# Asymmetric GARCH and the financial crisis: a preliminary study

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**Abstract**

*The paper deals with estimation of both general GARCH as well as asymmetric EGARCH and TGARCH models, used to model the leverage effect of good news and bad news on market volatility. We estimate the models using daily returns of S&P 500 stock index and describe the news impact curves (NICs) for these models. When estimating the crisis series, we show the possibility of using a news impact surface to describe the results from models of higher orders.*

**Keywords**
volatility modeling, financial crisis, asymmetric GARCH class models, news impact curve

**JEL Classifications**: G01, C22, C5, G15

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**Introduction**

In analyzing the financial time series are some interesting topics, which are worth to mention. Firstly, it is a well known property of such time series, i.e. *leptokurtosis* (Mandelbrot, 1963). With comparison to normal distribution, financial data tends to have fat tails. Secondly, it is a phenomenon called *volatility clustering*, which describes that large swings are followed by large changes, and small changes are followed by small changes. Such observations have implicated the widely use of ARCH and GARCH class of models in volatility modeling and forecasting the financial time series. As it is stated in Alberg – Shalit – Yosef (2008), both the ARCH and GARCH models capture volatility clustering and leptokurtosis, but as their distribution is symmetric, they fail to model the *leverage effect.* This is the last topic we would like to emphasize and it stands for different impacts of negative and positive shocks to conditional variance. This asymmetry problem resulted in a wide range of non-linear GARCH type models, which has been proposed during the last two decades.

The aim of this paper is to briefly introduce these models and show their applications on the American stock market index Standard and Poor's 500 (S&P500 henceforth). After estimating selected models (GARCH, EGARCH and TGARCH) we continue our study by comparing results using NIC – News Impact Curves.

This paper is organized as follows. Section 1 presents applied methodology, i.e. various GARCH class models. In Section 2 employed data are described and Section 3 provides empirical application. Section 4 is dedicated to conclude the results.

## 1 Methodology – GARCH Class Models

Since the paper of Engle (1982), autoregressive conditional heteroskedasticity models (ARCH henceforth) have been extensively used in the field of financial economics. Linear representation of conditional variance $\sigma_t^2$ and lagged values of error term $\varepsilon_t$ is defined in ARCH model as:

$$\sigma_t^2 = \omega + \alpha_1 \varepsilon_{t-1}^2 \tag{1}$$

This so called ARCH (1) process implies, that error terms are generated as:

$$\varepsilon_t = v_t \sqrt{\omega + \alpha_1 \varepsilon_{t-1}^2}\ , v_t \sim N(0,1) \tag{2}$$

while regression parameters $\omega, \alpha_1$ should satisfy conditions $\omega > 0, 0 \le \alpha_1 \le 1$, so the conditional variance is positive and the autoregressive process is stable.

Some generalizations of the ARCH model were provided by Bollerslev (1986):

$$\sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-i}^2 + \sum_{i=1}^{q} \beta_i \sigma_{t-i}^2 \tag{3}$$

where $\alpha_i \varepsilon_{t-i}^2$ is an ARCH component and $\beta_i \sigma_{t-i}^2$ is a GARCH component. Equation *(3)* is a general form of the GARCH (p,q) model.

We will further proceed to non-linear GARCH type models, which are somehow taking into account the different effects of positive or negative shocks on the conditional variance.

The **Exponential GARCH** (EGARCH henceforth) model proposed by Nelson (1991) is defined as:

$$\log \sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i \left| \frac{\varepsilon_{t-i}}{\sigma_{t-i}} \right| + \sum_{j=1}^{q} \gamma_j \frac{\varepsilon_{t-j}}{\sigma_{t-j}} + \sum_{k=1}^{z} \beta_k \log \sigma_{t-k}^2 \qquad (4)$$

The effect of a positive shock is given by the sum of parameters $\alpha_i + \gamma_i$ and the effect of a negative shock is given by a subtraction respectively. Since logarithms of the conditional variance could be negative, no further restrictions are necessary.

The **Threshold GARCH** (TGARCH henceforth) proposed by Rabemananjara – Zakoian (1993) and Zakoian (1994) divide error terms to a piecewise function $I(.)$ and can be written as:

$$\sigma_t^\delta = \omega + \underbrace{\sum_{i=1}^{p} \alpha_i \varepsilon_{t-i}^\delta}_{ARCH\ term} + \underbrace{\sum_{j=1}^{q} \gamma_j \varepsilon_{t-j}^\delta I(\varepsilon_{t-j} < 0)}_{assymetric\ component} + \underbrace{\sum_{k=1}^{z} \beta_k \sigma_{t-k}^\delta}_{GARCH\ term} \qquad (5)$$

With the indicator function $I(.) = 1\ if\ \varepsilon_{t-j} < 0$ or $I(.) = 0\ if\ \varepsilon_{t-j} > 0$ respectively. If $\gamma_i$ coefficients have positive values, it indicates a presence of the leverage effect. Note that in the TGARCH model an exponent $\delta$ equals 1. Model proposed by Glosten et al. (1993), which is well known as the **GJR-GARCH** model, replace $\delta = 2$. The difference lies in a fact that we are dealing with the conditional standard deviations in the TGARCH model or the conditional variance in the GJR-GARCH model.

The **Asymmetric Power ARCH** (APARCH henceforth) model by Ding – Granger – Engle (1993) is probably one of the most interesting ARCH type models[1]. It can be expressed as:

$$\sigma_t^\delta = \omega + \sum_{i=1}^{p} \alpha_i \left( |\varepsilon_{t-i}| - \gamma_i \varepsilon_{t-i} \right)^\delta + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^\delta \qquad (6)$$

If the parameter $\gamma_i$ equals 0 a positive shock $\varepsilon_t > 0$ has the same effect on analyzed volatility as negative one, i.e. $\varepsilon_t < 0$. Thus the $\gamma_i$ reflects the leverage effect.

## 2 Data

[1] With some particular modifications the APARCH actually includes seven other ARCH type models. For detailed discussion see original paper by Ding – Granger – Engle (1993) or e.g. Laurent (2004).

To fulfill our goal we decided to utilize daily closing prices of the S&P500 covering period from 1st July 2004 to 31st August 2009. This sample has been divided into two periods: the pre-crises and the crises period. Particularly it is interesting to analyze the effects of "bad and good" news in such different periods.

We are aware of the fact, that to determine an exact date of the beginning of the recent crises is not accurate. Thus as a starting point 1st February 2007 has been chosen, coinciding with the month when first problems in subprime mortgage market were announced by HSBC. The pre-crisis series was obtained to make it exactly the same in size of the sample (651 observations for each one).

Since the closing prices are non-stationary (ADF-GLS test was applied), daily returns are computed as follows:

$$r_{t+1} = \ln\left(\frac{p_{t+1}}{p_t}\right) = \ln(p_{t+1}) - \ln(p_t) \tag{7}$$

where $r_{t+1}$ are daily returns in time $t+1$, $p_{t+1}$ are closing prices in time $t+1$ and $p_t$ are closing prices in time $t$, $t = 1,2,\ldots,T-1$, where $T$ is the number of all observations. In the following figures are shown closing prices and returns respectively.

Figure 1

**Closing prices of the S&P500**

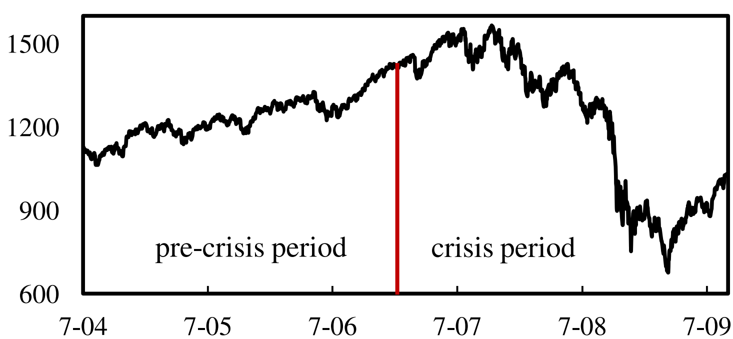
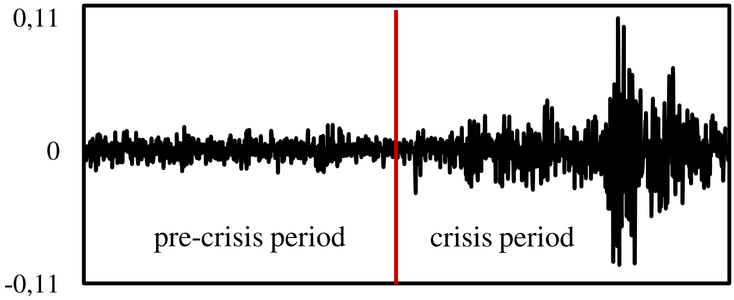


Figure 2

**Returns of the S&P500**




It can be seen in the Figure 2, that in the crises period wide swings of volatility are observed. In the following section are presented the results from modeling such volatility using asymmetric GARCH class models.

## 3 Results

At the first place, we estimate simple ARMA models. It is important to choose these models with respect to a presence of autocorrelation. Following a recommendation by Kočenda – Černý (2007), we control autocorrelation up to $T/4$ lags (using Ljung – Box test), where $T$ denotes the number of observations. In our case it makes 150 lags approximately.

Obtained results are presented in the following table. Note that for the crises period, ARMA of higher orders has to be estimated. In the pre-crises period, ARMA(2,0) has been chosen in consideration of the information criteria and the autocorrelation presence.

Table 1

**ARMA models - residual diagnostics**

| Type of | Q-stat (*p*-value) | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| model | lag 1 | lag 2 | lag 3 | lag 4 | lag 5 | lag 6 | lag 7 | lag 8 | lag 9 | lag 10 |
| ARMA(4,5) | 0,878 | 0,703 | 0,868 | 0,928 | 0,953 | 0,968 | 0,982 | 0,864 | 0,910 | 0,946 |
| ARMA(2,0) | 0,985 | 0,998 | 0,998 | 0,966 | 0,983 | 0,968 | 0,940 | 0,845 | 0,677 | 0,761 |

Note: autocorrelation was controlled up to 150 lags, these results are illustrative

Further we proceed with estimating selected GARCH type models. Summarized results are in the Table 2. Note that coefficients are labeled in the same way as they are in the models introduced in Section 1.

For the purpose of residual diagnostics is the standard LM test applied. In the Table 3 we can see, that no additional ARCH effects are observable.

Table 2

**GARCH models – estimation**

| Coefficients | PRE-CRISES | | | CRISES | | |
|---|---|---|---|---|---|---|
| | GARCH | EGARCH | TGARCH | GARCH | EGARCH | TGARCH |
| Omega | 0,0000 | -0,3052 | 0,0000 | 0,0000 | -0,3956 | 0,0000 |
| | *(0,1938)* | *(0,0041)* | *(0,0017)* | *(0,0000)* | *(0,0000)* | *(0,0000)* |
| Alpha_1 | 0,0438 | -0,0132 | -0,0490 | -0,0436 | 0,1251 | -0,0794 |
| | *(0,0296)* | *(0,4972)* | *(0,0000)* | *(0,0006)* | *(0,0003)* | *(0,0358)* |
| Alpha_2 | - | - | - | 0,1494 | - | 0,0690 |
| | - | - | - | *(0,0000)* | - | *(0,0355)* |
| Beta | 0,9105 | 0,9692 | 0,9913 | 0,8739 | 0,9635 | 0,9185 |
| | *(0,0000)* | *(0,0000)* | *(0,0000)* | *(0,0000)* | *(0,0000)* | *(0,0000)* |
| Gamma | - | -0,1092 | 0,0854 | - | -0,1078 | 0,0384 |
| | - | *(0,0000)* | *(0,0000)* | - | *(0,0000)* | *(0,3261)* |
| Gamma_2 | - | - | - | - | - | 0,1069 |
| | - | - | - | - | - | *(0,0127)* |

Note: *p*-values are in the parentheses

Table 3

**GARCH models – residual diagnostics**

| PRE-CRISES | | CRISES | |
|---|---|---|---|
| Type of model | LM test (*p*-value) | Type of model | LM test (*p*-value) |
| GARCH(1,1) | 0,752757 | GARCH(2,1) | 0,409555 |
| EGARCH(1,1) | 0,417449 | EGARCH(1,1) | 0,428714 |
| TGARCH(1,1) | 0,440764 | TGARCH(2,1) | 0,411825 |

Note: the presence of additional ARCH effects was controlled up to 150 lags, these results are illustrative using 10 lags in the LM test

To compare estimated models and particularly the asymmetry of the volatility response to news, we decided to apply the News Impact Curves (NIC henceforth), which are the functional relationship between conditional variance at time *t* and the error term at time *t-1*. The logic of using this curve in asymmetric GARCH models is straightforward – since a positive $\varepsilon_t$ (an unexpected increase in price) suggests the “good news”, while a negative $\varepsilon_t$ (an unexpected decrease in price) suggest the arrival of “bad news”. Further, a large value of $|\varepsilon_t|$ implies that the news is "significant" in the sense that it produce a large unexpected change in price (Engle – Ng, 1993).

We compute the NIC-s using following equations:

- For GARCH model

$$\sigma_t^2 = A^{GARCH} + \alpha\varepsilon_{t-1}^2, \tag{8}$$

where

$$A^{GARCH} = \omega + \beta\sigma^2. \tag{9}$$

- For EGARCH model

$$\sigma_t^2 = A^{EGARCH} \exp\left[\frac{(\gamma+\alpha)}{\sigma}\varepsilon_{t-1}\right], \text{for } \varepsilon_{t-1} > 0, \tag{10}$$

$$\sigma_t^2 = A^{EGARCH} \exp\left[\frac{(\gamma-\alpha)}{\sigma}\varepsilon_{t-1}\right], \text{for } \varepsilon_{t-1} < 0, \tag{11}$$

where

$$A^{EGARCH} = \sigma^{2\beta} \exp[\omega - \alpha]. \tag{12}$$

- For TGARCH model

$$\sigma_t^2 = A^{TGARCH} + \alpha\varepsilon_{t-1}^2, \text{for } \varepsilon_{t-1} > 0, \tag{13}$$

$$\sigma_t^2 = A^{TGARCH} + (\alpha+\gamma)\varepsilon_{t-1}^2, \text{for } \varepsilon_{t-1} < 0, \tag{14}$$

where

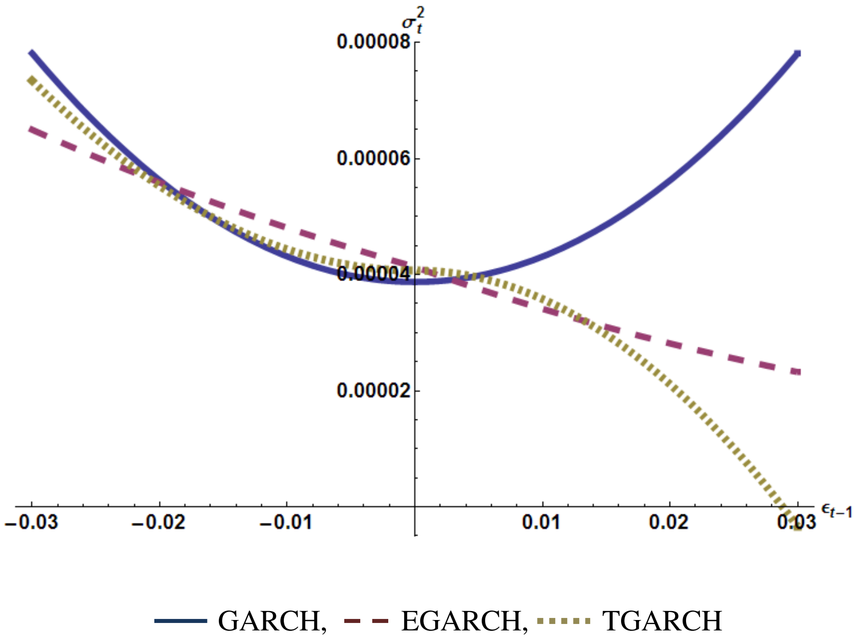
$$A^{TGARCH} = \omega + \beta\sigma^2. \tag{15}$$

The NIC equations are easy to obtain from the respective GARCH variant equations *(3)*, *(4)* and *(5)*. Essentially, one replaces the conditional GARCH terms ${\sigma_{t-1}}^2$ on the first lag with the unconditional return variance $\sigma^2$ (Engle – Ng, 1993).

By considering the range of values for $\varepsilon_t$, it is possible to obtain a graphical representation of the NIC, as depicted on the Figure 3.

Figure 3

**NIC-s in the pre-crises period**




The NIC of a fitted GARCH (1,1) model shows the symmetry typical for these models. As it does not account for the leverage effect, both good news and bad news are treated in the same way. Therefore, the model predicts higher volatility in case of "big" news, regardless of their nature.

The situation with the asymmetric models is rather different. Both EGARCH and TGARCH allow for asymmetric response to the news, measured by the residuals. Both of these models allow for different functional form of the NIC depending on the sign of $\varepsilon_{t-1}$, in case of TGARCH using a simple dummy variable. As such, the graphical representations of these models are expected to change for positive and negative news, usually following a U-shaped pattern.

There are several important observations that can be made by examining the Figure 3. First, we conclude that the asymmetric models do model the volatility differently from the widely used GARCH. It is quite clear that good news are according to TGARCH and EGARCH followed by significantly lower variance of returns as is predicted by the general GARCH model. This result questions the adequacy of this commonly used approach, as it seems to overestimate the true volatility in these situations.

Another interesting aspect seen on the Figure 3 is the shape of the asymmetric NICs. Instead of the expected U-curves, both EGARCH and TGARCH suggest volatility *decreasing* with an increasing $\varepsilon_{t-1}$ (i.e. „good news") during the sample period.

Figure 4

**NIC for the GARCH (2,1) model – a surface plot**

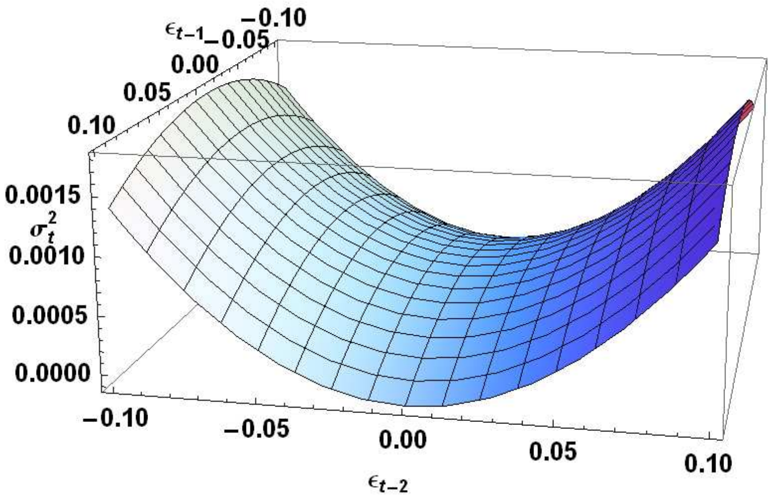
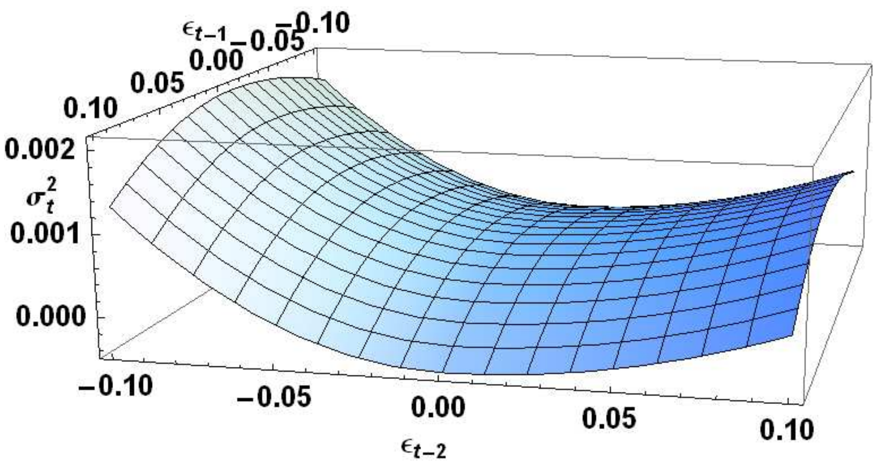



Figure 5

**NIC for the TGARCH (2,1) model – a surface plot**




The reason for this kind of NIC behavior can be explained by the examination of the estimation results summarized in the Table 2. If we consider the fitted TGARCH (1,1) model, we obtain this representation:

$$\sigma_t = \omega + \alpha_1 \varepsilon_{t-1} + \gamma_1 \varepsilon_{t-1} I(\varepsilon_{t-1} < 0) + \beta_1 \sigma_{t-1} \quad (16)$$

or if we substitute the estimated coefficient values

$$\sigma_t = 5{,}40\text{E-}07\ \text{-}0{,}0490\varepsilon_{t-1} + 0{,}0854\varepsilon_{t-1} I(\varepsilon_{t-1} < 0) + 0{,}9913\sigma_{t-1} \qquad (17)$$

We can see that while in the case of bad news, that is, when $I(\varepsilon_{t-1} < 0) = 1$ the impact of news on the modeled standard deviation is positive, as $\alpha_1 + \gamma_1 > 0$. However, in the case of positive values of $\varepsilon_{t-1}$ only the negative $\alpha_1$ plays a role in modeling the volatility, which leads to decreasing values of the NIC in the first quadrant.

Similarly for the estimated exponential GARCH model

$$\log\sigma_t^{\ 2} = \omega + \alpha_1 \left| \frac{\varepsilon_{t-1}}{\sigma_{t-1}} \right| + \gamma_1 \frac{\varepsilon_{t-1}}{\sigma_{t-1}} + \beta_1 \log\sigma_{t-1}^{\ 2} \qquad (18)$$

we can rewrite the equation using the sign function sgn as

$$\log\sigma_t^{\ 2} = \omega + [\alpha_1 \operatorname{sgn}(\varepsilon_{t-1}) + \gamma_1] \frac{\varepsilon_{t-1}}{\sigma_{t-1}} + \beta_1 \log\sigma_{t-1}^{\ 2} \qquad (19)$$

As the estimated coefficients $\alpha_1$ and $\gamma_1$ shown in the Table 2 are both negative, with $\gamma_1 < \alpha_1$, it follows that in this model the volatility is a decreasing function of $\varepsilon_{t-1}$, leading to the shape of its NIC shown in the Figure 3.

The next step in our analysis was the calculation of NIC for models from the crisis dataset. As in the case of GARCH and TGARCH models we have used ARCH order of 2, the news impact curve changes to news impact surface. This results from the fact, that the volatility is in this case dependent not only on the previous news $\varepsilon_{t-1}$, but also on the one before, i.e. $\varepsilon_{t-2}$. By allowing for different combinations of these lagged residuals we obtain surface plots on the Figures 4 and 5.

It can be seen that here, just like in the previous case, there is a marked difference between the GARCH and TGARCH surface. With GARCH surface being symmetric, in the TGARCH surface there is an asymmetry in reaction to good and bad news.

Figures 6 and 7 show an intersection of the news impact surfaces with the plains $\varepsilon_{t-2} = 0$ (solid line) and $\varepsilon_{t-1} = 0$ (dashed line). The nature of the concave shape of the solid lines can be followed to the negative estimated coefficient values in the Table 2. In case of the EGARCH we have obtained a NIC curve depicted on the Figure 8, having an expected shape, slowly increasing for positive values of $\varepsilon_{t-1}$.

Figure 6

**NIC for the GARCH (2,1) model – an intersection**

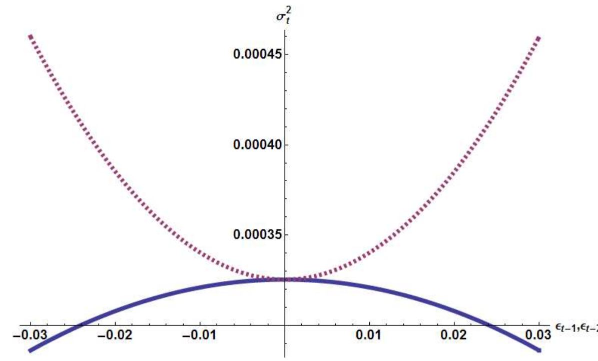



Figure 7

**NIC for the TGARCH (2,1) model – an intersection**

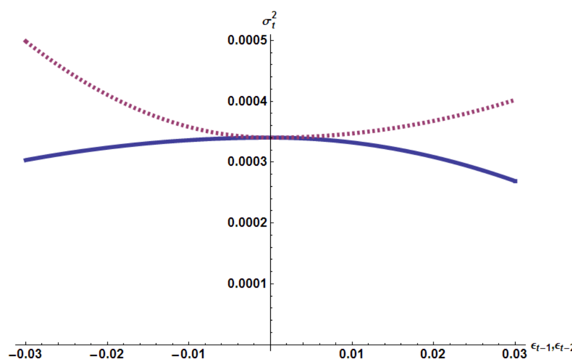
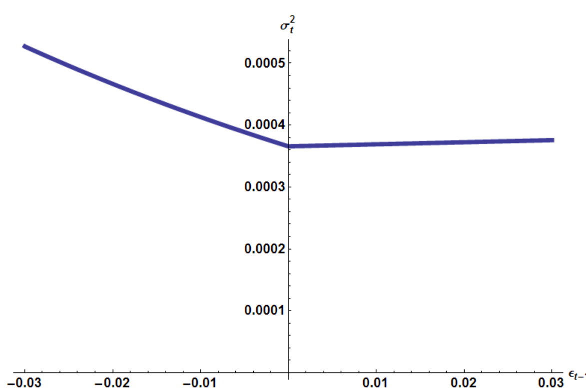



Figure 8

**NIC for the EGARCH (1,1) model**



## Conclusion

In this paper we have presented the results of an estimation of asymmetric GARCH models on two time series. The first series was made from daily returns of the S&P 500 stock index prior to the crisis, the other during the crisis. The main objective was to look for the presence of the leverage effect, influencing volatility by previous good or bad news.

The results obtained for both series were used to compute the news information curves, showing the inadequacy of using GARCH in the presence of asymmetric volatility effects, which treats all volatility equally.

The analysis of pre-crisis data showed a somewhat surprising result, where we obtained NICs from asymmetric GARCH models which were decreasing with positive lagged residuals (serving as proxy for good/bad news). The reasons for these results have been attributed to the estimated coefficients. Some of the coefficients were negative, thus producing decreasing slopes in the news impact curves.

In case of the crisis series, the interpretation was slightly more complicated, as the most common first order models were not sufficient to account for all heteroskedasticity effects present in the data. A model with the same order in both series could be only identified in the case of EGARCH. There is a marked difference in pre-crisis and crisis results, as the NIC in this case changes monotonicity for positive $\varepsilon_{t-1}$ and becomes increasing, in agreement with theory.

As most of the literature only describes the generally used GARCH (1,1) models and its NIC, it was necessary to take another approach for the higher order models. We expressed the news impact in a more general way by means of a surface instead of a curve. Even though most of the features sought – asymmetry and hence the leverage effect – could be identified, the sections of the surface describing individual impact of lagged residuals again show decreasing slopes with negative coefficients for the first ARCH term.

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