, *D. Makowiec*, *J.M. Amigó*, *J. Piskorsk*i, *K. Narkiewicz*, and *P. Guzik* [19] examine the ordinal pattern of short sequences of heartbeat durations with the aim of generalising the quantification and assessment of heart rate asymmetry in particular, and heart rate variability in general. Several previously published approaches for quantifying asymmetry are presented, followed by the method proposed by the authors. They applied their approach on 30 min interbeat interval data sets of 94 healthy subjects and found an uneven distribution of the different ordinal patterns for sequences of both 3 and 4 consecutive beats. The distributions differed from those found in shuffled data and from some of the patterns found in surrogate data. The authors conclude that this approach may be useful in the examination of heart rate regulation.

III-7) *H. Lange*, *O.A. Rosso*, and *M. Hauhs* [20] search for signatures of determinism in river daily stream flow data and use as a null hypothesis fully stochastic long-range correlated noise (k-noise). The basic tool is the existence of forbidden (ordinal) patterns in deterministic time series, as opposed to pure noise, which has no forbidden patterns. In practice though, true forbidden patterns can be supplanted by missing patterns due to a pure finite size effect. Although the processes considered by the authors closely resemble runoff series in their correlation behaviour, the ordinal patterns statistics reveals qualitative differences in terms of missing patterns behaviour or temporal asymmetry of the data. An index to quantify temporal asymmetry is proposed in the paper.

III-8) The paper coauthored by *U. Parlitz*, *H. Suetani*, and *S. Luther* [21] is devoted to the interesting question of whether the dynamics of a one-dimensional system can be reconstructed from the ordinal pattern distribution. The authors present their approach for a time series $x_{n+1} = ax_n(1 - x_n)$ generated by the logistic map with $a = 3.8$ and then distorted by a given transformation h. By comparing ordinal pattern distribution of the distorted time series $v_n = h(x_n)$ with experimentally obtained ordinal pattern distributions for the logistic maps with various a, the control parameter a is estimated. After that the transformation h is reconstructed. Furthermore, they compare the ordinal pattern distributions of time series from the Rössler model and the dripping faucet with the ordinal pattern distributions of the logistic maps for various a's.

III-9) *C. Rummel*, *E. Abela*, *M. Hauf*, *R. Wiest*, and *K. Schindler* [22] investigate the applicability of ordinal patterns and derive measures in time series analysis with the aim of quantifying the degree of signal determinism. They make use of three different measures that all quantify special aspects of the empirical frequency distribution of ordinal patterns: signal redundancy, the fraction of forbidden patterns, and, as a new measure, the fraction of under-represented patterns. These measures are applied to simulated data (logistic map in the chaotic regime), as well as to electroencephalographic data obtained in the context of epileptic research. Furthermore, the properties of the new defined measure in terms of a (technical) parameter scan are investigated and its performance compared with the two other measures when applied to a mixture of the deterministic logistic map and a purely stochastic process. The main part of the applications deals with case studies of two epileptic patients.

III-10) *M. Sinn*, *B. Chen*, and *K. Keller* [23] discuss two methods of time series analysis based on ordinal pattern distributions. The first one is change point detection. Here Maximum Mean Discrepancy of ordinal pattern distributions is used to investigate time series for locale changes. The other method applies cluster analysis to ordinal pattern distributions in order to segment and to classify time series. Both methods have already been considered in the authors' previous work, however, they are applied to new real-world data and interesting results are obtained.

Let us conclude these introductory lines by acknowledging the enthusiasm and support of all the contributors to this issue of the European Physical Journal/Special Topics. It is a great pleasure for us to express our gratitude to them all for the hard work and the excellent quality of their papers. We are also very much indebted to Dr. Christian Caron, Executive Publishing Editor of Springer Verlag, and Sabine Lehr, for their kind guidance through the publishing process.

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