

Neutrino oscillations and Lorentz invariance violation in a Finslerian geometrical model

Vito Antonelli^{1,a} , L. Miramonti¹, M. D. C. Torri^{1,2}

¹ Dipartimento di Fisica, Università degli Studi di Milano, Via Celoria 16, 20133 Milan, Italy

² INFN, Via Celoria 16, 20133 Milan, Italy

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Abstract Neutrino oscillations are one of the first evidences of physics beyond the Standard Model (SM). Since Lorentz invariance is a SM fundamental symmetry, recently also neutrino physics has been explored to verify this symmetry eventual modification and its potential magnitude. In this work we study the Lorentz invariance violation (LIV) introduction consequences in the high energy neutrinos propagation and evaluate the impact of this eventual violation on the oscillations predictions. An effective theory explaining these physical effects is introduced via modified dispersion relations. This approach, originally introduced by Coleman and Glashow, corresponds in our model to a special relativity geometry modification. Moreover, this perspective generalization leads to the introduction of a maximum attainable velocity which is specific of the particle. This can be formalized in Finsler geometry, a more general theory of space-time. In the present paper the impact of this kind of LIV on neutrino phenomenology is studied, in particular by analyzing the corrections introduced in neutrino oscillation probabilities for different values of neutrino energies and baselines of experimental interest. The possibility of further improving the present constraints on CPT-even LIV coefficients by means of our analysis is also discussed.

1 Introduction

Till now it is not possible to reach sufficiently high energies to probe the space-time structure at Planck scale, which is considered the separation point of standard gravitational theories from the quantized ones. Nevertheless, space-time quantum effects can possibly manifest as little deviations from the *standard physics* predictions. Hence, even if there are no definitive evidences to sustain departures from Lorentz invariance, it is possible that this symmetry emerges at “low”

energies as an effective symmetry, but is violated in a more energetic scenario, when the quantum effects start to be recognizable. Experimental observations, conducted on the propagation of high energy cosmic messengers, hint at the possibility that their propagation could be influenced by some violations from the standard physical theories (For a discussion about experimental tests of Lorentz invariance violation by means of ultra high energy cosmic rays analysis see, for instance, the following papers (and the references therein) [1], [2].

Coleman and Glashow [3] were the first to introduce the hypothesis of Lorentz invariance violation (LIV), as an attempt to justify such experimental observations, and to explore some consequences even in the neutrino sector. Other works [4,5] dealt with the LIV effects on neutrino physics, but they focused on posing constraints on the perturbations maximum magnitude for ultra-luminal neutrinos or investigated the possibility that the neutrino masses are generated in a modified relativity scenario. In the model we are going to discuss we consider, instead, Lorentz invariance violating effects as tiny deviations, that affect the oscillation sector without modifying the general pattern.

The existence of neutrino oscillations itself violates the original Standard Model predictions and seems, therefore, to require new physical theories introduction, beyond the “minimal version” of the Standard Model (in which neutrinos would be simply left handed massless Dirac fermions). Also for this reason, it is very interesting to explore the phenomenology introduced by LIV on very energetic particles, even in neutrinos oscillation sector. In this work we introduce the Lorentz symmetry violation from modified dispersion relations (MDRs), assumed as consequence of an underlying more general relativity theory, that modifies the kinematics. From this starting point, we show the need to resort to Finsler geometry [6], to construct an effective geometrical theory, which can account for LIV perturbations.

^a e-mail: vito.antonelli@mi.infn.it

Finally, we explore the phenomenological consequences introduced in neutrino oscillation physics by the LIV violating corrections presence and by the consequent modifications of dispersion relations. We focus, in particular, on the analysis of the way in which the oscillation probabilities, which rule the neutrino flavor transitions, get modified for different values of neutrino energies and baselines, with particular attention to the values relevant for long-baseline accelerator and atmospheric neutrinos and for high energy cosmic neutrinos. We also compare our analysis with similar studies developed in literature (even if in different kind of models in most cases) and we discuss the possibility of imposing more severe constraints on the LIV coefficients with a similar analysis applied to future neutrino experiments.

2 Modified dispersion relations introduced LIV and Finsler geometry

One simple way to introduce LIV consists in modifying the kinematics of the theory, that is the dispersion relations (DR). As shown in [3], imposing a maximum speed lower than the speed of light c , for a massive particle, implies a modification of the DR, given by $E^2 = (1 - \epsilon)^2 |\vec{p}|^2 + (1 - \epsilon)^4 m^2$. Reabsorbing the negligible correction term proportional to the mass, the relation can be written as:

$$E^2 - (1 - \epsilon)^2 |\vec{p}|^2 = m^2. \tag{1}$$

In the previous equations $\epsilon \ll 1$ indicates a constant, representing the maximum speed modification parameter.

Following the original idea proposed in [7], this type of MDR can be generalized in a form, which includes energy dependent corrections:

$$f_1^2 E^2 - f_2^2 |\vec{p}|^2 = m^2, \tag{2}$$

where f_i are functions of the quadrimomentum p , that can be written as $f_i = 1 - h_i$ and $h_i \ll 1$ are the velocity modification parameters. From this relation, it is possible to derive an explicit equality for the energy:

$$E = \sqrt{\frac{m^2}{f_1^2} + \frac{f_2^2}{f_1^2} |\vec{p}|^2} \simeq |\vec{p}| f_3, \quad \text{with } f_3 = \frac{f_2}{f_1}. \tag{3}$$

Using the Hamilton–Jacobi equation, one gets for the velocity:

$$\frac{\partial}{\partial p} E = f_3 + |\vec{p}| f_3'. \tag{4}$$

This means that every propagating lepton feels a local space-time foliation, parameterized by its momentum, that is by its energy. From this the necessity follows to resort to a more general geometry, that can account for this energy dependence, the Finsler geometry [6].

Following the work of [8], we introduce the LIV, even in neutrino sector, via the modified dispersion relations (MDRs) with the form:

$$E^2 - \left(1 - f \left(\frac{|\vec{p}|}{E} \right) - g \left(\frac{\vec{p}}{E} \right) \right) |\vec{p}|^2 = m^2. \tag{5}$$

The two perturbation functions

$$\begin{aligned} f \left(\frac{|\vec{p}|}{E} \right) &= \sum_{k=1}^{\infty} \alpha_k \left(\frac{|\vec{p}|}{E} \right)^k \\ g \left(\frac{\vec{p}}{E} \right) &= \sum_{k=1}^{\infty} \beta_k \left(\frac{\vec{p}}{E} \right)^k, \end{aligned} \tag{6}$$

are chosen homogeneous in order to guarantee the geometrical origin of the MDR, as it happens in special relativity, where the dispersion relations are written using the Minkowski metric as $E^2 - |\vec{p}|^2 = m^2 \Rightarrow \eta^{\mu\nu} p_\mu p_\nu = m^2$.

The perturbation f preserves the isotropy of space; this is not true, instead, for the function g , that introduces a preferred direction. Therefore, the form of the MDR can be chosen in such a way to preserve, or not, the idea of a privileged frame of reference. In this work, for simplicity, we assume the space to be isotropic, posing $g = 0$, but our main results are still valid even in the other case.

The adimensional coefficients α and β in (6) must be chosen in such a way to guarantee that the energy, as function of the momentum, assumes positive finite values. Therefore the ratio $\frac{|\vec{p}|}{E} \rightarrow 1 + \delta$ admits a limit for $p \rightarrow \infty$ and, consequently, even the perturbation functions have limits, $f(1 + \delta) = \epsilon$ and $g(1 + \delta) = \epsilon$, if not posed equal to zero. It is possible to reobtain the Coleman and Glashow’s “Very Special Relativity” (VRS) scenario, with the perturbation function f_3 that, for $p \rightarrow \infty$, tends to

$$\lim_{p \rightarrow \infty} f_3 = 1 - f(1 + \delta) = 1 - \epsilon.$$

Hence, from Eq. (4), one recovers for the “personal” *maximum attainable velocity* c' of a massive particle, the constant value different from the light speed:

$$c' = \left. \frac{\partial}{\partial p} E \right|_{max} = f_3 = 1 - \epsilon. \tag{7}$$

The hypotheses made on the perturbation functions permits to write the MDRs (5) as:

$$\tilde{g}(p)^{\mu\nu} p_\mu p_\nu = F^2(p) = m^2, \tag{8}$$

and, using the equation:

$$\tilde{g}(p)^{\mu\nu} = \frac{1}{2} \frac{\partial}{\partial p^\mu} \frac{\partial}{\partial p^\nu} F^2(E, \vec{p}), \tag{9}$$

we obtain the explicit metric form, defined in the momentum space (after eliminating a non-diagonal part, that gives no relevant contribution in computing the dispersion relations):

$$\tilde{g}(p)^{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -(1-f(p)) & 0 & 0 \\ 0 & 0 & -(1-f(p)) & 0 \\ 0 & 0 & 0 & -(1-f(p)) \end{pmatrix}. \tag{10}$$

The homogeneity of the perturbation f implies that the function F defined in (8) is homogeneous of degree 1, condition to be a Finsler norm; hence, even the derived metric is defined in a Finsler space. The properties of this geometry allows to define the *Legendre* transformation of the metric, as a bijection, to obtain the corresponding tensor in coordinate space and it results $g_{\mu\nu}(x) = \tilde{g}_{\mu\nu}(p)$. Therefore, we obtain the generic metric depending both on coordinates and momentum:

$$g(x, p)_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -(1+f(p)) & 0 & 0 \\ 0 & 0 & -(1+f(p)) & 0 \\ 0 & 0 & 0 & -(1+f(p)) \end{pmatrix}. \tag{11}$$

3 More on the geometry of space-time

In order to have a deeper insight in the introduced geometrical structure, we have to deal with the momentum magnitude depending foliation of the space-time. Therefore, it is necessary to introduce the Cartan formalism and resort to the *vierbein* or *tetrad*, whose form is given by:

$$[e]_{\mu}^a = \begin{pmatrix} 1 & \vec{0} \\ \vec{0}^T & \sqrt{1+f(p)}\mathbb{I} \end{pmatrix} \tag{12}$$

$$[e]_a^{\mu} = \begin{pmatrix} 1 & \vec{0} \\ \vec{0}^T & \sqrt{1-f(p)}\mathbb{I} \end{pmatrix},$$

where the dependence on the momentum is evident. Using the *tetrad*, it is possible to construct the explicit form of the modified Lorentz group:

$$\Lambda(x)_{\nu}^{\mu} = [e]_{\nu}^a \Lambda_b^a [e]_b^{\mu}, \tag{13}$$

obtaining a non-linear realization of this group, that preserves the form of the MDR and the homogeneity of degree 0 of the perturbation functions.

This implies that every particle lives in a section of the complete space-time, parameterized by its momentum. The *tetrad* can be used to project vectors from a tangent space identified by the metric $g_{\mu\nu}(x, v)$ to a space with another metric $\bar{g}(y, w)_{\mu\nu}$, as summarized in the scheme below:

$$\begin{array}{ccc} (TM, \eta_{ab}, v) & \xrightarrow{\Lambda} & (TM, \eta_{ab}, w) \\ \downarrow [e] & & \downarrow [\bar{e}] \\ (T_x M, g_{\mu\nu}(x, v)) & \xrightarrow{[\bar{e}] \circ \Lambda \circ [e]^{-1}} & (T_x M, \bar{g}_{\mu\nu}(y, w)). \end{array}$$

We can introduce the modified connections of the constructed geometry, starting from the definition of the *Christoffel* one:

$$\Gamma_{\mu\nu}^{\alpha} = \frac{1}{2} g^{\alpha\beta} (\partial_{\mu} g_{\beta\nu} + \partial_{\nu} g_{\mu\beta} - \partial_{\beta} g_{\mu\nu}). \tag{14}$$

Using the explicit form of the metric (11), it is simple to determine the connection components:

$$\Gamma_{\mu 0}^0 = \Gamma_{00}^i = \Gamma_{\mu\nu}^i = 0 \quad \forall \mu \neq \nu$$

$$\Gamma_{ii}^0 = -\frac{1}{2} \partial_0 f(p) \simeq 0$$

$$\Gamma_{0i}^0 = \Gamma_{i0}^0 = \frac{1}{2(1+f(p))} \partial_0 f(p) \simeq 0$$

$$\Gamma_{ii}^i = \frac{1}{2(1+f(p))} \partial_i f(p) \simeq 0$$

$$\Gamma_{jj}^i = -\frac{1}{2(1+f(p))} \partial_i f(p) \simeq 0 \quad \forall i \neq j$$

$$\Gamma_{ij}^i = \Gamma_{ji}^i = \frac{1}{2(1+f(p))} \partial_i f(p) \simeq 0 \quad \forall i \neq j. \tag{15}$$

In the previous equations the latin indices vary inside the set {1, 2, 3} and the greek ones inside {0, 1, 2, 3}. For the not null terms the approximation is possible, because the interaction of a massive particle with the background is assumed to be tiny and the derivative $|\partial_p f(p)| \ll 1$ is negligible, due to the form of the perturbation functions (6). The local covariant derivative can be introduced as:

$$\nabla_{\mu} v^{\nu} = \partial_{\mu} v^{\nu} + \Gamma_{\mu\alpha}^{\nu} v^{\alpha} \simeq \partial_{\mu} v^{\nu}. \tag{16}$$

In this way, we can compute the last connection that determines the space-time, the *Cartan* or *spinorial* one, defined as:

$$\omega_{\mu ab} = [e]_a^{\nu} \nabla_{\mu} [e]_{\nu a} \simeq [e]_a^{\nu} \partial_{\mu} [e]_{\nu a}. \tag{17}$$

Applying the first Cartan structural equation:

$$de = e \wedge \omega \tag{18}$$

to the external forms

$$e_0^{\mu} = dx^{\mu}$$

$$e_i^{\mu} = \sqrt{1-f(p)} dx^{\mu} \tag{19}$$

it is possible to show that, even for the spinorial connection, the not null elements are given by:

$$\frac{1}{2} \epsilon_{ijk} \omega^{ij} = \frac{1}{2} \frac{1}{1-f} \epsilon_{ijk} (\partial^i f dx^j - \partial^j f dx^i). \tag{20}$$

Since they are proportional to derivatives of the perturbation functions, they are negligible, as in the previous case. Hence, we can introduce the total covariant derivative of a tensor with a local index (greek) and a global one (latin):

$$D_{\mu} v_a^{\nu} = \partial_{\mu} v_a^{\nu} + \Gamma_{\mu\alpha}^{\nu} v_a^{\alpha} - \omega_{\mu\nu}^a v_b^{\nu} \simeq \partial_{\mu} v_b^{\nu}. \tag{21}$$

At this point we can conclude that the introduction of a geometrized interaction, for massive particles with the “quantized” background, identifies an asymptotically flat Finslerian structure.

4 Standard Model extension

The introduced geometry determines a change of the spinorial connection and, consequently, of the Dirac equation. These changes imply the need to modify the form of the Dirac matrices. These matrices have to satisfy the Clifford Algebra relation:

$$\{\Gamma_\mu, \Gamma_\nu\} = 2g^{\mu\nu} = 2[e]_\mu^a \eta_{ab} [e]_\nu^b, \tag{22}$$

that implies the following equality:

$$\Gamma^\mu = [e]^\mu_a \gamma^a. \tag{23}$$

From this, one obtains the explicit form of the modified Dirac matrices:

$$\begin{aligned} \Gamma_0 &= \gamma_0 & \Gamma_i &= \sqrt{1 + f(p(x, \dot{x}))} \gamma_i \\ \Gamma^0 &= \gamma^0 & \Gamma^i &= \sqrt{1 - f(p(x, \dot{x}))} \gamma^i, \end{aligned} \tag{24}$$

and of the Γ_5 matrix:

$$\begin{aligned} \Gamma_5 &= \frac{\epsilon^{\mu\nu\alpha\beta}}{4!} \Gamma_\mu \Gamma_\nu \Gamma_\alpha \Gamma_\beta = \frac{1}{\sqrt{\det g}} \Gamma_0 \Gamma_1 \Gamma_2 \Gamma_3 \\ &= \frac{1}{\sqrt{\det g}} \sqrt{\det g} \gamma_0 \gamma_1 \gamma_2 \gamma_3 = \gamma_5, \end{aligned} \tag{25}$$

where $\epsilon_{\mu\nu\alpha\beta}$ is the total antisymmetric tensor for curved space-time.

Finally, the explicit form of the modified Dirac equation can be written as:

$$(i \Gamma^\mu \partial_\mu - m) \psi = 0. \tag{26}$$

This modified equation admits solutions that can be developed in plane waves, as in the standard case. Resorting to the usual notation for spinors, we write these solutions in the form:

$$\begin{aligned} \psi^+(x) &= u_r(p) e^{-ip_\mu x^\mu} \\ \psi^-(x) &= v_r(p) e^{ip_\mu x^\mu}. \end{aligned} \tag{27}$$

The modified spinors can be easily computed, considering the associated equation in momentum space, applied to the generic positive energy spinor:

$$(i \Gamma^\mu \partial_\mu - m) u_r(p) e^{-ip_\mu x^\mu} \Rightarrow (\not{p} - m) u_r(p) = 0. \tag{28}$$

It is simple to derive the associated identity, for null momentum spinor $\vec{p} = 0$:

$$\begin{aligned} (\not{p} - m)(\not{p} + m) &= (p^\mu p_\mu) - m^2 = 0 \\ \Rightarrow (\not{p} - m)(\not{p} + m) u_r(m, \vec{0}) &= 0. \end{aligned} \tag{29}$$

This equation implies that the spinor of generic momentum \vec{p} can be obtained from the one with null momentum. Using the standard representation of the Dirac matrices, and of the null momentum positive energy spinor

$$u_r(m, \vec{0}) = \chi_r = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \tag{30}$$

one derives the modified positive energy not normalized spinor:

$$\begin{aligned} &(\Gamma^\mu p_\mu + m) \begin{pmatrix} \chi_r \\ 0 \end{pmatrix} \\ &\Rightarrow \left(p^0 \begin{pmatrix} \mathbb{I} & 0 \\ 0 & -\mathbb{I} \end{pmatrix} - p^i \begin{pmatrix} 0 & -\sigma^i \\ \sigma^i & 0 \end{pmatrix} \sqrt{1-f} \right) \begin{pmatrix} \chi_r \\ 0 \end{pmatrix} \\ &+ m \begin{pmatrix} \mathbb{I} & 0 \\ 0 & \mathbb{I} \end{pmatrix} \begin{pmatrix} \chi_r \\ 0 \end{pmatrix} = \begin{pmatrix} (E+m)\chi_r \\ \vec{p} \cdot \vec{\sigma} \sqrt{1-f} \chi_r \end{pmatrix}. \end{aligned} \tag{31}$$

Hence, the normalized spinor can be written, as:

$$u_r(m, \vec{p}) = \frac{1}{\sqrt{2m(E+m)}} \begin{pmatrix} (E+m)\chi_r \\ \vec{p} \cdot \vec{\sigma} \sqrt{1-f} \chi_r \end{pmatrix}. \tag{32}$$

All this derivation can be repeated, with a few changes, to obtain the negative energy spinors explicit form and the result is analogous to that of the positive energy ones.

Starting from the modified gamma matrices (24) and from the modified spinors ψ , defined according to (32), one can introduce a modified current:

$$J_\mu = \bar{\psi} \Gamma_\mu \psi. \tag{33}$$

In this current there is a simplification between the corrections originating from the Lorentz violating coefficients present in the new gamma matrices and the ones coming from the modified spinors. The current is, therefore, defined in the normal tangent space $(T_x M, \eta_{\mu\nu})$. This brings to the following interaction term:

$$J_\mu A^\mu = \bar{\psi} \Gamma_\mu \psi \eta^{\mu\nu} A_\nu, \tag{34}$$

describing the coupling with the electromagnetic field. The interaction takes place in the usual Minkowski tangent space $(T_x M, \eta_{\mu\nu})$.

Also the quark sector can be arranged, writing a modified effective Lagrangian of the form:

$$\frac{i}{2} \sum_j \bar{\psi}_j \Gamma_\mu \overleftrightarrow{D}^\mu \psi_j, \tag{35}$$

where D_μ represents the flat gauge covariant derivative of the Standard Model strong interaction sector. Even in this case, spinorial and Cartan connections are negligible and we can globally conclude that our modified version of the strong interactions lives in an asymptotically flat space-time. Moreover, to preserve the strong interaction $SU(3)$ internal symmetry, the gauge fields are supposed to be Lorentz invariant, as in the case of photons for QED.

It is remarkable that this kind of approach leads to the same results derived in ([9]), where the modified Dirac matrices are defined as $\Gamma_\mu = \gamma_\mu + c_{\mu\nu}\gamma^\nu$ and the authors adopt a particular choice of Lorentz violating CPT-even perturbation terms of the form:

$$\frac{i}{2} c_{\mu\nu} \bar{\psi} \gamma^\mu \overleftrightarrow{D}^\nu \psi. \tag{36}$$

It is possible to extend also the weak interaction sector (as shown for the strong interaction case) and derive an effective theory representing the modified minimal extension of the usual Standard Model Lagrangian. Using the modified Dirac matrices (24) and the modified spinors (32), it is simple to derive the explicit expression of the axial-vectorial current, which characterizes this interaction for a neutrino ν :

$$\bar{\nu} \Gamma_\mu (\mathbb{1} - \Gamma_5) \nu = \bar{\nu} \Gamma_\mu (\mathbb{1} - \gamma_5) \nu. \tag{37}$$

Since $\Gamma_5 = \gamma_5$ and the other modifications generated by LIV in spinors and Dirac matrices simplify, this current is defined in $(T_x M, \eta_{\mu\nu})$, as already shown in Eq. (33) for the QED case. Even in the more complete sector of electro-weak interactions, the correction terms arrange in a similar way to what happened in the previous cases, because only the fermionic fields are corrected, while the gauge fields are supposed to remain Lorentz invariant, to preserve the $SU(2) \times U(1)$ internal symmetry. Once more the corrections caused by the modified spinors simplify with the ones generated by the modified Dirac matrices.

Therefore LIV, as introduced in this work, only modifies the dynamics of massive particles, even neutrinos, without changing the interactions foreseen by the Standard Model. The modified spinors maintain the same chirality as their standard counterparts and only the left handed particles take an active part in the weak interaction.

5 LIV and neutrino oscillations in an Hamiltonian approach

Let's focus the attention on the central topic of this paper, that is the eventual Lorentz violation effects impact on neutrino phenomenology, caused by the modification of the flavor oscillation probabilities. This quantum phenomenon, which confirmed definitely that neutrino is a massive fermion, has been proved in a crystal-clear way both with natural neutrino sources (mainly solar [10–16] and atmospheric [17]) and with artificial neutrinos (short [18–23] and long-baseline [24,25] reactor antineutrinos, long-baseline [26–28] and, if one trusts the LSND [29,30] and MiniBOONE [31,32] results, also short-baseline accelerator neutrino beams). The evidences of oscillation from disappearance experiments have been further reinforced in the last decade by appearance experiments, like the ones using the CNGS beam [33] and like T2K [34]

and NovA [35] (which are collecting an increasing number of appearance signals of neutrinos with a flavor different from the production one).

As explained in the previous sections, the LIV perturbation introduced in our work can account just for tiny perturbative effects, with respect to the standard physics predictions. The presence of perturbative terms, violating Lorentz invariance, determines a modification of the Hamiltonian H that rules the evolution of neutrino wave function during its propagation, according to Schrödinger equation: $i \partial_t |\psi\rangle = H |\psi\rangle$.

As we have seen, in a more general approach to LIV, the extended Standard Model Lagrangian can be written in the general form [9]:

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_{LIV}, \tag{38}$$

with

$$\mathcal{L}_{LIV} = -(a_L)_\mu \bar{\psi}_L \gamma^\mu \psi_L - (c_L)_{\mu\nu} \bar{\psi}_L \gamma^\mu \partial^\nu \psi_L. \tag{39}$$

The first term in Eq. (39), proportional to (a_L) , violates CPT and, as a consequence, also Lorentz invariance, while the second contribution, proportional to (c_L) , breaks “only” Lorentz invariance¹. Consequently, it is possible to build the effective LIV Hamiltonian as the following sum:

$$H_{eff} = H_0 + H_{LIV}, \tag{40}$$

where H_0 denotes the usual Hamiltonian, conserving Lorentz invariance, and H_{LIV} indicates the corrections, introduced by the tiny LIV violating terms of (39). Using a perturbative approach and neglecting the part of H_0 that (for a fixed momentum neutrino beam) contributes identically to all the three mass eigenvalues and, therefore, do not influence the oscillation probability, the remaining part of the extended Hamiltonian can be written as:

$$H = \frac{1}{2E} \left(M^2 + 2(a_L)_\mu p^\mu + 2(c_L)_{\mu\nu} p^\mu p^\nu \right). \tag{41}$$

In the last equation M^2 is a 3×3 matrix, that in the mass eigenvalues basis assumes the form:

$$\begin{pmatrix} m_1^2 & 0 & 0 \\ 0 & m_2^2 & 0 \\ 0 & 0 & m_3^2 \end{pmatrix}. \tag{42}$$

Resorting to the quantum mechanic perturbation theory, the new eigenstates become:

$$|\tilde{\nu}_i\rangle = |\nu_i\rangle + \sum_{i \neq j} \frac{\langle \nu_j | H_{LIV} | \nu_i \rangle}{E_i - E_j} |\nu_j\rangle. \tag{43}$$

¹ It is well known from the work of Greenberg [36] that LIV does not imply CPT violation. The opposite was, instead, declared to be true in the same work [36]. However, the fact that CPT violation automatically brings to LIV was confuted in [37] (where a counterexample was found, by considering a nonlocal model) and the argument has been widely debated in literature [38–41].

The perturbed time evolution operator can be defined, as in the work [42]:

$$S(t) = \left(e^{-i(H_0+H_{LIV})t} e^{iH_0t} \right) e^{-iH_0t} = \left(e^{-i(H_0+H_{LIV})t} e^{iH_0t} \right) S^0(t), \tag{44}$$

and it is possible to evaluate the oscillation probability as:

$$P(v_\alpha \rightarrow v_\beta) = |\langle \beta(t) | \alpha(0) \rangle|^2 = \left| \sum_n \left[\langle \beta(t) | \left(|n_0\rangle \langle n_0| + \sum_{j \neq n} \frac{\langle j_0 | H_{LIV} | n_0 \rangle}{E_n^0 - E_j^0} |j_0\rangle \langle j_0| \right) | \alpha(0) \rangle + \dots \right] \right|^2 = P^0(v_\alpha \rightarrow v_\beta) + P^1(v_\alpha \rightarrow v_\beta) + \dots \tag{45}$$

In Eq. (45) $P^0(v_\alpha \rightarrow v_\beta)$ represents the usual foreseen oscillation probability and the other term is:

$$P^1(v_\alpha \rightarrow v_\beta) = \sum_{ij} \sum_{\rho\sigma} 2L \Re e \left(\left(S_{\alpha\beta}^0 \right)^* U_{\alpha i} U_{\rho i}^* H_{\rho\sigma}^{LIV} U_{\sigma j} U_{\beta j}^* \tau_{ij} \right), \tag{46}$$

with:

$$U_{\alpha i} = \langle \alpha | i \rangle, \tag{47}$$

where $|\alpha\rangle$ denotes a flavor eigenstate and, instead, $|j\rangle$ represents a H_0 one, that is a mass eigenstate. Moreover in (46):

$$\tau_{ij} = \begin{cases} (-i)e^{-iE_i t} & i = j \\ \frac{e^{-iE_i t} - e^{-iE_j t}}{E_i - E_j} & i \neq j, \end{cases} \tag{48}$$

with the constraints on the Hamiltonian matrix:

$$\begin{cases} H_{\alpha\beta}^{LIV} = \left(H_{\beta\alpha}^{LIV} \right)^* & \alpha \neq \beta \\ H_{\alpha\alpha}^{LIV} \in \mathbb{R}. \end{cases} \tag{49}$$

Hence, also the flavor transition probability can be expressed, as expected, in terms of a perturbative expansion. In case of a general treatment of H_{LIV} , assuming a direction depending perturbation, it would be necessary to specify a privileged frame of reference when reporting this kind of results.

6 LIV and neutrino oscillations in our model

An equivalent way to introduce LIV, even in neutrino oscillation sector, consists in using directly the modified dispersion relations. This approach corresponds, in our case, to geometrize the neutrino interactions with the background. In this work we assume that the MDRs of neutrinos are spherically symmetric (as already done in literature in the so called

“fried chicken models” [43]). Until now there are no experimental evidences against this assumption. In this way the Eq. (5), expressing MDRs, reduces to the form:

$$E^2 = |\vec{p}|^2 \left(1 - f \left(\frac{|\vec{p}|}{E} \right) \right) + m^2. \tag{50}$$

Furthermore, using the perturbation function f degree 0 homogeneity, we have shown that the MDR is originated by a metric in the momentum space and this guarantees the validity of Hamiltonian dynamics. The propagation in vacuum of an ultra-relativistic particle, such as a neutrino, is governed by the Schrödinger equation, whose solutions are in the form of generic plane waves:

$$e^{i(p_\mu x^\mu)} = e^{i(Et - \vec{p} \cdot \vec{x})} = e^{i\phi}. \tag{51}$$

The effects of the modified metric do not appear, because the correction terms simplify in the contraction between a covariant and a contravariant vector. The explicit form of the solution can be obtained starting from the MDR (50) and using the approximation of ultrarelativistic particle $|\vec{p}| \simeq E$:

$$|\vec{p}| \simeq \sqrt{|\vec{p}|^2 \left(1 - f \left(\frac{|\vec{p}|}{E} \right) \right) + m^2} \simeq E \left(1 - \frac{1}{2} f \left(\frac{|\vec{p}|}{E} \right) \right) + \frac{m^2}{2E}. \tag{52}$$

Adopting the natural measure units ($t = L$), the plane wave phase ϕ of Eq. (51), for a given mass eigenstate, becomes:

$$\phi = Et - EL + \frac{f}{2}EL - \frac{m^2}{2E}L = \left(fE - \frac{m^2}{E} \right) \frac{L}{2}. \tag{53}$$

Hence, the phase difference of two mass eigenstates for neutrinos with the same energy E can be written as:

$$\begin{aligned} \Delta\phi_{kj} &= \phi_j - \phi_k = \frac{(f_j - f_k)}{2}EL - \left(\frac{m_j^2}{2E} - \frac{m_k^2}{2E} \right) L \\ &= \left(\frac{\Delta m_{kj}^2}{2E} - \frac{\delta f_{kj}}{2} E \right) L. \end{aligned} \tag{54}$$

The oscillation probability depends on the phase differences $\Delta\phi_{kj}$, in addition to the elements of the usual 3×3 unitary matrix PMNS. The transition probability from a flavor $|\alpha\rangle$ to a flavor $|\beta\rangle$ (in the most general case, including even the CP violating phase in the mixing matrix) can be written in the usual form:

$$P(v_\alpha \rightarrow v_\beta) = \delta_{\alpha\beta} - 4 \sum_{i>j} \Re e \left(U_{\alpha i} U_{\beta i}^* U_{\alpha j}^* U_{\beta j} \sin^2(\Delta\phi_{ij}) \right) + 2 \sum_{i>j} \Im m \left(U_{\alpha i} U_{\beta i}^* U_{\alpha j}^* U_{\beta j} \sin^2(\Delta\phi_{ij}) \right). \tag{55}$$

Hence, the oscillation probability is modified, due to the appearance in the phase differences (defined in Eq. (54)) of the LIV violating correction term proportional to $\delta f_{kj} = f_k - f_j$. This term is different from zero only if the coefficients f_i , ruling the LIV violations, are not equal for all the three mass eigenstates; otherwise Eq. (55) reproduces the usual three flavor oscillation probability one gets in absence of Lorentz invariance violation.

Our model, considering CPT-even LIV terms (introduced starting from MDR), deals only with oscillation effects caused by the difference of LIV perturbations to different mass eigenstates ([44]). The fundamental assumption, that represents a reasonable physical hypothesis, is that every mass state presents a personal maximum attainable velocity, because it does not interact in the same way of the others with the background. It is even important to underline that the form of Lorentz invariance violation introduced in our model could not explain the neutrino oscillation without the introduction of masses. In fact the perturbative mass terms introduced by our LIV theory are proportional to the energy of the particle. This would be in contrast with the neutrino oscillations experimental evidences, which indicates a dominant mass term dependence. Neutrino oscillations are well described by models with the phase depending only on squared masses differences divided by the energy

$$\Delta\phi_{jk} = \left(\frac{m_j^2}{2E} - \frac{m_k^2}{2E} \right) = \frac{\Delta m_{jk}^2}{2E} L, \tag{56}$$

and LIV effects, of the type here introduced, could only appear at high energies as tiny perturbations (54). This theory, therefore, can only account for relatively little deviations from what is considered “standard physics” and, in the neutrino oscillation sector, can generate only little effects, at the highest observable energies. Nevertheless, experimentally these effects would be very interesting, because they could be observable as small deviations from the classical theory predictions for the spectrum.

Other LIV theories can even explain oscillations without resorting to the classical concept of mass [45]; in fact they usually insert terms in the Standard Model Lagrangian that introduce masses for neutrinos as caused by the interaction with background fields, as in [46], where the modified Dirac equation can be written using the modified Dirac matrices:

$$\Gamma_{AB}^\mu = \gamma^\mu \delta_{AB} + c_{AB}^{\mu\nu} \gamma_\nu + d_{AB}^{\mu\nu} \gamma_5 \gamma_\nu + e_{AB}^\mu + i f_{AB}^\mu \gamma_5 + \frac{1}{2} g_{AB}^{\mu\nu\tau} \sigma_{\nu\tau}, \tag{57}$$

and the modified mass matrix:

$$M_{AB} = m_{AB} + i m_{5AB} \gamma_5 + a_{AB}^\mu \gamma_\mu + b_{AB}^\mu \gamma_5 \gamma_\mu + \frac{1}{2} H_{AB}^{\mu\nu} \sigma_{\mu\nu}. \tag{58}$$

In the previous equations m and m_5 are Lorentz and CPT conserving masses. The CPT conserving Lorentz violating terms are: c, d, H , while a, b, e, f, g are CPT violating.

In this case, the LIV introduced mass terms would constitute a theoretical justification for the oscillations, but this kind of LIV (differently from the CPT even LIV corrections present in our model and also in [9]) would not spoil the general dependence of the oscillation probabilities on the neutrino energy. Therefore, it would not modify the “standard” oscillation pattern with the introduction of new effects that could be experimentally used to confirm or not the LIV hypothesis validity.

7 Phenomenological analysis of the LIV effects on neutrino oscillations

The experiments on neutrino physics cover a wide range of energies and baselines and, therefore, they offer an ideal playground to search for deviations from Lorentz invariance (see, for instance, [47] and the references therein, [48], [49]).

In order to evaluate in our model the Lorentz invariance violation impact on neutrino phenomenology, we compared the three oscillation probabilities (P_{ν_e, ν_μ} , P_{ν_e, ν_τ} and P_{ν_μ, ν_τ} , given by Eqs. (55) and (54)), ruling the neutrino oscillations in presence of LIV, with the standard oscillation probabilities (obtained preserving the Lorentz symmetry).

Differently from other previous studies, that adopted the two flavor oscillation approximation, our analysis has been pursued in the realistic three flavor scenario. The values of the Δm_{ij}^2 and of the various PMNS matrix elements ($U_{\alpha,i}$) have been taken by the most recent global fits, including all the different neutrino experiments [50, 51]. We assumed, for simplicity, the value $\delta = 0$ for the Dirac CP violation phase, because in our case we are not interested in the study of CP violating effects (which would not spoil our results), but the analysis could be easily modified to introduce also this effect.

The outcome of our study is reported in the following series of figures, where we draw the different oscillation probabilities $P_{\nu_\alpha, \nu_\beta}$, in absence and in presence of Lorentz violating terms, evaluated as a function of the baseline L (that is the distance between the neutrino production and detection points), for fixed values of the neutrino energy. The first two graphs report the probabilities for a muonic neutrino to oscillate, respectively, into an electronic and a tauonic one. The probability P_{ν_μ, ν_τ} is the most relevant one for the study of atmospheric neutrinos and it is important also for long-baseline accelerator neutrino experiments; the knowledge of P_{ν_μ, ν_e} over a wide range of L (from 1 up to 10^4 – 10^5 km) covers the regions of interest both for short- and long-baseline accelerator experiments and also for reactor antineutrino experiments (because $P_{\bar{\nu}_e, \bar{\nu}_\mu} = P_{\nu_\mu, \nu_e}$ under the assumption of CPT invariance). The remaining oscillation

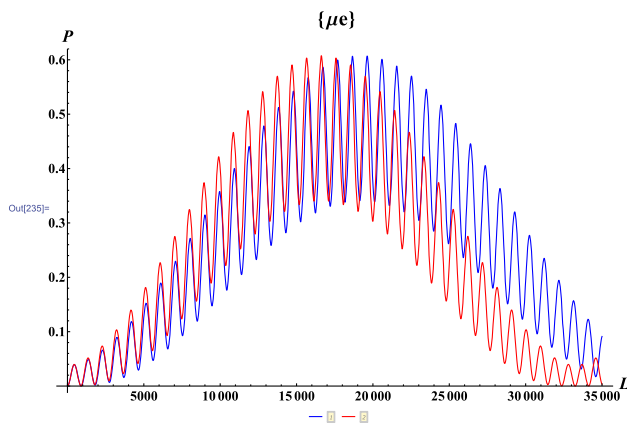


Fig. 1 Comparison of the oscillation probability from ν_μ to ν_e , computed (as a function of the baseline L) for neutrino energy $E = 1$ GeV, in the “standard theory” (red curve) and in presence of LIV (blue curve), for LIV parameters $\delta f_{32} = \delta f_{21} = 1 \times 10^{-23}$

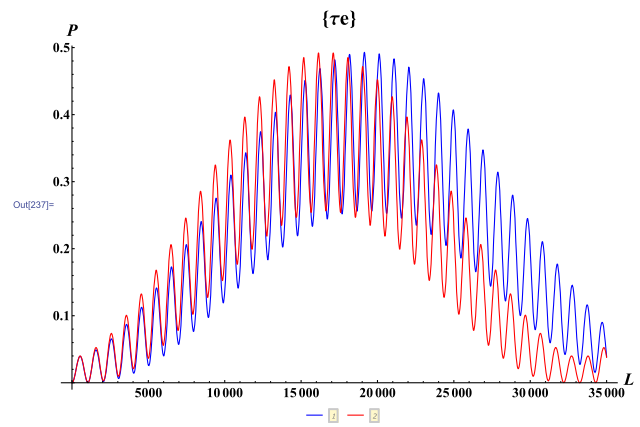


Fig. 3 Same analysis of Fig. 1, but for the oscillation probability from ν_e to ν_τ

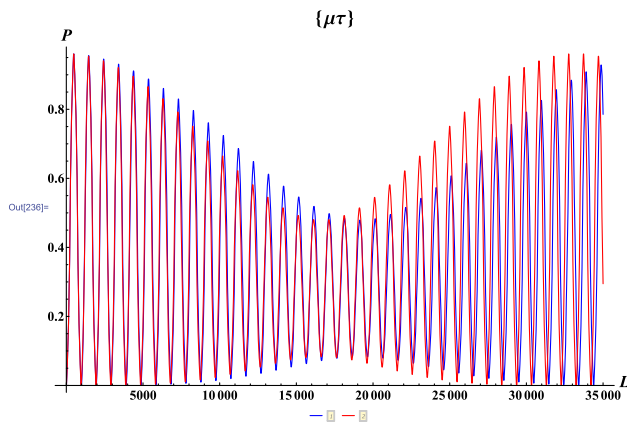


Fig. 2 Same analysis of Fig. 1, but for the oscillation probability from ν_μ to ν_τ

probability P_{ν_e, ν_τ} is shown, instead, in Fig. 3. The value ($E = 1$ GeV) considered for the energy in this series of 3 figures has been chosen having in mind the order of magnitude of the characteristic energies relevant for the oscillation studies, both for atmospheric and for long-baseline accelerator neutrino experiments.

The order of magnitude of the oscillation probability corrections induced by the Lorentz invariance violations is determined by the values chosen for the three parameters f_k and, consequently, for their differences δf_{kj} , as shown in Eq. (54). We assumed for simplicity that the 3 parameters f_k are of same order of magnitude and that they are ordered in a “natural” way, with the highest LIV parameter correction associated to the highest mass eigenvalue (that is: $f_1 < f_2 < f_3$ and $\delta f_{32} \simeq \delta f_{21}$). In Figs. 1, 2 and 3 we adopted the values $\delta f_{32} = \delta f_{21} = 1 \times 10^{-23}$, which are of the same order of magnitude of the limits derived for LIV violation in the phenomenological studies one could find in literature up to 2015 [52–57], or even more conservative than these lim-

its. As one can see clearly from Figs. Figs. 1, 2 and 3, for $\delta f_{ki} = 1 \times 10^{-23}$ the presence of LIV would modify in a visible way the oscillation probability patterns.

However, recently the SuperKamiokande collaboration performed a test of Lorentz invariance by analyzing atmospheric neutrino data. It derived more stringent constraints on the possible values of the coefficients for Lorentz invariant violating corrections to the Hamiltonian [58]. In particular, limits of the order of 10^{-26} to 10^{-27} were derived for the coefficient of the isotropic CPT even term, that introduces corrections to the oscillation probabilities proportional to $L \times E$ and would correspond to the kind of Lorentz invariance violation of our model. As a matter of fact, the comparison between our model and the Hamiltonian assumed as a reference for the SuperK analysis is not so immediate, because in that Hamiltonian are present also other kind of LIV violating corrections and in particular CPT odd terms (introducing corrections to $P_{\nu_\alpha, \nu_\beta}$ not proportional to the neutrino energy) of the order of 10^{-23} .

The Fig. 4 reports the comparison of the $\nu_\mu - \nu_e$ oscillation probabilities with and without LIV for values of our parameters $\delta f_{kj} = 10^{-25}$. In this case the two curves are practically superimposed and the situation is essentially the same also for P_{ν_μ, ν_τ} and for P_{ν_e, ν_τ} . The effects of LIV corrections are not anymore visible and the percentage variations of $P_{\nu_\alpha, \nu_\beta}$ are lower than 1% essentially in all the regions in which P is significantly different from zero.

Therefore, for values of the δf_{kj} coefficients of the same order derived by SuperKamiokande for the CPT even isotropic LIV corrections ($\delta f_{kj} \simeq 10^{-26}$ to 10^{-27}), the LIV effects on the oscillation probabilities are observable only for higher neutrino energies.

In the Figs. 5, 6, 7 and 8 we report the results we obtained for the three oscillation probabilities and for the total ν_μ survival probability ($1 - P_{\nu_\mu, \nu_e} - P_{\nu_\mu, \nu_\tau}$) in the case of a neutrino of 100 GeV, an energy studied, for instance, in

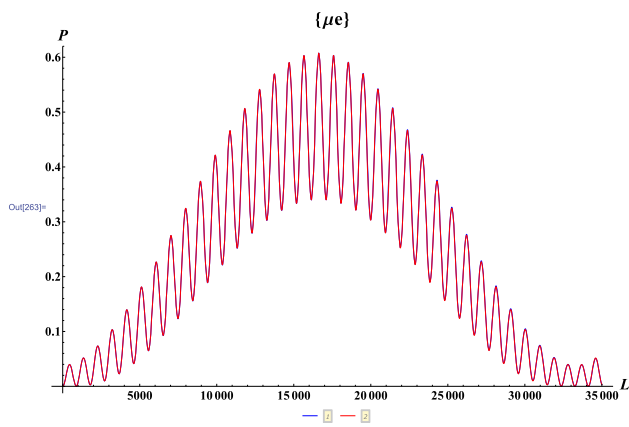


Fig. 4 Same analysis of Fig. 1, but for LIV parameters δf_{kj} of the order of 10^{-25}

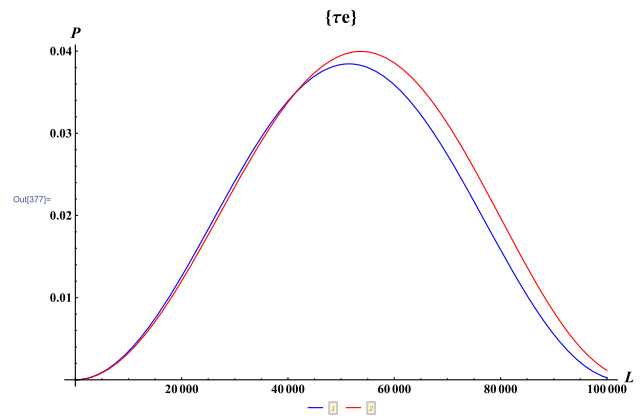


Fig. 7 Same of Fig. 5, but for P_{ν_e, ν_τ}

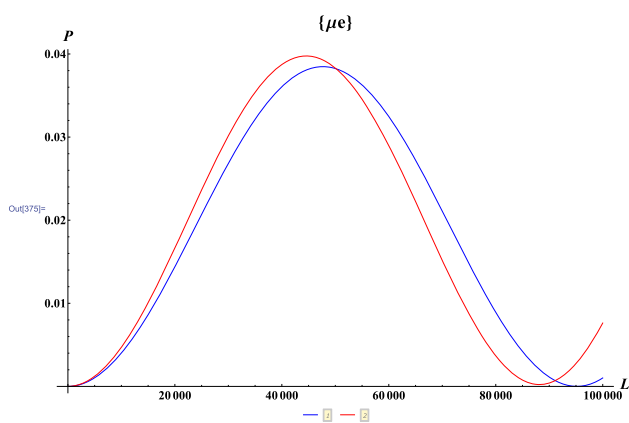


Fig. 5 Same analysis of Fig. 1, but for LIV parameters $\delta f_{32} = \delta f_{21} = 4.5 \times 10^{-27}$ and for neutrino energy $E = 100$ GeV

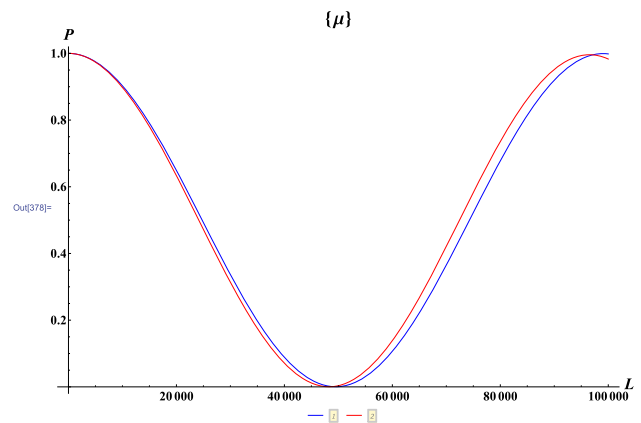


Fig. 8 Total survival probability for muonic neutrino, evaluated for the same conditions of Fig. 5

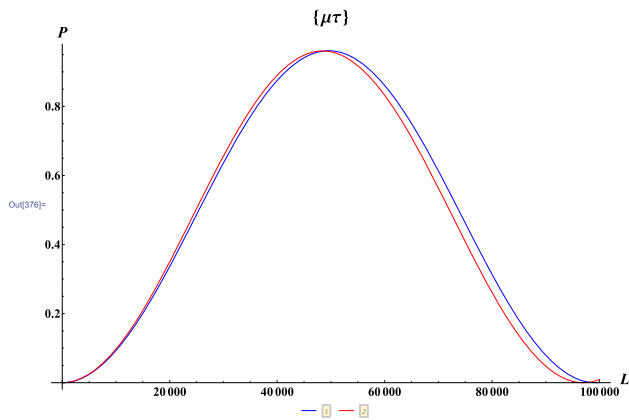


Fig. 6 Same of Fig. 5, but for the oscillation probability P_{ν_μ, ν_τ}

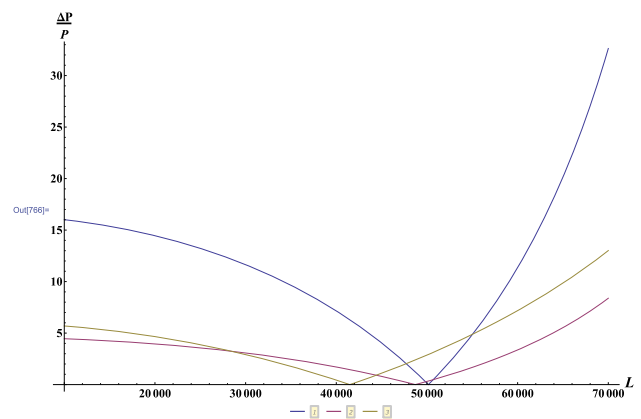


Fig. 9 Percentage variations induced in the neutrino oscillation probabilities by the LIV corrections. On the vertical axis we report, as a function of the baseline L , the percentage differences between the oscillation probabilities, for a 100 GeV neutrino, in presence and in absence of LIV, normalized with respect to their average value. The 3 different curves correspond to the percentage differences for the 3 oscillation probabilities: P_{ν_e, ν_μ} (blue), P_{ν_μ, ν_τ} (violet) and P_{ν_e, ν_τ} (green curve)

atmospheric neutrinos by SuperKamiokande and by the neutrino telescopes. In these graphs we assumed $\delta f_{32} = \delta f_{21} = 4.5 \times 10^{-27}$, that is of the same order of magnitude derived for the corresponding parameter by SuperKamiokande. For these coefficients values, LIV effects are visible and they induce variations of the oscillation and survival probabili-

ties of at least a few percent for most values of L , as one can see directly by Fig. 9. In this figure we represented

simultaneously, for all the 3 probabilities $P_{\nu_\alpha, \nu_\beta}$, the percentage variations due to LIV corrections (computed as $2 \frac{|P_{LIV} - P_{NOLIV}|}{P_{LIV} + P_{NOLIV}} \times 100$), evaluated over a restricted range of values for the baseline, $10,000 \text{ km} < L < 70,000 \text{ km}$, for which the oscillation probabilities are not too low. For most of the values considered for the baseline, the LIV induced percentage variations are higher than 5–10 % for P_{ν_μ, ν_e} and above 2–3 % for the two other oscillation probabilities. In the range considered, the LIV corrections become particularly significant for $L > 60,000 \text{ km}$ (more than 15 % for P_{ν_μ, ν_e})².

The impact of the LIV corrections increases if one considers higher energy neutrinos. One can consider, for instance, neutrino energies in the region from TeV to PeV, interesting for present and future neutrino telescopes like ANTARES [59], KM3NET [60] and (for the higher energies mainly) IceCube [61,62] (as analyzed also in [63]). Also the Ultra High Energy (above EeV) cosmic neutrinos, investigated, for instance, by Auger [64,65], are of great interest and they will play a more and more relevant role in a multimessenger approach, further stimulated by the recent discovery of gravitational waves [66–68].

Starting from the “lower” energies of this part of the spectrum, we analyzed the effect of Lorentz violation for a 1 TeV neutrino, considering 3 different sets of possible values for the δf_{kj} parameters: in the first case we assumed $\delta f_{32} = \delta f_{21} = 4.5 \times 10^{-27}$ (corresponding to the order of magnitude of the present limit derived by SuperKamiokande), while in the other 2 cases we explored values of the δf_{kj} lower, respectively, of one and two orders of magnitude. The results, reported in the series of graphs of Figs. 10, 11 and 12 for the 3 oscillation probabilities (P_{ν_μ, ν_e} , P_{ν_μ, ν_τ} , P_{ν_e, ν_τ}) and in Fig. 13 for the total survival probability of muonic neutrino, are promising. It is evident that the curves corresponding to the LIV expressions obtained for $\delta f_{32} = \delta f_{21} = 4.5 \times 10^{-27}$ (blue lines) are significantly different from the ones obtained in absence of LIV violations (orange curves). Moreover, the corrections due to LIV remain significant also for δf_{kj} parameters one order of magnitude lower (red) and they are in any case appreciable even for $\delta f_{32} = \delta f_{21} = 4.5 \times 10^{-29}$ (green curves), at least for values of the baseline sufficiently high (L above 400,000 km in the first oscillation cycle).

Hence, there is the hope that, by selecting the appropriate experimental context, in future it will be possible to use the detailed study of high energy neutrinos to further constrain the values of the coefficients controlling the candidate sources of Lorentz invariance violation.

² A word of caution must be spent about the interpretation of these percentage variations, that must be evaluated considering also the absolute value of the oscillation probability, used to “normalize” these variations. For some values of L , higher percentage variations sometimes are mainly due to the fact that the corresponding absolute value of $P_{\nu_\alpha, \nu_\beta}$ is extremely small.

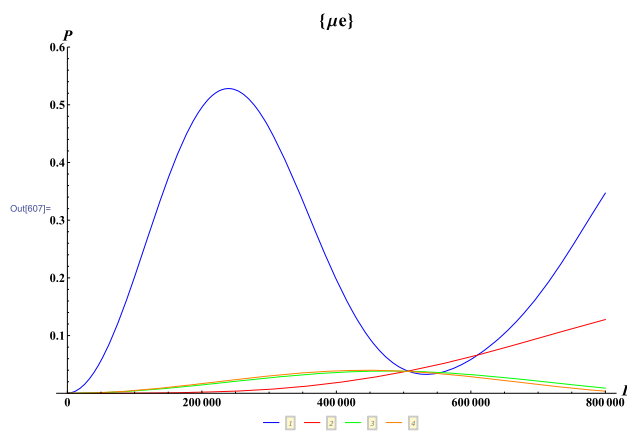


Fig. 10 Comparison of the P_{ν_μ, ν_e} oscillation probability, as a function of the baseline L , for neutrino energy $E = 1 \text{ TeV}$, for a “standard theory”, preserving Lorentz invariance (orange curve) and for different versions of models including LIV, with parameters equal, respectively, to $\delta f_{32} = \delta f_{21} = 4.5 \times 10^{-27}$ (blue), $\delta f_{32} = \delta f_{21} = 4.5 \times 10^{-28}$ (red) and $\delta f_{32} = \delta f_{21} = 4.5 \times 10^{-29}$ (green curve)

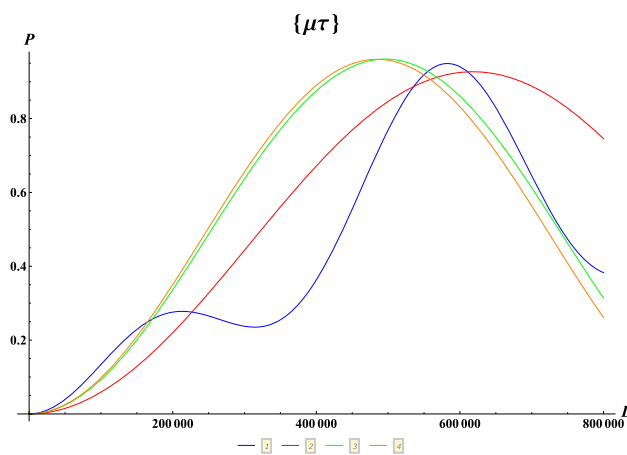


Fig. 11 Same analysis of Fig. 10, but for the case of P_{ν_μ, ν_τ}

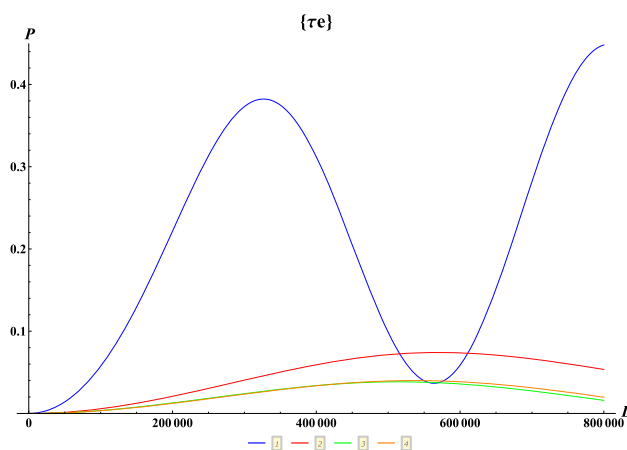


Fig. 12 Same analysis of Fig. 10, but for P_{ν_e, ν_τ}

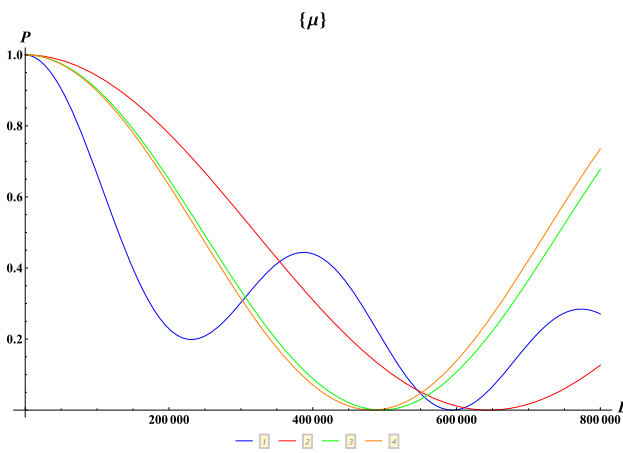


Fig. 13 Comparison of the results for the total muonic neutrino survival probability in a theory without LIV and in models with LIV corrections, corresponding to three different values of the δf_{kj} parameters, as illustrated in Fig. 10. Also the color code is the same adopted in Figs. 10, 11 and 12

In a full phenomenological analysis of any realistic experimental situation the information about the oscillation probability is central, but, obviously, it must be complemented by an expected fluxes accurate knowledge, in absence of oscillation, for every neutrino flavor and by a precise cross sections estimation for all the processes involved. The number $N_{\alpha,\beta}$ of detected transition events due to the $\nu_\alpha \rightarrow \nu_\beta$ flavor oscillation will be given, as a function of the energy E of the neutrinos that underwent the oscillation and of the distance L they travelled from the production to the detection point, by:

$$N_{\alpha,\beta} \propto \Phi_\alpha(L, E) P_{\nu_\alpha,\nu_\beta}(L, E) \sigma_\beta(E), \tag{59}$$

where Φ_α and σ_β represent, respectively, the predicted flux of neutrinos of flavor α , in absence of oscillation, and the cross section of interaction for the ν_β neutrinos with the detector, which depends upon the specific experiment studied. In most cases this information must be integrated over the neutrino energies (and eventually also over the distances L, that in many experiments are translated into angular bins) and the integrals have to be convoluted with functions describing the detector resolution and efficiency. Comparing, by means of statistical methods, the experimental results and the theoretical predictions one can extract the information about the impact of the eventual LIV violations, or put constraints on the order of magnitude of the coefficients ruling these effects in the model. We are performing such an analysis for different experimental situations of particular interest and the work describing the results of our study is in progress [69].

8 Conclusions

Lorentz covariance is one of the fundamental properties of space time in the standard version of relativity. Neverthe-

less, the possibility of small violations of this invariance has been explored in different extensions of the Standard Model and more generally in many exotic theories and a variety of possible experiments searching for signals of LIV (Lorentz invariance violations) have been proposed over the years. A significant numbers of these tests has to do with the study of neutrino properties, also because neutrino phenomenology is extremely reach and spans over a very wide range of energies.

In this paper we consider a class of models, more widely discussed in [8], in which the possibility of LIV is introduced starting from a modified version of dispersion relations and is founded on a more general geometrical description, making use of Finsler geometry. The choice of the particular form of the terms violating Lorentz invariance, represented as an homogeneous function of $\frac{|\vec{p}|}{E}$, guarantees the possibility of preserving a geometric derivation and, moreover, the LIV corrections are chosen in such a way to respect the isotropy of space time and the CPT invariance. The effect of the perturbative LIV corrections, that we introduce in our model, is that of modifying the kinematics, without changing the degrees of freedom of the theory and the interactions and preserving the internal $SU(3) \times SU(2) \times U(1)$ symmetry. Hence, the kind of model we obtain can be considered an extended version of the Standard Model, equivalent to models studied for instance in [9], in the case in which one restricts the LIV to CPT-even terms and wants to preserve isotropy.

We analyze the impact of our model LIV perturbative corrections on neutrino phenomenology, both in an hamiltonian approach and by means of an oscillation probabilities detailed study. The modification of the dispersion relations, with the introduction of Lorentz invariance violating terms (that can be treated with a sort of perturbative approach), imply a change in the form of the “phase differences” $\Delta\phi_{ij}$, which enter the calculation as arguments of $\sin^2(\Delta\phi_{ij})$, representing the contribution of the i, j mass eigenstates to the oscillation probability functions. As shown in Eq. (54), in addition to the usual term $\frac{\Delta m_{ij}^2 L}{E}$, another contribution appears in the expression for $\Delta\phi_{ij}$, proportional to $L E$ and dependent upon the differences δf_{ij} between the LIV coefficients for the different mass generations. This means that in our model the presence of LIV terms has an impact on the neutrino oscillation only if these terms are not identical for all the mass generations. Besides, the fact that the LIV corrections are proportional to E, instead of $\frac{1}{E}$, implies that, in order to be consistent with the data from the different oscillation experiments, these corrections must represent small perturbations which do not change the general “pattern” of neutrino oscillation. Nevertheless, they could be significant in particular experimental situations and, with an appropriate choice of the experimental tests, it could be possible to further constrain the possible values of the LIV coefficients.

We deeply investigated the impact of these LIV corrections, comparing all the different oscillation probabilities evaluated in presence of LIV with the analogous expressions in absence of LIV, for different fixed neutrino energy values (selected in such a way to cover different energy regions) and spanning over a wide range of values for the baseline between the neutrino production and detection points.

We showed that significant deviations from the “standard” values of oscillation probabilities could be present already for energies around 1 GeV, if one assumes LIV coefficients values of the same order of magnitude usually considered in literature [52–57] (around 10^{-23}). On the other hand, if one limits significantly the LIV corrections magnitude, considering the values recovered in a recent SuperKamiokande analysis [58] for the CPT-even LIV coefficients, the LIV effect on the oscillation probabilities starts to become evident for higher neutrino energies (around 100 GeV).

We studied in detail the situation for 1 TeV neutrinos, analyzing the improvement that, in this case, should be possible to obtain on the limits for the LIV coefficients and we also discussed the scenarios, that could be even more promising, of the future studies of ultra high energy neutrinos (like the cosmic ones). A series of real possible experimental situations, corresponding to various neutrino sources of different energies, for present and future experiments, are presently under investigation and will be discussed in a separate work [69].

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