ALEXANDER DUNN TO RECEIVE 2024 SASTRA RAMANUJAN PRIZE

The 2024 SASTRA Ramanujan Prize will be awarded to **Dr. Alexander Dunn** of the Georgia Institute of Technology (Georgia Tech), USA. This annual $10,000 prize is for outstanding contributions by individuals not exceeding the age of 32 in areas of mathematics influenced by Ramanujan in a broad sense. The age limit has been set at 32 because Ramanujan achieved so much in his brief life of 32 years. The prize will be awarded at an International Conference in Number Theory during December 20–22, 2024, at the Srinivasa Ramanujan Centre of SASTRA University in Kumbakonam (Ramanujan's hometown) in South India.

Dr. Alexander Dunn is a leading young researcher in the field of analytic number theory who has achieved several breakthroughs in the study of modular forms, half-integral weight forms, metaplectic forms and their connections to prime numbers and integer partitions. His sensational achievement, jointly with his post-doctoral advisor Maksym Radziwiłł at the California Institute of Technology (Caltech), is the resolution of the Patterson Conjecture related to the distribution of cubic Gauss sums — a problem with a 150-year illustrious history going back to Ernst Kummer (1810–1892). The Dunn–Radziwiłł paper has just been accepted for publication in the prestigious journal *Annals of Mathematics*.

The great mathematician Carl Friedrich Gauss (1777–1855) investigated certain exponential sums (now called quadratic Gauss sums) with respect to prime moduli. Quadratic Gauss sums have wonderful properties and have proved invaluable for tasks of counting solutions to various classes of equations. In the mid-nineteenth century, Kummer began studying a cubic analogue of the Gauss sums, motivated by the problem of studying solutions to the equation $X^3 + Y^3 + Z^3 = 0$ in finite fields of prime order. On the basis of computation, Kummer conjectured that the normalized cubic sums would have argument in the intervals $\pm[0, \pi/3]$, $\pm[\pi/3, 2\pi/3]$, $\pm[2\pi/3, \pi]$ according to the ratios $3 : 2 : 1$, and in particular, the normalized Gauss sum would not be equidistributed on the unit circle, thereby indicating a bias. A century later, Hermann Goldstine and John von Neumann, working on the new computer at the Institute for Advanced Study in Princeton, set out to test Kummer's conjecture. Their extensive calculations showed that the normalized values tended towards uniform distribution asymptotically, indicating that Kummer's conjecture was probably incorrect. In the 1970s, Samuel Patterson began a close study of cubic Gauss sums with the intention to resolve the mystery surrounding Kummer's conjecture. If the normalized values behaved like random complex numbers of absolute value 1, then by the law of of large numbers, the sum of the Gauss sums up to N ought to be of size $\sqrt{N}$, but based on his investigations, Patterson observed that the sum was much larger than $\sqrt{N}$, thereby indicating a bias, yet much smaller compared to N, hence suggesting uniform distribution! Around that time, Roger Heath-Brown, then a graduate student at Cambridge University, had developed new tools to treat cubic Gauss sums over primes. Patterson joined forces with Heath-Brown, and the two established the uniform distribution result in 1979, but their error bound was not strong enough to detect the bias. So the bias aspect of the problem remained unsolved until recently, when Dunn and Radziwiłł fully settled the bias conjecture of Patterson in a precise form in their breakthrough paper "Bias in cubic Gauss sums: Patterson's conjecture", assuming the Generalized Riemann Hypothesis (GRH).

The GRH is one of the most famous unsolved conjectures in mathematics. Most mathematicians believe the GRH to be true. Although the Dunn–Radziwiłł result is conditional, in that it depends on the GRH, the mathematical world has accepted their work as the long-awaited breakthrough. Indeed, their formidable paper is a masterpiece of analytic number theory, as it combines sieve methods, dispersion techniques, Heath-Brown's large sieve inequality for cubic symbols, and the analytic theory of cubic metaplectic forms. Dunn is an authority on metaplectic forms, and his expertise on metaplectic forms was crucial in the final resolution of the problem.

One important result that Dunn and Radziwiłł had to prove was that a certain term in the large sieve inequality of Heath-Brown is optimal, and this insight was due to Dunn. Dunn and Radziwiłł established a dispersion estimate for cubic Gauss sums valid for any sequence that is a convolution. The proof of this estimate relied on very careful estimates of several terms, some of them coming from the theory of metaplectic forms.

This amazing work of Dunn and Radziwiłł has more recently been extended in various directions by Dunn himself and in collaboration with other mathematicians. One such paper by Dunn, titled "Metaplectic cusp forms and the large sieve", concerns large sieve inequalities for Fourier coefficients of metaplectic cusp forms. The results in this paper, which is to appear in the journal *Algebra & Number Theory*, can be seen as an important extension of Heath-Brown's inequality.

Yet another seminal paper by Dunn, in collaboration with Alexander Zaharescu, titled "The twisted second moment of modular square roots and applications", is to appear in the *Journal of the European Mathematical Society*. Here Dunn and Zaharescu obtain an asymptotic formula for the second moment of L-functions of a certain half-integral weight form; this work was inspired by earlier fundamental results of Blomer, Fouvry, Kowalski, Michel, Milicevic, and Sawin, who treated the case of the fixed form being a classical form. Dunn and Zaharescu had to use totally different techniques (independent of the Ramanujan–Petersson conjecture, which is open for forms of half-integral weight), such as closed expressions for Salie sums, and utilize connections with Diophantine approximations and additive combinatorics. These techniques have further been improved by Dunn in collaboration with Kerr, Shparlinski, and Zaharescu, in the paper "Bilinear forms in Weyl sums for modular square roots and applications" that appeared in *Advances in Mathematics* in 2020.

Dunn's most recent advance is his work with David, Hamieh and Lin regarding the analogue of Heath-Brown's upper bound for quartic Gauss sums. Thus the results and techniques that Dunn and his collaborators have developed are continuing to make new inroads. With a broad spectrum of seminal results, and with the resolution of the Patterson Conjecture, Dunn has established himself, at this young age, as a leading figure in analytic number theory. The difficult field of metaplectic forms, which has largely been invigorated due to Dunn's efforts, is an area close to Ramanujan's work.

Dunn began studying metaplectic forms because of Scott Ahlgren, one of his two PhD advisors. Dunn and Ahlgren worked on half-integral weight modular forms for the rank partition statistic (coefficients of a Ramanujan mock theta function). In their paper "Maass forms and mock theta functions" that appeared in *Mathematische Annalen* in 2019, they use the spectral theory of automorphic forms to estimate the error term when one truncates the exact formula for the coefficients of a mock theta function of Ramanujan (the exact formula was a conjecture of George Andrews that Kathrin Bringmann and Ken Ono had proved in 2006). This paper of Ahlgren and Dunn closely ties with Ramanujan's work.

Alexander Dunn was born in Queensland, Australia. He graduated with BSc Honours in 2013 from the University of Queensland, and was awarded the University Medal for his meritorious perfomance. He then received his masters degree in 2015 from the University of Queensland, following which he joined the University of Illinois, Urbana-Champaign where he graduated with a PhD in 2020 under the joint supervision of Scott Ahlgren and Alexandru Zaharescu. He was awarded the Bateman Fellowship to write his dissertation, and the Bateman Prize in Number Theory for his thesis. After his PhD, he held the Olga Taussky and John Todd Instructorship at Caltech (2020–2023), during which time he was also on a Junior Fellowship at the Mittag-Leffler Institute in Sweden in Spring 2021. He is now a tenure-track assistant professor at the Georgia Institute of Technology.

The 2024 SASTRA Ramanujan Prize Committee consisted of Krishnaswami Alladi—Chair (University of Florida), Frank Calegari (University of Chicago), Shai Evra (Hebrew University, Jerusalem), Sergei Konyagin (Steklov Institute of Mathematics, Moscow), Bjorn Poonen (MIT), Dinakar Ramakrishnan (Caltech), and Wadim Zudilin (Radboud University Nijmegen). Alexander Dunn, who was enthusiastically endorsed by the prize committee as its top choice, joins the ranks of previous illustrious winners of the prize.