Asset allocation in extreme market conditions: a comparative analysis between developed and emerging economies

Montshioa, Keitumetse and Muteba Mwamba, John Weirstrass and Bonga-Bonga, Lumengo

University of Johannesburg, University of Johannesburg, University of Johannesburg

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ABSTRACT

This study makes use of the Extreme Value Theory, based on the Generalised Pareto Distribution and the Generalised Extreme Value Distribution, to construct efficient portfolios during periods of turmoil. The portfolios are constructed by combining different assets constituted by their positions in emerging and developed stock markets, with the aim of identifying which assets combinations provide optimal portfolio allocations during turmoil periods. For the developed stock markets, the study uses the French CAC 40, the Canadian S&P/TSX, the United Kingdom FTSE 100, the Japanese NIKKEI 225 and the United States S&P500 indices and returns. Five emerging stock markets indices are used, namely, the Brazilian BOVESPA, the Chinese SHCOMP the Indian S&P BSE SENEX, Indonesian JSI and the Turkish BIST 100. The data sample spans from August 1997 to August 2019 and include major economic and financial crises. Our findings show that for the different portfolios constructed, the estimated shape, location, and scale parameters differ depending on the Extreme Value Theory distribution under investigation. Moreover, based on the Generalised Pareto Distribution and the Generalised Extreme Value Distribution for portfolio optimisation, the results of the study show that during extreme conditions investors are prone to allocate more weight to developed stock market assets than to emerging markets. This confirms that developed economies are safe havens, especially during extreme market conditions. Moreover, the GPD is superior as it provides maximum risk-reward ratios.

1. INTRODUCTION

One of the most important tasks of asset managers is to efficiently select assets that could prevent large portfolio losses, especially during periods of financial distress. However, asset managers have been experiencing difficulties because of the significantly turbulent crises over the past decades, which have spread from country to country. These crises have negatively affected world equity markets and portfolio investments (Afzal & Ali, 2012; Khoon and Lim, 2010 & Samarakoon, 2017).

Several countrywide and regional financial and economic crises have had considerably contagious effects globally, with negative effects in equity markets. For example, the Asian financial Crisis, which occurred in 1997-1998. During the early 1990's South Asian economies achieved substantial economic growth however, significant risk was embedded to the growth. Economic developments were attributed to export growth and foreign investment. High interest rates and fixed currency exchange rates were implemented, as a consequence capital markets and corporates were exposed to foreign exchange risk. Preceding the US recession recovery the Federal Reserve raised interest rates against inflation. This resulted in massive capital inflow into the US and appreciation of the USD-currencies fixed to the USD appreciated, consequently negatively impacting export growth. Moreover, asset prices leveraged by huge amounts of credits collapsed and foreign investors withdrew capital investments. The capital outflow saw Asian currencies depreciating by approximately 38% and international stocks declining by nearly 60% (Corsetti, Pesenti & Roubini, 1999 and Cohen and Benjamin, 2008).

The dot-com bubble was a historic period of rapid increase in U.S. technology stock equity valuations fuelled by investment in internet based companies. During the dot-com bubble the values of equity markets grew exponentially. However March 2001 marked the burst of the dot-com companies resulting in the NASDAQ Composite declining by 78%. In the same year on September 11, the biggest terrorist attack was perpetrated against the United States. The terrorist attack led to approximately 3,000 deaths. The event prompted closure of the New York Stock Exchange and the NASDAQ composite saw a significant decline. (Junior and Franca, 2012).

The largest financial crisis began in 2007and reached its peak in 2008.This crisis was triggered by the default of a large number of mortgages in the USA. Loans were granted to borrowers with low credit scores. Most loans had small initial interest rates, adjustable for future payments, which led to many home foreclosures when rates increased. The loans were transformed in pools that were then resold to investors. Since the returns of such investments were high, a financial bubble was created that, inflated the subprime mortgage market until defaults started to pop up. Because of their underestimation of risk, financial institutions worldwide lost sizeable amount of money, and thus declared bankruptcy. This led to tightening of credit lines across the world and eventually wiped out global equity markets. Asian stock markets fell between ranges of 38% to 62%, with the largest market declines came from Singapore (27%), Thailand (21%) and the Philippines (21%). (Guinigundo & Paulson, 2010 and Junior & Franca, 2012).

At the start of the euro-zone debt crisis in 2009/2010, a number of European countries experienced a collapse of their financial institutions due to excessive government debt, which led to the increase in the bond yield spreads of government securities. This crisis triggered a fall in equity returns and the collapse of financial sectors in developed countries (Calice, Chen and Williams, 2013).

Bearing this in mind, it is evident that extreme conditions can be a fact of life in the operation of financial markets. The implications of financial turmoil can produce enormous mistrust in financial products and monetary system (Peterson & Weigelman, 2014). Hubbard (1991) further emphasises that financial crises can greatly affect investment opportunities and way in which funds should be allocated.
Durand, Lim & Yang (2014) note that equity returns are sensitive to during financial crisis. Smales (2014) finds that investors fear negatively related sentiment thus in presence of negative news-as in financial crisis investors fear increase, withdrawing capital from risky assets as equity markets causing equity market returns to plumage.
Ćvitanic, Polimenis & Zapatero (2008) warn that ignoring extreme risk leads to wealth loss by overinvesting in risky assets. Therefore, due to the frequency of financial crises and the magnitude of losses during crisis periods, it is important for asset managers

and investors to know how to apportion assets for efficient portfolio allocation during these periods (Kemp, 2011).

Risk-based approaches to asset allocation have been suggested to account for financial turbulances. For example, Briec, Kerstens, and Jokung (2007) suggest the use of a higher-order moment portfolio optimisation that creates a mean-variance skewness objective for portfolio allocation during turmoil periods. Naqvi,Mirza, et al. (2017) propose the construction of a mean-variance-skewness portfolio optimisation. However, these higher-order optimisation methods may be misleading due to misclassification of inefficient portfolios as efficient. It is in that context that DiTraglia and Gerlach (2007) remark that moment-based portfolio selection in the presence of heavy-tailed returns, especially methods based on the third moment, may be undefined or infinite.

Several authors suggest that the use of the extreme value theory (EVT) for risk management would be more effective than conventional methods for heavy tailed data (see Sullivan & Ge, 2011; Mwamba, Hammoudeh & Gupta, 2017). The superiority of the extreme value theory over other traditional models in modelling extreme events results in its capacity to use extreme observations that describe tail distributions, and to take into account skewness, kurtosis, persistence in volatility and correlation of risk asset returns in its modelling process (Wang, Sullivan & Ge, 2011). It is in this context that the study made use of the extreme value theory to construct and assess the effectiveness of the different portfolios obtained by combining assets from developed and emerging economies. In so doing, the study uncovered the combination of assets that will be effective during turmoil periods.

Assets should not be allocated in the same way during turmoil and calm periods. Given that most asset managers tend to reduce the risk inherent to crisis periods, it is necessary to accurately quantify and measure risk for optimal portfolio construction. In this regard, the use of a fat-tailed distribution asset allocation model improves the accuracy of measuring risk and reduces model risk. The study therefore employed the EVT framework in asset allocation to more accurately capture the downside risk. The three main advantages of EVT are: firstly, it focuses on extreme shocks and ignores the central observations; secondly, it makes no assumption regarding the underlying distribution of returns, so model risk is reduced. Specifically, it describes the dependence in the extreme lower tail of the joint distribution of a given portfolio's returns with those of the market. Lastly, it is better able to capture event risk such as crashes and currency devaluations (DiTraglia & Gerlach, 2012).

In times of extreme events and uncertainty, it is crucial for portfolio managers to successfully construct portfolios that yield high returns while minimising risk. Moreover, according to International financial theory, by spreading risk among different countries investors can minimise the effects of market volatility and ultimately yield increased long-term returns (Largoarde-Segot & Lucey, 2007). However, it is important to have insight into how different types of markets behave during turmoil periods. This study aims to provide insight into the treatment of risk and portfolio construction, with the specific refinements needed to handle extreme conditions. Furthermore, the study will add to the current literature on asset allocation using the EVT framework.

This study therefore aims to successfully construct an optimal portfolio using assets from different types of markets, such as developed and emerging markets, under extreme conditions and uncertainty with the aim of comparing the performance of the traditional mean-variance model and the EVT framework in portfolio optimisation. Focusing on which extreme value theory portfolio selection model is best suited for constructing a portfolio within extreme conditions when using assets from developed and emerging economies?

The investigation of portfolio optimisation within extreme events and uncertainty was conducted by analysing two portfolio selection methods under the EVT framework. The study employed the traditional Markowitz mean-variance model as a benchmark and compared the EVT framework in portfolio selection strategy, which focuses on the Peak Over Threshold (POT) method and the Block Maxima model (BMM). To this end, the study compared the Sharpe-ratio and Sortino-ratio to establish which portfolio outperformed the others. For the construction of the portfolio we use the returns of major equity indices in five developed markets and five emerging markets namely:the French CAC 40; Canadian S&P/TSX; the United Kingdom FTSE 100; Japanese NIKKEI 225; and the United States S&P 500 for developed economies. The study used the Brazilian BOVESPA; Chinese SHCOMP; Indian S&P BSE SENEX;

Indonesian JSI and Turkish BIST 100 for emerging economies. Data are sourced daily from Yahoo Finance. The sample covers a twenty-two year time horizon from August 1997 to August 2019, with a total number of 4261 observations, however .Chinese SHCOMP data spans from August 1997 to Sept 2017. The choice of the sample was informed by the inclusion of periods of major financial turmoil and stocks market capitalization.

The study is structured as follows: Chapter 2 presents a review of selected studies that focus on portfolio optimisation within extreme events. Chapter 3 explains how optimal portfolios were constructed using the EVT model, with a focus on GPD and GEV methods. Chapter 5 presents the data used in this study, the estimation of the traditional mean-variance model and the EVT model and the discussion of the results obtained. Chapter 6 concludes findings.

2. LITERATURE REVIEW

This chapter presents a survey of the previous literature focused on asset allocation under extreme conditions. While the literature on asset allocation and portfolio optimisation abounds, few studies have made use of extreme condition techniques.

Traditional optimal portfolio selection has relied on the Markowitz' (1952) modern portfolio theory (MPT). The theory postulates that an optimal investment portfolio is given by maximising returns at low-level risk. Although the theory has had a profound impact on financial economics and portfolio optimisation, it has been susceptible to empirical scrutiny. Sharpe (1964) and Sortino & van der Meer (1991), argue that the use of standard deviation as a measure of risk is inappropriate. Mandelbrot (1963) and Fama (1965) further point out that the theory assumes normally distributed asset returns while empirical evidence suggests that excess kurtosis and heavy tails exist in real financial data. Numerous empirical studies consolidate, that of, Bollerslev (1987), who analysed monthly returns of S&P 500 Index found leptokurtosis. Kariya, Maru, Matsu, Omaki & Tsukuda (1995) and Nagahara (1996) found fat-tailed and skewed return distribution in Japanese stocks.Using daily data, Mudakkar & Uppal (2013) found non-Gaussian distribution in Pakistan stocks. Therefore, it can be deduced that the MPT framework underestimates volatility and assets risk.

To remedy the shortcoming of traditional optimal portfolio selection, Arzac and Bawa (1977) analysed portfolio selection under extreme conditions when investor behaves according to the safety-first principle introduced by Roy (1952) and Tessler (1955). The results of their empirical analysis suggest that investors minimise the probability of losing more than a pre-specified amount of money. They further point out that in practice, it is extremely difficult to estimate the required probabilities using traditional techniques.

The problem of estimating tail probabilities has led to a growing interest in EVT in finance. In an early study, Koedijk, Schafgans & de Vries (1990) used EVT in analysing the effect of heavy tails on the bilateral EMS foreign exchange rates, which later led to Danielsson& De Vries (1997) and Kim (2015) using EVT to improve estimates of loss in foreign exchange markets. Susmel (2001) and Hyang & De Vries (2007) used the EVT approach to improve the Arzac and Bawa (1977) safety-first model, while Pownall and Koedijk (1999) use EVT to improve value at risk estimation(VaR) and Frey & McNeil(2000) included stochastic volatility in the EVT framework to improve VaR estimates .

Other than tail probability estimation, the EVT framework has been applied to study the distribution of asset returns. For instance, Jansen & De Vries (1991) employed EVT to measure univariate tail thickness using daily stock returns of ten stocks from the S&P 100 index and ten stocks from the S&P 500 index, within a sample period spanning February 1962 to December 1986. Results obtained report that distributions are consistent with Student-t distribution and ARCH classes. In related work, Malevergone, Pisarenko and Sornett (2006) used EVT to analyse the shape of stock returns using over 100 year's daily returns of the Dow Jones Industrial Average from May 1896 to May 2000. The study found rapidly varying behaviour of tails.

Studies such as these have proved that EVT is instrumental in statistically modelling unexpected events which are of considerable importance for aggregation of risk in an investment thus motivating interest in risk management within optimal portfolio selection.

Lhabitant (2001) investigated potential losses associated with a given investment strategy using 2 934 hedge funds over the period of 1994 to 2000 using EVT. The results of the in-sample and out-of-sample suggested that EVT better encapsulates high volatility and is valuable for assessing investment strategies and hedge funds risk.

Kabundi and Mwamba (2011) analysed the extreme losses likely to occur during market crashes. They used a portfolio that included stock indices from France, Germany, Japan, South Africa the United States, the United Kingdom and the South African rand and dollar exchange rates from January 2005 to December 2009. Dividing the sample period into two periods: pre-crisis and during the crisis, the study compared the estimates of value-at-risk using an EVT model (GEV distribution technique) and the traditional variance-covariance method. The results showed that the extreme value at risk -technique accurately measures downside risk, especially during downturns.

Mainik, Mitov and Rüschendrof (2015) used daily returns of the S&P 500 from 2001 to 2011 to evaluate -the performance of the extreme value technique in portfolio management. The study used a portfolio optimisation strategy based on the extreme risk index (ERI) using the EVT to minimise severe losses, benchmarked against the minimum variance portfolio and an equally weighted portfolio. Their results showed that the ERI significantly outperformed the minimum variance and equally weighted portfolio, as they reported the highest expected return even during the global financial crisis of 2008. Moreover, the strategy accounted for the special nature of diversification for heavy-tailed asset return.

Kiragu and Mung'atu (2016) evaluated the Nairobi Securities Exchange (NSE) All Share Index using the EVT-POT method in modelling extreme values to minimise losses. The authors utilised the daily closing prices of the NSE-All Share Index over an eight-year period from January 2008 to April 2016, and found that the probability of losses is lower compared to the possibility of gains.

Gilli (2006) used the EVT framework to assess the probability of extreme events in financial series in order to minimise market risk in financial portfolios. The data collected included daily returns of Dow Jones Euro Stoxx 50 over the period 02/01/1987 to 17/08/2004; the FTSE 100 from 05/01/1984 to 17/08/2004; the Hang Seng from 09/01/1981 to 17/08/2004; the Nikkei 225 from 07/01/1970 to 17/08/2004; Swiss Market Index 05/07/1988 to 17/08/2004; S&P 500 from 05/01/1960 to 16/08/2004. Comparing EVT BMM and the POT method results suggested that POT is superior as it exploited information in the data sample.

From the above studies it can be deduced that prior studies made use of EVT to model the risk associated with return distribution and using POT method. Studies comparing the performance of the POT method with other methods of -EVT in asset location, such as the BMM, are rare.

3. METHODOLOGY

This chapter presents the methodology used in this study. Section 3.1 outlines the two step procedure of how the data was filtered. Section 3.2 explains how the unique characteristics of financial data were accounted for using volatility models. Section 3.3 describes portfolio optimisation under the extreme value theory framework. Section 3.4 describes the traditional mean-variance optimisation model and lastly, section 3.5 provides a discussion on different portfolio performance measures.

3.1 Return Filtering

This section explains how a two-step procedure is used to filter each return series by fitting an –ARMA-GARCH process to remove serial correlation and standardise the daily returns residuals using the student's-t distribution to account for fat tails.

3.1.1 Conditional mean

Although we made use of an ARMA process to model the conditional mean, other models could be used as explained below.

3.1.1.1 Autoregressive model

The general formula of $AR(p)$ is written as follows:

$$Y_t = w + \sum_{i=1}^{p} \alpha_i Y_{t-i} + \varepsilon_t, \qquad \varepsilon_t \sim i.i.d\ N(0,1) \qquad (1)$$

Where $\alpha = (\alpha_1, \alpha_2, \dots, \alpha_p)$ is the vector of model coefficients and p is a non-negative integer. The AR model establishes that a realization at time t is a linear combination of the p previous realisation plus some noise term ε_t. For $p = 0, Y_t = \varepsilon_t$ and there is no autoregression term.

3.1.1.2 Moving Average model

A moving average(MA)model has a similar structure to an AR model but instead of using dependent variables as the independent variables, it uses past perturbations. A general form is $MA(q)$ and is written as:

$$Y_t = w + \sum_{j=1}^{q} \beta_j \varepsilon_{t-j} + \varepsilon_t, \qquad \varepsilon_t \sim i.i.d(0,1) \qquad (2)$$

where the Y_t is the dependent variable and ε_{t-j} $(j = 1,2 \ldots q)$ are independent variables with parameters $w, \beta_1, \beta_2, \ldots \beta_{q-1}, \beta_q$.

3.1.1.3 Autoregressive moving average model

An autoregressive moving average $(ARMA)$ model is a union of the two previously presented models. The general form $ARMA(p, q)$ is written as follows:

$$Y_t = w + \sum_{i=1}^{p} \alpha_i Y_{t-i} + \varepsilon_t + \sum_{j=1}^{q} \beta_j \varepsilon_{t-j} \qquad \varepsilon_t \sim i.i.d(0,1) \tag{3}$$

Following the conditional mean models is the second step to filtering the data using conditional volatility models to remove effects of heteroscedasticity. The section bellow briefly discusses conditional volatility models.

3.1.2 Conditional volatility

Financial time series have non-normal characteristics such as leptokurtosis, fat tails and volatility clustering. Thus volatility models were developed to account for these characteristics. The two types of volatility models are symmetric and asymmetric. The main difference of the symmetric volatility models is that they do not account for the leverage effect.

3.1.2.1 Symmetric volatility models

The Autoregressive Conditional Heteroscedasticity $(ARCH)$ model was pioneered by Engle (1982), which allows the conditional variance to change over time given as function of past error. The general formula of $ARCH(q)$ is expressed as follows:

$$\sigma_t^2 = w + \sum_{j=1}^{q} \alpha_j \varepsilon_{t-1}^2 \tag{4}$$

Where σ_t^2 is conditional variance. The model was then independently extended by Bollerslev (1986) and Taylor (1986), developing the generalised form of the model $ARCH$, thus the $GARCH$ model. The main advantage of the $GARCH$ model in time series is that the model is able to describe the observed volatility clustering and leptokurtosis. The general formula for $GARCH(p, q)$ is expressed as follows:

$$Y_t = \mu_t + \varepsilon_t \tag{5}$$

$$\varepsilon_t = \sigma_t . z_t, \qquad z_t \sim i.i.d\ N(0,1)$$

$$\sigma_t^2 = w + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-1}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-1}^2 \qquad (6)$$

Where μ represents the mean value and $w, \alpha(i = 1 \dots p), \beta(j = 1 \dots q)$ are parameters to be estimated. $GARCH(p, q)$ is stationary if $(\alpha + \beta) < 1$. It is important to note that $GARCH$ models are robust to misspecification and are able to estimate volatility in consistent manner (Nelson, 1992).

3.1.2.3 Asymmetric volatility models

The main benefit of the asymmetric model is that it accounts for the leverage effect. Asymmetric models t often used in literature are $APARCH, CSGARGH, EGARCH, GRJ-GARCH, IGARCH, LGARCH, NGARCH, QGARCH, TGARCH$ (Pellergrini, Ruiz and Espasa, 2010). For the purpose of this study we employed the $GRJ - GARCH$ which is expressed as follows:

$$Y_t = \mu_t + \varepsilon_t \qquad (7)$$

$$\varepsilon_t = \sigma_t . z_t, \qquad z_t \sim i.i.d\ N(0,1)$$

$$\sigma_t^2 = w + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-1}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-1}^2 + \sum_{i=1}^{p} \gamma \psi_{t-i} \varepsilon_{t-1}^2 \qquad (8)$$

Where μ represents the mean value and $w, \alpha(i = 1 \dots p), \beta(j = 1 \dots q)$and γ are volatility parameters. γ Indicates the leverage effect, if $\gamma = 0$ there is no asymmetric volatility; if $\gamma < 0$ negative shocks increase volatility if $\gamma > 0$ positive shocks increase volatility and ψ represents parameter affected by shocks.

3.3 The Mean-Variance Model

The study assumes the traditional mean-variance model as a benchmark portfolio. The portfolio consists of n assets with expected return μ_t, let w_t be weight of portfolio's value invested in asset i such that $\sum_i^n w_i = 1$ short selling is not allowed. Therefore, portfolio composition of n assets and mean asset returns is expressed as

$$w = [w_{1,} \dots , w_n]^T \text{ and} \qquad (9)$$

$$\mu = [\mu_{1,} \dots , \mu_{n,}] \qquad (10)$$

Portfolio expected return is formulated as:

$$\mu_{p,} = \mu^T . w \qquad (11)$$

The portfolio variance defined by:

$$\sigma_p^2 = w^T . \sum w \qquad . \qquad (12)$$

Where Σ is the covariance matrix of asset returns.

Given the targeted expected portfolio return $\mu_{p,}$, the mean-variance characterises an efficient portfolio by its weight vector w_{eff} that solves optimisation problem:

$$w_{eff} = \arg min\, w^T . \sum w \qquad (13)$$

Subject to

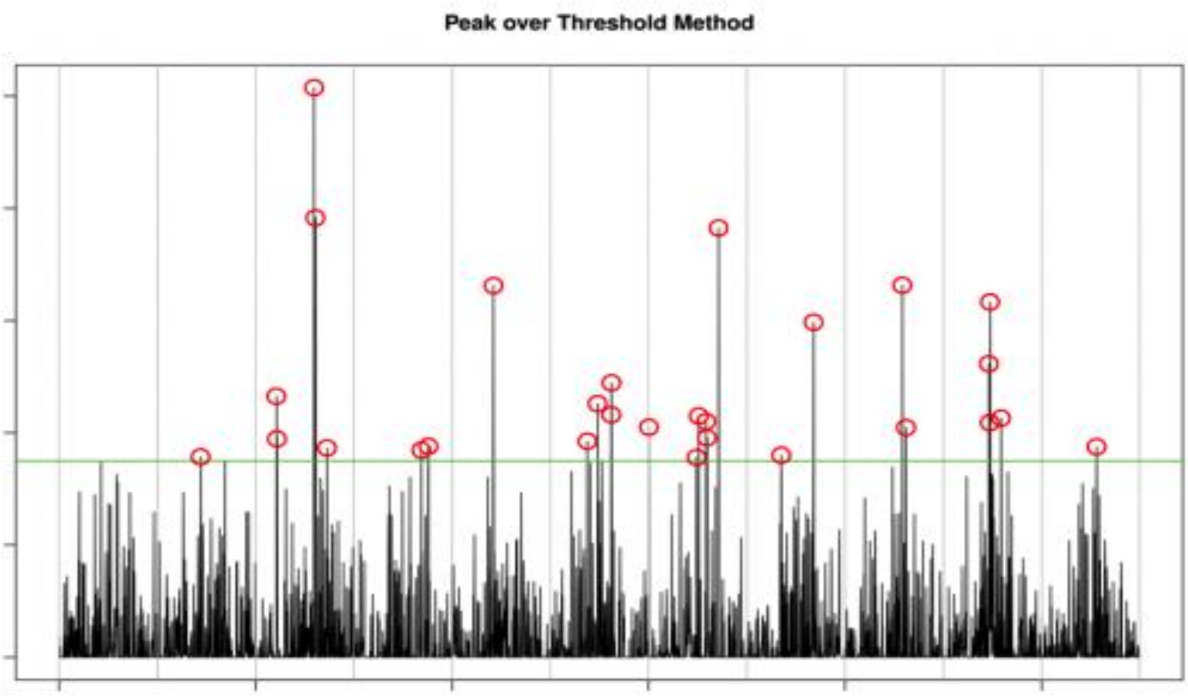
$$w^T . \mu = \mu \qquad w^T 1 = 1, w \geq 0 \qquad (16)$$

3.3 Extreme Value Theory

This study made use of the EVT to assess the performance of the different hypothetical portfolios made up of developed and emerging economies. EVT is an effective approach to estimate extreme cases of market risk. It focuses on the distribution of extreme values and can accurately describe the tail quantiles of distributions with the overall distribution unknown. EVT has two substantial ways of modelling results: the Peak Over Threshold model (POT) and Block Maxima Model (BMM). These methods are briefly discussed below.

3.4.1 Peak Over Threshold model

The POT model is built on the hypothesis that the distribution of returns over the threshold follows the General Pareto Distribution (GPD) which only models data returns over some high enough threshold. It is important to note that POT focuses only on the approximate description to tails,and not the overall distribution.



(Source: authors own)

Figure 1: Peak over Threshold

From figure 1, it can be seen that the green line illustrates the threshold, where the red dots represent the peak and everything below the threshold is considered as standard data.

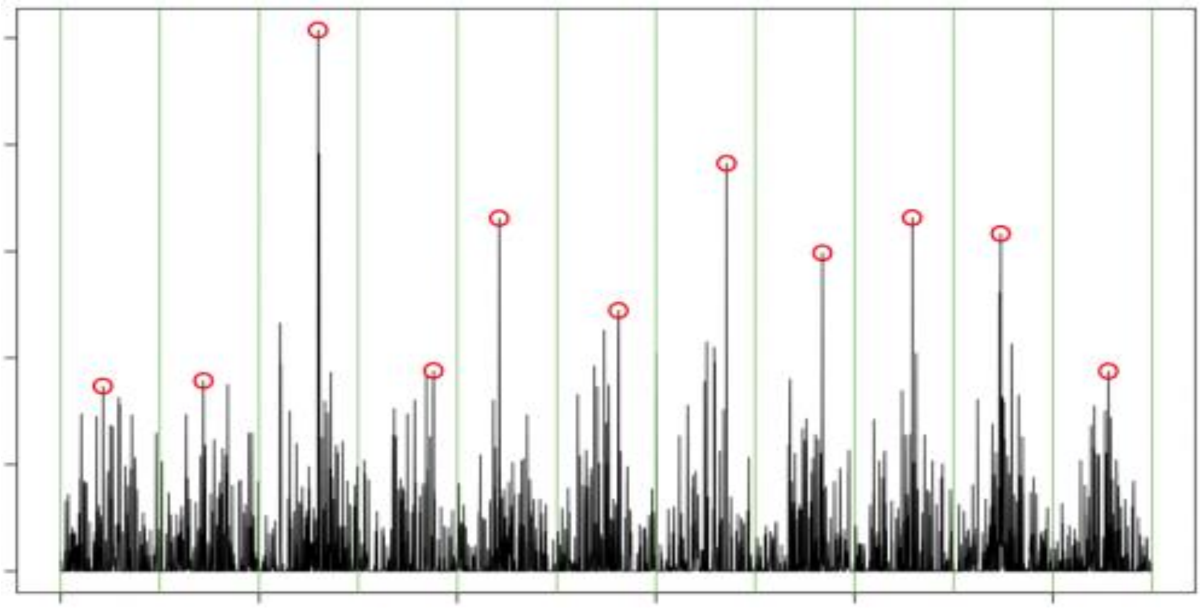
The general mathematical formula of GPD is written as follows:

$$G_{\xi,\beta} = \begin{cases} 1-\left(1+\frac{\xi x}{\beta}\right), & \xi \neq 0 \\ 1-\exp\left(-\frac{x}{\beta}\right), & \xi = 0 \end{cases} \qquad (17)$$

Where ξ denotes the shape parameter and β denotes the scale parameter. When $\xi < 0$ it represents a pareto distribution of type 2, when $\xi = 0$ it represents exponential distribution and when $\xi > 0$ it represents a reparametrised type of pareto distribution.

3.4.2 Block Maxima Model

The Block Maxima Model (BMM) is built on the hypothesis that the distribution of returns is characterised by Generalised Extreme Value (GEV) distribution. The application of BMM divides the sample into blocks, in the block split it then chooses the maximum value (maxima) for which the maxima are used in the tail distribution fit.



(Source: authors own)

Figure 2: Block Maxima

Figure 2 illustrates the BMM where the green lines divided samples into blocks and the values highlighted in red indicate the maximum extreme in sample.

The general mathematical formula of GEV may be written as follows:

$$G_{\xi}(x) = \begin{cases} \exp\left(-(1+\ \xi_{\ x})^{\frac{-1}{\xi}}\right), & \xi \neq 0 \\ \exp(-e^{-x}), & \xi = 0 \end{cases} \tag{18}$$

Where ξ denotes the shape parameter. When $\xi < 0$ it represents the Weibull distribution, when $\xi = 0$ it represents the Gumbel distribution and when $\xi > 0$ it represents the Frechet distribution. The factor $(1+\ \xi_{\ x})$ is always positive.

3.5 Portfolio Evaluation Methods

To evaluate the performance of the EVT methods in asset allocation we employed the Sharpe ratio and Sortino ratio. The superior model produces higher ratios.

3.5.1 The Sharpe Ratio (1966)

The Sharpe ratio is a commonly used performance measure in numerous studies that seeks to investigate the portfolio performance (Mwamba, 2012; Bhurjee, Kumar & Panda, 2014; Haugh, 2016).The Sharpe Ratio measures the expected portfolio return in excess of the risk free rate of return per unit of standard deviation and is often referred to as the reward-to-volatility ratio. It is formulated as follows:

$$S_P = \frac{r_i - r_f}{\sigma_i} \tag{19}$$

Where r_i denotes the return generated by the portfolio; r_f is the risk-free rate. The total portfolio volatility σ_i is measured by the standard deviation of the portfolio and captures both the systematic and unsystematic risk. Xiong and Idzorek (2011) point out that using tail risk or downside-risk would be more coherent way of defining risk we therefore use the Sortino ratio to measure portfolio performance.

3.5.2 The Sortino Ratio (1991)

The Sortino ratio is in line with the Post-Morden Portfolio Theory and ignores upside volatility. The ratio measures excess return against downside volatility in a portfolio. In any investment a minimum amount of return needs to be generated in order to accomplish certain goals or to keep project in operation. Therefore, investors are cautious about their returns falling below the desired level (Sortino & Van de Meer, 1991). This desired level is referred to as the Minimum Acceptable Return (MAR). An investment that generates a return greater than MAR is desirable because it is risk-free. However, returns that fall below MAR are undesirable because they expose the investor to greater risk. It is formulated as follows:

$$SoR = \frac{r_i - r_{MAR}}{DD} \quad (20)$$

Where

$$DD = \sqrt{\frac{1}{T}\sum_{t=0}^{T}(r_i - r_{MAR})^2} \quad (21)$$

r_{MAR} Denotes the minimum acceptable return and DD denotes downside deviation or semi standard deviation. The Sortino Ratio is thus a modification of the Sharpe Ratio; however, it replaces both the risk-free rate of return and standard deviation with the MAR and the downside deviation respectively. Mao (1970) highlights the use of semi-deviation as an alternative measure of risk.

4. DATA AND EMPIRICAL RESULTS

This chapter presents data used in the study, the estimation of MV, GPD and CEV methods in various portfolio optimisation and discussion of the results obtained. The chapter is divided into five sections. Section 1 presents data; section 2 descriptive statistics; section 3 presents return filtering process; section 4 steps taken for EVT analysis and section 5 discusses various portfolios constructed with different portfolio optimisations models while discussing investors risk preference.

4.1 Data Description

Given the aim of this study to construct a portfolio made of emerging and developing economies and to assess which performs better during turmoil periods, this study made use daily closing prices of major equity indices in five developed and five emerging markets. France (CAC 40), Canada (S&P/TSX), the United Kingdom (FTSE 100), Japan (NIKKEI 225), the United States (S&P500), Brazil (BOVESPA), China (SHCOMP) India (S&P BSE SENEX), Indonesia (JSI) and Turkey (BITS100) . The sample period spanned 05 August 1997 to 29 August 2019, with a total of 3 736 observations. However the Chinese SCHOMP spanned from 05 August 1997 to 25 August 2017. All data was sourced from yahoo finance.

Preliminary analysis is presented below as figure 3 illustrates the price of equity indices in the portfolio.

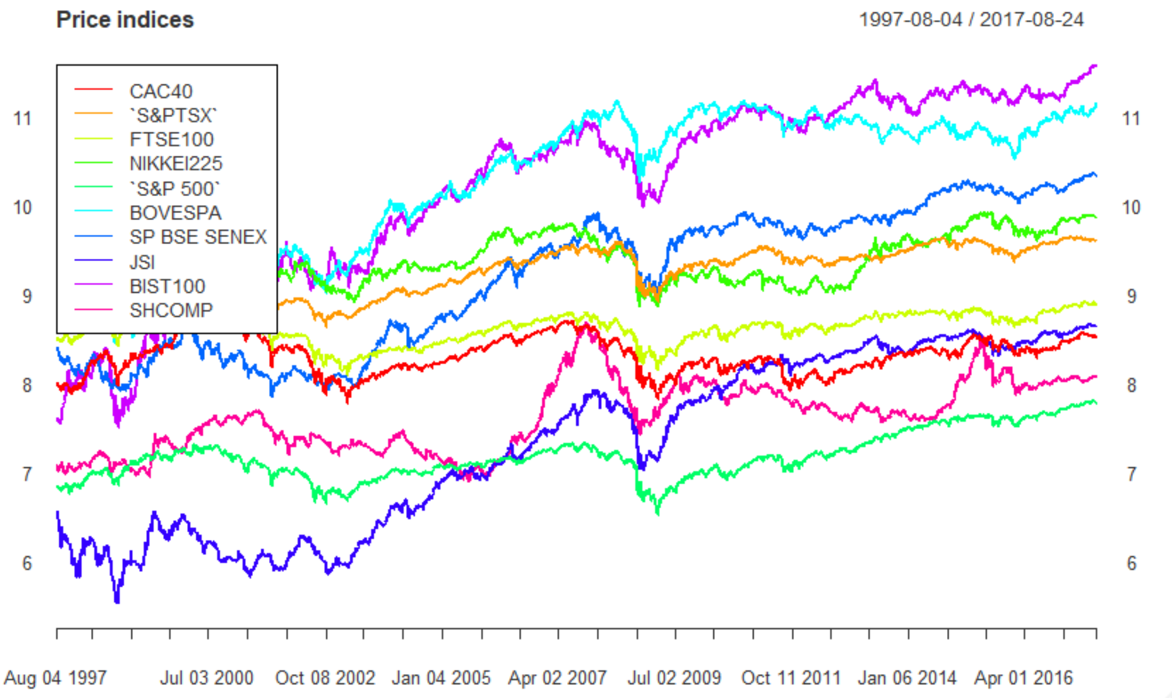


Figure 3: Price of equity indices

The Figure above illustrates that all equity indices in the portfolio experienced a price drop in 1998-Asian crisis and between 2008 and 2009-Global financial crisis. Vila (2000) suggests that. the rice drop in equity prices during financial periods is due to panic selling. Returns series are obtained as follows:

$$r_t = (\ln P_t - lnP_{t-1}) * 100 \qquad (22)$$

where r_t is the daily rate of return, *P* are the closing prices. *ln* is the natural logarithm. Preliminary and descriptive statistics of the daily equity returns series are reported below starting with figure 4.

Figure 4 below illustrates the daily equity returns series of all assets in the portfolio. From the figure it can be deduced that all series depict volatility clustering and heteroscedasticity. Moreover, crisis periods are characterised by high volatility translated by high spikes. These periods are between 1997 to 1999, 2001 to 2003 and 2008 to 2009 and slightly in 2011. These occurrences may be ascribed to the panic in the markets caused by the Asian crisis, the dotcom-bubble, the global financial crisis and Eur zone debt crisis, respectively. The Chinese SHCOMP depictd the largest jump in volatility during the 2008/2009 global financial crisis.

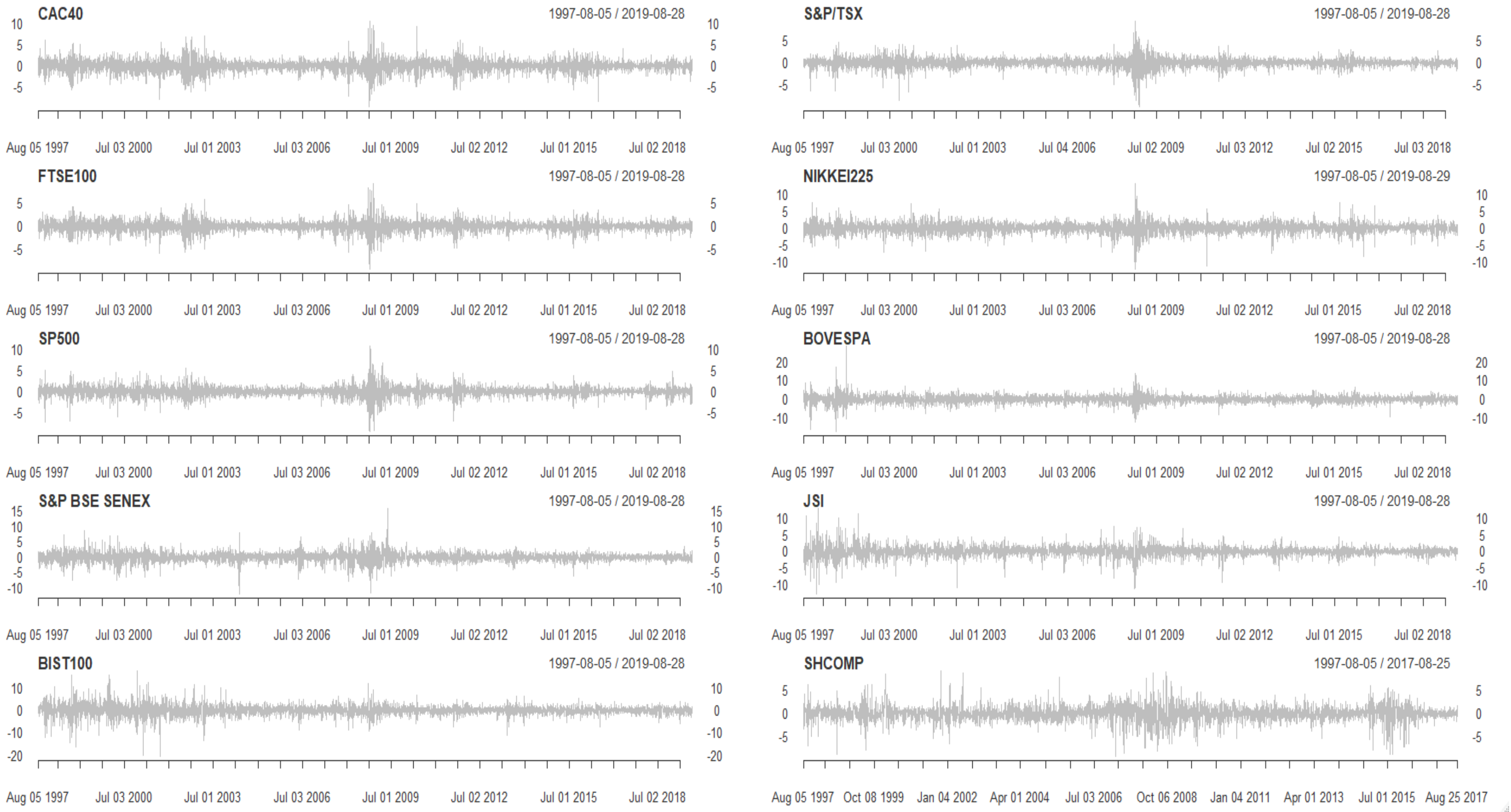


Figure 4: Log returns of equity indices

4.2 Descriptive Statistics

Table 1: Descriptive Statistics

Stock Market	Mean	Standard deviation	Skewness	Kurtosis
Developed Markets Indices				
CAC 40	-0.03	1.46	-0.29	3.57
S&P/TSX	-0.01	1.10	-0.55	8.18
FTSE 100	-0.03	1.19	-0.42	4.89
NIKKEI 225	-0.03	1.57	-0.44	5.88
S&P500	0.00	1.21	-0.53	5.40
Emerging Markets Indices				
BOVESPA	-0.03	1.98	-0.65	5.76
SCHOMP	0.02	1.62	-0.29	5.08
S&P BSE SENEX	0.01	1.52	-0.44	4.69
JSI	0.02	1.59	-0.25	7.83
BIST 100	0.06	2.45	-0.21	7.70

The return distribution of each of the series is presented in Table 1 above. The Table reports the first four moments of distribution over the entire sample period. Considering the mean in developed countries, found that all developed countries reported a negative mean value and are close to zero. In the emerging markets, only BOVESPA reported a negative mean of -0.03; the rest reported a positive mean, with BIST 100 reporting the highest mean of 0.06.

All markets are negatively skewed implying that the probability of getting negative returns is higher than getting positive returns. Moreover, the kurtosis of all the markets was found to be greater than 3, signalling the presence of leptokurtic distribution, and implying that in times of financial crises price drop occurs resulting in extreme losses.

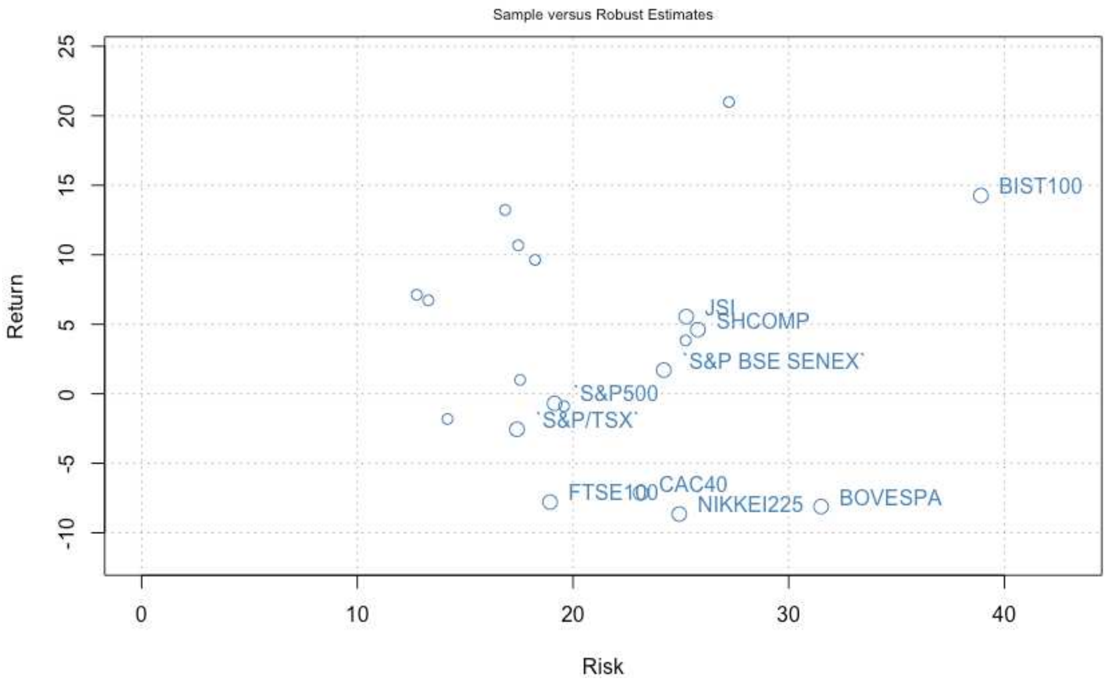


Figure 5: Risk Reward Plot

Figure 5 above illustrates the risk reward plot. The plot illustrates that the Turkish BIST 100 reports the highest return and highest risk. Chiang and Zhang(2018) point out that higher expected stock returns are assotiated with high expected variances. Although, BOVESPA also reports high risk – returns are negative.The FTSE 100, CAC 40, S&P/TSX and S&P 500 report moderate risk with low returns, while SHCOMP and JSI report moderate risk and moderate returns. Overall, emerging markets indices are more riskier and provide higher returns relative to developed markets indicies.

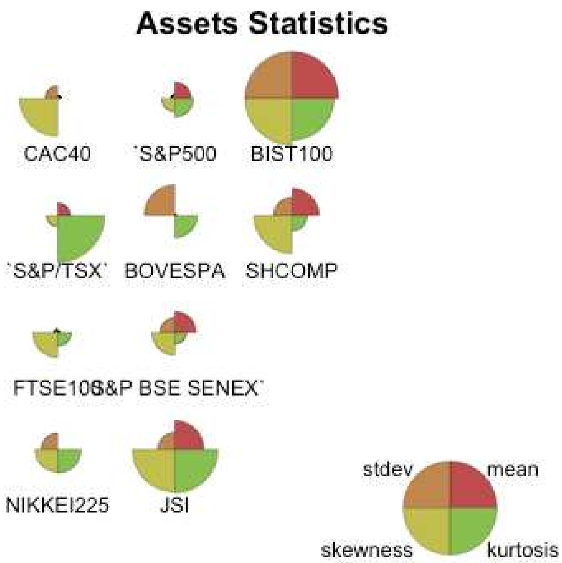


Figure 6: The first four moments of each series.

Note: the legend is represented at the extreme left

Figure 6 above is a graphical illustration of the first four moments of the return distribution of all equity indices in the portfolio; it displays the results of the descriptive statistics represented in Table 1. As discussed earlier, the BIST 100 and JSI report the highest mean, while the NIKKEI 225 and BOVESPA report minimal means. Concerning the standard deviation, BIST 100 and BOVESPA depict high risk and while the S&P/TSX and FTSE 100 report the lowest risk.

Regarding the skewness CAC40, the JSI, BIST 100 and SHCOMP show significant skewness asserting the likelihood of negative returns while the S&P/TSX, BIST 100 and JSI exhibit significant kurtosis confirming that in the occurrence of an extreme and or rare event index prices will drop immensely.

Given that all the returns series are negatively skewed with high kurtosis, the study employed the EVT models, specifically the POT method and BMM to model the behaviour of indices in the left tail, but first we filtered the return series data using the ARMA-GARCH process, removing the effect of auto-correlation and heteroscedasticity.

4.3 The ARMA-GARCH process

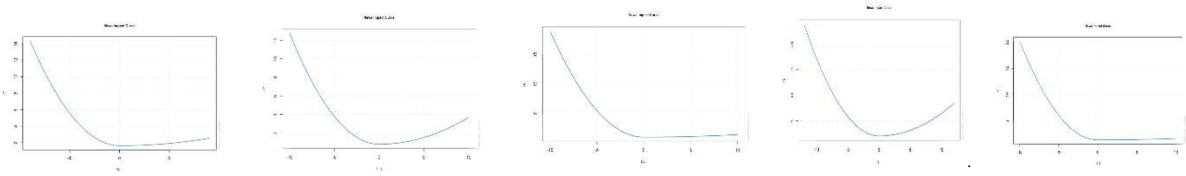
As explained earlier, a two-step process was used to filter each return series by fitting an ARMA-GARCH process to remove serial correlation and standardise the daily returns residuals using the student's-t distribution to account for fat tails. Table 2 shows the estimation of the ARMA-GJR-GARCH (1, 1), chosen based on the AIC criteria.

Table 2: Conditional volatility estimation using ARMA-GJR-GARCH (1,1) with student-t distribution

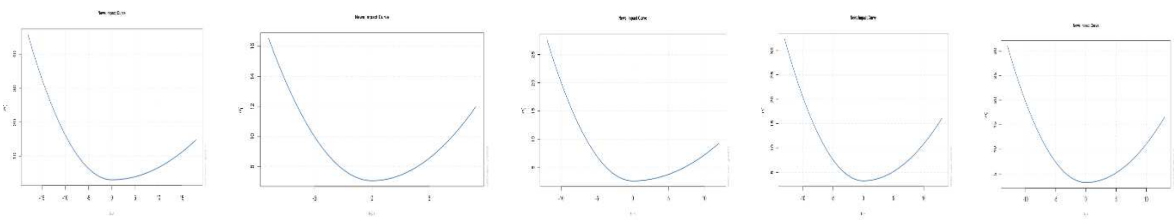
	CAC 40	S&P/TSX	FTSE 100	NIKKIE 225	S&P 500	BOVESPA	SHCOMP	S&P BSE SENEX	JSI	BIST 100
$ar1$	0.157 (17.23)	1.270 (6.04)	0.233 (13.56)	-0.773 (-3.55)	2.276 (101.9)	1.285 (13.41)	0.994 (405.33)	-0.544 (-3.91)	1.050 (1044)	1.349 (16.94)
$ar2$	1.012 (31.55)	-0.645 (-4.43)	-0.250 (-9.97)	-0.341 (-2.02)	-2.426 (-44.29)	-0.823 (-4.59)	-0.983		-0.216 (-3830)	-0.897 (-9.39)
$ar3$	0.275 (6.18)	-	0.263 (5.40)		1.375 (21.41)		0.029		0.104 (265)	
$ar4$	-0.859 (-88.08)	-	-0.947 (-201.1)		-0.279 (-9.182)	-			0.986 (-2046)	
$ar5$		-				-			-	
$ma1$	-0.169 (-59.27)	-1.242 (-5.58)	-0.226 (-7105)	0.749 (3.44)	-2.316 (-605)	-1.28 (-13.15)	-0.983 (-8371)	0.610 (4.64)	-0.986 (-2316)	-1.329 (-14.54)
$ma2$	-1.039 (-33.73)	0.593 (3.78)	0.250 (4754)	-	2.487 (129)	0.799 (4.29)	.		0.144 (1297)	0.867 (7.86)
$ma3$	-0.886 (-8.01)	-	-0.279 (-2175)	-	-1.401 (-423)	-			-0.11 (-2815)	
$ma4$	0.886 (-538.5)	-	0.942 (9623)	-	0.245 (22.02)	-			0.502 (2014)	
$ma5$		-		-	0.041 (3.72)	-			-	
ω	0.355 (4.89)	0.014 (4.38)	0.023 (5.089)	0.061 (5.14)	0.021 (4.50)	0.106 (3.79)	0.029 (3.58)	0.043 (4.41)	0.049 (2.07)	0.048 (3.59)
α	0.111 (1.25)	0.028 (2.63)	0.004 (0.41)	0.044 (4.19)	0.002 (4.50)	0.036 (3.45)	0.060 (6.14)	0.046 (4.21)	0.075 (4.76)	0.078 (5.16)
β	0.896 (76.76)	0.909 (78.62)	0.887 (73.42.)	0.875 (59.31)	0.891 (3.32)	0.883 (47.65)	0.907 (90.85)	0.874 (62.07)	0.862 (26.45)	0.865 (43.35)
γ	0.146 (7.72)	0.090 (5.42)	0.175 (8.48)	0.107 (5.82)	0.184 (7.33)	0.095 (3.92)	0.056 (3.25)	0.128 (5.78)	0.095 (3.0)	0.085 (3.81)
φ	10.38 (6.71)	9.045 (7.50)	11.75 (5.86)	8.906 (8.43)	7.191 (8.65)	10.08 (5.87)	4.251 (13.45)	7.139 (9.58)	5.084 (11.24)	5.181 (11.81)

(t-statistics reported in brackets. Source: own calculations

Table 2 above reports the conditional mean and conditional variance of all equity indices in the portfolio. All the conditional means of indices are statistically significant suggesting that auto-correlation was successfully removed. The conditional variance parameters of most indices are significant except for alpha (α) of CAC40, FTSE 100 and S&P 500 implying that previous shocks do not have impact on volatility where these indices are concerned. All β coefficients are statistically significant suggesting that previous volatility impacts current volatility. Furthermore, the condition $\alpha + \beta < 1$ is respected as there is no consistency in volatility, successfully removing heteroskedasticity. For all the markets the γ parameter is statistically significant and is greater than zero, negative shocks (bad news) increase the volatility of all equity indices more than positive shocks. This implies that a leverage effect for equity indices exists. This is further confirmed by the news curves in Figure 7 below.



CAC 40 S&P/TSX FTSE NIKKEI S&P 500



BOVESPA SHCOMP S&P BSE SENEX JSI BIST100

Figure 7: News Impact Curves

Figure 7 above depicts the News Impact Curve of all equity indices in the portfolio. From the figures it can be deduced that the volatility in all markets is highly impacted by bad news. Thus, there is a leverage effect in all markets.

4.4 Extreme Value Theory Application

As stated earlier, we used the POT-GPD and GEV to estimate the parameters of our series, based on the EVT technique. The table below presents maximum likelihood parameters for the POT-GPD. Coles (2001) points out that the POT method produces more efficient and reliable shape parameters and accurately models financial time series' tails.

Table 3: Generalized Pareto Distribution parameters

Stock Market	**Shape**	**Location**	**Scale**
Developed Markets Indices			
CAC 40	0.060 (0.096)	-0.029	0.967 (0.122)
S&P/TSX	0.299 (0.129)	-0.010	0.625 (0.098)
FTSE 100	0.046 (0.803)	-0.031	0.812 (0.094)
NIKKEI 225	0.278 (0.108)	-0.035	0.687 (0.092)
S&P500	-0.253 (0.086)	0.002	0.944 (0.113)
Emerging Markets Indices			
BOVESPA	0.140 (0.100)	-0.033	1.062 (0.138)
SHCOMP	0.1712 (0.098)	0.007	1.068 (0.131)
S&P BSE SENEX	0.059 (0.085)	0.006	1.014 (0.115)
JSI	0.258 (0.112)	0.021	1.061 (0.144)
BIST 100	0.211 (0.111)	0.022	1.156 (0.159)

(t-statistics reported in brackets, source: own calculations)

Similar to Mwamba, Hammoudeh and Gupta (2017) the GPD method produced negative and positive shape parameters. A positive parameter indicates that equity indices have fatter tails. Thus, the results reported in Table 3, indicate that most indices have fat tails except for the S&P 500, which indicates a negative shape parameter, implying that the distribution is thinner and possibly with minimal probability of extreme losses.

Table 4: Generalised Extreme Value Distribution Parameters

Stock Market	Shape	Location	Scale
Developed Markets Indices			
CAC 40	0.1132 (0.066)	1.906 (0.082)	1.906 (0.082)
S&P/TSX	0.283 (0.753)	1.359 (0.060)	0.691 (0.050)
FTSE 100	0.147 (0.606)	1.531 (0.065)	0.768 (0.050)
NIKKEI 225	0.128 (0.055)	2.098 (0.086)	1.024 (0.065)
S&P500	0.262 (0.072)	1.503 (0.063)	0.729 (0.052)
Emerging Markets Indices			
BOVESPA	0.219 (0.057)	2.605 (0.099)	1.181 (0.079)
SHCOMP	0.266 (0.070)	2.018 (0.096)	1.110 (0.079)
S&P BSE SENEX	0.230 (0.066)	1.895 (0.086)	1.003 (0.070)
JSI	0.219 (0.066)	1.916 (0.093)	1.082 (0.075)
BIST 100	0.223 (0.066)	1.933 (0.094)	1.095 (0.076)

The maximum likelihood parameters for generalised extreme values are reported above in Table 4. A shape parameter that is greater than zero implies that returns are fat-tailed, thus we would expect larger risk for indices that have a larger parameter. The results show that the developed markets indices are prone to less risk compared to emerging markets indices

Having estimated the parameters of EVT using the POT method and BMM, we simulated the returns using the GPD and GEV distribution results on estimated portfolios, which are reported below.

4.5 PORTFOLIO SELECTION

This section aims to successfully allocate assets to generate optimal portfolio using the different EVT -methods. Given that this study used the EVT method for portfolio allocation to account for portfolio allocation during periods of crisis, in comparison with the Markowitz (1952) Mean-Variance procedure, which assumes normal distribution and thus provided the estimate for portfolio allocation during normal or "quiet" periods.

4.5.1 The Markowitz Mean-Variance portfolio

Table 5: Markowitz Mean-variance international portfolio weights

	MEAN-VARIANCE	
Portfolio	**Optimal portfolio weights**	**Tangent portfolio weights**
Developed Markets Indices		
CAC 40	0.0000	0.0000
S&P/TSX	0.2457	0.0000
FTSE 100	0.1268	0.0000
NIKKEI 225	0.0854	0.0000
S&P 500	0.1581	0.0000
Emerging Markets Indices		
BOVESPA	0.0000	0.0000
SHCOMP	0.1770	0.0000
S&P BSE SENEX	0.0930	0.0000
JSI	0.1140	0.0000
BIST 100	0.0000	1
Expected Return ($\boldsymbol{E}[\boldsymbol{R}]$)	-0.0059	0.0224
Risk ($\boldsymbol{CVaR}$)	2.0391	3.8746
Sharpe Ratio	**-0.009**	**0.007**
Sortino Ratio(MAR=0)	**-0.004**	**0.003**

The table 5 above reports the traditional mean-variance optimal portfolio and tangent portfolio weights for a portfolio that is constituted of mixed assets of emerging and developing economies. The optimal portfolio weight based on efficient frontier shows that the a portfolio that combines emerging and developed market indices should allocate 61.56% to developed markets indices and the remaining to emerging economies. More weight is allocated to the Canadian S&P/TSX at 0.2457 and the least weight to the NIKKEI 225 at 0.0854. Whereas the tangent portfolio provided the highest Sharpe ratio and allocates 100% to emerging markets, the Turkish BIST 100 holds the largest weight of 1. This is evident from the results displayed in Figure 8, which show that the BIST 100 is the only asset that is on the efficient frontier, not far from the capital allocation line (CAL). With regard to the Sharpe and Sortino ratio, we found that the tangent portfolio presented a desirable portfolio relative to the optimal portfolio, based on the efficient frontier with a Sharpe ratio of 0.007and a Sortino ratio of 0.003.

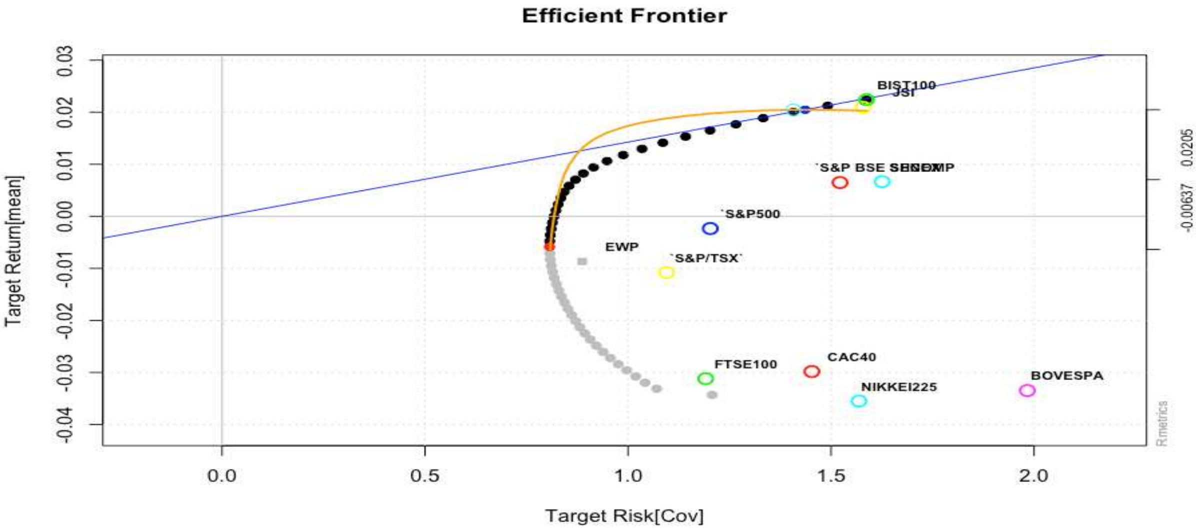


Figure 8: Efficient Portfolio Frontier for Mean-Variance international portfolio

Although the results reported in Table 5 show that the tangent portfolio allocation favours the use of emerging market assets only in a portfolio that is supposed to combine emerging and developed stock market assets, this may not always be the case when investors are risk averse. Figure 9 illustrates the relationship between asset weights in an optimal mean-variance portfolio and the risk aversion parameter. Figure 9 below shows that high risk aversion leads to a decrease in the holding of BIST 100 assets, given that they are exposed to major risk. This justifies the choice of optimal portfolio weights by investors, as reported in the second column of Table 5. Moreover, this asserts Riley Jr & Chow, (1992) findings that risk averse investors tend to invest in less risky assets.

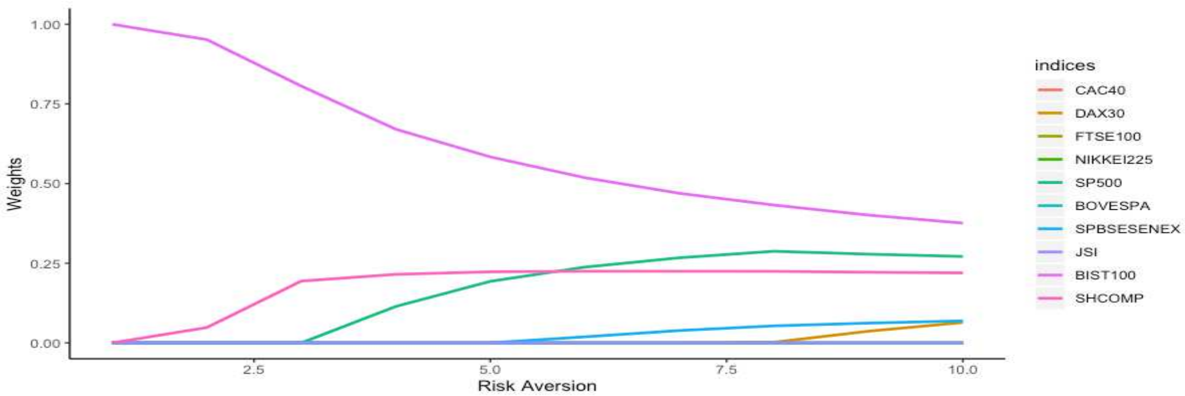


Figure 9: MV-Allocation of equity indices to different risk aversion levels

Assume the construction of a portfolio constituted by developed stock market indices only. Table 6 below describes the optimal and tangent portfolios of this portfolio. The optimal portfolio allocates more weight to the Canadian S&P/TSX, while the tangent portfolio allocates 100% to the S&P 500. Both portfolios have negative expected returns while they are exposed to inherently high risk-optimal portfolio reports $CVaR$ of. 2.33 and the tangent portfolio reports $CVaR$ of 2.9769. The Sharpe ratio based on the optimal portfolio weight is -0.0223.

Table 6: Mean-variance developed markets portfolio weights

	MEAN-VARIANCE	
Portfolio	**Optimal portfolio weights**	**Tangent portfolio weights**
Developed Markets Indices		
CAC 40	0.0000	0.0000
S&P/TSX	0.3664	0.0000
FTSE 100	0.2298	0.0000
NIKKEI 225	0.2236	0.0000
S&P 500	0.1801	1
Expected Return ($E[R]$)	-0.0195	-0.0023
Risk ($CVaR$)	2.3302	2.9769
Sharpe Ratio	**-0.0223**	**-0.0093**
Sortino Ratio(MAR=0)	**-0.0137**	**-0.0057**

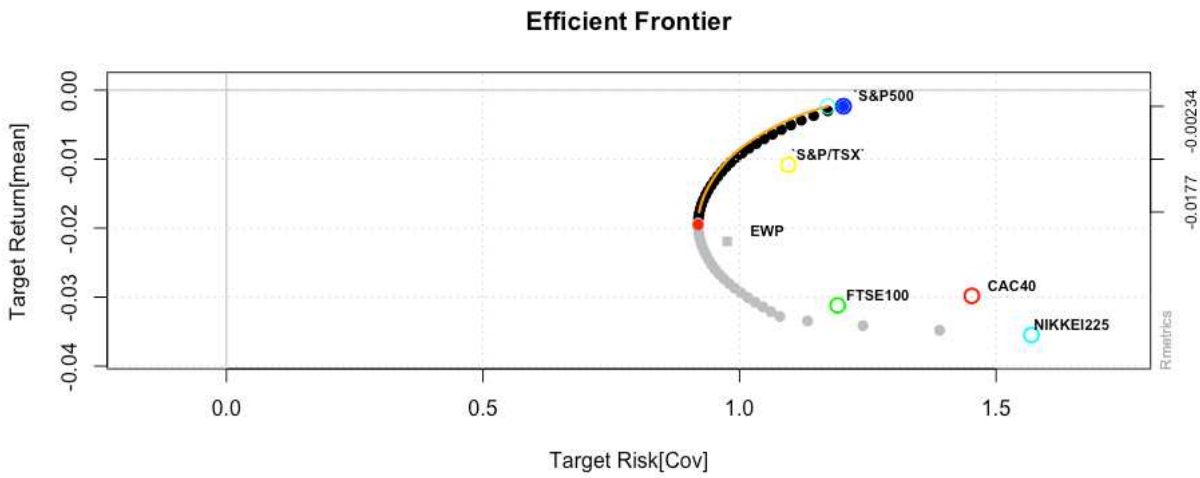


Figure 10: Efficient Portfolio Frontier for Mean-Variance developed markets portfolio

Table 7 below presents the results of the construction of a portfolio constituted by emerging stock market indices only. The optimal portfolio distribution allocates more weight to SCHOMP and the least weight to BIST 100. Contrary, the tangent portfolio allocates more all weight to BIST 100. The tangent portfolio reports a higher than expected return of 0.0224 relative to the optimal portfolio expected return of 0.0031 however, the risk of the tangent portfolio is high.

It is important to note that the Sharpe ratio for a portfolio constituted by emerging stock markets only is higher than one constituted by developed stock markets. This finding supports the fact that emerging stock markets provide the best distribution for asset managers in search of high yields (Violi & Camerini, 2016). However, given the risk aversion inclination of most investors, the norm during tranquil periods is a scenario where mixed assets are considered, with more weight allocated to developed economies.

Table 7: Mean-variance emerging markets portfolio weights.

	MEAN-VARIANCE	
Portfolio	**Optimal portfolio weights**	**Tangent portfolio weights**
Developed Markets Indices		
BOVESPA	0.1757	0.0000
SHCOMP	0.3040	0.0000
S&P BSE SENEX	0.2728	0.0000
JSI	0.2474	0.0000
BIST 100	0.0000	1
Expected Return ($E[R]$)	0.0031	0.0224
Risk ($CVaR$)	2.5398	3.8746
Sharpe Ratio	**-0.0036**	**0.0066**
Sortino Ratio(MAR=0)	**-0.0062**	**0.0091**

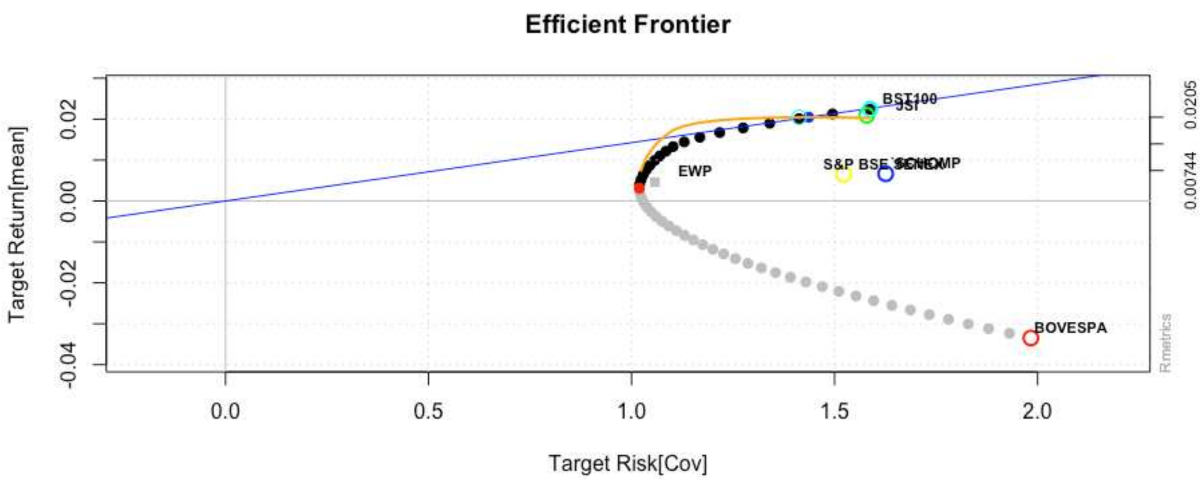


Figure 11: Efficient Portfolio Frontier for Mean-Variance emerging markets portfolio.

4.5.2 The Mean-Variance GPD portfolio

From this subsection onward the study makes use of EVT to assess the extent of efficient portfolio allocation with various portfolios-international portfolio, developed markets portfolio and emerging markets portfolio during turmoil periods. Table 8 below reports the mean-variance GPD portfolios of an international portfolio-mix of emerging and developed stock market assets, the portfolio weights to different indices, the expected return and the risk inherent to optimal and tangent portfolio respectively.

Table 8: Mean-Variance GPD international portfolio weights

	GENERALISED PARETO DISTRIBUTION	
Portfolio	**Optimal portfolio Weights**	**Tangent portfolio weights**
Developed Markets Indices		
CAC 40	0.1233	0.1311
S&P/TSX	0.0844	0.0676
FTSE 100	0.1865	0.1585
NIKKEI 225	0.0836	0.0879
S&P 500	0.1858	0.1614
Emerging Markets Indices		
BOVESPA	0.0783	0.1366
SHOMP	0.0586	0.0745
S&P BSE SENEX	0.1185	0.1243
JSI	0.0402	0.0551
BIST 100	0.0407	0.0589
Expected Return ($E[R]$)	3.2873	3.3670
Risk ($CVaR$)	-2.6836	-2.7534
Sharpe Ratio	**2.028**	**2.0773**
Sortino Ratio(MAR=0)	**3.133**	**3.2093**

The mean variance international portfolio under the GPD is reported in table 8, where the optimal portfolio is concerned, 66.3% is allocated to developed market indices, with 0.1858 allocated to the S&P 500 and 0.1865 allocated to the FTSE 100. This is parallel to the GPD S&P 500 shape parameter which implied that the distribution minimal probability of extreme losses. Similarly, the tangent portfolio allocates greater weight to developed markets, with only 44.94% allocated to emerging markets indices and only 0.0589 allocated to the BIST 100.

The expected return of the optimal portfolio is 3.15% while for the tangent portfolio it is 3.24%. According to Baily(2005) a rational investor would choose the portfolio with the higher expected return. Moreover, the risk of portfolios is inherently low which makes them more appealing. Both portfolios report positive and large Sharpe ratios and Sortino ratios, making them more appealing.

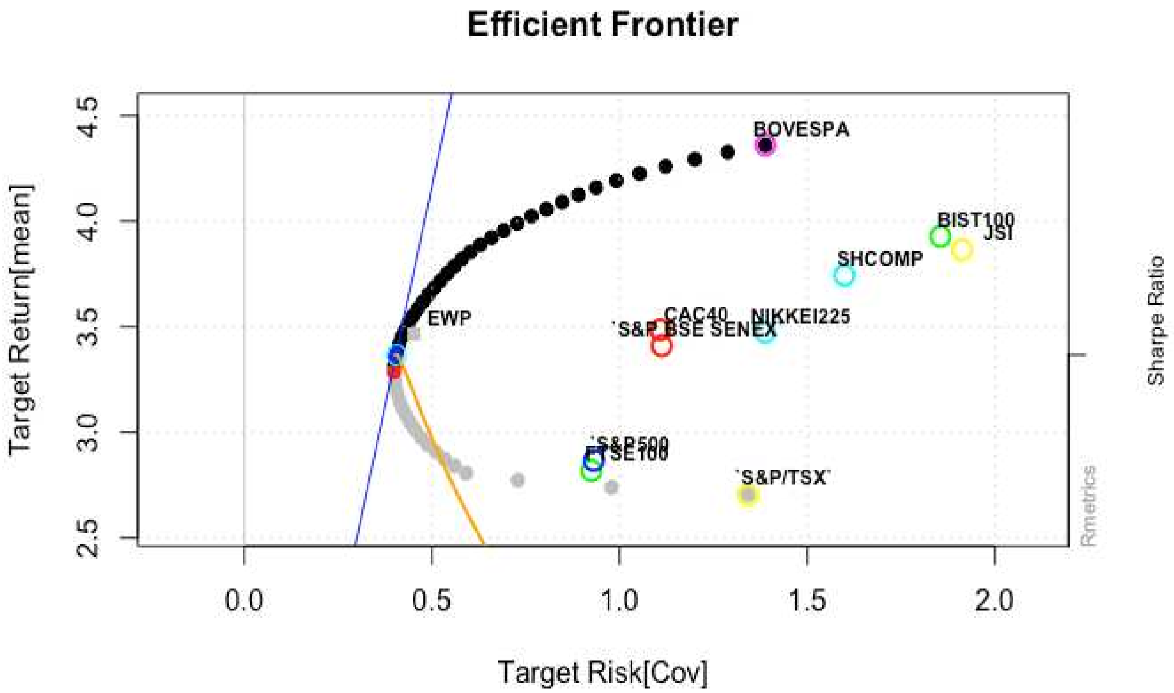


Figure 12: Efficient Portfolio Frontier for Generalised Pareto Distribution

The efficient portfolio frontier for generalised Pareto distribution is depicted above, by showing the mean-variance efficient frontier with a negatively sloped Sharpe Ratio (orange line). This implies that as targeted returns increase, the ratio of the mean return to risk decreases inversely. The equally weighted portfolio (EWP) indicates a return of 3.5, which is greater than the tangent portfolio-blue circle on efficient frontier, with a relatively low risk.

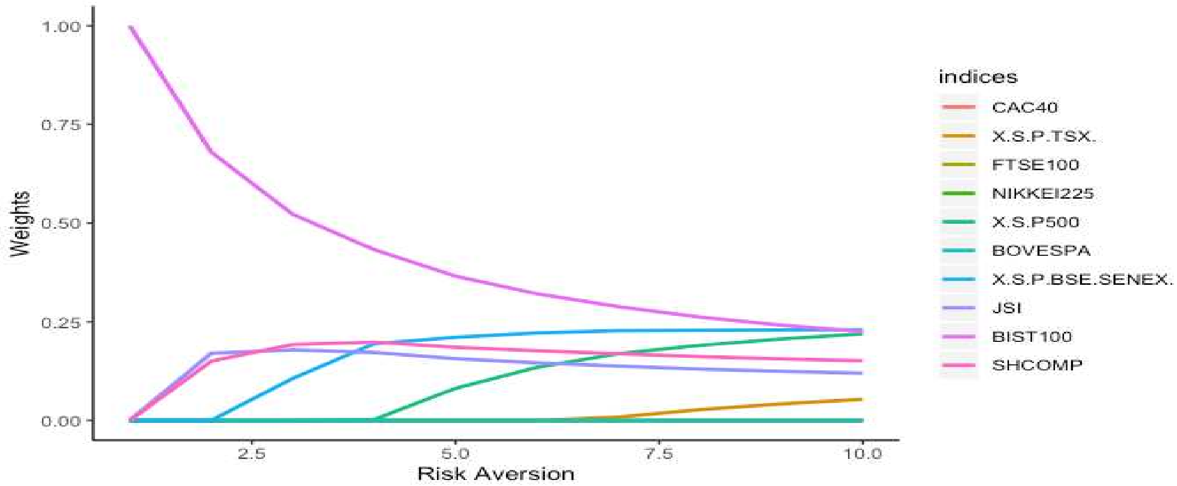


Figure 13: MV-GPD: Allocation of equity indices to different risk aversion levels

Figure 13 illustrates the relationship between asset weights in an optimally generalised Pareto distribution portfolio and the risk aversion parameter. Risk-aggressive investors only hold assets from emerging economies such as the BIST 100, JSI and SHCOMP- although they are risky they provide higher returns. All levels of risk-averse and risk-neutral investors hold a greater portion of the SHCOMP, Valukains (2013) attributes this to china being one the fastest growing emerging economy. All levels of investors disregard the FTSE 100, NIKKEI 225 and BOVESPA due to unsatisfactory returns.

Table 9 below reports the mean-variance GPD portfolios, which comprise only of developed country indices. Similarly, the FTSE 100 and S&P500 are allocated the largest portions. This is consistent with the preliminary analysis as they are low-risk indices. Uotila et al (2009) attributes investors popularity of indices to outperformance of indices over pas years.

The optimal portfolio reports risk of -2.2885 while the tangent portfolio reports expected risk of -2.3793. The risk in both portfolios is relatively low. Regarding the Sharp Ratio and Sortino Ratio, the tangent portfolio reports slightly higher ratios relative to the optimal portfolio,

Table 8: Mean-Variance developed countries portfolio weights

	MEAN-VARIANCE GPD	
Portfolio	**Optimal portfolio weights**	**Tangent portfolio weights**
Developed Markets Indices		
CAC 40	0.1873	0.2168
S&P/TSX	0.1304	0.1165
FTSE 100	0.2792	0.2596
NIKKEI 225	0.1252	0.1433
S&P 500	0.2779	0.2637
Expected Return ($E[R]$)	3.0240	3.0565
Risk ($CVaR$)	-2.3498	-2.3793
Sharpe Ratio	**2.2885**	**2.3132**
Sortino Ratio(MAR=0)	**3.3638**	**3.4001**

Figure 14: Efficient Portfolio Frontier for Generalised Pareto Distribution of developed countries portfolio

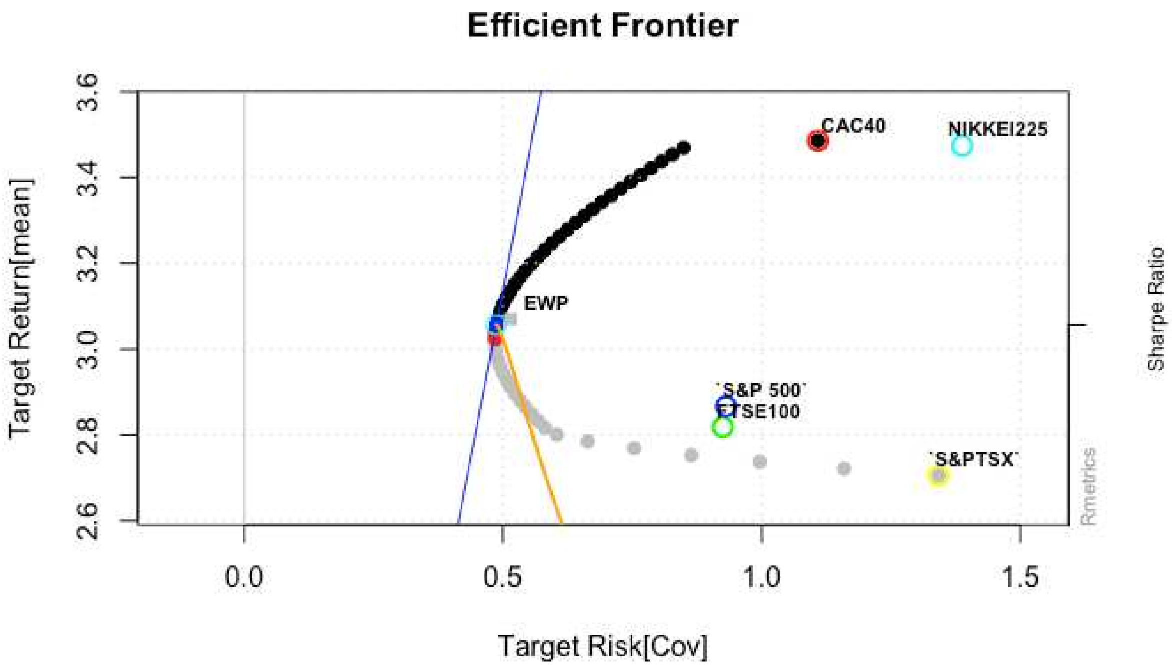


Table 9: Mean-Variance GDP of emerging countries portfolio weights

	MEAN-VARIANCE	
Portfolio	**Optimal portfolio weights**	**Tangent portfolio weights**
Developed Markets Indices		
BOVESPA	0.2275	0.2626
SHCOMP	0.1699	0.1670
S&P BSE SENEX	0.3554	0.3166
JSI	0.1186	0.1207
BIST 100	0.1286	0.1332
Expected Return ($E[R]$)	3.8037	3.8393
Risk ($CVaR$)	-2.8993	-2.9306
Sharpe Ratio	**2.3362**	**2.2942**
Sortino Ratio(MAR=0)	**3.5265**	**3.4633**

Table 10 above reports on the mean-variance GPD portfolios of emerging countries indices. The tangent portfolio is more desirable given the Sharp-ratio and Sortino ratio criteria.

Figure 15 below depicts the efficient frontier of the mean-variance GPD emerging countries portfolio. It illustrates that if an investor is willing to allow more risk, he/she should invest solely in the Turkish BIST 100 for the highest expected return

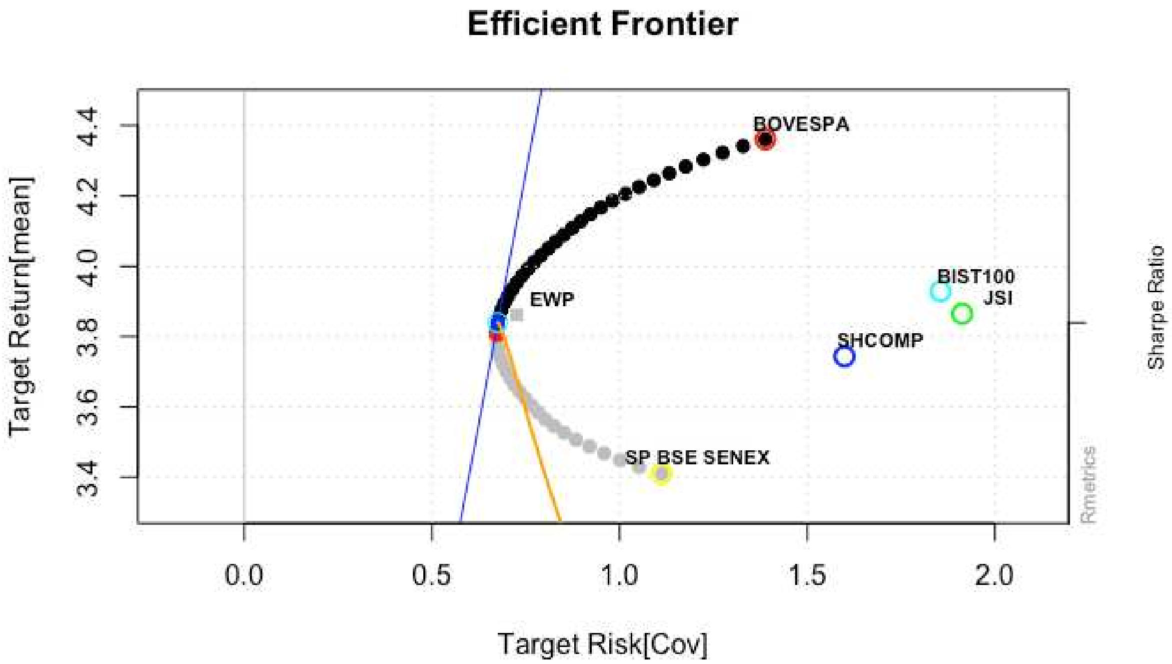


Figure 15: Efficient Portfolio Frontier for Generalised Pareto Distribution of emerging markets portfolio.

4.5.3 The Mean-Variance GEV distribution portfolio

Table 10: Mean Variance GEV international portfolio weights

	GENERALISED EXTREME VALUE DISTRIBURTION	
Portfolio	**Optimal portfolio Weights**	**Tangent portfolio weights**
Developed Markets Indices		
CAC 40	0.1472	0.1486
S&P/TSX	0.1170	0.0939
FTSE 100	0.1860	0.1536
NIKKEI 225	0.1103	0.1222
S&P 500	0.1092	0.0920
Emerging Markets Indices		
BOVESPA	0.0544	0.0784
SHCOMP	0.0498	0.0609
S&P BSE SENEX	0.0777	0.0844
JSI	0.0724	0.0803
BIST 100	0.0760	0.0857
Expected Return ($E[R]$)	2.5466	2.6201
Risk ($CVaR$)	-1.6817	-1.7357
Sharpe Ratio	**1.1569**	**1.6151**
Sortino Ratio(MAR=0)	**2.4250**	**2.4953**

Table 11 reports the mean-variance portfolios under the GEV distribution. The portfolios allocate less weightings to emerging markets indices, as they are more prone to risk. Satisfactory expected returns are reported, as the optimal portfolio reports an expected return of 2.54 and the tangent portfolio reports a 2.62 expected return with low risk. These results and those reported in Table 8, allude that in extreme conditions investors are prone to allocate more weight to developed stock markets. This shows that developed economies are safe havens for wealth protection, especially during extreme conditions (Tronzano, 2020). Figure 16 below further emphasises on findings depicts the efficient frontier of the international portfolio.

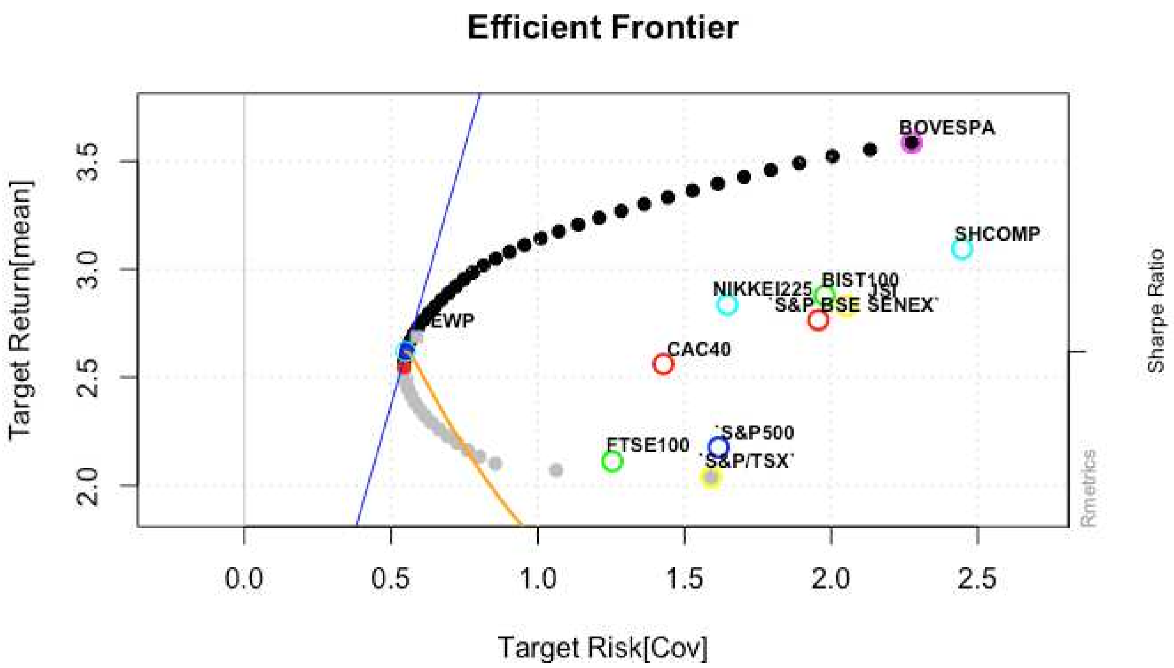


Figure 16: Efficient Portfolio Frontier for Generalised Pareto Distribution of international markets portfolio.

The Efficient portfolio frontier for generalised Pareto distribution is illustrated by Figure 16 above showing the mean-variance efficient frontier with a negatively sloped Sharpe Ratio (orange line). If the investor sets a risk target at 1, he/ she should invest only in an equally weighted portfolio or a tangent portfolio, which yield returns of 2.7 and 2.5 respectively.

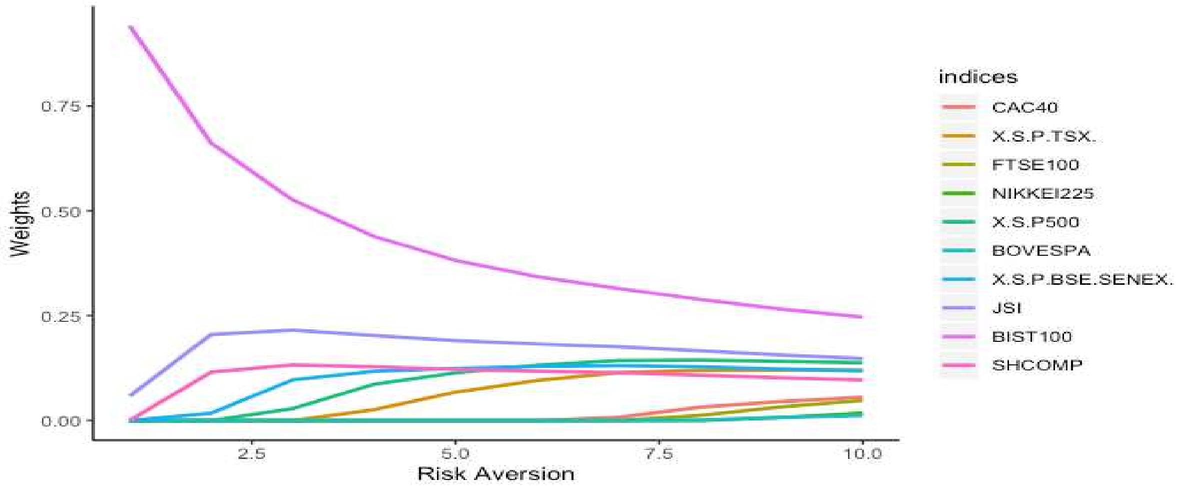


Figure 17: MV-GEV-Allocation of equity indices to different risk aversion levels

Figure 17 illustrates the relationship between asset weights in the optimal generalised extreme value distribution portfolio, and the risk aversion parameter. Risk-aggressive investors increase their holdings in S&P BSE SENEX as returns increase. The risk-neutral and risk-averse hold both developed and emerging market assets, leading to a diversified investment of their assets, as it is more secure to do so (Largoarde-Segot & Lucey, 2007).

Table 12 below details the mean-variance GEV developed markets portfolio. The FTSE 100 is allocated 28.13 of the optimal portfolio weight, and 25.63 of the tangent portfolio. With regard to the risk return ratios, both portfolios show satisfactory results

Table 11: Mean Variance GEV developed markets portfolio weights

	MEAN-VARIANCE	
Portfolio	**Optimal portfolio weights**	**Tangent portfolio weights**
Developed Markets Indices		
CAC 40	0.2165	0.2388
S&P/TSX	0.1711	0.1494
FTSE 100	0.2813	0.2563
NIKKEI 225	0.1659	0.2016
S&P 500	0.1652	0.1538
Expected Return ($E[R]$)	2.3275	2.364
Risk ($CVaR$)	-1.3341	-1.516
Sharpe Ratio	**1.759**	**1.787**
Sortino Ratio(MAR=0)	**2.585**	**2.6272**

The efficient frontier in Figure 18 below illustrates a set of optimal portfolios which an investor can undertake using the mean-variance GEV developed markets portfolio. If the investor undertakes a risk of 0.7 he/she will invest in the tangent of EWP.

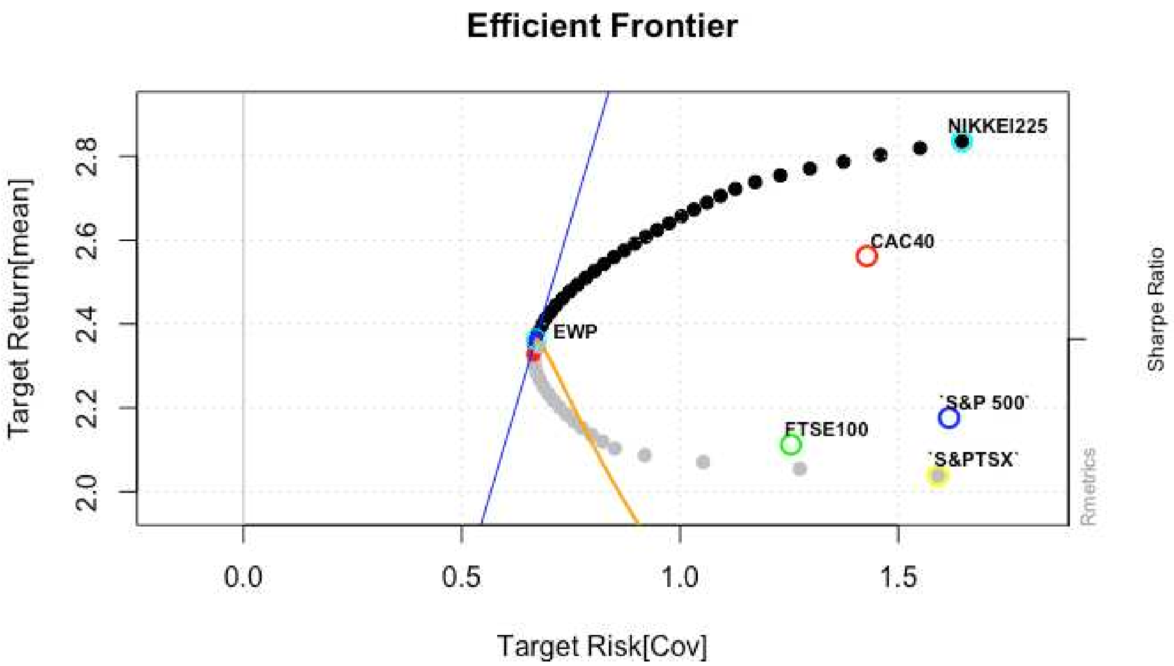


Figure 18: Efficient Portfolio Frontier for Generalised Pareto Distribution of developed markets portfolio.

Table 13 below records the mean-variance GEV emerging markets portfolio weights. The Indian S&P BSE SENEX is highly weighted in both portfolios. The tangent portfolio produces higher portfolio performance ratios, and more satisfactory results in terms of expected returns.

Table 12: Mean Variance GEV emerging markets portfolio weights

	MEAN-VARIANCE	
Portfolio	**Optimal portfolio weights**	**Tangent portfolio weights**
Developed Markets Indices		
BOVESPA	0.1748	0.2087
SHCOMP	0.1520	0.1570
S&P BSE SENEX	0.2322	0.2144
JSI	0.2133	0.2016
BIST 100	0.2276	0.2183
Expected Return ($\boldsymbol{E}[\boldsymbol{R}]$)	2.998	3.0259
Risk ($\boldsymbol{CVaR}$)	-1.641	-1.6664
Sharpe Ratio	**1.5995**	**1.6145**
Sortino Ratio(MAR=0)	**1.6695**	**1.6851**

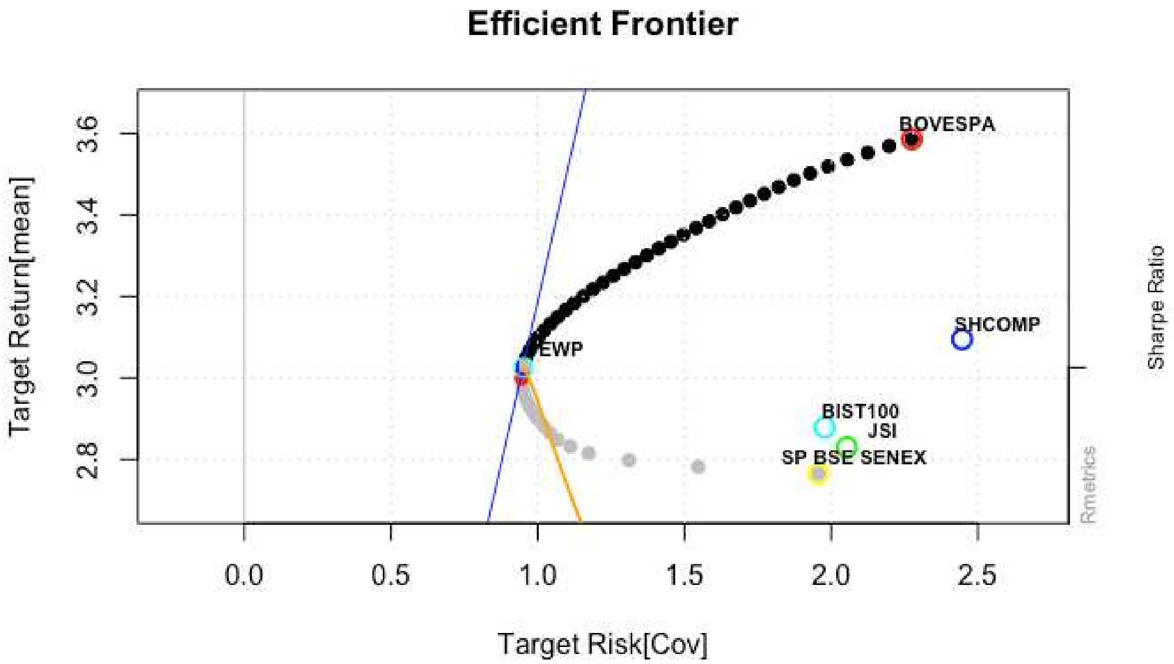


Figure 19: Efficient Portfolio Frontier for Generalised Pareto Distribution of emerging markets portfolio.

The efficient frontier above in figure 19 illustrates a set of optimal portfolios which an investor can undertake using the mean-variance GEV developed markets portfolio. If the investor undertakes a risk of 1, he/she will invest in the tangent of EWP for a return of 3.

The chapter analysed portfolio optimisation models suited for constructing a portfolio within extreme conditions –EVT GPD and GEV- using different portfolio compositions. The traditional MV asset allocation model was used as a benchmark model. The results suggest that the traditional MV model is inferior as produces portfolios with negative expected returns. Bollerslev (1987) attributes this to the method's inability to model fat-tails. The GPD method is superior as it produces favourable performance measure ratios.

5: CONCLUSION

This study aimed to compare the performance of the two EVT methods, namely the GPD and the GEV methods, for portfolio optimisation and for efficient portfolio allocation when considering the combination of developed and emerging stock market assets. The Sharpe ratio was used as the performance measure, based on mean-variance portfolio selection. Moreover, the traditional Markowitz mean-variance portfolio selection method was used as a benchmark for portfolio selection and to reflect the extent of portfolio allocation during turmoil periods. The data used were made up of ten equity indices from five developed markets, and five emerging markets.

The preliminary analysis and descriptive statistics illustrated and reported that equity prices of all markets dropped drastically during the economic and financial crises of 1998-1999 and 2008-2009. When employing the EVT, the study filtered returns using an ARMA-GJR-GARCH process to remove autocorrelation and heteroscedasticity. The results of the EVT for each of the markets reported that conditional variance and shape parameters exhibits a leverage effect in equity returns. These findings were confirmed by the News Impact Curves.

The portfolio selection results are similar to Zhang, Lu, Lu & Chen (2019) who found a risk managed strategy has superior results for portfolio construction as it reduces volatility while enhancing risk-reward ratio. The results show that the international portfolios are most suitable for investors to diversify risk during turmoil periods in every asset allocation method. Moreover when comparing method the EVT GPD method is superior as the international portfolio reports a sharp ratio of 2.028 and 2.0773 for optimal and tangent portfolio respectively. In addition the Sortino ratio of the optimal and tangent portfolio are reported as 3.133 and 3.2093 respectively. Findings are different from Mwamba et al (2017) who find GEV superior.

This study is important for both academics and asset managers. The findings provide support for investors' perception of risk in portfolio allocation where the stock market is concerned. Therefore, the is a need to discard traditional methods of asset allocation such as the Markowitz(1852) model and mean-variance high-order moment portfolio optimisation methods in portfolio allocation during turmoil periods as they could be misleading. For instance, the work of Briec, Keretens and Jokung (2007) who simply ass skewness in their mean-variance function show that risk was underestimated This study further demonstrated that modelling systematic tail risk is important.

In future research could investigate the extent to which dependence of stock markets are affected during times of turbulences or turmoil, given economies are becoming more integrated to each other.

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