# Robust Tests of the Lower Partial Moment Asset Pricing Model in Emerging Markets

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Abstract

This paper tests and compares the CAPM of Black (1972) and the Mean Lower Partial Moment (MLPM) Capital Asset Pricing Model of Bawa and Lindenberg (1977) and Harlow and Rao (1989) in the context of emerging markets. It is well known that returns in emerging markets are non-normal and have greater predictability than in the developed markets. Considering these stylized facts the paper extends the Harlow-Rao Likelihood Ratio test of a Black (1972) type version of the MLPM model and develops a Wald test that allow for non-normality of the returns. The paper also formulates a GMM test that is valid under the conditions of heteroskedasticity and serial dependence. For the test of the CAPM hypothesis against the MLPM alternative the paper remedies an econometric problem of testing in presence of a nuisance parameter. In the empirical application on an emerging market data it is shown that the conclusion on the validity of the asset pricing model are reversed when the correct p-values obtained through the bootstrap test are employed. We demonstrate that the empirical results appear to support both the Black version of the CAPM and the MLPM model when performed against unspecified alternative but the CAPM is supported when an MLPM alternative is specified.



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## I. Introduction

The capital asset pricing model of Sharp-Lintner-Black is the consequence of the Markowitz (1959) portfolio optimization theory which is based on the expected return and variance of the return distribution and assumes the multivariate normality of the joint return distribution of the underlying assets. The theory considers beta as the sole measure of the systematic risk in a diversified portfolio. The beta is estimated via a market model that assumes that the estimated beta is valid for all the market conditions. A plethora of empirical tests have been performed which implicitly assume the mean-variance based preference of the investors. Statistical tractability of mean-variance analysis based on multivariate normality was a more important consideration in the development of the theory than the explicit recognition of real world investor's preferences. The theory considers the volatility as undesirable by investors. In its treatment deviation both below and above the mean are penalized. It is intuitive that the variance above mean is not considered undesirable by the investors. Beginning primarily from the last quarter of the twentieth century alternative theories based on the better risk perception of the investors have challenged the dominance of the mean-variance theory. The most prominent of these is the asset pricing theory which is based on recognizing risk as the deviation below a critical target rate of return. These downside risk measures and associated asset pricing models are motivated both by the economic and statistical considerations-the investors psychology is consistent with asymmetric treatment of the variations in the returns; the empirical return distribution also appears to be non-normal.

Bawa (1975) advocates a mean lower partial moment as the appropriate risk measure and proves that the MLPM model will produce portfolios that will dominate all other portfolios according to the concept of stochastic dominance. Moreover the MLPM analysis is justified for a quite general set of utility function covering the quadratic utility as a special case. Bawa and Lindenberg (1977) developed an asset pricing model- the mean-lower partial moment model based on downside risk. For normal and Student-t-distributions of returns the LPM based model reduces to the conventional CAPM. They argue that their model must explain the data at least as well as the CAPM. In the MLPM model the lower partial moment based downside beta measure replaces the CAPM beta. In the Bawa-Lindenberg model the risk is defined as the deviation below the risk free rate. Harlow and Rao (1989) developed an asset pricing model that is more general in that the risk can be defined as the deviation below an arbitrary target rate. They demonstrate that because of this generality several risk measures can be expressed as a special case of their lower partial moment beta with an arbitrary target arte. The resulting asset pricing model is expressed as:

$$E(R_i) = R_f + \beta_i^{MLPM(\tau)}[E(R_m) - R_f]$$

Here the downside beta for an asset i with an arbitrary target rate $\tau$, the market return $R_m$ and the risk free rate $R_f$ is defined as

$$\beta_i^{MLPM} = \frac{E(R_i - R_f)\min(R_m - \tau, 0)}{E[\min(R_m - \tau, 0)]^2}$$

In this model a particular asset contributes to the risk only if its return and the market return are below the target rate $\tau$.

The non-normality of the asset return distribution is also well studied. As Kan and Zhou (2006) remark “the reason for the wide use of the normality assumption is not because it models financial data well, but due to its tractability that allows interesting economic questions to be asked and answered without substantial technical impediments”. Fama (1965), Kon (1984), Affleck-Graves and McDonald (1989), Richardson and Smith

(1993), and Dufour, Khalaf and Beaulieu (2003), among others provide strong evidence against normality of stock returns.

Asset pricing in emerging markets is of particular interest for this paper. It can be argued that if the mean-variance asset pricing appears to be incompatible with the theory and evidence in the developed markets it should be so in emerging markets to an even greater extent. With identical regulatory environment and taxes the extra utility of a dollar gain for the developed market investor who has higher initial wealth is lower compared to an emerging market investor with lower wealth endowment. Conversely the disutility of a dollar loss in investment is higher for the emerging market investor with lower initial wealth compared to developed market investors. Thus downside risk measures should better reflect the risk aversion in emerging markets. In addition with lower liquidity, infrequent trading and volatile political and macroeconomic conditions, the assumptions underlying smooth and symmetric behaviours of the security return is unlikely to be the case in emerging markets. These are precisely the characteristics of emerging markets. It is therefore a challenging task to model the investment risk in emerging equity markets and establish the risk return relationship.

The empirical evidence also appears to suggest inapplicability of the mean-variance based asset pricing model and the normality of the stock returns in emerging markets. Bekaert et al (1998) show that emerging market equities display significant skewness and kurtosis in their returns, while Bekaert and Harvey (1995, 1997) find that the degree of skewness and kurtosis changes over time. Eftekhari and Satchell (1996) and Claessens et al. (1995) also provide evidence of non-normality in emerging markets. Hwang and Pedersen (2004) linked the applicability of the CAPM and asymmetric pricing models with regional and timing effects in emerging markets and found that as the market mature over time the returns tend to be more normal. Harvey (2001) and Bekaert et al. (1998) among others suggest that empirical relationships between risk and stock returns in emerging markets are not appropriately described by the CAPM. Harvey (2000) and Estrada (2000, 2002) tested various equilibrium and non-equilibrium based risk measures and suggested that downside risk measures such as semi-standard deviation are relevant measures of risk for emerging market equity indices. Estrada (2000) argues that the costs of equity based on downside risk are consistent with partially-integrated emerging markets. Despite the intuitive appeal of the downside risk measures, there has been limited consideration in the empirical applicability of the downside models. For example Pedersen and Hwang (2003) conclude that although LPM-CAPM explains equity returns better than the conventional CAPM, the proportion of equities benefiting from using the downside beta is not large enough to improve asset pricing models significantly except for some smaller stocks.

This paper provides an empirical study of the Black-CAPM and Harlow and Rao (1989) MLPM asset pricing model using both the univariate Fama and Macbeth (1973) type of cross section methodology and the multivariate methodology actually employed by Harlow and Rao. Despite the sound theoretical foundation of Harlow-Rao model there has been no application of their asset pricing test in emerging markets. A possible reason is because Harlow and Row employed a Likelihood Ratio test in their study. This test is based on the normality, homoskedasticity and serial independence assumption of the returns of the asset and the residuals. The validity of these assumptions is questionable in emerging markets. The empirical evidence suggests that emerging markets returns may be predictable and non-normal. Harvey (1995) and Salomons and Grootveld (2003) among others provide empirical evidence that the emerging market returns show greater evidence of predictability than in the developed markets. If only non-normality is evident in the data then the Likelihood Ratio test can be replaced by a Wald test. The Likelihood Ratio test, the Lagrange Multiplier test and the Wald test are generally applied for testing non linear parametric hypothesis. Among the three asymptotic tests only the Wald test is robust to non-normality. Computation of this test requires only the unrestricted parameter estimates for which least squares estimates can be employed. If in addition the heteroskedasticity and serial dependence of the residuals are also of concern then the tests based the Generalized Method of Moments can be considered. The GMM based tests do not require strong distributional assumption regarding normality, heteroskedasticity and serial independence of the residuals. Therefore in addition to the Likelihood Ratio test we formulate the robust Wald and the GMM test for the Harlow-Rao MLPM model. The GMM test is based on specifying a set of orthogonality conditions between the residuals and the fragmented market portfolios return. The Black-CAPM and the Harlow-Rao asset pricing framework particularly suit an emerging market environment in that the empirical tests for this model do not require specification of a risk free rate. This is an advantage in empirical studies in emerging markets where the imperfect money markets make a suitable risk free rate difficult to obtain.

Harlow and Rao also conducted multivariate tests of the CAPM restriction against the MLPM alternative. Under the null hypothesis of the CAPM the beta on up and down markets are equal. In this case the critical parameter of their model i.e. the target rate becomes unidentified while under the alternative of the MLPM it continues to play its role. This is a non-standard hypothesis testing problem and the asymptotic Chi Square p-values are no longer valid in this case. The Harlow and Rao study does not address this issue and continued to use the asymptotic Chi Square p-values. The tests for the non standard problems are quite well studied in econometric literature for example Hansen (1996, 1997) discussed the testing problem and provide a bootstrap method to test the hypothesis of linearity in a threshold autoregressive model. Andrews and Ploberger (1994) developed tests for structural break for unknown change point in time series. Garcia (1998) developed a Likelihood Ratio test for testing linearity against a Markov switching alternative. Keeping the testing problem in mind this paper uses a Supremum Likelihood Ratio test and a robust Supremum Wald test with the p-values generated from the bootstrap methodology.

The tests are applied to Karachi stock market which is the largest stock exchange in Pakistan[1]. This market has received considerable attention in recent years when in 2002 it was declared the best performing stock market in the world in terms of the percent increase in the local market index value. The trading activity is also quite high therefore it will be interesting to study which, if any, of the alternative risk measures and the associated asset pricing model track the return behaviour in this emerging market. For greater details on the institutional features and some preliminary analysis for the market, see Iqbal and Brooks (2006b, 2007). The framework of the analysis for this paper, however, is quite general and can be applicable for asset pricing tests in emerging markets in general. Following this introduction the plan of the paper is as follows: Section II discusses multivariate tests of the CAPM and MLPM Model. In Section III robust tests for MLPM are discussed. The data and residual diagnostic tests are considered in section IV. Section V discusses empirical results and the conclusion is provided in section VI.

## II. Multivariate Tests of CAPM and MLPM Model

### A. Large Sample Tests of the Black's CAPM

Assume that the return generating process is the familiar market model

(1) $$R_t = \alpha + \beta R_{mt} + \varepsilon_t, \quad t = 1,2,\ldots,T$$

[1] Karachi Stock Exchange is the largest of the three stock markets in Pakistan. On April 17, 2006 the market capitalization was US$ 57 billion which is 46 percent of Pakistan's GDP for the Fiscal Year 2005-06. (Ref: Pakistan Economic Survey 2005-06)

Here $R_t = [R_{1t}\ R_{2t} \ldots R_{Nt}]'$ is the $N \times 1$ vector of raw returns on N portfolios, $\varepsilon_t$ is the $N \times 1$ vector of disturbances, $\alpha$ and $\beta$ are $N \times 1$ vector of the intercept and slope parameters respectively for each of the N market model time series regressions. The Black-CAPM specifies the following cross sectional relation.

(2) $$E(R_t) - \gamma 1_N = \beta(E(R_{mt}) - \gamma)$$

Here $\gamma$ is the parameter representing returns on a zero-beta portfolio. Applying expectations on (1) yields

(3) $$E(R_t) = \gamma(1 - \beta\ ) + \beta E(R_{mt})$$

The joint restrictions on the parameter imposed by the CAPM are expressed in the following non linear hypothesis.

(4) $$H_0 : \alpha_i = \gamma(1 - \beta_i),\ \ i = 1, 2, \ldots N$$

This is essentially a non-linear restriction on the system of the market model equations. Gibbons (1982) provides an iterative estimation and testing of a Likelihood Ratio test of the null hypothesis where

$$LR = T(\log|\hat{\Sigma}^*| - \log|\hat{\Sigma}|) \xrightarrow{d} \chi^2_{N-1}$$

Here $\hat{\Sigma}^*$ and $\hat{\Sigma}$ are the estimated restricted and unrestricted covariance matrices of the system of the market model (1) respectively. The test is derived under the assumption of multivariate normality of the returns. Chou (2000) developed a Wald test that permits the model to be estimated entirely in terms of alphas and betas by expressing the hypothesis as

$$H_0 : \frac{\alpha_i}{1 - \beta_i} = \gamma, \quad i = 1, 2, \ldots N$$

The resulting Wald test is

(5) $$W = g(\hat{\theta})' \left[ \left( \frac{\partial g}{\partial \theta'} \Big|_{\theta = \hat{\theta}} \right) \hat{\Sigma} \otimes (X'X)^{-1} \left( \frac{\partial g}{\partial \theta'} \Big|_{\theta = \hat{\theta}} \right)' \right]^{-1} g(\hat{\theta}) \xrightarrow{d} \chi^2_{N-1}$$

Here, $g(\theta) = [g_1 . \; . \; . g_{N-1}]^{/}$, $g_i = \frac{\alpha_i}{1-\beta_i} - \frac{\alpha_{i+1}}{1-\beta_{i+1}}$, $i = 1, 2, \ldots . N-1$,

$\theta = [\alpha_1 \quad \beta_1 \; . \; . \; . \quad \alpha_N \quad \beta_N]^{/}$ and X is the $T \times 2$ design matrix with a column of 1's and a column of return of the market portfolio. The partial derivatives $\frac{\partial g}{\partial \theta'}$ are evaluated at the OLS estimates from the unrestricted system. For extension to the GMM case see Chou (2000). Prompted by return predictability in emerging markets, Iqbal and Brooks (2006a) present a version of the GMM test that is more general and allows for the dynamics of the residuals dependence in addition to heteroskedasticity. Further, Iqbal and Brooks (2006a) provide an alternative formulation of the Wald test that is designed to achieve better small sample properties.

## B. Tests of MLPM Model

The market model (1) assumes that beta is valid for all market conditions. An alternative is to allow asymmetry of systematic risk; downside and upside deviations. The downside risk is measured as the deviation below a target rate $\tau$. To investigate asymmetry in systematic risk Bawa, Brown and Klein (1981) developed a data generating process called the asymmetric response model (ARM) which is expressed as,

(6) $$R_{it} = \alpha_i + \beta_i^- R^-{}_{mt} + \beta_i^+ R^+{}_{mt} + \delta_i (1 - D_t) + \varepsilon_{it}$$

where $$R^-{}_{mt} = \begin{cases} R_{mt} & if \;\; R_{mt} < \tau \\ 0 & otherwise \end{cases}$$

$$R^+{}_{mt} = \begin{cases} R_{mt} & if \;\; R_{mt} > \tau \\ 0 & otherwise \end{cases}$$

$$D_t = \begin{cases} 1 & if \;\; R_{mt} < \tau \\ 0 & otherwise \end{cases}$$

This model by construction creates a distinction between downside and upside movement in the market. The downside beta $\beta_i^-$ captures the co-movement of asset i with the market when the market return falls below the target rate of return, $\beta_i^+$ measures the co-movement of the asset with the market when the market return is above the target rate. Following Harlow and Rao (1989) we assume that $\delta_i = \varphi(\beta_i^- - \beta_i^+)$ where $\varphi = E(R^+{}_{mt}) / P(R_{mt} > \tau) = E(R_{mt} \mid R_{mt} > \tau)$. The ARM has been employed by Pederson and Hwang (2004) and Eftekari and Satchel (1996) among others for estimating downside beta in emerging markets. The ARM estimation is facilitated if expressed in a slightly different form which requires only one new variable $D_t$ to be created.

(7) $$R_{it} = \alpha_i + \beta_i^- D_t R_{mt} + \beta_i^+ (1 - D_t) R_{mt} + \delta_i (1 - D_t) + \varepsilon_{it}$$

Harlow and Rao (1989) show that $\beta_i^-$ is indeed the mean lower partial moment beta[2] and derived the following Gibbon (1982) type of restriction for testing a Black's version of the lower partial moment model.

(8) $$H_0 : \alpha_i = \gamma(1 - \beta_i^-)$$

They tested the restriction as a Bartlett factor corrected Likelihood Ratio test (eq 9) against an unspecified alternative assuming multivariate normality of the returns.

(9) $$LR = (T - N/2 - 5/2)(\log |\hat{\Sigma}^*| - \log |\hat{\Sigma}|) \xrightarrow{d} \chi^2_{N-1}$$

Here $\hat{\Sigma}^*$ and $\hat{\Sigma}$ are the estimated restricted and unrestricted covariance matrices of the system of ARM (6) respectively.

[2] Taking expectation on both sides of (6)

$$E(R_i) = \alpha_i + \beta_i^- E(R_m^-) + \beta_i^+ E(R_m^+) + \varphi(\beta_i^- - \beta_i^+) P(Rm > \tau)$$

substituting $\varphi = E(R^+{}_{mt}) / P(R_{mt} > \tau)$ and using the fact that $R^+ + R_m^- = R_m$ follows:

$$E(R_i) = \alpha_i + \beta_i^- E(R_m)$$

## III. Robust Testing of MLPM Model

### A. The test of MLPM Model

The Likelihood Ratio test employed by Harlow and Rao (1989) assumes that the returns are multivariate normal and the model disturbances are homoskedastic and serially independent. The validity of these assumptions is questionable in emerging markets. The empirical evidence suggests that emerging markets returns may be predictable and non-normal. Assuming that the model disturbances are i*id* a normality-robust Wald test similar to that in equation 5 can be established for the MLPM model with the following $4N \times 1$ vector of parameters and N-1 vector of restrictions.

(10) $$\theta = [\alpha_1 \;\; \beta_1^- \;\; \beta_1^+ \;\; \delta_1 \;\; \alpha_2 \;\; \beta_2^- \;\; \beta_2^+ \;\; \delta_2 \ldots \alpha_N \;\; \beta_N^- \;\; \beta_N^+ \;\; \delta_N]'$$

$$g_i = \frac{\alpha_i}{1-\beta_i^-} - \frac{\alpha_{i+1}}{1-\beta_{i+1}^-}, \; i = 1, 2, \ldots N-1$$

The $T \times 4$ design matrix X in this case is

$$X = [1 \quad D \circ R_m \quad (1-D) \circ R_m \quad (1-D)]'$$

The partial derivatives $\frac{\partial g}{\partial \theta'}$ (equation 12) are evaluated at the Seemingly Unrelated Regression (SUR) estimates from the unrestricted system. A more robust GMM based test that is valid under general return distribution, heteroskedasticity and serial dependence of the residuals can be established. If N assets and T time series observations on each asset are available the moment conditions vector on the disturbance of system (7) can be defined as

$$f_T(\theta) = \begin{bmatrix} \varepsilon_{1t}(\alpha_1, \beta_1^-, \beta_1^+, \delta_1) \\ \varepsilon_{1t}(\alpha_1, \beta_1^-, \beta_1^+, \delta_1)(D_t \circ R_{mt}) \\ \varepsilon_{1t}(\alpha_1, \beta_1^-, \beta_1^+, \delta_1)((1-D_t) \circ R_{mt}) \\ \varepsilon_{1t}(\alpha_1, \beta_1^-, \beta_1^+, \delta_1)(1-D_t) \\ . \\ . \\ . \\ \varepsilon_{Nt}(\alpha_N, \beta_N^-, \beta_N^+, \delta_N) \\ \varepsilon_{Nt}(\alpha_N, \beta_N^-, \beta_N^+, \delta_N)(D_t \circ R_{mt}) \\ \varepsilon_{Nt}(\alpha_N, \beta_N^-, \beta_N^+, \delta_N)((1-D)_t \circ R_{mt}) \\ \varepsilon_{Nt}(\alpha_N, \beta_N^-, \beta_N^+, \delta_N)(1-D_t) \end{bmatrix} = \varepsilon_t(\theta) \otimes x_t \ ,$$

The sample moment conditions are defined as $h_T(\theta) = \frac{1}{T}\sum_{t=1}^{T} \varepsilon_t(\theta) \otimes x_t$

Here $x_t = [1 \quad D_t \circ R_{mt} \quad (1-D_t) \circ R_{mt} \quad (1-D_t)]'$, $\varepsilon_t(\theta) = [\varepsilon_{1t} \ \varepsilon_{2t} \ \ldots \varepsilon_{Nt}]'$

and $\varepsilon_{it} = R_{it} - \alpha_i - \beta_i^- D_t R_{mt} - \beta_i^+ (1-D_t) R_{mt} - \delta_i (1-D_t)$. The notation '$\circ$' represents the element wise product of the vectors. The parameter vector is as in (10). There are 4N moment conditions and 4N parameters to be estimated therefore the multivariate system of equations is exactly identified. The GMM[3] parameters are estimated by minimizing a quadratic form of the sample moment restriction vector;

$$\hat{\theta}_{GMM} = \arg\min \quad h_T(\theta)' \, U_T \ h_T(\theta)$$

Here $U_T$ is a positive definite weighting matrix whose elements can be functions of parameters and data. Hansen (1982) shows that the optimal weighting matrix is

$$U_T = S^{-1} = \{Asy\ Var[\sqrt{T}\ h_T(\theta)]\}^{-1}$$

The asymptotic covariance matrix of the GMM estimator is

[3] The just identified system therefore leads to a simple method of moment estimator rather than a generalized method of moment estimator. We continue to use the term GMM following similar treatment of this case in literature.

$$V = [\Delta' S^{-1} \Delta]^{-1}$$

Where $\Delta = P\lim[\frac{\partial}{\partial\theta'} h_T(\theta)]$. In practice '$S$' and '$\Delta$' are unknown but the asymptotic results are valid for some consistent estimator '$S_T$' and '$\Delta_T$'. Following Mackinlay and Richardson (1991) portfolio efficiency testing case the MLPM hypothesis for this exactly identified case can be tested by first estimating the unrestricted system and then computing the test statistic of market efficiency hypothesis which involve these *unrestricted* estimates. In this case the GMM estimator is independent of the weighting matrix and is the same as the SUR estimator; however the covariance matrix must be adjusted to allow for heteroskedasticity and serial correlation. The GMM estimates are asymptotically normally distributed

$$\sqrt{T}\,(\theta - \hat{\theta}) \;\sim\; N(0, V\,)$$

Here V is as defined above. Therefore non-linear function g($\hat{\theta}$) of the parameter is also asymptotically normal

$$\sqrt{T}\,[g(\theta) - g(\hat{\theta})] \;\sim\; N[0, (\frac{\partial g}{\partial\theta'})\, V\, (\frac{\partial g}{\partial\theta'})']$$

In the test of the MLPM model with $g(\theta) = [g_1 . \; . \; . g_{N-1}]'$ where

$$g_i = \frac{\alpha_i}{1-\beta_i^-} - \frac{\alpha_{i+1}}{1-\beta_{i+1}^{-}{}_i},\; i = 1,2,\ldots.N-1$$

The GMM Wald test of the MLPM restrictions can be formulated as

(11) $$W = g(\hat{\theta})'\left[\left(\frac{\partial g}{\partial\theta'}\Big|_{\theta=\hat{\theta}}\right)\hat{V}_T\left(\frac{\partial g}{\partial\theta'}\Big|_{\theta=\hat{\theta}}\right)'\right]^{-1} g(\hat{\theta}) \quad \xrightarrow{d} \chi^2{}_{N-1}$$

Here (N-1)$\times$4N derivative matrix is

(12) $\frac{\partial g}{\partial\theta'} =$

$$
\begin{bmatrix}
\frac{1}{1-\bar{\beta_1}} & \frac{\alpha 1}{(1-\bar{\beta_1})^2} & 0 & 0 & \frac{-1}{1-\bar{\beta_2}} & \frac{-\alpha_2}{(1-\bar{\beta_2})^2} & 0 & 0 & 0 & 0 & . & . & . & . & 0 \\
0 & 0 & 0 & 0 & \frac{1}{1-\bar{\beta_2}} & \frac{\alpha_2}{(1-\bar{\beta_2})^2} & 0 & 0 & \frac{-1}{1-\bar{\beta_3}} & \frac{-\alpha_3}{(1-\bar{\beta_3})^2} & 0 & . & . & . & 0 \\
. & & & & & & & & & & & & & & \\
& . & & & & . & & & & . & & & & & \\
& & . & & & & . & & & . & & & & & \\
0 & 0 & 0 & . & . & . & & & 0 & \frac{1}{1-\bar{\beta_N}} & \frac{\alpha_{N-1}}{1-\bar{\beta_{N-1}}} & 0 & 0 & \frac{-1}{1-\bar{\beta_N}} & \frac{-\alpha_N}{(1-\bar{\beta_N})^2}
\end{bmatrix}
$$

The covariance matrix in this case is $V_T = [\Delta_T ' S_T^{-1} \Delta_T]^{-1}$. We estimate $\Delta_t$ and $S_t$ matrices as follows.

$$\Delta_T = \frac{1}{T}\sum_{t=1}^{T} I_N \otimes x_t x_t ' = I_N \otimes X' X$$

The matrix '$S_T$' is estimated by the Newey-West (1987) HAC covariance matrix, for details see Ray et al (1998).

$$S_T = \frac{1}{T}\sum_{t=1}^{T} \hat{\eta}_t \hat{\eta}'_t + \sum_{v=1}^{p} (1-\frac{v}{1+p})\ \frac{1}{T}\sum_{t=1}^{T} [\hat{\eta}_t \hat{\eta}'_{t-v} + \hat{\eta}_{t-v} \hat{\eta}'_t\ ]$$

Here $\eta_t = e_t \otimes x_t$ so that $\eta_t \eta_t = \varepsilon_t \varepsilon_t ' \otimes x_t x_t '$. The lag length 'p' in the auto-covariance matrices $(1/T)\sum \eta_t \eta'_{t-v}$ and $(1/T)\sum \eta_{t-v} \eta'_t$ can be specified by considering the time period beyond which we are willing to assume that the correlations between $\eta_t$ and $\eta_{t-v}$ are essentially zero. The data dependent Newey-West fixed bandwidth $p = \text{int}[4(\frac{T}{100})^{2/9}]$, can be employed where int[] denotes the integer part of the number in bracket. The auto-covariance matrices can be computed from the SUR residuals of the asymmetric response model (7)[4]. For example for the lag 1 we have

[4] The actual computations can be performed using commercially available softwares. The computations here are performed using Eviews with built-in Newey-West HAC covariance matrix.

$$\eta_t \eta'_{t-1} = \begin{bmatrix} \hat{\varepsilon}_{t,1}\hat{\varepsilon}_{t-1,1} & \hat{\varepsilon}_{t,1}\hat{\varepsilon}_{t-1,2} & 0 & 0 & . & . & . & . & . & 0 \\ \hat{\varepsilon}_{t,2}\hat{\varepsilon}_{t-1,1} & \hat{\varepsilon}_{t,2}\hat{\varepsilon}_{t-1,2} & \hat{\varepsilon}_{t,2}\hat{\varepsilon}_{t-1,3} & 0 & . & . & . & . & . & 0 \\ . & . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & . & . \\ 0 & 0 & 0 & 0 & . & . & . & . & \hat{\varepsilon}_{t,N}\hat{\varepsilon}_{t-1,N-1} & \hat{\varepsilon}_{t,N}\hat{\varepsilon}_{t-1,N} \end{bmatrix} \otimes x_t x'_{t-1}$$

Where $\hat{\varepsilon}_{t,i}$ is the SUR residual from the *ith* equation at time t. Note that in portfolio efficiency applications in Mackinlay and Richardson (1991) and Chou (2000) the covariance matrix '$S_T$' employed is the White (1980) covariance matrix. In other words these authors assume that the disturbances are heteroskedastic but serially independent. Considering the return predicability evidence from the emerging markets it is better to use the general robust covariance matrix. It can be noted that because of the nature of the explanatory variables in the asymmetric response model the parameter estimates may have high variances which can even results in a near singular covariance matrix making the estimates of the system difficult. It is therefore beneficial to use instruments for explanatory variables for precise estimation. In present case the estimation can be carried out with the instrument vector $Z_t = [1 \quad R_m \quad R_m{}^2 \quad D_t]$.

## B. The Test of CAPM against MLPM Alternative

The asymmetric response model subject to the MLPM restriction is

$$R_{it} = \gamma(1-\beta_i^-) + \beta_i^- R^-{}_{mt} + \beta_i^+ R^+{}_{mt} + \delta_i(1-D_t) + \varepsilon_{it} \tag{12}$$

The CAPM can be deduced from this model with the restrictions $\beta_i^- = \beta_i^+$ and $\delta_i = 0$ imposed[5]. Harlow and Rao (1989) test these restrictions as a Likelihood Ratio test with

[5] These restrictions reduce the asymmetric response model to the market model.

asymptotic Chi Square critical values[6]. The restricted model is the CAPM whereas the unrestricted model is the MLPM. They strongly reject the null of CAPM against the MLPM alternative. It is however evident that testing the null hypothesis of CAPM in this case is conditional on a specified target rate parameter $\tau$ which is not identified under the null hypothesis, while $\tau$ appears in the alternative. Therefore the problem of testing is non-standard and the asymptotic Chi Square distribution is not valid in this case. Tests for this non standard problem are well documented in econometric literature; see for example, Hansen (1997) for a discussion on the non-standard problem and a bootstrap method to test the hypothesis of linearity in a threshold autoregressive model. The appropriate test is a Sup Likelihood Ratio test whose sampling distribution is unknown. However the p-values can be generated from the bootstrap method as follows.

(1) Estimate the following system of market model subject to the null hypothesis of Black CAPM using time series regression by SUR; generate the parameter estimates and form a residual matrix.

$$R_{it} = \gamma(1-\beta_i) + \beta_i R_{mt} + \varepsilon_{it} \quad i = 1, 2, \ldots N$$

(2) Re-sample T rows of the residual matrix and using the parameter estimates obtained in step (1) above generate the return from the system

$$R_{it}^* = \hat{\gamma}(1-\hat{\beta}_i) + \hat{\beta}_i R_{mt} + \varepsilon_{it}^*$$

(3) Compute the Sup Likelihood Ratio statistics $LR^* = Sup_\tau\ LR(\tau)$

(4) Repeat steps (2) and (3) a large number B of times and compute the p-value of the test as the proportion of cases in which the bootstrap statistic exceed the Sup

[6] The degrees of freedom employed by Harlow and Rao (1989) table 4 page 303 are N-2. However in the testing of null hypothesis a total of 2N restrictions are imposed therefore the appropriate number of degrees of freedom with the asymptotic Chi square is 2N.

test statistics obtained using the real data. Reject the null hypothesis if this p-value is smaller than the level of significance specified.

Although the paper uses the bootstrap as a method of computing p-values of the non-standard test the superiority of the bootstrap based tests over the asymptotic tests in general is well established see for example MacKinnon (2002). Keeping in view the dependencies in time series of residual we have also employed the Sieve Bootstrap[7] of Buhlmann (1997). The results are however qualitatively not much different from the *iid* case and therefore not reported with the main results.

The robust Wald and GMM tests can also be constructed for this case. The 3N+1 parameter vector is

(13) $\theta = [\gamma \quad \beta_1^- \quad \beta_1^+ \quad \delta_1 \quad \beta_2^- \quad \beta_2^+ \quad \delta_2 \; . \; . \; . \; \beta_N^- \quad \beta_N^+ \quad \delta_N]'$

The $2N \times 1$ vector of null restrictions to be tested is

$$H_0 : g(\theta) = [\beta_1^- - \beta_1^+ \quad \delta_1 \quad \beta_2^- - \beta_2^+ \quad \delta_2 \; . \; . \; . \; \beta_N^- - \beta_N^+ \quad \delta_N]' = 0$$

The ($2N \times 3N+1$) matrix of derivatives simply contains 1,-1 and zeros and is given by

$$\frac{\partial g}{\partial \theta'} = \begin{bmatrix} 0 & 1 & -1 & 0 & 0 & 0 & 0 & & & . & . & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & & & . & . & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 & & & . & . & 0 \\ & . & & & . & & & & . & & & \\ & & . & & & . & & & & . & & \\ & & & . & & & . & & & & . & \\ 0 & 0 & . & . & . & & & . & . & 0 & 0 & 1 \end{bmatrix}$$

A Sup Wald test can be easily constructed which employs the parameter estimates under the unrestricted alternative model (12) and the null hypothesis can be tested using the similar bootstrap procedure as adopted in the Likelihood Ratio test.

[7] This method proceeds by assuming that the errors follow an autoregressive process. The appropriate order of the model is selected by AIC criteria. The resampling is done iteratively from the estimated autoregressive model.

## IV. The Data and the Residual Diagnostics Tests

The tests discussed in section III are applied to portfolios formed from a sample of stocks from the Karachi Stock Exchange. The monthly closing prices of 101 stocks and the Karachi Stock Exchange 100 index are collected from the DataStream database. The sample period covers 13 ½ years from September 1992 to April 2006. The criteria for the stocks selection was based on the availability of time series data on active stocks for which the prices have been adjusted for dividend, stock split, merger and other corporate actions. The KSE-100 index is a market capitalization weighted index. It comprises top companies from each sector of KSE in terms of their respective market capitalization. The rest of the companies are picked on the basis of market capitalization without considering their sector. This paper uses the KSE-100 index as a proxy for market portfolio. The 101 stocks in the sample comprise of about 80 % market capitalization of the entire market. Market capitalization data is not available historically for all firms in the database. However the financial daily Business Recorder[8] has some recent year data. We selected the market capitalization of all selected stocks at the beginning of July 1999 which roughly corresponds to the middle of the sample period considered in the study. The monthly raw returns are calculated assuming continuously compounding of the returns as, $R_{it} = \ln(P_t / P_{t-1}) \times 100$. The 30 day repurchase option rate was used as a proxy for the risk free rate of return. We formulate the size portfolios as equally weighted portfolios from the selected stocks. The market capitalization data of mid sample (July 1999) are used to rank the stocks into 17 portfolios from the lowest to the highest capitalized stocks. The first portfolio consists of 5 stocks while the rest comprise 6 stocks each. The portfolio return is calculated as the equally weighted average return of the stocks in the portfolio. For the industry portfolios the stocks are classified into sixteen major industrial sectors. The sector sizes

[8] www.businessrecorder.com.pk

range from two stocks in transport and communication to 13 stocks in textile sector and 13 in the investment banks and financial companies[9]. These sectors serve as natural portfolios. The beta portfolios are based on the CAPM beta estimates obtained through Least Square method for the entire sample period. The construction method of 17 beta portfolios is similar to the size portfolios. The choice of 16 and 17 portfolios is made by keeping a balance between the numbers of stocks in the portfolios and proving a reasonable number of observations for the cross section analysis. The three different portfolios procedure is employed to achieve more confident in empirical results. Some studies, such as Groenewold and Fraser (2001), report that the conclusion of the analysis may be different and even conflicting when different portfolios are employed. The size anomaly urges the need to control size so that the true price effect is discovered. The Harlow-Rao study used only LPM beta portfolios for their analysis except for the tests of CAPM against MLPM model which were conducted for portfolios based on both CAPM and LPM betas. It will be of interest to investigate the robustness of the results in this study if factors such as size and industry are controlled by constructing portfolios based on these characteristics.

[Table 1 Here]

All residual diagnostics and the asset pricing tests are performed for the three distinct sub-periods i.e. March 1992 to March 1997, April 1997 to October 1997 and November 1997 to April 2006. The objective here is to examine the stability of the risk return relationship in the three sub periods. This is important because the volatile political and macroeconomic scenario in emerging markets might make the risk return relationship

[9] The industry sectors employed are Auto and allied, Chemicals, Commercial Banks, Food products, Industrial Engineering, Insurance, Oil and Gas, Investment banks and other financial companies, Paper and board, Pharmacy, Power and utility, Synthetic and Rayon, Textile, Textile Spinning and Weaving, Transport and communication and Other /Miscellaneous firms that include tobacco, metal and building material companies.

non stationary and unstable. Each sub-period consist of 54 monthly observations that correspond to 4 ½ year of monthly data. This sample sub period is closer to 60 months sub period employed in most asset pricing studies in US markets. Panel A of Table 1 reports the Mardia (1970) test of multivariate normality of the residuals of the unrestricted asymmetric response model for the size, industry and beta sorted portfolios. This test is based on multivariate equivalents of skewness and kurtosis measures. The results are reported for the test based on skewness and kurtosis measures separately. The tests are performed for the risk free rate as the target returns. At 10 % level of significance the skewness test is significant for all cases with size and industry portfolios. The asymmetric risk measures might provide better measures of investment risk in this market. The kurtosis test is not significant in the first part of the sample period for the industry portfolios and the last two parts for the size portfolios. For size and industry portfolios the source of non-normality is primarily due to skewness whereas the excess kurtosis is responsible for non-normality for the beta portfolios. Panel B of Table 1 reports the Hosking (1980) multivariate portmanteau test of no autocorrelation for up to lag 3 in the asymmetric response model residuals. This test is a multivariate generalization of the univariate test[10] of Box and Pierce (1970). The results do not provide evidence of predictability in the residuals for both size and industry data when the risk free rate is specified as the target rate.

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[10] The univariate JB tests for normality and the LB test of autocorrelation are also performed which indicate that normality and serial independence is rejected for roughly 50% of the individual portfolios regressions. The results are not reported to save space but are available on request.

## V. Results of Empirical Analysis

### A. Univariate analysis

As a preliminary cross-section analysis, Fama-MacBeth (1973) type regressions are run with CAPM beta and MLPM beta separately as the only risk variable. Initially the CAPM beta and the MLPM beta are estimated using time series data over a 4 ½-year period. The CAPM beta is estimated through the market model whereas the MLPM beta is estimated by the asymmetric response model with average risk-free rate, average market return and zero rate of return as the target rate of return respectively. The portfolio returns in the subsequent testing period are then cross sectionally regressed on the portfolio beta risk estimated over the previous estimation period. These cross section tests are predictive in nature. The two steps are repeated over two sub-sample periods.

Panels A, B and C of Table 2 report the resulting average coefficients and an indication of statistical significance for size, industry and CAPM beta portfolios respectively. Although the regressions with the CAPM beta have slightly better explanatory power as measured by coefficient of determination, the CAPM beta risk does not appear to be priced in any of the sample periods for both size portfolios and industry portfolios[11]. The systematic risk measured by the CAPM beta is priced (at the 10 per cent level) for beta sorted portfolios in the most recent sub-sample period namely, November 2001 through to April 2006. Contrary to the theory, the sign of the premium for CAPM beta is negative in the testing period of April 1997 to October 2001. The downside risk measured by LPM beta is priced for all three target rates with size portfolios and for the target rate of average market returns in beta portfolios. With industry portfolios neither

[11] In an earlier study Iqbal and Brooks (2006b) show that the beta risk premium is positive and significant in the most recent sub period when other explanatory variables beside the beta are used in the cross section regression. The risk-return relationship is however nonlinear in beta.

risk measure appears to explain cross section variation in the returns. The downside risk premiums are in general higher for size portfolios compared to industry and beta portfolios. The risk premium for both CAPM beta and the downside beta are positive in the most recent sub period indicating that the market has became mature enough to reveal the anticipated direction of risk-return relationship.

[Table 2 here]

B. Multivariate analysis

Table 3 presents the multivariate test of the Black-CAPM. The diagnostic tests do not provide evidence of return predictability. However, normality is rejected and therefore only the results of the Likelihood Ratio test and the Wald test are reported.[12] Both Likelihood Ratio test and the Wald test fail to reject the market portfolio efficiency implied by the CAPM. The numerical values of the Likelihood Ratio test statistic are higher and the corresponding p-values are smaller compared to those of the Wald test. The tests imply that the Likelihood Ratio in this case appears to be robust enough to distributional features to yield results that are similar to those obtained in the robust Wald test.

[Table 3 here]

Table 4 reports the results of multivariate tests of the MLPM model. With size portfolios the Likelihood Ratio tests do not reject the restriction of the MLPM model in all sample periods considered. Similar results are observed with the CAPM beta and industry sorted portfolios. Under all three portfolios sorting schemes the conclusion in the Wald test is similar to that in the Likelihood Ratio test. The MLPM restriction is

[12] The GMM based test resulted in a qualitatively similar conclusion to the Wald test except that the numerical values of the GMM tests are higher compared to the Wald test. The detailed results are available from the authors upon request.

not rejected in any of the cases considered. In general the data do not reject the CAPM and the MLPM (with a pre-specified target rate) when tested separately.

[Table 4 here]

Multivariate tests can be sensitive to the specified alternative hypothesis. Therefore we test the null hypothesis of CAPM against the MLPM alterative as well. Table 5 reports the result of this test.[13] The bootstrap based Likelihood Ratio test and the Wald test provide strong evidence in favour of the mean-variance CAPM model as an alternative to the MLPM model.[14]

[Table 5 here]

C. Comparison of univariate and multivariate test results

In univariate Fama-MacBeth regressions, results of which are reported in Table 2, we observe weak evidence (at the 10 per cent level) that MLPM beta is priced in the last sub-sample period for size portfolios.  This is not supported by the multivariate test results reported in Table 5.  Several reasons may be advanced for this inconsistent result. Firstly, in the Fama-MacBeth type test current returns are cross sectionally regressed against the risk variables estimated in previous periods whereas the multivariate test use contemporaneous risk and return. Secondly, Fama-MacBeth type test employ beta and MLPM beta estimated from the data and therefore are subject to measurement error. The multivariate test does not employ beta as a risk variable, instead beta is used only as a parameter. Moreover, the Fama-MacBeth test implicitly assumes that beta is time varying while the multivariate tests assume that the systematic beta risk and downside beta risk is time invariant. Therefore it is probable that results in the two types of tests may differ in empirical studies. Moreover, the multivariate test

[13] The bootstrap tests are computed over 1000 simulations.

[14] An exception is with beta sorted portfolios in the most recent sub-sample period where the CAPM is rejected in favour of the MLPM at the 5 per cent level of significance.

may not be powerful enough to detect any deviation that might result when risk measures are generated from an asymmetric response model rather than the market model. The lack of power of multivariate Likelihood Ratio test and the Wald test for portfolio efficiency for emerging market data is reported in Iqbal and Brooks (2006a).

D. Choice of test procedure

Here we discuss the pros and cons of using different test procedures: Sup LR/Sup Wald with p-values from Chi Square (2N) and p-values via bootstrap. In Table 5 we report p-values from the asymptotic Chi Square test for comparison. In section IIIB we argue that the asymptotic p-value may not be valid since the Sup Likelihood Ratio test and the Sup Wald test do not have a known sampling distribution and the nuisance parameter problem makes inference difficult.

Fig 1 displays the Chi Square probability density function with 2N degrees of freedom and the kernel density fitted on the LR statistic obtained in the bootstrap simulation for size sorted portfolios in the first sub-sample period: April 1997 through to October 2001. The vertical line in Figure 1 (cuts the horizontal axis at 49.5) indicates the observed value of the Sup test statistic from the observed data. It is clear that for the asymptotic Chi Square distribution the value of 49.6 is unusual whereas in the bootstrap distribution 49.6 is a highly probable value with 53.5 per cent of the cases exceeding this value. The asymptotic distribution therefore does not appear to be valid for the Sup LR test considered. A similar observation is made with the Sup Wald test.

As evident in Table 5, only in two cases the Likelihood Ratio test fails to reject the CAPM against the MLPM alternative. This non-rejection is observed in industry and beta portfolios in the second sub-sample period. In general the asymptotic Chi-Square test rejects the CAPM as an alternative to the MLPM model while the bootstrap test does not. This observation highlights the importance of the choice of procedure when testing the CAPM against the MLPM alternative.

## VI. Conclusion

This paper compares the CAPM and the Mean Lower Partial Moment asset pricing model in the context of emerging markets. Keeping in consideration the stylized facts of emerging markets the paper extends the Harlow-Rao Likelihood Ratio test of the Black's version of the MLPM and develops a Wald test and a GMM based robust tests that allows for non-normality and serial dependence in the returns. Moreover for the test of the CAPM hypothesis against the MLPM alternative the paper remedies an econometric problem of testing in the presence of a nuisance parameter. It is shown that the conclusion on the validity of the asset pricing model with asymptotic test is reversed when the correct p-values obtained through bootstrap test are employed. We show that the univariate asset pricing tests of Fama MacBeth style may yield different conclusion than the multivariate tests which take into consideration the contemporaneous covariance among the assets under study. The multivariate tests appear to support both the Black version of the CAPM and the MLPM alternative model while the univariate test indicates that downside risk is priced in the most recent sample period from November 2001 to April 2006 for all target rates with size based portfolios and with beta portfolios when the average market return as the target rate is employed. To gain a better insight on whether the return for the emerging market under study are sensitive to the mean variance beta or the downside risk the paper conducts a multivariate test of CAPM against the MLPM alliterative. The results support the former asset pricing model. The results with the Likelihood Ratio test and the Wald test do not differ greatly. This is not surprising if we note that the multivariate diagnostic test does not reveal significant autocorrelations in the residuals in this study.

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Table 1: Diagnostics Tests for the residuals for the Multivariate Asymmetric Response model system

Panel A: Mardia (1970) Test of Multivariate Normality

| Sample Period | Size Portfolios | | Industry Portfolios | | Beta Portfolios | |
|---|---|---|---|---|---|---|
| | Skewness (P-value) | Kurtosis (P-value) | Skewness (P-value) | Kurtosis (P-value) | Skewness (P-value) | Kurtosis (P-value) |
| Sep 92 – Mar 97 | 123.633 (0.001) | 335.371 (0.036) | 108.644 (0.000) | 295.502 (0.125) | 108.408 (0.433) | 301.718 (0.001) |
| Apr 97 – Oct 01 | 115.509 (0.056) | 329.031 (0.192) | 112.931 (0.000) | 303.591 (0.008) | 94.336 (0.997) | 289.061 (0.000) |
| Nov 01 – Apr 06 | 121.991 (0.002) | 331.018 (0.123) | 103.887 (0.002) | 297.364 (0.075) | 98.339 (0.974) | 291.479 (0.000) |

Panel B: Hosking (1980) Multivariate portmanteau test of serial independence

| Sample Period | Size Portfolios | | | Industry Portfolios | | | Beta Portfolios | | |
|---|---|---|---|---|---|---|---|---|---|
| | Lag1 | Lag2 | Lag3 | Lag1 | Lag2 | Lag3 | Lag1 | Lag2 | Lag3 |
| Sep 92 – Mar 97 | 301.768 (0.291) | 599.747 (0.257) | 890.913 (0.279) | 274.839 (0.199) | 504.416 (0.586) | 756.258 (0.611) | 300.810 (0.304) | 583.848 (0.424) | 868.386 (0.528) |
| Apr 97 – Oct 01 | 286.209 (0.535) | 575.223 (0.524) | 840.541 (0.734) | 270.502 (0.255) | 522.747 (0.361) | 800.473 (0.202) | 277.710 (0.672) | 589.308 (0.363) | 904.630 (0.182) |
| Nov 01 – Apr 06 | 292.787 (0.426) | 585.957 (0.400) | 898.170 (0.224) | 269.122 (0.274) | 585.687 (0.226) | 790.612 (0.278) | 308.006 (0.211) | 620.570 (0.107) | 936.538 (0.050) |

This table reports the tests of multivariate normality and serial independence of the residuals of the asymmetric response model when the risk free rate is specified as the target rate of return. Panel A reports Mardia (1970) test of multivariate normality which is based on the multivariate skewness and kurtosis measures $D_1 = \frac{1}{T^2}\sum_{t=1}^{T}\sum_{s=1}^{T} r_{ts}^{\ 3}$

And $D_2 = \frac{1}{T}\sum_{t=1}^{T} r_{tt}^{\ 2}$ where $r_{ts} = (X_t - \overline{X})S^{-1}(X_s - \overline{X})$, $\overline{X}$ and $S$ are the sample mean and sample covariance matrix of the residuals respectively. $TD_1/6 \sim \chi^2(f)$ where $f = N(N+1)(N+2)/6$, and $D_2 - \frac{N(N+2)}{\sqrt{8N(N+2)/T}} \sim N(0,1)$

Panel B reports the Hosking (1980) multivariate portmanteau test which is a multivariate generalization of the univariate portmanteau test of Box and Pierce (1970). The test statistic at a lag length s is

$Q(s) = T^2 \sum_{j=1}^{s} \frac{1}{T-j} tr(C'_{0j} C_{00}^{\ -1} C_{oj} C_{00}^{\ -1}) \quad \sim \chi^2(N^2 s)$ where $C_{rs} = \frac{1}{T} U'_{-r} U_{-s}$, $U_{-i}$ is the $T \times N$ residual matrix lagged i periods. The test is performed for s=1 2, 3. The missing values are filled with zero.

Table 2: Cross Section Fama MacBeth Regressions

Panel A: Size Portfolios

| Test Period | CAPM | | | MLPM Model | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | Average Risk free rate | | | Average marker return | | | Zero return | | |
| | Intercept | $\beta$ | $R^2$ | Intercept | $\beta^-$ | $R^2$ | Intercept | $\beta^-$ | $R^2$ | Intercept | $\beta^-$ | $R^2$ |
| Apr 97 – Oct 01 | -0.429 | -0.503 | 0.165 | -0.672 | 0.112 | 0.139 | -0.714 | 0.195 | 0.141 | -0.734 | 0.231 | 0.137 |
| | (-0.663) | (-0.307) | | (-0.923) | (0.080) | | (-1.061) | (0.144) | | (-1.033) | (0.168) | |
| Nov 01 – Apr 06 | 2.888* | 1.588 | 0.147 | 2.174* | 1.768** | 0.101 | 2.314* | 1.572** | 0.111 | 2.177* | 1.755** | 0.096 |
| | (2.733) | (1.170) | | (2.633) | (1.649) | | (2.951) | (1.374) | | (2.699) | (1.620) | |

Panel B: Industry Portfolios

| Test Period | CAPM | | | MLPM Model | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | Average Risk free rate | | | Average marker return | | | Zero return | | |
| | Intercept | $\beta$ | $R^2$ | Intercept | $\beta^-$ | $R^2$ | Intercept | $\beta^-$ | $R^2$ | Intercept | $\beta^-$ | $R^2$ |
| Apr 97 – Oct 01 | -0.167 | -0.956 | 0.163 | -0.517 | -0.363 | 0.148 | -0.593 | -0.252 | 0.146 | -0.623 | -0.202 | 0.147 |
| | (-0.307) | (-0.630) | | (-0.918) | (-0.296) | | (-1.039) | (-0.206) | | (-1.059) | (-0.167) | |
| Nov01 – Apr 06 | 2.508* | 1.341 | 0.161 | 2.568* | 1.269 | 0.150 | 2.603* | 1.155 | 0.144 | 2.558* | 1.270 | 0.145 |
| | (3.122) | (0.947) | | (3.236) | (1.067) | | (3.069) | (1.032) | | (3.185) | (1.103) | |

Panel C: Beta Portfolios

| Test Period | CAPM | | | MLPM Model | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | Average Risk free rate | | | Average marker return | | | Zero return | | |
| | Intercept | $\beta$ | $R^2$ | Intercept | $\beta^-$ | $R^2$ | Intercept | $\beta^-$ | $R^2$ | Intercept | $\beta^-$ | $R^2$ |
| Apr 97 – Oct 01 | -0.267 | -0.595 | 0.199 | -0.426 | -0.321 | 0.175 | -0.513 | -0.207 | 0.173 | -0.569 | -0.143 | 0.176 |
| | (-0.510) | (-0.392) | | (-0.756) | (-0.206) | | (-0.899) | (-0.131) | | (-0.984) | (-0.089) | |
| Nov01 – Apr 06 | 2.040* | 1.857** | 0.177 | 2.263* | 1.578 | 0.145 | 2.284* | 1.509** | 0.144 | 2.333* | 1.482 | 0.141 |
| | (2.838) | (1.419) | | (3.538) | (1.289) | | (3.397) | (1.346) | | (3.675) | (1.254) | |

* Significantly different from zero at 5% level of significance

** Significantly different from zero at 10 % level of significance

This table presents the average coefficients from the Fama MacBeth cross section regressions. For the CAPM the risk variable beta is estimated from the market model using the previous 4 ½ year monthly data. In case of the Mean Lower Partial Moment model the risk variable downside beta is estimated from Asymmetric response model using previous 4 ½ year monthly data. The downside beta is estimated assuming that the target rate is average risk free rate, the average rate of return on the market portfolio and the zero return respectively. To make the analysis comparable with subsequent multivariate Black-CAPM tests raw returns are employed in the time series and cross section regressions. The t-statistics are reported in the parenthesis below each coefficient. The average risk free rate is 2.722%, 0.924% and 0.411% respectively for the three time periods and the average raw market return is 0.908%, -0.420% and 3.859% respectively in the three time periods. The test for beta coefficients are one tailed right sided tests.

Table 3: Likelihood Ratio and Wald tests of Black-CAPM for the multivariate system of market model equations

| | Size portfolios | | Industry Portfolios | | Beta Portfolios | |
|---|---|---|---|---|---|---|
| | Likelihood Ratio test | Wald Test | Likelihood Ratio test | Wald Test | Likelihood Ratio test | Wald Test |
| Sep 92 – Mar 97 | 8.145 (0.944) | 5.721 (0.990) | 11.232 (0.735) | 7.118 (0.954) | 8.160 (0.943) | 5.112 (0.995) |
| Apr 97 – Oct 01 | 9.105 (0.909) | 6.994 (0.973) | 7.736 (0.933) | 6.908 (0.960) | 10.066 (0.863) | 8.816 (0.920) |
| Nov 01 – Apr 06 | 14.182 (0.585) | 9.516 (0.890) | 12.799 (0.617) | 9.916 (0.825) | 13.072 (0.667) | 5.634 (0.991) |

This table presents the multivariate tests of the Black-CAPM. The Black-CAPM does not require risk free rate specification. The test statistics are the tests of the restrictions $H_0 : \alpha = \gamma(1-\beta)$ across each portfolio against the unspecified alternative. The Likelihood Ratio test assumes multivariate Normality of returns while the Wald test is robust to distributional specifications. The test statistics are asymptotically distributed as Chi square with N-1 degrees of freedom where N is the number of portfolios. The p-values appear in the parenthesis. To improve small sample performance of the asymptotic tests the Likelihood ratio test is adjusted by multiplying (T-5/2-N/2) and the Wald is adjusted by multiplying (T-N-1)/T. For detail, see Gibbons Ross and Shaken (1989) and Jobson and Korkie (1982)

Table 4: Multivariate Tests of Lower Partial Moment CAPM
Panel A: Size Portfolios

| | Likelihood Ratio Test | | | Wald Test | | |
|---|---|---|---|---|---|---|
| | Average Risk free rate | Average market return | Zero return | Average Risk free rate | Average market return | Zero return |
| Sep 92 – Mar 97 | 7.672<br>(0.958) | 12.148<br>(0.733) | 16.480<br>(0.419) | 1.151<br>(0.999) | 2.246<br>(0.999) | 2.804<br>(0.999) |
| Apr 97 – Oct 01 | 13.719<br>(0.619) | 16.628<br>(0.410) | 13.719<br>(0.619) | 3.913<br>(0.998) | 6.195<br>(0.985) | 3.913<br>(0.999) |
| Nov 01 – Apr 06 | 20.009<br>(0.219) | 18.105<br>(0.317) | 17.393<br>(0.360) | 7.503<br>(0.962) | 3.989<br>(0.998) | 8.605<br>(0.928) |

Panel B: Industry Portfolios

| | Likelihood Ratio Test | | | Wald Test | | |
|---|---|---|---|---|---|---|
| | Average Risk free rate | Average market return | Zero return | Average Risk free rate | Average market return | Zero return |
| Sep 92 – Mar 97 | 7.234<br>(0.950) | 5.169<br>(0.990) | 6.339<br>(0.973) | 1.307<br>(0.999) | 2.382<br>(0.999) | 3.726<br>(0.998) |
| Apr 97 – Oct 01 | 9.652<br>(0.841) | 13.294<br>(0.579) | 9.652<br>(0.841) | 2.346<br>(0.999) | 2.827<br>(0.999) | 2.346<br>(0.999) |
| Nov 01 – Apr 06 | 12.465<br>(0.643) | 12.247<br>(0.660) | 13.309<br>(0.578) | 2.482<br>(0.999) | 1.372<br>(0.999) | 2.534<br>(0.999) |

Panel C: Beta Portfolios

| | Likelihood Ratio Test | | | Wald Test | | |
|---|---|---|---|---|---|---|
| | Average Risk free rate | Average market return | Zero return | Average Risk free rate | Average market return | Zero return |
| Sep 92 – Mar 97 | 5.651<br>(0.991) | 10.084<br>(0.862) | 11.950<br>(0.747) | 2.380<br>(0.999) | 5.674<br>(0.991) | 6.940<br>(0.974) |
| Apr 97 – Oct 01 | 9.437<br>(0.894) | 10.215<br>(0.855) | 9.437<br>(0.894) | 1.290<br>(0.999) | 2.995<br>(0.999) | 1.290<br>(0.999) |
| Nov 01 – Apr 06 | 12.133<br>(0.734) | 7.959<br>(0.950) | 15.601<br>(0.481) | 7.503<br>(0.962) | 1.207<br>(0.999) | 6.229<br>(0.985) |

This table reports the likelihood Ratio and the Wald Test for the Harlow and Rao's (1989) Lower Partial Moment CAPM. The targe rates are the average risk free rate, the average market returns and the zero rates of returns respectively. In the second time period the average risk free rate is close to zero therefore the results for this target rate are identical to the zero target rates for this time period. The test statistics are the test of the non linear hypothesis $H_0 : \alpha = \gamma(1-\beta_i)$ across all portfolios against the unspecified alternative. With the pre-specified target rates the test statistics are asymptotically distributed as Chi-Square with N-1 degrees of freedom. The p-values appear in the parenthesis. As in table 3 the tests are adjusted by Bartlett factors.

Table 5: Multivariate tests of the null hypothesis of the Black CAPM against the alternative of the Lower Partial Moment CAPM

Panel A: Size Portfolios

| | Likelihood Ratio Test | | | Wald Test | | |
|---|---|---|---|---|---|---|
| | Sup LR | P-value Chi-Square (2N) | P-value Bootstrap | Sup Wald | P-value Chi-Square (2N) | P-value Bootstrap |
| Sep 92 – Mar 97 | 49.600 | 0.041 | 0.465 | 62.636 | 0.001 | 0.798 |
| Apr 97 – Oct 01 | 45.755 | 0.085 | 0.801 | 53.154 | 0.010 | 0.789 |
| Nov 01 – Apr 06 | 60.826 | 0.003 | 0.298 | 77.359 | 0.000 | 0.466 |

Panel B: Industry Portfolios

| | Likelihood Ratio Test | | | Wald Test | | |
|---|---|---|---|---|---|---|
| | Sup LR | P-value Chi-Square (2N) | P-value Bootstrap | Sup Wald | P-value Chi-Square (2N) | P-value Bootstrap |
| Sep 92 – Mar 97 | 60.663 | 0.002 | 0.296 | 78.866 | 0.000 | 0.559 |
| Apr 97 – Oct 01 | 41.456 | 0.122 | 0.796 | 45.803 | 0.054 | 0.864 |
| Nov 01 – Apr 06 | 45.256 | 0.060 | 0.766 | 49.363 | 0.025 | 0.914 |

Panel C: Beta Portfolios

| | Likelihood Ratio Test | | | Wald Test | | |
|---|---|---|---|---|---|---|
| | Sup LR | P-value Chi-Square (2N) | P-value Bootstrap | Sup Wald | P-value Chi-Square (2N) | P-value Bootstrap |
| Sep 92 – Mar 97 | 45.128 | 0.096 | 0.838 | 51.305 | 0.028 | 0.882 |
| Apr 97 – Oct 01 | 40.845 | 0.194 | 0.935 | 47.978 | 0.056 | 0.904 |
| Nov 01 – Apr 06 | 77.461 | 0.000 | 0.036 | 110.099 | 0.000 | 0.130 |

This table reports the test of the CAPM restrictions on the MLPM model. We test that in the asymmetric response model subject to MLPM restriction $\beta_i^- = \beta_i^+$ and $\delta_3 = 0$ against the alternative this is not the case. This results in a set of 2N restrictions. Note that under the null hypothesis the asymmetric response model which carries the MLPM restrictions collapses to the usual market model which carries Black' CAPM restrictions. The Chi square p-values are reported for only a comparison purpose. The correct p-values are obtained through bootstrap.

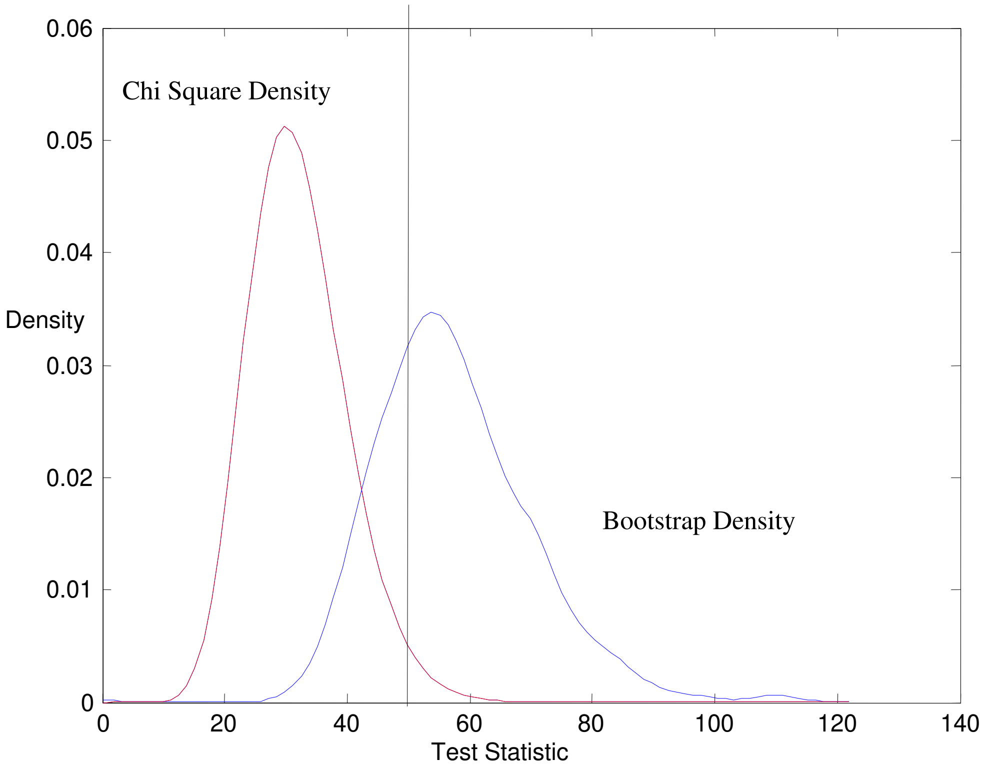


Fig 1: Chi Square (2N) pdf and the Bootstrap Kernel Density for the Likelihood Ratio Test of Null hypothesis of CAPM against the MLPM alternative for Size Portfolio in first sample period.