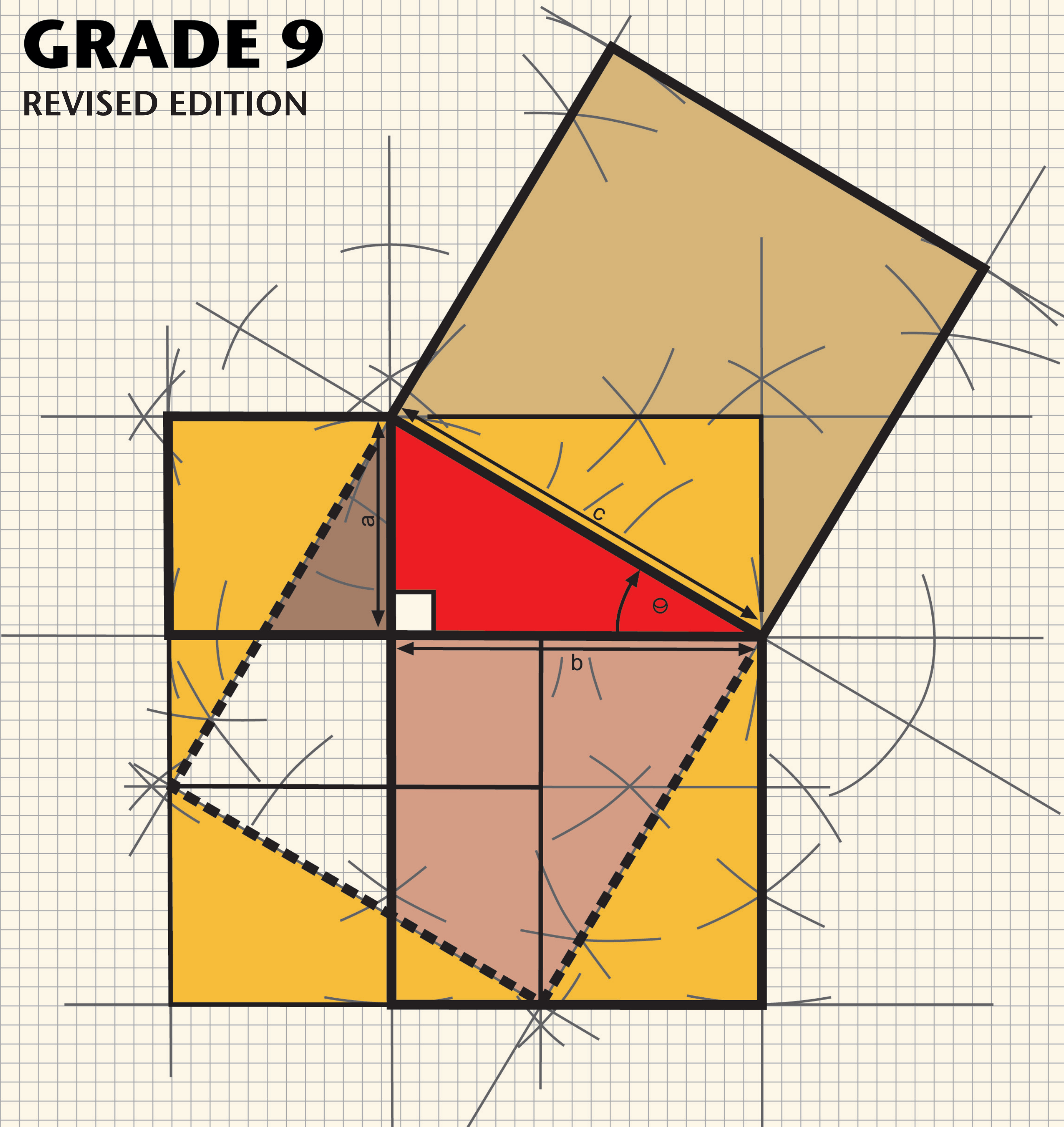
MATHEMATICS

GRADE 9

REVISED EDITION



basic education
Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

sasol

MATHEMATICS

Grade 9

CAPS

Learner Book

Revised edition

sasol inzalo foundation

UKUQONDA institute

Developed and funded as an ongoing project by the Sasol Inzalo Foundation in partnership with the Ukuqonda Institute.

Published by The Ukuqonda Institute
9 Neale Street, Rietondale 0084
Registered as a Title 21 company, registration number 2006/026363/08
Public Benefit Organisation, PBO Nr. 930035134
Website: http://www.ukuqonda.org.za

This edition published in 2017

ISBN: 978-1-4315-2881-3

This book was developed with the participation of the Department of Basic Education of South Africa with funding from the Sasol Inzalo Foundation.

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Printed by: [printer name and address]

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Chapter 1
Whole numbers

1.1 Properties of numbers

DIFFERENT TYPES OF NUMBERS

The natural numbers

The numbers that we use to count are called **natural numbers**:

1 2 3 4 5 6 7 8 9 10 11 12 13 14

Natural numbers have the following properties:

When you add two or more natural numbers, you get a natural number again.

When you multiply two or more natural numbers, you get a natural number again.

Mathematicians describe this by saying: The system of natural numbers is **closed under addition and multiplication**.

However, when a natural number is *subtracted* from another natural number, the answer is not always a natural number again. For example, there is no natural number that provides the answer to 5 – 20.

Similarly, when a natural number is *divided* by another natural number, the answer is not always a natural number again. For example, there is no natural number that provides the answer to 10 ÷ 3.

The system of natural numbers is **not closed under subtraction or division**.

When subtraction or division is done with natural numbers, the answers are not always natural numbers.

1. (a) Is there a smallest natural number, in other words, a natural number that is smaller than all other natural numbers? If so, what is it?
   (b) Is there a largest natural number, in other words, a natural number that is larger than all other natural numbers? If so, what is it?

2. In each of the following cases, say whether the answer is a natural number or not:
   (a) 100 + 400 (b) 100 – 400
   (c) 100 × 400 (d) 100 ÷ 400

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The whole numbers

Although we do not use 0 for counting, we need it to write numbers. Without 0, we would need a special symbol for 10, all multiples of 10 and some other numbers. For example, all the numbers that belong in the yellow cells below would need a special symbol.

	41	42	43	44	45	46	47	48	49
	51	52	53	54	55	56	57	58	59
	61	62	63	64	65	66	67	68	69
	71	72	73	74	75	76	77	78	79
	81	82	83	84	85	86	87	88	89
	91	92	93	94	95	96	97	98	99
	111	112	113	114	115	116	117	118	119

The natural numbers combined with 0 is called the system of **whole numbers**.

If you are working with natural numbers and you add two numbers, the answer will always be different from any of the two numbers added. For example: 21 + 25 = 46 and 24 + 1 = 25. If you are working with whole numbers, in other words including 0, this is not the case. When 0 is added to a number the answer is just the number you start with: 24 + 0 = 24.

For this reason, 0 is called the **identity element** for addition. In the set of natural numbers there is no identity element for addition.

3. Is there an identity element for multiplication in the whole numbers? Explain your answer.
4. (a) What is the smallest natural number?
   (b) What is the smallest whole number?

The integers

In the set of whole numbers, no answer is available when you subtract a number from a number smaller than itself. For example, there is no whole number that is the answer for 5 – 8. But there is an answer to this subtraction in the system of integers.

For example: 5 – 8 = –3. The number –3 is read as "negative 3" or "minus 3".

Whole numbers start with 0 and extend in one direction:

0 1 2 3 4 5 6 → → → ……

Integers extend in both directions:

…… ← ← ← –5 –4 –3 –2 –1 0 1 2 3 4 5 6 → → → ……

All whole numbers are also **integers**. The set of whole numbers forms part of the set of integers. For each whole number, there is a negative number that corresponds with it. The negative number –5 corresponds to the whole number 5 and the negative number –120 corresponds to the whole number 120.

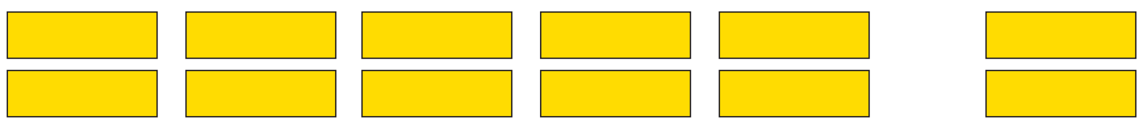
Within the set of integers, the sum of two numbers can be 0.

For example 20 + (–20) = 0 and 135 + (–135) = 0.

20 and –20 are called **additive inverses** of each other.

5. Calculate the following without using a calculator:
   (a) 100 – 165 (b) 300 – 700
6. You may use a calculator to calculate the following:
   (a) 123 – 765 (b) 385 – 723

The rational numbers



7. Five people share 12 slabs of chocolate equally among them.
   (a) Will each person get more or less than two full slabs of chocolate?
   (b) Can each person get another half of a slab?
   (c) How much more than two full slabs can each person get, if the two remaining slabs are divided as shown here?
   
   (d) Will each person get 2,4 or $2\frac{2}{5}$ slab?

The system of integers does not provide an answer for all possible division questions. For example, as we see above, the answer for 12 ÷ 5 is not an integer.

To have answers for all possible division questions, we have to extend the number system to include fractions and negative fractions, in other words, numbers of the form $\frac{\text{integer}}{\text{integer}}$. This system of numbers is called **rational numbers**. We can represent rational numbers as common fractions or as decimal numbers.

8. Express the answers for each of the following division problems in two ways. Firstly, using the common fraction notation and secondly, using the decimal notation for fractions.
   (a) 23 ÷ 10 (b) 23 ÷ 5
   (c) 230 ÷ 100 (d) 8 ÷ 10

9. Copy the table and answer the statement by writing "yes" or "no" in the appropriate cell.

Statement	Natural numbers	Whole numbers	Integers	Rational numbers
The sum of two numbers is a number of the same kind (closed under addition).				
The sum of two numbers is always bigger than either of the two numbers.				
When one number is subtracted from another, the answer is a number of the same kind (closed under subtraction).				
When one number is subtracted from another, the answer is always smaller than the first number.				
The product of two numbers is a number of the same kind (closed under addition).				
The product of two numbers is always bigger than either of the two numbers.				
The quotient of two numbers is a number of the same kind (closed under division).				
The quotient of two numbers is always smaller than the first of the two numbers.				

Irrational numbers

Rational numbers do not provide for all situations that may occur in Mathematics. For example, there is no rational number which will produce the answer 2 when it is multiplied by itself.

(number) × (same number) = 2

$2 \times 2 = 4$ and $1 \times 1 = 1$, so clearly, this number must be between 1 and 2.

But there is no number which can be expressed as a fraction, in either the common fraction or the decimal notation, which will solve this problem. Numbers like these are called **irrational numbers**.

Here are some more examples of irrational numbers:

$\sqrt{5}$ $\sqrt{10}$ $\sqrt{3}$ $\sqrt{7}$ π

Rational and irrational numbers together, are called **real numbers**.

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1.2 Calculations with whole numbers

Do **not** use a calculator in Section 1.2, unless told to do so.

ESTIMATING, ROUNDING OFF AND COMPENSATING

1. A shop owner wants to buy chickens from a farmer. The farmer wants R38 for each chicken. Answer the following questions without doing written calculations:
   (a) If the shop owner has R10 000 to buy chickens, do you think he can buy more than 500 chickens?
   (b) Do you think he can buy more than 200 chickens?
   (c) Do you think he can buy more than 250 chickens?

What you were trying to do in question 1 is called **estimation**. To estimate, when working with numbers, means to try to get close to an answer without actually doing the calculations. However, you can do other, simpler calculations to estimate.

When the goal is not to get an accurate answer, numbers may be rounded off. For example, the cost of 51 chickens at R38 each may be **approximated** by calculating 50 × 40. This is clearly much easier than calculating 51 × R38.

To approximate something means to try find out more or less how much it is, without measuring or calculating it precisely.

2. (a) How much is 5 × 4?
   (b) How much is 5 × 40?
   (c) How much is 50 × 40?

The cost of 51 chickens at R38 each is therefore, approximately R2 000.

This approximation was obtained by rounding both 51 and 38 off to the nearest multiple of 10, and then calculating with the multiples of 10.

3. In each case, estimate the cost by rounding off to calculate the approximate cost, without using a calculator. In each case, make two estimates. First make a rough estimate by rounding the numbers off to the nearest 100 before calculating. Then make a better estimate by rounding the numbers off to the nearest 10 before calculating.
   (a) 83 goats are sold for R243 each
   (b) 121 chairs are sold for R258 each
   (c) R5 673 is added to R3 277
   (d) R874 is subtracted from R1 234

Suppose you have to calculate R823 – R273.
An estimate can be made by rounding the numbers off to the nearest 100:
R800 – R300 = R500.

4. (a) By working with R800 instead of R823, an error was introduced into your answer. How can this error be corrected: by adding R23 to the R500, or by subtracting it from R500?

(b) Correct the error to get a better estimate.
(c) Now also correct the error that was made by subtracting R300 instead of R273.

What you did in question 4 is called **compensating for errors**.

5. Estimate each of the following by rounding off the numbers to the nearest 100:

(a) 812 – 342	(b) 2 342 – 1 876
(c) 812 + 342	(d) 2 342 + 1 876
(e) 9 + 278	(f) 3 231 – 1 769
(g) 8 234 – 2 776	(h) 5 213 – 3 768

6. Find the exact answer for each of the calculations in question 5, by working out the errors caused by rounding, and compensating for them.

ADDING IN COLUMNS

1. (a) Write 8 000 + 1 100 + 130 + 14 as a single number.
   (b) Write 3 000 + 700 + 50 + 8 as a single number.
   (c) Write 5 486 in expanded notation, as shown in question 1(b).

You can calculate 3 758 + 5 486 as shown on the left below.

	3 758
	5 486
Step 1	8 000
Step 2	1 100
Step 3	130
Step 4	14
	9 244

You can do this in short, as shown on the right. This is a bit harder on the brain, but it saves paper!

3 758
5 486
9 244

2. Explain how the numbers in each of Steps 1 to 4 are obtained.

It is only possible to use the shorter method if you add the units first, then add the tens, then the hundreds and finally, the thousands. You can then do what you did in question 1(a), without writing the separate terms of the expanded form.

3. Calculate each of the following:

(a) 3 878 + 3 784	(b) 298 + 8 594
(c) 10 921 + 2 472	(d) 1 298 + 18 782

4. A farmer buys a truck for R645 840, a tractor for R783 356, a plough for R83 999 and a bakkie for R435 690.

(a) Estimate to the nearest R100 000 how much these items will cost altogether.
(b) Use a calculator to calculate the total cost.

5. An investor makes R543 682 in one day on the stock market and then loses R264 359 on the same day.
   (a) Estimate to the nearest R100 000 how much money she has made in total on that day.
   (b) Use a calculator to determine how much money she has made.

MULTIPLYING IN COLUMNS

1. (a) Write 3 489 in expanded notation.
   (b) Write an expression without brackets that is equivalent to
   $7 \times (3\ 000 + 400 + 80 + 9)$.

$7 \times 3\ 489$ may be calculated as shown on the left below.

A shorter method is shown on the right.

	3 489		3 489
	× 7		× 7
Step 1	63		24 423
Step 2	560		
Step 3	2 800		
Step 4	21 000		
	24 423		

2. Explain how the numbers in each of Steps 1 to 4 on the above left are obtained.

$47 \times 3\ 489$ may be calculated as shown on the left below.

A shorter method is shown on the right.

	3 489		3 489
	× 47		× 47
Step 1	63		24 423
Step 2	560		139 560
Step 3	2 800		163 983
Step 4	21 000		
Step 5	360		
Step 6	3 200		
Step 7	16 000		
Step 8	120 000		
	163 983		

3. Explain how the numbers in each of Steps 5 to 8 on the left on page 7 are obtained.
4. Explain how the number 139 560 that appears in the shorter form on the right on page 7 is obtained.

SUBTRACTING IN COLUMNS

1. Write each of the following as a single number:
   (a) 8 000 + 400 + 30 + 2
   (b) 7 000 + 1 300 + 120 + 12
   (c) 3 000 + 900 + 50 + 7
2. If you worked correctly you should have obtained the same answers for questions 1(a) and 1(b). If this was not the case, redo your work.

The expression 7 000 + 1 300 + 120 + 12 was formed from 8 000 + 400 + 30 + 2 by:

- taking 1 000 away from 8 000 and adding it to the hundreds term to get 1 400
- taking 100 away from 1 400 and adding it to the tens term to get 130
- taking 10 away from 130 and adding it to the units term to get 12.

3. Form an expression like the expression in question 1(b) for each of the following:
   (a) 8 000 + 200 + 100 + 4 (b) 3 000 + 400 + 30 + 1
4. Write expressions like in question 1(b) for the following numbers:
   (a) 7 214 (b) 8 103

8 432 – 3 957 can be calculated as shown below:

	8 432
	– 3 957
Step 1	5
Step 2	70
Step 3	400
Step 4	4 000
Step 5	4 475

To do the subtraction in each column, you need to think of 8 432 as 8 000 + 400 + 30 + 2; in fact, you have to think of it as 7 000 + 1 300 + 120 + 12.

In Step 1, the 7 of 3 957 is subtracted from 12.

5. (a) How is the 70 in Step 2 obtained?
   (b) How is the 400 in Step 3 obtained?
   (c) How is the 4 000 in Step 4 obtained?
   (d) How is the 4 475 in Step 5 obtained?

Because of the zeros obtained in Steps 2, 3 and 4, the answers need not be written separately as shown above. The work can actually be shown in the short way on the right.

  8 432
– 3 957
  4 475

6. Calculate each of the following:
   (a) 9 123 – 3 784 (b) 8 284 – 3 547

7. Use a calculator **only** to check your answers. If your answers are wrong, try again.

8. Calculate each of the following:
   (a) 7 243 – 3 182 (b) 6 221 – 1 888

You may use a calculator to do the questions below.

9. Bettina has R87 456 in her savings account. She withdraws R44 800 to buy a car. How much money is left in her savings account?

10. Liesbet starts a savings account by making a deposit of R40 000. Over a period of time she does the following transactions on the savings account:
    - a withdrawal of R4 000
    - a withdrawal of R2 780
    - a deposit of R1 200
    - a deposit of R7 550
    - a withdrawal of R5 230
    - a deposit of R8 990
    - a deposit of R1 234

    How much money does she have in her savings account now?

11. (a) R34 537 – R13 267 (b) R135 349 – R78 239

LONG DIVISION

Study this method for calculating 13 254 ÷ 56:

	13 254	
200 × 56 = 11 200	11 200	(200 is a rough estimate of the answer for 13 254 ÷ 56)
	2 054	(2 054 remains after 11 200 is taken from 13 254)
30 × 56 = 1 680	1 680	(30 is a rough estimate of the answer for 2 054 ÷ 56)
	374	(374 remains after 1 680 is taken from 2 054)
6 × 56 = 336	336	(6 is an estimate of the answer for 374 ÷ 56)
236 × 56 = 13 216	38	(38 remains)

So, 13 254 ÷ 56 = 236 remainder 38, or 13 254 ÷ 56 = $236\frac{38}{56} = 236\frac{19}{28}$, which can also be written as 236,68 (correct to two decimal figures).

The work can also be set out as follows:

$$\begin{array}{r|r} & 6 \\ & 30 \\ & 200 \\ \hline 56 & 13\,254 \\ & \underline{11\,200} \\ & 2\,054 \\ & \underline{1\,680} \\ & 374 \\ & \underline{336} \\ & 38 \end{array} \quad \text{or more briefly as} \quad \begin{array}{r|r} & 236 \\ \hline 56 & 13\,254 \\ & \underline{11\,200} \\ & 2\,054 \\ & \underline{1\,680} \\ & 374 \\ & \underline{336} \\ & 38 \end{array}$$

1. (a) Mlungisi's work to do a certain calculation is shown on the right. What is the question that Mlungisi tries to answer?
   (b) Where does the number 31 200 in Step 1 come from? How did Mlungisi obtain it, and for what purpose did he calculate it?
   (c) Explain Step 2 in the same way as you explained Step 1.
   (d) Explain Step 3.

$$\begin{array}{lr|r} & & 463 \\ \hline & 78 & 36\,177 \\ \textbf{Step 1} & & \underline{31\,200} \\ \textbf{Step 2} & & 4\,977 \\ \textbf{Step 3} & & \underline{4\,680} \\ \textbf{Step 4} & & 297 \\ \textbf{Step 5} & & \underline{234} \\ & & 63 \end{array}$$

2. Calculate each of the following without using a calculator:
   (a) 33 030 ÷ 63　　(b) 18 450 ÷ 27

3. Use a calculator to check your answers to question 2. If your answers are wrong, try again. It is important that you learn to do long division correctly.

4. Calculate each of the following:
   (a) 76 287 ÷ 287　　(b) 65 309 ÷ 44

Use your calculator to do questions 5 and 6 below.

5. A municipality has budgeted R85 000 for putting up new street name boards. The street name boards cost R72 each. How many new street name boards can be put up, and how much money will be left in the budget?

6. A furniture dealer quoted R840 000 for supplying 3 450 school desks. A school supply company quoted R760 000 for supplying 2 250 of the same desks. Which provider is cheapest, and what do the two providers actually charge for one school desk?

1.3 Multiples and factors

LOWEST COMMON MULTIPLES AND PRIME FACTORISATION

1. Consecutive multiples of 6, starting at 6 itself, are shown in the following table:

6	12	18	24	30	36	42	48	54	60
66	72	78	84	90	96	102	108	114	120
126	132	138	144	150	156	162	168	174	180
186	192	198	204	210	216	222	228	234	240

(a) The following table also shows multiples of a number. What is the number?

15	30	45	60	75	90	105	120	135	150
165	180	195	210	225	240	255	270	285	300
315	330	345	360	375	390	405	420	435	450
465	480	495	510	525	540	555	570	585	600

(b) Copy both tables. Draw rough circles around all the numbers that occur in both tables.

(c) What is the smallest number that occurs in both tables?

90 is a multiple of 6; it is also a multiple of 15.

90 is called a **common multiple** of 6 and 15; it is a multiple of both.

The smallest number that is a multiple of both 6 and 15 is the number 30.

30 is called the **lowest common multiple** or **LCM** of 6 and 15.

2. Calculate, without using a calculator:

(a) 2 × 3 × 5 × 7 × 11

(b) 2 × 2 × 5 × 7 × 13

(c) 2 × 3 × 3 × 3 × 5 × 13

(d) 3 × 5 × 5 ×17

Check your answers by using a calculator or by comparing with some classmates.

The number 2 is a factor of each of the numbers 2 310, 1 820 and 3 510.

Another way of saying this is: 2 is a **common factor** of 2 310, 1 820 and 3 510.

3. (a) Is 2 × 3, in other words, 6, a common factor of 2 310 and 3 510?

(b) Is 2 × 3 × 5, in other words, 30, a common factor of 2 310 and 3 510?

(c) Is there any bigger number than 30 that is a common factor of 2 310 and 3 510?

30 is called the **highest common factor** or **HCF** of 2 310 and 3 510.

In question 2 you can see the list of **prime factors** of the numbers 2 310, 1 820, 3 510 and 1 275.

The LCM of two numbers can be found by multiplying all the prime factors of both numbers, without repeating (except where a number is repeated as a factor in one of the numbers).

The HCF of two numbers can be found by multiplying the factors that are common to the two numbers, i.e. in the list of prime factors of both numbers.

4. In each case, find the HCF and LCM of the numbers:

(a) 1 820 and 3 510
(b) 2 310 and 1 275
(c) 1 820 and 3 510 and 1 275
(d) 2 310 and 1 275 and 1 820
(e) 780 and 7 700
(f) 360 and 1 360

1.4 Solving problems about ratio, rate and proportion

RATIO AND RATE PROBLEMS

You **may** use a calculator in this section.

1. Moeneba collects apples in the orchard. She picks about five apples each minute. Approximately how many apples will Moeneba pick in each of the following periods of time?

(a) eight minutes
(b) 11 minutes
(c) 15 minutes
(d) 20 minutes

In the situation described in question 1, Moeneba picks apples **at a rate of** about five apples **per minute**.

2. Garth and Kate also collect apples in the orchard, but they both work faster than Moeneba. Garth collects at a rate of about 12 apples per minute, and Kate collects at a rate of about 15 apples per minute. Copy and complete the following table to show approximately how many apples they will each collect in different periods of time:

Period of time in min	1	2	3	8	10	20
Moeneba	5			40		
Garth	12					
Kate	15					
The three together	32					

In this situation, the number of apples picked is **directly proportional** to the time taken.

If you filled the table in correctly, you will notice that during any period of time, Kate collected three times as many apples as Moeneba. We can say that during any time interval, the **ratio** between the numbers of apples collected by Moeneba and Kate is **3 to 1**, which can be written as **3 : 1**. For any period of time, the ratio between the numbers of apples collected by Garth and Moeneba is 12 : 5.

3. (a) What is the ratio between the numbers of apples collected by Kate and Garth during a period of time?
   (b) Would it be correct to also say that the ratio between the numbers of apples collected by Kate and Garth is 5 : 4? Explain your answer.

4. To make biscuits of a certain kind, five parts of flour are to be mixed with two parts of oatmeal, and one part of cocoa powder. How much oatmeal and how much cocoa powder must be used if 500 g of flour is used?

5. A motorist covers a distance of 360 km in exactly four hours.
   (a) Approximately how far did the motorist drive in one hour?
   (b) Do you think the motorist covered exactly 90 km in each of the four hours? Explain your answer briefly.
   (c) Approximately how far will the motorist drive in seven hours?
   (d) Approximately how long will the motorist need to travel 900 km?

Some people use these formulae to do calculations like those in question 5:

$$\textbf{average speed} = \frac{\textbf{distance}}{\textbf{time}}$$, which means distance ÷ time

distance = average speed × time

$$\textbf{time} = \frac{\textbf{distance}}{\textbf{average speed}}$$, which means distance ÷ average speed

6. For each of questions 5(c) and 5(d), state which formula will produce the correct answer.

7. A motorist completes a journey in three sections, making two long stops to eat and relax between sections. During section A he covers 440 km in four hours. During section B he covers 540 km in six hours. During section C he covers 280 km in four hours.
   (a) Calculate his average speed over each of the three sections.
   (b) Calculate his average speed for the journey as a whole.
   (c) On the next day, the motorist has to travel 874 km. How much time (stops excluded) will he need to do this? Justify your answer with calculations.

8. Different vehicles travel at different average speeds. A large transport truck with a heavy load travels much slower than a passenger car. A small bakkie is also slower than a passenger car. In the table on the following page, some average speeds and the

times needed are given for different vehicles that all have to be driven for the same distance of 720 km. Copy and complete the table:

Time in hours	12	9	8	6	5
Average speed in km/h	60				

9. Look at the table you have just completed.
   (a) What happens to the time needed if the average speed increases?
   (b) What happens to the average speed if the time is reduced?
   (c) What can you say about the product average speed × time, for the numbers in the table?

In the situation above, the average speed is said to be **indirectly proportional** to the time needed for the journey.

1.5 Solving problems in financial contexts

You **may** use a calculator in this section.

DISCOUNT, PROFIT AND LOSS

1. (a) R12 800 is divided equally between 100 people. How much money does each person get?
   (b) How much money do eight of the people together get?

Another word for hundredths is **per cent**.

Instead of $\frac{5}{100}$ we can write 5%. The symbol % means exactly the same as $\frac{}{100}$.
In question 1(a) you calculated $\frac{1}{100}$ or 1% of R12 800, and in question 1(b) you calculated $\frac{8}{100}$ or 8% of R12 800.

The amount that a dealer pays for an article is called the **cost price**. The price marked on the article is called the **marked price** and the price of the article after the discount is the **selling price**.

2. The marked prices of some articles are given below. A discount of 15% is offered to customers who pay cash. In each case, calculate how much a customer who pays cash will actually pay:
   (a) R850
   (b) R140
   (c) R32 600
   (d) R138

Lina bought a couch at a sale. It was marked R3 500 but she paid only R2 800.

She was given a discount of R700.

What percentage discount was given to Lina?

This question means:

How many hundredths of the marked price were taken off?

To answer the question we need to know how much $\frac{1}{100}$ (one hundredth) of the marked price is.

3. (a) How much is $\frac{1}{100}$ of R3 500?
   (b) How many hundredths of R3 500 is the same as R700?
   (c) What percentage discount was given to Lisa: 10% or 20%?

4. The cost price, marked price and selling price of some articles are listed below:

   Article A: Cost price = R240; marked price = R360; selling price = R324.

   Article B: Cost price = R540; marked price = R700; selling price = R560.

   Article C: Cost price = R1 200; marked price = R2 000; selling price = R1 700.

   The profit is the difference between the cost price and the selling price.

   For each of the above articles, calculate the percentage discount and profit.

5. Remey decided to work from home and bought herself a sewing machine for R750. She planned to make 40 covers for scatter cushions. The fabric and other items she needed cost her R3 600. Remey planned to sell the covers at R150 each.
   (a) How much profit could Remey make if she sold all 40 covers at this price?
   (b) Remey managed to sell only 25 of the covers and decided to sell the rest at R100 each. Calculate her percentage profit.

6. Zadie bakes and sells pies to earn some extra income. The cost of the ingredients for one chicken pie comes to about R68. She sold the pies for R60 each. Did she make a profit or a loss? Calculate the percentage loss or profit.

HIRE PURCHASE

Sometimes you need an item but do not have enough money to pay the full amount immediately. One option is to buy the item on **hire purchase** (**HP**). You will have to pay a deposit and sign an agreement in which you undertake to pay monthly instalments until you have paid the full amount. Therefore:

HP price = deposit + total of instalments

The difference between the HP price and the cash price is the interest that the dealer charges you for allowing you to pay off the item over a period of time.

1. Sara buys a flat screen television on HP. The cash price is R4 199. She has to pay a deposit of R950 and 12 monthly instalments of R360.

(a) Calculate the total HP price.
(b) How much interest does she pay?

2. Susie buys a car on HP. The car costs R130 000. She pays a 10% deposit on the cash price and will have to pay monthly instalments of R4 600 for a period of three years. David buys the same car, but chooses another option where he has to pay a 35% deposit on the cash price and monthly instalments of R3 950 for two years.
   (a) Calculate the HP price for both options.
   (b) Calculate the difference between the total price paid by Susie and by David.
   (c) Calculate the interest that Susie and David have to pay as a percentage of the cash price.

SIMPLE INTEREST

When interest is calculated for a number of years on an amount (i.e. a fixed deposit), without the interest being added to the amount each year for the purpose of later interest calculations, it is referred to as **simple interest**. If the amount is invested for part of a year, the time must be written as a fraction of a year.

Example:

R2 000 invested at 10% per annum simple interest over 2 years:

End of first year: Amount = R2 000 + R200 interest of original amount = R2 200

End of second year: Amount = R2 200 + R200 interest of original amount = R2 400

1. Interest rates are normally expressed as percentages. This makes it easier to compare rates. Express each of the following as a percentage:
   (a) A rate of R5 for every R100
   (b) A rate of R7,50 for every R50
   (c) A rate of R20 for every R200
   (d) A rate of x rands for every a rands

2. Annie deposits R8 345 into a savings account at Bonus Bank. The interest rate is 9% per annum.
   (a) How much interest will she have earned at the end of the first year?
   (b) Annie decides to leave the deposit of R8 345 with the bank for an indefinite period, and to withdraw only the interest at the end of every year. How much interest does she receive over a period of five years?

Per annum means "per year".

3. Maxi invested R3 500 at an interest rate of 5% per annum. Her total interest was R875. For what period did she invest the amount?

4. Money is invested for one year at an interest rate of 8% per annum. Copy and complete the table of equivalent rates:

Sum invested (R)	1 000	2 500	8 000	20 000	90 000	x
Interest earned (R)						

5. Interest on overdue accounts is charged at a rate of 20% per annum. Calculate the interest due on an account that is ten days overdue if the amount owing is R260. (Give your answer to the nearest cent.)

6. A sum of money invested in the bank at 5% per annum, i.e. simple interest, amounted to R6 250 after five years. This final amount includes the interest. Thuli figured out that the final amount is $(1 + 0{,}05 \times 5) \times$ amount invested.

   (a) Explain Thuli's thinking.
   (b) Calculate the amount that was invested.

COMPOUND INTEREST

When the interest earned each year is added to the original amount, and the interest for the following year is calculated on this new amount, the result is known as **compound interest**.

Example:

R2 000 is invested at 10% per annum compound interest:

End of first year:	Amount = R2 000 + R200 interest = R2 200
End of second year:	Amount = R2 200 + R220 interest = R2 420
End of third year:	Amount = R2 420 + R242 interest = R2 662

1. An amount of R20 000 is invested at 5% per annum compound interest.
   (a) What is the total value of the investment after one year?
   (b) What is the total value of the investment after two years?
   (c) What is the total value of the investment after three years?

2. Bonus Bank is offering an investment scheme over two years at a compound interest rate of 15% per annum. Mr Pillay wishes to invest R800 in this way.

   (a) How much money will be due to him at the end of the two-year period?
   (b) How much interest will have been earned during the two years?

3. Andrew and Zinzi are arguing about interest on money that they have been given for Christmas. They each received R750. Andrew wants to invest his money in ABC Building Society for two years at a compound interest rate of 14% per annum, while Zinzi claims that she will do better at Bonus Bank, earning 15% simple interest per annum over two years. Who is correct?

4. Mr Martin invests an amount (P) of R12 750 at 5,3% (r) compound interest over a period (n) of four years. Use the formula: $A = P(1 + \frac{r}{100})^n$ and calculate the final amount (A) that his investment will be worth after four years.
   (a) How many conversion periods will his investment have altogether?
   (b) How much is his investment worth after four years?
   (c) Calculate the total interest that he earns on his initial investment.
5. Calculate the interest generated by an investment (P) of R5 000 at 10% (r) compound interest over a period (n) of three years. A is the final amount. Use the formula: $A = P(1 + \frac{r}{100})^n$ to calculate the interest.

EXCHANGE RATE AND COMMISSION

1. (a) Tim bought £650 at the foreign exchange desk at Gatwick Airport in the UK at a rate of R15,66 per £1. The desk also charged 2,5% commission on the transaction. How much did Tim spend to buy the pounds?
   (b) What was the value of R1 in British pounds on that day?
2. Mandy wants to order a book from the internet. The price of the book is $25,86. What is the price of the book in rands? Say, for example, that the exchange rate is R9,95 for $1.
3. Bongani is a car salesperson. He earns a commission of 3% on the sale of a car with the value of R220 000. Calculate how much commission he earned.

CHAPTER 2
Integers

2.1 Which numbers are smaller than 0?

WHY PEOPLE DECIDED TO HAVE NEGATIVE NUMBERS

Numbers such as –7 and –500, the additive inverses of whole numbers, are included with all the whole numbers and are called **integers**.
Fractions can be negative too, for example: $-\frac{3}{4}$ and –3,46.

Natural numbers are used for counting and fractions (rational numbers) are used for measuring. Why do we also have negative numbers?

When a larger number is subtracted from a smaller number, the answer may be a negative number: 5 – 12 = –7. This number is called **negative 7**.

One of the most important reasons for inventing negative numbers was to provide solutions for equations like the following:

Equation	Solution	Required property of negative numbers
$17 + x = 10$	$x = -7$ because $17 + (-7) = 17 - 7$ $= 10$	1. Adding a negative number is just like subtracting the corresponding positive number
$5 - x = 9$	$x = -4$ because $5 - (-4) = 5 + 4 = 9$	2. Subtracting a negative number is just like adding the corresponding positive number
$20 + 3x = 5$	$x = -5$ because $3 \times (-5) = -15$	3. The product of a positive number and a negative number is a negative number

PROPERTIES OF INTEGERS

1. In each case, state what number will make the equation true. Also state which of the properties of integers in the table above, is demonstrated by the equation:
   (a) $20 - x = 50$
   (b) $50 + x = 20$
   (c) $20 - 3x = 50$
   (d) $50 + 3x = 20$

2.2 Adding and subtracting with integers

Addition and subtraction of negative numbers

Examples: (–5) + (–3) and (–20) – (–7)

This is done in the same way as the addition and subtraction of positive numbers.

(–5) + (–3) can also be written as –5 + (–3) or as –5 + –3

$(-5) + (-3) = -8$ and $-20 - (-7) = -13$

This is just like 5 + 3 = 8 and 20 – 7 =13, or R5 + R3 = R8, and R20 – R7 = R13.

Subtraction of a larger number from a smaller number

Examples: 5 – 9 and 29 – 51

Let us first consider the following:

$5 + (-5) = 0$ $10 + (-10) = 0$ and $20 + (-20) = 0$

If we subtract 5 from 5, we get 0, but then we still have to subtract 4:

$$\begin{aligned} 5 - 9 &= \underbrace{5 - 5} - 4 \\ &= 0 - 4 \\ &= -4 \end{aligned}$$

We know that –9 = (–4) + (–5)

Suppose the numbers are larger, for example 29 – 51:

$$29 - 51 = 29 - 29 - 22$$

–51 = (–29) + (–22)

How much will be left of the 51, after you have subtracted 29 from 29 to get 0?

How can we find out? Is it 51 – 29?

Addition of a positive and a negative number

Examples: 7 + (–5); 37 + (–45) and (–13) + 45

The following statement is true if the unknown number is 5:

20 – (a certain number) = 15

We also need numbers that will make sentences like the following true:

20 + (a certain number) = 15

But to go from 20 to 15 you have to subtract 5.

The number we need to make the sentence *20 + (a certain number) = 15* true, must have the following strange property:

If you **add** this number, it should have the **same effect** as **subtracting 5**.

So, mathematicians agreed that the number called negative 5 will have the property that if you add it to another number, the effect will be the same as subtracting the natural number 5.

This means that mathematicians agreed that 20 + (–5) is equal to 20 – 5.

In other words, instead of adding *negative 5* to a number, you may subtract 5.

> Adding a negative number has the same effect as subtracting a corresponding natural number.
> For example: 20 + (–15) = 20 – 15 = 5.

Subtraction of a negative number

We have dealt with cases like –20 – (–7) on the previous page.

The following statement is true if the number is 5:

25 + (a certain number) = 30

We also need a number to make this statement true:

25 – (a certain number) = 30

If you subtract this number, it should have the same effect as adding 5.

It was agreed that: *25 – (–5) is equal to 25 + 5*.

Instead of subtracting the negative number, you add the corresponding positive number (the additive inverse):

$$\begin{aligned} 8 - (-3) &= 8 + 3 \\ &= 11 \\ -5 - (-12) &= -5 + 12 \\ &= 7 \end{aligned}$$

> We may say that for each "positive" number there is a **corresponding** or **opposite** negative number.
> Two positive and negative numbers that correspond, for example 3 and (–3), are called **additive inverses**.

Subtraction of a positive number from a negative number

For example: –7 – 4 actually means (–7) – 4.

Instead of subtracting a positive number, you add the corresponding negative number.

For example: –7 – 4 can be seen as (–7) + (–4) = –11.

CALCULATIONS WITH INTEGERS

Calculate each of the following:

1. –7 + 18
2. 24 – 30 – 7
3. –15 + (–14) – 9
4. 35 – (–20)
5. 30 – 47
6. (–12) – (–17)

2.3 Multiplying and dividing with integers

MULTIPLICATION WITH INTEGERS

1. Calculate each of the following:
   (a) $-7 + -7 + -7 + -7 + -7 + -7 + -7 + -7 + -7 + -7$
   (b) $-10 + -10 + -10 + -10 + -10 + -10 + -10$
   (c) $10 \times (-7)$ (d) $7 \times (-10)$

2. Say whether you agree (✓) or (✗) disagree with each statement:
   (a) $10 \times (-7) = 70$ (b) $9 \times (-5) = (-9) \times 5$
   (c) $(-7) \times 10 = 7 \times (-10)$ (d) $9 \times (-5) = -45$
   (e) $(-7) \times 10 = 10 \times (-7)$ (f) $5 \times (-9) = 45$

Multiplication of integers is commutative:
$(-20) \times 5 = 5 \times (-20)$

THE DISTRIBUTIVE PROPERTY

1. Calculate each of the following. Note that brackets are used for two purposes in these expressions, i.e. to indicate that certain operations are to be done first, and to show the integers.
   (a) $20 + (-5)$ (b) $4 \times \mathbf{(}20 + (-5)\mathbf{)}$ (c) $4 \times 20 + 4 \times (-5)$
   (d) $(-5) + (-20)$ (e) $4 \times \mathbf{(}(-5) + (-20)\mathbf{)}$ (f) $4 \times (-5) + 4 \times (-20)$

2. If you worked correctly, your answers for question 1 should be 15; 60; 60; –25; –100 and –100. If your answers are different, check to see where you went wrong and correct your work.

3. Calculate each of the following where you can:
   (a) $20 + (-15)$ (b) $4 \times \mathbf{(}20 + (-15)\mathbf{)}$ (c) $4 \times 20 + 4 \times (-15)$
   (d) $(-15) + (-20)$ (e) $4 \times \mathbf{(}(-15) + (-20)\mathbf{)}$ (f) $4 \times (-15) + 4 \times (-20)$
   (g) $10 + (-5)$ (h) $(-4) \times \mathbf{(}10 + (-5)\mathbf{)}$ (i) $(-4) \times 10 + \mathbf{(}(-4) \times (-5)\mathbf{)}$

4. What property of integers is demonstrated in your answers for questions 3(a) and (g)? Explain your answer.

In question 3(i) you had to multiply two negative numbers. What was your guess?
We can calculate $(-4) \times \mathbf{(}10 + (-5)\mathbf{)}$ as in (h). It is $(-4) \times 5 = -20$.

If we want the distributive property to be true for integers, then $(-4) \times 10 + (-4) \times (-5)$ must be equal to –20.

$(-4) \times 10 + (-4) \times (-5) = -40 + (-4) \times (-5)$

Then (–4) × (–5) must be equal to 20.

5. Calculate each of the following:
   (a) 10 × 50 + 10 × (–30)
   (b) 50 + (–30)
   (c) 10 × **(**50 + (–30)**)**
   (d) (–50) + (–30)
   (e) 10 × (–50) + 10 × (–30)
   (f) 10 × **(**(–50) + (–30)**)**

- The product of two positive numbers is a positive number, for example: 5 × 6 = 30.
- The product of a positive number and a negative number is a negative number, for example: 5 × (–6) = –30.
- The product of a negative number and a positive number is a negative number, for example: (–5) × 6 = –30.

6. (a) Write out only the numerical expressions below which you would expect to have the same answers. Do not do the calculations.

   16 × (53 + 68)   53 × (16 + 68)   16 × 53 + 16 × 68   16 × 53 + 68

   (b) What property of operations is demonstrated by the fact that two of the above expressions have the same value?

7. Consider your answers for question 5.
   (a) Does multiplication distribute over addition in the case of integers?
   (b) Illustrate your answer with two examples.

8. Write out only the numerical expression below which you would expect to have the same answers. Do not do the calculations now.

   10 × **(**(–50) – (–30)**)**   10 × (–50) – (–30)   10 × (–50) – 10 × (–30)

9. Do the three sets of calculations given in question 8.

10. Calculate (–10) × **(**5 + (–3)**)**.

11. Now consider the question of whether or not multiplication by a negative number distributes over addition and subtraction of integers. For example, would (–10) × 5 + (–10) × (–3) also have the answer of –20, like (–10) × **(**5 + (–3)**)**?

To make sure that multiplication distributes over addition and subtraction in the system of integers, we have to agree that:

(a negative number) × (a negative number) is a positive number.

For example: (–10) × (–3) = 30.

12. Calculate each of the following:

(a) $(-20) \times (-6)$
(b) $(-20) \times 7$
(c) $(-30) \times (-10) + (-30) \times (-8)$
(d) $(-30) \times \big((-10) + (-8)\big)$
(e) $(-30) \times (-10) - (-30) \times (-8)$
(f) $(-30) \times \big((-10) - (-8)\big)$

Here is a summary of the properties of integers that make it possible to do calculations with integers:

- When a number is added to its additive inverse, the result is 0. For example, $(+12) + (-12) = 0$.
- Adding an integer has the same effect as subtracting its additive inverse. For example, $3 + (-10)$ can be calculated by doing $3 - 10$, and the answer is -7.
- Subtracting an integer has the same effect as adding its additive inverse. For example, $3 - (-10)$ can be calculated by calculating $3 + 10$ is 13.
- The product of a positive and a negative integer is negative. For example, $(-15) \times 6 = -90$.
- The product of a negative and a negative integer is positive. For example, $(-15) \times (-6) = 90$.

DIVISION WITH INTEGERS

1. Calculate each of the following:

(a) $5 \times (-7)$
(b) $(-3) \times 20$
(c) $(-5) \times (-10)$
(d) $(-3) \times (-20)$

2. Use your answers in question 1 to determine the following:

(a) $(-35) \div 5$
(b) $(-35) \div (-7)$
(c) $(-60) \div 20$
(d) $(-60) \div (-3)$
(e) $50 \div (-5)$
(f) $50 \div (-10)$
(g) $60 \div (-20)$
(h) $60 \div (-3)$

- The quotient of a positive number and a negative number is a negative number.
- The quotient of two negative numbers is a positive number.

MIXED CALCULATIONS WITH INTEGERS

1. Calculate each of the following:

(a) $20(-50 + 7)$
(b) $20 \times (-50) + 20 \times 7$
(c) $20(-50 + -7)$
(d) $20 \times (-50) + 20 \times -7$
(e) $-20(-50 + -7)$
(f) $-20 \times -50 + -20 \times -7$

2. Calculate each of the following:
   (a) $40 \times (-12 + 8) - 10 \times (2 + -8) - 3 \times (-3 - 8)$
   (b) $(9 + 10 - 9) \times 40 + (25 - 30 - 5) \times 7$
   (c) $-50(40 - 25 + 20) + 30(-10 + 7 + 13) - 40(-16 + 15 - 2)$
   (d) $-4 \times (30 - 50) + 7 \times (40 - 70) - 10 \times (60 - 100)$
   (e) $-3 \times (-14 - 6 + 5) \times (-13 - 7 + 10) \times (20 - 10 - 15)$

2.4 Powers, roots and word problems

Answer all questions in this section **without** using a calculator.

1. Copy and complete the following tables:

(a)

x	1	2	3	4	5	6	7	8	9	10	11	12
x^2												
x^3												

(b)

x	–1	–2	–3	–4	–5	–6	–7	–8	–9	–10	–11	–12
x^2												
x^3												

3^2 is 9 and $(-3)^2$ is also 9.
3^3 is 27 and $(-5)^3$ is -125.
Both (–3) and 3 are **square roots** of 9.
3 may be called the **positive square root** of 9, and (–3) may be called the **negative square root** of 9.
3 is called the **cube root** of 27, because $3^3 = 27$.
–5 is called the cube root of –125 because $(-5)^3 = -125$.
10^2 is 100 and $(-10)^2$ is also 100.
Both 10 and (–10) are called **square roots** of 100.

The symbol $\sqrt{\ }$ means that you must take the **positive square root** of the number.

2. Calculate each of the following:
   (a) $\sqrt{4} - \sqrt{9}$
   (b) $\sqrt[3]{27} + (-\sqrt[3]{64})$
   (c) $-(3^2)$
   (d) $(-3)^2$
   (e) $4^2 - 6^2 + 1^2$
   (f) $3^3 - 4^3 - 2^3 - 1^3$
   (g) $\sqrt{81} - \sqrt{4} \times \sqrt[3]{125}$
   (h) $-(4^2)(-1)^2$
   (i) $\dfrac{(-5)^2}{\sqrt{37-12}}$
   (j) $\dfrac{-\sqrt{36}}{-1^3 - 2^3}$

3. Determine the answer to each of the following:
   (a) The overnight temperature in Polokwane drops from 11 °C to –2 °C. By how many degrees has the temperature dropped?
   (b) The temperature in Escourt drops from 2 °C to –1 °C in one hour, and then another two degrees in the next hour. How many degrees in total did the temperature drop over the two hours?
   (c) A submarine is 75 m below the surface of the sea. It then rises by 21 m. How far below the surface is it now?
   (d) A submarine is 37 m below the surface of the sea. It then sinks a further 15 m. How far below the surface is it now?

CHAPTER 3
Fractions

3.1 Equivalent fractions

THE SAME NUMBER IN DIFFERENT FORMS

1. How much money is each of the following amounts?

   (a) $\frac{1}{5}$ of R200 (b) $\frac{2}{10}$ of R200 (c) $\frac{4}{20}$ of R200

Did you notice that all the answers are the same? That is because $\frac{1}{5}$, $\frac{2}{10}$ and $\frac{4}{20}$ are **equivalent fractions**. They are different ways of writing the same number.

Consider this bar. It is divided into five equal parts.

Each piece is **one fifth** of the whole bar.

2. Now copy the bar and draw lines on the bar so that it is approximately divided into ten equal parts.

   (a) What part of the whole bar is each of your ten parts?
   (b) How many tenths is the same as one fifth?
   (c) How many tenths is the same as two fifths?
   (d) How many fifths is the same as eight tenths?

3. Copy the bar below and draw lines on the bar below so that it is approximately divided into 25 equal parts.

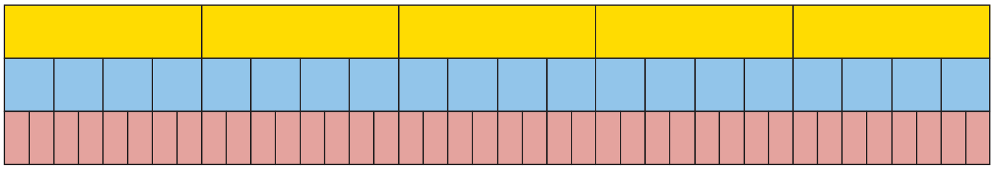
   (a) How many twenty-fifths is the same as two fifths?
   (b) How many fifths is the same as 20 twenty-fifths?

In question 3(b) you found that $\frac{4}{5}$ is equivalent to $\frac{20}{25}$: these are just two different ways to describe the same part of the bar.

This can be expressed by writing $\frac{4}{5} = \frac{20}{25}$ which means that $\frac{4}{5}$ and $\frac{20}{25}$ are equivalent to each other.

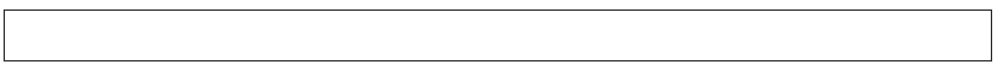
4. Write down all the other pairs of equivalent fractions which you found while doing questions 2 and 3.

The yellow bar is divided into fifths.



5. (a) Into what kind of fraction parts is the blue bar divided?
   (b) Into what kind of fraction parts is the red bar divided?
   (c) If you want to mark the yellow bar in twentieths (like the blue bar), into how many parts do you have to divide each of the fifths?
   (d) If you want to mark the yellow bar in fortieths (like the red bar), into how many parts do you have to divide each of the fifths?
   (e) If you want to mark the yellow bar in eightieths, into how many parts do you have to divide each of the fifths?
   (f) If you want to mark the blue bar in eightieths, into how many parts do you have to divide each of the twentieths?

6. Suppose this bar is divided into four equal parts, in other words, quarters.



   (a) If the bar is also divided into 20 equal parts, how many of these smaller parts will there be in each quarter?
   (b) If each quarter is divided into six equal parts, what part of the whole bar will each small part be?

7. Copy and complete this table of equivalent fractions, as far as you can using whole numbers. All the fractions in each column must be equivalent.

sixteenths	8	4	2	10	14	12
eighths						
quarters						
twelfths						
twentieths						

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Equivalent fractions can be formed by multiplying the numerator and denominator by the same number. For example: $\frac{1}{5} = \frac{4 \times 1}{4 \times 5} = \frac{4}{20}$

8. Write down five different fractions that are equivalent to $\frac{3}{4}$.
9. Express each of the following numbers as twelfths:

   (a) $\frac{2}{3}$ (b) $\frac{3}{4}$ (c) $\frac{5}{6}$ (d) $\frac{1}{6}$

You may divide the numerator and denominator by the same number, instead of multiplying the numerator and denominator by the same number. This gives you a simpler fraction.

The **simplest form** of a fraction has no common factors. For example, you find the simplest form of the fraction $\frac{4}{12}$ is $\frac{1}{3}$ by dividing both the numerator and denominator by the common factor of 4.

10. Convert each of the following fractions to their simplest form:

   (a) $\frac{40}{100}$ (b) $\frac{4}{16}$

   (c) $\frac{5}{25}$ (d) $\frac{6}{30}$

   (e) $\frac{6}{24}$ (f) $\frac{8}{88}$

CONVERTING BETWEEN MIXED NUMBERS AND FRACTIONS

Numbers that have both whole number and fraction parts are called **mixed numbers**.

Examples of mixed numbers: $3\frac{4}{5}$, $2\frac{7}{8}$ and $8\frac{3}{10}$

Mixed numbers can be written in expanded notation, for example:

$3\frac{4}{5}$ means $3 + \frac{4}{5}$ $2\frac{7}{8}$ means $2 + \frac{7}{8}$ $8\frac{3}{10}$ means $8 + \frac{3}{10}$.

To add and subtract mixed numbers, you can work with the whole number parts and the fraction parts separately, for example:

$3\frac{4}{5} + 13\frac{3}{5}$

$= 16\frac{7}{5}$

$= 17\frac{2}{5}$

$13\frac{3}{5} - 3\frac{4}{5}$ (we need to "borrow" a unit from 13, because we cannot subtract $\frac{4}{5}$ from $\frac{3}{5}$)

$= 12\frac{8}{5} - 3\frac{4}{5}$

$= 9\frac{4}{5}$

However, this method can be difficult to do with some examples, and it does not work with multiplication and division.

An alternative and preferred method is to convert the mixed number to an **improper fraction**, as shown in the example below:

$3\frac{4}{5}$

$= 3 + \frac{4}{5}$

$= \frac{15}{5} + \frac{4}{5}$

$= \frac{19}{5}$

NOTE
You can obtain the numerator of 19 in one step by multiplying the denominator (5) by the whole number (3), and then adding the numerator (4).

So, you can calculate $3\frac{4}{5} + 13\frac{3}{5}$ using this method:

$3\frac{4}{5} + 13\frac{3}{5}$

$= \frac{19}{5} + \frac{68}{5}$

$= \frac{87}{5}$

The answer must be converted to a mixed number again: $\frac{87}{5} = 17\frac{2}{5}$

1. Convert each of the following mixed numbers to improper fractions:
   (a) $5\frac{3}{5}$ (b) $2\frac{3}{8}$ (c) $3\frac{4}{7}$ (d) $4\frac{5}{12}$
2. Convert each of the following improper fractions to mixed numbers:
   (a) $\frac{32}{5}$ (b) $\frac{25}{8}$ (c) $\frac{24}{9}$ (d) $\frac{37}{20}$

3.2 Adding and subtracting fractions

To add or subtract two fractions, they have to be expressed with the *same* denominators first. To achieve that, one or more of the given fractions may have to be replaced with equivalent fractions.

$\frac{3}{20} + \frac{2}{5}$

$= \frac{3}{20} + \frac{2 \times 4}{5 \times 4}$

$= \frac{3}{20} + \frac{8}{20}$

$= \frac{11}{20}$

We will refer to this as the LCM method.

$\frac{5}{12} + \frac{7}{20}$

$= \frac{5 \times 20}{12 \times 20} + \frac{7 \times 12}{20 \times 12}$

$= \frac{100}{240} + \frac{84}{240}$

$= \frac{184}{240}$

$= \frac{23}{30}$

We will later refer to this method of adding or subtracting fractions as Method A.

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In the case of $\frac{5}{12} + \frac{7}{20}$, multiplying by 20 and by 12 was a sure way of making equivalent fractions of the same kind, in this case two hundred-and-fortieths. However, the numbers became quite big. Just imagine how big the numbers will become if you use the same method to calculate $\frac{17}{75} + \frac{13}{85}$!

Fortunately, there is a method of keeping the numbers smaller (in many cases) when making equivalent fractions, so that fractions can be added or subtracted. In this method you first calculate the **lowest common multiple** or LCM of the denominators. In the case of $\frac{5}{12} + \frac{7}{20}$, the smaller multiples of the denominators are:

12:	12	24	36	48	60	72	84
20:	20	40	60	80	100	120	140

The smallest number that is a multiple of both 12 and 20 is 60.

Both $\frac{5}{12}$ and $\frac{7}{20}$ can be expressed in terms of sixtieths:

$\frac{5}{12} = \frac{5 \times 5}{12 \times 5} = \frac{25}{60}$ because to make twelfths into sixtieths you have to divide each twelfth into five equal parts, to get 12 × 5 = 60 equal parts, i.e. sixtieths.

Similarly, $\frac{7}{20} = \frac{7 \times 3}{20 \times 3} = \frac{21}{60}$.

Hence $\frac{5}{12} + \frac{7}{20} = \frac{25}{60} + \frac{21}{60} = \frac{46}{60} = \frac{23}{30}$

We may call this method the LCM method of adding or subtracting fractions.

ADDING AND SUBTRACTING FRACTIONS

1. Which method of adding and subtracting fractions do you think will be the easiest and quickest for you, Method A or the LCM method? Explain.

2. Calculate each of the following:

   (a) $\frac{3}{8} + \frac{2}{5}$ (b) $\frac{3}{10} + \frac{7}{8}$

   (c) $3\frac{2}{5} + 2\frac{3}{10}$ (d) $7\frac{3}{8} + 3\frac{11}{12}$

3. Calculate each of the following:

   (a) $\frac{13}{20} - \frac{2}{5}$ (b) $\frac{7}{12} - \frac{1}{4}$

   (c) $5\frac{1}{2} - 3\frac{3}{8}$ (d) $4\frac{1}{9} - 5\frac{2}{3}$

4. Paulo and Sergio buy a pizza. Paulo eats $\frac{1}{3}$ of the pizza and Sergio eats two fifths. How much of the pizza is left over?

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5. Calculate each of the following. State whether you use Method A or the LCM method.

   (a) $\frac{7}{15} + \frac{11}{24}$
   (b) $\frac{73}{100} - \frac{7}{75}$
   (c) $\frac{3}{25} + \frac{13}{40}$
   (d) $\frac{9}{16} - \frac{3}{10}$
   (e) $\frac{1}{18} + \frac{7}{20}$
   (f) $\frac{11}{35} - \frac{3}{14}$
   (g) $\frac{5}{8} + \frac{5}{8} + \frac{5}{8} + \frac{5}{8} + \frac{5}{8} + \frac{5}{8} + \frac{5}{8} + \frac{5}{8} + \frac{5}{8} + \frac{5}{8}$

3.3 Multiplying and dividing fractions

THINK ABOUT MULTIPLICATION AND DIVISION WITH FRACTIONS

1. Read the questions below, but do not answer them now. Just describe in each case what calculations you think must be done to find the answer to the question. You can think later about how the calculations may be done.

   (a) Ten people come to a party and each of them must get $\frac{5}{8}$ of a pizza. How many pizzas must be bought to provide for all of them?
   (b) $\frac{5}{8}$ of the cost of a new clinic must be carried by the ten doctors who will work there. What part of the cost of the clinic must be carried by each of the doctors, if they have agreed to share the cost equally?
   (c) If a whole pizza costs R10, how much does $\frac{5}{8}$ of a pizza cost?
   (d) The owner of a spaza shop has ten whole pizzas. How many portions of $\frac{5}{8}$ of a pizza each can he make up from the ten pizzas?

2. Look at the different sets of calculations shown below.

   (a) Which set of calculations is a correct way to find the answer for question 1(a)?
   (b) Which set of calculations is a correct way to find the answer for question 1(b)?
   (c) Which set of calculations is a correct way to find the answer for question 1(c)?
   (d) Which set of calculations is a correct way to find the answer for question 1(d)?

Set A: $\frac{10}{10} \times \frac{5}{8} = \frac{50}{80}$

Set B: $\frac{5}{8} = \frac{50}{80}$. 50 eightieths ÷ 10 = $\frac{5}{80}$

Set C: How many eighths in ten wholes? 80 eighths. How many five-eighths in 80?
80 ÷ 5 = 16

Set D: $\frac{5}{8}$ is five eighths. 10 × five eighths = $\frac{50}{8}$

Set E: $\frac{5}{8} \div 10 = \frac{5}{8} \times \frac{10}{1} = \frac{50}{8}$

Multiply a fraction by a whole number

Example:

$8 \times \frac{3}{5} = 8 \times 3 \text{ fifths} = 24 \text{ fifths} = \frac{24}{5} = 4\frac{4}{5}$

Divide a fraction by a whole number

You can divide a fraction by converting it to an equivalent fraction with a numerator that is a multiple of the divisor.

Example:

$\frac{2}{3} \div 5 = \frac{10}{15} \div 5 = 10 \text{ fifteenths} \div 5 = 2 \text{ fifteenths} = \frac{2}{15}$

A fraction of a whole number, and a fraction of a fraction

Examples:

A $\frac{7}{12}$ of R36.

$\frac{1}{12}$ of R36 is the same as R36 ÷ 12 = R3, so $\frac{7}{12}$ of R36 is 7 × R3 = R21.

B $\frac{7}{12}$ of 36 fiftieths.

$\frac{1}{12}$ of 36 fiftieths is the same as 36 fiftieths ÷ 12 = 3 fiftieths,

so $\frac{7}{12}$ of 36 fiftieths is 7 × 3 fiftieths = 21 fiftieths.

$\frac{7}{12} \times \frac{36}{50}$ means $\frac{7}{12}$ of $\frac{36}{50}$, it is the same.

$\frac{1}{12}$ of $\frac{36}{50}$ is the same as $\frac{36}{50} \div 12 = \frac{3}{50}$, so $\frac{7}{12}$ of $\frac{36}{50}$ is $7 \times \frac{3}{50} = \frac{21}{50}$.

3. (a) You calculated $\frac{7}{12} \times \frac{36}{50}$ in the example above. What was the answer?

   (b) Calculate $\frac{7 \times 36}{12 \times 50}$, and simplify your answer.

Example:

$\frac{2}{3} \times \frac{5}{8} = \frac{2}{3} \text{ of } \frac{15}{24} = \frac{1}{3} \text{ of } \frac{30}{24} = \frac{10}{24} = \frac{5}{12}$

The same answer is obtained by calculating $\frac{2 \times 5}{3 \times 8}$.

To multiply two fractions, you may simply multiply the numerators and the denominators.

$\frac{2}{3} \times \frac{9}{20} = \frac{2 \times 9}{3 \times 20} = \frac{18}{60} = \frac{3}{10}$

Division by a fraction

When we divide by a fraction, we have a very different situation. Think about this:

If you have 40 pizzas, how many learners can have $\frac{3}{5}$ a pizza each?

To find the number of fifths in 40 pizzas: 40 × 5 = 200 fifths of a pizza.

To find the number of three fifths: 200 ÷ 3 = 66 portions of $\frac{3}{5}$ pizza and two fifths of a pizza is left over.

Since the portion for each learner is three fifths, the two fifths of a pizza that remains is two thirds of a portion.

So, to calculate $40 \div \mathbf{\frac{3}{5}}$, we multiplied by **5** and divided by **3**, and that gave us 66 and two thirds of a portion.

In fact, we calculated $40 \times \frac{5}{3}$.

Division is the inverse of multiplication.
So, to divide by a fraction, you multiply by its inverse.

Example:

$$\frac{18}{60} \div \frac{2}{3} = \frac{18}{60} \times \frac{3}{2} = \frac{54}{120} = \frac{9}{20}$$

MULTIPLYING AND DIVIDING FRACTIONS

1. Calculate each of the following:
   (a) $\frac{3}{4}$ of $\frac{12}{25}$
   (b) $\frac{3}{4} \times \frac{12}{100}$
   (c) $\frac{3}{4}$ of $\frac{13}{25}$
   (d) $\frac{3}{4} \times 1\frac{1}{2}$
   (e) $\frac{3}{20} \times \frac{5}{6}$
   (f) $\frac{3}{20}$ of $\frac{3}{20}$
2. A small factory manufactures copper pans for cooking. Exactly $\frac{3}{50}$ kg of copper is needed to make one pan.
   (a) How many pans can they make if $\frac{18}{50}$ kg of copper is available?
   (b) How many pans can they make if $\frac{20}{50}$ kg of copper is available?
   (c) How many pans can they make if $\frac{2}{5}$ kg of copper is available?
   (d) How many pans can they make if $\frac{3}{4}$ kg of copper is available?

(e) How many pans can be made if $\frac{144}{50}$ kg of copper is available?

(f) How many pans can be made if 5 kg of copper is available?

3. Calculate each of the following:

(a) $\frac{18}{50} \div \frac{3}{50}$

(b) $\frac{9}{25} \div \frac{3}{50}$

(c) $\frac{144}{50} \div \frac{3}{50}$

(d) $2\frac{44}{50} \div \frac{3}{50}$

(e) $2\frac{22}{25} \div \frac{3}{50}$

(f) $\frac{5}{8} \div \frac{3}{50}$

(g) $20 \div \frac{3}{50}$

(h) $2 \div \frac{3}{50}$

(i) $1 \div \frac{3}{50}$

(j) $\frac{1}{2} \div \frac{3}{50}$

4. A rectangle is $3\frac{5}{8}$ cm long and $2\frac{3}{5}$ cm wide.

(a) What is the area of this rectangle?
(b) What is the perimeter of this rectangle?

5. A rectangle is $5\frac{5}{6}$ cm long and its area is $8\frac{1}{6}$ cm^2.

How wide is this rectangle?

6. Calculate each of the following:

(a) $2\frac{3}{8}$ of $5\frac{4}{5}$

(b) $3\frac{2}{7} \times 2\frac{7}{12}$

(c) $8\frac{2}{5} \div 3\frac{3}{10}$

(d) $3\frac{3}{10} \times 3\frac{3}{10}$

(e) $2\frac{5}{8} \div 5\frac{7}{10}$

(f) $\frac{3}{5} \times 1\frac{2}{3} \times 1\frac{3}{4}$

7. Calculate each of the following:

(a) $\frac{2}{3}(\frac{3}{4} + \frac{7}{10})$

(b) $\frac{2}{3} \times \frac{3}{4} + \frac{2}{3} \times \frac{7}{10}$

(c) $\frac{5}{8}(\frac{4}{5} - \frac{1}{3})$

(d) $\frac{5}{8} \times \frac{4}{5} - \frac{5}{8} \times \frac{1}{3}$

8. A piece of land with an area of 40 ha is divided into 30 equal plots. The total price of the land is R45 000. Remember that "ha" is the abbreviation for hectares.

(a) Jim buys $\frac{2}{5}$ of the land.

(i) How many plots is this and how much should he pay?

(ii) What is the area of the land that Jim buys?

(b) Charlene buys $\frac{1}{3}$ of the land. How many plots is this and how much should she pay?

(c) Bongani buys the rest of the land. Determine the fraction of the land that he buys.

SQUARES, CUBES, SQUARE ROOTS AND CUBE ROOTS

1. Calculate each of the following:

(a) $\frac{3}{4} \times \frac{3}{4}$

(b) $\frac{7}{10} \times \frac{7}{10}$

(c) $2\frac{5}{8} \times 2\frac{5}{8}$

(d) $1\frac{5}{12} \times 1\frac{5}{12}$

(e) $3\frac{5}{7} \times 3\frac{5}{7}$

(f) $10\frac{3}{4} \times 10\frac{3}{4}$

$\frac{9}{16}$ is the square of $\frac{3}{4}$, because $\frac{3}{4} \times \frac{3}{4} = \frac{9}{16}$. $\frac{3}{4}$ is the square root of $\frac{9}{16}$.

2. Find the square root of each of the following numbers:

(a) $\sqrt{\frac{25}{49}}$

(b) $\sqrt{\frac{36}{121}}$

(c) $\sqrt{\frac{64}{25}}$

(d) $\sqrt{2\frac{46}{49}}$

3. Calculate each of the following:

(a) $\frac{3}{4} \times \frac{3}{4} \times \frac{3}{4}$

(b) $\frac{7}{10} \times \frac{7}{10} \times \frac{7}{10}$

(c) $\frac{9}{10} \times \frac{9}{10} \times \frac{9}{10}$

(d) $\frac{5}{8} \times \frac{5}{8} \times \frac{5}{8}$

4. Find the cube root of each of the following numbers:

(a) $\sqrt[3]{\frac{27}{1\,000}}$

(b) $\sqrt[3]{\frac{125}{216}}$

(c) $\sqrt[3]{\frac{1\,000}{216}}$

(d) $\sqrt[3]{15\frac{5}{8}}$

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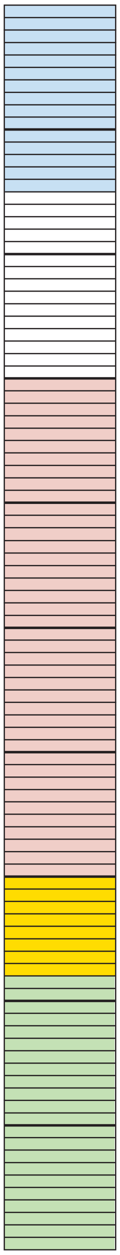
3.4 Equivalent forms

FRACTIONS, DECIMALS AND PERCENTAGE FORMS



1. The rectangle on the right is divided into small parts.
   (a) How many of these small parts are there in the rectangle?
   (b) How many of these small parts are there in one tenth of the rectangle?
   (c) What fraction of the rectangle is blue?
   (d) What fraction of the rectangle is pink?

Instead of "six hundredths" we may say "6 per cent" or, in short, "6%". It means the same thing.

15 per cent of the rectangle on the right is blue.

2. (a) What percentage of the rectangle is green?
   (b) What percentage of the rectangle is pink?

0,37 and 37% and $\frac{37}{100}$ are different ways of writing the same value (**37 hundredths**).

3. Express each of the following in three ways, namely as a decimal, a percentage and a fraction (in simplest form):
   (a) three tenths
   (b) seven hundredths
   (c) 37 hundredths
   (d) seven tenths
   (e) two fifths
   (f) seven twentieths

4. Copy the table and fill in the missing values.

Decimal	Percentage	Common fraction (simplest form)
0,2		
	40%	
		$\frac{3}{8}$
0,05		

5. (a) Jannie eats a quarter of a watermelon. What percentage of the watermelon is this?
   (b) Sibu drinks 75% of the milk in a bottle. What fraction of the milk in the bottle has he drunk?
   (c) Jem used 0,18 of the paint in a tin. If he uses half of the remaining amount the next time he paints, what fraction (in simplest form) is left over?

CHAPTER 4
The decimal notation for fractions

4.1 Equivalent forms

Decimal fractions and common fractions are simply different ways of expressing the same number. They are different **notations** showing the same value.

To write a decimal fraction as a common fraction: Write the decimal with a denominator that is a power of ten (10, 100, 1 000, etc.) and then simplify it if possible.

For example: $0{,}35 = \frac{35}{100} = \frac{7}{20} \times \frac{5}{5} = \frac{7}{20}$

To write a common fraction as a decimal fraction: Change the common fraction to an equivalent fraction with a power of ten as a denominator.

For example: $\frac{3}{4} = \frac{3}{4} \times \frac{25}{25} = \frac{75}{100} = 0{,}75$

If you are permitted to use your calculator, simply type in 3 ÷ 4 and the answer will be given as 0,75. On some calculators you will need to press an additional button to convert the exact fraction to a decimal.

Notation means a set of symbols that are used to show a special thing.

COMMON FRACTIONS, DECIMAL FRACTIONS AND PERCENTAGES

Do *not* use a calculator in this exercise.

1. Write the following decimal fractions as common fractions in their simplest form:

   (a) 0,56 (b) 3,87
   (c) 1,9 (d) 5,205

2. Write the following common fractions as decimal fractions:

(a) $\frac{9}{20}$ (b) $\frac{7}{5}$

(c) $\frac{24}{25}$ (d) $2\frac{3}{8}$

3. Write the following percentages as common fractions in their simplest form:

(a) 70% (b) 5% (c) 12,5%

4. Write the following decimal fractions as percentages:

(a) 0,6 (b) 0,43 (c) 0,08

(d) 0,265 (e) 0,005

5. Write the following common fractions as percentages:

(a) $\frac{7}{10}$ (b) $\frac{3}{4}$ (c) $\frac{33}{50}$

(d) $\frac{60}{60}$ (e) $\frac{2}{25}$ (f) $\frac{29}{50}$

6. Jane and Devi are in different schools. At Jane's school the year mark for Mathematics was out of 80, and Jane got 60 out of 80. At Devi's school the year mark was out of 50 and Devi got 40 out of 50.
   (a) What fraction of the total marks, in its simplest form, did Devi obtain at her school?
   (b) What percentage did Devi and Jane get for Mathematics?
   (c) Who performed better, Jane or Devi?

7. During a basketball game, Lebo tried to score 12 times. Only four of her attempts were successful.
   (a) What fraction of her attempts was successful?
   (b) What percentage of her attempts was not successful?

4.2 Calculations with decimals

When you **add** and **subtract** decimal fractions:

- Add tenths to tenths.
- Subtract tenths from tenths.
- Add hundredths to hundredths.
- Subtract hundreds from hundredths.

And so on!

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When you **multiply** decimal fractions, you change the decimals to whole numbers, do the calculation and lastly, change them back to decimal fractions.

Example: To calculate 13,1 × 1,01, you first calculate 131 × 101 (which equals 13 231). Then, since you have multiplied the 13,1 by 10, and the 1,01 by 100 in order to turn them into whole numbers, you need to divide this answer by 10 × 100 (i.e. 1 000). Therefore, the final answer is 13,231.

When you **divide** decimal fractions, you can use equivalent fractions to help you.

Example: $21{,}7 \div 0{,}7 = \frac{21{,}7}{0{,}7} = \frac{21{,}7}{0{,}7} \times \frac{10}{10} = \frac{217}{7} = 31$

Notice how you multiply both the numerator and denominator of the fraction by the same number (in this case, 10). Always multiply by the *smallest* power of ten that will convert both values to whole numbers.

CALCULATIONS WITH DECIMALS

Do *not* use a calculator in this exercise. Ensure that you show all steps of your working.

1. Calculate the value of each of the following:
   (a) 3,3 + 4,83
   (b) 0,6 + 18,3 + 4,4
   (c) 9,3 + 7,6 – 1,23
   (d) (16,0 – 7,6) – 0,6
   (e) 9,43 – (3,61 + 1,14)
   (f) 1,21 + 2,5 – (2,3 – 0,23)
2. Calculate the value of each of the following:
   (a) 4 × 0,5
   (b) 15 × 0,02
   (c) 0,8 × 0,04
   (d) 0,02 × 0,15
   (e) 1,07 × 0,2
   (f) 0,016 × 0,02
3. Calculate the value of each of the following:
   (a) 7,2 ÷ 3
   (b) 12 ÷ 0,3
   (c) 0,15 ÷ 0,5
   (d) 10 ÷ 0,002
   (e) 0,3 ÷ 0,006
   (f) 0,024 ÷ 0,08
4. Write down the value that is equal to or closest to the answer to each calculation:
   (a) 3 × 0,5
   A: 6
   B: 1,5
   C: 0,15
   (b) 4,4 ÷ 0,2
   A: 8,8
   B: 2,2
   C: 22

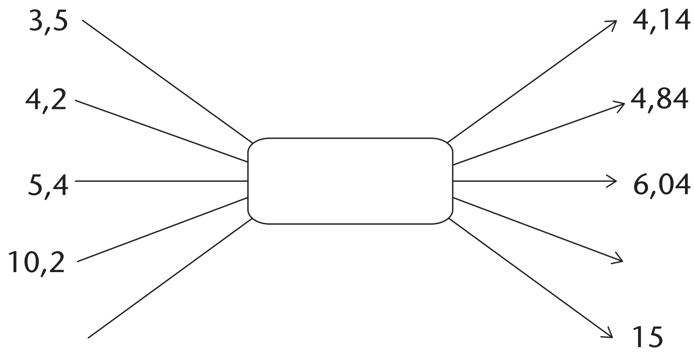
(c) $56 \times 1{,}675$

A: more than 56

B: more than 84

C: more than 112

5. Copy the diagram. Determine the operator and the unknown numbers and fill them in:



6. Calculate each of the following:

(a) $(0{,}1)^2$ (b) $(0{,}03)^2$ (c) $(2{,}5)^2$

(d) $\sqrt{0{,}04}$ (e) $\sqrt{0{,}16}$ (f) $\sqrt{0{,}49}$

(g) $(0{,}2)^3$ (h) $(0{,}4)^3$ (i) $(0{,}03)^3$

(j) $\sqrt[3]{0{,}064}$ (k) $\sqrt[3]{0{,}125}$ (l) $\sqrt[3]{0{,}216}$

7. Calculate each of the following:

(a) $2{,}5 \times 2 \div 10$ (b) $4{,}2 - 5 \times 1{,}2$

(c) $\dfrac{5{,}4 + 7{,}35}{0{,}05}$ (d) $4{,}2 \div 0{,}21 + 0{,}45 \times 0{,}3$

4.3 Solving problems

ALL KINDS OF PROBLEMS

Do *not* use a calculator in this exercise. Ensure that you show all steps of working.

1. Is $6{,}54 \times 0{,}81 = 0{,}654 \times 8{,}1$? Explain your answer.

2. You are given that $45 \times 24 = 1\ 080$. Use this result to determine:

(a) $4{,}5 \times 2{,}4$ (b) $4{,}5 \times 24$ (c) $4{,}5 \times 0{,}24$

(d) $0{,}045 \times 24$ (e) $0{,}045 \times 0{,}024$ (f) $0{,}045 \times 24$

3. Without actually dividing, choose which answer in brackets is the correct answer, or the closest to the correct answer.
   (a) $14 \div 0{,}5$ (7; 28; 70)
   (b) $0{,}58 \div 0{,}7$ (8; 80; 0,8)
   (c) $2{,}1 \div 0{,}023$ (10; 100; 5)

4. (a) John is asked to calculate $6{,}5 \div 0{,}02$. He does the following:
   Step 1: $6{,}5 \div 2 = 3{,}25$
   Step 2: $3{,}25 \times 100 = 325$
   Is he correct? Why?
   (b) Use John's method in part (a) to calculate:
   (i) $4{,}8 \div 0{,}3$
   (ii) $21 \div 0{,}003$

5. Given: $0{,}174 \div 0{,}3 = 0{,}58$. Using this fact, write down the answers for the following without doing any further calculations:
   (a) $0{,}3 \times 0{,}58$
   (b) $1{,}74 \div 3$
   (c) $17{,}4 \div 30$
   (d) $174 \div 300$
   (e) $0{,}0174 \div 0{,}03$
   (f) $0{,}3 \times 5{,}8$

4.4 More problems

MORE PROBLEMS AND CALCULATIONS

You *may* use a calculator for this exercise.

1. Calculate the following, rounding off all answers correct to two decimal places:
   (a) $8{,}567 + 3{,}0456$
   (b) $2{,}781 - 6{,}0049$
   (c) $1{,}234 \times 4{,}056$
   (d) $\dfrac{5{,}678 + 3{,}245}{1{,}294 - 0{,}994}$

2. What is the difference between 0,890 and 0,581?

3. If a rectangle is 12,34 cm wide and 31,67 cm long:
   (a) What is the perimeter of the rectangle?
   (b) What is the area of the rectangle? Round off your answer to two decimal places.

4. Alison buys a cooldrink for R5,95, a chocolate for R3,25 and a packet of chips for R4,60. She pays with a R20 note.
   (a) How much did she spend?
   (b) How much change did she get?

5. A tractor uses 11,25 ℓ of fuel in 0,75 hours. How many litres does it use in one hour?

6. Mrs Ruka received her municipal bill.
   (a) Her water consumption charge for one month is R32,65. The first 5,326 kℓ are free, then she pays R5,83 per kilolitre for every kilolitre thereafter.
   How much water did the Ruka household use?
   (b) The electricity charge for Mrs Ruka for the same month was R417,59. The first 10 kWh are free. For the next 100 kWh the charge is R1,13 per kWh, and thereafter for each kWh the charge is R1,42.
   How much electricity did the Ruka household use?
7. A roll of material is 25 m long. To make one dress, you need 1,35 m of material. How many dresses can be made out of a roll of material and how much material is left over?
8. If one litre of petrol weighs 0,679 kg, what will 28,6 ℓ of petrol weigh?
9. The reading on a water meter at the beginning of the month is 321,573 kℓ. At the end of the month the reading is 332,523 kℓ. How much water (in ℓ) was used during this month?

4.5 Decimals in algebraic expressions and equations

DECIMALS IN ALGEBRA

1. Simplify the following:
   (a) $\sqrt{0{,}09x^{36}}$
   (b) $7{,}2x^3 - 10{,}4x^3$
   (c) $(2{,}4x^2y^3)(10y^3x)$
   (d) $11{,}75x^2 - 1{,}2x \times 5x$
   (e) $\dfrac{3{,}4x - 1{,}2x}{1{,}1x \times 4}$
   (f) $\sqrt[3]{0{,}008x^{12}} + \sqrt{0{,}16x^8}$
   (g) $3x^2 + 0{,}1x^2 - 45{,}6 + 3{,}9$
   (h) $\dfrac{0{,}4y + 1{,}2y}{0{,}6x - 3x}$
2. Simplify the following:
   (a) $\dfrac{0{,}5x^9}{0{,}02x^3}$
   (b) $\dfrac{0{,}325}{x^2} - \dfrac{1{,}675}{x^2}$
   (c) $\dfrac{3{,}6x}{1{,}5y^3} \times \dfrac{5y}{0{,}6x}$
   (d) $\dfrac{9{,}5x^2}{1{,}2y^2} \div \dfrac{0{,}05x}{0{,}04y^8}$
3. Solve the following equations:
   (a) $0{,}24 + x = 0{,}31$
   (b) $x + 5{,}61 = 7{,}23$
   (c) $x - 3{,}14 = 9{,}87$
   (d) $4{,}21 - x = 2{,}74$
   (e) $0{,}96x = 0{,}48$
   (f) $x \div 0{,}03 = 1{,}5$

WORKSHEET

You are *not* permitted to use a calculator in this exercise, *except* for question 5. Ensure that you show all steps of working, where relevant.

1. Copy and complete the following table:

Percentage	Common fraction	Decimal fraction
2,5%		
	$\frac{15}{250}$	
		0,009

2. Calculate each of the following:

   (a) $6{,}78 - 4{,}92$ (b) $1{,}7 \times 0{,}05$ (c) $7{,}2 \div 0{,}36$

   (d) $4{,}2 - 0{,}4 \times 1{,}2 + 7{,}37$ (e) $(0{,}12)^2$ (f) $\frac{3\sqrt{0{,}04}}{\sqrt[3]{0{,}027}}$

3. $36 \times 19 = 684$. Use this result to determine:

   (a) $3{,}6 \times 1{,}9$ (b) $0{,}036 \times 0{,}19$ (c) $68{,}4 \div 0{,}19$

4. Simplify:

   (a) $(4{,}95x - 1{,}2) - (3{,}65x + 3{,}1)$ (b) $\frac{2{,}75x^{50}}{0{,}005x^{25}}$

5. Mulalo went to the shop and purchased two tubes of toothpaste for R6,98 each; three cans of cooldrink for R6,48 each, and five tins of baked beans for R7,95 each. If he pays with a R100 note, how much change should he get?

Chapter 5
Exponents

5.1 Revision

Remember that exponents are a shorthand way of writing repeated multiplication of the same number by itself. For example: $5 \times 5 \times 5 = 5^3$. The **exponent**, which is 3 in this example, stands for however many times the value is being multiplied. The number that is being multiplied, which is 5 in this example, is called the **base**.

If there are mixed operations, then the powers should be calculated before multiplication and division. For example: $5^2 \times 3^2 = 25 \times 9$.

You learnt these laws for working with exponents in previous grades:

Law	Example
$a^m \times a^n = a^{m+n}$	$3^2 \times 3^3 = 3^{2+3} = 3^5$
$a^m \div a^n = a^{m-n}$	$5^4 \div 5^2 = 5^{4-2} = 5^2$
$(a^m)^n = a^{m \times n}$	$(2^3)^2 = 2^{2 \times 3} = 2^6$
$(a \times t)^n = a^n \times t^n$	$(3 \times 4)^2 = 3^2 \times 4^2$
$a^0 = 1$	$32^0 = 1$

THE EXPONENTIAL FORM OF A NUMBER

1. Write the following in exponential notation:

   (a) $2 \times 2 \times 2 \times 2 \times 2$ (b) $s \times s \times s \times s$ (c) $(-6) \times (-6) \times (-6)$

   (d) $2 \times 2 \times 2 \times 2 \times s \times s \times s \times s$ (e) $3 \times 3 \times 3 \times 7 \times 7$ (f) $500 \times (1{,}02) \times (1{,}02)$

2. Write each of the numbers in exponential notation in some different ways, if possible:

   (a) 81 (b) 125 (c) 1 000

   (d) 64 (e) 216 (f) 1 024

ORDER OF OPERATIONS

1. Calculate the value of $7^2 - 4$.

   Bathabile did the calculation like this: $7^2 - 4 = 14 - 4 = 10$

   Nathaniel did the calculation differently: $7^2 - 4 = 49 - 4 = 45$

   Which learner did the calculation correctly? Give reasons for your answer.

2. Calculate: $5 + 3 \times 2^2 - 10$, with explanations.

3. Explain how to calculate $2^6 - 6^2$.

4. Explain how to calculate $(4 + 1)^2 + 8 \times \sqrt[3]{64}$.

LAWS OF EXPONENTS

1. Use the laws of exponents to simplify the following (leave answer in exponential form):
   (a) $2^2 \times 2^4$ (b) $3^4 \div 3^2$ (c) $3^0 + 3^4$
   (d) $(2^3)^2$ (e) $(2 \times 5)^2$ (f) $(2^2 \times 7)^3$

2. Copy and complete the table. Substitute the given number for y. The first column has been done as an example.

	y	2	3	4	5
(a)	$y \times y^4$	2×2^4 $= 2^{1+4}$ $= 2^5$ $= 32$			
(b)	$y^2 \times y^3$	$2^2 \times 2^3$ $= 2^{2+3}$ $= 4 \times 8$ $= 32$			
(c)	y^5	$2^5 = 32$			

3. Are the expressions $y \times y^4$, $y^2 \times y^3$ and y^5 equivalent? Explain.

4. Copy and complete the table. Substitute the given number for y.

	y	2	3	4	5
(a)	$y^4 \div y^2$	$2^4 \div 2^2$ $= 16 \div 4$ $= 4$			
(b)	$y^3 \div y^1$	$2^3 \div 2^1$ $= 8 \div 2$ $= 4$			
(c)	y^2	$2^2 = 4$			

5. (a) From the table, is $y^4 \div y^2 = y^3 \div y^1 = y^2$? Explain.
   (b) Evaluate $y^3 \div y^1$ for $y = 15$.

6. Copy and complete the following table:

	x	2	3	4	5
(a)	2×5^x	2×5^2 $= 2 \times 25$ $= 50$			
(b)	$(2 \times 5)^x$	$(2 \times 5)^2$ $= 10^2$ $= 100$			
(c)	$2^x \times 5^x$	$2^2 \times 5^2$ $= 4 \times 25$ $= 100$			

7. (a) From the table above, is $2 \times 5^x = (2 \times 5)^x$? Explain.
   (b) Which expressions in question 6 are equivalent? Explain.

8. Below is a calculation that Wilson did as homework. Mark each problem correct or incorrect and explain the mistakes.
   (a) $b^3 \times b^8 = b^{24}$
   (b) $(5x)^2 = 5x^2$
   (c) $(-6a) \times (-6a) \times (-6a) = (-6a)^3$

5.2 Integer exponents

5^4 means $5 \times 5 \times 5 \times 5$. The exponent 4 indicates the number of appearances of the repeated factor.

What may a negative exponent mean, however? For example, what may 5^{-4} mean?

Mathematicians have decided to use negative exponents to indicate repetition of the multiplicative inverse of the base, for example 5^{-4} is used to indicate $\frac{1}{5} \times \frac{1}{5} \times \frac{1}{5} \times \frac{1}{5}$ or $(\frac{1}{5})^4$, and x^{-3} is used to indicate $(\frac{1}{x})^3$, which is $\frac{1}{x} \times \frac{1}{x} \times \frac{1}{x}$.

This decision was not taken blindly – mathematicians were well aware that it makes good sense to use negative exponents in this meaning. One major advantage is that the negative exponents, when used in this meaning, have the same properties as positive exponents, for example:

$2^{-3} \times 2^{-4} = 2^{(-3)+(-4)} = 2^{-7}$ because $2^{-3} \times 2^{-4}$ means $(\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}) \times (\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2})$ which is $(\frac{1}{2})^7$.

$2^{-3} \times 2^4 = 2^{(-3)+4} = 2^1$ because $2^{-3} \times 2^4$ means $(\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}) \times (2 \times 2 \times 2 \times 2)$ which is 2.

NEGATIVE EXPONENTS

1. Express each of the following in the exponential notation in two ways: with positive exponents and with negative exponents:
   (a) $\frac{1}{5} \times \frac{1}{5} \times \frac{1}{5} \times \frac{1}{5} \times \frac{1}{5} \times \frac{1}{5}$
   (b) $\frac{1}{3} \times \frac{1}{3} \times \frac{1}{3} \times \frac{1}{3}$
2. In each case, check whether the statement is true or false. If it is false, write a correct statement. If it is true, give reasons why you say so.
   (a) $10^{-3} = 0{,}001$
   (b) $3^{-5} \times 9^{2} = 3$
   (c) $25^{2} \times 10^{-6} \times 2^{6} = 5$
   (d) $(\frac{1}{5})^{-4} = 5^{4}$
3. Calculate each of the following, without using a calculator:
   (a) $10^{-3} \times 20^{4}$
   (b) $(\frac{1}{5})^{-4}$
4. (a) Use a scientific calculator to determine the decimal values of the given powers.

   Example: To find 3^{-1} on your calculator, use the key sequence: 3 y^{x} 1 ± =

Power	2^{-1}	5^{-1}	$(-2)^{-1}$	$(0{,}3)^{-1}$	0^{-1}	10^{-1}	10^{-2}
Decimal value							

   (b) Explain the meaning of 10^{-3}.
5. Determine the value of each of the following in two ways:
   A. By using the definition of powers (for example, $5^{2} \times 5^{0} = 25 \times 1 = 25$).
   B. By using the properties of exponents (for example, $5^{2} \times 5^{0} = 5^{2+0} = 5^{2} = 25$).

   (a) $(3^{3})^{-2}$
   (b) $4^{2} \times 4^{-2}$
   (c) $5^{-2} \times 5^{-1}$
6. Calculate the value of each of the following. Express your answers as common fractions.
   (a) 2^{-3}
   (b) $3^{2} \times 3^{-2}$
   (c) $(2 + 3)^{-2}$
   (d) $3^{-2} \times 2^{-3}$
   (e) $2^{-3} + 3^{-3}$
   (f) 10^{-3}
   (g) $2^{3} + 2^{-3}$
   (h) $(3^{-1})^{-1}$
   (i) $(2^{-3})^{2}$
7. Which of the following are true? Correct any false statement.
   (a) $6^{-1} = -6$
   (b) $3x^{-2} = \frac{1}{3x^{2}}$
   (c) $3^{-1}x^{-2} = \frac{1}{3x^{2}}$
   (d) $(ab)^{-2} = \frac{1}{a^{2}b^{2}}$
   (e) $\left(\frac{2}{3}\right)^{-2} = \left(\frac{3}{2}\right)^{2}$
   (f) $\left(\frac{1}{3}\right)^{-1} = 3$

5.3 Solving simple exponential equations

An exponential equation is an equation in which the variable is in the exponent. So, when you solve exponential equations, you are solving questions of the form; *"To what power must the base be raised for the statement to be true?"*

To solve this kind of equation, remember that:

If $a^m = a^n$, then $m = n$.

In other words, if the base is the same on either side of the equation, then the exponents are the same.

Example:

$3^x = 243$

$3^x = 3^5$ (rewrite using the same base)

$x = 5$ (since the bases are the same, we equate the exponents)

Some exponential equations are slightly more complex:

Examples:

$3^{x+3} = 243$

$3^{x+3} = 3^5$ (rewrite using the same base)

$x + 3 = 5$ (equate the exponents)

$x = 2$

$3^{x+3} = 1$ (remember $1 = 3^0$)

$3^{x+3} = 3^0$ (rewrite using the same base)

$x + 3 = 0$ (equate the exponents)

$x = -3$

Check: LHS $3^{2+3} = 3^5 = 243$

Remember that the exponent can also be negative. However, you follow the same method to solve these kinds of equations.

Example:

$2^x = \frac{1}{32}$

$2^x = 2^{-5}$ (rewrite using the same base)

$x = -5$ (equate the exponents)

SOLVING EXPONENTIAL EQUATIONS

1. Use the following table to answer questions that follow:

x	2	3	4	5
2^x	4	8	16	32
3^x	9	27	81	243
5^x	25	125	625	3 125

Find the value of x:

(a) $2^x = 32$ (b) $3^x = 81$ (c) $5^x = 3\,125$

(d) $2^x = 8$ (e) $5^x = 625$ (f) $3^x = 9$

(g) $5^{x+1} = 25$ (h) $3^{x+2} = 27$ (i) $2^{x-1} = 8$

2. Solve these exponential equations. You may use your calculator if necessary.

(a) $4^x = \frac{1}{64}$ (b) $6^{2x} = 1\,296$ (c) $2^{x-1} = \frac{1}{8}$

(d) $3^{x+2} = \frac{1}{729}$ (e) $5^{x+1} = 15\,625$ (f) $2^{x+3} = \frac{1}{4}$

(g) $4^{x+3} = \frac{1}{256}$ (h) $3^{2-x} = 81$ (i) $5^{3x} = \frac{1}{125}$

5.4 Scientific notation

Scientific notation is a way of writing numbers that are too big or too small to be written clearly in decimal form. The diameter of a hydrogen atom, for example, is a very small number. It is 0,000000053 mm. The distance from the sun to the earth is, on average, 150 000 000 km.

In scientific notation, the diameter of the hydrogen molecule is written as $5,3 \times 10^{-8}$ and the distance from the sun to the earth is written as $1,5 \times 10^8$. It is easier to compare and to calculate numbers like these, as it is very cumbersome to count the zeros when you work with these numbers.

Look at more examples below:

Decimal notation	Scientific notation
6 130 000	$6,13 \times 10^6$
0,00001234	$1,234 \times 10^{-5}$

A number written in scientific notation is written as the product of two numbers, in the form $\pm\, a \times 10^n$. Here, a is a decimal number between 1 and 10, and n is an integer.

Any number can be written in scientific notation, for example:

$40 = 4,0 \times 10$

$2 = 2 \times 10^0$

The decimal number 324 000 000 is written as $3,24 \times 10^8$ in scientific notation, because the decimal comma is moved eight places to the left to form 3,24.

The decimal number 0,00000065 written in scientific notation is $6,5 \times 10^{-7}$, because the decimal point is moved seven places to the right to form the number 6,5.

WRITING VERY SMALL AND VERY LARGE NUMBERS

1. Express the following numbers in scientific notation:
   (a) 134,56
   (b) 0,0000005678
   (c) 876 500 000
   (d) 0,0000000000321
   (e) 0,006789
   (f) 89 100 000 000 000
   (g) 0,001
   (h) 100
2. Express the following numbers in ordinary decimal notation:
   (a) $1{,}234 \times 10^{6}$
   (b) 5×10^{-1}
   (c) $4{,}5 \times 10^{5}$
   (d) $6{,}543 \times 10^{-11}$
3. Why do we say that 34×10^{3} is not written in scientific notation? Rewrite it in scientific notation.
4. Is each of these numbers written in scientific notation? If not, rewrite it so that it is in scientific notation.
   (a) $90{,}3 \times 10^{-5}$
   (b) 100×10^{2}
   (c) $1{,}36 \times 10^{5}$
   (d) $2{,}01 \times 10^{-2}$
   (e) $0{,}01 \times 10^{3}$
   (f) $0{,}6 \times 10^{8}$

CALCULATIONS USING SCIENTIFIC NOTATION

Example: $123\ 000 \times 4\ 560\ 000$

$= 1{,}23 \times 10^{5} \times 4{,}56 \times 10^{6}$ (write in scientific notation)

$= 1{,}23 \times 4{,}56 \times 10^{5} \times 10^{6}$ (multiplication is commutative)

$= 5{,}6088 \times 10^{11}$ (Use a calculator to multiply the decimals, but add the powers mentally.)

1. Use scientific notation to calculate each of the following. Give the answer in scientific notation.
   (a) $135\ 000 \times 246\ 000\ 000$
   (b) $987\ 654 \times 123\ 456$
   (c) $0{,}000065 \times 0{,}000216$
   (d) $0{,}000000639 \times 0{,}0000587$

Example: $5 \times 10^{3} + 4 \times 10^{4}$

$= 0{,}5 \times 10^{4} + 4 \times 10^{4}$ (Form like terms)

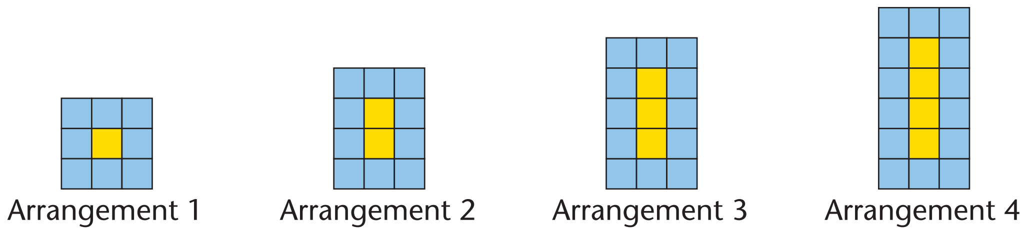
$= 4{,}5 \times 10^{4}$ (Combine like terms)

2. Calculate the following. Leave the answer in scientific notation.
   (a) $7{,}16 \times 10^{5} + 2{,}3 \times 10^{3}$
   (b) $2{,}3 \times 10^{-4} + 6{,}5 \times 10^{-3}$
   (c) $4{,}31 \times 10^{7} + 1{,}57 \times 10^{6}$
   (d) $6{,}13 \times 10^{-10} + 3{,}89 \times 10^{-8}$

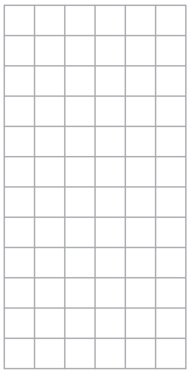
Chapter 6
Patterns

6.1 Geometric patterns

INVESTIGATING AND EXTENDING



1. Blue and yellow square tiles are combined to form the above arrangements.
   (a) How many yellow tiles are there in each arrangement?
   (b) How many blue tiles are there in each arrangement?
   (c) If more arrangements are made in the same way, how many blue tiles and how many yellow tiles will there be in arrangement 5? Check your answer by drawing the arrangement onto grid paper.

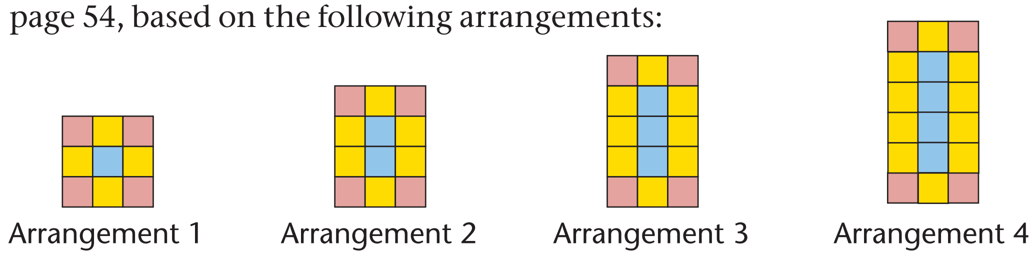


   (d) Copy and complete the following table:

Number of yellow tiles	1	2	3	4	5	8
Number of blue tiles						

   (e) How many blue tiles will there be in a similar arrangement with 26 yellow tiles?
   (f) How many blue tiles will there be in a similar arrangement with 100 yellow tiles?
   (g) Describe how you thought to produce your answer for (f)?

2. (a) In these arrangements there are red tiles too. Copy and complete the table on page 54, based on the following arrangements:



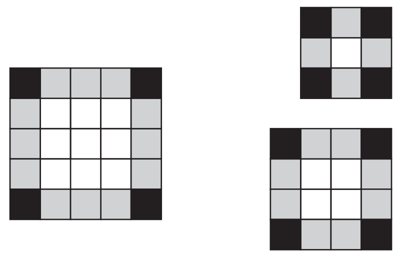
Number of blue tiles	1	2	3	4	5	6	7
Number of yellow tiles							
Number of red tiles							

(b) How many red tiles are there in each arrangement?
(c) How many yellow tiles are there in each arrangement?

The number of red tiles in arrangements like those in question 2, is **constant.** It is always four, no matter how many blue and yellow tiles there are.

The number of blue tiles is different for different arrangements. We can say the number of blue tiles **varies**. We can also say the number of blue tiles is a **variable**.

3. Is the number of yellow tiles in the above arrangements a constant or is it a variable?



4. Look at the arrangements on the right. They consist of black squares, grey squares and white squares.

   (a) Draw another arrangement of the same kind, but with a different length, on grid paper.
   (b) Describe what is constant in these arrangements.
   (c) What are the variables in these arrangements?

The smallest arrangement above may be called arrangement 1, the next bigger one may be called arrangement 2, and so on.

5. (a) Copy and complete the table for arrangements like those in question 4.

Arrangement number	1	2	3	4	5	6	7	10	20
Number of black squares									
Number of grey squares									
Number of white squares									

   (b) How many grey squares do you think there will be in arrangement 15? Explain your answer.
   (c) How many black squares do you think there will be in arrangement 15? Explain your answer.

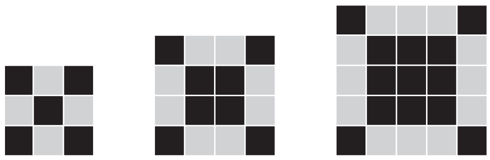
(d) How many white squares do you think there will be in arrangement 15? Explain your answer.

The numbers of grey squares in the different arrangements in question 4 form a pattern: 4; 8; 12; 16; 20; 24; . . . , and so on.

The numbers of white squares in the different arrangements also form a pattern: 1; 4; 9; 16; 25; 36; 49; . . . , and so on.

6. What are the next five numbers in each of the above patterns?

7. (a) On grid paper, draw the next arrangement that follows the same pattern:



(b) How many black tiles are there in the arrangement you have drawn?

(c) How many black tiles will there be in each of the next four arrangements?

DO SOMETHING MORE

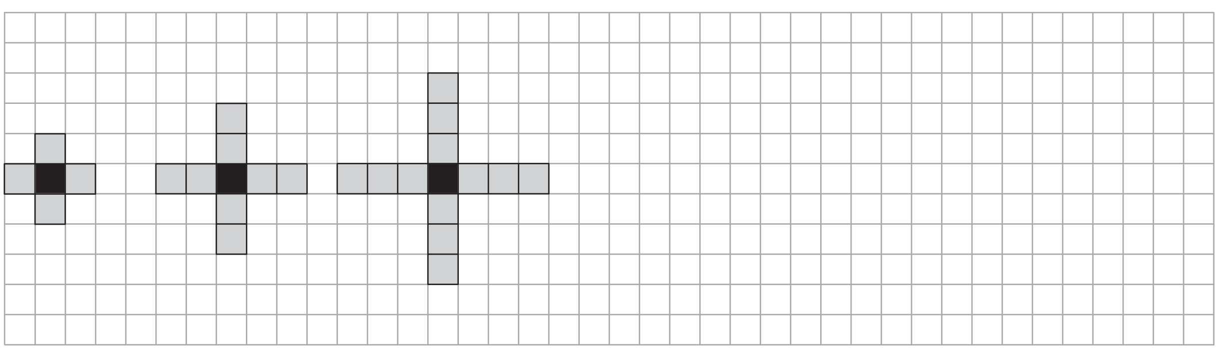
Consider the arrangements in question 4 on page 54 again. If there are 20 grey tiles in such an arrangement, how many white tiles are there? Copy and complete the table, entering your answer in the table.

Number of grey squares	20	36	52			
Number of white squares				256	225	625

6.2 More patterns

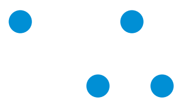
DRAWING AND INVESTIGATING

1. (a) On grid paper, make two more arrangements of black and grey squares so that a pattern is formed.



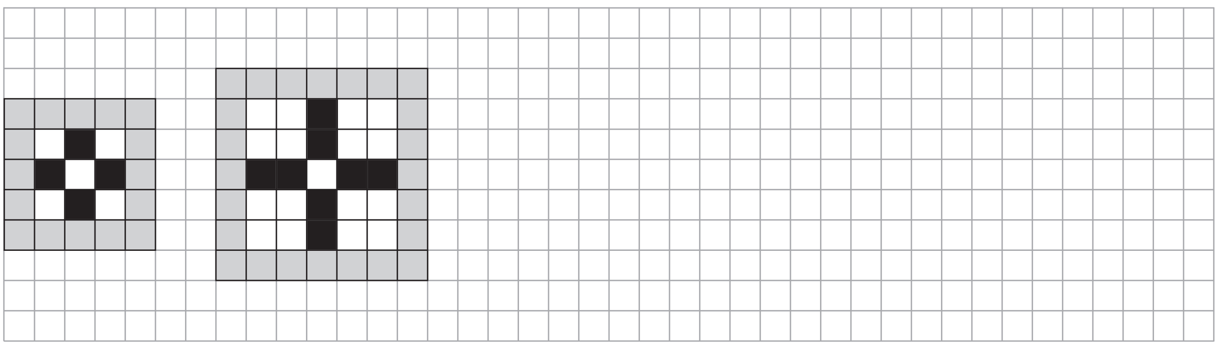
(b) Is there a constant in your pattern? If yes, what is its value?
(c) Is there a variable in your pattern? If yes, give the values of the variable.

2. (a) Make three more arrangements with dots to form the sequence 1; 3; 6; 10; 15 . . .



(b) How many dots will there be in the sixth and seventh arrangements? Explain how you got your answer.
(c) How many dots are there in arrangements 1 and 2 together?
(d) How many dots are there in arrangements 2 and 3 together?
(e) How many dots are there in arrangements 3 and 4 together?
(f) How many dots are there in arrangements 4 and 5 together?
(g) Describe the pattern in your answers for (c), (d), (e) and (f).

3. (a) On grid paper, draw two more arrangements to make a pattern.



(b) What are the variables in your pattern?
(c) The number of black squares is a variable in these arrangements. The value of this variable is four in the first arrangement and eight in the second arrangement. What is the value of this variable in the third arrangement?
(d) What are the values of each of the variables in the fifth arrangement in your pattern? Explain your answers.

4. (a) Now, on grid paper, make a pattern of your own.

(b) Copy this table and use it to describe the variables in your pattern, and their values:

Arrangement number	1	2	3	4	5	6

6.3 Different kinds of patterns in sequences

DO THE SAME THING REPEATEDLY

1. (a) Write the next three numbers in each of the sequences below.

Sequence A: 5 9 13 17 21 ....

Sequence B: 5 10 20 40 80 ....

Sequence C: 5 10 17 26 37 ....

(b) Describe the differences in the ways in which the three sequences are formed.

2. You will now make a sequence with the first term 5. Write 5 on the left on the line below. Then add 8 to the first term (5) to form the second term of your sequence. Write the second term next to the first term (5) in the line below. Now add 8 to the second term to form the third term. Continue like this to form ten more terms.

The numbers in a sequence are also called the **terms** of the sequence.

A sequence can be formed by repeatedly adding or subtracting the same number. In this case the **difference** between consecutive terms in a sequence is **constant**.

A sequence can be formed by repeatedly multiplying or dividing. In this case the **ratio** between consecutive terms is **constant**.

A sequence can also be formed in such a way that neither the difference nor the ratio between consecutive terms is constant.

To write more terms of sequence A in question 1(a), you **added** 4 **repeatedly**.

To write more terms of sequence B in question 1(a), you **multiplied** by 2 **repeatedly**.

To write more terms of sequence C in question 1(a) you did not add the same number each time, nor did you multiply by the same number.

3. Write the next three terms of each sequence. In each case also describe what the pattern is, for example "there is a constant difference of –5 between consecutive terms".

(a) 16; 8; 0; –8; ...
(b) 1; 4; 9; 16; ...
(c) 2; 8; 18; 32; ...
(d) 3; 6; 11; 18; ...
(e) 640; 320; 160; ...
(f) 1; 2; 4; 7; 11; ...

4. In each case, follow the instruction to make a sequence with eight terms.
   (a) Start with 1 and multiply by 2 repeatedly.
   (b) Start with 256 and subtract 32 repeatedly.
   (c) Start with 256 and divide by 2 repeatedly.

The sequence that you made in question 2 can be represented with a table like the one shown below:

Term number	1	2	3	4	5	6	7	8	9	10
Term value	5	13	21	29	37	45	53	61	69	77

5. In each case make a sequence by following the instructions. Copy the tables and write the term numbers and the term values in the tables.
   (a) Term 1 = 10. Add 15 repeatedly.

Term number								
Term value								

   (b) Term 1 = 10. Term value = 15 × term number – 5.

Term number								
Term value								

   (c) Term 1 = 10. Multiply by 2 repeatedly.

Term number						
Term value						

   (d) Term 1 = 20. Term value = $10 \times 2^{\text{term number}}$

Term number						
Term value						

(e) Term 1 = 10. Term value = $10 \times 2^{\text{term number} - 1}$

Term number						
Term value						

(f) Term 4 = 30. Add 5 repeatedly.

Term number								
Term value								

6. Instructions for forming a sequence are given in two different ways in question 5. How would you describe the two different ways for giving instructions to form a sequence?

6.4 Formulae for sequences

The formula for a number sequence can be written in two different ways:

- A description of the **relationship between consecutive terms**: In other words, the calculations that you do to a term to produce the next term, as in questions 5(a), (c) and (f) on the previous page. The first (or another) term must be given. This kind of formula has two parts: the first term and the relationship between terms.
- A description of the **relationship between the value of the term and its position in the sequence**: This relationship describes the calculations that can be done **on the term number** to produce the **term value**, as in question 5(b), (d) and (e) on the previous page.

MAKE TWO FORMULAE FOR THE SAME SEQUENCE

1. Choose any whole number smaller than 10 as the first term of a sequence.
   (a) Copy the table. Use your chosen first term to form a sequence by adding 5 repeatedly.
   (b) Multiply each term number below by 5 to form a sequence:

Term number	1	2	3	4	5	6	7	8
Term value								

   (c) What is similar about the two sequences you have formed?

(d) Now fill in your own sequence in the same table:

Term number	1	2	3	4	5	6	7	8
Term value in (b)								
Term value of your own sequence in (a)								

(e) What must you add to or subtract from each term value in (b) to get the same sequence as the one you made in (a)?

(f) Copy and fill in the following to write a formula for each sequence:

For the sequence in (b): Term value = ... (term number)

For the sequence in (a): Term value = ... (term number)

2. Now you are going to repeat what you did in question 1, with a different set of sequences. In this sequence, the term number is multiplied by 3 to get the term value.

Term number	1	2	3	4	5	6	7	8
Term value	3	6	9	12	15	18	21	24

Now make a formula describing the relationship of the **term value** to the **term number** for each of these sequences:

(a) The sequence that starts with 8 and is formed by adding 3 repeatedly.

(b) The sequence that starts with 12 and is formed by adding 3 repeatedly.

(c) The sequence that starts with 2 and is formed by adding 3 repeatedly.

3. Copy the tables. Write the first eight terms of each of the following sequences and in each case, describe how each term can be calculated from the previous term.

(a) Term value = 10 × term number + 5

Term number	1	2	3	4	5	6	7	8
Term value								

(b) Term value = 5 × term number – 3

Term number	1	2	3	4	5	6	7	8
Term value								

4. For each sequence, write a formula to obtain each term from the previous term. Try to write a formula which relates each term to its position in the sequence. Check both your formulae by applying them, and write the results in a table like the one below.

(a) 7 11 15 19 23 27 31 35 39 43

A. Relationship between consecutive terms

B. Relationship between term value and its position in sequence

Term number	1	2	3	4	5
Term value using A					
Term value using B					

(b) 60 57 54 51 48 45 42 39 36

A. Relationship between consecutive terms

B. Relationship between term value and its position in sequence

Term number	1	2	3	4	5
Term value using A					
Term value using B					

(c) 1 2 4 8 16 32 64 128

A. Relationship between consecutive terms

B. Relationship between term value and its position in sequence

Term number	1	2	3	4	5
Term value using A					
Term value using B					

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CHAPTER 7
Functions and relationships

7.1 Find output numbers for given input numbers

TWO DIFFERENT SETS OF INPUT NUMBERS

In this activity you will do some calculations with:

- Set A: the natural numbers smaller than 10, i.e. 1, 2, 3, 4, 5, 6, 7, 8 and 9.
- Set B: multiples of 10 that are bigger than 10 but smaller than 100, i.e. the numbers 20, 30, 40, 50, 60, 70, 80 and 90.

1. You are going to choose a number, multiply it by 5, and subtract the answer from 50.
   (a) Choose any number from set A and do the above calculations.
   (b) Choose any number from set B and do the above calculations.
   (c) If you choose any other number from set B, do you think the answer will also be a negative number?

2. (a) Write down all the different output numbers that will be obtained when the calculations $50 - 5x$ are performed on the different numbers in set A.
   (b) Write down the output numbers that will be obtained when the formula $50 - 5x$ is applied to set B.

Output numbers are numbers that you obtain when you apply the rule to the input numbers.

3. (a) Copy and complete the following table for set A:

Input numbers	1	2	3	4	5	6	7	8	9
Values of $50 - 5x$									

   (b) Copy and complete the following table for set B:

Input numbers	20	30	40	50	60	70	80	90
Values of $50 - 5x$								

4. In this question your set of input numbers will be the even numbers: 2; 4; 6; 8; 10; ...
   (a) What will all the output numbers be if the rule $2n + 1$ is applied to the set of even numbers? Write a list.

(b) What will the output numbers be if the rule $2n - 1$ is applied?
(c) What will the output numbers be if the rule $2n + 5$ is applied?
(d) What will the output numbers be if the rule $3n + 1$ is applied?

5. (a) What kind of output numbers will be obtained by applying the rule $x - 1\,000$ to natural numbers smaller than 1 000?
(b) What kind of output numbers will be obtained by applying the rule $\frac{x}{10} + 10$ to natural numbers smaller than 10?
(c) If you use the rule $30x + 2$, and use input numbers that are positive fractions with denominators 2, 3 and 5, what kind of output numbers will you obtain?

7.2 Different ways to represent the same relationship

Consider the work that you did in Section 6.4 of Chapter 6. In each question, there were two variable quantities.

A quantity that changes is called a **variable quantity** or just a **variable**.

If one variable quantity is influenced by another, we say there is a **relationship** between the two variables. You can sometimes work out which number is linked to a specific value of the other variable.

An algebraic expression, such as $10x + 5$, describes what calculations must be done to find the output number that corresponds to a given input number.

The output number can also be called the output value, or the value of the expression, which is $10x + 5$ in this case.

For any input number, application of the rule $10x + 5$ produces only one output number, and it is very clear what that number is. For instance, if the formula is applied to $x = 3$, the output number is 35.

A relationship between two variables in which there is only one output number for each input number, is called a **function**.

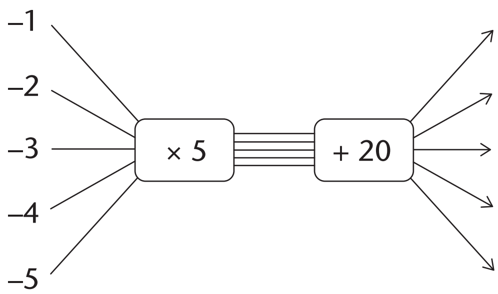
Functions can be represented in:

- a table that shows some values of the two variables as it clearly shows which value of the output variable corresponds to each particular value of the input variable
- a flow diagram, which shows what calculations are to be done to calculate the output number that corresponds to a given input variable

- a formula, which also describes what calculations are to be done to calculate the output number that corresponds to a given input variable
- a graph.

Examples of these four ways of describing a function are given on the next two pages.

1. Copy and complete the following flow diagram:



A completed flow diagram shows two kinds of information:

- It shows what calculations are done to produce the output numbers.
- It shows which output number is connected to which input number.

The flow diagram that you completed shows:

- that each input number is multiplied by 5, then 20 is added, to produce the output numbers
- which output numbers correspond to which input numbers.

The calculations that need to be done can also be described with an expression. The expression $5x + 20$ describes the calculations that you did in question 1. You can also write this as a formula:

- A **verbal formula**:
  output number = 5 × input number + 20
- An **algebraic formula**:
  output number = $5x + 20$

The output numbers of a function are also called **function values**. Hence the formula can also be written as *function value* = $5x + 20$.

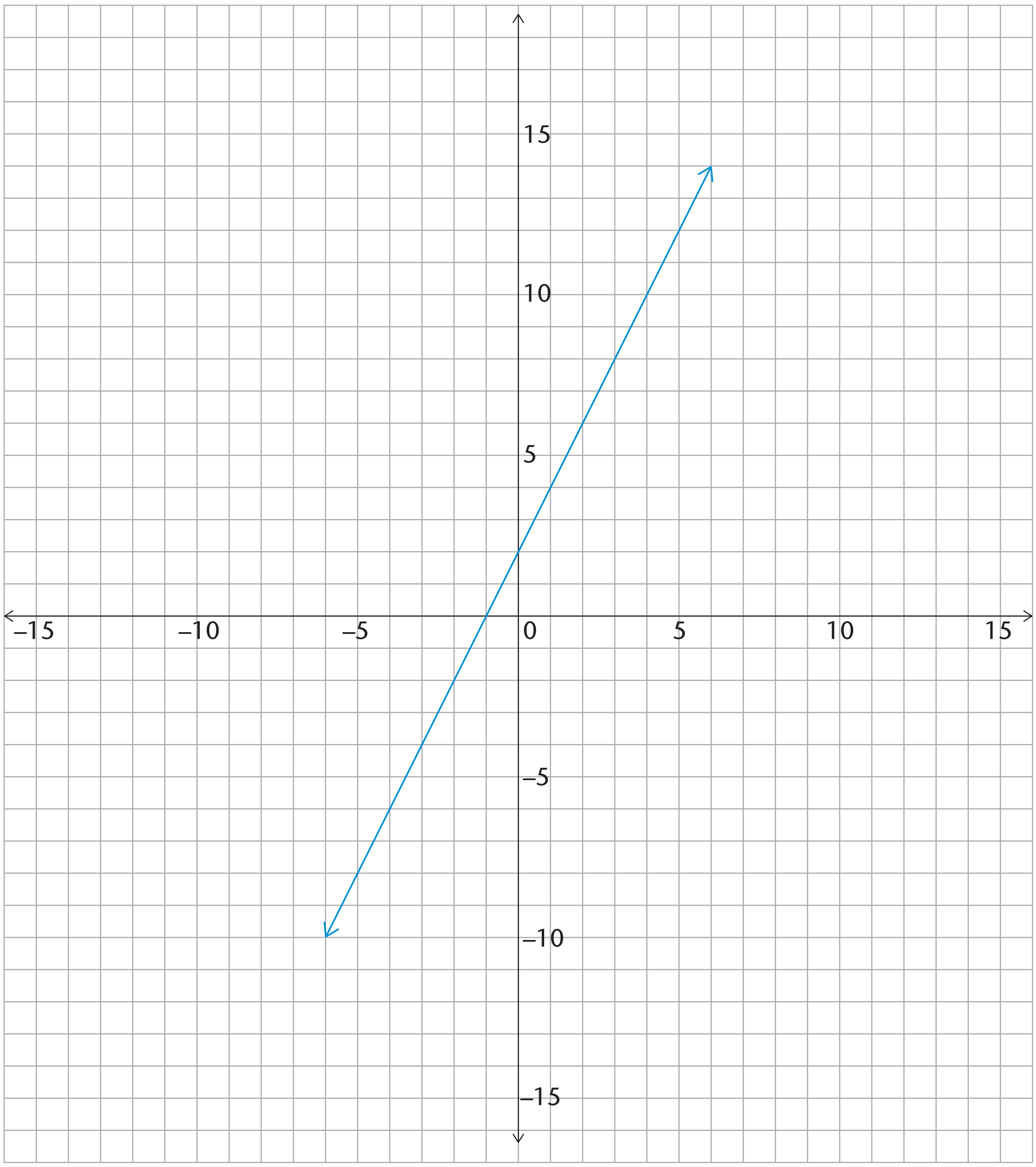
2. Copy and complete this table for the function described by $5x + 20$:

Input numbers	–1	–2	–3	–4	–5
Function values					

3. Draw a graph of this function discussed in question 1 and 2 on graph paper.

4. A graph of a certain function is given below. Copy and complete the table for this function:

Input numbers					
Function values					



7.3 Different representations of the same relationship

On separate pages, represent each of the following functions with the following:

(a) a flow diagram
(b) a table of values for the set of integers from –5 to 5
(c) a graph

1. The relationship described by the expression $3x + 4$.
2. The relationship described by the expression $2x - 5$.
3. The relationship described by the expression $\frac{1}{2}x + 2$.
4. The relationship described by the expression $-3x + 4$.
5. The relationship described by the expression $2{,}5x + 1{,}5$.
6. The relationship described by the expression $0{,}2x + 1{,}4$.
7. The relationship described by the expression $-2x - 4$.

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Chapter 8
Algebraic expressions

8.1 Algebraic language

WORDS, DIAGRAMS AND EXPRESSIONS

1. Copy and complete the following table:

	Words	Flow diagram	Expression
	Multiply a number by 5 and then subtract 3 from the answer.	— [× 5] — [– 3] →	$5x - 3$
(a)	Add 5 to a number and then multiply the answer by 3.	— [ ] — [ ] →	
(b)		— [– 3] — [× 5] →	
(c)			$3(2x + 3)$

> An **algebraic expression** indicates **a sequence of operations** that can also be described in words. In some cases they can be described with flow diagrams.

Expressions in brackets should always be calculated first. If there are no brackets in an algebraic expression, it means that multiplication and division must be done first, and addition and subtraction afterwards.

For example, if $x = 5$ the expression $12 + 3x$ means "multiply 5 by 3, then add 12". It does **not** mean "add 12 and 3, then multiply by 5".

If you wish to say "add 12 and 3, then multiply by 5", the numerical expression should be $5 \times (12 + 3)$ or $(12 + 3) \times 5$.

2. Describe each of these sequences of calculations with an algebraic expression:
   (a) Multiply a number by 10, subtract 5 from the answer, and multiply the answer by 3.

(b) Subtract 5 from a number, multiply the answer by 10, and multiply this answer by 3.

3. Evaluate each of these expressions for $x = 10$:

(a) $200 - 5x$

(b) $(200 - 5)x$

(c) $5x + 40$

(d) $5(x + 40)$

(e) $40 + 5x$

(f) $5x + 5 \times 40$

SOME WORDS WE USE IN ALGEBRA

- An expression with one term only, like $3x^2$, is a **monomial**.
- An expression which is a sum of two terms, like $5x + 4$, is called a **binomial**.
- An expression which is a sum of three terms, like $3x^3 + 2x + 9$, is called a **trinomial**.

The symbol x is often used to represent the **variable** in an algebraic expression, but other letter symbols may also be used.

In the monomial $3x^2$, the 3 is called the **coefficient** of x^2.

In the binomial $5x + 4$, and the trinomial $3x^2 + 2x + 9$, the numbers 4 and 9 are called **constants**.

1. Copy and complete the table, using the completed first row as an example:

Expression	Type of expression	Symbol used to represent the variable	Constant	Coefficient of ...
$x^2 + 6x + 10$	Trinomial	x	10	the second term is: 6
$6s^3 + s^2 + 5$				s^2 is:
$\frac{k}{3} + 12$				the first term is:
$4p^{10}$				p^{10} is:

2. Consider the polynomial pattern starting with $7x^5 + 5x^4 + 3x^3 + x^2 + ...$

(a) What is the coefficient of the fourth term?

(b) What is the exponent value of the fifth term?

(c) Do you think the sixth term will be a constant? Why?

EQUIVALENT ALGEBRAIC EXPRESSIONS

1. Copy and complete the table on page 69 by doing the necessary calculations. Calculate the numerical value of the expressions for the various values of x.

	x	–2	–1	0	1	2
(a)	$3x + 2$					
(b)	$2x - 3$					
(c)	$3x + 2 + 2x - 3$					
(d)	$2x - 3 + 3x + 2$					
(e)	$5x - 1$					
(f)	$(3x + 2)(2x - 3)$					
(g)	$3x(2x - 3) + 2(2x - 3)$					
(h)	$6x^2 - 5x - 6$					
(i)	$\frac{(3x+2)(2x-3)}{3x+2}$					
(j)	$\frac{6x^2-5x-6}{3x+2}$					

2. Although they may look different, make a list of all the algebraic expressions above which have the same numerical value for the same value of x.

Equivalent expressions are algebraic expressions that have different sequences of operations, but have the same numerical value for any given value of x.

It is often convenient not to work with a given expression, but to **replace** it with an equivalent expression.

3. Copy and complete the following table:

x	2	3	5	10	–5	–10
$12x - 7 + 3x + 10 - 5x$						

4. Copy and complete the following table:

x	2	3	5	10	–5	–10
$10x + 3$						

5. (a) Is $10x + 3$ equivalent to $12x - 7 + 3x + 10 - 5x$? Explain your answer.

(b) Suppose you need to know how much $12x - 7 + 3x + 10 - 5x$ is for $x = 37$ and $x = -43$. What do you think is the easiest way to find out?

CONVENTIONS FOR WRITING ALGEBRAIC EXPRESSIONS

Here are some things that mathematicians have agreed upon, and it makes mathematical work much easier if all people stick to these agreements.

A **convention** is something that people have agreed to do in the same way.

The multiplication sign is often omitted in algebraic expressions: We normally write $4x$ instead of $4 \times x$, and $4(x - 5)$ instead of $4 \times (x - 5)$.

It is a convention to write a known number first in a product, i.e. we write $3 \times x$ rather than $x \times 3$, and we write $3x$ but **not** $x3$.

1. Rewrite each of the following in the normal way of writing algebraic expressions:

   (a) $x \times 4 + x \times y - y \times 3$

   (b) $7 \times (10 - x) + (5 \times x + 3)10$

People all over the world have agreed that, in expressions that do not contain brackets, addition and subtraction should be performed as they appear from left to right.

According to this convention, $x - y + z$ means that you first have to subtract y from x, then add z. For example if $x = 10$, $y = 5$ and $z = 3$, $x - y + z$ is $10 - 5 + 3$ and it means $10 - 5 = 5$, then $5 + 3 = 8$. It does not mean $5 + 3 = 8$, then $10 - 8 = 2$.

2. Calculate $50 - 20 + 30$ and $50 + 30 - 20$ and $50 - 30 + 20$.

3. Evaluate each of the following expressions for $x = 10$, $y = 5$ and $z = 2$:

   (a) $x + y - z$

   (b) $x - z + y$

   (c) $10y - 3x + 5z - 4y$

   (d) $10y - 3x - 5z + 4y + 3x$

People have also agreed that, in expressions that do not contain brackets, we should do multiplication (and division) **before** addition and subtraction.

Hence, $5 + 3 \times 4$ should be understood as "multiply 4 by 3, then add the answer to 5"; not as "add 5 and 3 then multiply the answer by 4".

Also, $3 \times 4 + 5$ should be understood as "multiply 4 by 3, then add 5 to the answer"; not as "add 4 and 5 then multiply the answer by 3".

4. Do each of the following calculations:
   (a) Multiply 4 by 3, then add 5 to the answer.
   (b) Add 4 and 5, then multiply the answer by 3.
   (c) Multiply 4 by 3, then add the answer to 5.
   (d) Add 5 and 3, then multiply the answer by 4.
5. Rewrite the instructions in 4(a) and 4(c) without using words.
6. Calculate each of the following:
   (a) $10 \times 5 + 30$
   (b) $30 + 10 \times 5$
   (c) $10 \times 5 - 30$
   (d) $30 - 10 \times 5$
7. (a) Add 4 and 5, then subtract the answer from 20.
   (b) Subtract 4 from 20 and then add 5.
   (c) Add 4 and 5, then multiply the answer by 3.
   (d) Multiply 3 by 5 and then add the answer to 4.

If we want to specify the calculations in 7(a) and 7(c) without using words, we will face challenges.

We cannot write $20 - 4 + 5$ for "*add 4 and 5 then subtract the answer from 20*", because that would mean "*subtract 4 from 20, then add 5*". We need a way to indicate, without using words, that we want the addition to be performed before the subtraction in this case.

Similarly, we cannot write $4 + 5 \times 3$ for "*add 4 and 5 then multiply the answer by 3*", because that would mean "*multiply 3 by 5 and then add the answer to 4*". We need a way to indicate, without using words, that we want the addition to be performed *before* the multiplication in this case.

Mathematicians have agreed to use brackets to address the above challenges. The following convention is used all over the world:

> Whenever there are brackets in an expression, the calculations within the brackets should be performed first.

Hence, $20 - (4 + 5)$ means *"add 4 and 5 then subtract the answer from 20"*, but $20 - 4 + 5$ means "*subtract 4 from 20, then add 5"*.

$(4 + 5) \times 3$ or $3 \times (4 + 5)$ means "*add 4 and 5 then multiply the answer by 3"*, but $4 + 5 \times 3$ means "*multiply 3 by 5, then add the answer to 4"*.

$10 + 2(5 + 9)$ means "*add 5 and 9, multiply the answer by 2, then add this answer to 10"*:

$5 + 9 = 14$ $\quad$ $14 \times 2 = 28$ $\quad$ $28 + 10 = 38$

8. Calculate each of the following:

(a) $100 + 50 - 30$
(b) $100 + (50 - 30)$
(c) $100 - 50 + 30$
(d) $100 - (50 + 30)$
(e) $3(10 - 4) + 2$
(f) $10(5 + 7) + 3(18 - 8)$
(g) $250 - 10 \times (18 + 2) + 35$
(h) $(20 + 20) \times (20 - 10)$
(i) $(250 - 10) \times (18 + 2) + 35$
(j) $20 + 20 \times (20 - 10)$
(k) $200 + \mathbf{(}100 \times 2(15 + 5)\mathbf{)}$
(l) $(200 + 100) \times 2 \times 15 + 5$

In algebra, we normally write $3(x + 2y)$ instead of $(x + 2y) \times 3$, and we write $3(x - 2y)$ instead of $(x - 2y) \times 3$. Do not let this conventional way of writing in algebra confuse you. The expression $3(x + 2y)$ does not mean that multiplication by 3 is the first thing you should do when you evaluate the expression for certain values of x and y. The first thing you should do is to add the values of x and y. That is what the brackets tell you!

However, performing the instructions $3(x + 2y)$ is not the only way in which you can find out how much $3(x + 2y)$ is for any given values of x and y. Instead of working out $3(x + 2y)$, you may work out $3x + 6y$. In this case you will multiply each term before you add them together.

9. Evaluate each of the following expressions for $x = 10$, $y = 5$ and $z = 2$:

(a) $xy + z$
(b) $x(y + z)$
(c) $x + yz$
(d) $xy + xz$
(e) $xy - z$
(f) $x(y - z)$
(g) $x - yz$
(h) $xy - yz$
(i) $x + (y - z)$
(j) $x - (y - z)$
(k) $x - (y + z)$
(l) $x - y - z$
(m) $x + y - z$
(n) $x - y + z$

8.2 Properties of operations

1. Calculate each of the following:

(a) $5(3 + 4)$
(b) $5 \times 3 + 5 \times 4$
(c) $6 \times 3 + (4 + 6)$
(d) $(6 + 4) + 3 \times 6$
(e) $3 \times (4 \times 5)$
(f) $(3 \times 4) \times 5$

You should have noticed that for each row the results are the same. This is because operations with numbers have certain properties, namely the **distributive**, **commutative** and **associative** properties.

The **distributive** property is used each time you multiply a number in parts. For example:

The number thirty-four is actually 30 + 4. You may calculate 5 × 34 by calculating 5 × 30 and 5 × 4, and then adding the two answers:

$$5 \times 34 = 5 \times 30 + 5 \times 4$$

The word "distribute" means to spread out. The distributive property may be described as follows:

$a(b + c) = ab + ac$

where a, b and c can be any numbers.

We may say: "multiplication distributes over addition".

2. Calculate each of the following:
   (a) $5(x - y)$ for $x = 10$ and $y = 8$
   (b) $5x - 5y$ for $x = 10$ and $y = 8$
   (c) $5(x - y)$ for $x = 100$ and $y = 30$
   (d) $5x - 5y$ for $x = 100$ and $y = 30$
   (e) $5(x - y + z)$ for $x = 10$, $y = 3$ and $z = 2$
   (f) $5x - 5y + 5z$ for $x = 10$, $y = 3$ and $z = 2$

3. We say "multiplication distributes over addition". Does multiplication also distribute over subtraction? Give examples to support your answer.

For any values of x and y:

- $x + y$ and $y + x$ give the same answers, and
- xy and yx give the same answers.

This is called the **commutative property** of addition and multiplication.

4. We say "addition is commutative" and "multiplication is commutative".
   Is subtraction also commutative? Demonstrate your answer with an example.

The **associative property** allows you to arrange three or more numbers in any sequence when adding or multiplying. For any values of x, y and z, the following expressions all have the same answer:

$$x + y + z \qquad y + x + z \qquad z + y + x$$

5. Calculate 16 + 33 + 14 + 17 in the easiest possible way.

The associative property of multiplication allows you to simplify something like the following:

$$abc + bca + cba$$

Because the order of multiplication does not change the result we can rewrite this expression as: $abc + abc + abc$.

This then can be simplified by adding like terms to be $3abc$. You will be able to use these properties throughout this chapter and when you do algebraic manipulations.

When you form an expression that is equivalent to a given expression, you say that you *manipulate* the expression.

6. Replace each of the following expressions with a simpler expression that will give the same answer. **Do not do any calculations now**. In each case, state why your replacement will be easier to do.
   (a) $17 \times 43 + 17 \times 57$
   (b) $7 \times 5 \times 8 \times 4 + 12 \times 8 \times 4 \times 7 - 9 \times 4 \times 5 \times 8$
   (c) $43 \times 17 + 57 \times 17$
   (d) $43x + 57x$ (for $x = 213$ or any other value)

7. Which properties of operations did you use in each part of question 6?

8.3 Combining like terms in algebraic expressions

REARRANGE TERMS, THEN COMBINE LIKE TERMS

To check whether two expressions are possibly equivalent, you can evaluate both expressions for several different values of the variable.

1. In each case below, copy the tables, then predict whether the two expressions are equivalent. Check by evaluating both for $x = 1$, $x = 10$, $x = 2$ and $x = -2$ in the tables.
   (a) $x(x + 3)$ and $x^2 + 3$

   (b) $x(x + 3)$ and $x^2 + 3x$

Some expressions can be simplified by rearranging the terms and combining "like terms".

In the expression $5x^2 + 13x + 7 + 2x^2 - 8x - 12$, the terms $5x^2$ and $2x^2$ are like terms.

> Two or more like terms can be combined to form a single term.

$5x^2 + 2x^2$ can be replaced by $7x^2$ because for any value of x, for example $x = 2$ or $x = 10$, calculating $5x^2 + 2x^2$ and $7x^2$ will produce the same output value. Try it!

2. Copy and complete the following table:

x	10	2	5	1
$5x^2 + 2x^2$				
$7x^2$				
$13x - 8x$				
$5x$				

It is difficult to see the like terms in a long expression like $3x^2 + 13x + 7 + 2x^2 - 8x - 12$. Fortunately, you can rearrange the terms in an expression so that the like terms are next to each other.

3. (a) Copy the table and complete the second and third rows of the table. You will complete the next two rows when you do question 3(g).

x	10	2	5	1
$3x^2 + 13x + 7 + 2x^2 - 8x - 12$				
$3x^2 + 2x^2 + 13x - 8x + 7 - 12$				

   (b) What do you observe?
   (c) How does the one expression in the above table differ from the other one?
   (d) Combine like terms in $3x^2 + 2x^2 + 13x - 8x + 7 - 12$ to make a shorter equivalent expression.
   (e) Evaluate your shorter expression for $x = 10$, $x = 2$ and $x = 5$.
   (f) Is your shorter expression equivalent to $3x^2 + 13x + 7 + 2x^2 - 8x - 12$? Explain how you know whether it is or is not.
   (g) Evaluate $5x^2 + 5x - 5$ and $5(x^2 + x - 1)$ for $x = 10$, $x = 2$, $x = 5$ and $x = 1$, and write your answers in the last two rows of the table.

4. Simplify each expression:
   (a) $(3x^2 + 5x + 8) + (5x^2 + x + 4)$
   (b) $(7x^2 + 3x + 5) + (2x^2 - x - 2)$
   (c) $(6x^2 - 7x - 4) + (4x^2 + 5x + 5)$
   (d) $(2x^2 - 5x - 9) - (5x^2 - 2x - 1)$
   (e) $(-2x^2 + 5x - 3) + (-3x^2 - 9x + 5)$
   (f) $(y^2 + y + 1) + (y^2 - y - 1)$

5. Copy and complete the table. (Hint: Save yourself some work by simplifying first!)

x	2,5	3,7	6,4	12,9	35	–4,7	–0,04
$(3x + 6{,}5) + (7x + 3{,}5)$							
$(13x - 6) + (26 - 12x)$							

6. Simplify:
   (a) $(2r^2 + 3r - 5) + (7r^2 - 8r - 12)$
   (b) $(2r^2 + 3r - 5) - (7r^2 - 8r - 12)$
   (c) $(2x + 5xy + 3y) - (12x - 2xy - 5y)$
   (d) $(2x + 5xy + 3y) + (12x - 2xy - 5y)$

7. Evaluate the following expressions for $x = 3$, $x = -2$, $x = 5$ and $x = -3$:
   (a) $2x(x^2 - x - 1) + 5x(2x^2 + 3x - 5) - 3x(x^2 + 2x + 1)$
   (b) $(3x^2 - 5x + 7) - (7x^2 + 3x - 5) + (5x^2 - 2x + 8)$

8. Write equivalent expressions without brackets:
   (a) $3x^4 - (x^2 + 2x)$
   (b) $3x^4 - (x^2 - 2x)$
   (c) $3x^4 + (x^2 - 2x)$
   (d) $x - (y + z - t)$

9. Write equivalent expressions without brackets, rearrange so that like terms are grouped together, and then combine the like terms:
   (a) $2y^2 + (y^2 - 3y)$
   (b) $3x^2 + (5x + x^2)$
   (c) $6x^2 - (x^4 + 3x^2)$
   (d) $2t^2 - (3t^2 - 5t^3)$
   (e) $6x^2 + 3x - (4x^2 + 5x)$
   (f) $2r^2 - 5r + 7 + (3r^2 - 7r - 8)$
   (g) $5(x^2 + x) + 2(x^2 + 3x)$
   (h) $2x(x - 3) + 5x(x + 2)$

10. Write equivalent expressions without brackets and simplify these expressions as far as possible.

    Example: $5r^2 - 2r(r + 5) = 5r^2 - 2r^2 - 10r$
    $= 3r^2 - 10r$

    (a) $3x^2 + x(x + 3)$
    (b) $5x + x(7 - 2x)$
    (c) $6r^2 - 2r(r - 5)$
    (d) $2a(a + 3) + 5a(a - 2)$
    (e) $6y(y + 1) - 3y(y + 2)$
    (f) $4x(2x - 3) - 3x(x + 2)$
    (g) $2x^2(x - 5) - x(3x^2 - 2)$
    (h) $x(x - 1) + x(2x + 3) - 2x(3x + 1)$

8.4 Multiplication of algebraic expressions

MULTIPLY POLYNOMIALS BY MONOMIALS

1. (a) Calculate 3 × 38 and 3 × 62, and add the two answers.
   (b) Add 38 and 62, then multiply the answer by 3.

(c) If you do not get the same answer for (a) and (b), you have made a mistake. Rework until you get it right.

The fact that if you work correctly, you get the same answer in questions 1(a) and (b), is a demonstration of the **distributive property**.

The distributive property may be described as follows:
$a(b + c) = ab + ac$ and
$a(b - c) = ab - ac$,
where a, b and c can be any numbers.

What you saw in question 1 was that:
$3 \times 100 = 3 \times 38 + 3 \times 62$.

This can also be expressed by writing $3(38 + 62) = 3 \times 38 + 3 \times 62$.

2. (a) Calculate 10×56.
   (b) Calculate $10 \times 16 + 10 \times 40$.
3. (a) Write down any two numbers smaller than 100. Let us call them x and y. Add your two numbers and multiply the answer by 3.
   (b) Calculate $3 \times x$ and $3 \times y$, and add the two answers.
   (c) If you do not get the same answers for (a) and (b), you have made a mistake somewhere. Correct your work.
4. Copy and complete the following table:

x	12	50	5
y	4	30	10
$5x - 5y$			
$5(x - y)$			
$5x + 5y$			
$5(x + y)$			

Performing the instructions $5(x + y)$ is not the only way in which you can find out how much $5(x + y)$ is for any given values of x and y. Instead of doing $5(x + y)$, you may do $5x + 5y$. In this case you will multiply first, and again, before you add.

5. (a) For $x = 10$ and $y = 20$, evaluate $8(x + y)$ by first adding 10 and 20, and then multiplying by 8.
   (b) Now evaluate $8(x + y)$ by doing $8x + 8y$; in other words, first calculate 8×10 and 8×20.

6. In question 5 you evaluated $8(x + y)$ in two different ways for the given values of x and y. Now also evaluate $20(x - y)$ in two different ways, for $x = 5$ and $y = 3$.

7. Use the distributive property in each of the following cases to make a different expression that is equivalent to the given expression:

   (a) $a(b + c)$ (b) $a(b + c + d)$
   (c) $x(x + 1)$ (d) $x(x^2 + x + 1)$
   (e) $x(x^3 + x^2 + x + 1)$ (f) $x^2(x^2 - x + 3)$
   (g) $2x^2(3x^2 + 2)$ (h) $3x^3(2x^2 + 4x - 5)$
   (i) $-2x^4(x^3 - 2x^2 - 4x + 5)$ (j) $a^2b(a^3 - a^2 + a + 1)$
   (k) $x^2y^3(3x^2y + xy^2 - y)$ (l) $-2x(x^3 - y^3)$
   (m) $2a^2b(3a^2 + 2a^2b^2 + 4b^2)$ (n) $2ab^2(3a^3 - 1)$

What you do in this question is sometimes called "multiplication of a polynomial by a monomial". One may also say that in each case you **expand** the expression, or you write an equivalent expression in **expanded form**.

8. Expand the parts of each expression and simplify. Then evaluate the expression for $x = 5$.

   (a) $5(x - 2) + 3(x + 4)$ (b) $x(x + 4) - 4 (x + 4)$
   (c) $x(x - 4) + 4(x - 4)$ (d) $x(x^2 + 3x + 9) - 3(x^2 + 3x + 9)$
   (e) $x(x^2 - 3x + 9) + 3(x^2 - 3x + 9)$ (f) $x^2(x^2 - 3x + 4) - x(x^3 + 4x^2 + 2x + 3)$

9. Write in expanded form:

   (a) $x(x^2 + 2xy + y^2) + y(x^2 + 2xy + y^2)$
   (b) $x^2y(x^2 - 2xy + y^2) - xy^2(2x^2 - 3xy - y^2)$
   (c) $ab^2c(b^2c^2 - ac) + b^2c^4(a^2 + abc^2)$
   (d) $p^2q(pq^2 + p + q) + pq(p - q^2)$

SQUARES AND CUBES AND ROOTS OF MONOMIALS

1. Evaluate each of the following expressions for $x = 2$, $x = 5$ and $x = 10$:

   (a) $(3x)^2$ (b) $9x^2$
   (c) $(2x)^2$ (d) $4x^2$
   (e) $(2x)^3$ (f) $8x^3$
   (g) $(2x + 3x)^2$ (h) $(10x - 7x)^2$

2. In each case, write an equivalent monomial without brackets:

   (a) $(5x)^2$ (b) $(5x)^3$
   (c) $(20x)^2$ (d) $(10x)^3$
   (e) $(2x + 7x)^2$ (f) $(20x - 13x)^3$

The square root of $16x^2$ is $4x$, because $(4x)^2 = 16x^2$.

3. Write down the square root of each of the following expressions:

(a) $\sqrt{(7x)^2}$ (b) $\sqrt{9x^2}$

(c) $\sqrt{(20x)^2}$ (d) $\sqrt{100x^2}$

(e) $\sqrt{(20x - 15x)^2}$ (f) $\sqrt{16x^2 + 9x^2}$

(g) $\sqrt{(21x - 16x)^2}$ (h) $\sqrt{(5x)^2}$

The cube root of $64x^3$ is $4x$, because $(4x)^3 = 64x^3$.

4. Write down the cube root of each of the following expressions:

(a) $\sqrt[3]{(7x)^3}$ (b) $\sqrt[3]{27x^3}$

(c) $\sqrt[3]{(20x)^3}$ (d) $\sqrt[3]{1\ 000x^3}$

(e) $\sqrt[3]{(20x - 15x)^3}$ (f) $\sqrt[3]{125x^3}$

8.5 Dividing polynomials by integers and monomials

1. Copy and complete the following table:

x	20	10	5	–5	–10	–20
$(100x - 5x^2) \div 5x$						
$20 - x$						

Can you explain your observations?

2. (a) R240 prize money must be shared equally between 20 netball players. How much should each one get?
   (b) Mpho decided to do the calculations below. Do not do Mpho's calculations, but think about this: Will Mpho get the same answer that you got for question (a)?
   $(140 \div 20) + (100 \div 20)$
   (c) Gert decided to do the calculations below. Without doing the calculations, say whether or not Gert will get the same answer that you got for question (a).
   $(240 \div 12) + (240 \div 8)$

3. Do the necessary calculations to find out whether the following statements are true or false:
   (a) $(140 + 100) \div 20 = (140 \div 20) + (100 \div 20)$
   (b) $240 \div (12 + 8) = (240 \div 12) + (240 \div 8)$
   (c) $(300 - 60) \div 20 = (300 \div 20) - (60 \div 20)$

Division is **right-distributive** over addition and subtraction, for example:
$(2 + 3) \div 5 = (2 \div 5) + (3 \div 5)$.

The division symbol is to the right of the brackets; it is not left-distributive, for example:
$10 \div (2 + 4) \neq (10 \div 2) + (10 \div 4)$.

For example: $(200 + 40) \div 20 = (200 \div 20) + (40 \div 20) = 10 + 2 = 12$, and
$(500 + 200 - 300) \div 50 = (500 \div 50) + (200 \div 50) - (300 \div 50)$

4. Evaluate each expression for $x = 2$ and $x = 10$:
   (a) $(10x^2 + 5x) \div 5$
   (b) $(10x^2 \div 5) + (5x \div 5)$
   (c) $2x^2 + x$
   (d) $(10x^2 + 5x) \div 5x$
   (e) $(10x^2 \div 5x) + (5x \div 5x)$
   (f) $2x + 1$

The distributive property of division can be expressed in the following way:
$(x + y) \div z = (x \div z) + (y \div z)$
$(x - y) \div z = (x \div z) - (y \div z)$

5. (a) Do not do any calculations. Which of the following expressions do you *think* will have the same value as $(10x^2 + 20x - 15) \div 5$, for $x = 10$ as well as $x = 2$?

   $2x^2 + 20x - 15$ $\quad$ $10x^2 + 20x - 3$ $\quad$ $2x^2 + 4x - 3$

   (b) Do the necessary calculations to check your answer.

6. Simplify:
   (a) $(2x + 2y) \div 2$
   (b) $(4x + 8y) \div 4$
   (c) $(20xy + 16x) \div 4x$
   (d) $(42x - 6) \div 6$
   (e) $(28x^4 - 7x^3 + x^2) \div x^2$
   (f) $(24x^2 + 16x) \div 8x$
   (g) $(30x^2 - 24x) \div 3x$

7. Simplify:
   (a) $(9x^2 + xy) \div xy$
   (b) $(48a - 30ab + 16ab^2) \div 2a$
   (c) $(3a^3 + a^2) \div a^2$
   (d) $(13a - 17ab) \div a$
   (e) $(3a^2 + 5a^3) \div a$
   (f) $(39a^2b + 13ab + ab^2) \div ab$

The instruction $72 \div 6$ may also be written as $\frac{72}{6}$.

This notation, which looks just like the common fraction notation, is often used to indicate division.

Hence, instead of $(10x^2 + 20x - 15) \div 5$, we may write $\frac{10x^2 + 20x - 15}{5}$.

Since $(10x^2 + 20x - 15) \div 5$ is equivalent to $(10x^2 \div 5) + (20x \div 5) - (15 \div 5)$,
$\frac{10x^2 + 20x - 15}{5}$ is equivalent to $\frac{10x^2}{5} + \frac{20x}{5} - \frac{15}{5}$.

8. Find a simpler equivalent expression for each of the following expressions (clearly, these expressions do not make sense if $x = 0$):

(a) $\frac{16x^2 - 12x}{4x}$

(b) $\frac{16x^3 - 12x}{4x}$

(c) $\frac{16x^3 - 12x^2}{4x}$

(d) $\frac{16x^3 - 12x^2}{4x^2}$

(e) $\frac{16x^3 - 12x^2}{2x}$

(f) $\frac{16x^3 - 12x}{8x}$

9. In each case check if the statement is true for $x = 10$, $x = 100$, $x = 5$, $x = 1$ and $x = -2$.

(a) $\frac{x^2}{x} = x$

(b) $\frac{x^3}{x} = x^2$

(c) $\frac{x^3}{x^2} = x$

(d) $\frac{5x^3}{x} = 5x^2$

(e) $\frac{5x^3}{x} = 5^3$

(f) $\frac{5x}{x^2} = \frac{5}{x}$

10. Explain why the equations below are true:

(a) $\frac{100x - 5x^2}{5x} = 20 - x$ for all values of x, except $x = 0$.

(b) $\frac{15x^2 - 10x}{5x}$ is equivalent to $3x - 2$, excluding $x = 0$.

11. Copy and complete the following table:

x	1,5	2,8	–3,1	0,72
$\frac{3x+12}{3}$				
$\frac{18x^2+6}{6}$				
$\frac{5x^2+7x}{x}$				

(Hint: Simplify the expressions first to save yourself some work!)

12. Simplify each expression to the equivalent form requiring the fewest operations:

(a) $\frac{3a + a^2}{a}$

(b) $\frac{x^3 + 2x^2 - x}{x}$

(c) $\frac{2a + 12ab}{2a}$

(d) $\frac{12x^2 + 10x}{2x}$

(e) $\frac{21ab - 14a^2}{7a}$

(f) $\frac{15a^2b + 30ab^2}{5ab}$

(g) $\frac{7x^3 + 21x^2}{7x^2}$

(h) $\frac{3x^2 + 9x}{3x}$

13. Solve the equations:

(a) $\dfrac{3x^2+15x}{3x}=20$ (b) $\dfrac{30x-18x^2}{6x}=2$

14. Copy and complete the following table:

	x	1,1	1,2	1,3	1,4	1,5
(a)	$\dfrac{x^3+2x^2-x}{x}$					
(b)	$\dfrac{7x^3+21x^2}{7x^2}$					
(c)	$\dfrac{50x^2+5x}{5x}$					

15. Simplify the following expressions:

(a) $\dfrac{3x(5x+4)+6x(5x+3)}{5x}$ (b) $\dfrac{14x^2-28x}{7x}+\dfrac{24x-18x^2}{3x}$

8.6 Products and squares of binomials

How can we obtain the expanded form of $(x + 2)(x + 3)$?

In order to expand $(x + 2)(x + 3)$, you can first keep $(x + 2)$ as it is, and apply the distributive property:

$(\boldsymbol{x+2})(x+3)$
$= (\boldsymbol{x+2})x + (\boldsymbol{x+2})3$
$= x^2 + 2x + 3x + 6$
$= x^2 + 5x + 6$

1. Describe how can you check if $(x + 2)(x + 3)$ is actually equivalent to $x^2 + 5x + 6$.

To expand $(x - y)(x + 3y)$ it can be written as $(x - y)x + (x - y)3y$, and the two parts can then be expanded.

$(x - y)(x + 3y)$
$= (x - y)x + (x - y)3y$
$= x^2 - xy + 3xy - 3y^2$
$= x^2 + 2xy - 3y^2$

2. Do some calculations to check whether $(x - y)(x + 3y)$ and $x^2 + 2xy - 3y^2$ are equivalent. Write the results of your calculations in a table like the one on page 83.

x					
y					

3. Expand each of these expressions:

(a) $(x + 3)(x + 4)$
(b) $(x + 3)(4 - x)$
(c) $(x + 3)(x - 5)$
(d) $(2x^2 + 1)(3x - 4)$
(e) $(x + y)(x + 2y)$
(f) $(a - b)(2a + 3b)$
(g) $(k^2 + m)(k^2 + 2m)$
(h) $(2x + 3)(2x - 3)$
(i) $(5x + 2)(5x - 2)$
(j) $(ax - by)(ax + by)$

4. Expand each of these expressions:

(a) $(a + b)(a + b)$
(b) $(a - b)(a - b)$
(c) $(x + y)(x + y)$
(d) $(x - y)(x - y)$
(e) $(2a + 3b)(2a + 3b)$
(f) $(2a - 3b)(2a - 3b)$
(g) $(5x + 2y)(5x + 2y)$
(h) $(5x - 2y)(5x - 2y)$
(i) $(ax + b)(ax + b)$
(j) $(ax - b)(ax - b)$

5. Can you guess the answer to each of the following questions without working it out as you did in question 3? Try them out and then check your answers.
Expand the following expressions:

(a) $(m + n)(m + n)$
(b) $(m - n)(m - n)$
(c) $(3x + 2y)(3x + 2y)$
(d) $(3x - 2y)(3x - 2y)$

All the expressions in questions 4 and 5 are **squares of binomials**, for example $(ax + b)^2$ and $(ax - b)^2$.

6. Expand:

(a) $(ax + b)^2$
(b) $(ax - b)^2$
(c) $(2s + 5)^2$
(d) $(2s - 5)^2$
(e) $(ax + by)^2$
(f) $(ax - by)^2$
(g) $(2s + 5r)^2$
(h) $(2s - 5r)^2$

7. Expand and simplify:

(a) $(4x + 3)(6x + 4) + (3x + 2)(8x + 5)$
(b) $(4x + 3)(6x + 4) - (3x + 2)(8x + 5)$

8.7 Substitution into algebraic expressions

1. In question 2 you have to find the values of different expressions, for some given values of x. Look carefully at the different expressions in the table. Do you think some of them may be equivalent?

   Simplify the longer expression to check whether you end up with the shorter expression.

2. Copy and complete the following table:

	x	13	–13	2,5	10
(a)	$(2x + 3)(3x - 5)$				
(b)	$10x^2 + 5x - 7 + 3x^2 - 4x - 3$				
(c)	$3(10x^2 - 5x + 2) - 5x(6x - 4)$				
(d)	$13x^2 + x - 10$				
(e)	$6x^2 - x - 15$				
(f)	$5x + 6$				

3. Copy and complete the following table:

	x	1	2	3	4
(a)	$(2x + 3)(5x - 3) + (10x + 9)(1 - x)$				
(b)	$\frac{9x^2 + 30x}{3x}$				
(c)	$3x(10x - 5) - 5x(6x - 4)$				
(d)	$5x(4x + 3) - 2x(7 + 13x) + 2x(3x + 2)$				

4. Describe any patterns that you observe in your answers for question 3.

5. Copy and complete the following table:

	x	1,5	2,5	3,5	4,5
(a)	$(2x + 3)(5x - 3) + (10x + 9)(1 - x)$				
(b)	$\frac{9x^2 + 30x}{3x}$				
(c)	$3x(10x - 5) - 5x(6x - 4)$				
(d)	$5x(4x + 3) - 2x(7 + 13x) + 2x(3x + 2)$				

Chapter 9
Equations

9.1 Solving equations by inspection

1. Six equations are listed in the table below. Use the table to find out for which of the given values of x will be true that the left-hand side of the equation is equal to the right-hand side.

"Searching" for the solution of an equation by using tables is called **solution by inspection**.

x	–3	–2	–1	0	1	2	3	4
$2x + 3$	–3	–1	1	3	5	7	9	11
$x + 4$	1	2	3	4	5	6	7	8
$9 - x$	12	11	10	9	8	7	6	5
$3x - 2$	–11	–8	–5	–2	1	4	7	10
$10x - 7$	–37	–27	–17	–7	3	13	23	33
$5x + 3$	–12	–7	–2	3	8	13	18	23
$10 - 3x$	19	16	13	10	7	4	1	–2

(a) $2x + 3 = 5x + 3$ (b) $5x + 3 = 9 - x$ (c) $2x + 3 = x + 4$
(d) $10x - 7 = 5x + 3$ (e) $3x - 2 = x + 4$ (f) $9 - x = 2x + 3$

Two equations can have the same solution. For example, $5x = 10$ and $x + 2 = 4$ have the same solution; $x = 2$ is the solution for both equations.

Two equations are called **equivalent** if they have the same solution.

2. Which of the equations in question 1 have the same solutions? Explain.

9.2 Solving equations using additive and multiplicative inverses

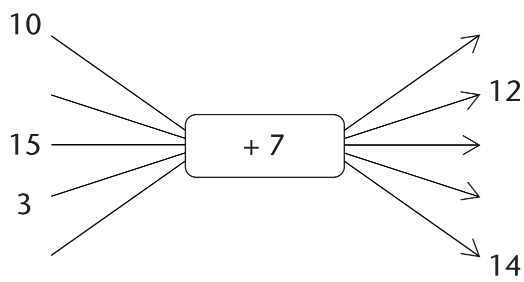
1. In each case find the value of x:

(a) x → [+ 7] → 10

(b) x → [× 2] → [+ 3] → 13

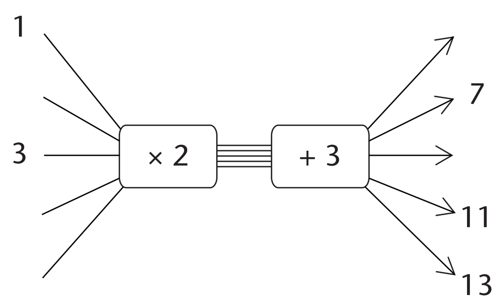
2. Copy and complete the flow diagrams. Fill in all the missing numbers.

(a)



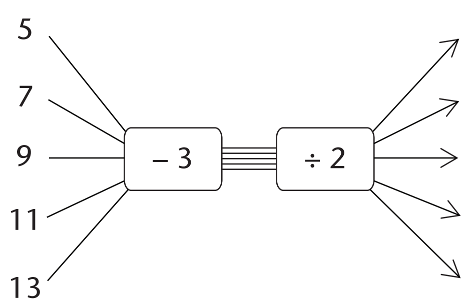
To find the second input number you may say to yourself, *"After I added 7, I had 12. What did I have before I added 7?"*

(b)



To find the input number that corresponds to 13, you may ask yourself, "*What did I have before I added 3?*" and then, "*What did I have before I multiplied by 2?*"

3. Use your answers for question 2 to check your answers for question 1.
4. Describe the instructions in flow diagram 2(b) in words, and also with a symbolic expression.
5. Copy and complete the following flow diagram:



This flow diagram is called the **inverse** of the flow diagram in question 2(b).

6. Compare the input numbers and the output numbers of the flow diagrams in question 2(b) and question 5. What do you notice?
7. (a) Add 5 to any number and then subtract 5 from your answer. What do you get?
   (b) Multiply any number by 10 and then divide the answer by 10. What do you get?

If you add a number and then subtract the same number, you are back where you started. This is why addition and subtraction are called **inverse operations**.

If you multiply by a number and then divide by the same number, you are back where you started. This is why multiplication and division are called **inverse operations**.

The expression $5x - 3$ says *"multiply by 5 then subtract 3"*. This instruction can also be given with a flow diagram: $\longrightarrow \boxed{\times 5} \longrightarrow \boxed{-3} \longrightarrow$

The equation $5x - 3 = 47$ can also be written as a flow diagram:

$$\longrightarrow \boxed{\times 5} \longrightarrow \boxed{-3} \longrightarrow 47$$

8. Solve the equations below. You may do this by using the inverse operations. You may write a flow diagram to help you to see the operations.
   (a) $2x + 5 = 23$
   (b) $3x - 5 = 16$
   (c) $5x - 60 = -5$
   (d) $\frac{1}{3}x + 11 = 19$
   (e) $10(x + 3) = 88$
   (f) $2(x - 13) = 14$

9.3 Setting up equations

CONSTRUCTING EQUATIONS

You can easily make an equation that has 5 as the solution. Here is an example:

Start by writing the solution	$x = 5$
Add 3 to both sides	$x + 3 = 8$
Multiply both sides by 5	$5x + 15 = 40$

1. What is the solution of the equation $5x + 15 = 40$?
2. Make your own equation with the solution $x = 3$.
3. Bongile worked like this to make the equation $2(x + 8) = 30$, but he rubbed out part of his work:

Start by writing the solution	$x =$
Add 8 to both sides	$= 15$
Multiply both sides by 2	$2(x + 8) = 30$

   Copy and complete Bongile's writing to solve the equation $2(x + 8) = 30$.

4. This is how Bongile made a more difficult equation:

Start by writing the solution	$x =$
Multiply by 3 on both sides	$3x =$
Subtract 9 from both sides	$3x - 9 = 6$
Add $2x$ to both sides	$5x - 9 = 2x + 6$

   (a) What was on the right-hand side before Bongile subtracted 9?
   (b) What is the solution of $5x - 9 = 2x + 6$?

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5. Bongile started with a solution and he ended up with an equation. Write down the steps that Bongile took to make the equation, and solve the equation:

$$x =$$
$$8x =$$
$$8x + 3 =$$
$$3x + 3 = 35 - 5x$$

SOLVING EQUATIONS

To make an equation, you can apply the same operation on both sides.

To solve an equation, you can apply the inverse operation on both sides.

Make (↓)	Equation	Solve (↑)
	$x = 4$	
Multiply by 8	$8x = 32$	Divide by 8
Add 3	$8x + 3 = 35$	Subtract 3
Subtract $5x$	$3x + 3 = 35 - 5x$	Add $5x$

Use any appropriate method to solve the following equations:

1. (a) $5x + 3 = 24 - 2x$ (b) $2x + 4 = -9$
   (c) $3 - x = x - 3$ (d) $6(2x + 1) = 0$
2. (a) $4(1 - 2x) = 12 - 7x$ (b) $8(1 - 3x) = 5(4x + 6)$
   (c) $7x - 10 = 3x + 7$ (d) $1{,}6x + 7 = 3{,}5x + 3{,}2$

NUMBER PATTERNS AND EQUATIONS

1. (a) Which of the following rules will produce the number pattern given in the second row of the table below?
   A. Term value = $8n$ where n is the term number
   B. Term value = $6n - 1$ where n is the term number
   C. Term value = $6n + 2$ where n is the term number
   D. Term value = $10n - 2$ where n is the term number
   E. Term value = $5n + 3$ where n is the term number

Term number	1	2	3	4	5	6	7	8	9
Term value	8	13	18	23	28	33	38	43	48

   (b) The sixth term of the sequence has the value 33. Which term will have the value 143? You may set up and solve an equation to find out.
   (c) Apply rule E to your answer, to check if your answer is correct.

2. (a) Write the rule that will produce the number pattern in the second row of this table. You may have to experiment to find out what the rule is.

Term number	1	2	3	4	5	6	7	8	9
Term value	5	8	11	14	17	20	23	26	29

(b) Which term will have the value 221?

3. The rule for number pattern A is $4n + 11$, and the rule for pattern B is $7n - 34$.
   (a) Copy and complete the following table for the two patterns:

Term number	1	2	3	4	5	6	7	8	9
Pattern A									
Pattern B									

   (b) For which value of n are the terms of the two patterns equal?

9.4 Equation and situations

1. Consider this situation:
   To rent a room in a certain building, you have to pay a deposit of R400 and then R80 per day.
   (a) How much money do you need to rent the room for ten days?
   (b) How much money do you need to rent the room for 15 days?

2. Which of the following best describes the method that you used to do question 1(a) and (b)?
   A. Total cost = R400 + R80
   B. Total cost = 400(number of days + 80)
   C. Total cost = 80 × number of days + 400
   D. Total cost = (80 + 400) × number of days

3. For how many days can you rent the room described in question 1, if you have R2 800 to pay?

If you want to know for how many days you can rent the room if you have R720, you can set up an equation and solve it.

Example: You know the total cost is R720 and you know that you can work out the total cost like this:

Total cost = $80x + 400$, where x is the number of days.

So, $80x + 400 = 720$ and x = four days.

In each of the cases on page 90 (given in questions 4 to 7), find the unknown number by setting up an equation and solving it.

4. To rent a certain room, you have to pay a deposit of R300 and then R120 per day.
   (a) For how many days can you rent the room if you can pay a total of R1 740? (If you experience trouble in setting up the equation, it may help you to decide first how you will work out what it will cost to rent the room for six days.)
   (b) What will it cost to rent the room for ten days, 11 days and 12 days?
   (c) For how many days can you rent the room if you have R3 300 available?
   (d) For how many days can you rent the room if you have R3 000 available?

5. Ben and Thabo decide to do some calculations with a certain number. Ben multiplies the number by 5 and adds 12. Thabo gets the same answer as Ben when he multiplies the number by 9 and subtracts 16. What is the number they worked with?

6. The cost of renting a certain car for a period of x days can be calculated with the following formula:

   Rental cost in rands = $260x + 310$

   What information about renting this car will you get, if you solve the equation $260x + 310 = 2\,910$?

7. Sarah paid a deposit of R320 for a stall at a market, and she also pays R70 per day rental for the stall. She sells fruit and vegetables at the stall, and finds that she makes about R150 profit each day. After how many days will she have earned as much as she has paid for the stall, in total?

9.5 Solving equations by using the laws of exponents

You may need to look back at Chapter 5 to remember the laws of exponents.

One kind of exponential equation that you deal with in Grade 9 has one or more terms with a base that is raised to a power containing a variable.

Example: $2^x = 16$

When we need to find the unknown value, we are asking the question: *"To what power must the base be raised for the statement to be true?"*

Example:	$2^x = 16$	Make sure that the terms with x are on their own on one side.
	$2^x = 2^4$	Write the known term in the same base as the term with the exponent.
	$x = 4$	Equate the exponents.

In the example above, we can equate the exponents because the two numbers are equal only when they are raised to the same power.

1. Solve for x:

(a) $5^{x-1} = 125$
(b) $2^{x+3} = 8$
(c) $10^x = 10\ 000$
(d) $4^{x+2} = 64$
(e) $7^{x+1} = 1$
(f) $x^0 = 1$

Example: Solve for x: $3^x = \frac{1}{27}$

$3^x = 3^{-3}$ (Rewrite $\frac{1}{27}$ as a number to base 3.)

$x = -3$ (Equate the exponents.)

2. Solve for x:

(a) $7^x = \frac{1}{49}$
(b) $10^x = 0{,}001$
(c) $6^x = \frac{1}{216}$
(d) $10^{x-1} = 0{,}001$
(e) $4^{-x} = \frac{1}{16}$
(f) $7^x = 7^{-3}$

In another kind of equation involving exponents, the variable is in the base.

> When we need to find the unknown value, we ask the question: *"Which number must be raised to the given power for the statement to be true?"*

For these equations, you should remember what you know about the powers of numbers such as 2, 3, 4, 5 and 10.

SOLVING EQUATIONS WITH A VARIABLE IN THE BASE

1. Copy and complete the table below and answer the questions that follow:

	x	2	3	4	5
(a)	x^3	$2^3 = 8$			
(b)	x^5	$2^5 = 32$			
(c)	x^4	$2^4 = 16$			

For what value of x is:

(a) $x^3 = 64$
(b) $x^5 = 32$
(c) $x^4 = 256$
(d) $x^3 = 8$
(e) $x^4 = 16$
(f) $x^5 = 3\ 125$

2. Solve for x and give a reason:

(a) $x^3 = 216$
(b) $x^2 = 324$
(c) $x^4 = 10\ 000$
(d) $8^x = 512$
(e) $18^x = 324$
(f) $6^x = 216$

WORKSHEET

1. Ahmed multiplied a number by 5, added 3 to the answer, and then subtracted the number he started with. The answer was 11. What number did he start with?

2. Use any appropriate method to solve the equations:

(a) $3(x - 2) = 4(x + 1)$

(b) $5(x + 2) = -3(2 - x)$

(c) $1{,}5x = 0{,}7x - 24$

(d) $5(x + 3) = 5x + 12$

(e) $2{,}5x = 0{,}5(x + 10)$

(f) $7(x - 2) = 7(2 - x)$

(g) $\frac{1}{2}(2x - 3) = 5$

(h) $2x - 3(3 + x) = 5x + 9$

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CHAPTER 10

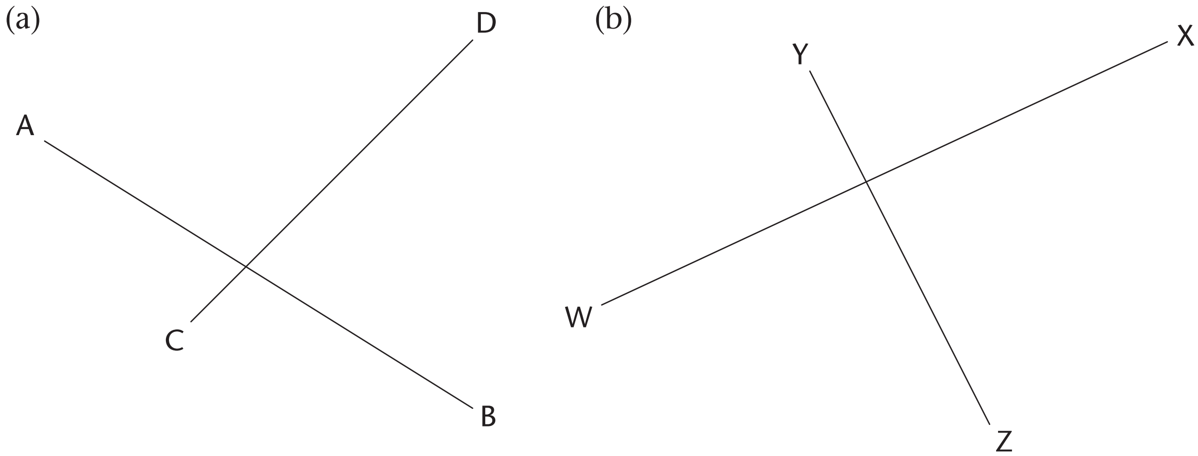
Construction of geometric figures

10.1 Constructing perpendicular lines

REVISING PERPENDICULAR LINES

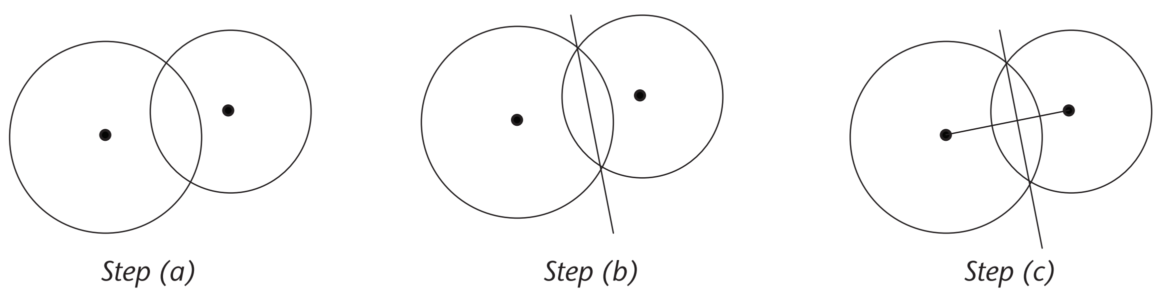
In Grade 8, you learnt about **perpendicular lines**.

1. What does it mean if we say that two lines are perpendicular?
2. Use your protractor to measure the angles between the following pairs of lines. Then state whether they are perpendicular or not.



LINES THAT FORM WHEN CIRCLES INTERSECT

1. Do the following:
   (a) Use a compass to draw two overlapping circles of different sizes.
   (b) Draw a line through the points where the circles intersect (overlap).
   (c) Draw a line to join the centres of the circles.



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(d) Use your protractor to measure the angles between the intersecting lines.
(e) What can you say about the intersecting lines?

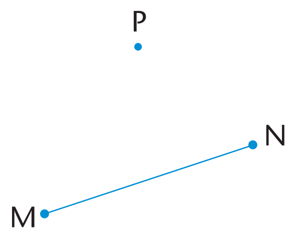
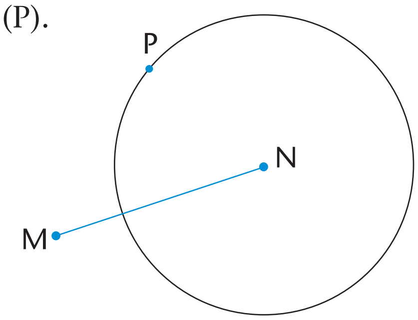
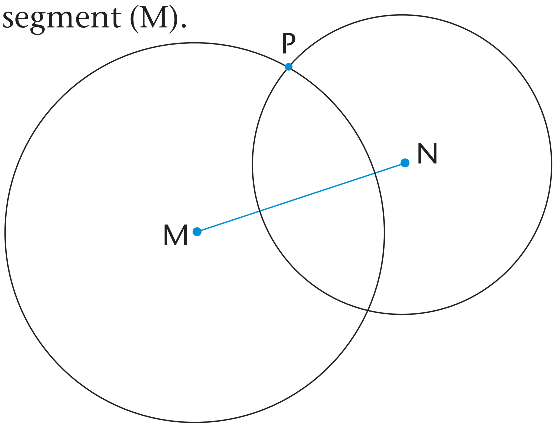
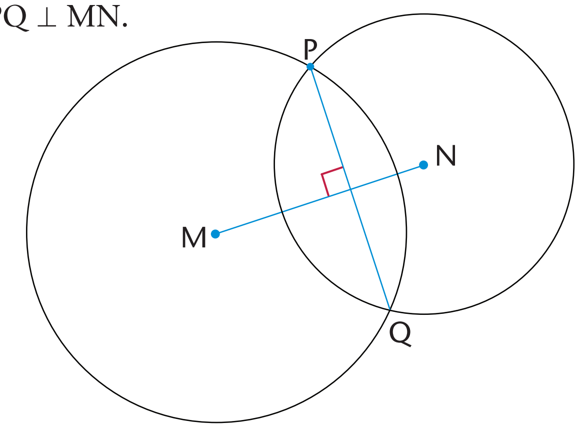
2. Repeat questions 1(a) to (e) with circles that are the same size.

3. What conclusion can you make about a line drawn between the intersection points of two overlapping circles and a line through their centres?

USING CIRCLES TO CONSTRUCT PERPENDICULAR LINES

Case 1: A perpendicular through a point that is not on the line segment

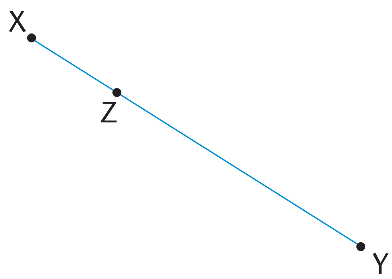
Copy the steps below:

You are given line segment MN with point P at a distance from it. You must construct a line that is perpendicular to MN, so that the perpendicular passes through point P. 	**Step 1** Use your compass to draw a circle whose centre is the one end point of the line segment (N) and passes through the point (P). 
Step 2 Repeat step 1, but make the centre of your circle the other end point of the line segment (M). 	**Step 3** Join the points where the circles intersect: PQ $\perp$ MN. 

Case 2: A perpendicular at a point that is on the line segment

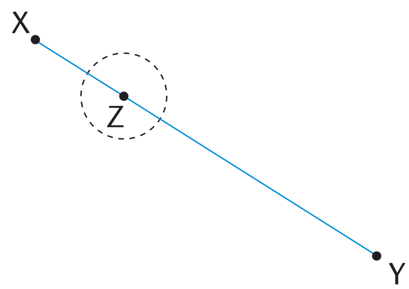
Copy the steps below:

You are given line segment XY with point Z on it. You must construct a perpendicular line passing through Z.



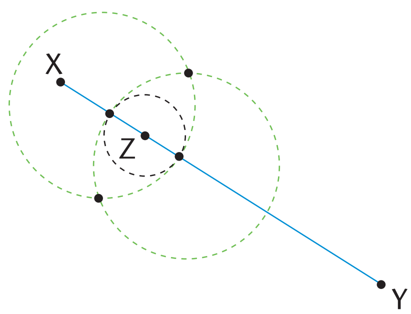
Step 1

Use your compass to draw a circle whose centre is Z. Make its radius smaller than ZX. Note the two points where the circle intersects XY.



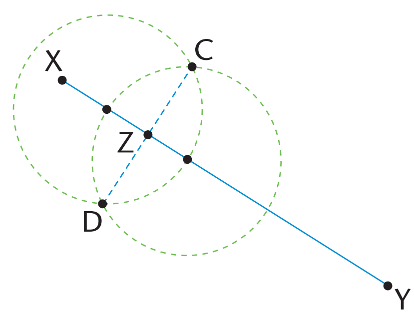
Step 2

Set your compass wider than it was for the circle with centre Z. Draw two circles of the same size whose centres are at the two points where the first (black) circle intersects XY. The two (green) circles will overlap.



Step 3

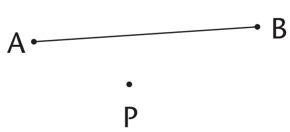
Join the intersection points of the two overlapping circles. Mark these points C and D: CD $\perp$ XY and passes through point Z.



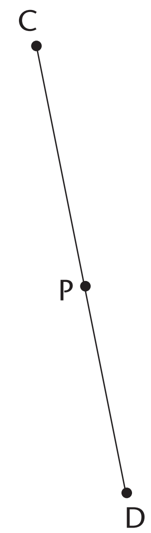
PRACTISE USING CIRCLES TO CONSTRUCT PERPENDICULAR LINES

In each of the following two cases, copy the line segment, and draw a line that is perpendicular to the segment and passes through point P.

1.



2.

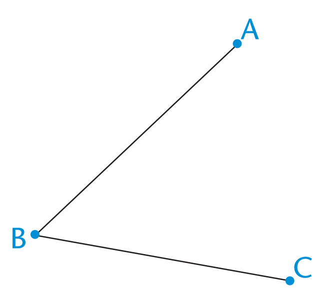
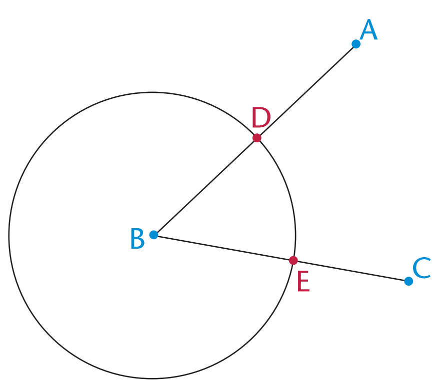


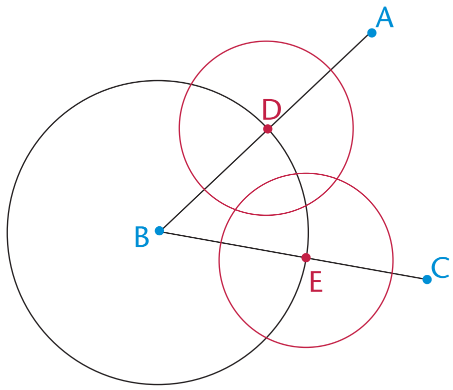
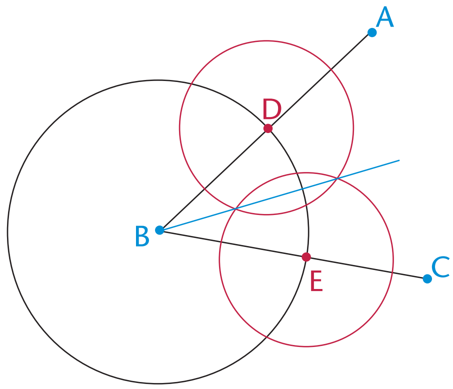
10.2 Bisecting angles

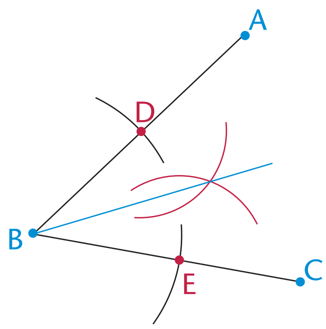
USING CIRCLES TO BISECT ANGLES

Work through the following example of using intersecting circles to **bisect** an angle. Do the following steps yourself.

To bisect something means "to cut in half".

<table>
<tr>
<td>You are given $A\hat{B}C$. You must bisect the angle.

</td>
<td>Step 1
Draw a circle with centre B to mark off an equal length on both arms of the angle. Label the points of intersection D and E: DB = BE.

</td>
</tr>
</table>

Step 2	**Step 3**
Draw two equal circles with centres at D and at E. Make sure the circles overlap. 	Draw a line from B through the points where the two equal circles intersect. This line will bisect the angle. 



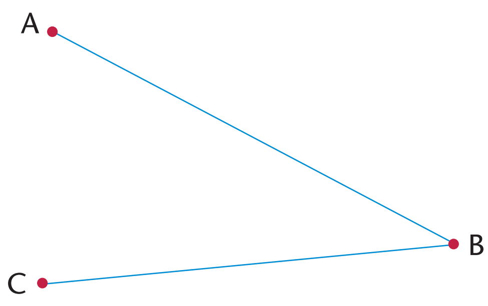
Same construction as in step 3 above

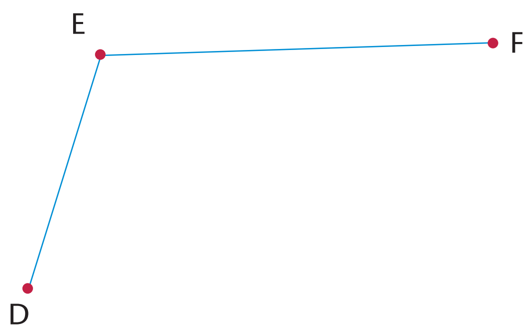
Can you explain why the method above works to bisect an angle?

Can you also see that we need not draw full circles, but can merely use parts of circles (arcs) to do the above construction?

PRACTISE BISECTING ANGLES

Copy the following angles and then bisect them without using a protractor:





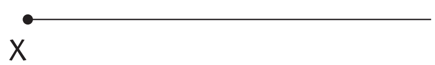
10.3 Constructing special angles without a protractor

Angles of 30°, 45°, 60° and 90° are known as **special angles**. You must be able to construct these angles without using a protractor.

CONSTRUCTING A 45° ANGLE

You have learnt how to draw a 90° angle and how to bisect an angle, without using a protractor. Copy the line below and use your knowledge on angles and bisecting angles to draw a 45° angle at point X on the line.

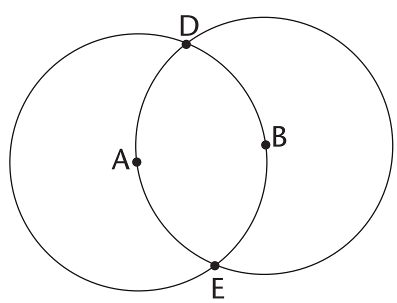
Hint: Extend the line to the left of X.



CONSTRUCTING 60° AND 30° ANGLES

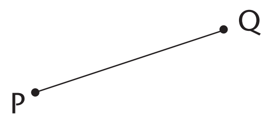
1. What do you know about the sides and angles in an equilateral triangle?
2. Draw two circles with the following properties:
   - The circles are the same size.
   - Each circle passes through the other circle's centre.
   - The centres of the circles are labelled A and B.
   - The points of intersection of the circles are labelled D and E.

   An example is shown on the right.



3. Draw in the following line segments: AB, AD and DB.
4. What can you say about the lengths of AB, AD and DB?

5. What kind of triangle is ABD?
6. Therefore, what do you know about $\hat{A}$, $\hat{B}$ and $\hat{D}$?
7. Use your knowledge of bisecting angles to create an angle of 30° on the construction you made in question 2.
8. Copy the line segment below and use what you have learnt to construct an angle of 60° at point P on the line segment.



CONSTRUCTING THE MULTIPLES OF SPECIAL ANGLES

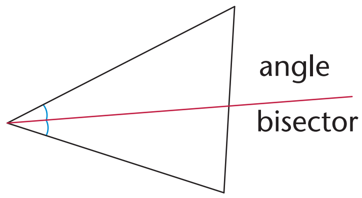
1. Copy and complete the table below. The first one has been done for you.

Angle	Multiples below 360°	Angle	Multiples below 360°
30°	30°; 60°; 90°; 120°; 150°; 180°; 210°; 240°; 270°; 300°; 330°	45°	
60°		90°	

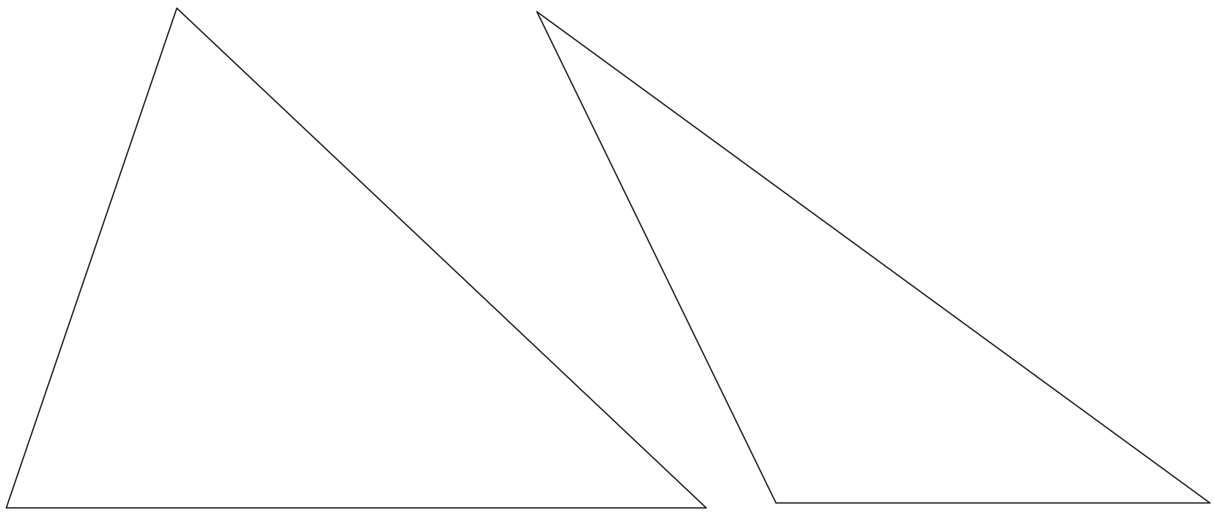
2. Construct the following angles without using a protractor. You will need to do more than one construction to create each angle.
   (a) 120° (b) 135° (c) 270° (d) 240° (e) 150°

10.4 Angle bisectors in triangles

You learnt how to bisect an angle in Section 10.2.
Now you will investigate the angle bisectors in a triangle.
An **angle bisector** is a line that cuts an angle in half.



1. (a) Copy the acute triangle below. Bisect each of the angles of the acute triangle.
   (b) Extend each of the bisectors to the opposite side of the triangle.
   (c) What do you notice?
2. (a) Copy the obtuse angle below. Do the same with the obtuse triangle.
   (b) What do you notice?



3. Compare your triangles with those of two classmates. You should have the same results.

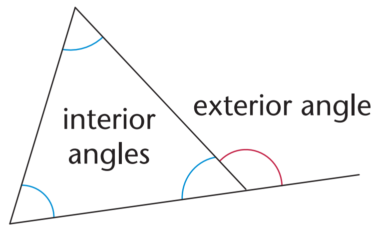
You should have found that the three **angle bisectors** of a triangle **intersect at one point**. This point is the same distance away from each side of the triangle.

10.5 Interior and exterior angles in triangles

WHAT ARE INTERIOR AND EXTERIOR ANGLES?

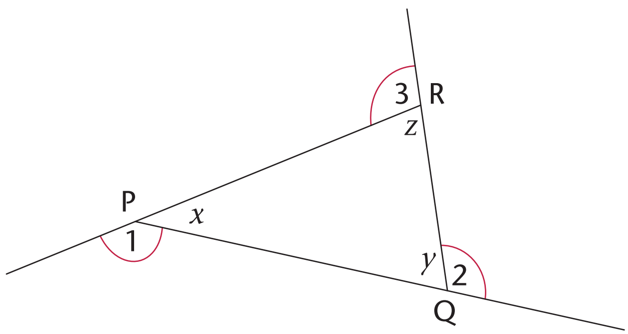
An **interior angle** is an angle that lies between two sides of a triangle. It is inside the triangle. A triangle has three interior angles.

An **exterior angle** is an angle between a side of a triangle and another side that is extended. It is outside the triangle.



Look at ΔPQR. Its three sides are extended to create three exterior angles.

Each exterior angle has one interior adjacent angle (next to it) and two **interior opposite angles**, as described in the following table:

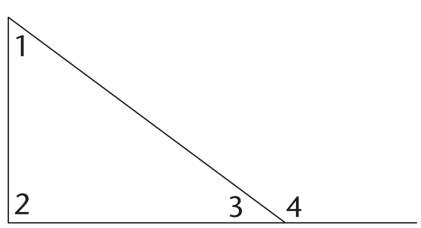


Exterior angle	Interior adjacent angle	Interior opposite angles
1	x	z and y
2	y	x and z
3	z	x and y

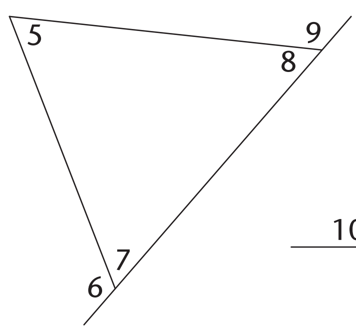
IDENTIFYING EXTERIOR ANGLES AND INTERIOR OPPOSITE ANGLES

1. Copy the following table and name each exterior angle and its two interior opposite angles below.

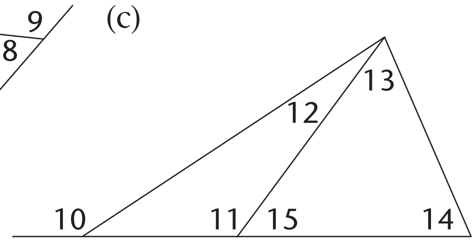
(a)



(b)

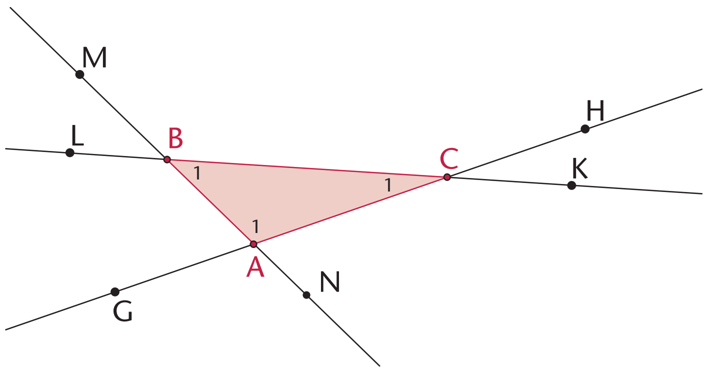


(c)



Ext. ∠					
Int. opp. ∠s					

2. ΔABC below has each side extended in both directions to create six exterior angles.

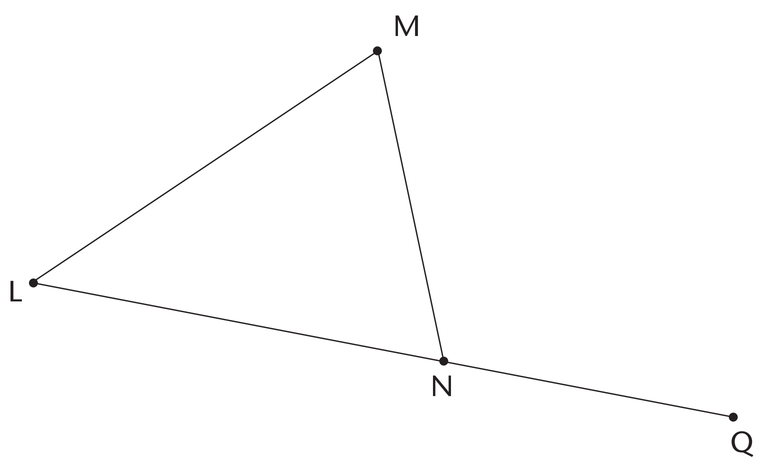


(a) Write down the names of the interior angles of the triangle.
(b) Since a triangle has three sides that can be extended in both directions, there are two exterior angles at each vertex. Write down the names of all the exterior angles of the triangle.
(c) Explain why $M\hat{B}L$ is not an exterior angle of ΔABC.
(d) Write down two other angles that are neither interior nor exterior.

INVESTIGATING THE EXTERIOR AND INTERIOR ANGLES IN A TRIANGLE

1. Consider ΔLMN. Write down the name of the exterior angle.
2. Use a protractor to measure the interior angles and the exterior angle. Copy the drawing and write the measurements on the drawing.
3. Use your findings in question 2 to complete the following sum:

   $L\widehat{M}N + M\hat{L}N = \ldots$

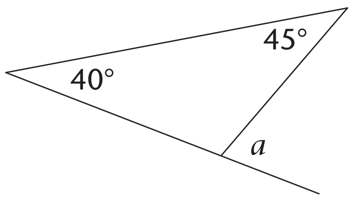


4. What is the relationship between the exterior angle of a triangle and the sum of the interior opposite angles?

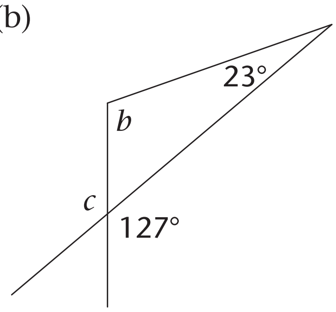
The **exterior angle** of a triangle is equal to the sum of the interior opposite angles.

5. Work out the sizes of angles a to f below, without using a protractor. Give reasons for the statements you make as you work out the answers.

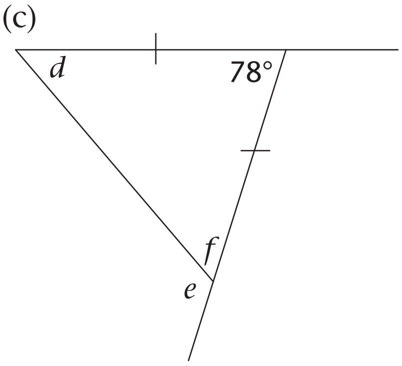
(a)



(b)

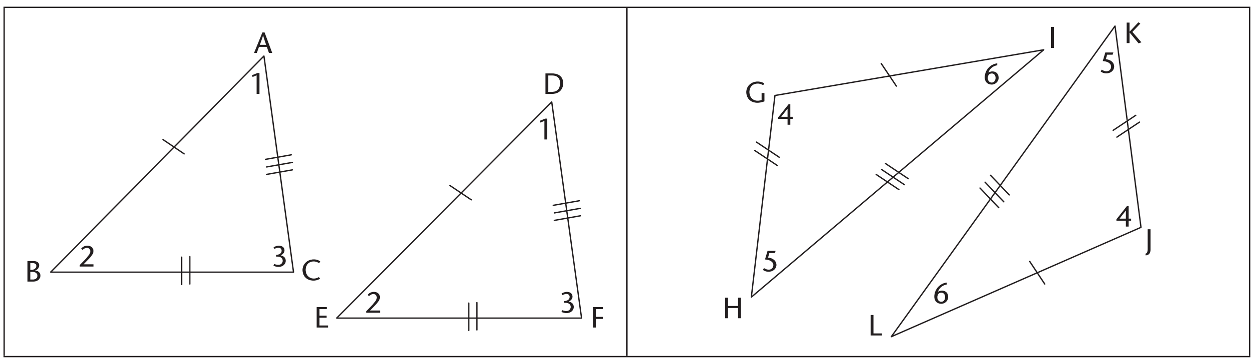


(c)



10.6 Constructing congruent triangles

Two triangles are **congruent** if they have exactly the **same shape** and **size**, i.e. they are able to fit exactly on top of each other. This means that all three corresponding sides and three corresponding angles are equal, as shown in the following two pairs:



ΔABC ≡ ΔDEF and ΔGHI ≡ ΔJKL. In each pair, the corresponding sides and angles are equal.

MINIMUM CONDITIONS FOR CONGRUENCY

To determine if two triangles are congruent, we need a certain number of measurements, but not all of these. Let's investigate which measurements give us only one possible triangle.

1. Use a ruler, compass and protractor to construct the following triangles. Each time minimum measurements are given.
   (a) Given three sides: side, side, side (SSS):
   ΔDEF with DE = 7 cm, DF = 6 cm and EF = 5 cm.
   (b) Given three angles: angle, angle, angle (AAA):
   ΔABC with $\hat{A} = 80°$, $\hat{B} = 60°$ and $\hat{C} = 40°$.

(c) Given one side and two angles: side, angle, angle (SAA):
ΔGHI with GH = 8 cm, $\hat{G} = 60°$ and $\hat{H} = 30°$.

(d) Given two sides and an included angle: side, angle, side (SAS):
ΔJKL with JK = 9 cm, $\hat{K} = 130°$ and KL = 7 cm.

(e) Given two sides and an angle that is not included: side, side, angle (SSA):
ΔMNP with MN = 10 cm, $\hat{M} = 50°$ and PN = 8 cm.

(f) Given a right angle, the hypotenuse and a side (RHS):
ΔTRS with $TR \perp RS$, RS = 7 cm and TS = 8 cm.

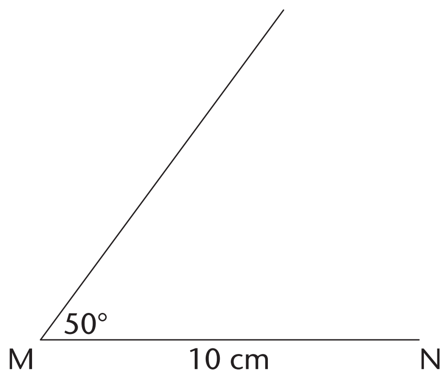
(g) Triangle UVW with UV = 6 cm and VW = 4 cm.

2. Compare your triangles with those of three classmates. Which of your triangles are congruent to theirs? Which are not congruent?

3. Go back to ΔMNP (question 1e). Did you find that you can draw two different triangles that both meet the given measurements? One of the triangles will be obtuse and the other acute. Follow the construction steps below to see why this is so.

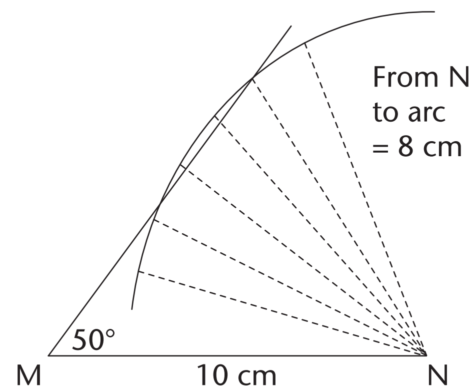
Step 1

Construct MN = 10 cm and the 50° angle at M, even though you do not know the length of the unknown side (MP).



Step 2

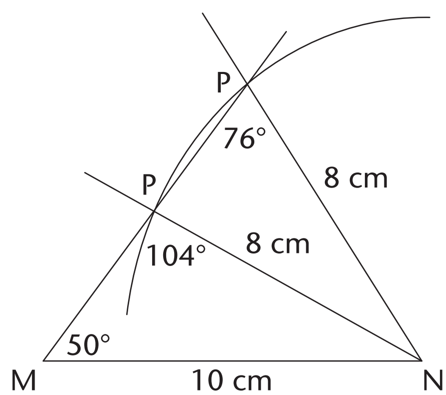
$\hat{N}$ is unknown, but NP = 8 cm. Construct an arc 8 cm from N. Every point on the arc is 8 cm from N.



Step 3

Point P must be 8 cm from N and fall on the unknown side of the triangle. The arc intersects the third side at two points, so P can be at either point.

So two triangles are possible, each meeting the conditions given, i.e. MN = 10 cm, NP = 8 cm and $\hat{M} = 50°$.



4. Copy and complete the table. Write down whether or not we can construct a congruent triangle when the following conditions are given.

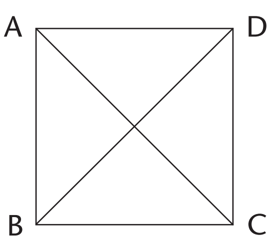
Conditions	**Congruent?**
Three sides (SSS)	
Two sides (SS)	
Three angles (AAA)	
Two angles and a side (AAS)	
Two sides and an angle not between the sides (SSA)	
Two sides and an angle between the sides (SAS)	
Right-angled with the hypotenuse and a side (RHS)	

10.7 Diagonals of quadrilaterals

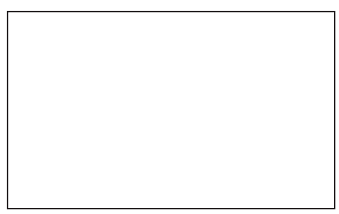
DRAWING DIAGONALS

A **diagonal** is a straight line inside a figure that joins two vertices of the figure, where the vertices are not next to each other.

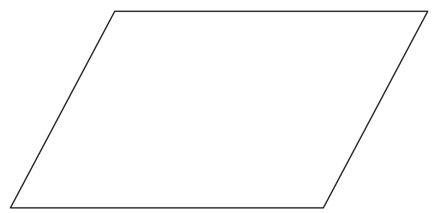
1. Look at the quadrilaterals below. The two diagonals of the square have been drawn in: AC and BD.
2. Copy the quadrilaterals below and draw in the diagonals.



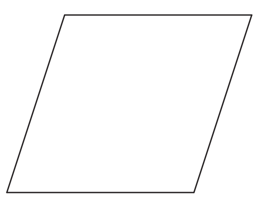
Square



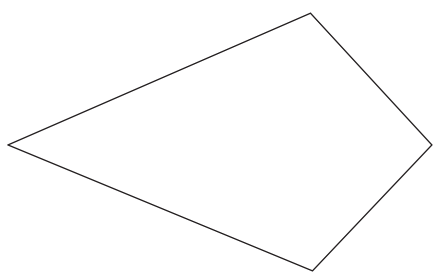
Rectangle



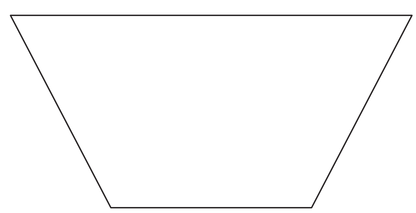
Parallelogram



Rhombus



Kite



Trapezium

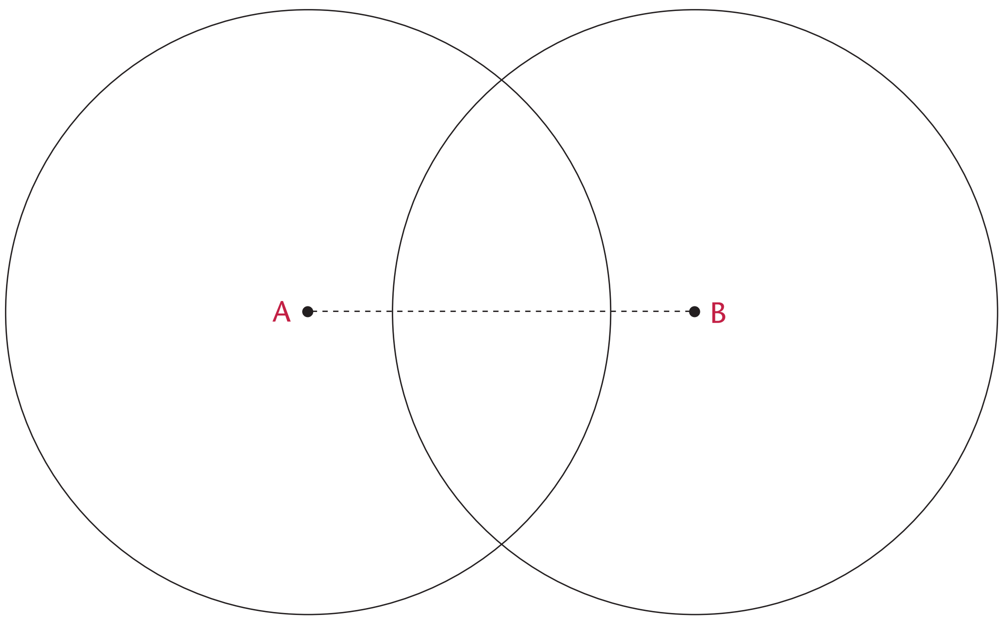
3. How many sides does a quadrilateral have?
4. How many angles does a quadrilateral have?
5. How many diagonals does a quadrilateral have?

DIAGONALS OF A RHOMBUS

Below are two overlapping circles with centres A and B. The circles are the same size.

1. Construct a rhombus inside the circles by joining the centre of each circle with the intersection points of the circles. Join AB.
2. Copy the circles and construct the perpendicular bisector of AB. (Go back to Section 10.1 if you need help.) What do you find?

A **perpendicular bisector** is a line that cuts another line in half at a right angle (90°).

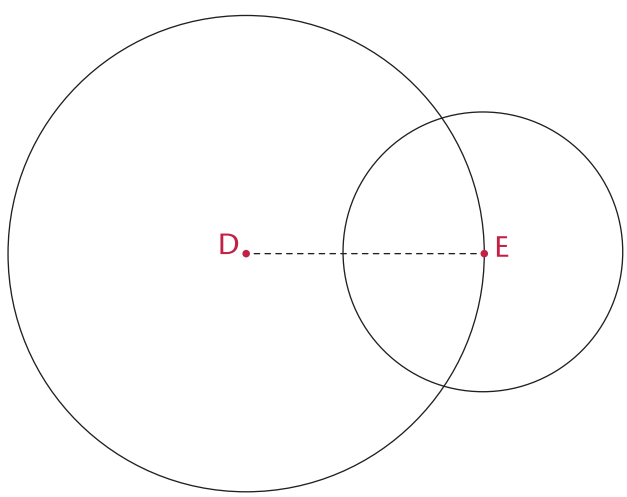


3. Do the diagonals bisect each other?
4. Copy and complete the sentence: The diagonals of a rhombus will always . . .

DIAGONALS OF A KITE

Below are two overlapping circles with centres D and E. The circles are different sizes.

1. Copy the circles and construct a kite by joining the centre points of the circles to the intersection points of the circles.
2. Draw in the diagonals of the kite.
3. Mark all lines that are of the same length.



4. Are the diagonals of the kite perpendicular?
5. Do the diagonals of the kite bisect each other?
6. What is the difference between the diagonals of a rhombus and those of a kite?

DIAGONALS OF PARALLELOGRAMS, RECTANGLES AND SQUARES

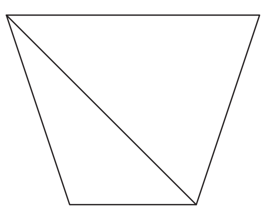
1. Draw a parallelogram, rectangle and square onto grid paper.
2. Draw in the diagonals of the quadrilaterals.
3. Indicate on each shape all the lengths in the diagonals that are equal. (Use a ruler.)
4. Use the information you have found to copy and complete the table below. Fill in "yes" or "no".

Quadrilateral	Diagonals equal	Diagonals bisect	Diagonals meet at 90°
Parallelogram			
Rectangle			
Square			

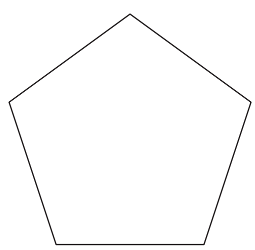
10.8 Angles in polygons

USING DIAGONALS TO INVESTIGATE THE SUM OF THE ANGLES IN POLYGONS

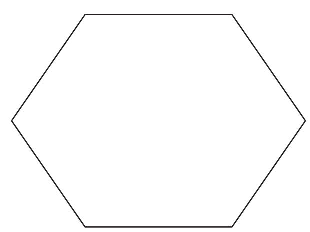
1. We can divide a quadrilateral into two triangles by drawing in one diagonal.
   (a) Copy the polygons below and draw in diagonals to divide each of the polygons into as few triangles as possible.
   (b) Write down the number of triangles in each polygon.



Quadrilateral

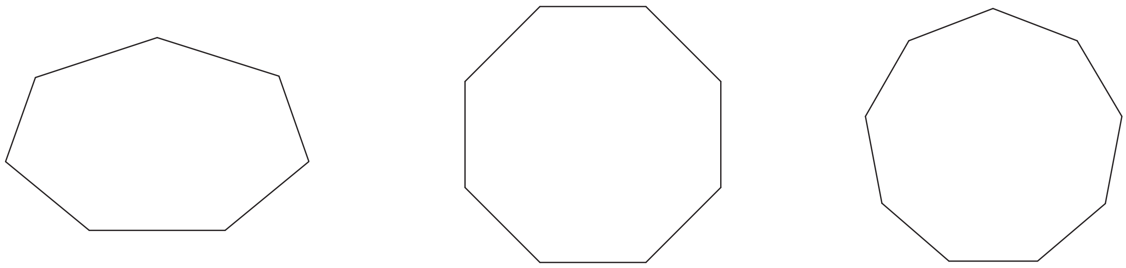


Pentagon



Hexagon

No. of Δs	2		
Sum of ∠s	$2 \times 180° = 360°$		



Heptagon *Octagon* *Nonagon*

No. of Δs			
Sum of ∠s			

2. The sum of the angles of one triangle = 180°. A quadrilateral is made up of two triangles, so the sum of the angles in a quadrilateral = 2 × 180° = 360°. Work out the sum of the interior angles of each of the other polygons above.

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WORKSHEET

1. Match the words in the column on the right with the definitions on the left. Write the letter of the definition next to the matching word.

(a) A quadrilateral that has diagonals that are perpendicular and bisect each other	Kite
(b) A quadrilateral that has diagonals that are perpendicular to each other, and only one diagonal bisects the other	Congruent
(c) A quadrilateral that has equal diagonals that bisect each other	Exterior angle
(d) Figures that have exactly the same size and shape	Rhombus
(e) Divides into two equal parts	Perpendicular
(f) An angle that is formed outside a closed shape: it is between the side of the shape and a side that has been extended	Bisect
(g) Lines that intersect at 90°	Special angles
(h) 90°, 45°, 30° and 60°	Rectangle

2. Copy and complete the sentence: The exterior angle in a triangle is equal to ...

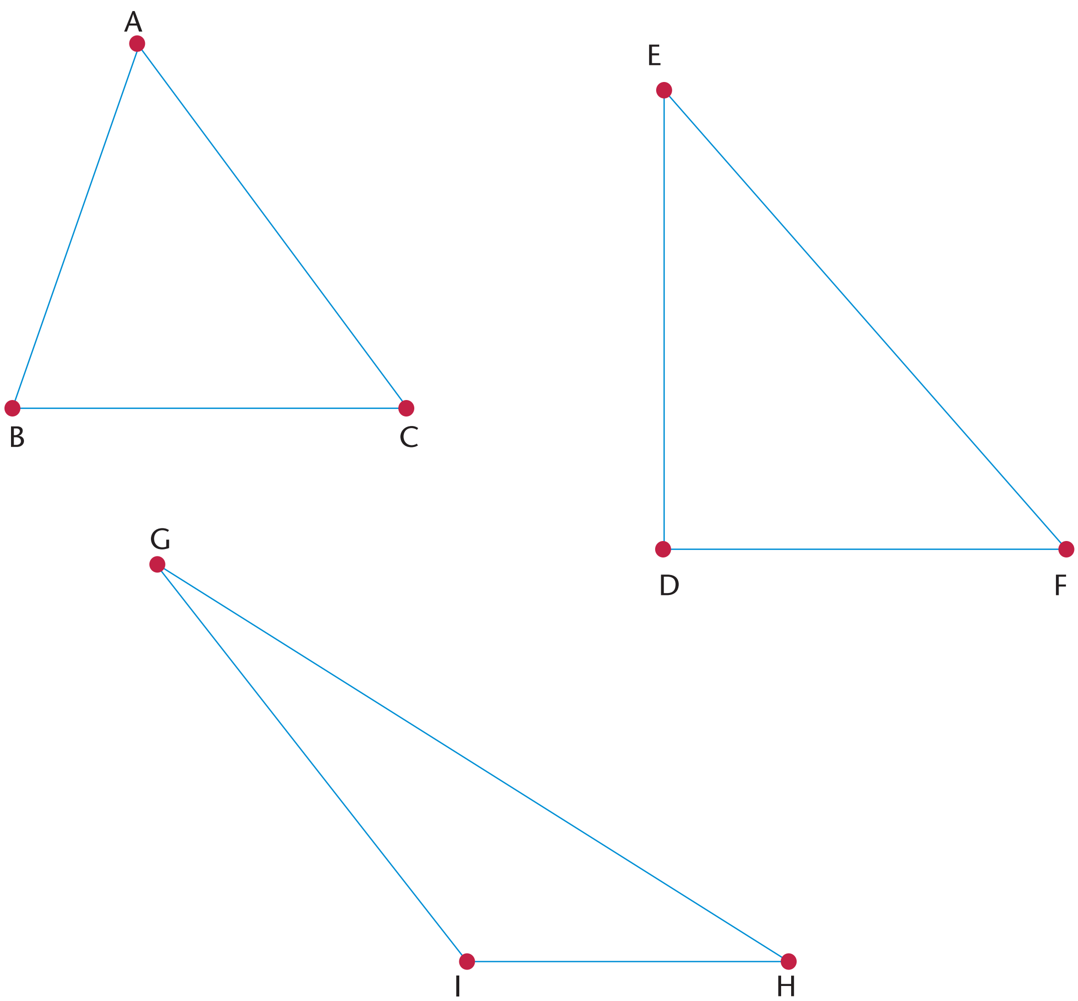
3. (a) Construct ΔPQR with angles of 30° and 60°. The side between the angles must be 8 cm. You may use only a ruler and a compass.

   (b) Will all triangles with the same measurements above be congruent to $\triangle PQR$? Explain your answer.

Chapter 11
Geometry of 2D shapes

11.1 Revision: Classification of triangles

1. Use a protractor to measure the interior angles of each of the following triangles. Write down the sizes of the angles.



2. Classify the triangles in question 1 according to their angle properties. Copy and complete the following statements by choosing from the following types of triangles: **acute-angled**, **obtuse-angled** and **right-angled**.

   (a) ΔABC is an .... triangle, because ....

(b) ΔEDF is a .... triangle, because ....

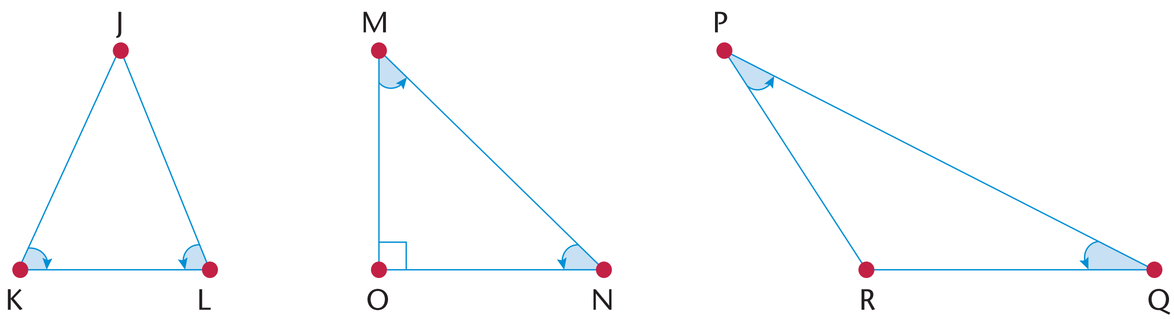
(c) ΔGHI is an obtuse-angled triangle, because ....

3. The marked angles in each triangle below are equal. Copy and complete the following statements and classify the triangles according to angle and side properties.

   (a) Δ....... is an acute isosceles triangle, because .... and ....

   (b) Δ....... is a right-angled isosceles triangle, because .... and ....

   (c) Δ....... is an obtuse isosceles triangle, because .... and ....

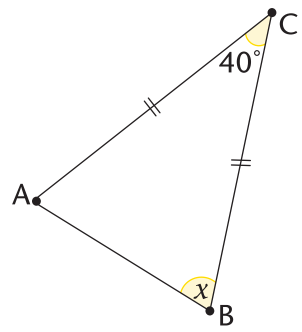


4. Copy the table below. Say for what kind of triangle each statement is true. If it is true for all triangles, then write "All triangles".

	Statement	True for:
(a)	Two sides of the triangle are equal.	
(b)	One angle of the triangle is obtuse.	
(c)	Two angles of the triangle are equal.	
(d)	All three angles of the triangle are equal to 60°.	
(e)	The size of an exterior angle is equal to the sum of the opposite interior angles.	
(f)	The longest side of the triangle is opposite the biggest angle.	
(g)	The sum of the two shorter sides of the triangle is bigger than the length of the longest side.	
(h)	The square of the length of one side is equal to the sum of the squares of the other sides.	
(i)	The square of the length of one side is bigger than the sum of the squares of the other sides.	
(j)	The sum of the interior angles of the triangle is 180°.	

11.2 Finding unknown angles in triangles

When you have to determine the size of an unknown angle or length of a shape in geometry, you must give a reason for each statement you make. Complete the example below.

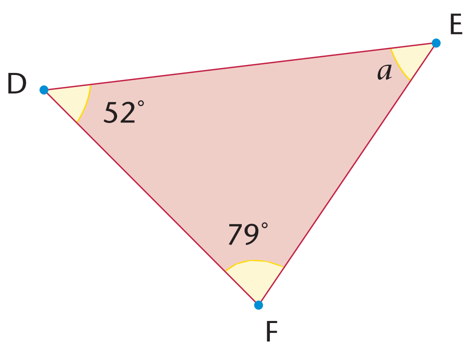


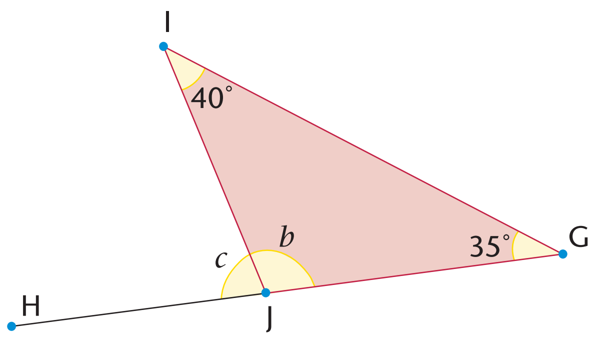
In ΔABC, AC = BC and $\hat{C} = 40°$. Find the size of $\hat{B}$ (shown in the diagram as x).

Statement	Reason
AC = BC	Given
$\therefore \hat{A} = \hat{B}$	
$180° = 40° + x + x$	Sum $\angle$s Δ
$180° - 40° = 2x$	
$\therefore x =$	

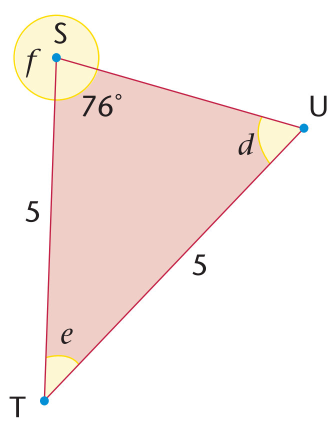
FINDING UNKNOWN LENGTHS AND ANGLES

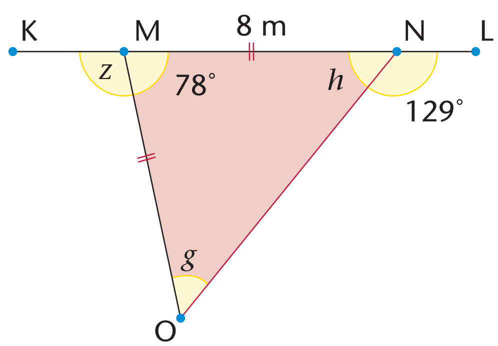
1. Calculate the sizes of the unknown angles.



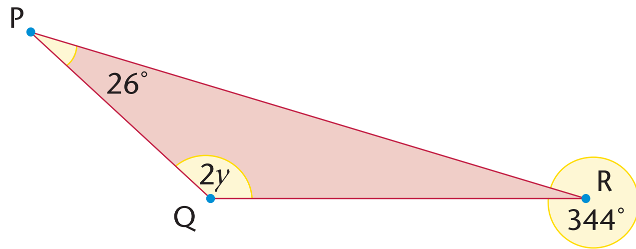


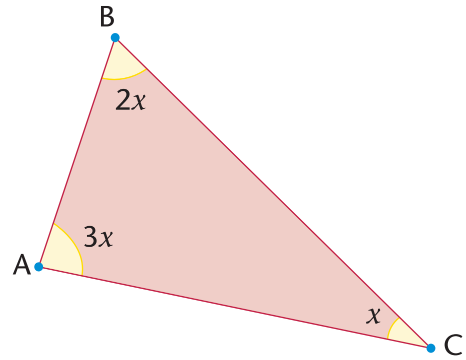
2. Determine the sizes of the unknown angles and the length of MO.





3. Calculate the sizes of y and x.

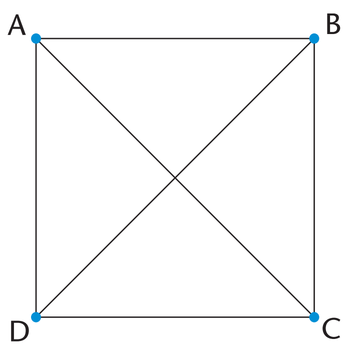


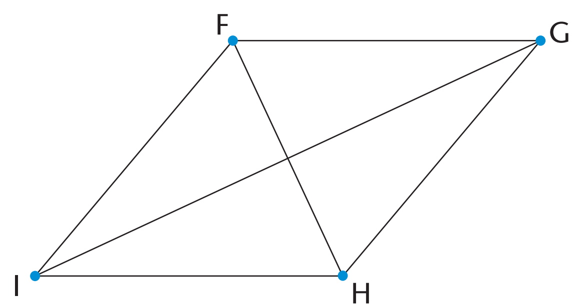


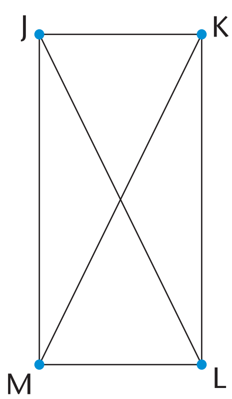
11.3 Quadrilaterals

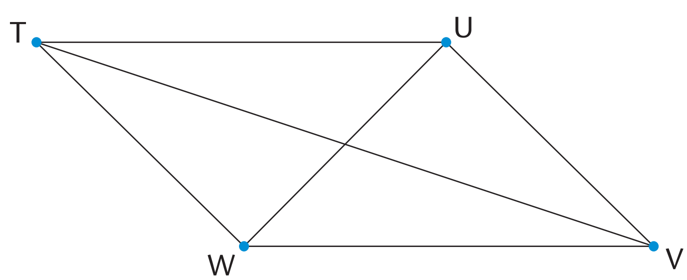
PROPERTIES OF QUADRILATERALS

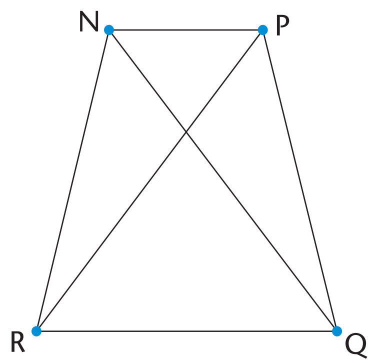
1. Name the following quadrilaterals. Copy the quadrilaterals and mark equal angles and equal sides in each figure. Use your ruler and protractor to measure angle sizes and lengths where necessary.

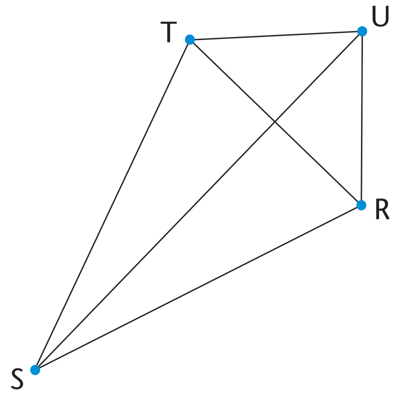












2. Copy and complete the following table:

Properties	True for the following quadrilaterals					
	Square	Rhombus	Rectangle	Parallelogram	Kite	Trapezium
At least one pair of opposite angles is equal.	yes	yes	yes	yes	yes	no
Both pairs of opposite angles are equal.						
At least one pair of adjacent angles is equal.						
All four angles are equal.						
Any two opposite sides are equal.						
Two adjacent sides are equal and the other two adjacent sides are also equal.						
All four sides are equal.						
At least one pair of opposite sides is parallel.						
Any two opposite sides are parallel.						
The two diagonals are perpendicular.						
At least one diagonal bisects the other one.						

The two diagonals bisect each other.						
The two diagonals are equal.						
At least one diagonal bisects a pair of opposite angles.						
Both diagonals bisect a pair of opposite angles.						
The sum of the interior angles is 360°.						

3. Look at the properties of a square and a rhombus.
   (a) Are all the properties of a square also the properties of a rhombus? Explain.
   (b) Are all the properties of a rhombus also the properties of a square? Explain.
   (c) Which statement is true? Write down the statement.

   A square is a special kind of rhombus.

   A rhombus is a special kind of square.

4. Look at the properties of rectangles and squares.
   (a) Are all the properties of a square also the properties of a rectangle? Explain.
   (b) Are all the properties of a rectangle also the properties of a square? Explain.
   (c) Which statement is true? Write down the statement.

   A square is a special kind of rectangle.

   A rectangle is a special kind of square.

5. Look at the properties of parallelograms and rectangles.
   (a) Are all the properties of a parallelogram also the properties of a rectangle? Explain.
   (b) Are all the properties of a rectangle also the properties of a parallelogram? Explain.
   (c) Which statement is true? Write down the statement.

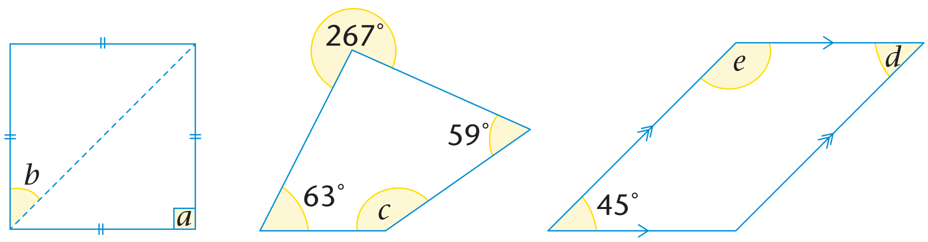
   A rectangle is a special parallelogram.

   A parallelogram is a special rectangle.

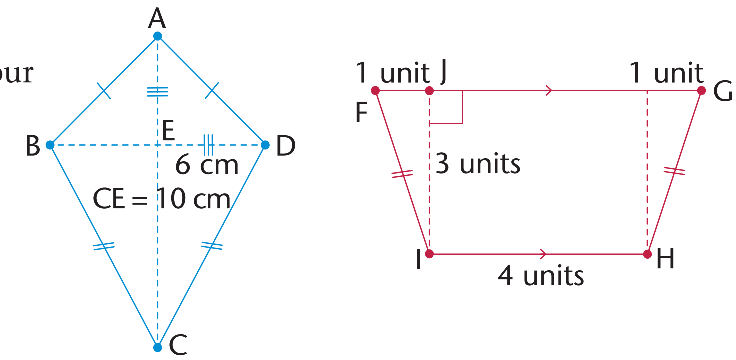
6. Look at the properties of a rhombus and a parallelogram. Is a rhombus a special kind of parallelogram? Explain.
7. Compare the properties of a kite and a parallelogram. Why is a kite not a special kind of parallelogram?
8. Compare the properties of a trapezium and a parallelogram. Why is a trapezium not a special kind of parallelogram?

UNKNOWN SIDES AND ANGLES IN QUADRILATERALS

1. Determine the sizes of angles *a* to *e* in the quadrilaterals below. Give reasons for your answers.



2. Calculate the perimeters of the quadrilaterals on the right. Give your answers to two decimal places.

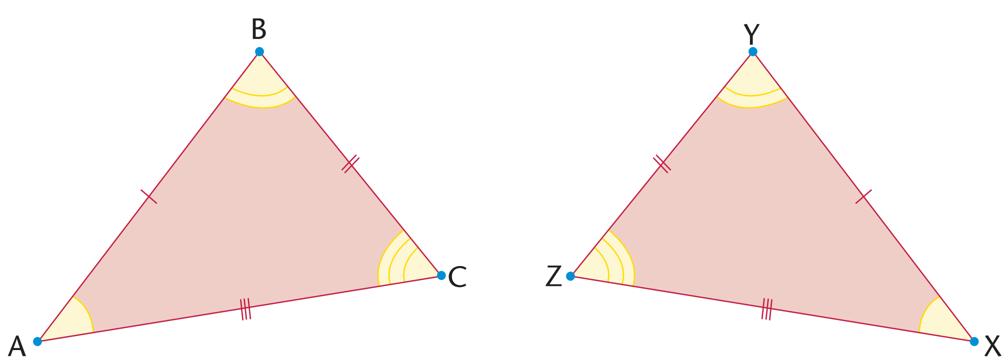


11.4 Congruent triangles

DEFINITION AND NOTATION OF CONGRUENT TRIANGLES

If two triangles are congruent, then they have exactly the same size and shape. In other words, if you cut out one of the triangles and place it on the other, they will match exactly.

> If you know that two triangles are congruent, then each side in the one triangle will be equal to each corresponding side in the second triangle. Also, each angle in the one triangle will be equal to each corresponding angle in the second triangle.



In the triangles on the previous page, you can see that $\Delta ABC \equiv \Delta XYZ$.

> **Congruency symbol**
>
> $\equiv$ means "is congruent to".

The order in which you write the letters when stating that two triangles are congruent is very important. The letters of the corresponding vertices between the two triangles must appear in the same position in the notation. For example, the notation for the triangles on the previous page should be: $\Delta ABC \equiv \Delta XYZ$, because it indicates that $\hat{A} = \hat{X}$, $\hat{B} = \hat{Y}$, $\hat{C} = \hat{Z}$, AB = XY, BC = YZ and AC = XZ.

It is incorrect to write $\Delta ABC \equiv \Delta ZYX$. Although the letters refer to the same triangles, this notation indicates that $\hat{A} = \hat{Z}$, $\hat{C} = \hat{X}$, AB = ZY and BC = YX, and these statements are not true.

Write down the equal angles and sides according to the following notations:

1. $\Delta KLM \equiv \Delta PQR$
2. $\Delta FGH \equiv \Delta CST$

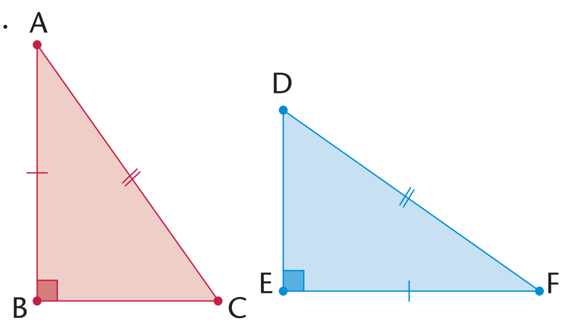
MINIMUM CONDITIONS FOR CONGRUENT TRIANGLES

Earlier in this chapter, you investigated the minimum conditions that must be satisfied in order to establish that two triangles are congruent.

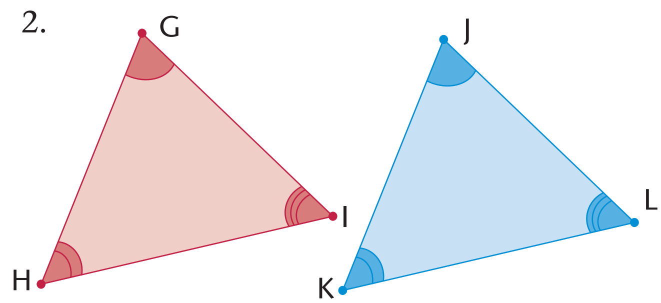
> The conditions for congruency consist of:
> - SSS (all corresponding sides are equal)
> - SAS (two corresponding sides and the angle between the two sides are equal)
> - AAS (two corresponding angles and any corresponding side are equal)
> - RHS (both triangles have a 90° angle and have equal hypotenuses and one other side equal).

Decide whether or not the triangles in each pair below are congruent. For each congruent pair, write the notation correctly and give a reason for congruency.

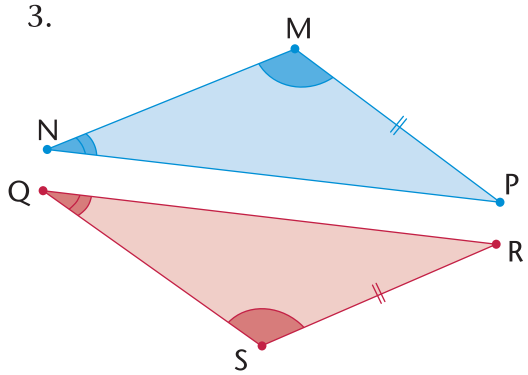
1.



2.



3.



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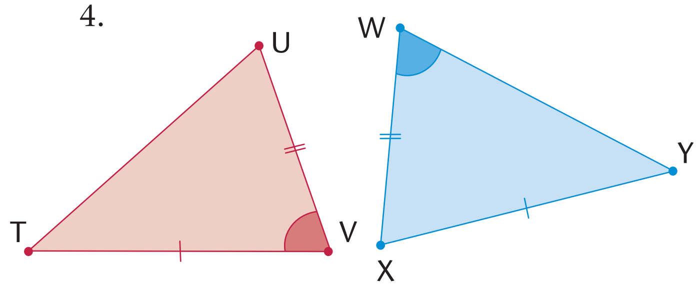
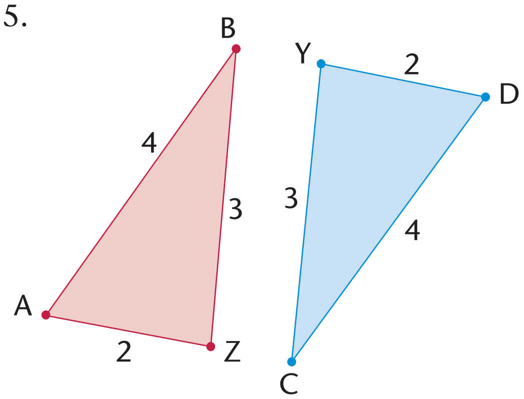


5.



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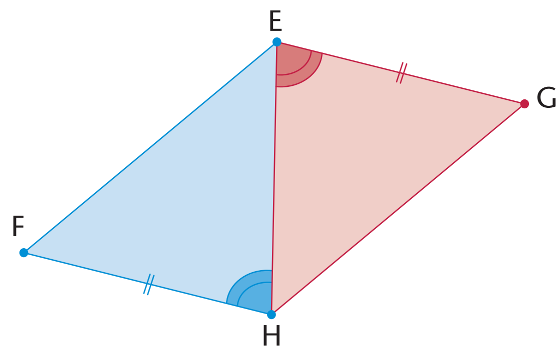


PROVING THAT TRIANGLES ARE CONGRUENT

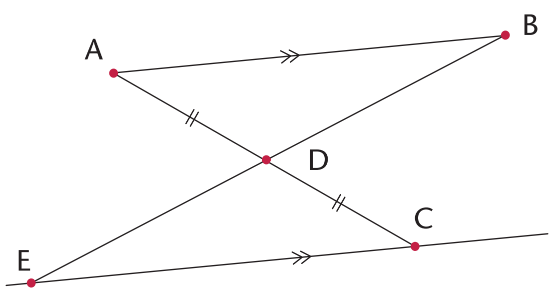
You can use what you know about the minimum conditions for congruency to prove that two triangles are congruent.

When giving a proof for congruency, remember the following:

- Each statement you make needs a reason.
- You must give three statements to prove any two triangles congruent.
- Give the reason for congruency.

Example:

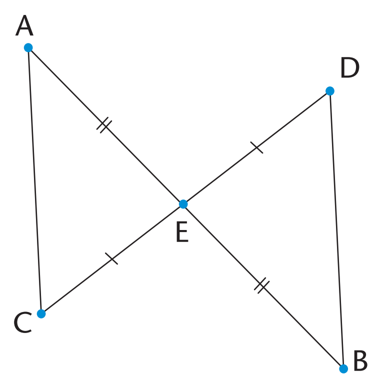
In the sketch on the right: AB || EC and AD = DC.
Prove that the triangles are congruent.



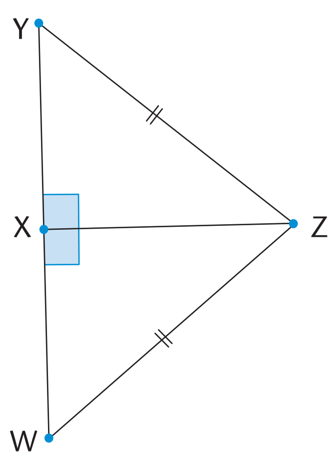
Solution:

Statement	Reason
In ΔABD and ΔCED:	
1) $AD = DC$	Given
2) $A\hat{D}B = C\hat{D}E$	Vert. opp. $\angle$s
3) $B\hat{A}D = E\hat{C}D$	Alt. $\angle$s (AB || EC)
$\therefore \Delta ABD \equiv \Delta CED$	AAS

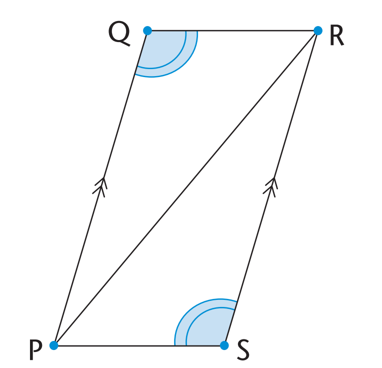
1. Copy the table with the sketch, and prove that $\Delta ACE \equiv \Delta BDE$.

	Statement	Reason
		

2. Copy the table with the sketch, and prove that $\Delta WXZ \equiv \Delta YXZ$.

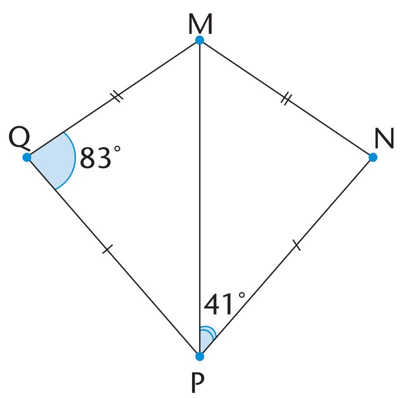
	Statement	Reason
		

3. Copy the table with the sketch, and prove that QR = SP. (Hint: First prove that the triangles are congruent.)



Statement	Reason

4. Copy the table with the sketch, and prove that the triangles below are congruent. Then find the size of $Q\hat{M}P$.

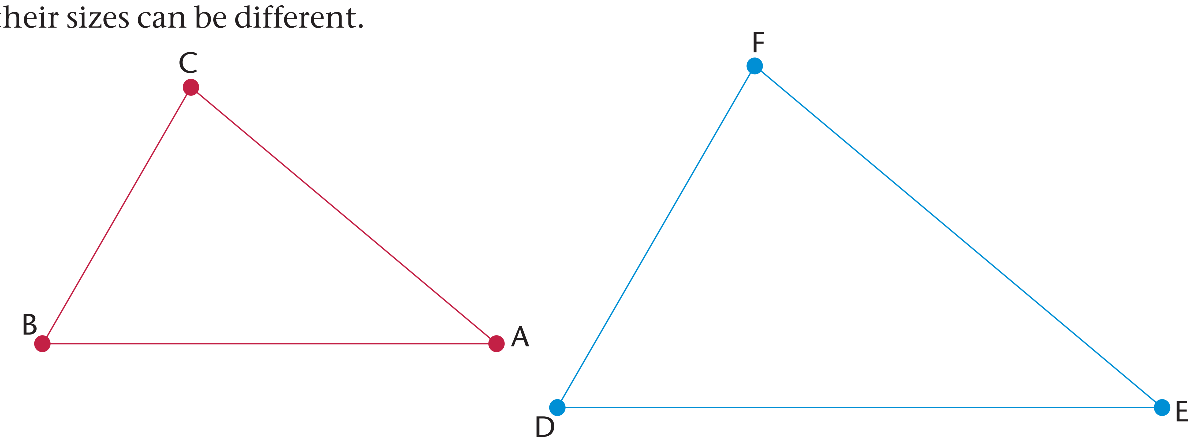


Statement	Reason

11.5 Similar triangles

PROPERTIES OF SIMILAR TRIANGLES

ΔBAC and ΔDEF below are similar to each other. Similar figures have the same shape, but their sizes can be different.



1. (a) Use a protractor to measure the angles in each triangle on the previous page. Then copy and complete the following table:

Angle	Angle	What do you notice?
$\hat{B}$ =	$\hat{D}$ =	
$\hat{A}$ =	$\hat{E}$ =	
$\hat{C}$ =	$\hat{F}$ =	

(b) What can you say about the sizes of the angles in similar triangles?

2. (a) Use a ruler to measure the lengths of the sides in each triangle in question 1. Then copy and complete the following table:

Length (cm)	Length (cm)	Ratio
BA =	DE =	BA : DE = $= 1 : 1\frac{1}{3}$
BC =	DF =	BC : DF = =
CA =	FE =	CA : FE = =

(b) What can you say about the relationship between the sides in similar triangles?

Ratio reminder

You read 2 : 1 as "two to one".

3. The following notation shows that the triangles are similar: ΔBAC ||| ΔDEF. Why do you think we write the first triangle as ΔBAC and not as ΔABC?

The properties of similar triangles:

- The corresponding angles are equal.
- The corresponding sides are in proportion.

Notation for similar triangles:

If ΔXYZ is similar to ΔPQR, then we write: ΔXYZ ||| ΔPQR.

As for the notation of congruent figures, the order of the letters in the notation of similar triangles indicates which angles and sides are equal.

For ΔXYZ ||| ΔPQR:

Angles: $\hat{X} = \hat{P}$, $\hat{Y} = \hat{Q}$ and $\hat{Z} = \hat{R}$

Sides: XY : PQ = XZ : PR = YZ : QR

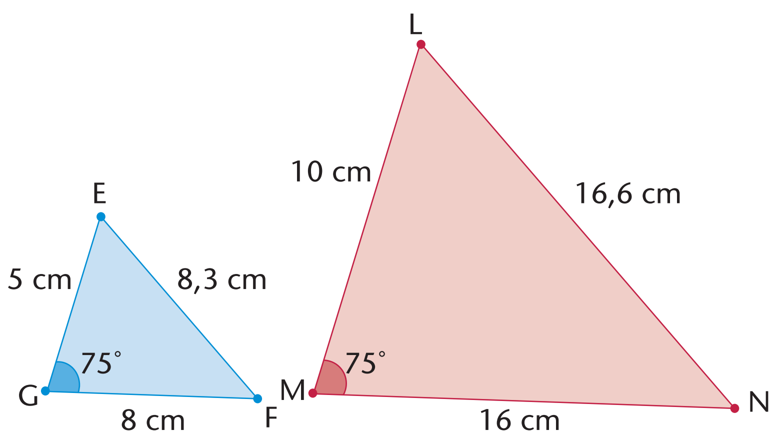
If the triangles' vertices were written in a different order, then the statements above would not be true.

When proving that triangles are similar, you either need to show that the corresponding angles are equal, or you must show that the sides are in proportion.

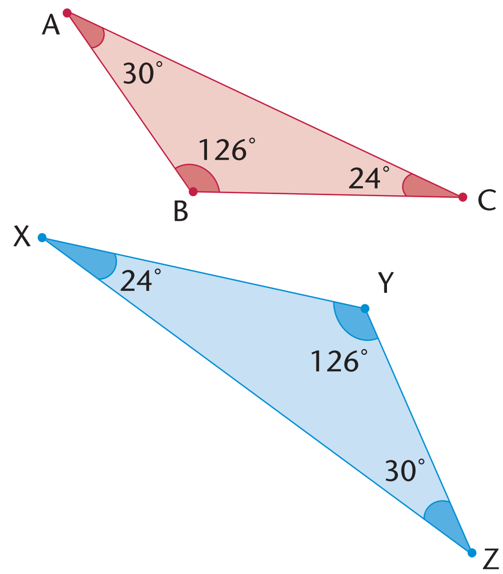
WORKING WITH PROPERTIES OF SIMILAR TRIANGLES

1. Decide if the following triangles are similar to each other:

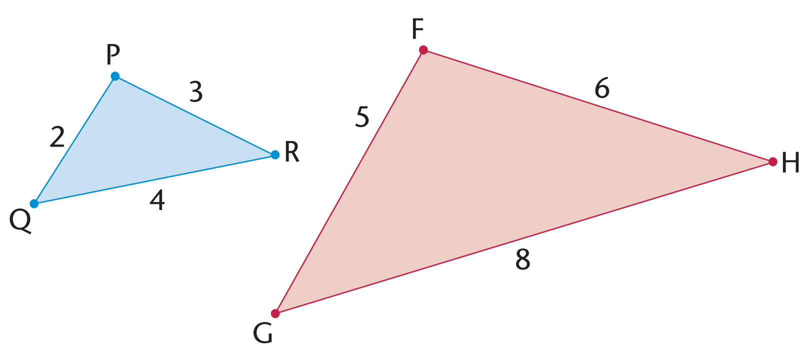
(a)



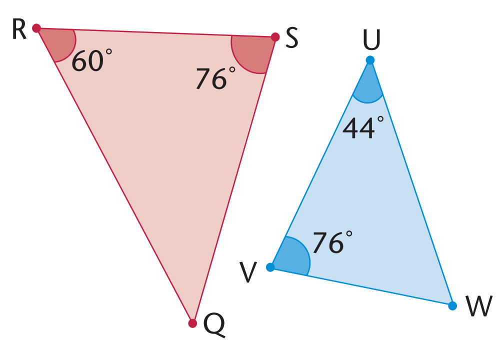
(b)



(c)



(d)



2. Do the following task:
   - Use a ruler and protractor to construct the triangles described in (a) to (d) on the next page.
   - Use your knowledge of similarity to draw the second triangle in each question.
   - Indicate the sizes of the corresponding sides and angles on the second triangle.

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(a) In ΔEFG, $\hat{G} = 75°$, EG = 4 cm and GF = 5 cm.
ΔABC is an enlargement of ΔEFG, with its sides three times longer.

(b) In ΔMNO, $\hat{M} = 45°$, $\hat{N} = 30°$ and MN = 5 cm.
ΔPQR is similar to ΔMNO. The sides of ΔMNO to ΔPQR are in proportion 1 : 3.

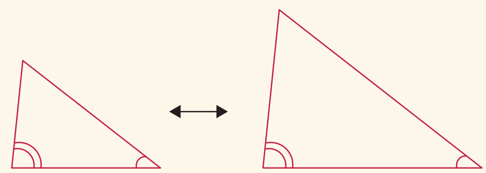
(c) ΔRST is an isosceles triangle. $\hat{R} = 40°$, RS is 10 cm and RS = RT.
ΔVWX is similar to ΔRST. The sides of ΔRST to ΔVWX are in proportion $1 : \frac{1}{2}$.

(d) ΔKLM is right-angled at $\hat{L}$, LM is 7 cm and the hypotenuse is 12 cm.
ΔXYZ is similar to ΔKLM, so that the sides are a third of the length of ΔKLM.

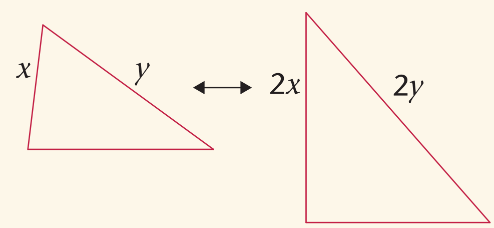
INVESTIGATION: MINIMUM CONDITIONS FOR SIMILARITY

Which of the following are minimum conditions for similar triangles?

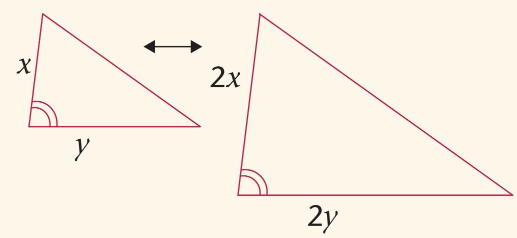
(a) Two angles in one triangle are equal to two angles in another triangle.



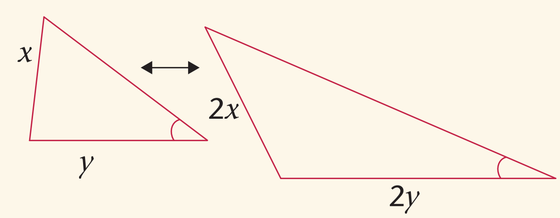
(b) Two sides of one triangle are in the same proportion as two sides in another triangle.



(c) Two sides of one triangle are in the same proportion as two sides in another triangle, and the angle between the two sides is equal to the angle between the corresponding sides.

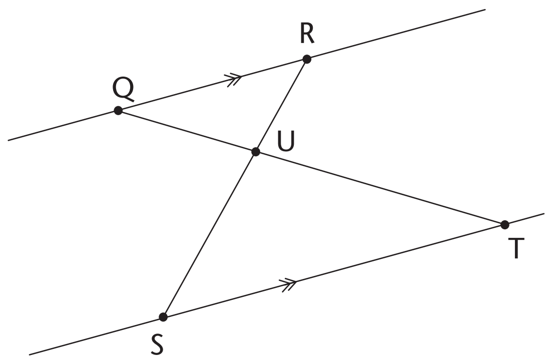


(d) Two sides of one triangle are in the same proportion as two sides in another triangle, and one angle not between the two sides is equal to the corresponding angle in the other triangle.



SOLVING PROBLEMS WITH SIMILAR TRIANGLES

1. Line segment QR is parallel to line segment ST.

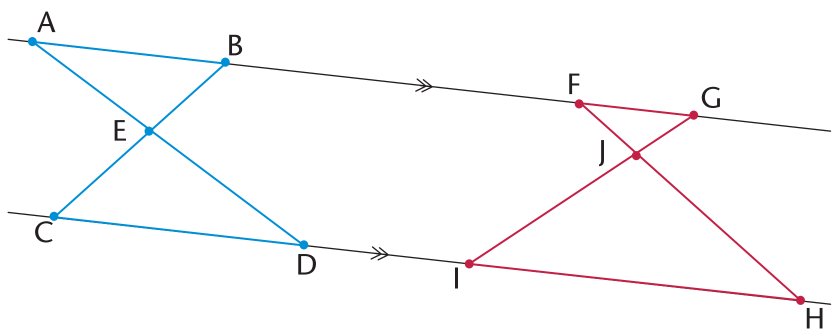


Parallel lines never meet. Two lines are parallel to each other if the distance between them is the same along the whole length of the lines.

Copy and complete the following proof that ΔQRU ||| ΔTSU:

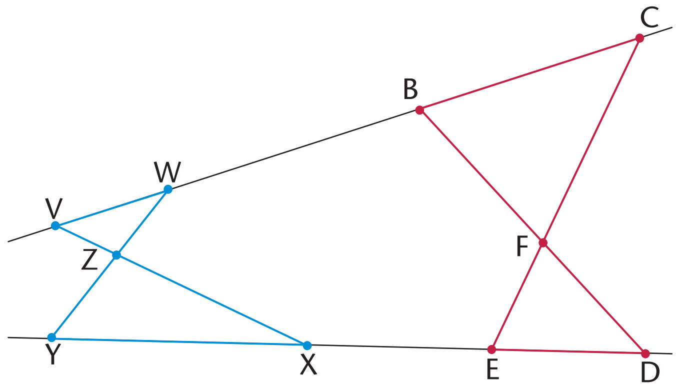
Statement	**Reason**
$R\hat{Q}T = Q\hat{T}S$	Alt. ∠s (QR || ST)
$Q\hat{R}S =$	
=	Vert. opp. ∠s
∴ ΔQRU ||| ΔTSU	Equal ∠s (or AAA)

2. The following intersecting line segments form triangle pairs between parallel lines.

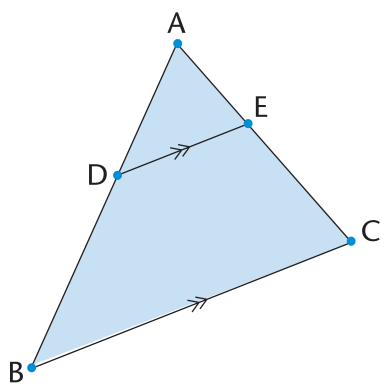


(a) Are the triangles in each pair similar? Explain.
(b) Write down pairs of similar triangles.
(c) Are triangles like these always similar? Explain how you can be sure without measuring every possible triangle pair.

3. The intersecting lines on the right form triangle pairs between the line segments that are not parallel. Are these triangle pairs similar? Explain why or why not.

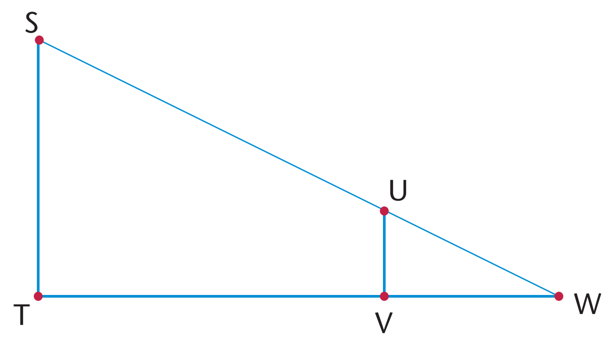


4. Consider the triangles below. DE || BC. Copy the table with the sketch, and prove that ΔABC ||| ΔADE.

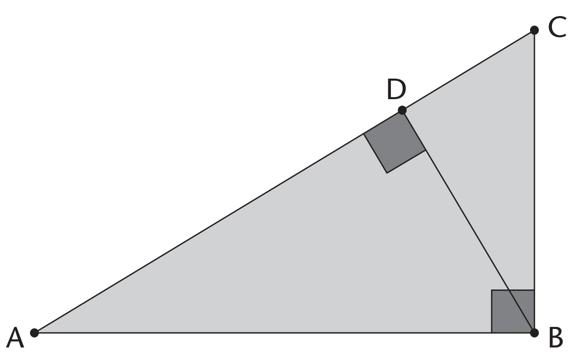


Statement	Reason

5. In the diagram on the right, ST is a telephone pole and UV is a vertical stick. The stick is 1 m high and it casts a shadow of 1,7 m (VW). The telephone pole casts a shadow of 5,1 m (TW). Use similar triangles to calculate the height of the telephone pole.

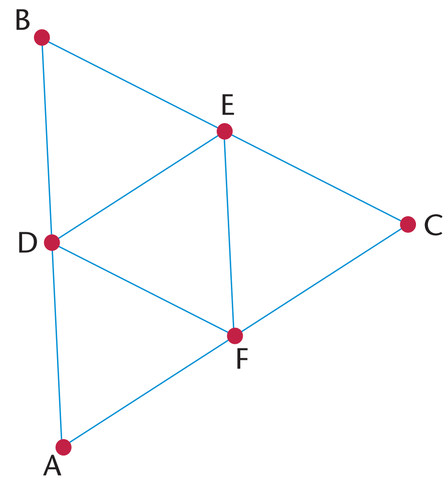


6. How many similar triangles are there in the diagram below? Explain your answer.



11.6 Extension questions

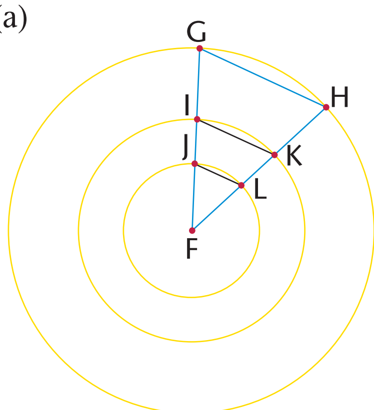
1. ΔABC on the right is equilateral. D is the midpoint of AB, E is the midpoint of BC and F is the midpoint of AC.



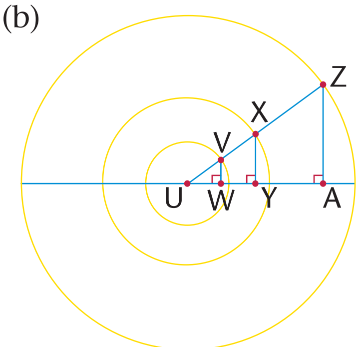
   (a) Prove that ΔBDE is an equilateral triangle.
   (b) Find all the congruent triangles. Give a proof for each.
   (c) Name as many similar triangles as you can. Explain how you know they are similar.
   (d) What is the proportion of the corresponding sides of the similar triangles?
   (e) Prove that DE is parallel to AC.
   (f) Is DF parallel to BC? Is EF parallel to BA? Explain.

2. Consider the similar triangles drawn below using concentric circles. Explain why the triangles are similar in each diagram.

   (a)



   (b)



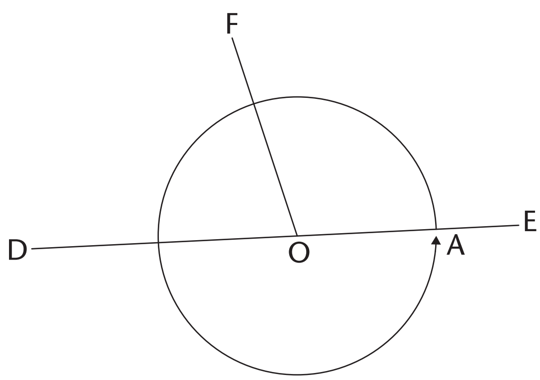
Chapter 12
Geometry of straight lines

12.1 Angle relationships

Remember that 360° is one full revolution.

If you look at something and then turn all the way around so that you are looking at it again, you have turned through an angle of 360°. If you turn only halfway around so that you look at something that was right behind your back, you have turned through an angle of 180°.

1. Answer the questions about the figure below.

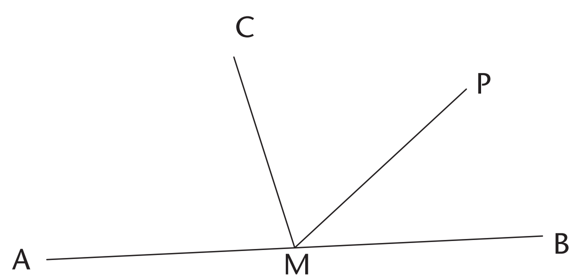


   (a) Is angle FOD in the figure smaller or bigger than a right angle?
   (b) Is angle FOE in the above figure smaller or bigger than a right angle?

In the figure above, $F\hat{O}D + F\hat{O}E$ = half of a revolution = 180°.

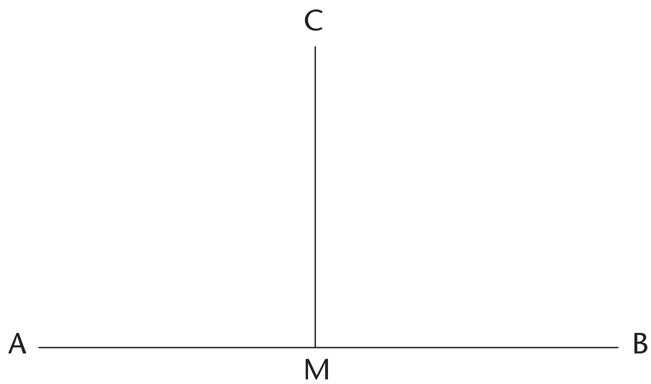
The sum of the angles on a straight line is 180°.
When the sum of angles is 180°, the angles are called **supplementary**.

2. $C\hat{M}A$ in the figure on the right is 75°.
   AMB is a straight line.
   (a) How big is $C\hat{M}B$?
   (b) Why do you say so?



3. $P\hat{M}B$ in the figure in question 2 is 40°.
   (a) How big is $C\hat{M}P$?
   (b) Explain your reasoning.

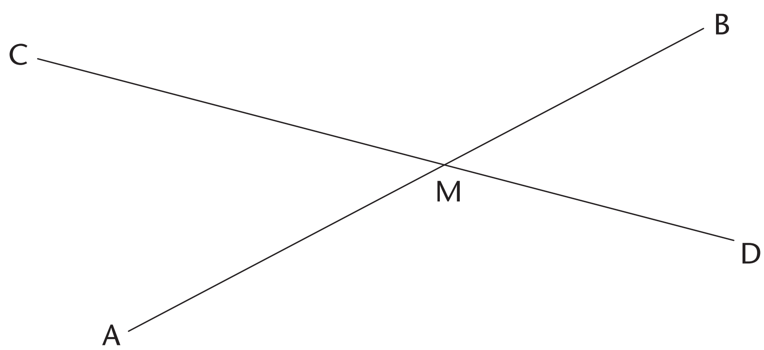
4. In the figure below, AMB is a straight line and $A\hat{M}C$ and $B\hat{M}C$ are equal angles.
   (a) How big are these angles?
   (b) How do you know this?



When one line forms two equal angles where it meets another line, the two lines are said to be **perpendicular**.

Because the two equal angles are angles on a straight line, their sum is 180°, hence each angle is 90°.

5. In the figure below, lines AB and CD intersect at point M.



   (a) Does it look as if $C\hat{M}A$ and $B\hat{M}D$ are equal?
   (b) Can you explain why they are equal?
   (c) What does $C\hat{M}A + D\hat{M}A$ equal? Why do you say so?
   (d) What is $C\hat{M}A + C\hat{M}B$? Why do you say so?

In this chapter, you are required to give good reasons for every statement you make.

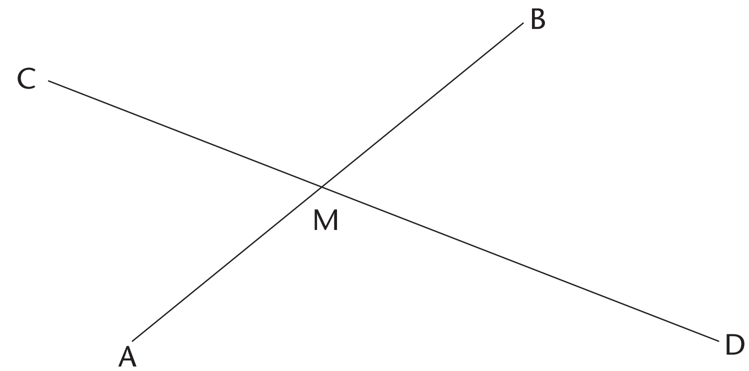
(e) Is it true that $C\hat{M}A + D\hat{M}A = C\hat{M}A + C\hat{M}B$?

(f) Which angle occurs on both sides of the equation in (e)?

6. Look carefully at your answers to questions 5(c) to (e).
   Now try to explain your observation in question 5(a).

7. In the figure below, AB and CD intersect at M. Four angles are formed. Angle CMB and angle AMD are called **vertically opposite** angles. Angle CMA and angle BMD are also **vertically opposite**.

When two straight lines intersect, the vertically opposite angles are equal.

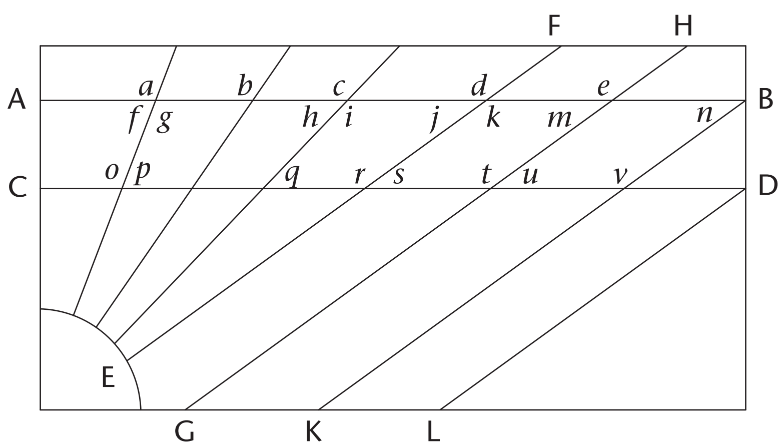


(a) If angle BMC = 125°, how big is angle AMD?

(b) Why do you say so?

LINES AND ANGLES

A line that intersects other lines is called a **transversal**.



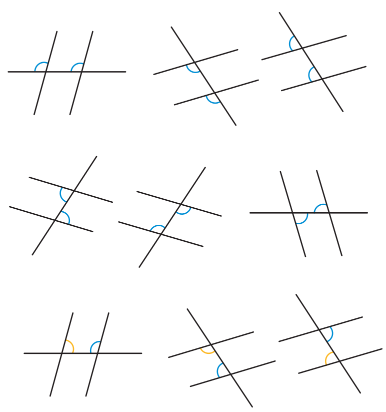
In the above pattern, AB is parallel to CD and EF || GH || KB || LD.

1. Angles *a*, *b*, *c*, *d* and *e* are **corresponding angles**. Do the corresponding angles appear to be equal?
2. Investigate whether or not the corresponding angles are equal by using tracing paper. Trace the angle you want to compare to other angles and place it on top of the other angle to find out if they are equal. What do you notice?
3. Angles *f*, *h*, *j*, *m* and *n* are also corresponding angles. Identify all the other groups of corresponding angles in the pattern.
4. Describe the position of corresponding angles that are formed when a transversal intersects other lines.
5. The following are pairs of **alternate angles**: *g* and *o*; *j* and *s*; and *k* and *r*. Do these angles appear to be equal?
6. Investigate whether or not the alternate angles are equal by using tracing paper. Trace the angle you want to compare and place it on top of the other angle to find out if they are equal. What do you notice?
7. Identify two more pairs of alternate angles.
8. Clearly describe the relative position of alternate angles that are formed when a transversal intersects other lines.
9. Did you notice anything about some of the pairs of corresponding angles when you did the investigation in question 6? Describe your finding.
10. Angles *f* and *o*, *i* and *q* and *k* and *s* are all pairs of **co-interior angles**. Identify three more pairs of co-interior angles in the pattern.

The angles in the same relative position at each intersection where a straight line crosses two others are called **corresponding angles**.

Angles on different sides of a transversal and between two other lines are called **alternate angles**.

Angles on the same side of the transversal and between two other lines are called **co-interior angles**.

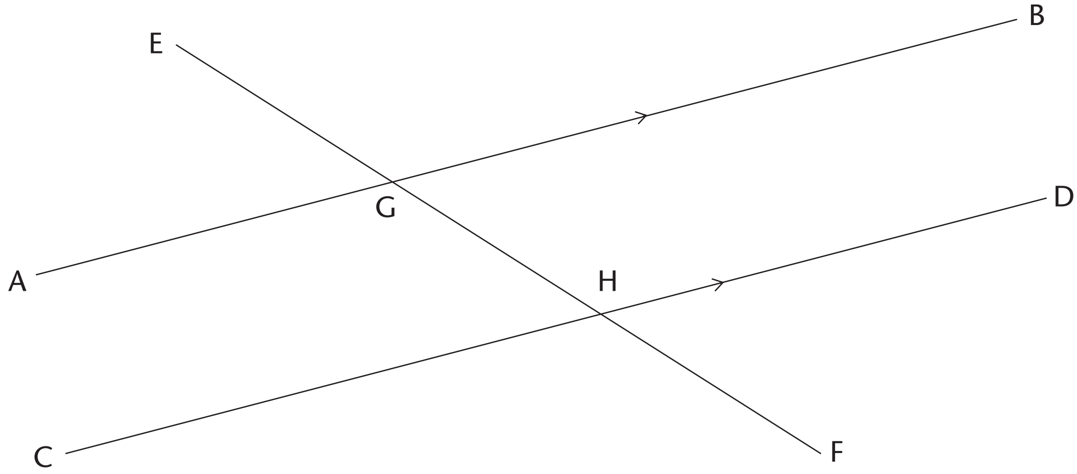


ANGLES FORMED BY PARALLEL LINES

Corresponding angles

The lines AB and CD shown on the following page never meet. Lines that never meet and are at a fixed distance from one another are called parallel lines. We write AB || CD.

Parallel lines have the same direction, i.e. they form **equal corresponding angles** with any line that intersects them.



The line EF cuts AB at G and CD at H.
EF is a transversal that cuts parallel lines AB and CD.

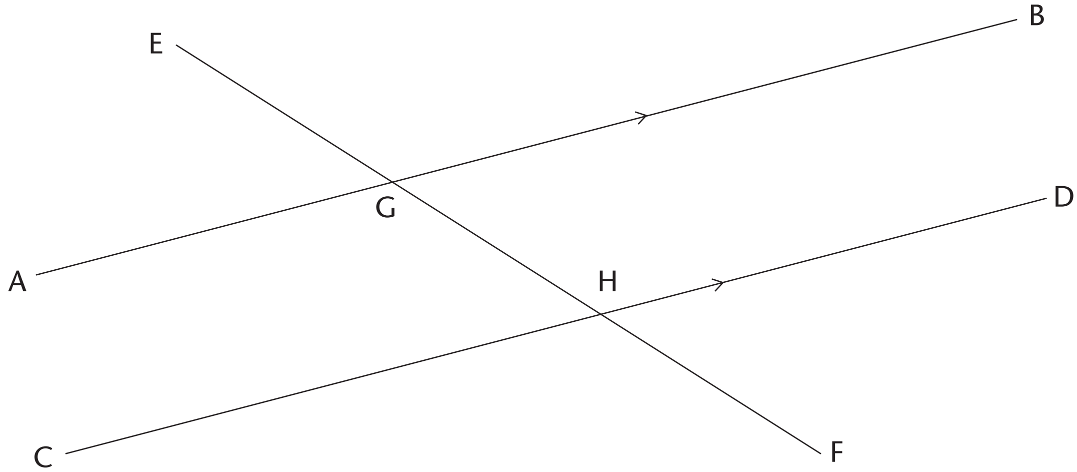
1. (a) Look carefully at the angles EGA and EHC in the above figure. They are called corresponding angles. Do they appear to be equal?
   (b) Measure the two angles to check if they are equal. What do you notice?

2. Suppose $E\hat{G}A$ and $E\hat{H}C$ are really equal. Would $E\hat{G}B$ and $E\hat{H}D$ then also be equal? Give reasons to support your answer.

When two parallel lines are cut by a transversal, the corresponding angles are equal.

Alternate angles

The angles $B\hat{G}F$ and $C\hat{H}E$ below are called alternate angles. They are on opposite sides of the transversal.



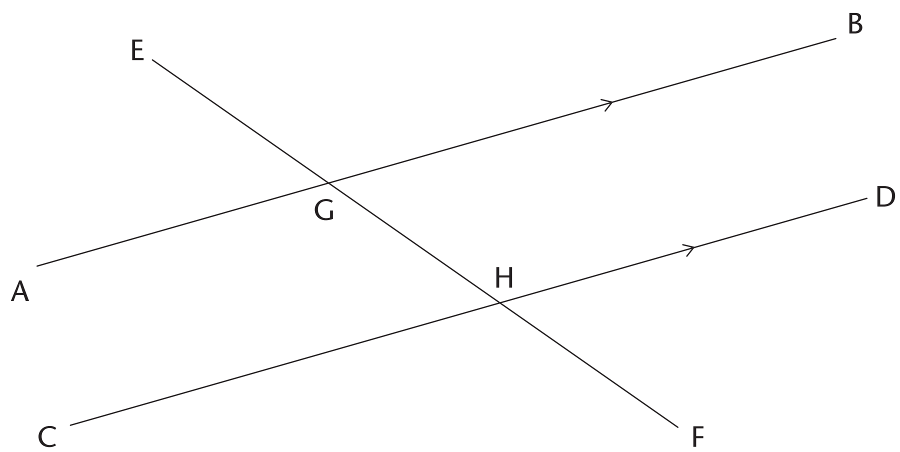
3. Do you think angles AGF and DHE should also be called alternate angles?
4. Do you think alternate angles are equal? Investigate by using the tracing paper like you did previously, or measure the angles accurately with your protractor. What do you notice?

When parallel lines are cut by a transversal, the alternate angles are equal.

5. Try to explain why alternate angles are equal when the lines that are cut by a transversal are parallel, keeping in mind that corresponding angles are equal.

By answering the following questions, you should be able to see how you can explain why alternate angles are equal when parallel lines are cut by a transversal.

6. Are angles $B\hat{G}H$ and $D\hat{H}F$ in the figure corresponding angles? What do you know about corresponding angles?

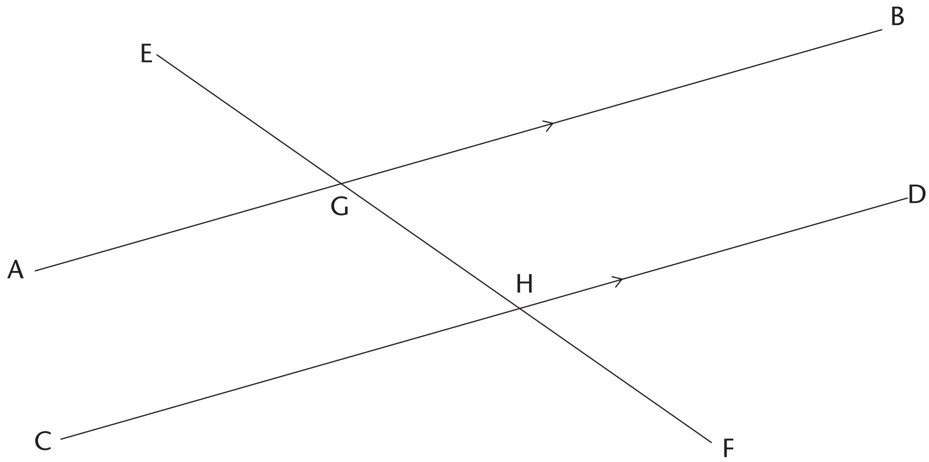


7. (a) What can you say about $B\hat{G}H + A\hat{G}H$? Give a reason.
   (b) What can you say about $D\hat{H}G + C\hat{H}G$? Give a reason.
   (c) Is it true that $B\hat{G}H + A\hat{G}H = D\hat{H}G + C\hat{H}G$? Explain.
   (d) Will the equation in (c) still be true if you replace angle $B\hat{G}H$ on the left-hand side with angle $C\hat{H}G$?
8. Look carefully at your work in question 7 and write an explanation why alternate angles are equal, when two parallel lines are cut by a transversal.

Co-interior angles

The angles $A\hat{G}H$ and $C\hat{H}G$ in the figure on the following page are called co-interior angles. They are on the same side of the transversal.

The prefix "co-" means together. The word "co-interior" means on the same side.



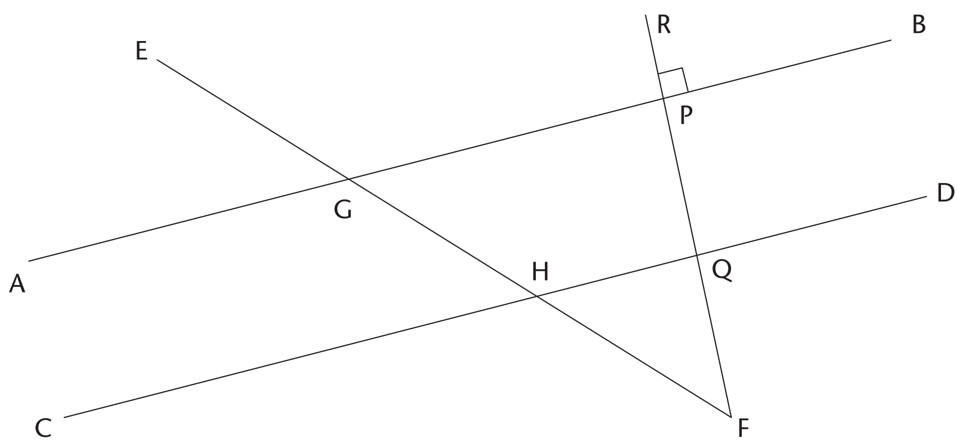
9. (a) What do you know about $C\hat{H}G + D\hat{H}G$? Explain.
   (b) What do you know about $B\hat{G}H + A\hat{G}H$? Explain.
   (c) What do you know about $B\hat{G}H + C\hat{H}G$? Explain.
   (d) What conclusion can you draw about $A\hat{G}H + C\hat{H}G$? Give detailed reasons for your conclusion.

When two parallel lines are cut by a transversal, the sum of two co-interior angles is 180°.

Another way of saying this is to say that the two co-interior angles are **supplementary**.

12.2 Identify and name angles

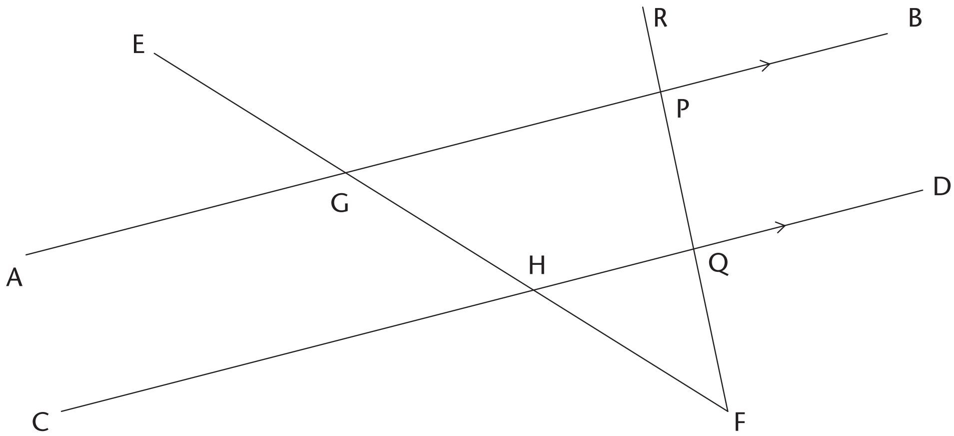
1. In the figure below, the line RF is perpendicular to AB.



   (a) Is RF also perpendicular to CD? Justify your answer.
   (b) Name four pairs of supplementary angles in the figure. In each case, say how you know that the angles are supplementary.

(c) Name four pairs of co-interior angles in the figure.
(d) Name four pairs of corresponding angles in the figure.
(e) Name four pairs of alternate angles in the figure.

2. Now you are given that AB and CD in the figure below are parallel.

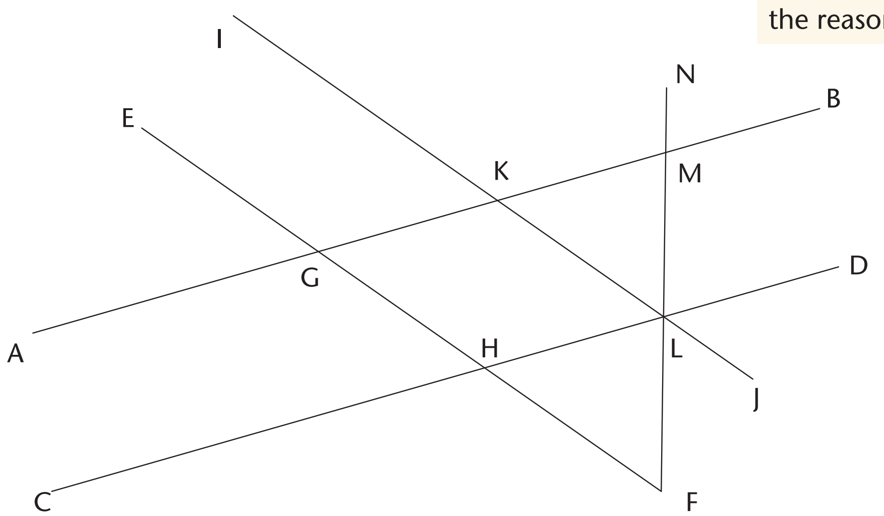


(a) If it is also given that RF is perpendicular to AB, will RF also be perpendicular to CD? Justify your answer.
(b) Name all pairs of supplementary angles in the figure. In each case, say how you know that the angles are supplementary.
(c) Suppose $E\hat{G}A = x$. Give the size of as many angles in the figure as you can, in terms of x. Each time give a reason for your answer.

12.3 Solving problems

When you solve problems in geometry you can use a short-hand way to write your reasons. For example, if two angles are equal because they are corresponding angles, then you can write (corr ∠s, AB || CD) as the reason.

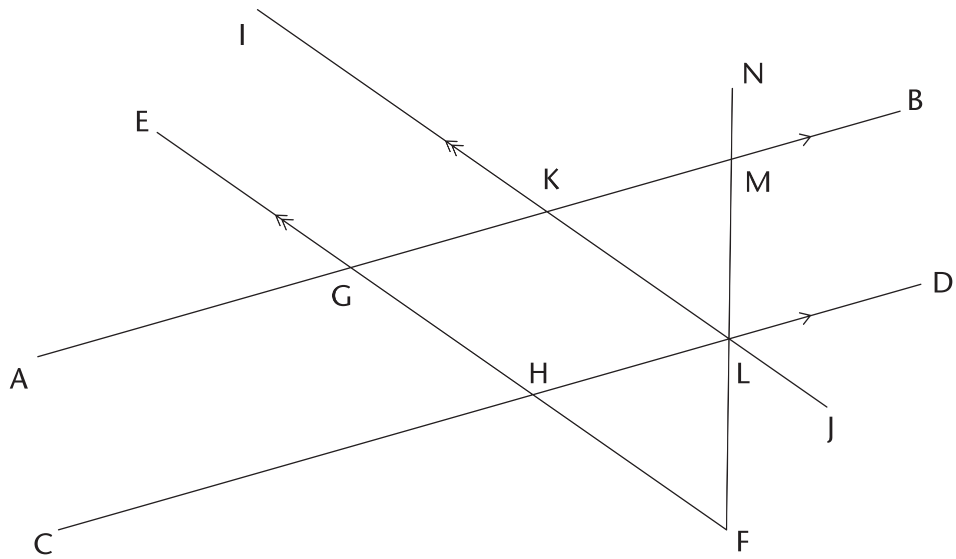
1. Line segments AB and CD in the figure below are parallel. EF and IJ are also parallel. Copy the figure and mark these facts on the figure, and then answer the questions.



(a) Name five angles in the figure that are equal to $G\hat{H}D$. Give a reason for each of your answers.

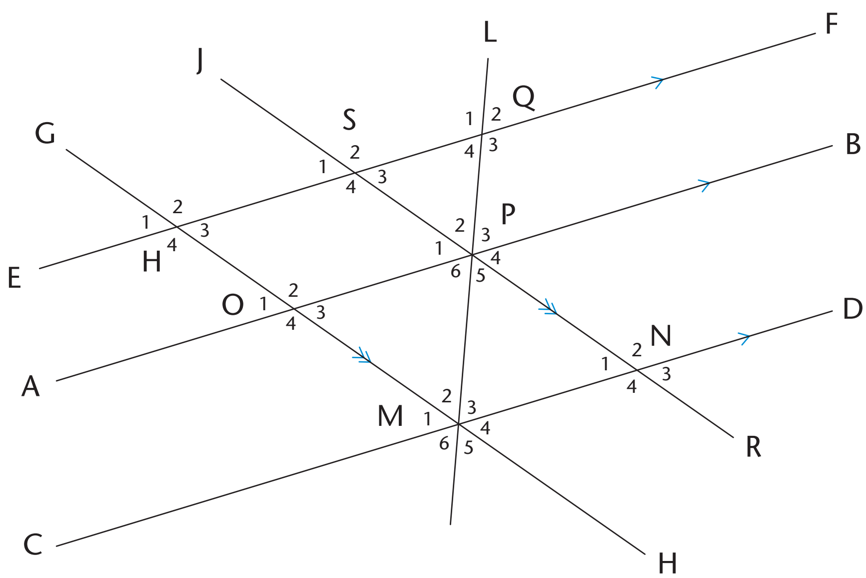
(b) Name all the angles in the figure that are equal to $A\hat{G}H$. Give a reason for each of your answers.

2. AB and CD in the figure below are parallel. EF and IJ are also parallel. $N\hat{M}B = 80°$ and $J\hat{L}F = 40°$.



Find the sizes of as many angles in the figure as you can, giving reasons.

3. In the figure below, AB || CD; EF || AB; JR || GH. You are also given that $P\hat{M}N = 60°$, $R\hat{N}D = 50°$.



(a) Find the sizes of as many angles in the figure as you can, giving reasons.

(b) Are EF and CD parallel? Give reasons for your answers.

CHAPTER 13

Pythagoras' Theorem

13.1 Investigating the sides of a right-angled triangle

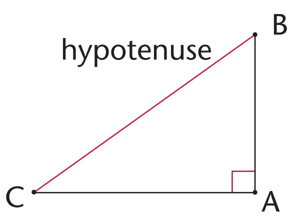
A **theorem** is a rule or a statement that has been proved through reasoning. **Pythagoras' Theorem** is a rule that applies only to **right-angled triangles**. The theorem is named after the Greek mathematician, Pythagoras.

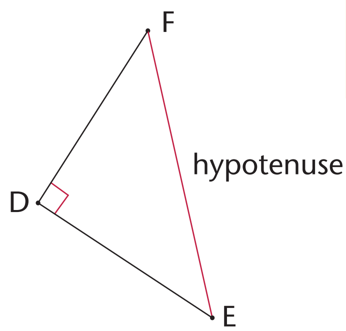
A right-angled triangle has one 90° angle. The longest side of the right-angled triangle is called the **hypotenuse**.

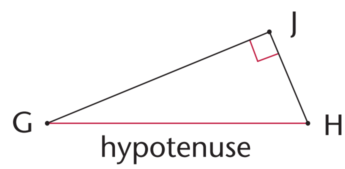
Pythagoras (569–475 BC)

Pythagoras was an influential mathematician. Like many Greek mathematicians of 2 500 years ago, he was also a philosopher and a scientist. He formulated the best-known theorem, today known as Pythagoras' Theorem.

However, the theorem had already been in use 1 000 years earlier, by the Chinese and the Babylonians.







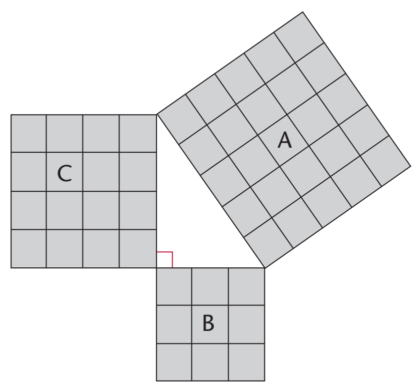
The hypotenuse is the side opposite the 90° angle in a right-angled triangle. It is always the longest side.

How to say it:

"high - *pot* - eh - news"

INVESTIGATING SQUARES ON THE SIDES OF RIGHT-ANGLED TRIANGLES

1. The figure shows a right-angled triangle with squares on each of the sides.
   (a) Write down the areas of the following:
   Square A
   Square B
   Square C
   (b) Add the area of square B and the area of square C.
   (c) What do you notice about the areas?



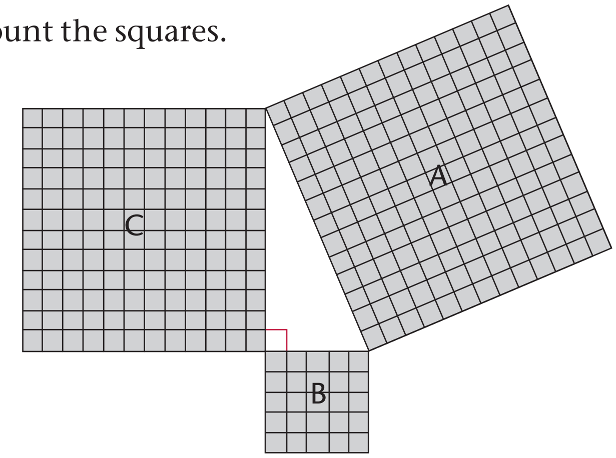
2. The figure below is similar to the one in question 1. The lengths of the sides of the right-angled triangle are 5 cm and 12 cm.
   (a) What is the length of the hypotenuse? Count the squares.
   (b) Use the squares to find the following:

      Area of A

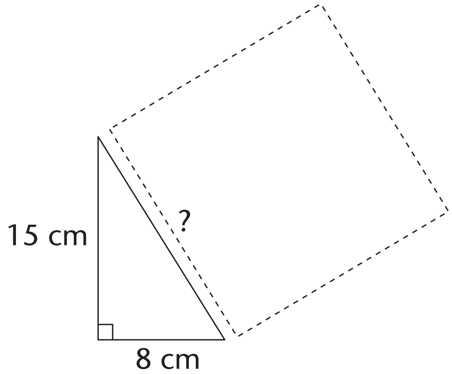
      Area of B

      Area of C

      Area of B + Area of C
   (c) What do you notice about the areas? Is it similar to your answer in 1(c)?



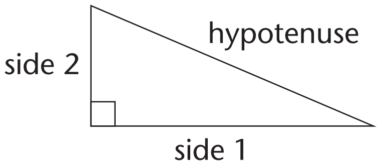
3. A right-angled triangle has side lengths of 8 cm and 15 cm. Use your findings in the previous questions to answer the following questions:
   (a) What is the area of the square drawn along the hypotenuse?
   (b) What is the length of the triangle's hypotenuse?



In the previous activity, you should have discovered Pythagoras' Theorem for right-angled triangles.

Pythagoras' Theorem says:
In a right-angled triangle, a square formed on the hypotenuse will have the same area as the sum of the area of the two squares formed on the other sides of the triangle. Therefore:

$(\text{Hypotenuse})^2 = (\text{Side } 1)^2 + (\text{Side } 2)^2$



13.2 Checking for right-angled triangles

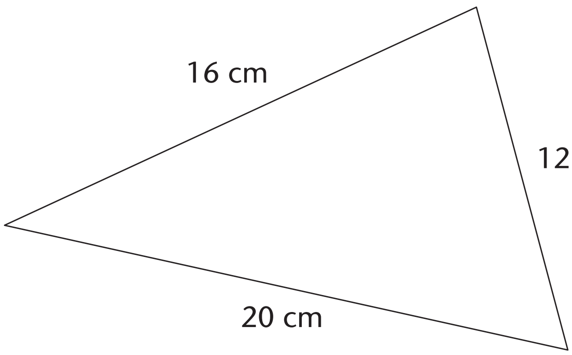
Pythagoras' Theorem applies in two ways:

- If a triangle is right-angled, the sides will have the following relationship: $(\text{Hypotenuse})^2 = (\text{Side } 1)^2 + (\text{Side } 2)^2$.
- If the sides have the relationship: $(\text{Longest side})^2 = (\text{Side } 1)^2 + (\text{Side } 2)^2$, then the triangle is a right-angled triangle.

So, we can test if any triangle is right-angled without using a protractor.

Example:

Is a triangle with sides 12 cm, 16 cm and 20 cm right-angled?



$(\text{Longest side})^2 = 20^2 = 400 \text{ cm}^2$

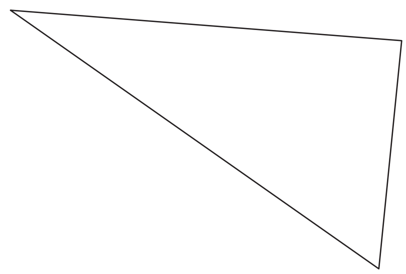
$(\text{Side } 1)^2 + (\text{Side } 2)^2 = 12^2 + 16^2 = 144 + 256 = 400 \text{ cm}^2$

$(\text{Longest side})^2 = (\text{Side } 1)^2 + (\text{Side } 2)^2$

$\therefore$ The triangle is right-angled.

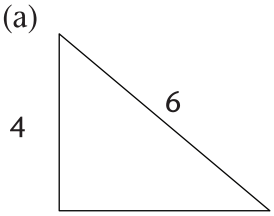
ARE THESE RIGHT-ANGLED TRIANGLES?

1. This triangle's side lengths are 29 mm, 20 mm and 21 mm.
   (a) Prove that it is a right-angled triangle.
   (b) Copy the triangle and mark the right angle in the diagram.

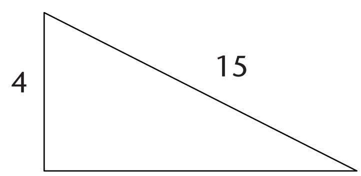


2. Use Pythagoras' Theorem to determine whether these triangles are right-angled. All values are in the same units.

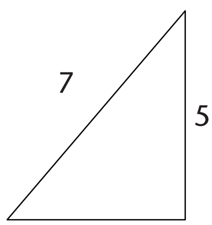
(a)



(b)



(c)



3. Determine whether the following side lengths would form right-angled triangles. All values are in the same units.
   (a) 7, 9 and 12
   (b) 7, 12 and 14
   (c) 16, 8 and 10
   (d) 6, 8 and 10
   (e) 8, 15 and 17
   (f) 16, 21 and 25

13.3 Finding missing sides

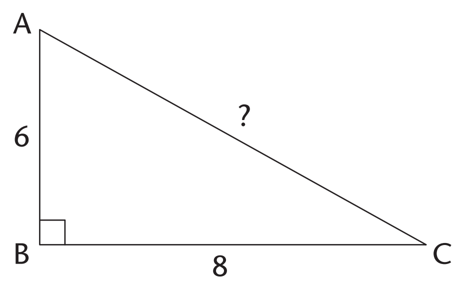
You can use the Pythagoras' Theorem to find the lengths of missing sides if you know that a triangle is right-angled.

FINDING THE MISSING HYPOTENUSE

Example: Calculate the length of the hypotenuse if the lengths of the other two sides are six units and eight units.

ΔABC is right-angled, so:

$$\begin{aligned} AC^2 &= AB^2 + BC^2 \\ &= (6^2 + 8^2) \text{ units}^2 \\ &= 36 + 64 \text{ units}^2 \\ &= 100 \text{ units}^2 \\ AC &= \sqrt{100} \text{ units} \\ &= 10 \text{ units} \end{aligned}$$



Sometimes the square root of a number is not a whole number or a simple fraction. In these cases, you can leave the answer under the square root sign. This form of the number is called a **surd**.

Surd form

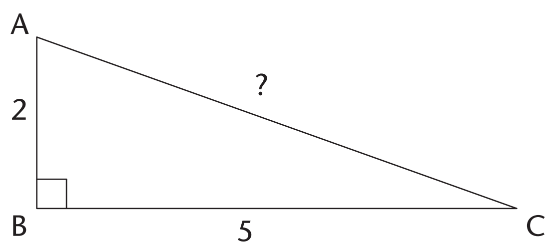
You pronounce *surd* so that it rhymes with *word*.

$\sqrt{5}$ is an example of a number in surd form.

$\sqrt{9}$ is not a surd because you can simplify it:

$\sqrt{9} = 3$

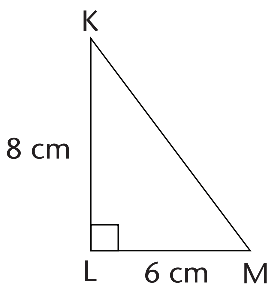
Example: Calculate the length of the hypotenuse of ΔABC if $\hat{B} = 90°$, AB = two units and BC = five units. Leave your answer in surd form, where applicable. Remember when taking the square root that length is always positive.



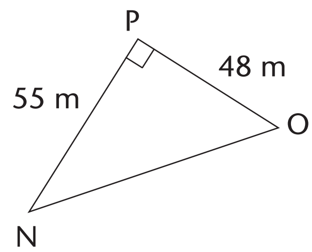
$$\begin{aligned} AC^2 &= AB^2 + BC^2 \\ &= 2^2 + 5^2 \text{ units}^2 \\ &= 4 + 25 \text{ units}^2 \\ &= 29 \text{ units}^2 \\ AC &= \sqrt{29} \text{ units} \end{aligned}$$

1. Find the length of the hypotenuse in each of the triangles shown on the following page. Leave the answers in surd form where applicable.

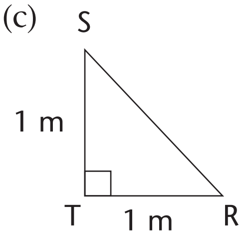
(a)



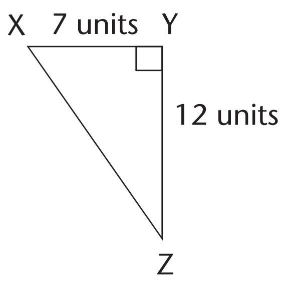
(b)



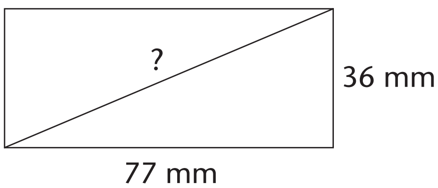
(c)



(d)

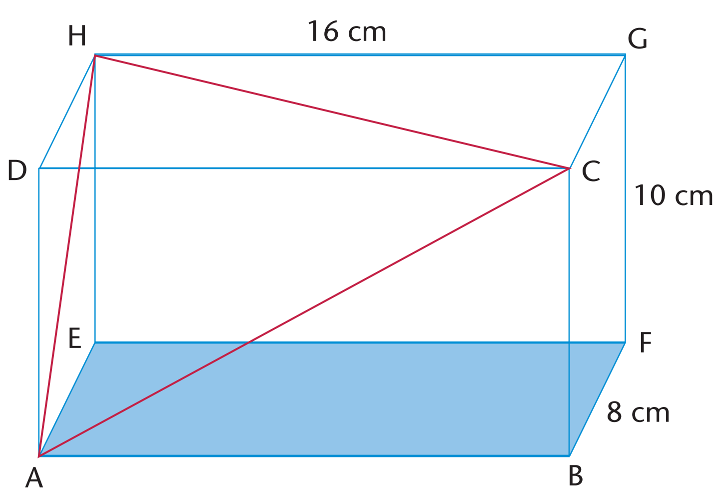


2. A rectangle has sides with lengths of 36 mm and 77 mm. Find the length of the rectangle's diagonal.



3. ΔABC has $\hat{A} = 90°$, AB = 3 cm and AC = 5 cm. Make a rough sketch of the triangle, and then calculate the length of BC.

4. A rectangular prism is made of glass. It has a length of 16 cm, a height of 10 cm and a breadth of 8 cm. ABCD and EFGH are two of its faces. ΔACH has been drawn inside the prism. Is ΔACH right-angled? Answer the questions to find out.

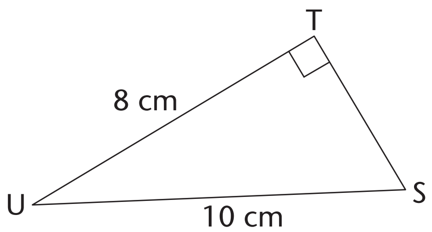


(a) Calculate the length of the sides of ΔACH. Note that all three sides of the triangles are diagonals of rectangles. AC is in rectangle ABCD, AH is in ADHE and HC is in HDCG.

(b) Is ΔACH right-angled? Explain your answer.

FINDING ANY MISSING SIDE IN A RIGHT-ANGLED TRIANGLE

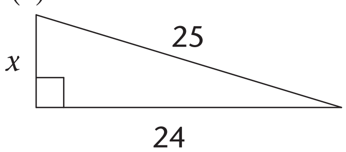
Example: Find the length of TS in the triangle below.



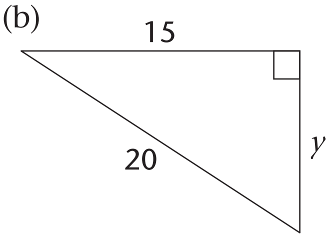
$$\begin{aligned} US^2 &= TU^2 + TS^2 \\ 10^2 &= 8^2 + TS^2 \\ 100 &= 64 + TS^2 \\ 36 &= TS^2 \\ \sqrt{36} &= TS \\ \therefore TS &= 6 \text{ cm} \end{aligned}$$

1. In the right-angled triangles below, calculate the length of the sides that have not been given. Leave your answers in surd form where applicable.

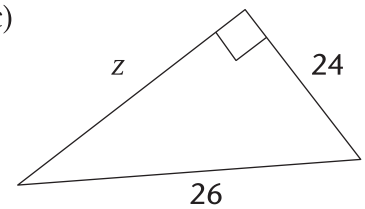
(a)



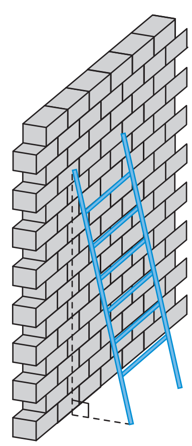
(b)



(c)



2. Calculate the length of the third side of each of the following right-angled triangles. First draw a rough sketch of each of the triangles before you do any calculations. Round off your answers to two decimal places.
   (a) ΔABC has AB = 12 cm, BC = 18 cm and $\hat{A}$ = 90°. Calculate AC.
   (b) ΔDEF has $\hat{F}$ = 90°, DE = 58 cm and DF = 41 cm. Calculate EF.
   (c) ΔJKL has $\hat{K}$ = 90°, JK = 119 m and KL = 167 m. Calculate JL.
   (d) ΔPQR has PQ = 2 cm, QR = 8 cm and $\hat{Q}$ = 90°. Calculate PR.



3. (a) A ladder with a length of 5 m is placed at an angle against a wall. The bottom of the ladder is 1 m away from the wall. How far up the wall will the ladder reach? Round off to two decimal places.
   (b) If the ladder reaches a height of 4,5 m against the wall, how far away from the wall was it placed? Round off to two decimal places.

PYTHAGOREAN TRIPLES

Sets of **whole numbers** that can be used as the sides of a right-angled triangle are known as **Pythagorean triples**, for example:

3-4-5 5-12-13 7-24-25 16-30-34 20-21-29

You extend these triples by finding multiples of them. For examples, triples from the 3-4-5 set include the following:

3-4-5 6-8-10 9-12-15 12-16-20

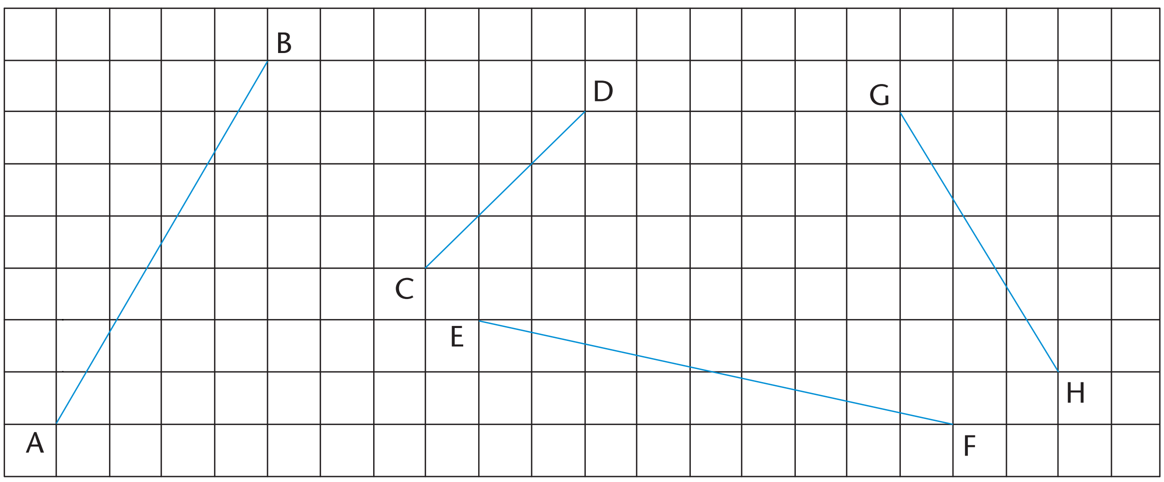
There are many old writings that record Pythagorean triples. For example, from 1900 to 1600 BC, the Babylonians had already calculated very large Pythagorean triples, such as:

1 679-2 400-2 929.

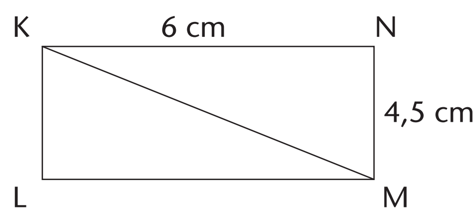
How many Pythagorean triples can you find? What is the largest one you can find that is not a multiple of another one?

13.4 More practice using Pythagoras' Theorem

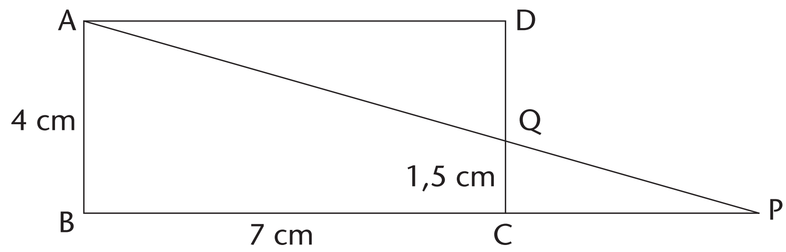
1. Four lines have been drawn on the grid below. Each square is one unit long. Calculate the lengths of the lines: AB, CD, EF and GH. Do the calculations and write the answers in surd form.



2. (a) Calculate the area of rectangle KLMN.
   (b) Calculate the perimeter of ΔKLM.

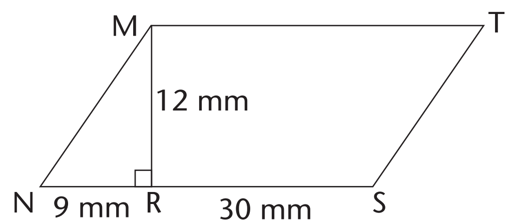


3. ABCD is a rectangle with AB = 4 cm, BC = 7 cm and CQ = 1,5 cm. Round off your answers to two decimal places if the answers are not whole numbers.



(a) What is the length of QD?
(b) If CP = 4,2 cm, calculate the length of PQ.
(c) Calculate the length of AQ and the area of ΔAQD.

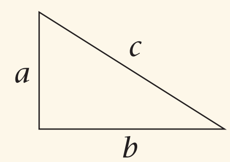
4. MNST is a parallelogram. NR = 9 mm and MR = 12 mm.
   (a) Calculate the area of ΔMNR.
   (b) Calculate the perimeter of MNST.



PYTHAGORAS' THEOREM AND OTHER TYPES OF TRIANGLES

Pythagoras' Theorem works only for right-angled triangles. But we can also use it to find out whether other triangles are acute or obtuse.

- If the square of the longest side is *less* than the sum of the squares of the two shorter sides, the *biggest angle is acute*.
  For example, in a 6-8-9 triangle: $6^2 + 8^2 = 100$ and $9^2 = 81$.
  81 is less than 100 ∴ the 6-8-9 triangle is acute.
- If the square of the longest side is *more* than the sum of the squares of the two shorter sides, the *biggest angle is obtuse*.
  For example, in a 6-8-11 triangle: $6^2 + 8^2 = 100$ and $11^2 = 121$.
  121 is more than 100 ∴ the 6-8-11 triangle is obtuse.



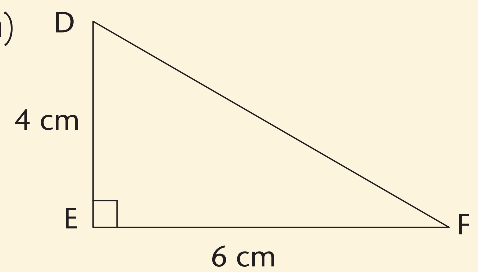
Copy and complete the following table. It is based on the triangle on the right. Decide whether each triangle described is right-angled, acute or obtuse.

a	*b*	c	$a^2 + b^2$	c^2	Fill in =, > or <	Type of triangle
3	5	6	$3^2 + 5^2 = 9 + 25 = 34$	$6^2 = 36$	$a^2 + b^2 < c^2$	Acute
2	4	6			$a^2 + b^2$ ..... c^2	
5	7	9			$a^2 + b^2$ ..... c^2	
12	5	13			$a^2 + b^2$ ..... c^2	
12	16	20	$12^2 + 16^2 = 144 + 256 = 400$	$20^2 = 400$	$a^2 + b^2 = c^2$	Right-angled
7	9	11			$a^2 + b^2$ ..... c^2	
8	12	13			$a^2 + b^2$ ..... c^2	

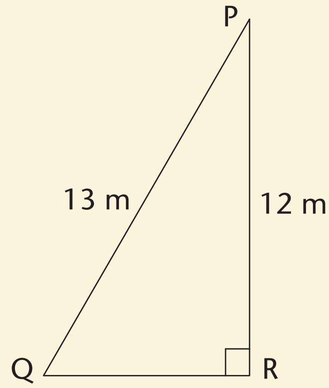
WORKSHEET

1. Write down Pythagoras' Theorem in the way that you best understand it.
2. Calculate the lengths of the missing sides in the following triangles. Leave the answers in surd form if necessary.

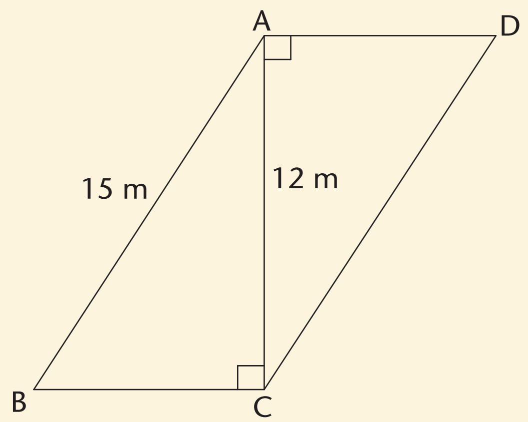
(a)



(b)



3. ABCD is a parallelogram.
   (a) Calculate the perimeter of ABCD.
   (b) Calculate the area of ABCD.



Chapter 14
Area and perimeter of 2D shapes

14.1 Area and perimeter of squares and rectangles

REVISING CONCEPTS

The **perimeter** (*P*) of a shape is the distance along the sides of the shape. The **area** (*A*) of a figure is the size of the flat surface enclosed by the figure.

1. Each block in figures A to F below measures 1 cm × 1 cm. What is the perimeter and area of each of the figures?
   Copy and complete the table below.

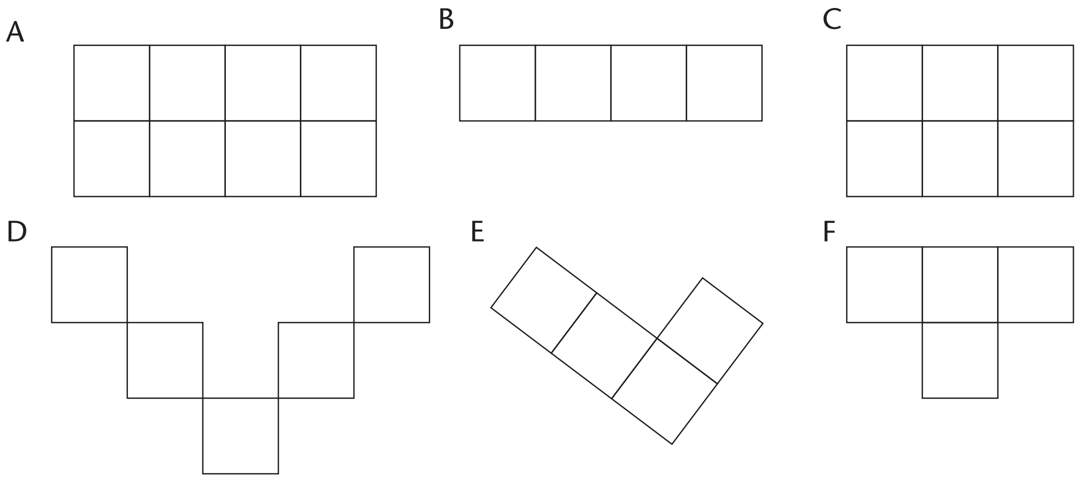
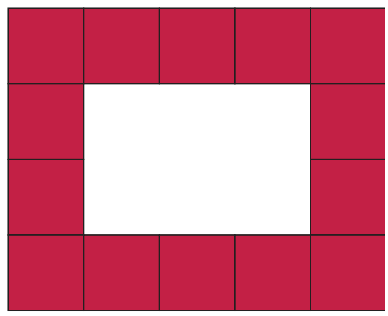


Figure	Perimeter	Area	Number of 1 cm × 1 cm squares
A			
B			
C			
D			

Figure	Perimeter	Area	Number of 1 cm × 1 cm squares
E			
F			
G			
H			

2. Consider the rectangle below on the right-hand side. It is formed by tessellating identical squares that are 1 cm by 1 cm each. The white part has squares that are hidden.
   (a) Write down, without counting, the total number of squares that form this rectangle, including those that are hidden. Explain your reasoning.
   (b) What is the area of the rectangle, including the white part?

To **tessellate** means to cover a surface with identical shapes in such a way that there are no gaps or overlaps. Another word for tessellating is **tiling**.



Area of a rectangle = length × breadth
= $l \times b$

Area of a square = $l \times l$
= l^2

Both length (l) and breadth (b) are expressed in the same unit.

3. Sipho and Theunis each paint a wall to earn some money during the school holidays. Sipho paints a wall 4 m high and 10 m long. Theunis's wall is 5 m high and 8 m long. Who should be paid more? Explain.

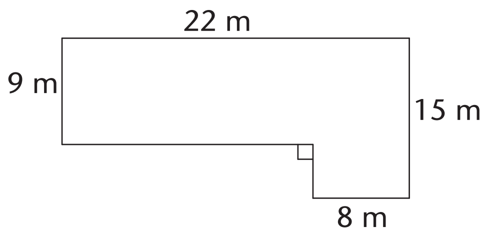
4. What is the area of a square with a length of 12 mm?

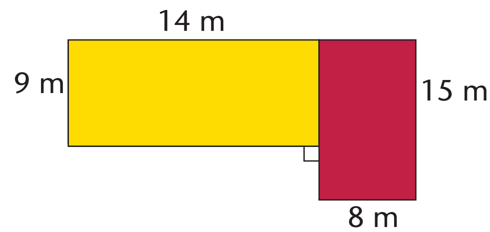
5. The area of a rectangle is 72 cm^2 and its length is 8 cm. What is its breadth?

14.2 Area and perimeter of composite figures

BREAKING UP FIGURES AND PUTTING THEM BACK TOGETHER AGAIN

1. The diagram on the left on the following page shows the floor plan of a room.
   We can calculate the area of the room by dividing the floor into two rectangles, as shown in the diagram on the right on the following page.



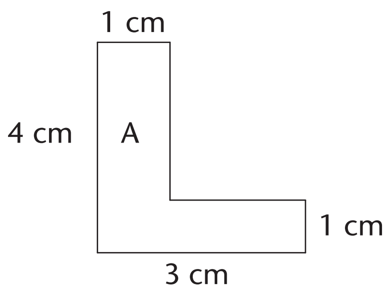


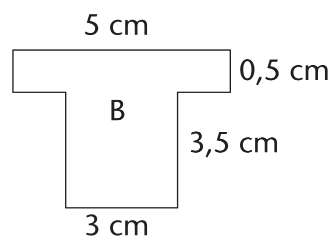
Area of the room	=	Area of yellow rectangle + Area of red rectangle
	=	$(l \times b) + (l \times b)$
	=	$(14 \times 9) + (15 \times 8)$
	=	$126 + 120$
	=	246 m^2

(a) The yellow part of the room has a wooden floor and the red part is carpeted. What is the area of the wooden floor? What is the area of the carpeted floor?

(b) Calculate the area of the room dividing the floor into two other shapes. Draw a sketch.

2. Calculate the area of the figures below.





3. Which of the following rules can be used to calculate the perimeter (P) of a rectangle? Explain.
   - Perimeter = $2 \times (l + b)$
   - Perimeter = $l + b + l + b$
   - Perimeter = $2l + 2b$
   - Perimeter = $l + b$

l and *b* refer to the length and the breadth of a rectangle.

The following are equivalent expressions for perimeter:
$P = 2l + 2b$ and $P = 2(l + b)$ and $P = l + b + l + b$

4. Check with two classmates that the rule or rules you have chosen above are correct; then apply it to calculate the perimeter of figure A. Think carefully!

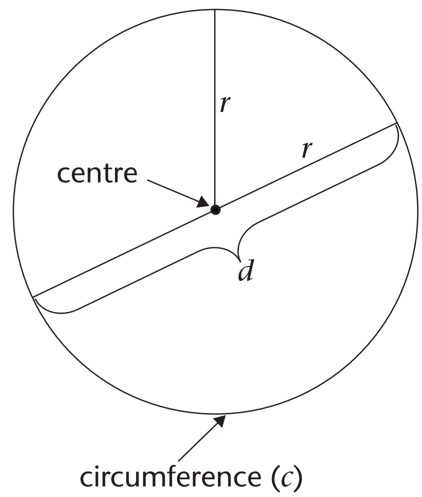
5. The perimeter of a rectangle is 28 cm and its breadth is 6 cm. What is its length?

14.3 Area and perimeter of circles

REVISING CONCEPTS FROM PREVIOUS GRADES

The perimeter of a circle is called the **circumference** of a circle. You will remember the following about circles from previous grades:

- The distance across the circle through its centre is called the **diameter** (d) of the circle.
- The distance from the centre of the circle to any point on the circumference is called the **radius** (r).
- The circumference (c) of a circle divided by its diameter is equal to the irrational value we call **pi** (π). To simplify calculations, we often use the approximate values:
  $\pi \approx 3{,}14$ or $\frac{22}{7}$.



The following are important formulae to remember:

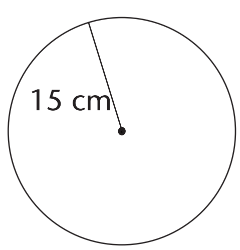
- $d = 2r$ and $r = \frac{1}{2}d$
- Circumference of a circle $(c) = 2\pi r$
- Area of a circle $(A) = \pi r^2$

CIRCLE CALCULATIONS

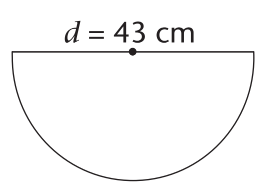
In the following calculations, use $\pi = 3{,}14$ and round off your answers to two decimal places. If you take a square root, remember that length is always positive.

1. Calculate the perimeter and area of the following circles:
   (a) A circle with a radius of 5 m
   (b) A circle with a diameter of 18 mm
2. Calculate the radius of a circle with:
   (a) a circumference of 53 cm
   (b) a circumference of 206 mm
3. Work out the area of the following shapes:

A

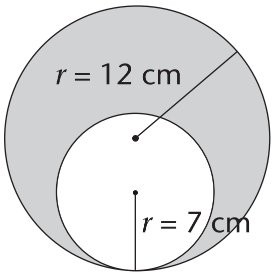


B



4. Calculate the radius and diameter of a circle with:
   (a) an area of 200 m^2
   (b) an area of 1 000 m^2

5. Calculate the area of the shaded part.



14.4 Converting between units

CONVERTING BETWEEN UNITS USED FOR PERIMETER AND AREA

Always make sure that you use the correct units in your calculations. Practise the conversions below.

Remember:
1 cm = 10 mm 1 mm = 0,1 cm
1 m = 100 cm 1 cm = 0,01 m
1 km = 1 000 m 1 m = 0,001 km

1. Copy and complete the following conversions:
   (a) 34 cm = ... mm
   (b) 501 m = ... km
   (c) 226 m = ... cm
   (d) 0,58 km = ... m
   (e) 1,9 cm = ... mm
   (f) 73 mm = ... cm
   (g) 924 mm = ... m
   (h) 32,23 km = ... m

Remember, to convert between square units, you can use method shown below:

To convert cm^2 to m^2:

$$\begin{aligned} 1\ cm^2 &= 1\ cm \times 1\ cm \\ &= 0{,}01\ m \times 0{,}01\ m \\ &= 0{,}0001\ m^2 \end{aligned}$$

Example

Convert 50 cm^2 to m^2

$$\begin{aligned} 1\ cm^2 &= 0{,}0001\ m^2 \\ \therefore 50\ cm^2 &= 50 \times 0{,}0001\ m^2 \\ &= 0{,}005\ m^2 \end{aligned}$$

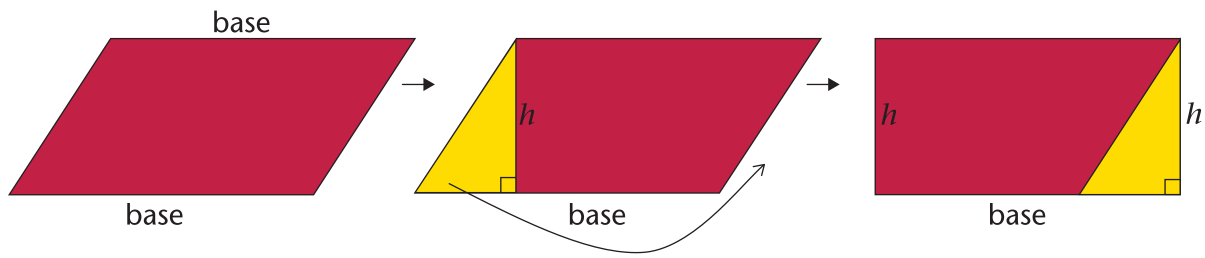
2. Convert to cm^2:
   (a) 650 mm^2
   (b) 1 200 mm^2
   (c) 18 m^2
   (d) 0,045 m^2
   (e) 93 mm^2
   (f) 177 m^2

3. (a) Convert 93 mm^2 to m^2.
   (b) Convert 0,017 km^2 to m^2.

14.5 Area of other quadrilaterals

PARALLELOGRAMS

As shown below, a parallelogram can be made into a rectangle if a right-angled triangle from one side is cut off and moved to its other side.



So we can find the area of a parallelogram using the formula for the area of a rectangle:

Area of rectangle $= l \times b$

$=$ (base of parallelogram) × (perpendicular height of parallelogram)

Area of parallelogram = Area of rectangle

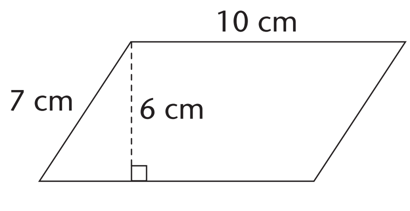
∴ Area of parallelogram = base × perp. height

We can use any side of the parallelogram as the base, but we must use the perpendicular height on the side we have chosen.

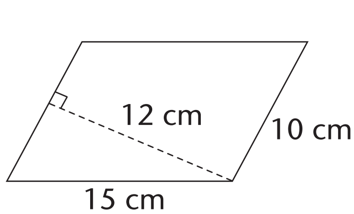
1. (a) Copy the parallelogram above.
   (b) Using the shorter side as the base of the parallelogram, follow the steps above to derive the formula for the area of a parallelogram.

2. Work out the area of the following parallelograms using the formula:

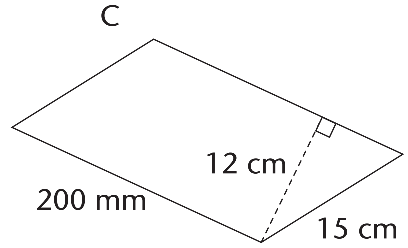
A



B

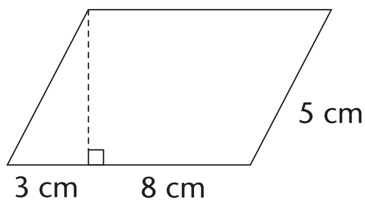


C

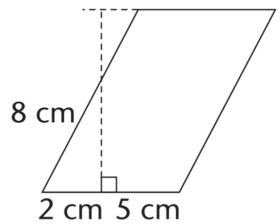


3. Work out the area of the parallelograms. Use the Pythagoras' Theorem to calculate the unknown sides you need. Remember to use the pre-rounded value for height and then round the final answer to two decimal places where necessary.

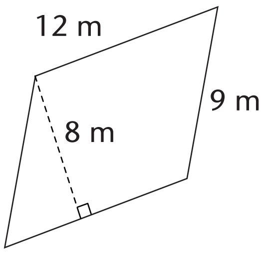
A



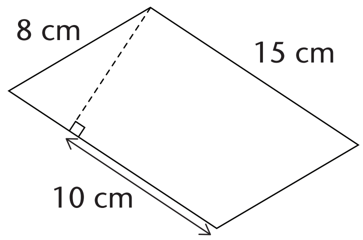
B



C



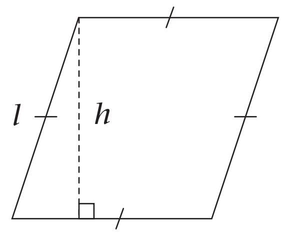
D



RHOMBI

A rhombus is a parallelogram with all its sides equal.

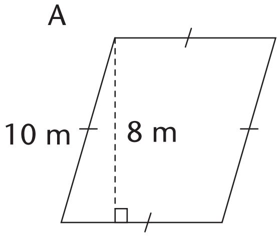
In the same way we derived the formula for the area of a parallelogram, we can show the following:



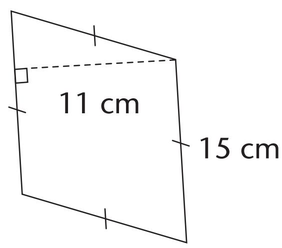
- Area of a rhombus = length × perp. height

1. Show how to derive the formula for the area of a rhombus.
2. Calculate the areas of the following rhombi. Round off answers to two decimal places where necessary.

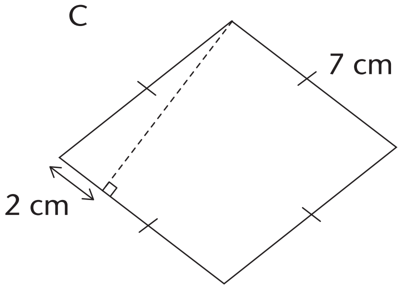
A



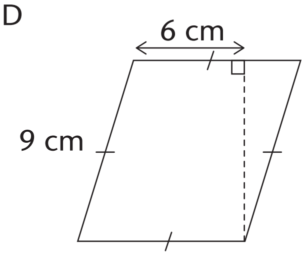
B



C

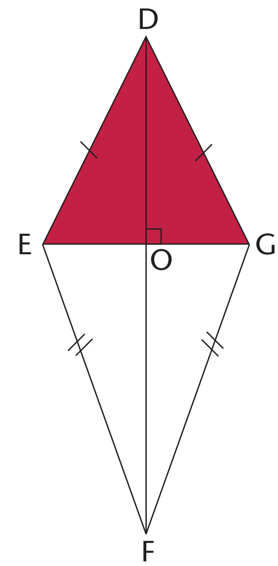


D



KITES

To calculate the area of a kite, you use one of its properties, namely that the diagonals of a kite are perpendicular.



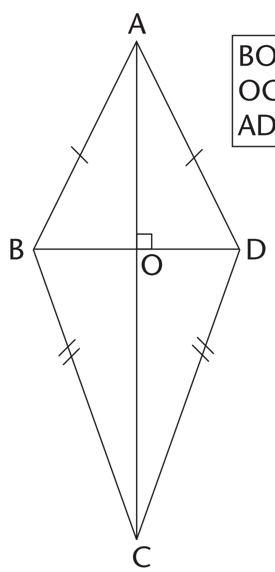
$$\begin{aligned}\text{Area of kite DEFG} &= \text{Area of } \Delta\text{DEG} + \text{Area of } \Delta\text{EFG}\\ &= \frac{1}{2}(b \times h) + \frac{1}{2}(b \times h)\\ &= \frac{1}{2}(\text{EG} \times \text{OD}) + \frac{1}{2}(\text{EG} \times \text{OF})\\ &= \frac{1}{2}\text{EG}(\text{OD} + \text{OF})\\ &= \frac{1}{2}\text{EG} \times \text{DF}\end{aligned}$$

Notice that EG and DF are the diagonals of the kite.

$\therefore$ Area of a kite $= \frac{1}{2}(\text{diagonal}_1 \times \text{diagonal}_2)$

1. Calculate the area of kites with the following diagonals. Give your answers in m².
   (a) 150 mm and 200 mm
   (b) 25 cm and 40 cm

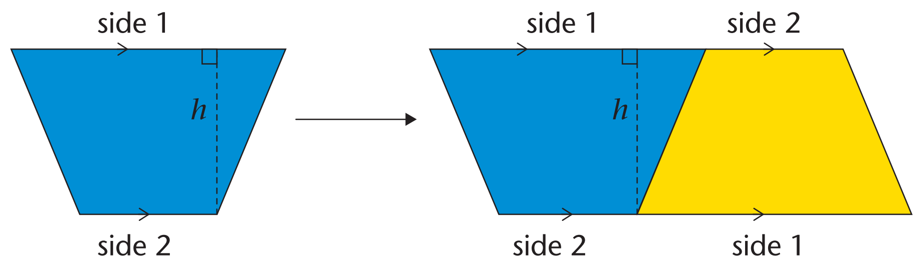
2. Calculate the area of the kite.



BO = OD = 6 cm
OC = 15 cm
AD = 10 cm

TRAPEZIUMS

A trapezium has two parallel sides. If we tessellate (tile) two trapeziums, as shown in the diagram on the following page, we form a parallelogram. (The yellow trapezium is the same size as the blue one. The base of the parallelogram is equal to the sum of the parallel sides of the trapezium.)



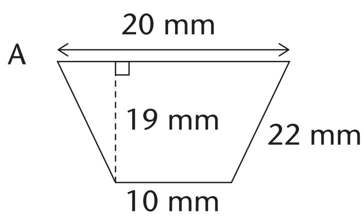
We can use the formula for the area of a parallelogram to work out the formula for the area of a trapezium as follows:

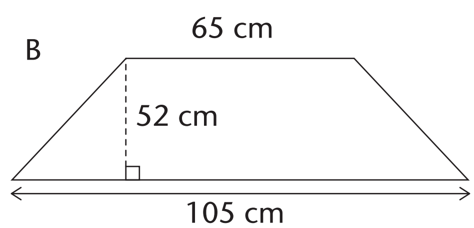
$$\begin{aligned}\text{Area of parallelogram} &= \text{base} \times \text{height}\\ &= (\text{side 1} + \text{side 2}) \times \text{height}\end{aligned}$$

$$\begin{aligned}\text{Area of trapezium} &= \frac{1}{2}\ \text{area of parallelogram}\\ &= \frac{1}{2}(\text{side 1} + \text{side 2}) \times \text{height}\end{aligned}$$

$\therefore$ Area of a trapezium $= \frac{1}{2}$ (sum of parallel sides) × perp. height

Calculate the area of the following trapeziums:

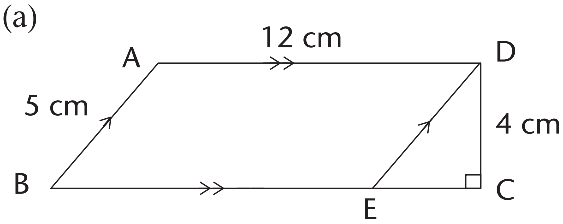




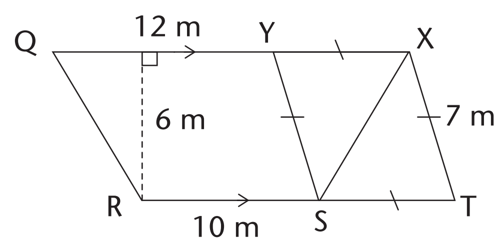
AREAS OF COMPOSITE SHAPES

Calculate the areas of the following 2D shapes. Round off your answers to two decimal places where necessary.

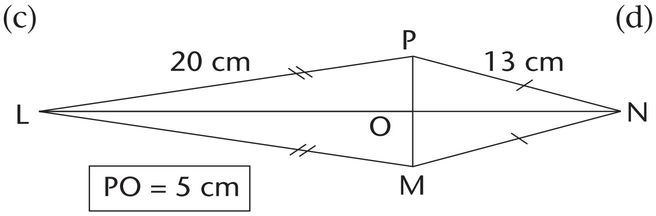
(a)



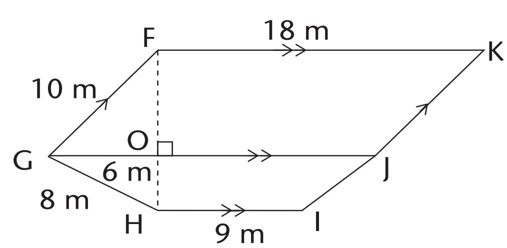
(b)



(c)



(d)



14.6 Doubling dimensions of a 2D shape

Remember that a 2D shape has two dimensions, namely length and breadth. You have used length and breadth in different forms, to work out the perimeters and areas of shapes, for example:

- length and breadth for rectangles and squares
- bases and perpendicular heights for triangles, rhombi and parallelograms
- two diagonals for kites.

But how does doubling one or both of the dimensions of a figure affect the figure's perimeter and area?

"Doubling" means to multiply by 2.

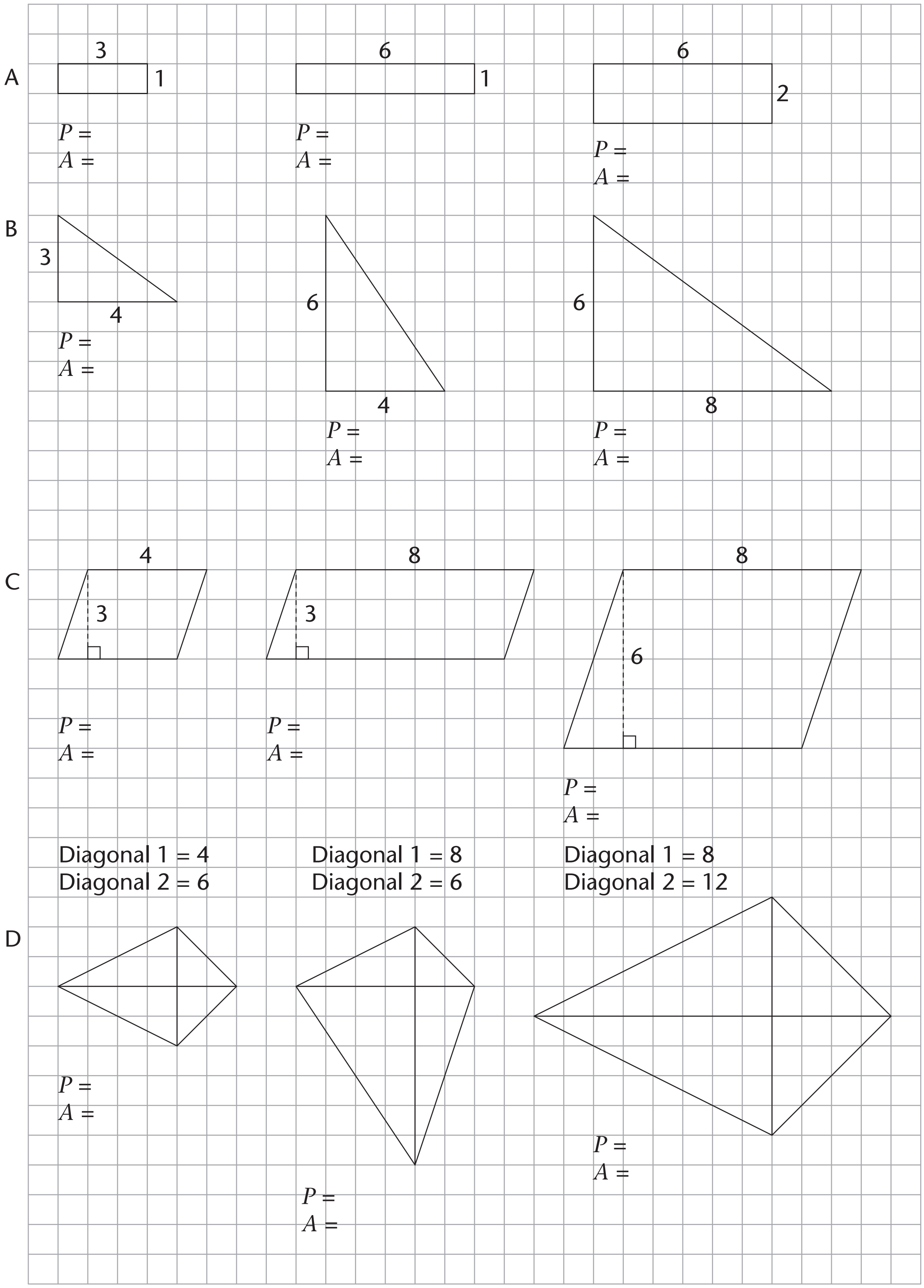
The four sets of figures on the next page are drawn on a grid of squares. Each row shows an original figure; the figure with one of its dimensions doubled, and the figure with both of its dimensions doubled. Each square has a side of one unit.

1. Work out the perimeter and area of each shape. Round off your answers to two decimal places where necessary.
2. Which figure in each set is congruent to the original figure?
3. Copy the table below and fill in the perimeter (*P*) and area (*A*) of each figure:

Figure	Original figure	Figure with both dimensions doubled
A	P = A =	P = A =
B	P = A =	P = A =
C	P = A =	P = A =
D	P = A =	P = A =

4. Look at the completed table. What patterns do you notice? Choose one:
   - When both dimensions of a shape are doubled, its **perimeter is doubled** and its **area is doubled**.
   - When both dimensions of a shape are doubled, its **perimeter is doubled** and its area is **four times bigger**.

Original figure	One dimension doubled	Both dimensions doubled

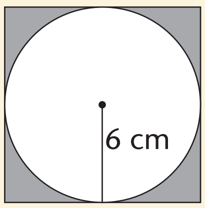


WORKSHEET

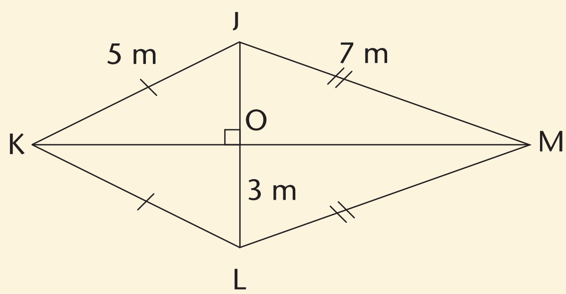
1. Write down the formulae for the following:

Perimeter of a square	
Perimeter of a rectangle	
Area of a square	
Area of a rectangle	
Area of a triangle	
Area of a rhombus	
Area of a kite	
Area of a parallelogram	
Area of a trapezium	
Diameter of a circle	
Circumference of a circle	
Area of a circle	

2. (a) Calculate the perimeter of the square and the area of the shaded parts of the square:



(b) Calculate the area of the kite:



Chapter 15
Functions

15.1 From formulae to words, tables and graphs

THE SAME INSTRUCTIONS IN WORDS AND IN SYMBOLS

1. Each formula below indicates a relationship between two sets of numbers that may be called the *input numbers* and the *output numbers*. For each formula, calculate the output numbers that correspond to the input numbers 0, 1, 2 and 10.

   (a) $y = 3x + 5$ (b) $y = 3(x + 5)$ (c) $y = 3x + 5x$
   (d) $y = 3x^2 + 5$ (e) $y = 3x^2 + 5x$ (f) $y = 3x(x + 5)$

2. The information provided in the formula $y = 5x^2 - 3x$ can also be represented in words, for example: *To get the output number, you have to subtract three times the input number from five times the square of the input number*.

   Represent each of the formulae in question 1 in words:

   (a) $y = 3x + 5$ (b) $y = 3(x + 5)$ (c) $y = 3x + 5x$
   (d) $y = 3x^2 + 5$ (e) $y = 3x^2 + 5x$ (f) $y = 3x(x + 5)$

3. For each set of instructions, write a formula that provides the same information.

   (a) *multiply the input number by 10, then subtract 3 to get the output number*
   (b) *subtract 3 from the square of the input number, then multiply by 10 to get the output number*
   (c) *multiply the square of the input number by 10, then add 5 times the input number to get the output number*
   (d) *subtract 7 times the square of the input number from 100, then multiply by 3 to get the output number*
   (e) *add 4 to the input number, then subtract the answer from 50 to get the output number*
   (f) *multiply the input number by 3, then subtract the answer from 15 to get the output number*

4. Copy and complete the table on the following page to check your answers for question 3. First apply the verbal instructions for the input numbers 1, 5 and 10 in each case. Then choose another input number and do the same thing. Next, use the formula you have written to calculate the output numbers. Do corrections where there are differences.

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		1	5	10	
(a)	verbal description				
	formula				
(b)	verbal description				
	formula				
(c)	verbal description				
	formula				
(d)	verbal description				
	formula				
(e)	verbal description				
	formula				
(f)	verbal description				
	formula				

5. In certain cases, flow diagrams can be used to provide instructions on how output numbers can be calculated. For each flow diagram below, represent the information in a formula and also in words:

(a) → × 3 → + 17 →

(b) → + 5 → × 3 → + 2 →

(c) → − 2 → × 3 → + 23 →

(d) → × 2 → + 3 → × 5 → + 4 →

(e) → + 3 → × 2 → + 4 → × 5 →

(f) → × 10 → + 19 →

(g) → + 5 → × 10 →

6. (a) Copy and complete the following table:

x	0	1	2	3
y according to your formula for 5(a)				
y according to your formula for 5(b)				
y according to your formula for 5(c)				

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(b) If your output numbers for 5(a), 5(b) and 5(c) are not the same, you have made a mistake somewhere. If this is the case, find your mistake and correct it.

7. (a) Copy and complete the following table:

x	–3	–2	–1	0
y according to your formula for 5(d)				
y according to your formula for 5(e)				
y according to your formula for 5(f)				
y according to your formula for 5(g)				

(b) If your output numbers for 5(d) and 5(f) are not the same, you have made a mistake somewhere. If this is the case, find your mistake and correct it.

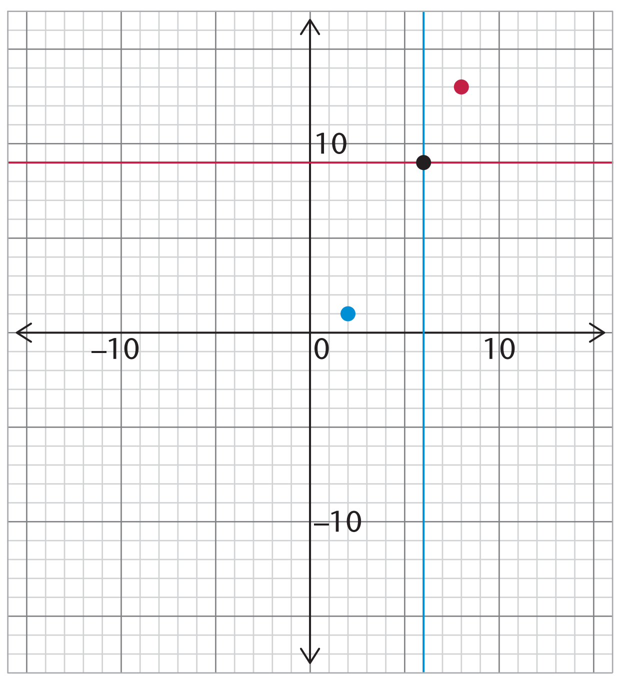
(c) If your output numbers for 5(e) and 5(g) are not the same, you have made a mistake somewhere. If this is the case, find your mistake and correct it.

8. Explain why the output numbers in 5(a), (b) and (c) are the same.

15.2 Tables and graphs

1. Copy and complete the table to show some of the input and output numbers of the relationship described by the formula $y = 2x - 3$.

Input numbers	–5	0	2	4	6	8
Output numbers						



The vertical blue line on this graph represents the input number 6.

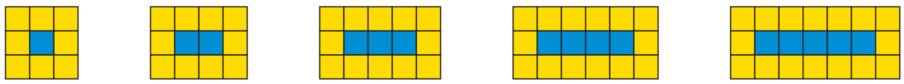
The heavy horizontal red line represents the output number 9.

The black point where the blue and red lines intersect indicates that the input number 6 is associated with the output number 9.

We also say the black point represents the **ordered number pair** (6; 9).

2. (a) Which ordered number pair does the red point on the graph represent?

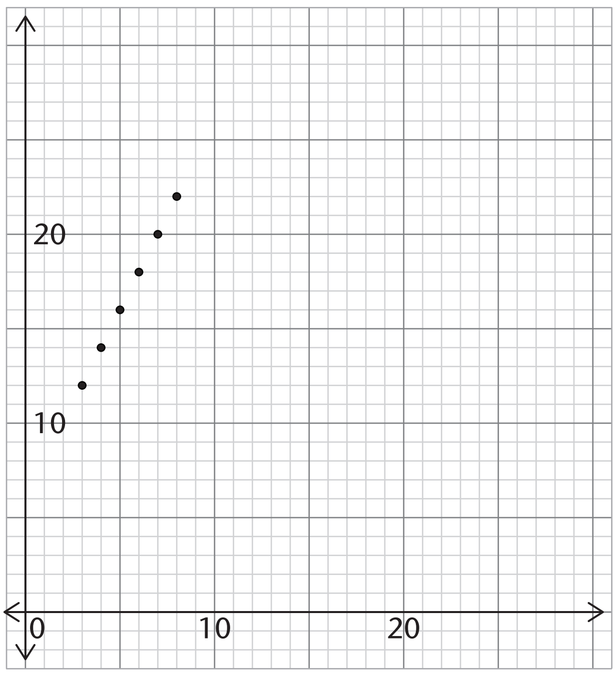
(b) Which ordered number pair does the blue point on the graph represent?

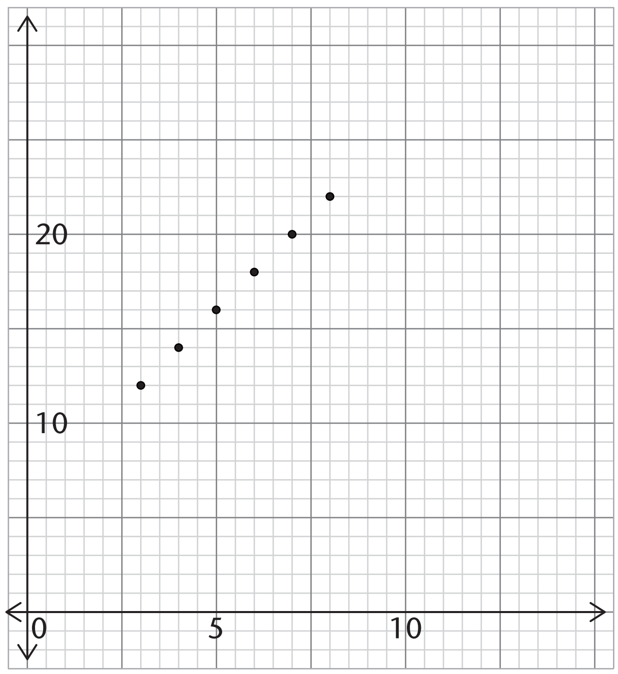


A relationship between two variables can be represented by a table that shows the values of the independent and dependent variables (input and output numbers):

Values of the independent variable	3	4	5	6	7	8
Values of the dependent variable	12	14	16	18	20	22

The same information can also be shown on a graph:





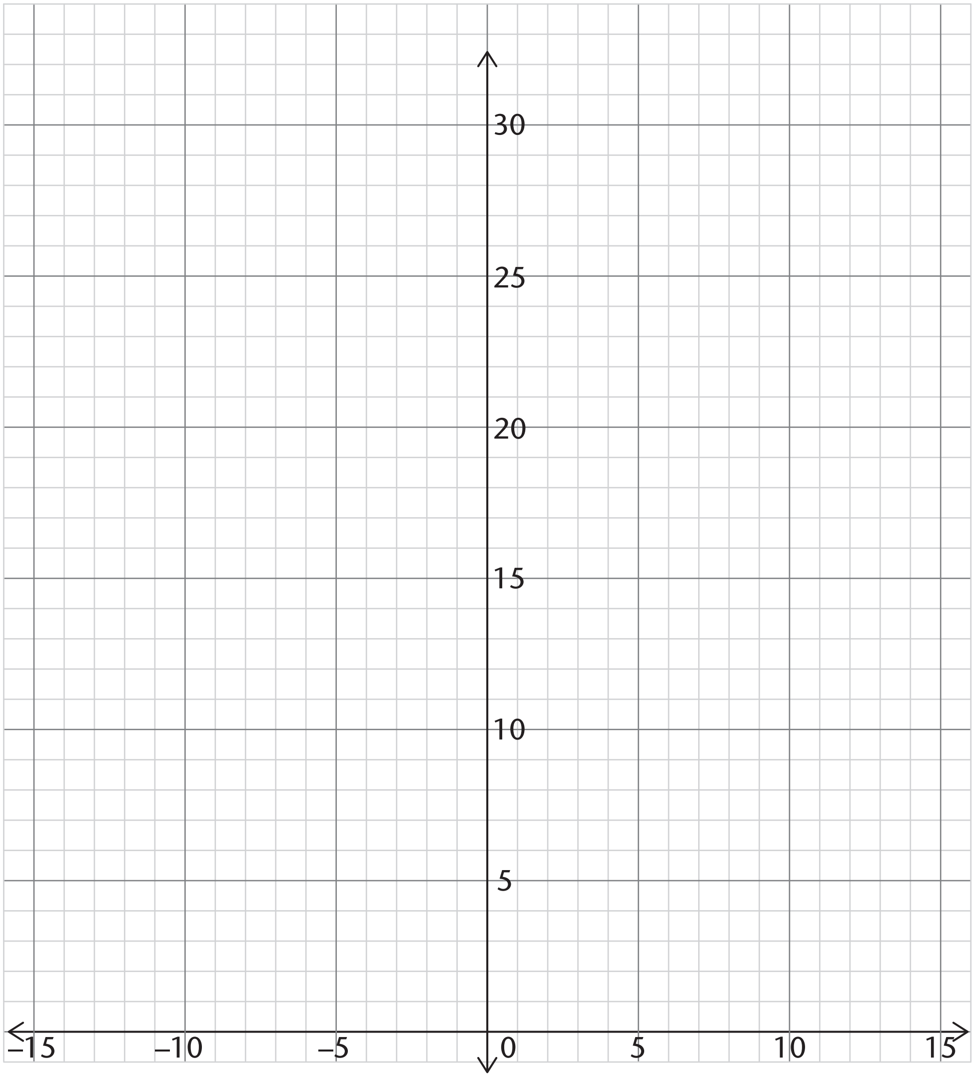
3. Do the two graphs show the same relationship, or different relationships between two variables?
4. How do the two graphs differ?
5. Use one of the graphs to find out how many yellow squares there will be, in an arrangement like those at the top of the page, with 12 blue squares.
6. Does the table below represent the same relationship as the table at the top of the page? Explain your answer.

Values of the independent variable	0	5	10	15	20	25
Values of the dependent variable	8	18	28	38	48	58

7. (a) Copy and complete the following table for the relationship described by $y = x^2$:

x	–5	–4	–3	–2	–1	0	1	2	3	4	5
y											

(b) On a graph sheet, copy the axis as below and represent the ordered number pairs in the table.



8. Copy and complete the table for the relationship $y = 15 + x$. Represent the ordered number pairs on the graph sheet you used in question 7(b) above.

x	–15	–10	–5	0	5	10	15
$15 + x$							

9. Copy and complete the table for the relationship $y = 15 – x$. Represent the ordered number pairs on the graph sheet you used in question 7(b) above.

x	–15	–10	–5	0	5	10	15
$15 – x$							

10. (a) The output values for $y = x^2$ and $y = 15 + x$ show patterns. Describe, in words, how the patterns differ. Use the words *increase* and *decrease* in your description.
(b) Describe, in words, how the graphs of $y = x^2$ and $y = 15 + x$ differ.

11. (a) Describe, in words, how the patterns in the output values for $y = 15 + x$ and $y = 15 - x$ differ. Use the words *increase* and *decrease* in your description.
(b) Describe, in words, how the graphs of $y = 15 + x$ and $y = 15 - x$ differ.

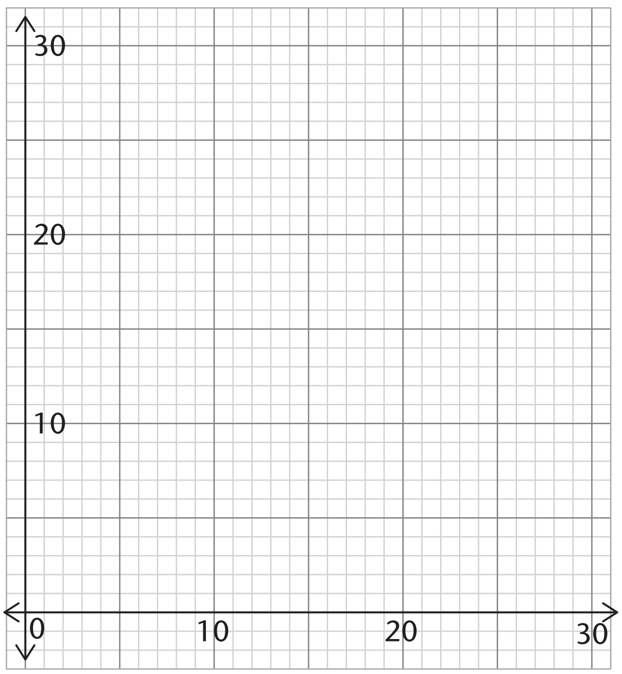
12. Copy and complete each of the following tables by extending the pattern in the output numbers. Also represent the relationship on graph sheets as the two shown below.

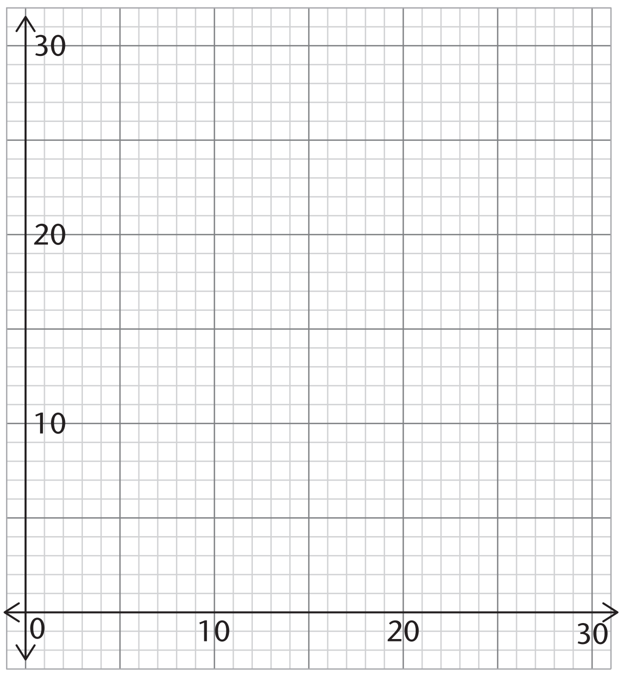
(a)

Input numbers	0	5	10	15	20	25	30
Output numbers	0	4	8	12			

(b)

Input numbers	0	5	10	15	20	25	30
Output numbers	0	2	4	6			





13. How do the patterns in 12(a) and (b) differ, and how do the graphs differ?

14. Each table on the following page shows some values for a relationship represented by one of these rules:

$y = -2x + 3$	$y = 2x - 5$	$y = -3x + 5$	$y = -3(x + 2)$
$y = 3x + 2$	$y = 5(x - 2)$	$y = 2x + 3$	$y = 2x + 5$
$y = -3x + 6$	$y = 5x + 10$	$y = 5x - 10$	$y = -x + 3$

(a) Copy and complete the tables by extending the patterns in the output values.
(b) For each table, describe what you did to produce more output values. Also write down the rule (formula) that corresponds to the table.

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A.

x	0	1	2	3	4	5	6	7
y	2	5	8					

B.

x	0	1	2	3	4	5	6	7
y	3	1	–1	–3				

C.

x	0	1	2	3	4	5	6	7
y	–10	–5	0	5				

D.

x	0	1	2	3	4	5	6	7
y	–5	–3	–1					

E.

x	0	1	2	3	4	5	6	7
y	6	3	0					

F.

x	0	1	2	3	4	5	6	7
y	3	2	1	0				

G.

x	0	1	2	3	4	5	6	7
y	3	5	7					

AN INVESTIGATION: PATTERNS IN DIFFERENCES

1. Copy and complete the tables for $y = x^2$, $z = x^2 + 1^2$, $w = x^2 + 2^2$ and $s = x^2 + 3^2$:

x	1	2	3	4	5	6	7	8	9	10
y										
z										
w										
s										

2. Copy and complete the tables for $y = x^2$, $p = (x + 1)^2$, $q = (x + 2)^2$ and $r = (x + 3)^2$:

(a)

x	1	2	3	4	5	6	7	8	9	10
p										
y										
$p - y$										

(b)

x	1	2	3	4	5	6	7	8	9	10
q										
y										
$q - y$										

(c)

x	1	2	3	4	5	6	7	8	9	10
r										
y										
$r - y$										

3. In each of the following cases, you should have different output values for the two relationships. If your output values are the same, find your mistakes and correct your work.
   (a) $z = x^2 + 1^2$ and $p = (x + 1)^2$
   (b) $w = x^2 + 2^2$ and $q = (x + 2)^2$
   (c) $s = x^2 + 3^2$ and $r = (x + 3)^2$

4. Copy and complete the tables, for $y = x^2$, $p = (x + 1)^2$, $q = (x + 2)^2$ and $r = (x + 3)^2$:

(a)

x	1	2	3	4	5	6	7	8	9	10
$p - y$										
$q - y$										
$r - y$										

(b)

x	10	11	12	13	14	15	16	17
$p - y$								
$q - y$								
$r - y$								

5. (a) Copy and complete the following table:

x	1	2	3	4	5	6	7	8	9	10
$2x + 1$										
$4x + 4$										
$6x + 9$										

   (b) What are the constant differences in the sequences of values of $2x + 1$, $4x + 4$ and $6x + 9$, for $x = 1;\ 2;\ 3;\ 4 \ldots\ldots$?
   (c) Do you have an idea whether or not the corresponding sequence for $12x + 36$ will also have a constant difference and what the constant difference may be?
   (d) There are certain patterns in the coefficients and constant terms in the expressions in the table above. Copy the table below and continue the patterns to make some more similar expressions for your expressions.

x	1	2	3	4	5	6	7	8	9	10

6. (a) If your answers for the tables in 4(a) and 5(a) are correct, they will be the same. Try to explain why they are the same.
   (b) What expressions, similar to those in question 5(a), may have the same values as $(x + 4)^2 - x^2$ and $(x + 5)^2 - x^2$, respectively?

Chapter 16
Algebraic expressions

16.1 Introduction

MANIPULATING EXPRESSIONS

The process of writing a polynomial as a product is called **factorisation**. This is the inverse of expansion.

$$x^2 + 5x + 6 \underset{\text{expansion}}{\overset{\text{factorisation}}{\rightleftarrows}} (x + 2)(x + 3).$$

A numerical or algebraic expression that requires multiplication as a last step, is called a **product**. For example, 12(37 + 63), $2x(x - 5)$ and xyz are called products. A product is a monomial.

Each part of a product is called a **factor** of the expression. If $c = ab$, then a and b are factors of c. $x + 2$ and $x + 3$ are the factors of $(x + 2)(x + 3)$. Since $x^2 + 5x + 6 = (x + 2)(x + 3)$, $x + 2$ and $x + 3$ are the factors of $x^2 + 5x + 6$.

1. Calculate the value of each of the following expressions for $x = 12$:

   (a) $\frac{(x+2)(x+5)}{x+2}$ (b) $\frac{(x-3)(x-4)}{x-4}$

   (c) $\frac{x(2x+1)}{2x+1}$ (d) $\frac{(x+5)(x-5)}{x-5}$

2. Check if the following statements are identities by expanding the expressions on the right.

   A statement like $2(x + 3) = 2x + 6$, which is true for all values of x you can think of, is called an **identity**.

   (a) $x^2 - 9 = (x + 3)(x - 3)$ (b) $x^2 + x - 6 = (x + 3)(x - 2)$

   (c) $x^2 + 4x + 3 = (x + 3)(x + 1)$ (d) $x^2 + 3x = x(x + 3)$

3. Write down the factors of each of the following expressions:

   (a) $x^2 + x - 6$ (b) $x^2 + 3x$

   (c) $x^2 + 4x + 3$ (d) $x^2 - 9$

4. Simplify the following quotients (algebraic fractions):

   (a) $\frac{x^2-9}{x+3}$ (b) $\frac{x^2+x-6}{x+3}$ (c) $\frac{x^2+x-6}{x-2}$ (d) $\frac{x^2+4x+3}{(x+3)(x+1)}$

5. (a) Suppose you have to find the value of the expression for $x = 15$.
Which expression will be the least amount of work: $\frac{x^2 - 9}{x + 3}$ or $x - 3$?
(b) Are you sure that you will get the same answers for the two expressions?

In the following sections you will learn how to factorise certain types of expressions. The following identities are useful for the purposes of factorisation:

$a(b + c) = ab + ac$ $\quad$ $(x + a)(x + b) = x^2 + (a + b)x + ab$ $\quad$ $(a + b)(a - b) = a^2 - b^2$

16.2 Factors of expressions of the form *ab* + *ac*

THE GREATEST COMMON FACTOR

1. (a) Is 5 a factor of 20?
(b) Is 5 a factor of 30?
(c) Is 5 a factor of 30 + 20?
(d) Is 5 a factor of 30 – 20?

2. (a) Is a a factor of ab?
(b) Is a a factor of ac?
(c) Is a a factor of $ab + ac$?
(d) Find another factor of $ab + ac$.
(e) Now try and simplify: $\frac{ab + ac}{a}$.

Suppose you have to factorise $4x^3 + 2x^2 - 6x$: It is clear that $2x$ is a factor of every term, hence it is a factor of $4x^3 + 2x^2 - 6x$.

By division we get $\frac{4x^3 + 2x^2 - 6x}{2x} = 2x^2 + x - 3$. Hence $4x^3 + 2x^2 - 6x = 2x(2x^2 + x - 3)$.

It is always a good idea to check factorisation by expanding the answer and making sure that the result is equal to the original expression.

3. Copy and complete the following table:

	$3x + 6y$	$4a^3 + 2a$	$5x - 2x^2$	$ax^2 - bx^3$	$12a^2b + 18ab^2$
(a) For each expression, find:					
the factors of the first term	3; x				
the factors of the second term	2; 3; y				
the greatest common factor of the two terms	3				
(b) Write the expression in factor form	$3(x + 2y)$				

4. Study the example and then factorise the expressions that follow.

$$(a - b)x + (b - a)y = (a - b)x - (a - b)y$$
$$= (a - b)(x - y)$$

Note that:
$b - a = -a + b = -(a - b)$

(a) $(a - b)x + a - b$
(b) $(a - b)x - a + b$
(c) $(a + b)^2 - (a + b)$
(d) $(a + b)x - a - b$
(e) $3x(2x - 3) - (3 - 2x)$
(f) $(y^2 - 4y) + (3y - 12)$

SOMETHING IN BETWEEN

1. By copying and completing the tables below you will learn something that will help you to find the factors of expressions of the form $x^2 + (b + c)x + bc$, for example: $x^2 + 17x + 30$.

b	1	2	3	5	–1	–2	–3	–5
c	30	15	10	6	–30	–15	–10	–6
$b + c$								
bc								

b	–1	–2	–3	–5	1	2	3	5
c	30	15	10	6	–30	–15	–10	–6
$b + c$								
bc								

2. For each case below, find two numbers x and y so that their product xy is 30 and their sum $x + y$ is the given number:
   (a) $xy = 30$ and $x + y = 13$
   (b) $xy = 30$ and $x + y = -17$
   (c) $xy = 30$ and $x + y = -11$
   (d) $xy = 30$ and $x + y = 11$

3. Find x and y in each case:
   (a) $xy = -30$ and $x + y = -13$
   (b) $xy = 30$ and $x + y = -13$
   (c) $xy = -30$ and $x + y = 13$
   (d) $xy = -30$ and $x + y = -1$
   (e) $xy = -30$ and $x + y = 1$

You may use the tables you completed in question 1 to find the answers to some of these questions.

4. Find x and y in each case:
   (a) $xy = 36$ and $x + y = 15$
   (b) $xy = 40$ and $x + y = 22$
   (c) $xy = 36$ and $x + y = 20$
   (d) $xy = -40$ and $x + y = 18$
   (e) $xy = 36$ and $x + y = -20$
   (f) $xy = -40$ and $x + y = -18$

5. Evaluate each expression for $x = 2$. Also expand each expression.
   (a) $(x + 5)(x - 2)$
   (b) $(x + 5)(x + 2)$
   (c) $(x - 5)(x - 2)$
   (d) $(x - 5)(x + 2)$

6. Evaluate each polynomial you formed in question 5 for $x = 2$. Compare the answers with the values of the corresponding product expressions in question 1. In cases where the values differ, you have made a mistake somewhere. Sort out any mistakes before you continue with question 7.

7. Expand each product:
   (a) $(x + 3)(x + 8)$
   (b) $(x + 2)(x + 12)$
   (c) $(x + 4)(x + 6)$
   (d) $(x + 1)(x + 24)$
   (e) $(x + 3)(x - 8)$
   (f) $(x + 2)(x - 12)$
   (g) $(x + 4)(x - 6)$
   (h) $(x + 1)(x - 24)$

16.3 Factors of expressions of the form $x^2 + (b + c)x + bc$

The expanded form of a product of two linear binomials, such as $(x + 3)(x + 8)$, or $(x + 3)(x - 8)$, is a **quadratic trinomial**, such as $x^2 + 11x + 24$ or $x^2 - 5x - 24$ with:

- a term in x^2
- a term in x that is called the **middle term**, which is $+11x$ in $x^2 + 11x + 24$ and $-5x$ in $x^2 - 5x - 24$
- a constant term also called the **last term**, which is $+24$ in $x^2 + 11x + 24$, and -24 in $x^2 - 5x - 24$.

To factorise an expression, such as $x^2 + 5x + 6$, means to reverse the process of expansion. This means that we have to find out which binomials will produce the trinomial when the product of the binomials is expanded, for example:

$$x^2 + 5x + 6 = (\mathbf{?} + \mathbf{?})(\mathbf{?} + \mathbf{?})$$

expansion $(x + 2)(x + 3)$ factorisation

$= x^2 + 5x + 6$

TRY TO FIND THE FACTORS

1. Write the following out and fill in the missing parts of the factors in each case:
   (a) $(x + 3)(x \ldots\ldots) = x^2 + 9x + 18$
   (b) $(x + 2)(x \ldots\ldots) = x^2 + 11x + 18$
   (c) $(x + 3)(x - \ldots\ldots) = x^2 + 9x - 18$
   (d) $(\ldots\ldots + \ldots\ldots)(x + 2) = x^2 + 5x + 6$
   (e) $x^2 - x - 6$

2. Expand each product:
   (a) $(x + p)(x + q)$
   (b) $(x + p)(x - q)$
   (c) $(x - p)(x + q)$
   (d) $(x - p)(x - q)$

The product of the first terms of the factors must be equal to the x^2 term of the trinomial.

$$x^2 + 5x + 6 = (x + 2)(x + 3)$$

Meaning: $x \times x = x^2$

Meaning: $2 \times 3 = 6$

The product of the last terms of the factors must be equal to the last term (the constant term) of the trinomial. The sum of the inner and outer products must be equal to the term in x (the middle term) of the trinomial.

$$x^2 + 5x + 6 = (x + 2)(x + 3)$$

Meaning: $2x + 3x = (2 + 3)x = 5x$

$(x + a)(x + b) = x \times x + ax + bx + a \times b = x^2 + (a + b)x + ab$

3. Try to factorise the following trinomials:
   (a) $x^2 + 8x + 12$
   (b) $x^2 - 8x + 12$

PRACTICE MAKES PERFECT

1. Factorise the following trinomials. (Remember to check your answer by expanding the factors to test if you do get the original expression.)

   (a) $a^2 + 9a + 14$
   (b) $x^2 + 3x - 18$
   (c) $x^2 - 18x + 17$
   (d) $y^2 + 17y + 30$
   (e) $y^2 - 13y - 30$
   (f) $y^2 + 7y - 30$
   (g) $x^2 + 2x - 15$
   (h) $m^2 + 4m - 21$
   (i) $x^2 - 6x + 9$
   (j) $b^2 + 15b + 56$
   (k) $a^2 - 2a - 63$
   (l) $a^2 - ab - 30b^2$
   (m) $x^2 - 5xy - 24y^2$
   (n) $x^2 - 13x + 40$

An alternative method

2. Study the example and then factorise the expressions on page 170.

Example: Factorise $ac + bc + bd + ad$

$ac + bc + bd + ad$	$= (ac + bc) + (bd + ad)$	(Order and group terms with common factors)
	$= c(a + b) + d(b + a)$	(Take out the common factor)
	$= (a + b)(c + d)$	(Write expression as a product)

(a) $px + py + qx + qy$

(b) $9x^3 - 27x^2 + x - 3$

(c) $4a + 4b + 3ap + 3bp$

(d) $a^4 + a^3 + 3a + 3$

(e) $xy + x + y + 1$

(f) $ac - ad - bc + bd$

Another method

Example 1:	**Example 2:**	**Action:**
$x^2 + 4x + 3$	$x^2 + 3x - 4$	
$= x^2 + x + 3x + 3$	$= x^2 - x + 4x - 4$	(Re-write middle term as sum of two terms)
$= (x^2 + x) + (3x + 3)$	$= (x^2 - x) + (4x - 4)$	(Group)
$= x(x + \mathbf{1}) + 3(x + \mathbf{1})$	$= x(x - \mathbf{1}) + 4(x - \mathbf{1})$	(Take out the GCF of each group)
$= (x + \mathbf{1})(x + 3)$	$= (x - \mathbf{1})(x + 4)$	(Write it as a product)

3. Factorise:

   (a) $x^2 + 7x + 12$

   (b) $x^2 - 7x + 12$

The challenge is to re-write the middle term as the sum of two terms in a way that you are able to take out the common factor.

16.4 Factors of expressions of the form $a^2 - b^2$

PRELIMINARY WORK

1. Copy and complete the following table and see if you can notice a pattern (rule) whereby you can predict the answers to the first column's calculations without squaring it:

(a)	$3^2 - 2^2$	$3 + 2$	$3 - 2$	$(3 + 2)(3 - 2)$
(b)	$4^2 - 3^2$	$4 + 3$	$4 - 3$	$(4 + 3)(4 - 3)$
(c)	$6^2 - 4^2$	$6 + 4$	$6 - 4$	$(6 + 4)(6 - 4)$
(d)	$9^2 - 3^2$	$9 + 3$	$9 - 3$	$(9 + 3)(9 - 3)$

2. Do you notice a pattern (rule) whereby you can predict the answers to such calculations?
3. Now predict the answers to each of the following without squaring. Check your answers where necessary. Does the rule that you discovered in question 2 also hold for the following cases?
   (a) $17^2 - 13^2$ (b) $54^2 - 46^2$ (c) $28^2 - 22^2$
4. Formulate your rule in symbols:
   $a^2 - b^2 = \ldots$
5. Can you explain why factors of $a^2 - b^2$ have this form?

Stated differently: If p and q are perfect squares, also "algebraic squares", then:

$$p - q = (\sqrt{p} + \sqrt{q})(\sqrt{p} - \sqrt{q})$$

$$9x^4 - 4y^2 = (\sqrt{9x^4} + \sqrt{4y^2})(\sqrt{9x^4} - \sqrt{4y^2})$$

$$= (3x^2 + 2y)(3x^2 - 2y)$$

(Note the operations within the brackets differ.)

An expression of the form $a^2 - b^2$ is called the **difference between two squares**.
To factorise a difference between squares, we use the identity: $a^2 - b^2 = (a + b)(a - b)$ where a and b represent numbers or algebraic expressions.

FACTORISING DIFFERENCE BETWEEN TWO SQUARES EXPRESSIONS

1. Use the skills you learnt in the previous exercises to factorise the following:
   (a) $4a^2 - b^2$ (b) $m^2 - 9n^2$
   (c) $25x^2 - 36y^2$ (d) $121x^2 - 144y^2$
   (e) $16p^2 - 49q^2$ (f) $64a^2 - 25b^2c^2$
   (g) $x^2 - 4$ (h) $16x^2 - 36y^2$

Always factorise completely.
Always take out the greatest common factor if there is one.
One is a perfect square: $1 = 1^2$ and $1^m = 1$.
The exponential law: $a^m.a^n = a^{m+n}$.

2. Factorise:
   (a) $x^4 - 1$ (b) $16a^4 - 81$
   (c) $1 - a^2b^2c^2$ (d) $25x^{10} - 49y^8$
   (e) $2x^2 - 18$ (f) $200 - 2b^2$
   (g) $3xy^2 - 48xa^2$ (h) $5a^4 - 20b^2$

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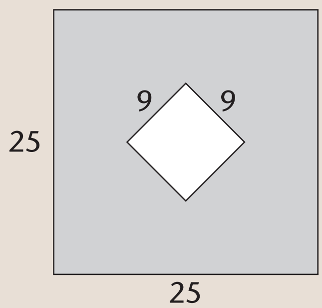
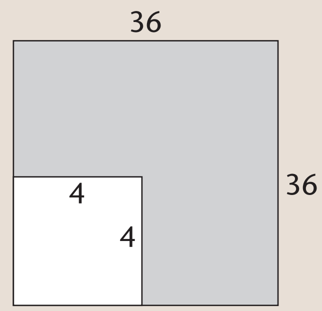
FACTORISATION CAN MAKE CALCULATION EASY

In each case, calculate the area of the shaded part. Use the shortest possible method.

(a)



(b)



THIS IS HOW FACTORISATION CAN MAKE CALCULATION EASY!

16.5 Simplification of algebraic fractions

WORKING WITH ALGEBRAIC FRACTIONS

Liza and Madodo have to determine the value of $\frac{x^2 - 2x - 3}{x - 3}$ for $x = 4{,}6$.

Liza's solution:	Madoda's solution:
$\frac{x^2 - 2x - 3}{x - 3}$ $= \frac{(4{,}6)^2 - 2(4{,}6) - 3}{4{,}6 - 3}$ (Substitute $x = 4{,}6$) $= \frac{21{,}16 - 9{,}2 - 3}{4{,}6 - 3}$ $= \frac{8{,}96}{1{,}6}$ $= 5{,}6$	$\frac{x^2 - 2x - 3}{x - 3}$ $= \frac{(x - 3)(x + 1)}{x - 3}$ (Factorise the numerator) $= x + 1$ (Simplify the expression) $= 4{,}6 + 1$ (Substitute $x = 4{,}6$) $= 5{,}6$

1. Which solution do you prefer? Why?

It is useful to manipulate quotient expressions, such as $\frac{x^2 + 5x + 6}{x + 2}$, into simpler but equivalent sum expressions, like $x + 3$ in this case. It makes substitution and the solving of equations easier.

2. Solve the following problems:
   (a) Evaluate $\frac{x^2 + 5x + 6}{x + 2}$ if $x = 23$.
   (b) Solve $\frac{x^2 + 5x + 6}{x + 2} = 19$.
3. Determine the value of each of the expressions on page 173 if $x = 36$. See if you can use the shortest possible method.

(a) $\frac{x^2 - 9}{x + 3}$

(b) $\frac{x^2 + x - 6}{x + 3}$

HOW IS IT POSSIBLE THAT 2 = 1?

What went wrong in the following argument?

Let:		$a = b$	(If: $b \neq 0$)
$\times\, a$:	$\Leftrightarrow$	$a^2 = ab$	
$-\, b^2$:	$\Leftrightarrow$	$a^2 - b^2 = ab - b^2$	
Factorise:	$\Leftrightarrow$	$(a + b)(a - b) = b(a - b)$	
$\div\, (a - b)$:	$\Leftrightarrow$	$a + b = b$	
But $a = b$:	$\Leftrightarrow$	$b + b = b$	
Add terms:	$\Leftrightarrow$	$2b = b$	
$\div\, b$:	$\Leftrightarrow$	$2 = 1$	

Explain what went wrong and why it is wrong?

DIVIDING BY ZERO CANNOT BE DONE

1. Copy and complete the following table by evaluating the value of the expression $\frac{x + 2}{x - 2}$ for the x-values given in the top row.

x	-2	0	2	4
$\frac{x + 2}{x - 2}$				

2. If $x = 2$, then $\frac{x + 2}{x - 2}$ will have the value $\frac{4}{0}$. What is the value of $\frac{4}{0}$?
3. One way to determine the value of $\frac{4}{0}$, is to set it as $\frac{4}{0} = a$. Then $4 = 0 \times a$. Which values of a will make this statement true?
4. What is the result of the calculation of $4 \div 0$ on your calculator? Can you explain the message on your calculator?

Division by 0 is not possible. The algebraic fraction $\frac{x + 2}{x - 2}$ cannot have a value if the denominator $(x - 2)$ is equal to 0. We may say the expression $\frac{x + 2}{x - 2}$ is **undefined** for $x - 2 = 0$, i.e. for $x = 2$. We also say $x = 2$ is an **excluded value** of x for $\frac{x + 2}{x - 2}$.

DEFINING THE UNDEFINED

1. Are the following statements true? If not, correct the statement.
   (a) $\frac{x}{x} = 1$ for all values of x.

(b) $\frac{x^3}{x^2} = x$ for all values of x.

(c) $\frac{x-3}{x-3} = 1$ for all values of x.

(d) $\frac{x^2+x}{x(x+1)} = 1$ for all values of x.

2. For which values of the variables will each expression be undefined?

(a) $\frac{7(y+5)}{y+2}$

(b) $\frac{3x+2}{x+4}$

(c) $\frac{2x+1}{x^2-1}$

(d) $\frac{2x^2-1}{(x-2)(x+3)}$

SIMPLIFYING ALGEBRAIC FRACTIONS

To simplify an algebraic fraction that contains a polynomial as a numerator or denominator, the polynomial should be factorised first.

To prevent division by zero, the excluded values must be stated.

1. Simplify each of the following algebraic fractions by factorising the numerator and then using the property $\frac{ax}{a} = x$ if $a \neq 0$. Give the excluded values.

(a) $\frac{3xy+y^2}{3x+y}$

(b) $\frac{a^2b+ab^2}{a+b}$

(c) $\frac{3x^2y-6x^2y^2}{3xy}$

(d) $\frac{10x^4+15x^3}{5x^2}$

2. Simplify each of the following algebraic fractions by factorising the numerator and then using the property $\frac{ax}{a} = x$ if $a \neq 0$. (See if you can factorise the trinomials.)

(a) $\frac{x^2+5x+6}{x+2}$

(b) $\frac{x^2+2x-8}{x-2}$

(c) $\frac{x^2-5x-50}{x+5}$

(d) $\frac{x^2-16x+15}{x-15}$

3. Simplify each of the following algebraic fractions by factorising the numerator and then using the property $\frac{ax}{a} = x$ if $a \neq 0$:

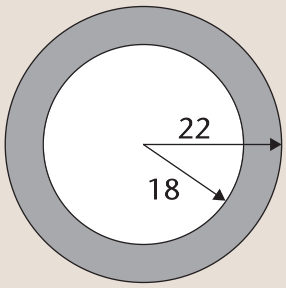
(a) $\frac{x^2-4}{x-2}$

(b) $\frac{4x^2-1}{2x+1}$

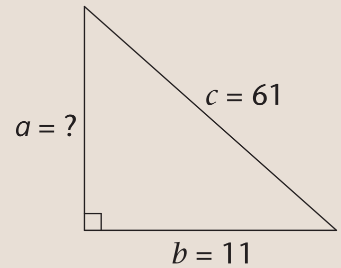
FACTORISATION CAN REDUCE CALCULATIONS

In each case, use the shortest possible method to get to your answer.

(a) Calculate the shaded area.
(Area = πr^2 and use $\pi = 3.142$)



(b) Calculate the length of side a.
(Pythagoras: $c^2 = a^2 + b^2$)



THIS IS HOW FACTORISATION CAN SAVE YOU TIME!

MORE PRACTICE

1. Factorise the following expressions completely:

(a) $4a + 6b$
(b) $x^2 + 8x + 7$
(c) $c^2 - 9$
(d) $y^2 - 8y + 15$
(e) $-3ab + b$
(f) $-3a(b - 1) + (b - 1)$
(g) $dfg^2 + d^2g - df^2g$
(h) $x^2 + 6x + 8$
(i) $a^2 + 5a + 6$
(j) $x^2 - 8x - 20$
(k) $x^5y^3 - x^3y^5$
(l) $x^3y - xy^3$
(m) $4 - 4y + y^2$
(n) $3a^2 + 18a - 21$
(o) $6a^2 - 54$
(p) $-a^2 - 11a - 30$
(q) $2a^2 + 10a - 72$
(r) $5x^3 - 15x^2 - 200x$
(s) $(x + 2)^2 - y^2$
(t) $(x + y)^2 - a^2$
(u) $(a^2 - 2a + 1) - b^2$
(v) $1 - (a^2 - 2ab + b^2)$
(w) $(a - b)x + (b - a)y$
(x) $a(2x - y) + (y - 2x)$
(y) $2x^2y^{10} - 8x^{10}y^2$
(z) $(a + b)^3 - 4(a + b)$
(aa) $(a + b)^2 - a - b$
(ab) $(x + y)(a - b) + (-x - y)(b - a)$

2. Simplify each of the following algebraic fractions as far as possible:

(a) $\dfrac{16 - 9x^2}{4 + 3x}$
(b) $\dfrac{25x^2 - 36}{5x^2 + 6x}$
(c) $\dfrac{x^3 + x^2 - 30x}{x + 6}$
(d) $\dfrac{2x^2 + 5x + 3}{2x + 3}$
(e) $\dfrac{ab + bc}{abc}$
(f) $\dfrac{pa + pb}{a + b}$

Chapter 17
Equations

17.1 Introduction

SOLUTION BY INSPECTION

1. Copy and complete the following table. Substitute the given x-values into the equation until you find the value that makes the equation true.

You can read the solutions of an equation from a table.

	Equation	LHS if $x = 4$	Is LHS = RHS ?	LHS if $x = 5$	Is LHS = RHS ?	LHS if $x = 6$	Is LHS = RHS ?	Correct solution
(a)	$3x - 4 = 11$							$x =$
(b)	$2x + 7 = 19$							$x =$
(c)	$13 - 5x = -7$							$x =$

(LHS = left-hand side and RHS = right-hand side)

2. In the following table, you are given equations with their solutions. Copy the table and insert **+** or **–** or **=** signs between each term to make the equations true for the solution given.

The "searching" for the solution of an equation is referred to as solving the equation by **inspection**.

	Equation	Solution
(a)	$2x \quad 7 = 15$	$x = 4$
(b)	$3 \quad 2x = 11$	$x = -4$
(c)	$-x \quad 7 = 3$	$x = 4$
(d)	$28 \quad 5x = 3$	$x = 5$

Statements like $21 - x = 2x + 3$ and $(x - 3)(x - 5) = 0$, which are true for only some values of x, are called **equations**.

A statement like $2(x + 3) = 2x + 6$, which is true for all values of x you can think of, is called an **identity**.

A statement like $2(x + 3) = 2x + 3$, where there are no values of x for which it is true, is called an **impossibility**.

SOLVING EQUATIONS THROUGH INVERSE OPERATIONS

In this section you are going to explore a different way of solving equations.

1. Complete the following calculations:
   (a) $3 - 3$ (b) $-9\,765 + 9\,765$
   (c) $-a + a$ (d) $13a - 13a$
2. What do you notice?
3. Complete the following calculations:
   (a) $3 \div 3$ (b) $3 \times \frac{1}{3}$
   (c) $\frac{1}{x} \times x$ (d) $\frac{x}{3} \times \frac{3}{x}$
4. What do you notice?

We can start with a solution as an equation and then apply some operations to it to turn it into an equivalent but more complicated equation.

Two equations are **equivalent** if they have the same solution.

Building an equation

Action on both sides	Equivalent equations
Solution (1)	$x = 3$
$\times 3$	$3x = 9$
$+ 2$	$3x + 2 = 11$
Solution (2)	$x = -9$
$\times 2$	$2x = -18$
$+ 6$	$2x + 6 = -12$
$+ x$	$3x + 6 = x - 12$
factorise	$3(x + 2) = x - 12$

Solving an equation

Action on both sides	Equivalent equations
Equation (1)	$3x + 2 = 11$
$- 2$	$3x = 9$
$\div 3$	$x = 3$
Equation (2)	$3(x + 2) = x - 12$
remove brackets	$3x + 6 = x - 12$
$- x$	$2x + 6 = -12$
$- 6$	$2x = -18$
$\div 2$	$x = -9$

Building an equation		Solving an equation	
Action on both sides	Equivalent equations	Action on both sides	Equivalent equations
Solution (3)	$x = 1$	Equation (3)	$\frac{(x+3)}{2} = 1 + x$
$\times -1$	$-x = -1$	$\times 2$	$x + 3 = 2 + 2x$
$+ 3$	$-x + 3 = 2$	$-2x$	$-x + 3 = 2$
$+ 2x$	$+x + 3 = 2 + 2x$	-3	$-x = -1$
$\div 2$	$\frac{(x+3)}{2} = 1 + x$	$\div -1$	$x = 1$

Try making up your own equations and then solving them. Did you get the "solution" with which you started?

When you solve an equation, you actually reverse the making of the equation.

5. Solve for x:
   (a) $2(x + 4) + 9 = 15$
   (b) $5(x - 2) = 7(2 - x)$
   (c) $\frac{2x}{3} - 2 = 12$
   (d) $\frac{3y-3}{2} + \frac{5}{2} = \frac{5y}{3}$

Up to now you have only dealt with equations of the **first degree**. That means they contained only *first powers* of the unknown (x), for example $3x - 2 = 5x + 7$. In the following sections you will solve equations of the **second degree**, where the expressions contain second powers. This is an equation of the second degree:

$x^2 + 1 = x + 13$.

When the expression part of the equation is written as the product of a monomial and a binomial (e.g. $x(x - 2) = 0$); or the product of two binomials (e.g. $(x - 2)(x + 3) = 0$), the result is also an equation of the second degree.

17.2 Solving by factorisation (Part 1)

DEVELOPING A STRATEGY: MULTIPLYING BY ZERO

1. Can you find two numbers x and y so that if you multiply them the answer is 0, i.e. $xy = 0$?

Each part of a product is called a **factor** of the expression.
If $c = ab$, then a and b are factors of c.
If $x^2 + 5x + 6 = (x + 2)(x + 3)$, then $x + 2$ and $x + 3$ are factors of $x^2 + 5x + 6$.

2. Copy and complete the following table:

	Equation	Factors	Product	First possible solution	Second possible solution
Example	$x(x-2)=0$	x and $(x-2)$	0	$x=0$	$x-2=0$ $x=2$
(a)	$x(x+5)=0$				
(b)	$2x(3x-12)=0$				
(c)	$0=(x+2)(x-2)$				

You can rewrite an equation so that it is in the form *expression* = 0; for example you can write $x^2-2x=3x+6$ as $x^2-5x-6=0$.

You can factorise x^2-5x-6 and then use the zero-product property to solve the equation, as shown below:

$x^2-5x-6=0$
$(x-6)(x+1)=0$
$x=6$ or $x=-1$

Zero-product property
If: $a \times b=0$
Then: $a=0$ or
$b=0$ or
$a=0$ and $b=0$

In a later section you will solve equations like the above example. You have to write the equation in the form, *expression* = 0, then factorise the left-hand side and then use the zero-product property.

TAKING OUT THE HIGHEST COMMON FACTOR

The process of writing a sum expression (polynomial) as a product (monomial) is called **factorisation**. This is the inverse of **expansion**.

Look at the expression $2x^2-6x$.

$2x$ is a factor of both terms, therefore it is a factor of $2x^2-6x$.

By division we get $\frac{2x^2-6x}{2x}=x-3$.

Hence $2x^2-6x=2x(x-3)$.

It is unnecessary to write out the division step of this method. After finding the common factor, we write down the product form directly:
$2x^2-6x=2x(x-3)$

Determine the values of x which will make the following statements true:

1. $x^2 = -3x$
2. $x^2 + 2x^2 = 6x$
3. $\frac{6x}{3} + x = -4x^2$
4. $x = x(2 - x)$

17.3 Solving by factorisation (Part 2)

SOLVING BY FACTORISING TRINOMIALS

The product of the first terms of the factors must be equal to the x^2 term of the trinomial. The product of the last terms of the factors must be equal to the last term (the constant term) of the trinomial.

$$x^2 + 5x + 6 \quad = \quad (x + 2)(x + 3)$$

Meaning: $x.x = x^2$

Meaning: $2.3 = 6$

The sum of the inner and outer products must be equal to the x term of the trinomial.

$$x^2 + 5x + 6 \quad = \quad (x + 2)(x + 3)$$

Meaning: $(2 + 3)x = 5x$

The factors are of the form: $(x.x) + (a + b)x + (a.b) = (x + a)(x + b)$.

Determine the values of x which will make the following statements true.
Remember to write the equation in the form *expression* = 0 so that you can use the zero-product property.

1. $x^2 + 9x = -14$
2. $x^2 + 3x = 18$
3. $x^2 - 18x = -17$
4. $x^2 + 30 = 11x$
5. $x^2 = 13x + 30$
6. $x^2 + 7x = 30$

SOLVING BY FACTORISING THE DIFFERENCE BETWEEN TWO SQUARES

Remember from the previous chapter:
If p and q are perfect squares, also "algebraic squares", then:

$$p - q = (\sqrt{p} + \sqrt{q})(\sqrt{p} - \sqrt{q})$$

$$9x^4 - 4y^2 = (\sqrt{9x^4} + \sqrt{4y^2})(\sqrt{9x^4} - \sqrt{4y^2})$$
$$= (3x^2 + 2y)(3x^2 - 2y)$$

An expression of the form $a^2 - b^2$ is called the **difference between two squares**.
To factorise a difference between squares, we use the identity: $a^2 - b^2 = (a + b)(a - b)$ where a and b represent numbers or algebraic expressions.

Determine the values of the unknown (x or a or n, etc.) which will make the statements on the next page true.

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Remember to write the equation in the form *expression* = 0 so that you can use the zero-product property.

1. $x^2 = 4$
2. $x^2 = 16$
3. $4a^2 = 9$
4. $81 = 9n^2$
5. $25x^2 = 36$
6. $121x^2 = 144$
7. $16p^2 = 49$
8. $64a^2 = 25$

17.4 Solving by factorisation (Part 3)

SOLVING BY USING PROPERTIES OF EXPONENTS

1. Write the following numbers as the product of their prime factors:
   (a) 128
   (b) 243
   (c) 125
   (d) 2 401

All numbers can be written as the product of their prime factors:
16 $= 4 \times 4 = 2 \times 2 \times 2 \times 2 =$ $\mathbf{2^4}$
Factorise the number until all the factors are prime numbers.

2. Determine the values of *x* which will make the following statements true:
   (a) $2^x = 2^7$
   (b) $3^x = 3^5$
   (c) $5^x = 5^3$
   (d) $7^x = 7^4$

If the base of the LHS is the same as the base of the RHS, then the exponent on the LHS must be equal to the exponent on the RHS.
If $a^x = a^y$, then $x = y$.

3. Determine the values of *x* which will make the following statements true:
   (a) $2^x = 128$
   (b) $3^x = 243$
   (c) $5^x = 125$
   (d) $7^x = 2\ 401$
   (e) $2^x + 9 = 25$
   (f) $27(3^x) = 3$

In the equation $2^x = 16$, the letter symbol (x) is the exponent. Equations with the letter symbol as an exponent are referred to as **exponential equations**.

MIXED EXERCISES FOR MORE PRACTICE

Determine the values of the unknown (*x* or *m* or *b*, etc.) which will make the following statements true:

1. $\frac{6x}{3} + x = -4x^2$
2. $x = x(2 - x)$
3. $x^2 + 2x = 15$
4. $m^2 + 4m = 21$
5. $x^2 + 3 = 4x$
6. $b^2 - 16b = -15$

7. $1 = a^2$

8. $25x^2 = 49$

9. $2^x - 25 = -9$

10. $81(3^x) = 3$

17.5 Set up equations to solve problems

THE MATHEMATICAL MODELLING PROCESS

Consider this problem involving a practical situation:

Printing shop A charges 45c per page and R12 for binding a book.

Printing shop B charges 35c per page and R15 for binding a book.

For a book with the same amount of pages, will the two shops charge the same?

You can write an equation to describe the problem.

Let the number of pages for which the work costs the same be x. Then:

$45x + 1\,200 = 35x + 1\,500$.

The equation represents a mathematical problem that can be solved without necessarily keeping the practical situation in mind. It is called a **mathematical model** of the practical situation.

Now solve the equation.

$$\begin{aligned} 45x + 1\,200 &= 35x + 1\,500 \\ 45x - 35x &= 1\,500 - 1\,200 \\ 10x &= 300 \\ x &= 30 \end{aligned}$$

We describe this as **analysing** the mathematical model to produce a **mathematical solution**.

We may now ask what the solution to the mathematical problem ("$x = 30$") means in terms of the practical situation. When the equation was set up above, the symbol x was used as a placeholder for the number of pages in a book for which the two shops would charge the same. So, what does the solution tell you?

The mathematical solution may be interpreted to establish what it means in the practical situation.

Now check to see if the two shops will charge the same for a book with 30 pages.

At shop A, 30 pages will cost 30 × 45c = 1 350c = R13,50. Binding is R12; total cost is R25,50.

At shop B, 30 pages will cost 30 × 35c = 1 050c = R10,50. Binding is R15; total cost is R25,50.

The solution to the mathematical problem is also a solution to the practical problem.

The mathematical solution should be tested in the practical situation, because mistakes may have been made.

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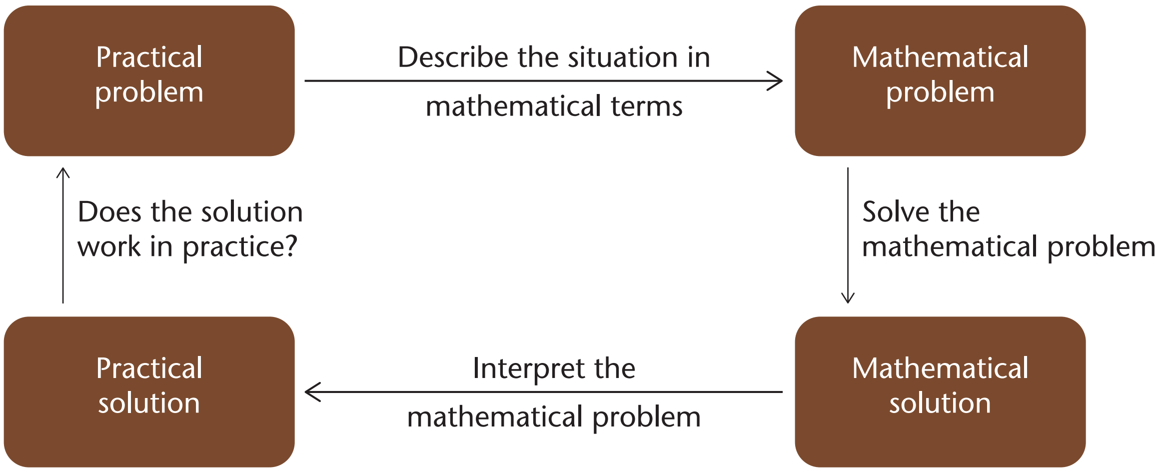


When people work like this, we say they do **mathematical modelling**.

PRACTISE YOUR MODELLING SKILLS

For each situation in questions 1 to 3, the mathematical model is outlined and some clues are provided. Fill in the missing information.

1. Louis is six years older than Karin and Karin is four years older than Heidi. The sum of their ages is 53 years. How old is Heidi?

Model:	Let x be:	Heidi's age
	Then:	Karin's age will be
	And:	
	Hence:	... = 53
Analysis:	$x + (x + 4) + (x + 10) = 53$	
Interpretation:	So Heidi is:	

2. The sum of two numbers is 15. Three times the smaller number is 5 more than the larger number. Calculate the two numbers. (**Hint:** Let the smaller number be x.)

Model:	Let x be:	
	Then:	is the larger number
	Hence:	
Analysis:		
Interpretation:	So the smaller number is:	
	And the larger number is:	

3. The sum of three consecutive even numbers is 108. What are the numbers?
   Hint: Consecutive numbers are numbers that follow on from each other.
   We define an even number as a number of the form $2n$ where n is a counting number.

   Model: Let the first number be:

   Then:

   Hence:

   Analysis:

   Interpretation: So the first number is:

   The second number is:

   And the third number is:

4. Firm A calculates the cost of a job using the formula: Cost = $500 + 30t$, where t is the number of days it takes to complete the job.
   Firm B calculates the cost of the same job using the formula: Cost = $260 + 48t$, where t is the number of days needed to complete the job.
   (a) What would Firm A charge for a job that takes ten days?
   (b) How long would Firm B take to complete a job for which their charge is R596?
   (c) Here is a specific job for which firms charge the same and take the same time to complete. How long does this job take?

17.6 Equations and ordered pairs

WHEN UNKNOWNS BECOME VARIABLES

In the previous sections we dealt with equations which had fixed or limited solutions. They only had one letter symbol, which in this case acted as a placeholder for the value/s which will make the statement true.

Study the equation: $y = 5x + 2$.

1. How many letter symbols does the equation have? (List them.)
2. Is it possible to solve this "equation"?
3. Copy and complete the following table:

x	12	10	20	5	6	–5	–10
$5x + 2$							

FUNCTIONS AS SETS OF ORDERED PAIRS

A specific input number, for example 10, and the output number associated with it (52 in the case of the function described by $y = 5x + 2$), is called an **ordered pair**. Ordered pairs can be represented in a table like the one you completed in question 3 on the previous page.

Ordered pairs can also be written in brackets: (input number; output number).

For example, the ordered pairs you entered into the table in question 3 can be written as: (12; 62), (10; 52), (20; 102), (5; 27), (6; 32), (–5; –23), (–10; –48)

In the function indicated by $y = 5x + 2$, the letter symbol in the formula (x) represents the **input** or **independent** variable while the other letter symbol (y) represents the **output** or **dependent** variable.

If there is precisely one value of y for each value of x, we say that y is a **function** of x.

1. Copy and complete each table by writing the ordered pairs in brackets below the table, in the table, as shown in the example. Then choose two more input numbers and write down two additional ordered pairs that belong to each given function.

For the function with the rule $y = 4x + 5$:

x	–2	0	1	2	5	10	20
y	–3	5	9	13	25	45	85

(–2; –3), (0; 5), (1; 9), (2; 13), (5; 25), and (10; 45) and (20; 85)

(a) For the function with the rule $y = x^2 + 9$:

x	5		0	–3	
y		18			34

(5;34), (3; 18), (0; 9), (–3; 18), (–5; 34), and (. . ; . . .), and (. . ; . . .)

(b) For the function with the rule $y = 3x - 2$:

x	5	1	0	–3	
y					–17

(5; 13), (1; 1), (0; –2), (–3; –11), (–5; –17), and (. . . ; . . .) and (. . ; . . .)

(c) For the function with the rule $y = 5x - 4$:

x	–5	–3	1	2	
y					21

(–5; –29), (–3; –19), (1; 1), (2; 6), (5; 21), and (. . . ; . . .) and (. . ; . . .)

(d) For the function with the rule $y = 12 - 3x$:

x	1	2	3	4	
y					–3

(1; 9), (2; 6), (3; 3), (4; 0), (5; –3), and (. . .; . . .) and (. .; . . .)

(e) For the function with the rule $y = x^2 + 2$:

x	–12	–7	–2	3	
y					102

(–12; 146), (–7; 51), (–2; 6), (3; 11), (10; 102), and (. . .; . . .) and (. .; . . .)

(f) For the function with the rule $y = 2^x + 2$:

x	0	1	2	3	
y					18

(0; 3), (1; 4), (2; 6), (3; 10), (4; 18) and (. . .; . . .) and (. .; . . .)

2. (a) Which ordered pair belongs to both $y = 3x - 2$ and $y = 5x - 4$?
   (b) Which ordered pair belongs to both $y = 12 - 3x$ and $y = 5x - 4$?

3. Which ordered pair belongs to both $y = 5x + 7$ and $y = 3x + 25$?

CHAPTER 18
Graphs

18.1 Global graphs

DISCRETE AND CONTINUOUS VARIABLES

Sibongile collects honey on his farm and puts it in large jars to sell. His business is doing so well that he can no longer do all the work himself. He needs to get some help. Sibongile knows that one person can normally fill two jars in three days. He sets up this table to help him determine how many full-time workers he should employ to fill different numbers of jars in a five-day week.

Jars per week	$3\frac{1}{3}$	$6\frac{2}{3}$	10	$13\frac{1}{3}$	$16\frac{2}{3}$	20	$23\frac{1}{3}$
Workers	1	2	3	4	5	6	7

1. (a) If Sibongile needs to produce 40 jars a week, how many workers does he need?
   (b) How many jars can nine workers fill in a week?
   (c) How many workers does Sibongile need to produce 15 jars per week?
   (d) What are the two variables in the above situation?

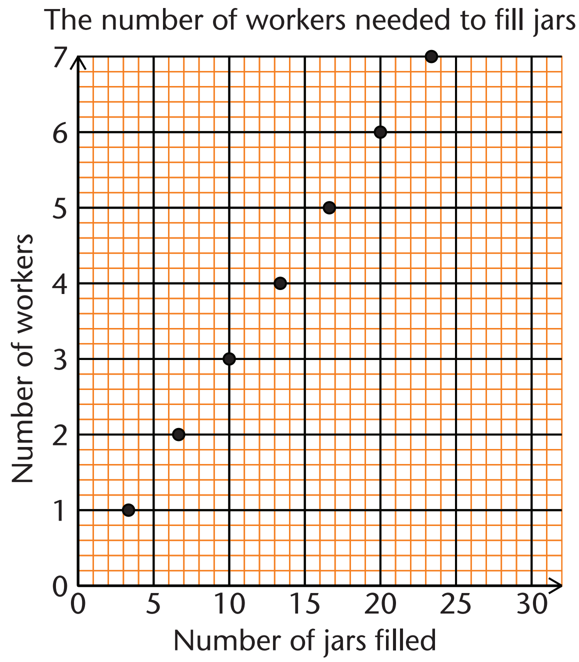
In a situation like the one above, one can have any number of jars, as well as fractions of a jar. One can have a whole number of jars (for example, four jars) or a fractional quantity of jars (for example, $6\frac{2}{3}$ or 4,45 jars). The other variable in the above situation, the number of full-time employees, is different. Only whole numbers of people are possible.

Quantities like the quantity of jars of honey, which can include any fraction, are sometimes called "continuous quantities" or "continuous variables". Quantities that can be counted, like a number of people or a number of motor cars or rivers or towns, are sometimes called "discrete quantities" or "discrete variables".

When a graph of a discrete variable is drawn, it does not normally make sense to join the dots with a line, but for some purposes it may be useful.

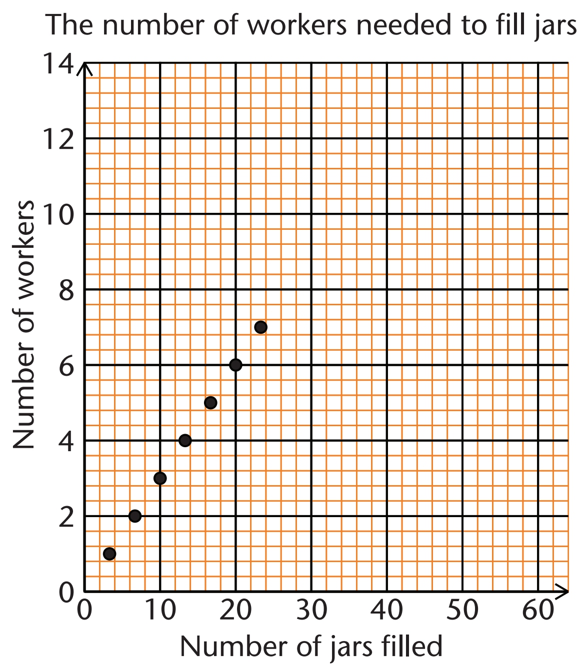
2. Can you use the second graph on the next page to find out how many workers are needed to fill 30 jars in a week, and how many to fill 40 jars? Check your answers by doing calculations.

Here is a graph with the information in Sibongile's table:



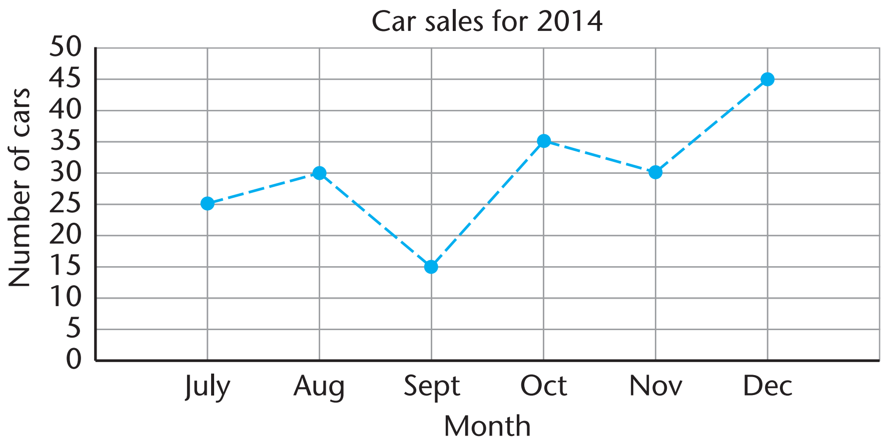
Here is another graph with the same information:

3. In what way are these two graphs different?



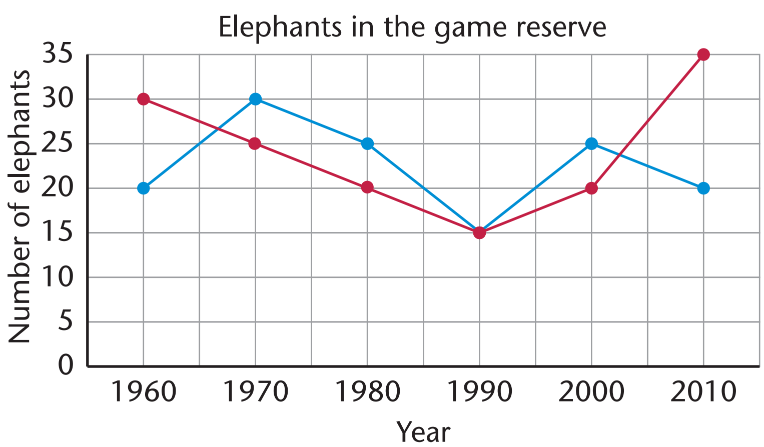
4. In each case, say whether the variables are "discrete" or "continuous".
   (a) You order pizzas for a class party and you need one pizza for every three learners.
   (b) Your height measured at different stages as you grew up.
   (c) The speed the car is travelling as you drive to town.

5. The following line graph shows the number of cars that a company sold between July and December of 2014:



(a) Is the data shown in the graph discrete or continuous? Explain your answer.
(b) How many cars were sold in August?
(c) During which months were the maximum and minimum number of cars sold?
(d) How many more cars were sold in November than in July?
(e) During which months did the car sales decrease?
(f) Would you say that the car sales generally improved over the six months? Explain your answer.

6. The graph below shows the population of elephants at a game reserve in South Africa between 1960 and 2010. Study the graph and answer the questions that follow.



(a) Did the elephant population increase or decrease between 1970 and 1990?
(b) Between which years did the elephant population increase?

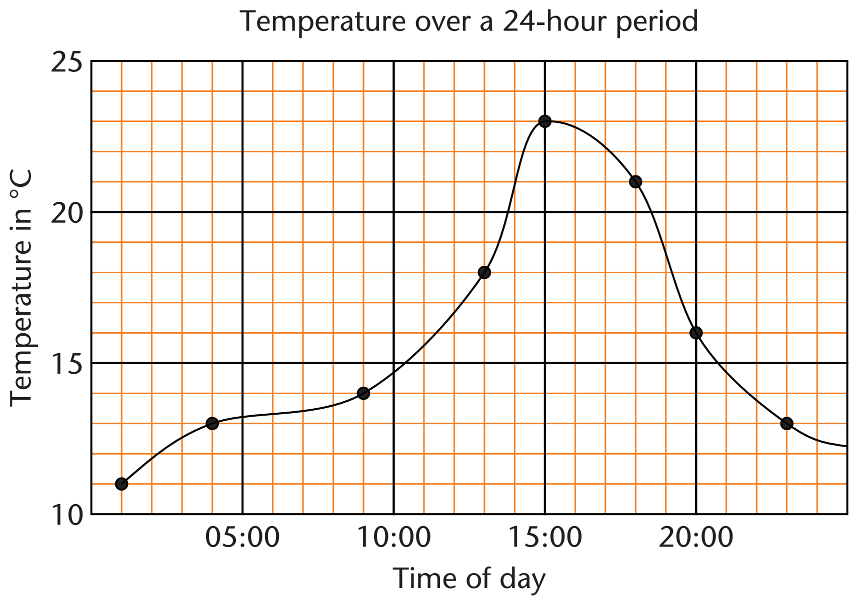
(c) In which year were there the most elephants on the game farm?
(d) Is the data in this graph discrete or continuous?
(e) How many elephants do you think there were on the game reserve in 1995?
(f) The following data shows the number of elephants at a different game reserve. Copy the graph from page 189 on grid paper and plot this information on the grid:

Year	1960	1970	1980	1990	2000	2010
Elephants	30	25	20	15	20	35

(g) Would you say that the second game reserve had more elephants than the first game reserve between 1960 and 2010? Explain your answer.

SHOWING INCREASE AND DECREASE ON GRAPHS

The graph below shows the temperature over a 24-hour period in a town in the Free State. The graph was drawn by connecting the points that show actual temperature readings.



1. (a) Do you think the above temperatures were recorded on a summer day or a winter day?
   (b) At what time of the day was the highest temperature recorded, and what was this temperature?
   (c) During what part of the day did the temperature rise, and during what part did the temperature drop?
   (d) During what part of the period when the temperature was rising, did it rise most rapidly?
   (e) During what part of the day did the temperature drop most rapidly?

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2. Here are descriptions of the temperature changes on five different days:

Day A: It is already warm early in the morning. The temperature does not change much during the day but late in the afternoon a breeze causes the temperature to drop quite sharply.

Day B: It is very cold early in the morning but it gets quite hot soon after the sun rises. By midday a cold wind comes up and the temperature drops until late in the afternoon. The wind then stops and it gets warmer again into the evening.

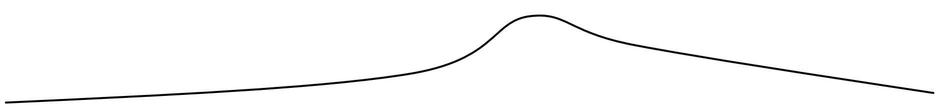
Day C: It is warm in the early morning and the temperature remains about the same until midday. Then the temperature drops slowly during the afternoon.

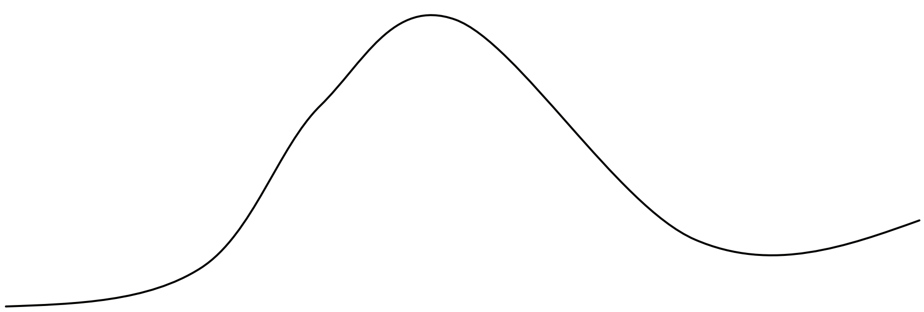
Day D: It is cold in the early morning and it remains cold for the whole day, except for a short time after lunch when the sun comes out for a while.

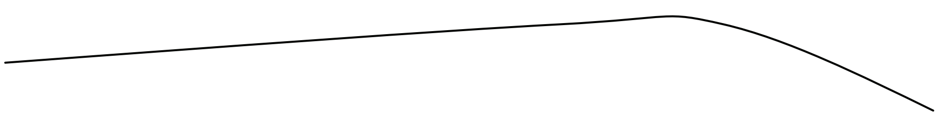
Day E: It is warm early in the morning, but the temperature drops sharply soon after sunrise and remains low until mid-afternoon, when it slowly warms up a little.

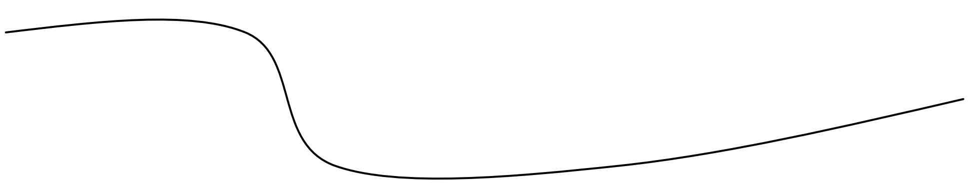
The shapes of some temperature graphs for 24-hour periods, starting early in the morning, are given below. Write which of the above days is possibly represented by the graph.

(a) 

(b) 

(c) 

(d) 

(e) 

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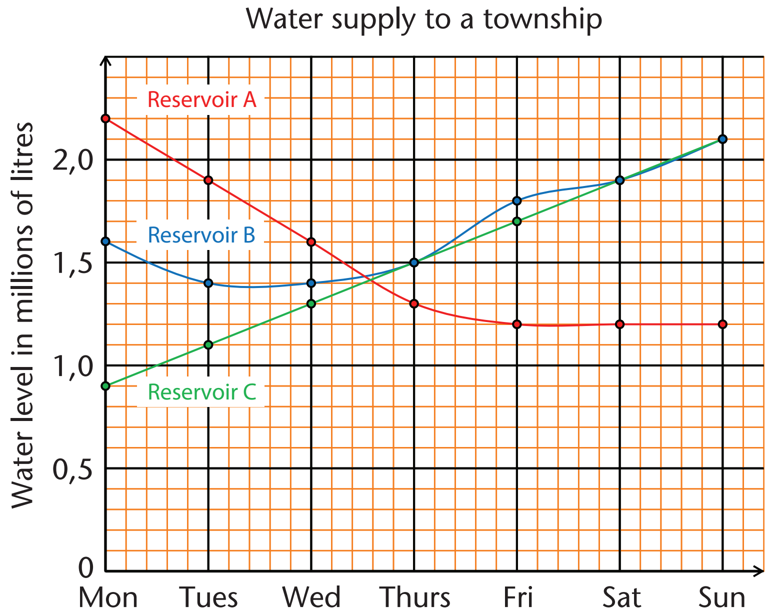
Water is supplied to a township from three reservoirs. The amount of water in each reservoir is measured each day at 08:00. The water level in reservoir A is represented in red on the graph below, and the water levels in reservoirs B and C are represented in blue and green, respectively.



The daily water levels in the three reservoirs, in millions of litres, are also given in the following table:

	Mon.	Tue.	Wed.	Thu.	Fri.	Sat.	Sun.
Reservoir A	2,2	1,9	1,6	1,3	1,2	1,2	1,2
Reservoir B	1,6	1,4	1,4	1,5	1,8	1,9	2,1
Reservoir C	0,9	1,1	1,3	1,5	1,7	1,9	2,1

3. You may use the graph or the table, or both, to find the answers to the questions below.
   (a) On which days does the water level in reservoir B increase from one day to the next?
   (b) On which of these days does the water level in reservoir B increase most, and by how much does it increase from that day to the next?
   (c) By how much does the water level in reservoir B change each day?
   (d) By how much does the water level in reservoir C change each day?
   (e) Describe the water level situation from Friday to Sunday, in reservoir A.

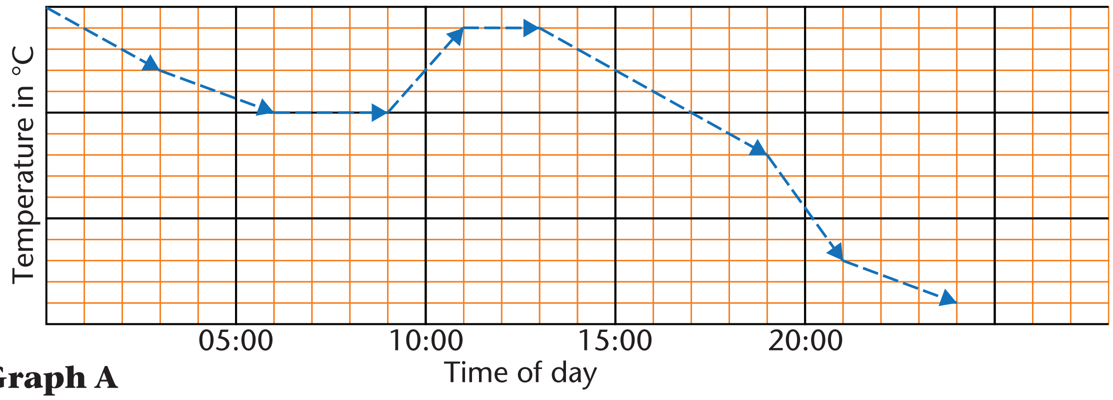
4. During a certain day, these changes occur in the temperature at a certain place:
   Between 00:00 and 03:00, the temperature drops by 2 °C.
   Between 03:00 and 06:00, the temperature drops by 3 °C.
   Between 06:00 and 10:00, the temperature remains constant.
   Between 10:00 and 12:00, the temperature rises by 3 °C.
   Between 12:00 and 16:00, the temperature remains constant.
   Between 16:00 and 18:00, the temperature drops by 4 °C.

Between 18:00 and 22:00, the temperature drops by 5 °C.

Between 22:00 and 24:00, the temperature remains constant.

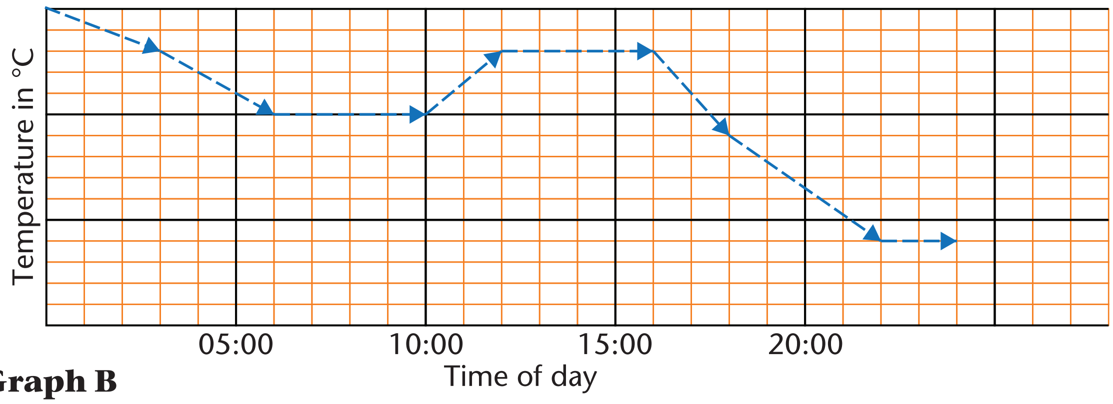
Which of the graphs below show the above temperature changes?

Temperature changes during a certain day



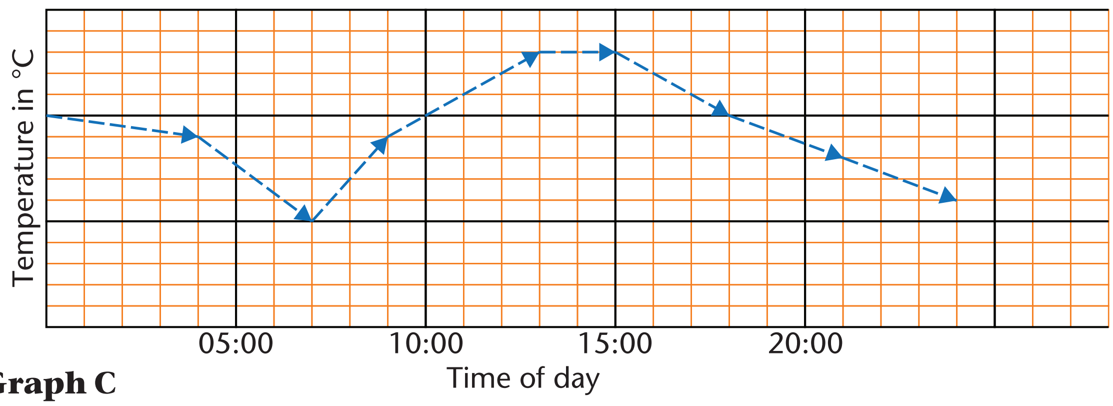
Graph A

Temperature changes during a certain day



Graph B

Temperature changes during a certain day



Graph C

5. Write a verbal description, like in question 4, of the temperature changes shown in graph A in question 4, by copying and completing the following descriptions:

   Between .... and ...., the temperature ....

   Between .... and ...., the temperature ....

Between .... and ...., the temperature ....

Between .... and ...., the temperature ....

Between .... and ...., the temperature ....

Between .... and ...., the temperature ....

Between .... and ...., the temperature ....

Between .... and ...., the temperature ....

6. Write a verbal description, like in question 4, of the temperature changes shown in graph C in question 4, by copying and completing the following descriptions:

   Between .... and ...., the temperature ....

   Between .... and ...., the temperature ....

   Between .... and ...., the temperature ....

   Between .... and ...., the temperature ....

   Between .... and ...., the temperature ....

   Between .... and ...., the temperature ....

   Between .... and ...., the temperature ....

   Between .... and ...., the temperature ....

7. Look at graph A in question 4.
   (a) By how much does the temperature drop from 13:00 to 19:00?
   (b) By how much does the temperature drop from 19:00 to 21:00?
   (c) When does the temperature drop most rapidly, from 13:00 to 19:00 or from 19:00 to 21:00? Explain your answer.

8. Look at graph C in question 4.
   (a) By how much does the temperature increase from 07:00 to 09:00?
   (b) By how much does the temperature increase from 09:00 to 13:00?
   (c) When does the temperature increase more rapidly, from 07:00 to 09:00 or from 09:00 to 13:00? Explain your answer.

9. Look at graph B in question 4.
   (a) By how much does the temperature drop from 16:00 to 18:00?
   (b) By how much does the temperature drop from 18:00 to 22:00?
   (c) When does the temperature drop more rapidly, from 16:00 to 18:00 or from 18:00 to 22:00? Explain your answer.

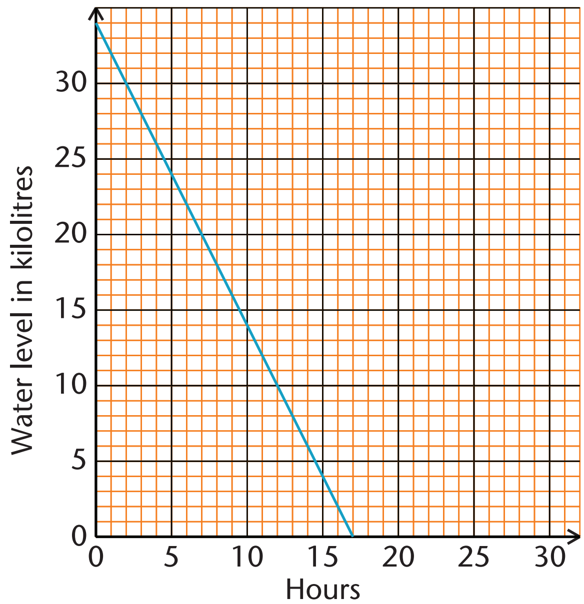
18.2 Change at different rates

The water levels in kilolitres (kl) in different water storage tanks over a period of 30 hours are represented on the graphs below and on the next page.

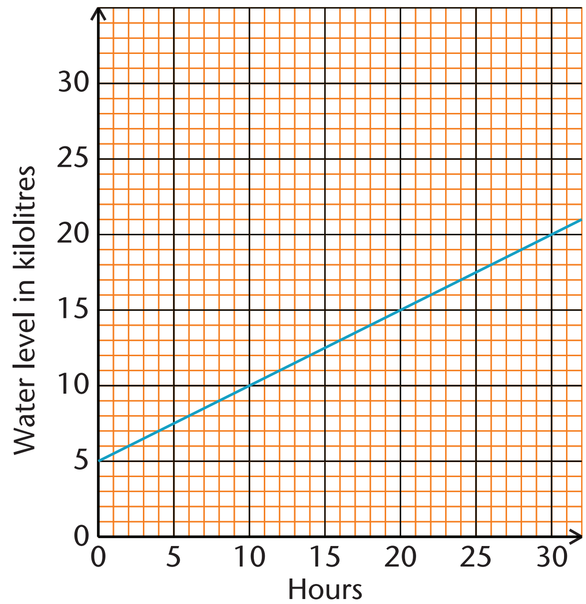
1 kilolitre = 1 000 litres

1. (a) In which tanks does the water level rise during the 30-hour period?
   (b) In which tanks does the water level drop during the 30-hour period?
2. How much water is there at the start of the 30-hour period, in each of the tanks?
3. (a) Which tank is losing water most rapidly? Explain your answer.
   (b) Which tank is gaining water most slowly? Explain your answer.

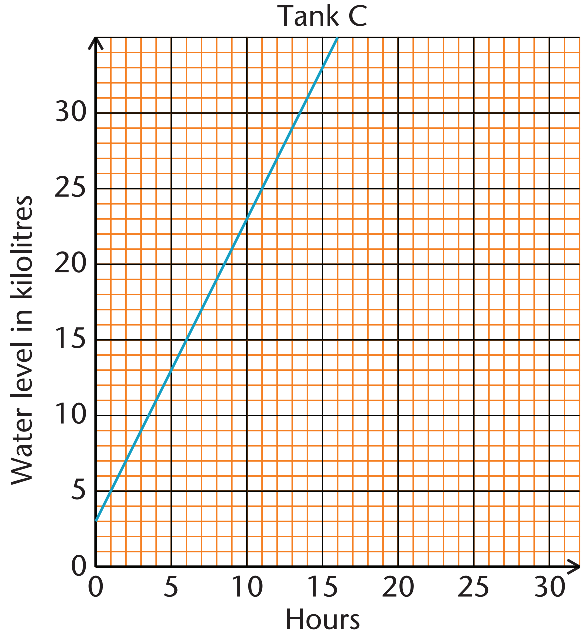
Tank A



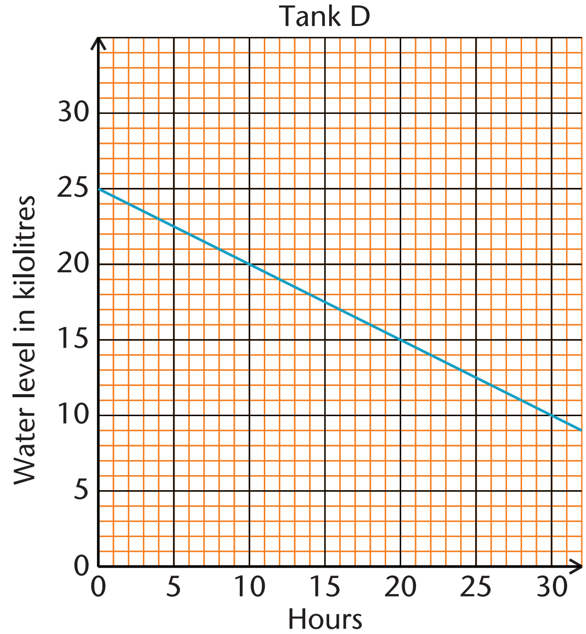
Tank B

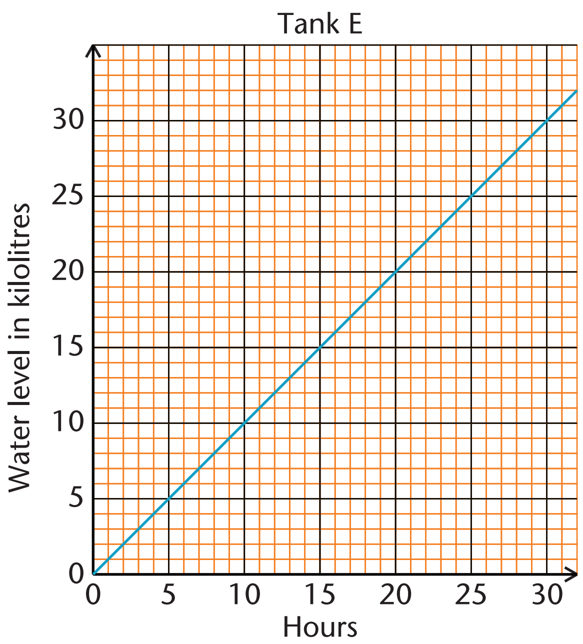


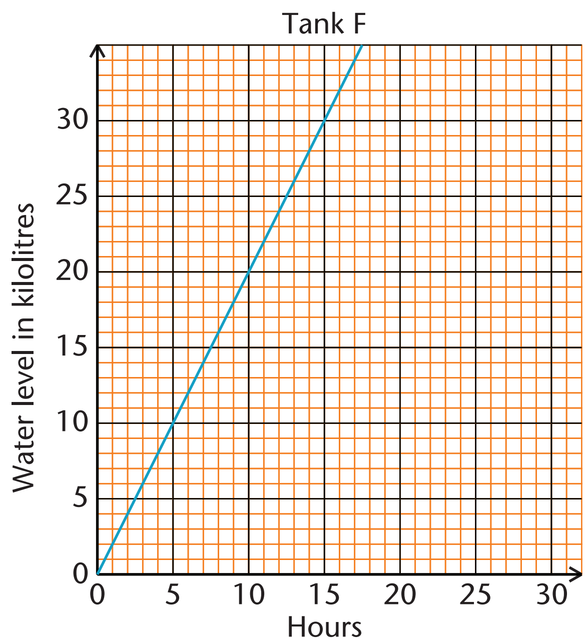
Tank C



Tank D







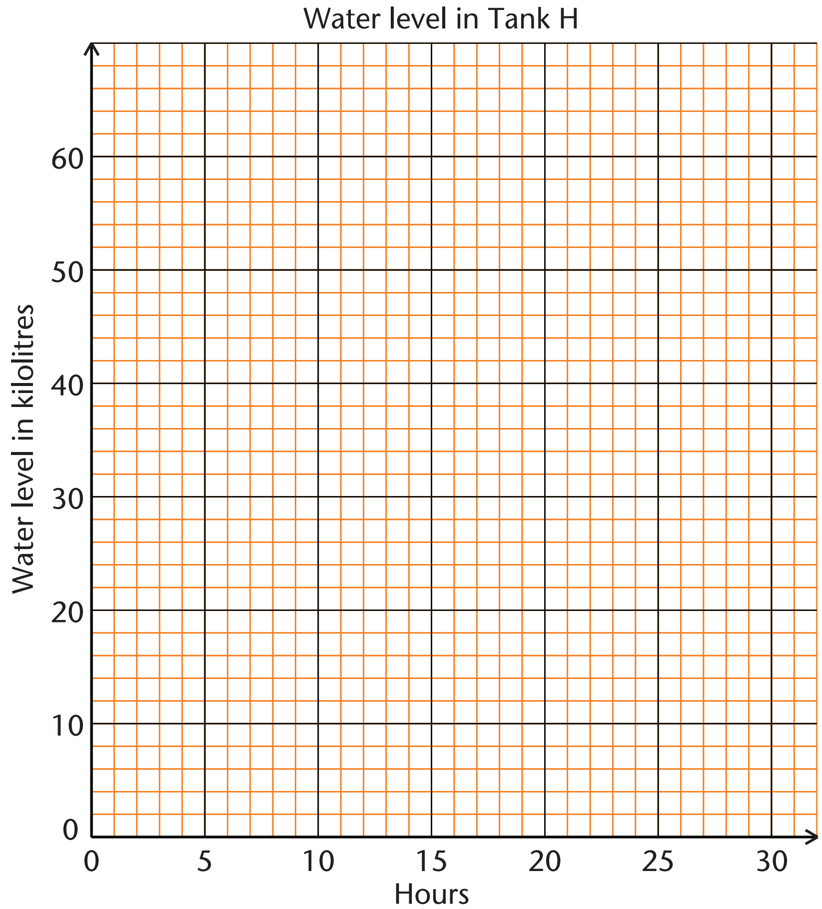
4. Copy and complete the table. Use negative numbers for decreases.

	Change over each hour	Change over any period of five hours
Tank A		
Tank B		
Tank C		
Tank D		
Tank E		
Tank F		

If a constant stream of water is pumped into a tank so that the water level is increased by 3 kilolitres in each hour, we say:

Water is pumped into the tank at a **constant rate** of **3 kilolitres per hour**.

5. (a) Tank G contains 12 kilolitres at the beginning of a 30-hour period. Water is then pumped into it at a constant rate of 3 kilolitres per hour. On graph paper, draw a dotted line graph to show the water level in Tank G.
   (b) Tank H also contains 12 kilolitres at the beginning of a 30-hour period. Water is then pumped into it at a constant rate of 1,5 kilolitres per hour. Return to the graph you drew for question 5(a) and draw a solid line graph to show the water level in Tank H.



6. Copy and complete the table for Tanks G and H over the 30-hour period:

Hours	0	5	10	15	20	25	30
Kilolitres in Tank G	12						
Kilolitres in Tank H	12						

18.3 Draw graphs from tables of ordered pairs

A "coordinate" graph shows the relationship between two variables; the dependent and independent variable in a function. The value of the dependent variable depends on the value given to the independent variable, hence its name. Sometimes there is no pattern to the relationship between the two variables and sometimes there is. In Grade 9 we will focus on graphs where there is a pattern to the relationship. Specifically, we will focus on graphs of linear functions. The graph of a linear function is a straight line.

GRAPHS OF FUNCTIONS WITH CONSTANT DIFFERENCES

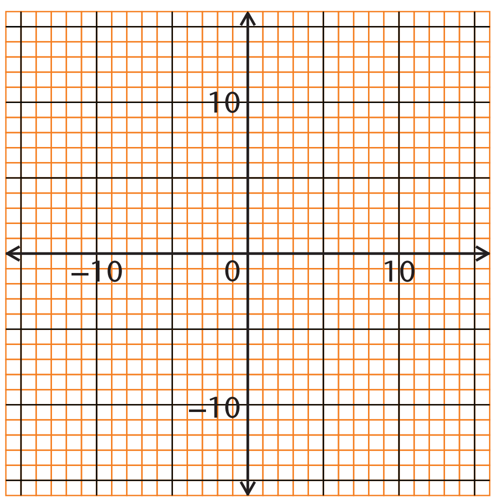
1. Copy and complete the following table:

x	0	1	2	3	4	5	6	7	8	9
Function A	8	$8\frac{1}{2}$	9	$9\frac{1}{2}$						
Function B	4	5	6	7	8	9	10	11	12	13
Function C	0	$1\frac{1}{2}$	3	$4\frac{1}{2}$						
Function D	–4	–2	0	2						

2. Represent each of the functions in question 1 with a graph (like the ones shown below), by plotting the points on grid paper. You may join the points in each case and write down the constant difference between the function values.

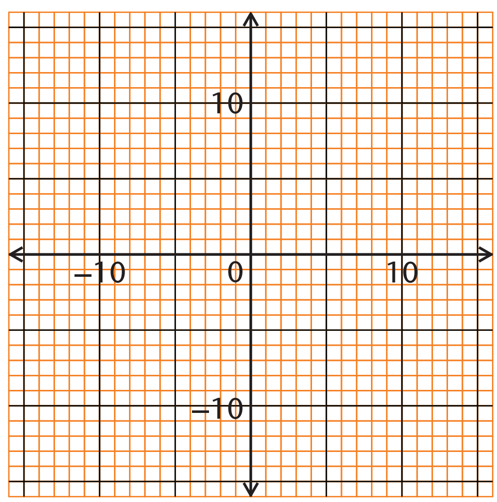
Function A

Constant difference =



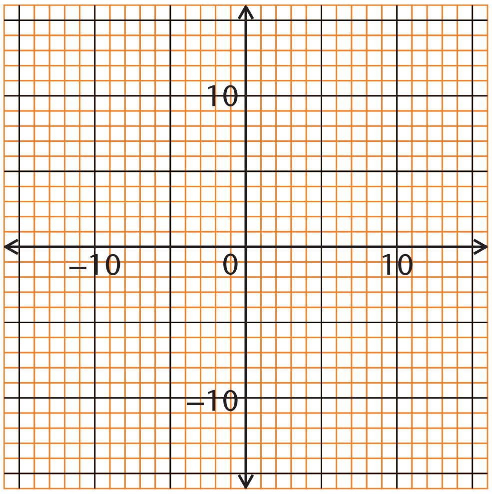
Function B

Constant difference =



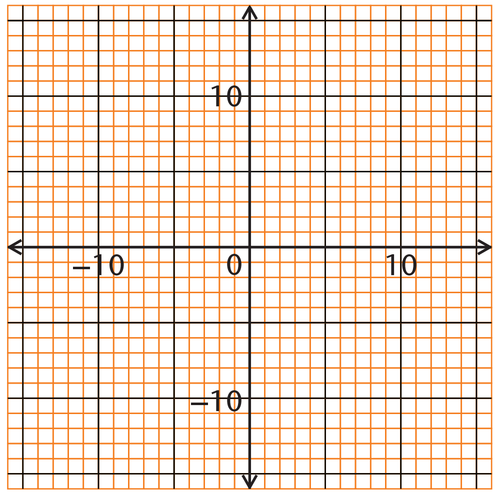
Function C

Constant difference =



Function D

Constant difference =



3. Some of the graphs you have drawn "go upwards" (or downwards) quickly, like a steep hill or mountain; others "go up" (or down) slowly.
   (a) Is there a link between the constant difference and the "steepness" of the graph?
   (b) Try to explain why this is the case.

4. (a) Copy and complete the following table:

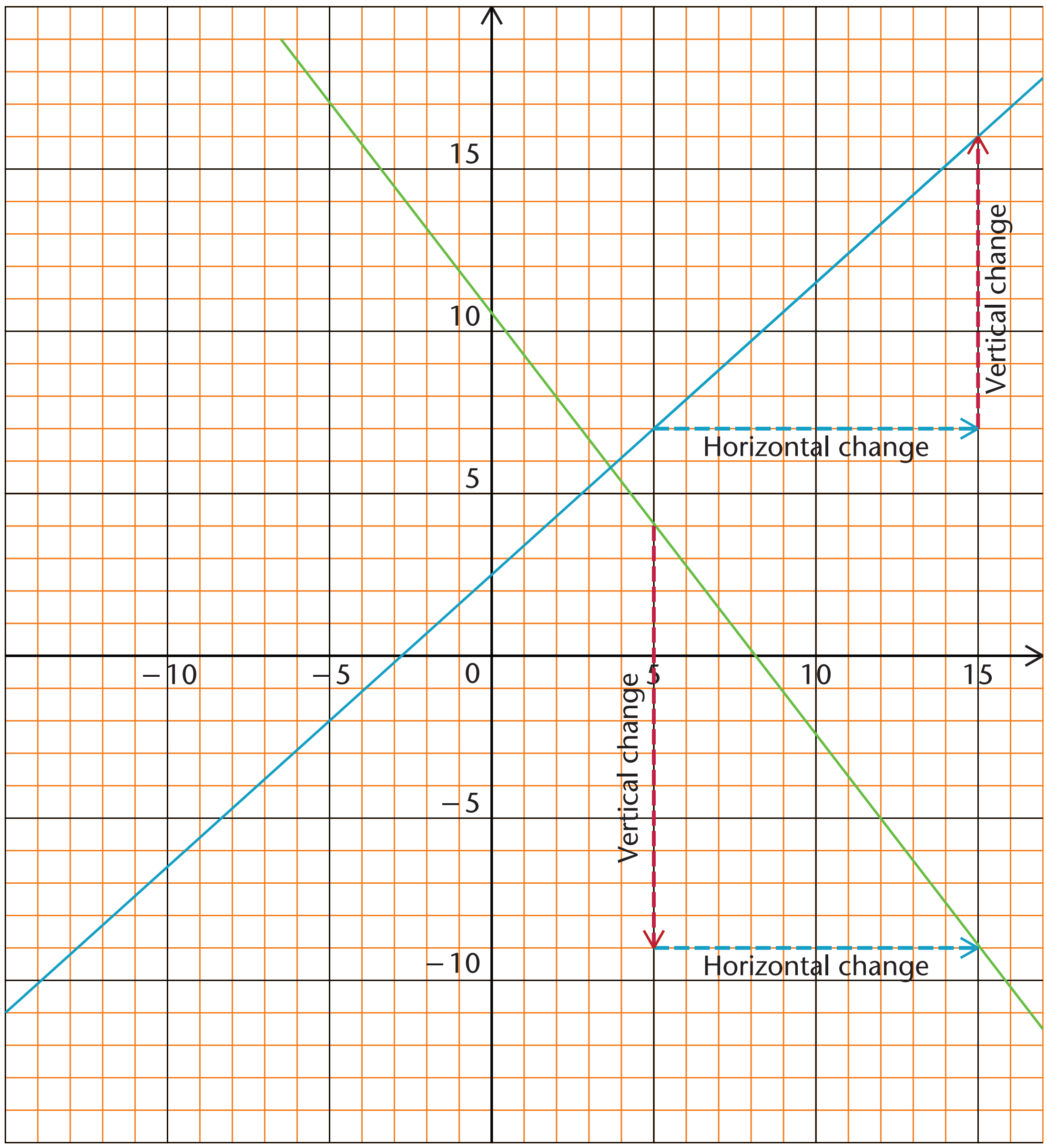
x	1	2	3	4	5	6	7	8	9	10
$2x + 3$										
$5x + 4$										
$3x + 3$										

   (b) Determine the difference between consecutive terms in each of the above three number sequences. What do you notice about this difference?
   (c) What difference between consecutive terms would you expect in the output numbers for $4x + 5$, if the input numbers are the natural numbers 1; 2; 3; . . . . .?

18.4 Gradient

The "steepness" or **slope** of a line can be indicated by a number, as described below. This number is called the **gradient** of the line.

> The gradient is the vertical change divided by the horizontal change as you move from left to right on the line.
>
> $$\text{Gradient} = \frac{\text{vertical change}}{\text{horizontal change}}$$



The gradient of the blue line above is $\frac{9}{10} = 0{,}9$.

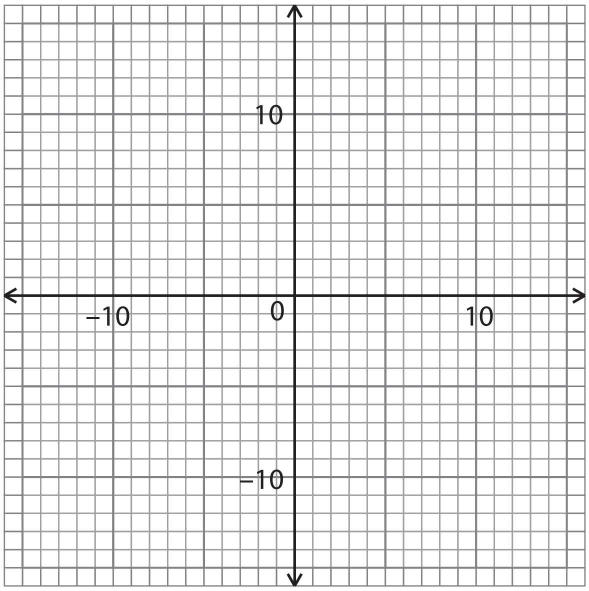
The gradient of the green line is $\frac{-13}{10} = -1{,}3$.

Note that the horizontal change is always taken to be positive (moving to the right), but the vertical change can be positive (if it is upwards) or negative (if it is downwards).

1. A certain line passes through the points (2; 3) and (8; 15). A straight line is drawn through the two points.

(a) Try to think of a way in which you can work out the gradient of the line that passes through the two points.

(b) On a graph sheet, plot the two points shown on the graph sheet below.

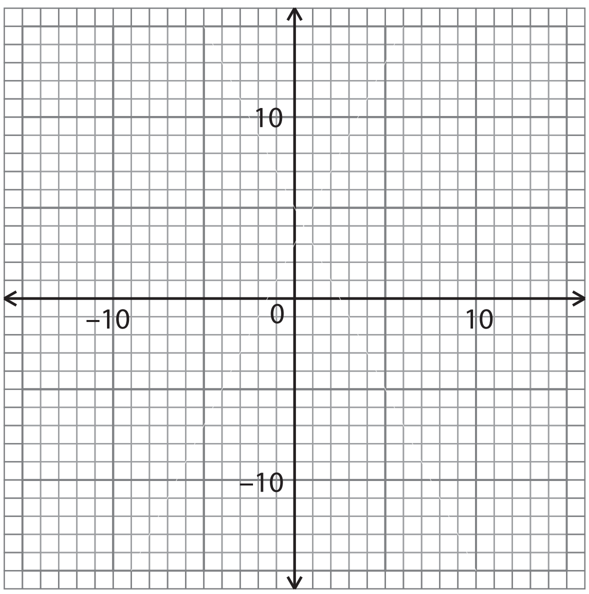


(c) What horizontal change and vertical change is needed to move from the point (2; 3) to the point (8; 15)? You may draw arrows on your graph to help you to think clearly about this.

(d) Work out the gradient of the line that passes through the two points.

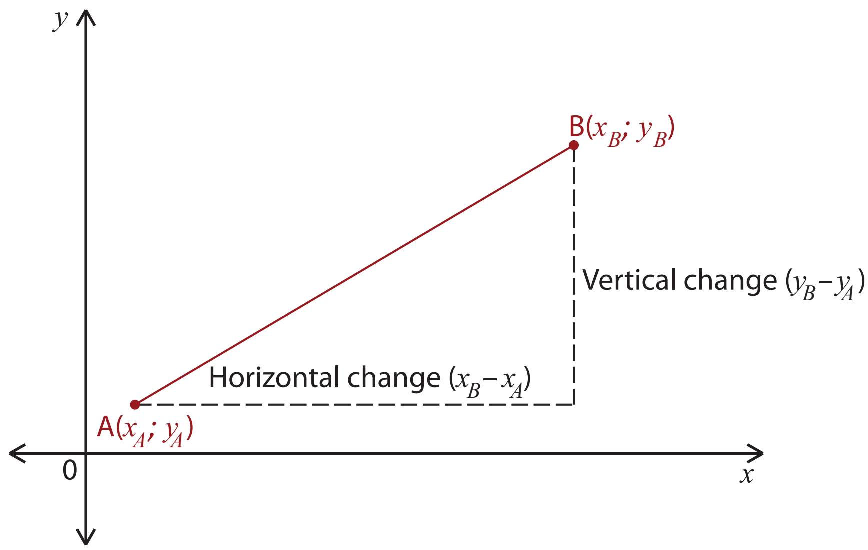
2. Copy and complete the table and plot graphs of $y = 2x + 3$ and $y = -2x + 5$ on a graph sheet.

x	-3	1	3	5
$2x + 3$				
$-2x + 5$				



3. Work out the gradients of the graphs of $y = 2x + 3$ and $y = -2x + 5$. You may use the coordinates of any of the points you have plotted.

Suppose the coordinates of point A are $(x_A; y_A)$ and the coordinates of B are $(x_B; y_B)$.



The gradient of line AB is: $m_{AB} = \dfrac{\textit{Vertical change}}{\textit{Horizontal change}} = \dfrac{y_A - y_B}{x_A - x_B}$

In summary:
If you have two points A $(x_A; y_A)$ and B $(x_B; y_B)$ then the formula for the gradient is: $m = \dfrac{y_A - y_B}{x_A - x_B}$

Examples of finding the gradient between two points

Calculate the gradient of the line that goes through the points:

(a) A(2; 5) and B(4; 1)

$$\begin{aligned} m &= \frac{y_A - y_B}{x_A - x_B} \\ &= \frac{5-1}{2-4} \\ &= \frac{4}{-2} \\ &= -2 \end{aligned}$$

(b) C(2; 2) and D(–6; 0)

$$\begin{aligned} m &= \frac{y_C - y_D}{x_C - x_D} \\ &= \frac{2-0}{2-(-6)} \\ &= \frac{2}{8} \\ &= \frac{1}{4} \end{aligned}$$

(c) A(0; –1) and B(1; 1)

$$\begin{aligned} m &= \frac{y_A - y_B}{x_A - x_B} \\ &= \frac{-1-1}{0-1} \\ &= \frac{-2}{-1} \\ &= 2 \end{aligned}$$

The gradient of a straight line is the same everywhere, so it does not matter which two points you use to determine the gradient.

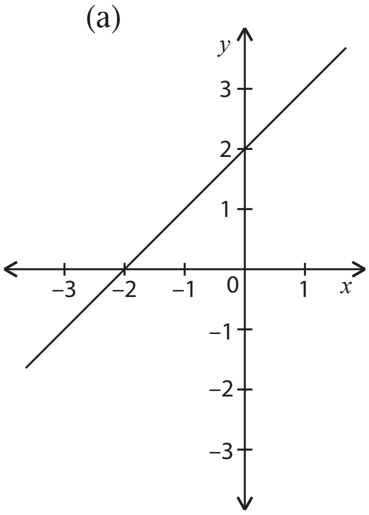
DETERMINE THE GRADIENT

Do the following task:

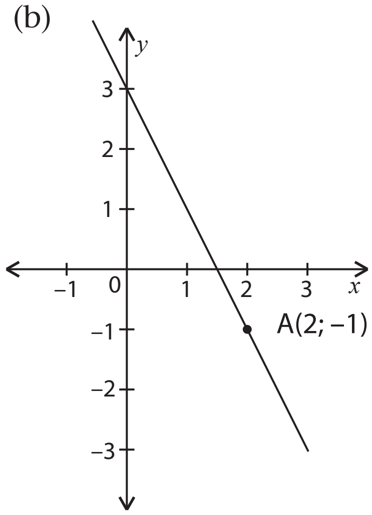
1. Determine the gradient of the lines that go through the following points:
   (a) A(2; 10) and B(6; 12)
   (b) C(1; 3) and D(–2; –3)
   (c) E(0; 3) and F(4; –1)
   (d) G(5; 2), H(4; 4) and I(2; 8)

2. Determine the gradient of the following lines:

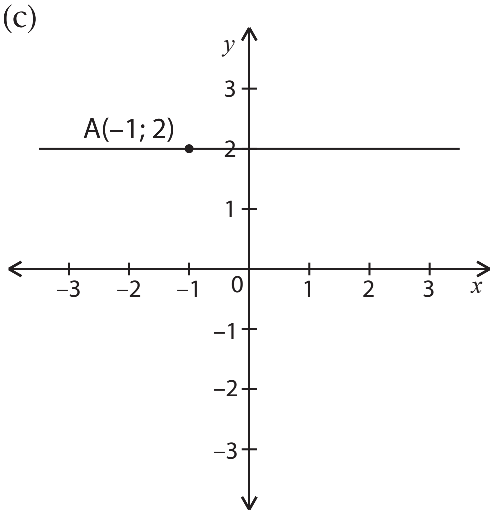
(a)



(b)



(c)



18.5 Finding the formula for a graph

TABLES AND FORMULAE

1. Each table below and on the next page shows values for a relationship represented by one of these rules:

$y = -2x + 3$	$y = 2x - 5$	$y = -3x + 5$	$y = -3(x + 2)$
$y = 3x + 2$	$y = 5(x - 2)$	$y = 2x + 3$	$y = 2x + 5$
$y = -3x + 6$	$y = 5x + 10$	$y = 5x - 10$	$y = -x + 3$

   (a) Copy and complete the following tables by extending the patterns in the output values:

A.

x	0	1	2	3	4	5	6	7
y	2	5	8					

B.

x	0	1	2	3	4	5	6	7
y	3	1	–1	–3				

C.

x	0	1	2	3	4	5	6	7
y	–10	–5	0	5				

D.

x	0	1	2	3	4	5	6	7
y	–5	–3	–1					

E.

x	0	1	2	3	4	5	6	7
y	6	3	0					

F.

x	0	1	2	3	4	5	6	7
y	3	2	1	0				

G.

x	0	1	2	3	4	5	6	7
y	3	5	7					

(b) For each table, describe what you did to produce more output values. Also write down the rule (formula) that corresponds to the table.

You may have noticed that the equations of straight lines look similar.

The equation of a straight line is $y = mx + c$, where:

- m tells us the **gradient** of the line
- c tells us where the line crosses the y-axis
- it is called the **y-intercept** and it has the coordinates $(0; c)$
- the line $y = 3x + 4$ has a **gradient of 3** and the y-intercept is **(0; 4)**
- the equation of a line with a **gradient of –2** and y-intercept of **(0;10)** is $y = -2x + 10$
- the line $y = 2x$ has a **gradient of 2** and the y-intercept is **(0; 0)**
- the line $y = 5$ has a **gradient of 0** and the y-intercept is **(0; 5)**

Gradient means the steepness or slope of the line.

The point where a line crosses one of the axes is called the **intercept**.

What is the gradient and y-intercept of the line $2y = 6x + 10$?

If you said $m = 6$ and $c = 10$, you would be wrong. The equation is not in standard form. The equation must be written in standard form before you can read off the values of the gradient and the y-intercept.

$$2y = 6x + 10 \quad \rightarrow \text{Divide both sides by 2}$$
$$y = 3x + 5$$

Therefore, the **gradient is 3** and the y-intercept is **(0; 5)**.

The **standard form** of a straight line graph is $y = mx + c$.
On one side there should only be a "y" (with a coefficient of 1).

If $m > 0$, the line will be increasing.
If $m < 0$, the line will be decreasing.
If the line is horizontal, $m = 0$.
If the line is vertical, m is undefined.

2. Copy and complete the following table:

Equation	Gradient	y-intercept
$y = 3x + 5$		
$y = \frac{x}{2} - 7$		
$y = 2 - 3x$		
$-y = 5x - 10$		
$y = 3$		
	1	(0; 0)
	–2	(0; –7)

3. Write each of the following equations in standard form and then determine the gradient and y-intercept:

(a) $2y + 4x = 10$
(b) $-3x = y + 4$
(c) $3x - 4 = y$
(d) $3y + 6 = x$
(e) $y = -3x + 4y - 12$
(f) $y = 3x - 2$
(g) $y = \frac{1}{4}x + 6$
(h) $y = -12$
(i) $x = 15$

DETERMINE THE EQUATION OF A STRAIGHT LINE

The equation of a straight line is $y = mx + c$. If you need to determine the equation of a straight line, then all you need to know are the values of m and c.

If you know the values of two points on the graph, then you can determine the gradient using the formula: $m = \frac{y_A - y_B}{x_A - x_B}$

Once you know the gradient you can calculate the value of the y-intercept using substitution.

Example 1: Determine the equation of the straight line that goes through (1; 1) and (5; 13).

Step 1: Calculate the gradient.

$$m = \frac{y_A - y_B}{x_A - x_B} = \frac{1 - 13}{1 - 5} = \frac{-12}{-4} = 3$$

Step 2: Since you now know $m = 3$ you can substitute it into the equation $y = mx + c$.
Therefore, $y = 3x + c$.

Step 3: To determine c you need to substitute the coordinates of a point on the line into the equation. (It can be either of the points that were given, so choose the easier one.)

Substitute (5; 13) into $y = 3x + c$

$$\begin{aligned} (13) &= 3(5) + c \\ 13 &= 15 + c \\ 13 - 15 &= c \\ -2 &= c \end{aligned}$$

Step 4: Write down the equation: $y = 3x - 2$.

Example 2: Determine the equation of the line that passes through (4; –1) and (7; 5).

Information	m (Gradient)	c (y-intercept)	$y = mx + c$ (Equation)
(4; –1) **(7; 5)**	$m = \frac{y_A - y_B}{x_A - x_B}$ $= \frac{-1-5}{4-7}$ $= \frac{-6}{-3}$ $= 2$	Substitute $m = 2$ and (7; 5) $y = mx + c$ $y = 2x + c$ $(5) = 2(7) + c$ $5 = 14 + c$ $-9 = c$	$y = 2x - 9$

Example 3: Determine the equation of the line with a gradient of 4 passing through (2; 6).

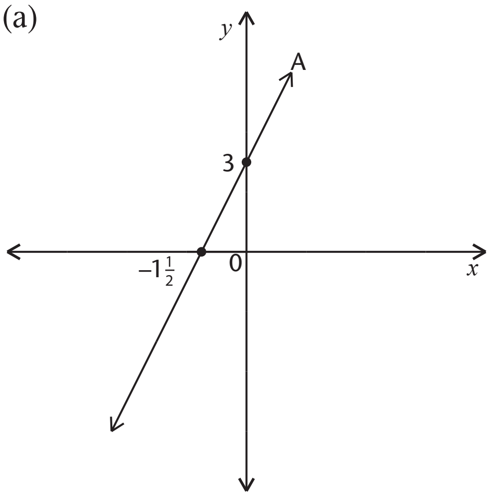
Information	m (gradient)	c (y-intercept)	$y = mx + c$ (equation)
$\boldsymbol{m = 4}$ **(2; 6)**	$m = 4$	Substitute $m = 4$ and (2; 6) $y = mx + c$ $y = 4x + c$ $6 = 4(2) + c$ $-2 = c$	$y = 4x - 2$

You may want to set your work out as shown in Examples 2 and 3 above.

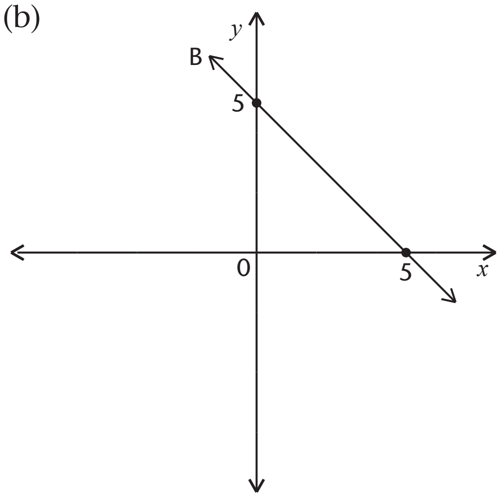
1. Determine the equation of the each of the straight lines passing through the points given.
   (a) (3; 10) and (2; 5)
   (b) (–4; 5) and (2; 5)
   (c) (0; 0) and (4; –8)
   (d) $(1\frac{1}{2}; 4)$ and $(-\frac{1}{2}; 12)$
   (e) (3; 4) and (–7; –1)
   (f) (0; 3) and (–14; –4)
2. Determine the equation of the straight line with:
   (a) a gradient of 5 and passing through the point (1; –3)
   (b) a gradient of –2 passing through the point (0; 0)
   (c) a y-intercept of 7 passing through the point (1; –3)

3. Determine the equations of the straight lines. Question (d) is a challenge.

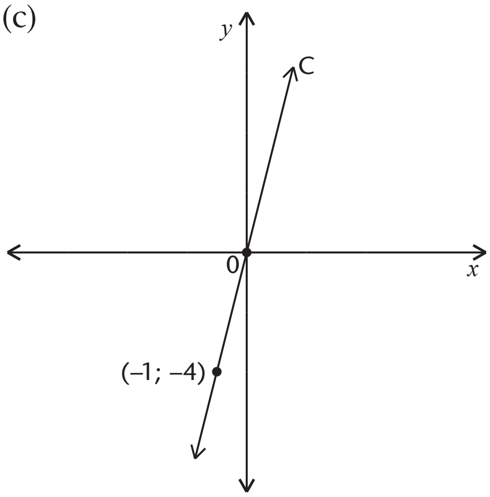
(a)



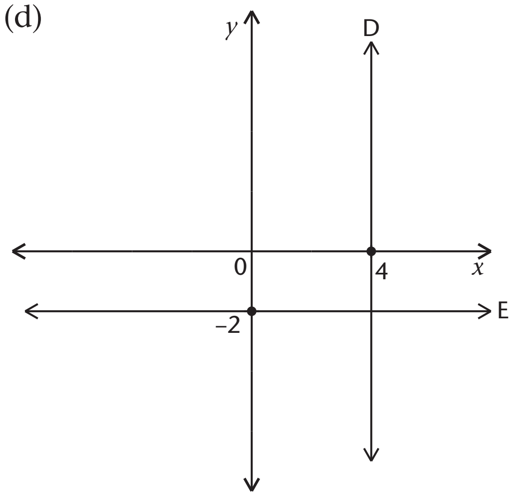
(b)



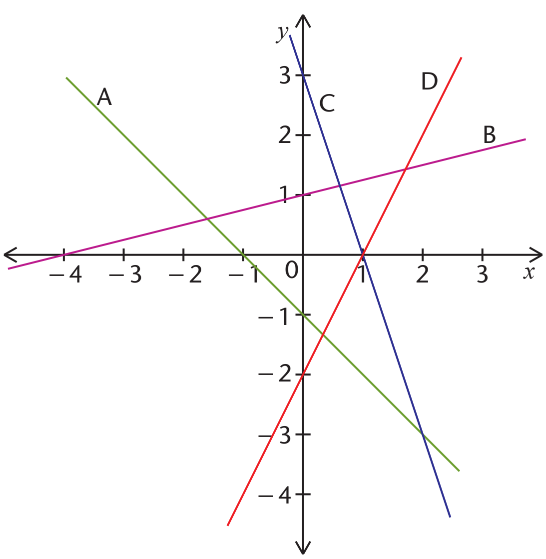
(c)



(d)



18.6 *x*- and *y*-intercepts



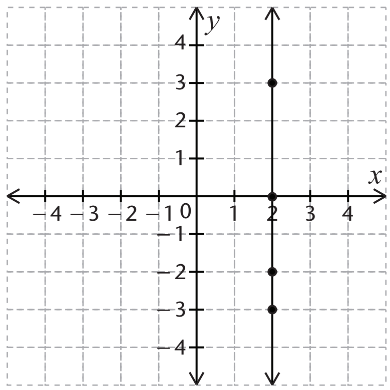
1. Copy the table and write down the coordinates of the points where each line cuts the two axes:

	x-intercept	*y*-intercept
A		
B		
C		
D		

2. What do all the *x*-intercepts have in common?

3. What do all the y-intercepts have in common?
4. Determine the coordinates of the intercepts of the following straight line graphs:
   (a) $y = 3x + 12$
   (b) $y = x - 3$
   (c) $y = -2x - 4$
   (d) $2y = 6x + 12$
   (e) $4x + 2y = 20$
   (f) $13 - y = -26x$

VERTICAL AND HORIZONTAL LINES



Some special lines are so easy that you do not need any fancy methods to draw them or get their equation; you can just look at them.

1. What do the following coordinate pairs have in common?
   (2; 3), (2; –2), (2; 0) and (2; –3)
2. Write down two more points that have an x-coordinate of 2.

If you plot these points on a set of axes you will see that they form a **vertical line**. The equation of the line is $x = 2$.

3. Will the two extra points you wrote down (question 2) also be on the line?
4. Write down five coordinate pairs with $x = -1$.

18.7 Graphs of non-linear functions

Some of the following relationships are represented by graphs on the next page. Identify which of the relationships are represented by which set of points on the graph. You may use the table below to help you to answer this question. For example, you may calculate some output numbers by using the formulas and record this in tables.

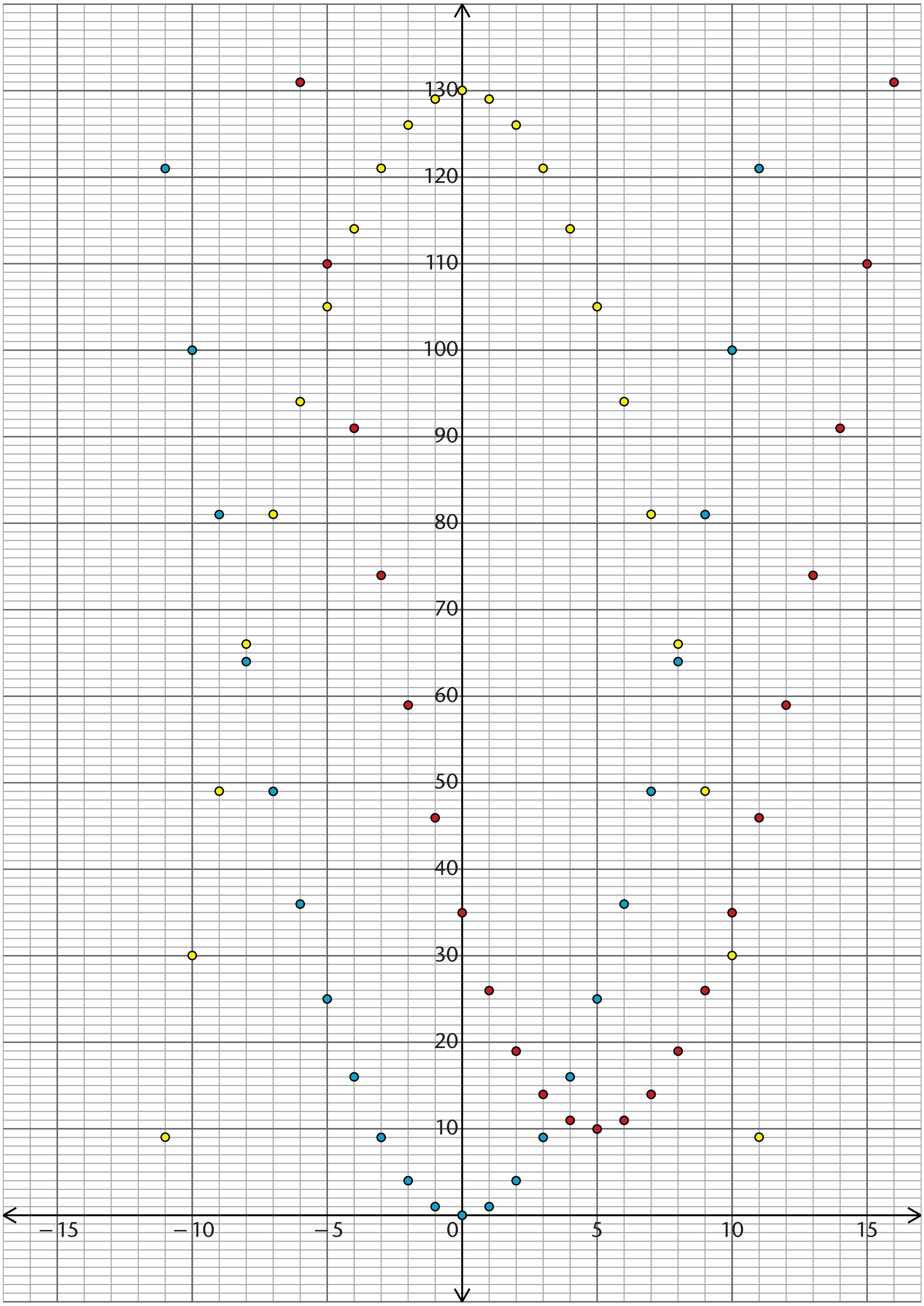
$y = -x^2$ $\quad$ $y = (-x)^2$ $\quad$ $y = x^2 + 130$ $\quad$ $y = (x - 5)^2 + 10$

$y = x^2$ $\quad$ $y = -x^2 + 130$ $\quad$ $y = 130 - x^2$ $\quad$ $y = x^2 - 10x + 35$

(a) Set of points in yellow

(b) Set of points in blue

(c) Set of points in red


130
120
110
100
90
80
70
60
50
40
30
20
10
– 15
– 10
– 5
0
5
10
15

CHAPTER 19
Surface area, volume and capacity of 3D objects

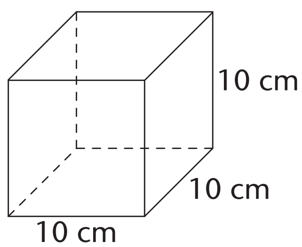
19.1 Surface area

SURFACE AREA OF PRISMS

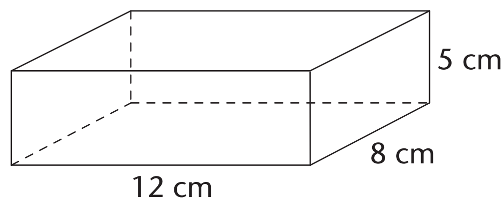
The **surface area** of an object is the total area of all of its faces added together. You learnt the following formula in previous grades:

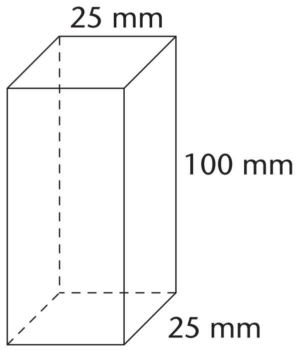
- Surface area of a prism = Sum of the areas of all its faces

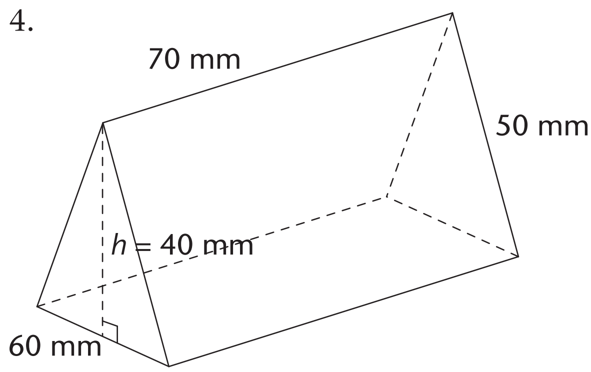
Calculate the surface area of the following objects to revise what you should already know:

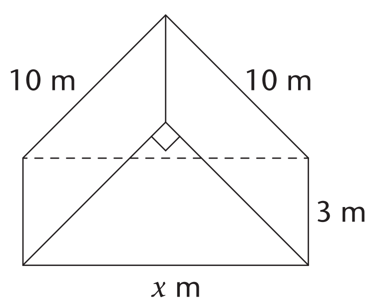
1.


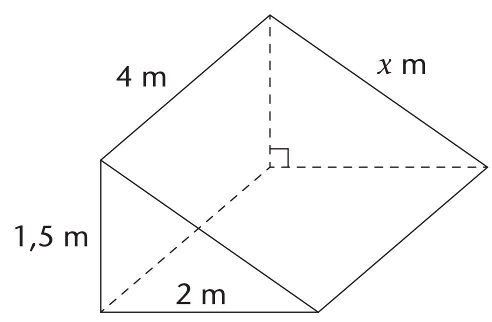
(We use *SA* for surface area.)

2.


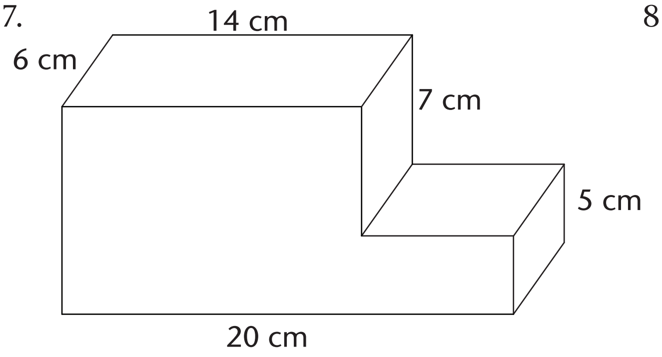
3.


4.


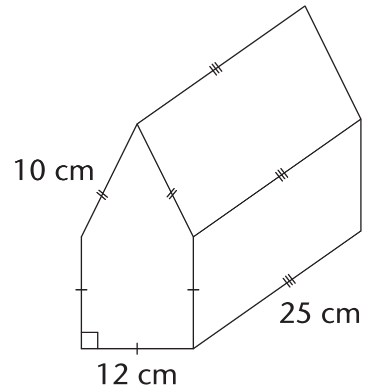
5.


6.


7.



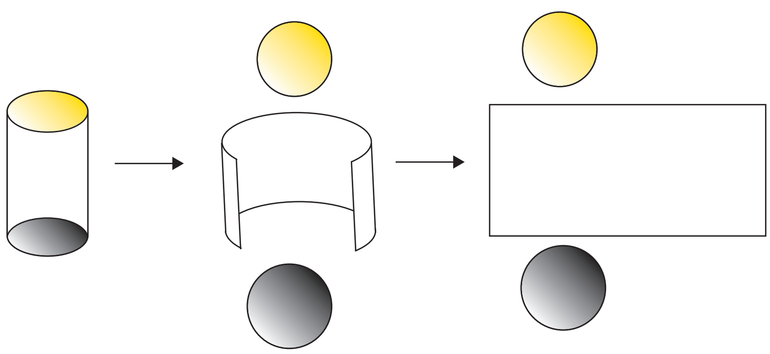
8.



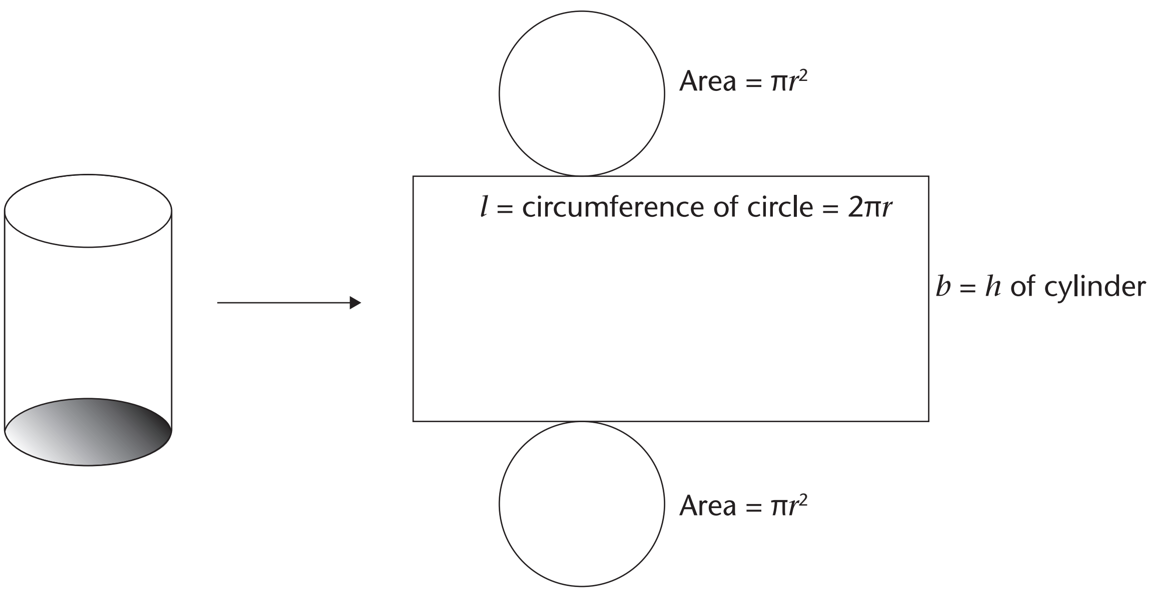
INVESTIGATING THE SURFACE AREA OF CYLINDERS

In order to calculate the surface area of a cylinder, you need to know what shape the surfaces of the cylinder are.

The surfaces of the top and base of a cylinder are made up of circles. The curved surface between the top and base of a cylinder can be unrolled to create a rectangle.



So, the net of a cylinder looks like this:



Surface area of a cylinder = Area of all its surfaces

$$= \text{Area of top} + \text{Area of base} + \text{Area of curved surface}$$
$$= \pi r^2 + \pi r^2 + (l \times b)$$
$$= 2\pi r^2 + (2\pi r \times h)$$
$$= 2\pi r(r + h)$$

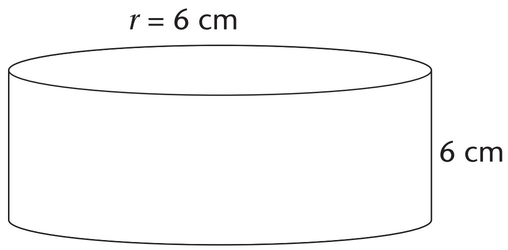
Can you explain why the length of the rectangle is equal to the circumference of the top or base of the cylinder?

CALCULATING THE SURFACE AREA OF CYLINDERS

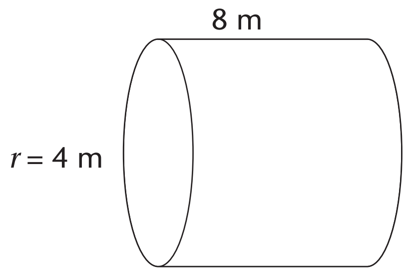
From the formula above, you can see that we need only know the radius (r) and the height (h) of a cylinder in order to work out its surface area.

1. Calculate the surface areas of the following objects. Use $\pi = 3{,}14$ and round off all your answers to two decimal places.

A.



B.



2. Calculate the surface area of a cylinder if its height is 60 cm and the circumference of its base is 25,12 cm.
3. Calculate the surface area of a cylinder if its height is 5 m and the circumference of its base is 12,56 m.
4. The outside of a cylindrical structure at a factory must be painted. Its radius is 3,5 m and its height is 8 m. How many litres of paint must be bought if 1 litre covers 10 m^2? (The bottom of the structure will not be painted.)

19.2 Volume

The **volume** of an object is the amount of space it occupies. We usually measure volume in cubic units, such as mm^3, cm^3 and m^3.

To convert between cubic units, remember:
1 cm^3 = 1 000 mm^3
1 m^3 = 1 000 000 cm^3

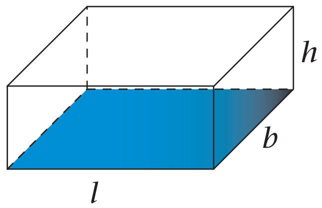
FORMULAE FOR VOLUME OF PRISMS

The general formula for the volume of a prism is:
Volume of a prism = Area of base × height.

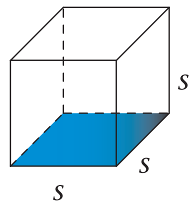
In case of a triangular prism, do not confuse the height of the base of the triangle (h_b) with the height of the prism (h_p).

Therefore, the formulas to work out the volumes of the following prisms are:

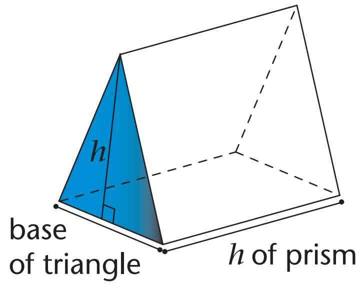
Rectangular prism



Cube



Triangular prism



$V = (l \times b) \times h$

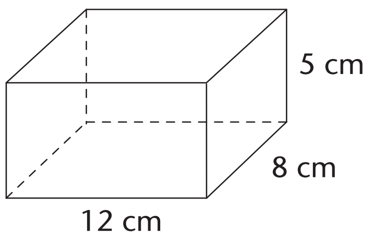
$V = (s \times s) \times s$
$= s^3$

$V = (\frac{1}{2} \text{ base} \times h_b) \times h_p$

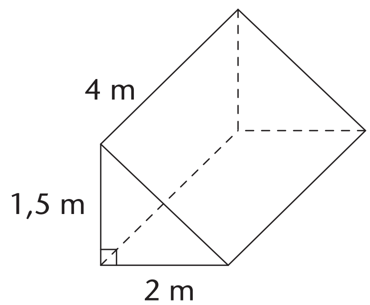
CALCULATING THE VOLUME OF PRISMS

1. Calculate the volumes of the following prisms:

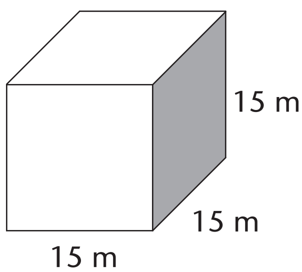
A.



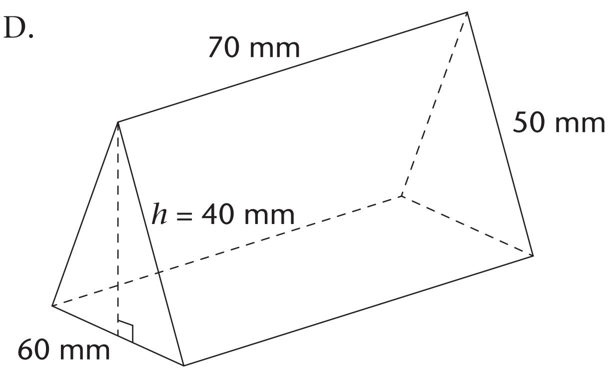
B.



C.



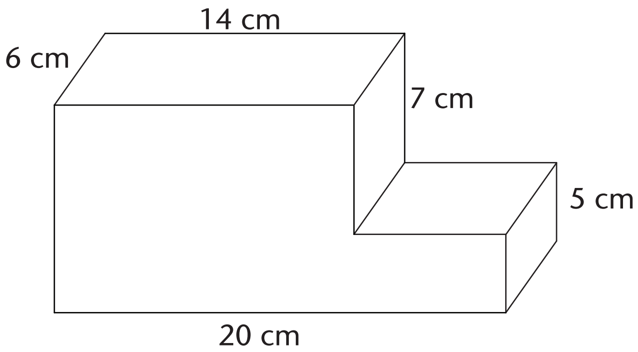
D.



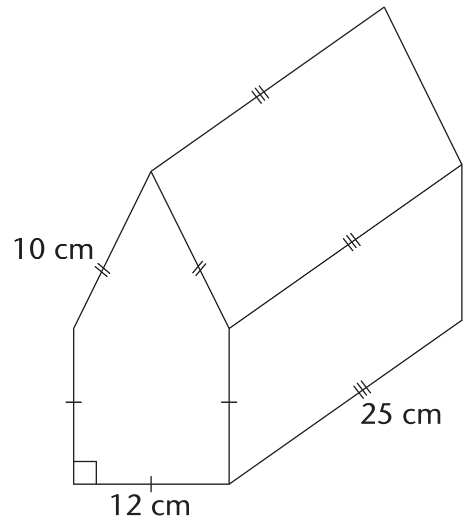
2. (a) The area of the base of a rectangular prism is 32 m^2 and its height is 12 m. What is its volume?
   (b) The volume of a cube is 216 m^3. What is the length of one of its edges?

3. Calculate the volume of the following objects:

A.

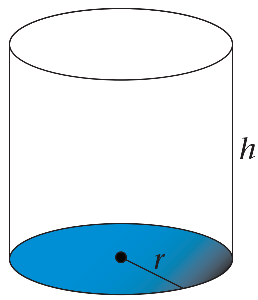


B.



VOLUME OF CYLINDERS

You also calculate the volume of a cylinder by multiplying the area of the base by the height of the cylinder. The base of a cylinder is circular, therefore:

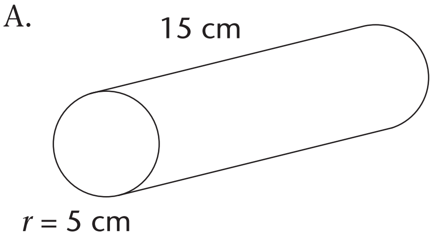


Area of circle = πr^2

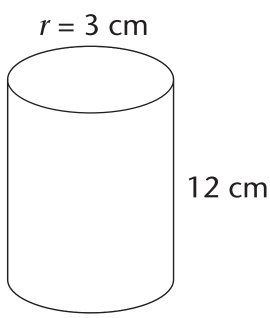
$$\text{Volume of a cylinder} = \text{Area of base} \times h$$
$$= \pi r^2 \times h$$

1. Calculate the volume of the following cylinders. Use $\pi = 3{,}14$ and round off all answers to two decimal places.

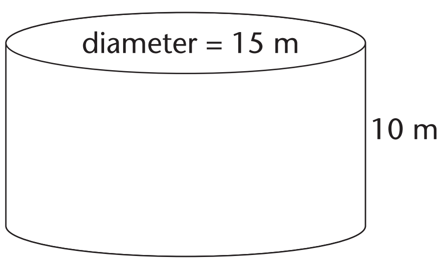
A.



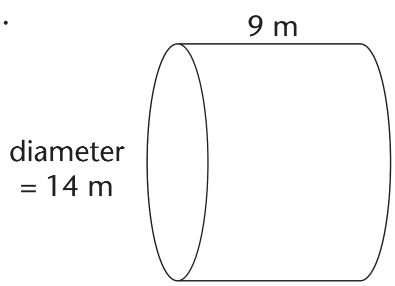
B.



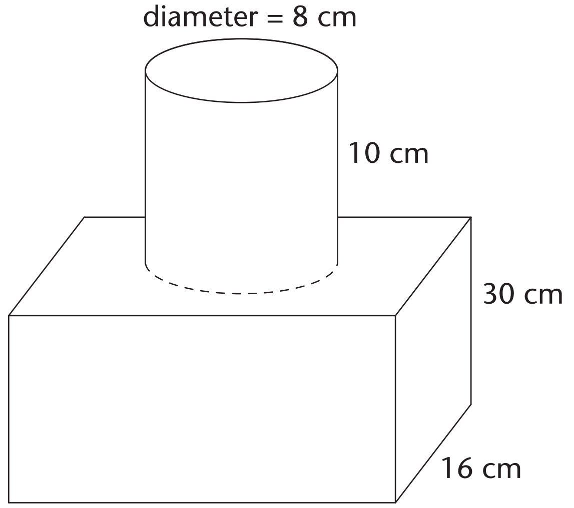
C.



D.



2. Without using a calculator, calculate the volume of cylinders with the measurements given below. Use $\pi = \frac{22}{7}$.
   (a) $r = 14$ cm; $h = 20$ cm
   (b) $r = 7$ cm; $h = 35$ cm
   (c) diameter = 28 cm; $h = 50$ cm
   (d) diameter = 7 cm; $h = 10$ cm
3. Calculate the volume of the following object. Use a calculator and round off all answers to two decimal places.



19.3 Capacity

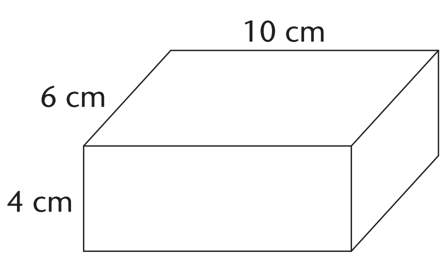
Remember that the **capacity** of an object is the amount of space *inside* the object. You can think of the capacity of an object as the amount of liquid that the object can hold.

The **volume** of an object is the amount of space that the object itself takes up.

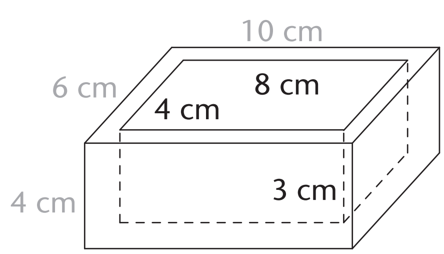
The volume of a solid block of wood is 10 cm × 6 cm × 4 cm = 240 cm^3.

The same block of wood is carved out to make a hollow container with inside measurements of 8 cm × 4 cm × 3 cm. (Its walls are 1 cm thick.) The amount of space inside the container must be calculated using the *inside* measurements. So, the capacity of the container is 8 cm × 4 cm × 3 cm = 96 cm^3.

A. Solid block with outside measurements



B. Hollowed block with inside measurements



1. Write, in ml, the volume of water that would fill container B.
2. If the walls and bottom of container B were 0,5 cm thick, what would its capacity be? Write the answer in ml.

Remember:
$1 \text{ cm}^3 = 1 \text{ ml}$
$1 \text{ m}^3 = 1 \text{ kl}$

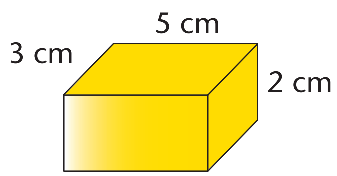
3. The inside measurements of a swimming pool are 9 m × 4 m × 2 m. What is the capacity of the pool in kl?

19.4 Doubling dimensions and the effect on volume

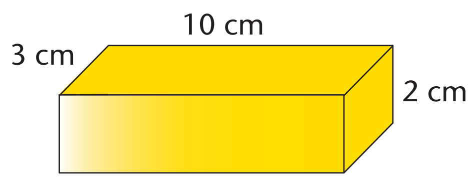
DOUBLING THE DIMENSIONS OF A PRISM

The first prism below measures 5 cm × 3 cm × 2 cm. The other diagrams show the prism with one or more of its dimensions doubled.

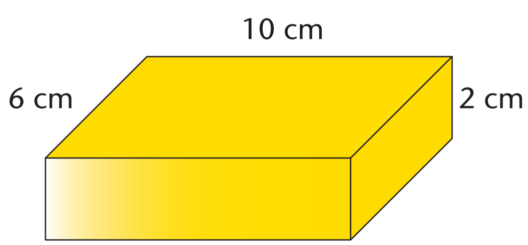
1. Work out the volume of each prism.



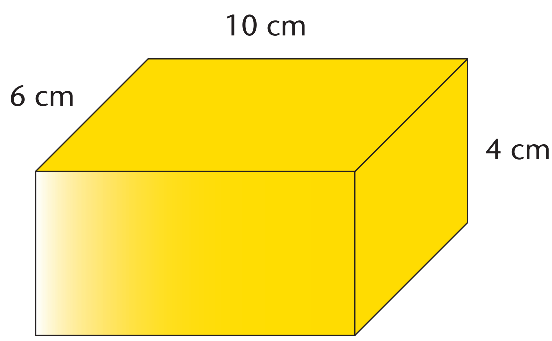
One dimension doubled



Two dimensions doubled



Three dimensions doubled



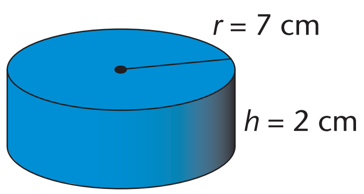
2. Copy and complete the following:
   (a) When one dimension of a prism is doubled, the volume ....
   (b) When two dimensions of a prism are doubled, the volume increases by .... times.
   (c) When all three dimensions of a prism are doubled, the volume increases by .... times.

3. The volume of a prism is 80 cm^3. What is its volume if:
   (a) its length is doubled?
   (b) its length and breadth are doubled?
   (c) its length, breadth and height are doubled?

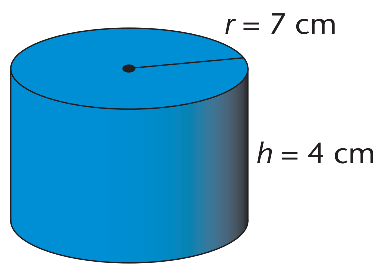
DOUBLING THE DIMENSIONS OF A CYLINDER

The first cylinder below has a radius of 7 cm and a height of 2 cm. The other diagrams show the cylinder with one or more of its dimensions doubled.

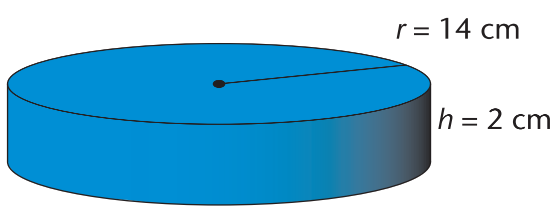
1. Work out the volume of each cylinder:



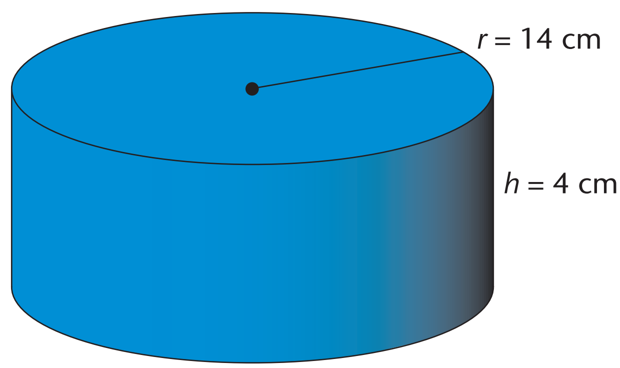
Only height doubled



Only radius doubled



Radius and height doubled



2. Copy and complete the following:
   (a) When the height of a cylinder is doubled, the volume ....
   (b) When the radius of a cylinder is doubled, the volume increases by .... times.
   (c) When height and radius of cylinder are doubled, the volume increases by .... times.

3. The volume of a cylinder is 462 cm^3. What is its volume if:
   (a) its height is doubled?
   (b) its radius is doubled?
   (c) its height and radius are doubled?

4. (a) Study the following tables and copy them. Without using the formulas to calculate volume, complete the last column in each table. (Hint: Identify which dimensions are doubled each time, then work out the volume accordingly.)

Rectangular prism			
Length (l) in m	**Breadth (b) in m**	**Height (h) in m**	**Volume (V) in m³**
4	2	1	
4	4	1	
8	2	1	
8	2	2	
8	4	2	

Cylinder		
Radius (r) in m	**Height (h) in m**	**Volume (V) in m³**
3,5	4	
7	4	
3,5	8	
7	8	

(b) Explain how you worked out the answers in the tables.

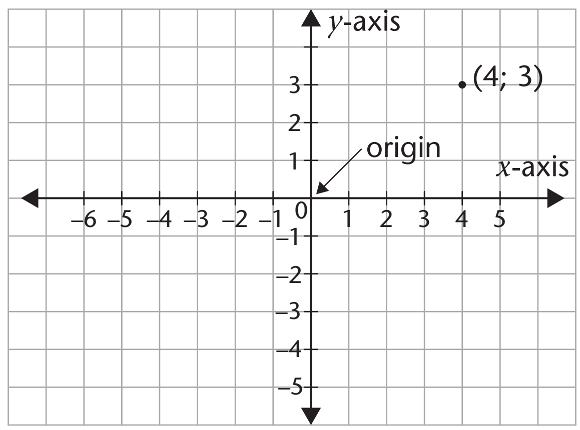
Chapter 20
Transformation geometry

20.1 Points on a coordinate system

A rectangular coordinate system is also called a **Cartesian coordinate system**. It consists of a horizontal x-axis and a vertical y-axis.

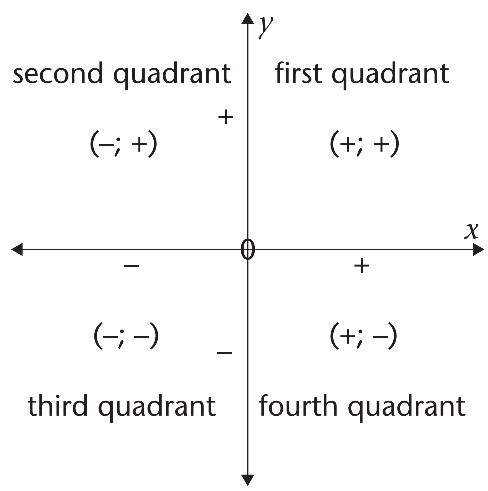
The intersection of the axes is called the **origin**, and represents the point (0; 0).

Any point can be represented on a coordinate system using an x-value and a y-value. These numbers are called **coordinates**, and describe the position of the point with reference to the two axes.



The coordinates of a point are always written in a certain order:

- The horizontal distance from the origin (x-coordinate) is written first.
- The vertical distance from the origin (y-coordinate) is written second.
- These numbers, called an **ordered pair**, are separated by a semi-colon (;) and are placed between brackets. Here is an example of an ordered pair: (4; 3) (see on the coordinate system above).
- The x-axis and y-axis divide the coordinate system into four sections called **quadrants**. The diagram alongside shows how the quadrants are numbered, and also whether the x- and y-coordinates are negative or positive in each quadrant.



1. In which quadrant will the following points be plotted?
   (a) (–4; 1)　　(b) (–1; –5)
   (c) (4; –3)　　(d) (5; 2)
2. Copy the first coordinate system given on the previous page and plot the points in question 1 on it.

When a point is translated to a different position on a coordinate system, the new position is called the image of the point. We use the prime symbol (') to indicate an image. For example, the image of A is indicated by A' (read as "A prime"). If the coordinates of A are labelled as $(x; y)$, the coordinates of A' can be labelled as $(x'; y')$.

We write A → A' and $(x; y) \rightarrow (x'; y')$ to indicate that A is mapped to A'.

20.2 Reflection (flip)

The **mirror image** or **reflection** of a point is on the opposite side of a **line of reflection**.

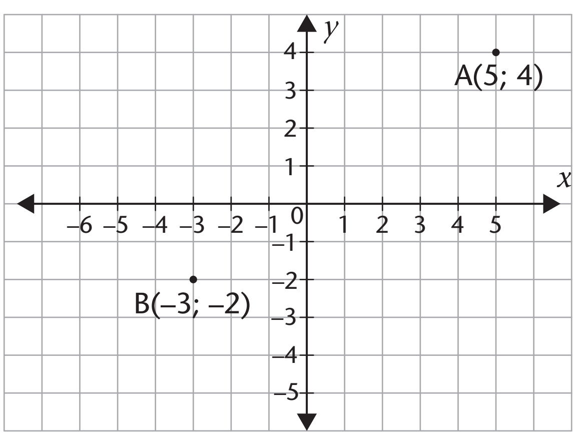
"Reflecting a point in the x-axis" means that the x-axis is the line of reflection.

The original point and its mirror image are the same distance away from the line of reflection, and the line that joins the point and its image is perpendicular to the line of reflection.

Any line on the coordinate system can be a line of reflection, including the x-axis, the y-axis and the line $y = x$.

REFLECTING POINTS IN THE x-AXIS, y-AXIS AND THE LINE $y = x$

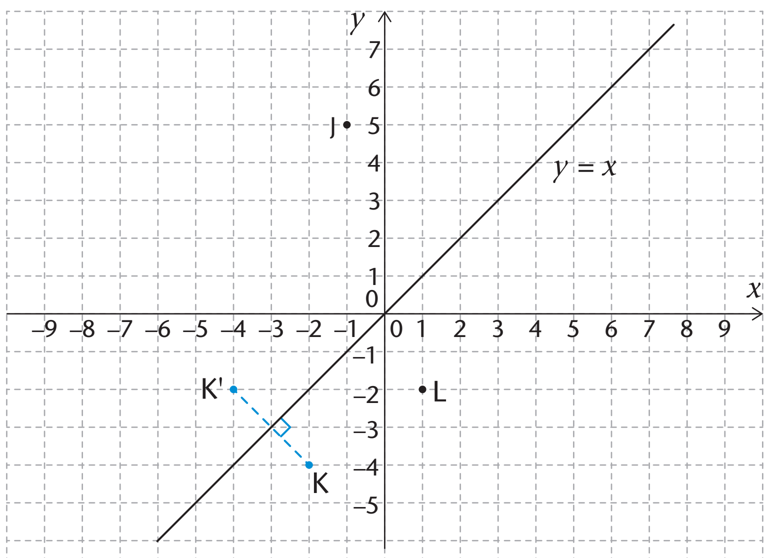
1. The points A(5; 4) and B(–3; –2) are plotted on a coordinate system. Copy the coordinate system.
   (a) Reflect points A and B in the x-axis and write down the coordinates of the images.
   (b) Reflect points A and B in the y-axis and write down the coordinates of the images.
   (c) Compare the coordinates of the original points with those of its images. What do you notice?



2. Copy the table and write down the coordinates of the images of the following reflected points:

Point	Reflection in the *x*-axis	Reflection in the *y*-axis
(–131; 24)		
(–459; –795)		
$(x; y)$		

3. The points J(–1; 5), K(–2; –4) and L(1; –2) are plotted on the coordinate system. K' is the reflection of point K in the line $y = x$. This means that the line $y = x$ is the line of reflection.
   (a) Reflect J and L in the line $y = x$.
   (b) Write down the coordinates of the images of the points.
   (c) What do you notice about the coordinates of the images of the points in (b) above?
   (d) Use your observation in (c) above to copy and complete this table.



Point	Coordinates of the image of the point reflected in $y = x$
(–1 001; –402)	
(459; –795)	
(–342; 31)	
(21; 67)	
$(x; y)$	

While doing the previous activity, you may have noticed the following:

- For a reflection in the *y*-axis, the sign of the *x*-coordinate changes and the *y*-coordinate stays the same: $(x; y) \rightarrow (-x; y)$ or $x' = -x$ and $y' = y$, for example: (–3; 4)→ (3; 4)

- For a reflection in the x-axis, the sign of the y-coordinate changes and the x-coordinate stays the same: $(x; y) \rightarrow (x; -y)$ or $x' = x$ and $y' = -y$, for example: $(-3; 4) \rightarrow (-3; -4)$
- For a reflection in the line $y = x$, the values of the x- and y-coordinates are interchanged:
  $(x; y) \rightarrow (y; x)$ or $x' = y$ and $y' = x$, for example: $(-3; 4) \rightarrow (4; -3)$.

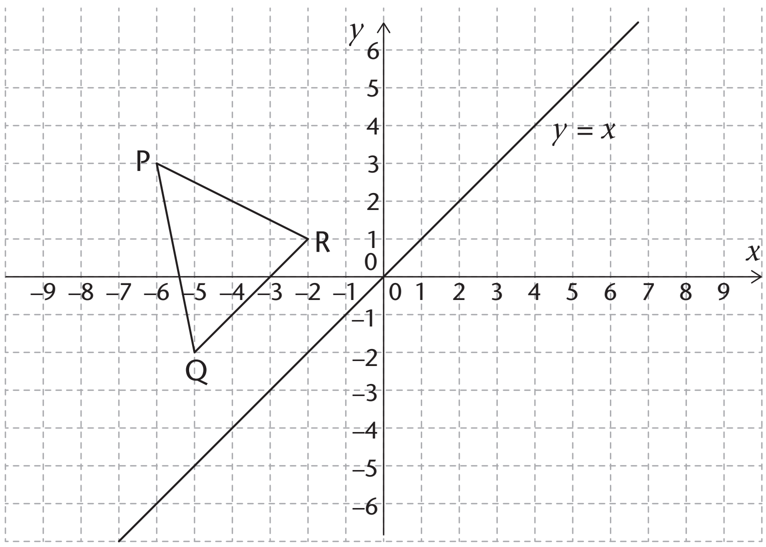
4. Investigate the effect of reflection in the line $y = -x$ on the coordinates of a point.
5. A is the point $(5; -2)$. Write the coordinates of the mirror images of A if the point is reflected in:

   (a) the y-axis (b) the line $y = -x$
   (c) the line $y = x$ (d) the x-axis

REFLECTING GEOMETRIC FIGURES

The same principles, as above, apply when reflecting geometric figures.

1. (a) On graph paper, reflect ΔPQR in the x-axis, in the y-axis and in the line $y = x$ in the coordinate system (first reflect the vertices and then join the reflected points).

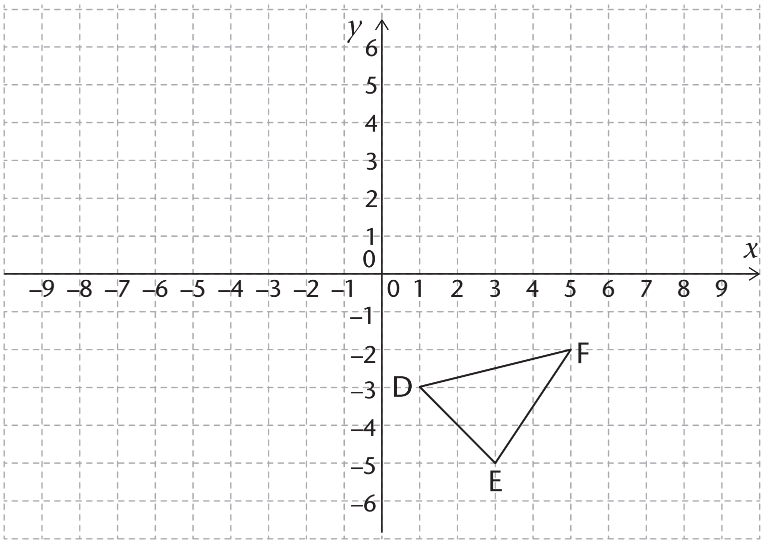


(b) Copy the table below. Look at your completed reflections in question 1(a), and write down the coordinates of the image points in the table.

Vertices of triangle	Reflection in the x-axis	Reflection in the y-axis	Reflection in the line $y = x$
P(–6; 3)			
Q(–5; –2)			
R(–2; 1)			

(c) What do you notice about ΔPQR, ΔP'Q'R', ΔP''Q''R'' and ΔP'''Q'''R'''?

2. On graph paper, reflect ΔDEF in the x-axis, in the y-axis and in the line $y = x$.



3. A quadrilateral has the following vertices: A(1; 4), B(–6; 1), C(–2; –1) and D(7; 2). Without performing the actual reflections, write down the coordinates of the vertices of the image when the quadrilateral is:
   (a) reflected in the x-axis
   (b) reflected in the y-axis
   (c) reflected in the line $y = x$

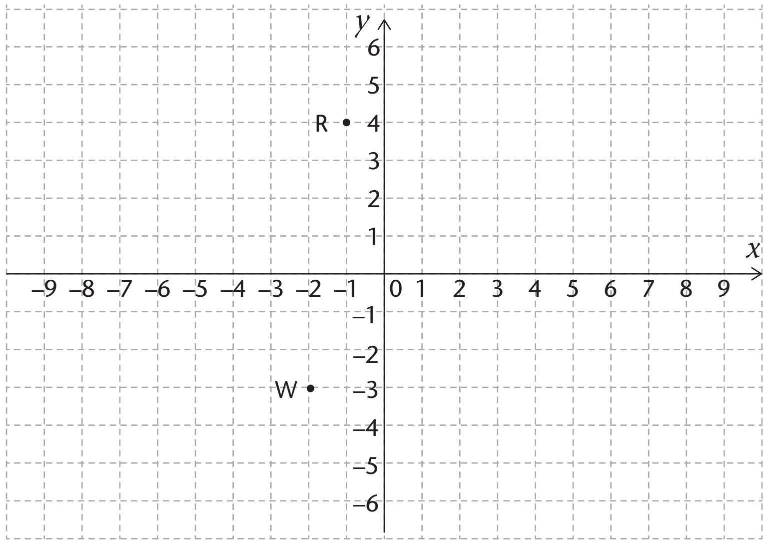
4. In each case, state around which line the point was reflected.
   (a) (–4; 5) → (–4; –5)
   (b) (2; –3) → (–2; –3)
   (c) (–13; –3) → (–3; –13)
   (d) (1; 16) → (16; 1)
   (e) (12; –8) → (–12; –8)
   (f) (–7; –5) → (–5; –7)
   (g) (2; –3) → (–2; –3)

20.3 Translation (slide)

Remember: A translation of a point or geometric figure on a coordinate system means moving or sliding the point in a vertical direction, in a horizontal direction, or in both a vertical and horizontal direction.

TRANSLATING POINTS HORIZONTALLY OR VERTICALLY ON A COORDINATE SYSTEM

1. Points R and W are plotted on a coordinate system.



   (a) On graph paper, plot the image of point R after a translation of:
   - five units to the right
   - five units to the left
   - two units up
   - two units down

   (b) On the same graph paper, plot the image of point W after a translation of:
   - four units to the right
   - four units to the left
   - three units up
   - three units down

   (c) Look at your completed translations in (a) and (b) above. Copy and complete the following table by writing down the coordinates of the original points and their images after each translation.

Coordinates of original points	**R(–1; 4)**	**W(–2; –3)**
Coordinates of image after a translation to the **right**		
Coordinates of image after a translation to the **left**		
Coordinates of image after a translation **up**		
Coordinates of image after a translation **down**		

   (d) Look at your completed table in (c) above. Choose the correct answers below to make each statement true:
   - For translations to the **right or left**, the (x-value/y-value) changes and the (x-value/y-value) stays the same.
   - For translations **up or down**, the (x-value/y-value) changes and the (x-value/y-value) stays the same.
   - For translations to the **right**, (add/subtract) the number of translated units (to/from) the x-value.

- For translations to the **left**, (add/subtract) the number of translated units (to/from) the x-value.
- For translations **up**, (add/subtract) the number of translated units (to/from) the y-value.
- For translations **down**, (add/subtract) the number of translated units (to/from) the y-value.

2. Copy the table and write down the coordinates of each image after the following translations:

Point	Three units to the right	Four units to the left	Two units up	Five units down
(3; 5)				
(–13; 42)				
(–59; –95)				
$(x; y)$				

3. Copy the table and write down the coordinates of each image after the following translations:

Point	Four units to the right and three units up	Two units to the left and one unit up	One unit to the right and five units down	Six units to the left and two units down
(4; 2)				
(–32; 21)				
(–68; –57)				
$(x; y)$				

While doing the previous activity, you may have noticed the following:

- For a horizontal translation through the distance p, the x-coordinate increases by the distance p if the slide is to the right, and decreases by the distance p if the slide is to the left. We may write $x' = x + p$, with $p > 0$ for a translation to the right, and $p < 0$ for a translation to the left. The y-coordinate remains the same, so $(x; y) \rightarrow (x + p; y)$.
- For a vertical translation through the distance q, the y-coordinate increases by the distance q if the slide is upwards, and decreases by the distance q if the slide is downwards. We may write $y' = y + q$, with $q > 0$ for a translation vertically upwards, and $q < 0$ for a translation vertically downwards. The x-coordinate remains the same, so $(x; y) \rightarrow (x; y + q)$.

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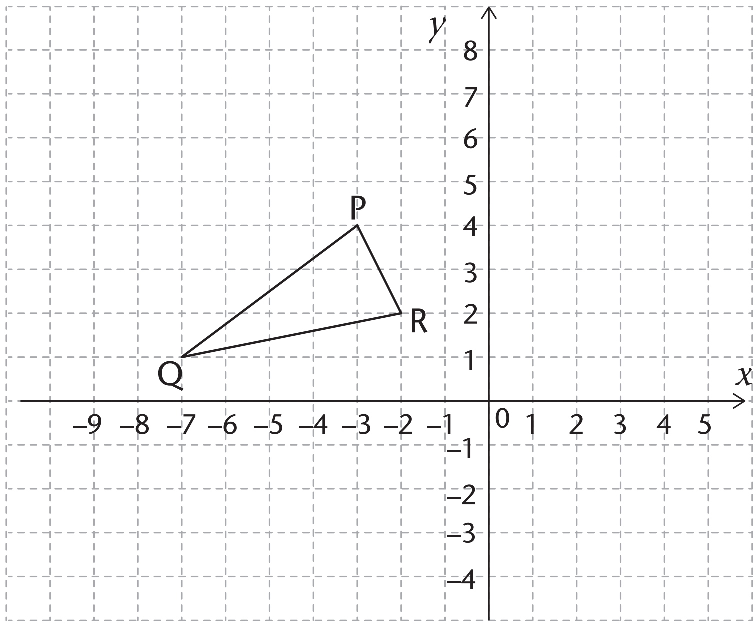
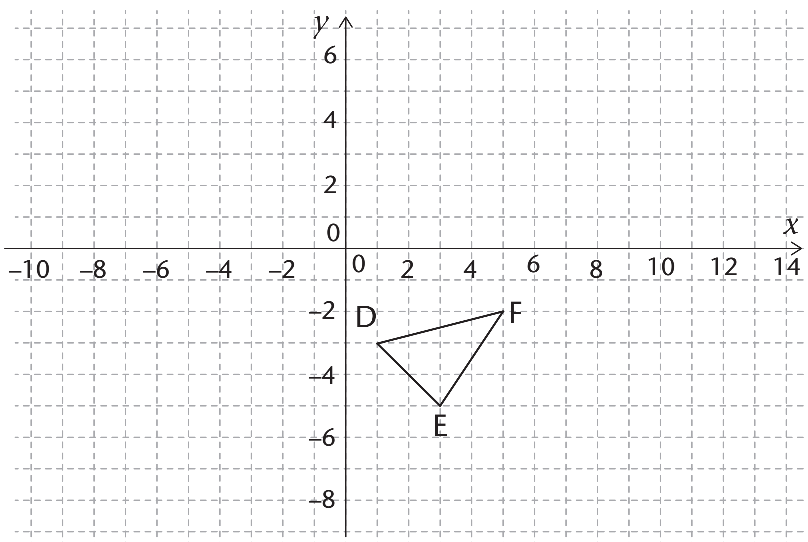
TRANSLATION OF GEOMETRIC FIGURES ON A COORDINATE SYSTEM

1. (a) On graph paper, translate ΔPQR five units to the right and three units down.
   (b) Translate ΔPQR two units to the left and three units up.
   (c) Are all the triangles congruent?



2. (a) On graph paper, translate ΔDEF four units to the left and two units down.
   (b) Translate ΔDEF three units to the right and four units up.
   (c) Are all the triangles congruent?



3. The vertices of a quadrilateral have the following coordinates: K(–5; 2), L(–4; –2), M(1; –3) and N(4; 3). Write down the coordinates of the image of the quadrilateral after the following translations:
   (a) seven units to the right and two units up
   (b) five units to the right and two units down
   (c) four units to the left and three units down
   (d) two units to the left and seven units up

4. Describe the translation if the coordinates of the original point and the image point are:

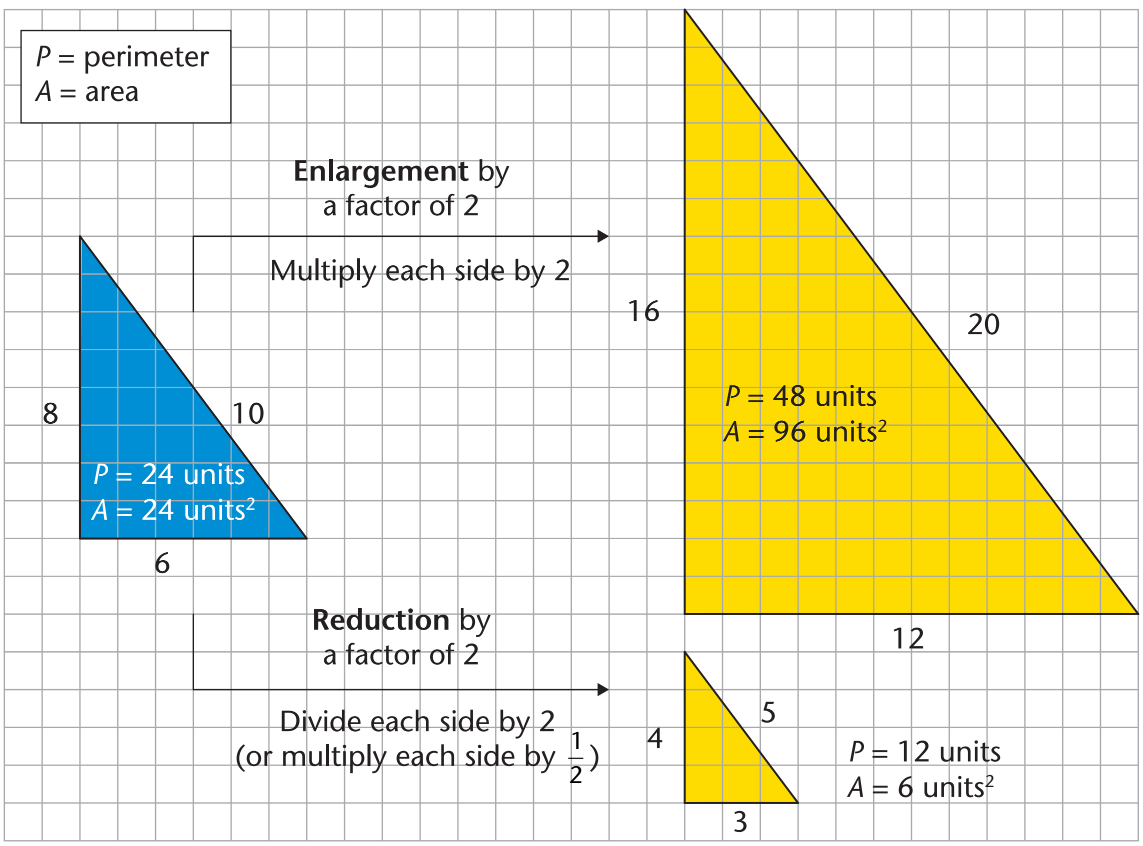
(a) (–2; –3) → (–2; –5)
(b) (4; –7) → (–6; 0)
(c) (3; 11) → (16; 20)
(d) (–1; –2) → (5; –4)
(e) (8; –11) → (–2; –3)

20.4 Enlargement (expansion) and reduction (shrinking)

WHAT ARE ENLARGEMENTS AND REDUCTIONS?

You will remember the following from Grade 8:

- An image is an enlargement or reduction of the original figure only if all the corresponding sides between the two figures are in **proportion**. This means that *all* the sides of the original figure are multiplied by the same number (the **scale factor**) to produce the image.
- Scale factor = $\dfrac{\text{side length of image}}{\text{length of corresponding side of original figure}}$
  - If the scale factor is > 1, the image is an enlargement.
  - If the scale factor is < 1, the image is a reduction.
- The original figure and its enlarged or reduced image are **similar**.
- Perimeter of image = Perimeter of original figure × scale factor
- Area of image = Area of original figure × (scale factor)^2



Sometimes the terminology used for enlargements and reductions can be confusing. Make sure you understand the following examples. Refer to the diagram on the previous page.

"**Enlarge** a figure by a scale factor of 2" means:

- $\frac{\text{side length of image}}{\text{length of corresponding side of original figure}} = 2.$
- Each side of the original figure must be *multiplied* by 2.
- Each side of the image will be two times *longer* than its corresponding side in the original figure.
- The perimeter of the image will be two times *longer* than the perimeter of the original figure.
- The area of the image will be 2^2 times ($2 \times 2 = 4$ times) *bigger* than the area of the original figure.

"**Reduce** a figure by a scale factor of 2" means:

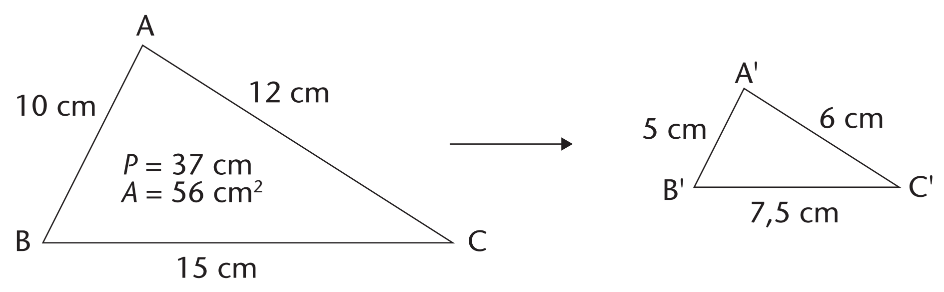
- $\frac{\text{side length of image}}{\text{length of corresponding side of original figure}} = 0{,}5.$
- Each side of the original figure must be *multiplied* by $\frac{1}{2}$ (or *divided* by 2).
- Each side of the image will be two times *shorter* than its corresponding side in the original figure.
- The perimeter of the image will be two times *shorter* than the perimeter of the original figure.
- The area of the image will be 2^2 times ($2 \times 2 = 4$ times) *smaller* than the area of the original figure. (Or, area of image = $(\frac{1}{2})^2 = \frac{1}{4}$ of the area of the original figure.)

Note that the multiplicative inverse of 2 is $\frac{1}{2}$.

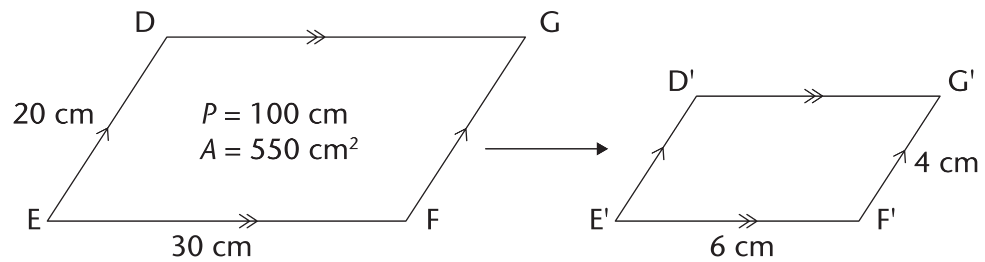
PRACTISE WORKING WITH ENLARGEMENTS AND REDUCTIONS

1. Work out the scale factor of each original figure and its image:

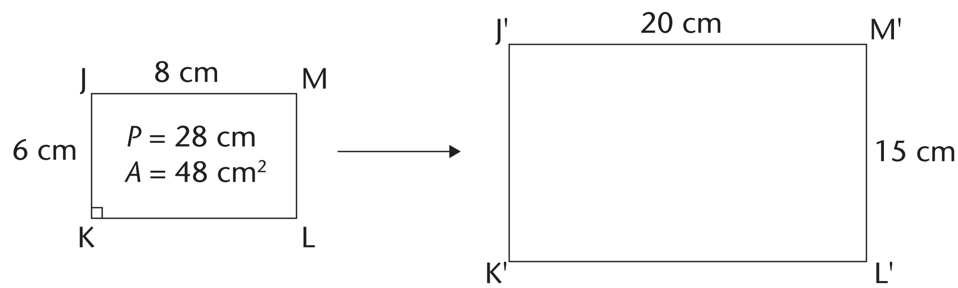
(a)



(b)



(c)



2. For each set of figures in question 1, write down by how many times the *perimeter* of each image is longer or shorter than the perimeter of the original image. Also write down the perimeter of each image.

3. For each set of figures in question 1, write down by how many times the *area* of each image is bigger or smaller than the area of the original image. Also write down the area of each image.

4. The perimeter of rectangle DEFG = 20 cm. Write down the perimeter of the rectangle D'E'F'G' if the scale factor is 3.

5. The perimeter of quadrilateral PQRS = 30 cm and its area is 50 cm^2.
   (a) Find the perimeter of P'Q'R'S' if the scale factor is $\frac{1}{5}$.
   (b) Determine the area of quadrilateral P'Q'R'S' if the scale factor is $\frac{1}{5}$.

6. The perimeter of ΔDEF = 17 cm and the perimeter of ΔD'E'F' = 25,5 cm.
   (a) What is the scale factor of enlargement?
   (b) What is the area of ΔD'E'F' if the area of ΔDEF = 14 cm^2?

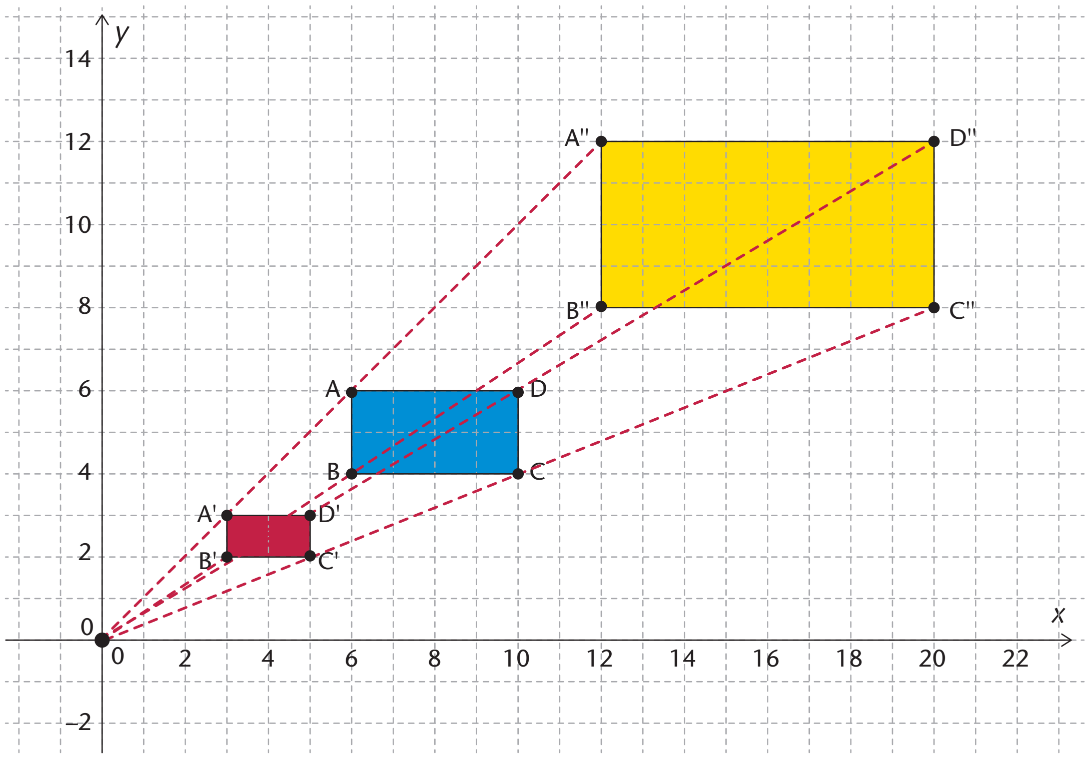
7. The area of ΔABC = 20 cm^2 and the area of ΔA'B'C' = 5 cm^2.
   (a) What is the scale factor of reduction?
   (b) What is the perimeter of the image if the perimeter of ΔABC = 22 cm?

INVESTIGATING ENLARGEMENT AND REDUCTION

When we do enlargements or reductions on a coordinate system, we use one point from which to perform the enlargement or reduction. This point is known as the **centre of enlargement or reduction.**

The centre of enlargement or reduction can be any point on the coordinate system. In this chapter, we will always use the **origin** as the centre of enlargement or reduction.

Rectangle ABCD, rectangle A'B'C'D' and rectangle A"B"C"D" are plotted on a coordinate system as shown below:



1. (a) Is rectangle A"B"C"D" an enlargement of rectangle ABCD? Explain your answer.
   (b) Is rectangle A'B'C'D' a reduction of rectangle ABCD? Explain your answer.
2. (a) The origin is the centre of enlargement and reduction. Draw four line segments to join the origin with A", B", C" and D".
   (b) What do you notice about these line segments?

3. (a) Copy the following table and list the coordinates of the images to complete it:

Vertices of ABCD	Vertices of A'B'C'D'	Vertices of A''B''C''D''
A(6; 6)		
B(6; 4)		
C(10; 4)		
D(10; 6)		

(b) What do you notice about the coordinates of the vertices of the original rectangle and the coordinates of the vertices of the image?

From the previous activity, you should have found the following:

On a coordinate system, the line that joins the centre of an enlargement or reduction to a vertex of the original figure also passes through the corresponding vertex of the enlarged or reduced image.

The coordinates of a vertex of the enlarged or reduced image are equal to the scale factor × the coordinates of the corresponding vertex of the original figure.

For example:

B(6; 4) → B' (3; 2): The coordinates of B' are $\frac{1}{2}$ the coordinates of B. Note that the scale factor is $\frac{1}{2}$.

B(6; 4) → B'' (12; 8): The coordinates of B'' are two times the coordinates of B. Note that the scale factor is 2.

In general, we therefore use the following notation to describe the enlargement or reduction with respect to the origin:

$(x; y) \rightarrow (kx; ky)$ or $(x'; y') = (kx; ky)$ where k is the scale factor.

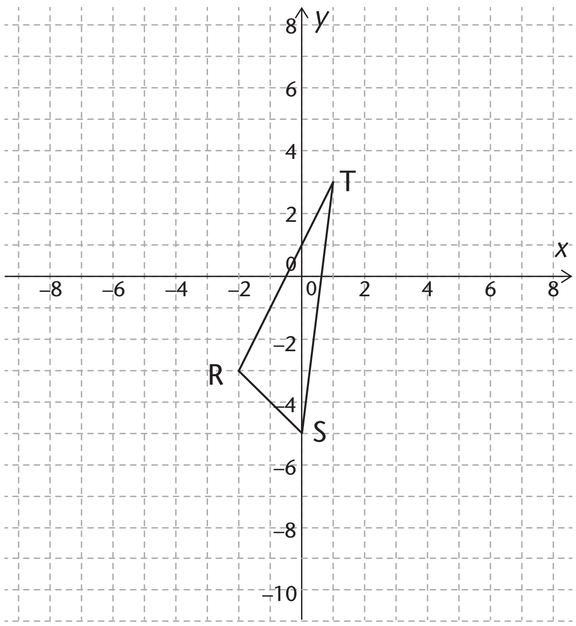
If $0 < k < 1$, the image is a reduction.

If $k > 1$, the image is an enlargement.

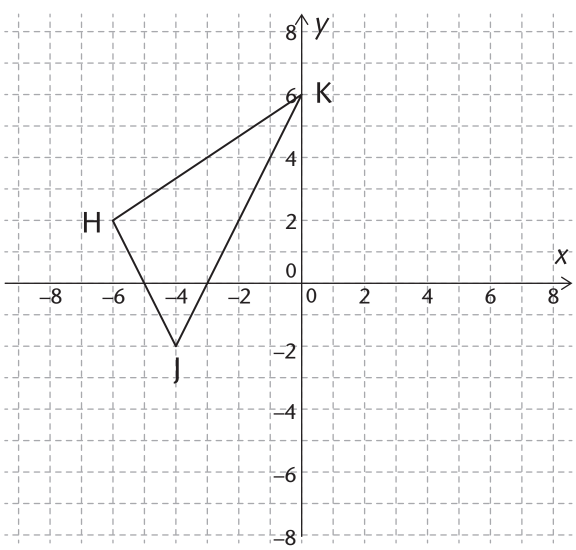
PRACTISE

1. On grid paper, draw the enlarged or reduced images of the following figures according to the scale factor given. In each case, use the **origin as the centre of enlargement or reduction**.

   (a) Scale factor = 2



   (b) Scale factor = $\frac{1}{2}$



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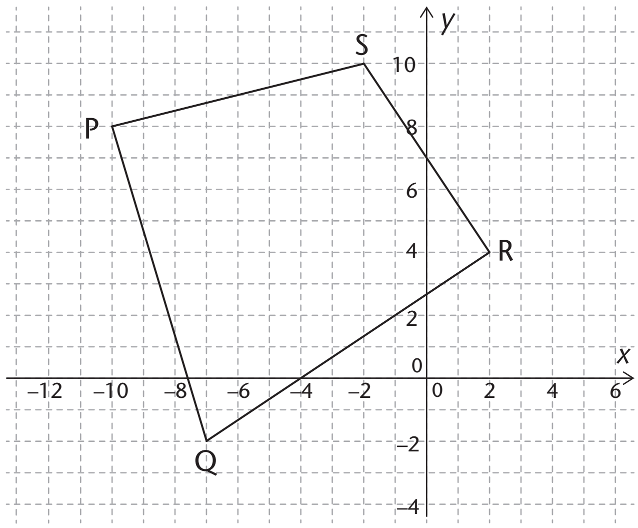
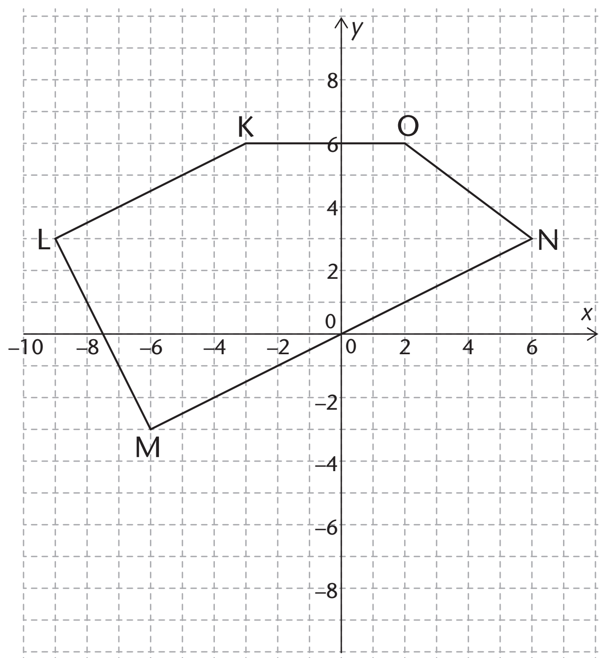
(c) Scale factor = $\frac{1}{2}$



(d) Scale factor = $\frac{1}{3}$



2. A quadrilateral has the following vertices: A(–2; 4), B(–4; –2), C(4; –3) and D(2; 1). Determine the coordinates of the enlarged image if the scale factor = 2.

3. A quadrilateral has the following vertices: P(–4; 0), Q(2,5; 4,5), R(6; –2,25) and S(2; –4). Determine the coordinates of the enlarged image if the scale factor = 4.

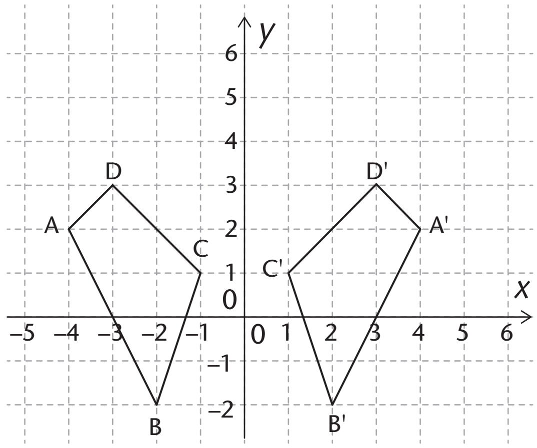
4. A quadrilateral has the following vertices: D(6; –4), E(4; –6), F(–4; 2) and G(–2; –2). Determine the coordinates of the reduced image if the scale factor = $\frac{1}{2}$.

5. A quadrilateral has the following vertices: K(8; –2), L(4; –6), M(–8; –4) and N(–6; 10). Determine the coordinates of the reduced image if the scale factor = $\frac{1}{4}$.

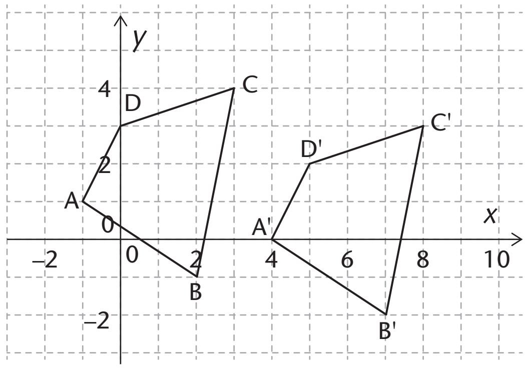
6. Describe the following transformations:
   (a) A(7; –5) → A'(9; 0)
   (b) A(–4; 6) → A'(4; 6)
   (c) A(–3; –2) → A'(–2; –3)
   (d) A(8; 1) → A'(8; –1)
   (e) A(4; –2) → A'(8; –4)
   (f) A(12; –16) → A'(3; –4)
   (g) A(2; –1) → A'(–3; –5)

7. Describe each of the following transformations:

(a)



(b)

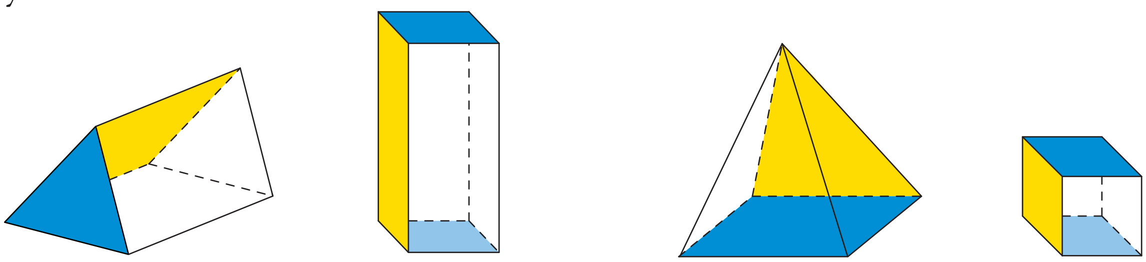


Chapter 21
Geometry of 3D objects

21.1 Classifying 3D objects

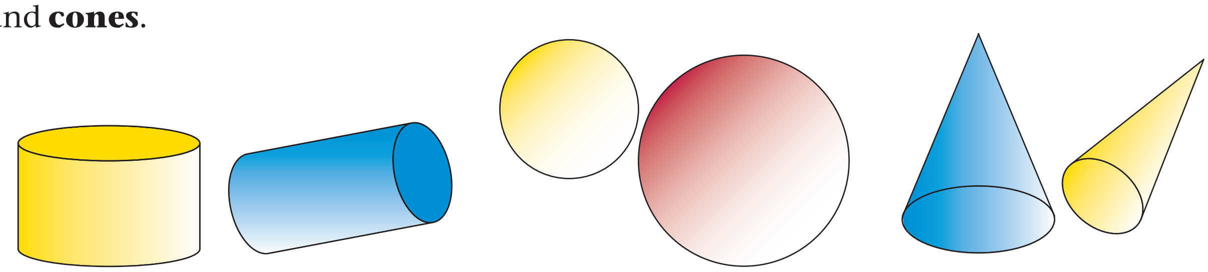
3D objects with only flat faces are called **polyhedra**. Prisms and pyramids are two types of polyhedra.

A polyhedron is a 3D object with only flat faces.



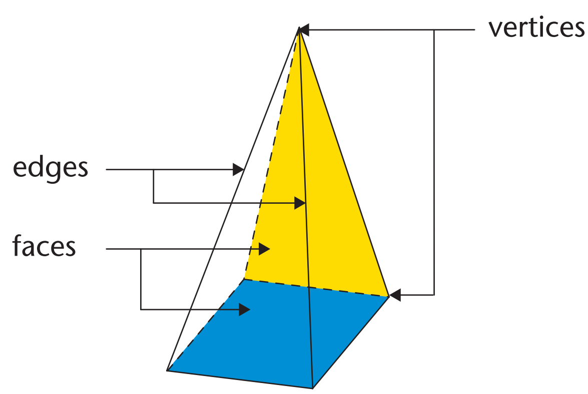
Triangular prism | Rectangular prism | Rectangular-based pyramid | Cube

Examples of 3D objects that have at least one curved surface are **cylinders**, **spheres** and **cones**.



Cylinders | Spheres | Cones

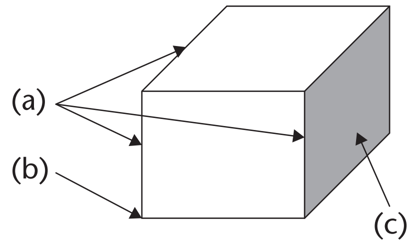
When we study the properties of a 3D object, we investigate the shapes of its faces, its number of faces, its number of vertices and its number of edges. For example, the pyramid alongside has one square face and four triangular faces, five vertices and eight edges.



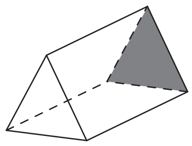
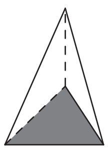
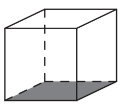
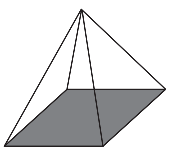
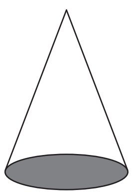
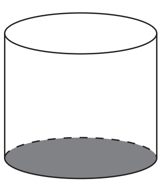
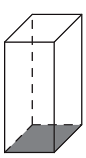
Square-based pyramid

CLASSIFYING AND DESCRIBING 3D OBJECTS

1. Write down the labels for parts (a) to (c) on the prism.



2. Copy and complete the following table:

	3D object	Name of the object	Number of faces and shape of faces	Number of vertices
(a)		Triangular prism	two triangles and three rectangles	6
(b)				
(c)			six squares	8
(d)			one rectangle and four triangles	5
(e)				
(f)				
(g)				

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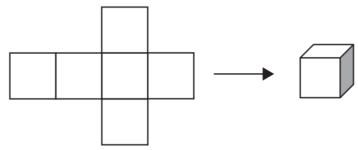
3. Say whether each statement below is true or false:
   (a) A cylinder is a polyhedron.
   (b) A triangular-based pyramid has four triangular faces.
   (c) A cube is also known as a hexahedron.
   (d) A triangular-based pyramid has six vertices.
   (e) A pyramid is a 3D object.

21.2 Nets and models of prisms and pyramids

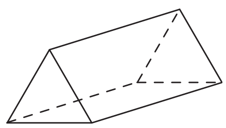
A **net** is a flat pattern that can be used to represent a 3D object. The net can be folded up to create a model of the 3D object.



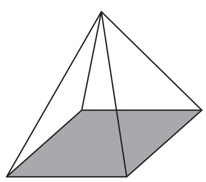
Net of a cube

1. Name each object below and match it with its net.

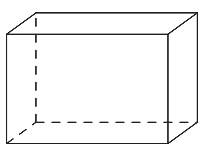
(a)



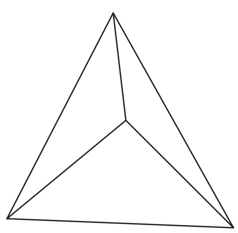
(b)



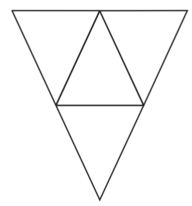
(c)



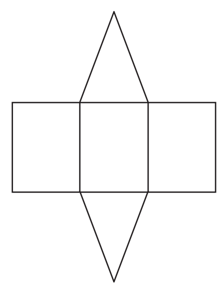
(d)



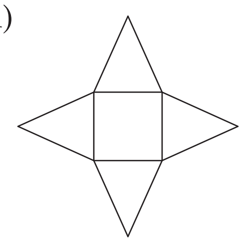
(i)



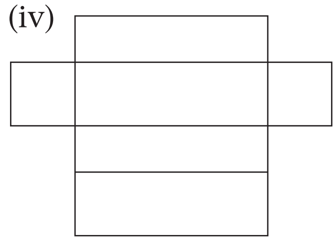
(ii)



(iii)

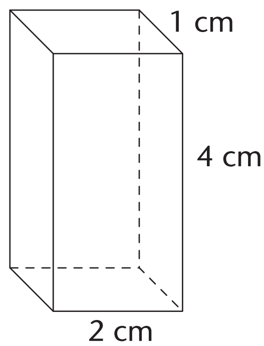


(iv)

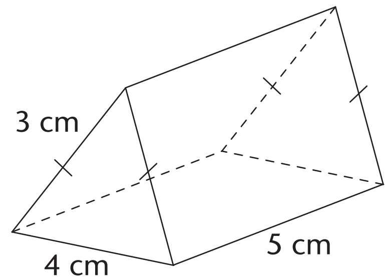


2. Construct an accurate net for each of the following 3D objects:

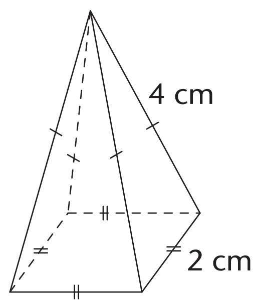
(a)



(b)



(c)

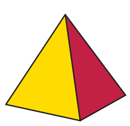


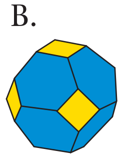
3. Construct models of the objects in question 2, but double all the measurements.

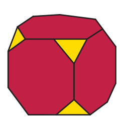
21.3 Platonic solids

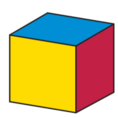
A **Platonic solid** is a 3D object which has identical faces, and all of the faces are identical regular polygons. This means that all its faces are the same shape and size and all the vertices are identical.

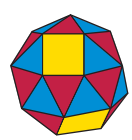
1. Which of the following objects are Platonic solids?

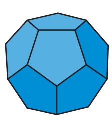
A.


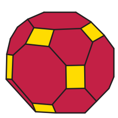
B.


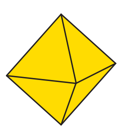
C.


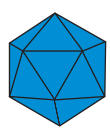
D.


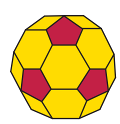
E.


F.


G.


H.


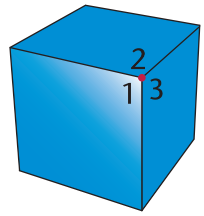
I.


J.


2. How many Platonic solids are there in question 1?

ONLY FIVE PLATONIC SOLIDS?

You can use your knowledge about angles to prove that the five Platonic solids are the only 3D objects that can be made from identical regular polygons. Keep the following facts in mind:



- A 3D object has *at least* three faces that meet at each vertex.
- The sum of the angles that meet at a vertex must be less than 360°. If it is equal to 360°, it will form a flat surface. If it is greater than 360°, the faces will overlap.
- Each Platonic solid is made up of one type of regular polygon only.

What 3D objects can you make from equilateral triangles?

We use the following reasoning:

Size of each interior angle = 180° ÷ 3 = 60°

$\therefore$ three triangles = 3 × 60° = 180° [< 360°]

four triangles = 4 × 60° = 240° [< 360°]

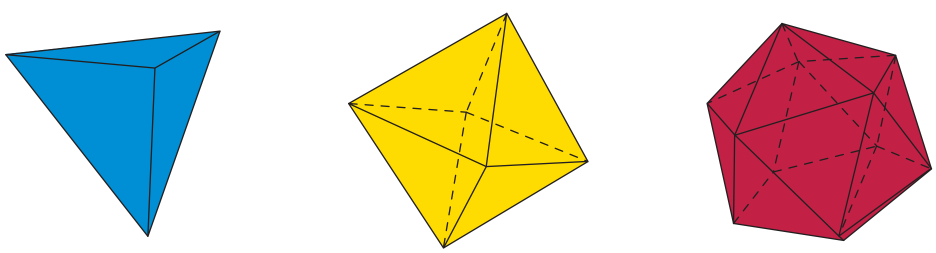
five triangles = 5 × 60° = 300° [< 360°]

six triangles = 6 × 60° = 360°

Any more than five triangles will be equal to or more than 360° and will therefore form a flat surface or overlap.

This means that we can make three 3D objects from equilateral triangles:

- If three triangles are at each vertex, it will form a **tetrahedron**.
- If four triangles are at each vertex, it will form an **octahedron**.
- If five triangles are at each vertex, it will form an **icosahedron**.



Tetrahedron Octahedron Icosahedron

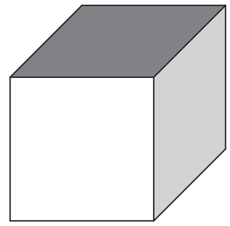
What 3D objects can you make from squares?

Complete the statements:

Size of each interior angle

∴ three squares = 3 ×

four squares = 4 ×



Hexahedron (cube)

Therefore, we can make only one 3D object using squares. This 3D object is called a **hexahedron** (**or cube**).

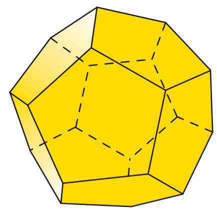
What 3D objects can you make from regular pentagons?

Complete the statements:

Size of each interior angle

∴ three pentagons =

four pentagons =



Dodecahedron

Therefore, we can make only one 3D object using regular pentagons. This 3D object is called a **dodecahedron**.

What 3D objects can you make from regular hexagons?

Complete the statements:

Size of each interior angle

∴ three hexagons =

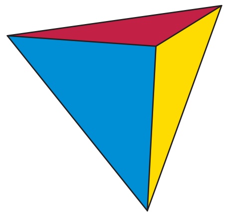
Three hexagons will already form a flat surface. Therefore, it is impossible to make a 3D object from regular hexagons.

Also, the interior angles of polygons with more than six sides are bigger than those of a hexagon, so it is not possible to make 3D objects from any other regular polygons.

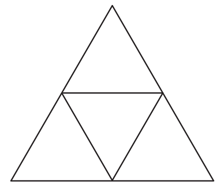
Therefore, the five Platonic solids already mentioned (tetrahedron, octahedron, icosahedron, hexahedron and dodecahedron) are the only ones that can be made of identical regular polygons. Each of these solids is named after the number of faces it has.

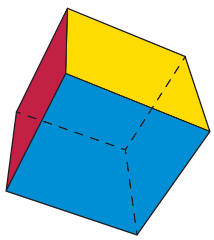
PROPERTIES OF THE PLATONIC SOLIDS

Copy and complete the information about each of the following Platonic solids:

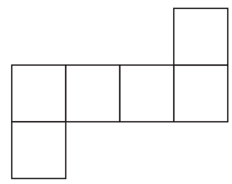
1. 

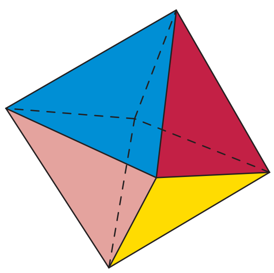
Name:
Shape of the faces:
Number of faces:
Number of edges:
Number of vertices:



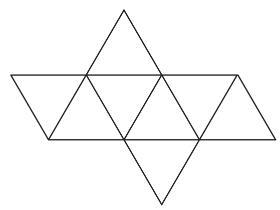
2. 

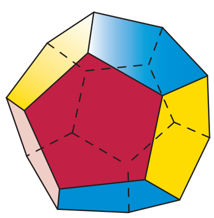
Name:
Shape of the faces:
Number of faces:
Number of edges:
Number of vertices:



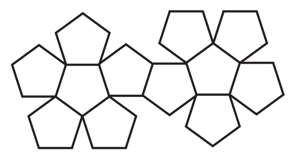
3. 

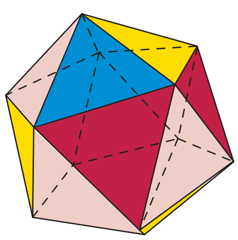
Name:
Shape of the faces:
Number of faces:
Number of edges:
Number of vertices:



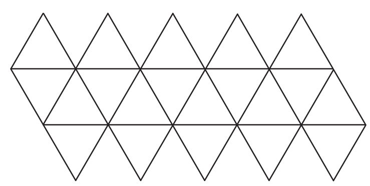
4. 

Name:
Shape of the faces:
Number of faces:
Edges:
Vertices:



5. 

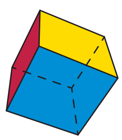
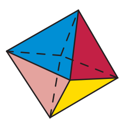
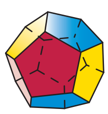
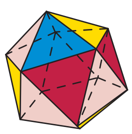
Name:
Shape of the faces:
Number of faces:
Edges:
Vertices:



21.4 Euler's formula

EULER'S FORMULA AND PLATONIC SOLIDS

1. You learnt about Euler's formula in Grade 8. Copy and complete the following table to investigate whether or not Euler's formula holds true for Platonic solids:

	Name	Shape of faces	No. of faces (F)	No. of vertices (V)	No. of edges (E)	F + V – E
						
						
						
						
						

2. Complete Euler's formula for polyhedra: F +

3. Apply Euler's formula to each of the following:
   (a) A polyhedron has 25 faces and 13 vertices. How many edges will it have?
   (b) A polyhedron has 11 vertices and 23 edges. How many faces does it have?
   (c) A polyhedron has eight faces and 12 edges. How many vertices does it have?

EULER'S FORMULA AND OTHER POLYHEDRA

1. Is each of the following statements true or false?
   (a) A polyhedron with ten vertices and 15 edges must have seven faces.
   (b) A polyhedron will always have more edges than either faces or vertices.

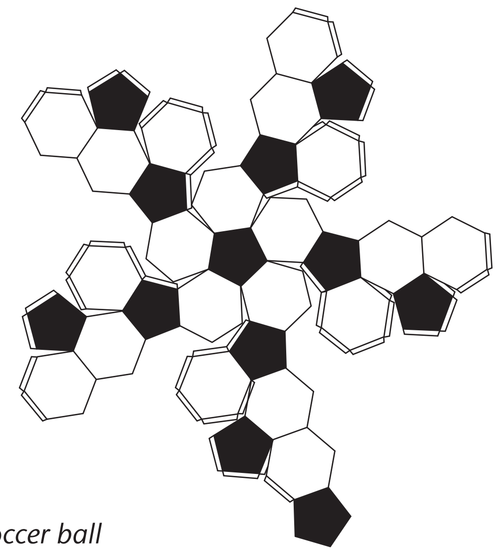
(c) A polyhedron with five faces must have six edges.

(d) A pyramid will always have the same number of faces and vertices.

2. Copy and complete the following table:

	No. of faces (F)	No. of vertices (V)	No. of edges (E)	Name of polyhedron	Shapes of faces
(a)	6		12		Rectangles
(b)		7		Hexagonal pyramid	
(c)	4	4			
(d)	5	6	9		Triangles and rectangles

3. A soccer ball consists of pentagons and hexagons.
   (a) How many pentagons does it consist of?
   (b) How many hexagons does it consist of?
   (c) How many edges does it have?
   (d) How many vertices does it have?
   (e) Does Euler's formula apply to soccer balls too?

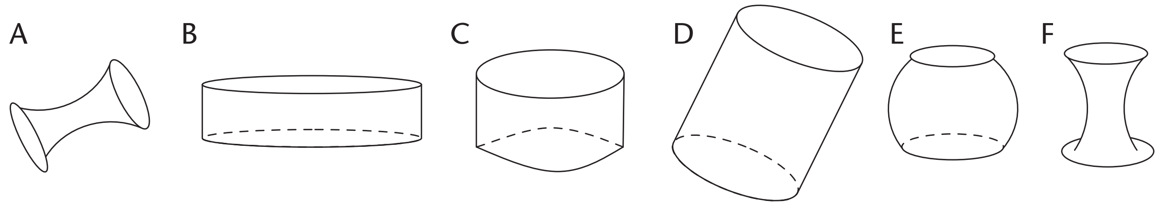


Net of a soccer ball

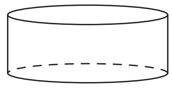
21.5 Cylinders

PROPERTIES OF CYLINDERS

1. Which of the following 3D objects are cylinders?

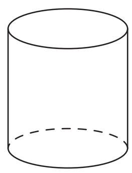


2. Write down the statement or statements below that are true only for cylinders and not for the other objects shown in question 1:



- ☐ It is a 3D object.
- ☐ It has a curved surface.
- ☐ It has two circular bases that are parallel to each other.
- ☐ It has two flat circular bases and a curved surface.
- ☐ The radius of its curved surface is equal from the top to the bottom between the bases.
- ☐ It has two circular bases opposite each other, joined by a curved surface whose radius is equal from the top to the bottom between the bases.

3. Look at the cylinder alongside and write down the:



   (a) number and shape of faces:

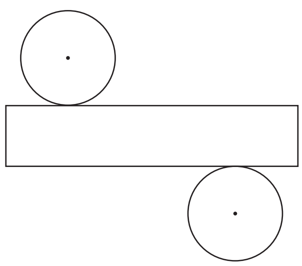
   (b) number of vertices:

   (c) number of edges:

NETS OF CYLINDERS

In Chapter 19, you learnt about the net of a cylinder. If you cut the curved surface of a cylinder vertically and flatten it, it will be the shape of a rectangle.

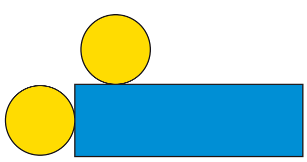
1. Explain why the length of the rectangular face is equal to the circumference of the base.



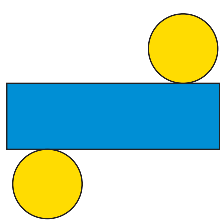
2. Will each of the following nets form a cylinder?

A.

B.



C.



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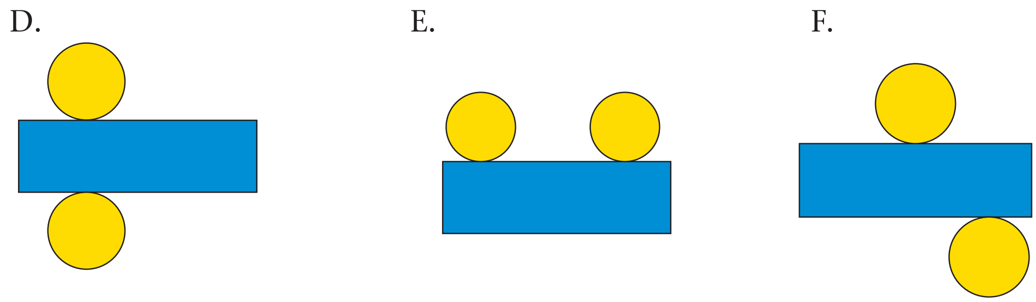


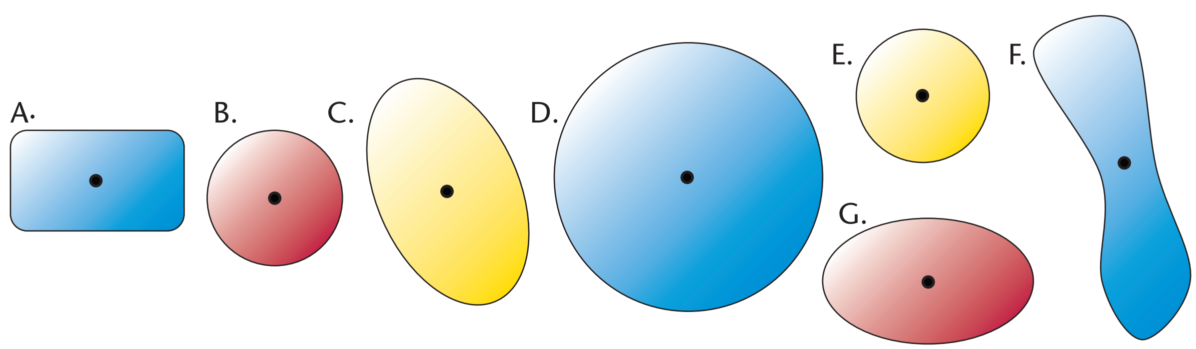
3. In each of the following questions, use $\pi = \frac{22}{7}$ and round off your answer to two decimal places to do the calculations.
   (a) If the radius of a cylinder is 3 cm, what is the length of the rectangular surface of the cylinder?
   (b) If the radius of a cylinder is 5 cm, what is the length of the rectangular surface of the cylinder?
   (c) If the diameter of a cylinder is 8 cm, what is the length of the rectangular surface of the cylinder?
   (d) If the diameter of a cylinder is 9 cm, what is the length of the rectangular surface of the cylinder?

4. Use a ruler and a set of compasses to construct the following nets as accurately as possible. Show the measurements on each net.
   (a) Net of a cylinder with a radius of 1 cm and a height of 4 cm
   (b) Net of a cylinder with a radius of 1,5 cm and a height of 3 cm

5. Construct models of the cylinders in question 4 but double the measurements.

21.6 Spheres

1. Which of the following 3D objects are spheres?

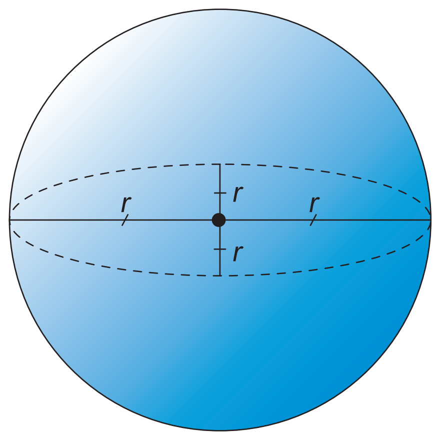


2. Write down the property or properties below that are true for spheres only and not for the other objects shown in question 1:
   - ☐ It is a 3D object.
   - ☐ It has one curved surface.
   - ☐ It has no bases.
   - ☐ It has no vertices.
   - ☐ It has no edges.
   - ☐ The distance from its centre to any point on its surface is always equal.

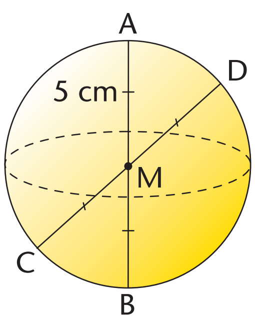
3. Complete the following information for a sphere:
   (a) Number and shape of faces:
   (b) Number of vertices:
   (c) Number of edges:

From your study of spheres in the above activity, you should have found the following:

A **sphere** is a round 3D object with only one curved surface and the distance from its centre to any point on its surface is always equal. It has no vertices or edges.

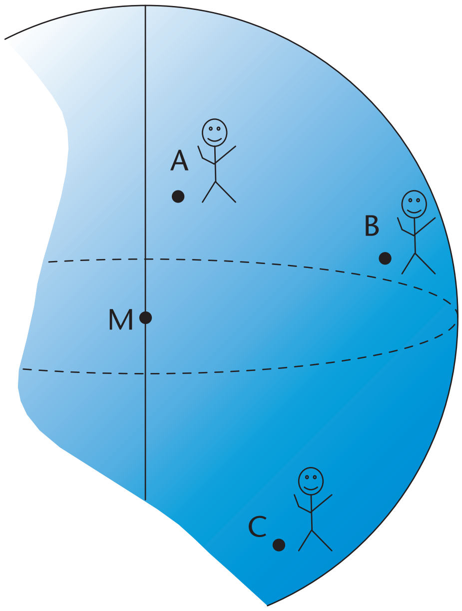


4. Trace the sphere alongside, and write down the length of:
   (a) the radius:
   (b) the diameter:
   (c) MD:
   (d) CD:



5. The drawing on the following page shows part of a sphere with a diameter of 100 km. Imagine that you are at point M, at the centre inside the sphere. People A, B and C are all at different places on the surface of the sphere.

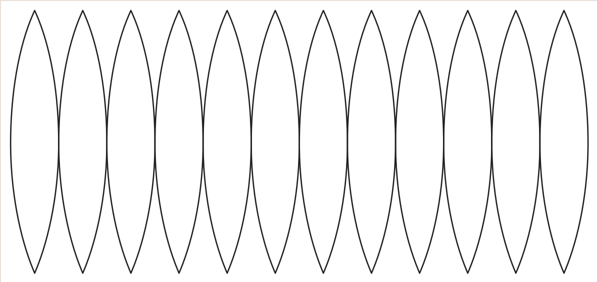
(a) Which of the people – A, B or C – is closest to you?
(b) How far away is person C from you?



NET OF A SPHERE

It is impossible to make a perfect sphere (ball or globe) from a flat sheet of paper. Paper can curve in one direction, but cannot curve in two directions at the same time. So, all spheres made from paper or card will be approximations. This is the best net we can make of a sphere.

Can you make your own paper model of a sphere?

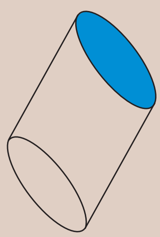


WORKSHEET

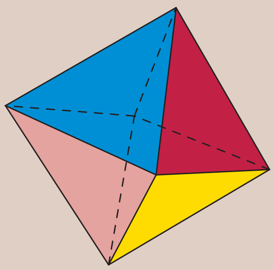
1. Grade 9 learners were asked to represent a 3D object and give the class clues as to which polyhedron they represent. Name their objects:
   (a) Amy: I have six faces and they are all the same size.
   (b) John: I have six faces and 12 edges. I am not a cube.
   (c) Onke: I have three faces. I also have two edges.
   (d) Tessa: I have eight edges and I have five vertices.
   (e) Mandlakazi: I have six edges and four vertices.
   (f) Chiquita: I have eight faces and I am a Platonic solid.
   (g) Seni: I do not have any edges.
   (h) Mpu: My faces are made only of regular pentagons.

2. Copy the table and write down the required information about each object below.

A.



B.



	Object A	Object B
Name		
Number of faces		
Shape/s of faces		
Number of edges		
Number of vertices		
Does Euler's formula work?		
Is it a Platonic solid?		

3. (a) On a separate sheet of paper, construct a net of a cylinder with a diameter of 7 cm and a height of 10 cm.
   (b) Fold your net to make a model of the cylinder.

Chapter 22
Collect, organise and summarise data

22.1 Collecting data

Avoiding bias when selecting a sample

The methods that we use to collect data must help us to make sure that the data is reliable. This means that it is data that we can trust.

Data cannot be trusted unless it has been collected in a way that makes sure that every member of the population had the same chance of being selected in the sample.

It is not practical to taste all the oranges on a tree to know whether the oranges are sweet. Only a small number of oranges can be tested, otherwise the farmer would have too few oranges to sell. The oranges that are tested are called a **sample**, and all the oranges harvested from the tree are called the **population**.

Sample bias occurs when the particular section of the population from which the sample is drawn does not represent that population. The way to avoid sample bias is to take a **random** sample. A sample is random if **every member of the population has the same chance** of being selected. A random sample of the orange trees means that every tree should have a chance of being selected for the sample. Every person in the country should have a chance of being selected for the housing survey in a random sample.

An example of sample bias would be to survey only the people in Limpopo about their views on housing provision when you want to know the views of the whole country. For the sample to provide information on the population as a whole, each person in the country should have the same chance of being part of the survey.

Data can be collected through questionnaires, through observation and through access to databases.

How to develop a good questionnaire

The questionnaire also has an important role in making sure that the information you collect is reliable. You should aim to get a high number of respondents and accurate information. If not enough people fill in the questionnaire, then you won't know whether the information you get reflects the real situation. Sampling techniques and rules developed by statisticians determine the numbers needed.

There are some important points to consider when designing a questionnaire. Two of the most important points are that the questions are **clear and accurate** and that people find the questionnaire relatively **easy to complete**.

1. Keep in mind the length of the questionnaire and the time that it takes to complete. Your participants are more likely to complete a short questionnaire that is quick and easy to complete. Exclude unnecessary information.
2. Write down a selection of questions that you think will provide the information that you want.
3. Check the wording for each question.
4. Order the items so that they are in a logical sequence. It might make sense to have the easiest questions first, but in some cases the more general questions should come first and the more specific questions towards the end of the questionnaire.
5. Then try the questionnaire out on a partner. Ask the following questions:
   - Is this question necessary? What information will be provided by the answer?
   - How easy will it be for the respondent to answer this question? How much time will it take to answer the question?
   - Do the questions ask for sensitive information? Will people want to answer the question? Will the respondent answer the question honestly?
   - Can the question be answered quickly?
6. Decide how the answers should be provided. Questions may require **open-ended** responses or **closed-ended** responses, as described below.

In an **open-ended** question, the person responds in his or her own words. Through his or her own words important information can be gained; the person is therefore free to write what he or she likes. A disadvantage is that you might not get the information you want and that it might take a long time to answer.

In a **closed-ended** question, the respondents are given some options to choose from. They tick the box which most closely represents their response. These options can be constructed in categories. For example, age may be categorised as follows:

Under 10 ☐   From 10 to 14 ☐   From 15 to 19 ☐   20 and older ☐

THINK ABOUT DATA COLLECTION AND DEVELOP A QUESTIONNAIRE

1. Which method for collecting data would be most appropriate for each of the cases below? Give reasons for your choice.
   (a) The number of learners who bring lunch to school. What are the contents of the school lunch?
   (b) Whether or not the tellers at a supermarket chain are happy with their conditions of work.
   (c) Whether or not the clients of a clinic are satisfied with the professional conduct of the medical staff.
   (d) The types of activities preschool children choose during their free time.
   (e) The number of Grade 9 learners in the Gauteng North district.

2. You are doing some market research for a new fast-food shop near the high school. The owners of the shop want to find out what kind of food and music the target market likes. The target market is learners from the high school. Develop a questionnaire to collect this information.

22.2 Organising data

There is a difference between **data** and **information**. Data is unorganised facts. When data is organised and analysed so that people can make decisions, it may be called information. Data can be organised in many different ways. Some methods are described below.

Data can be organised by making a **tally table**. Here is an example of a tally table showing the numbers of learners in a class that participate in different sports:

Sport	Tally marks
Soccer	卌 卌 卌 卌 卌
Athletics	卌 ///
Netball	卌 卌 卌 卌 /
Chess	卌 /

The above data can also be organised in a **frequency table**:

Sport	Frequency
Soccer	25
Athletics	8
Netball	21
Chess	6

Numerical data sets with many items are often grouped into equal **class intervals** and represented in a table of frequencies for the different class intervals. This is very useful since it makes it easy to see how the data is spread out.

Here is an example of grouped data showing the heights of all the learners in a school. To make a frequency table for numerical data, the data has to be arranged from smallest to biggest first.

Height in m	Number of learners (frequency)
< 1,20 m	13
1,20 m – 1,30 m	28
1,30 m – 1,40 m	57
1,40 m – 1,50 m	164
1,50 m – 1,60 m	274
1,60 m – 1,70 m	198
1,70 m – 1,80 m	73
> 1,80 m	13

A value equal to the **lower boundary** of a class interval is counted in that interval.
For example, a length of 1,60 m is counted in the interval 1,60 – 1,70, and not in the interval 1,50 – 1,60 m.
However, 1,599 m is less than 1,60 m, so it belongs in the interval 1,50 m – 1,60 m.

A **stem-and-leaf display** is a useful way to organise numerical data. It also shows you what the "shape" of the data is like. Here is an example of a stem-and-leaf display:

Key: 35 | 4 means 354

34	0 4
35	4 8 8
36	0 1 6 8
37	1 3 5 8 8 8 9
38	2 4 9
39	0 3 4 4 5 6 9
40	0 3 7
41	1

The above stem-and-leaf display represents the following data about the masses in grams of the chickens in a sample of six-week-old chickens on a chicken farm:

399	378	382	360	396	389	344	411	378	394
394	354	375	378	400	371	379	358	366	403
358	395	390	340	393	384	361	407	373	368

To make a stem-and-leaf display, it helps to first arrange the data from smallest to largest, as shown on the next page, for the above data set.

340	344	354	358	358	360	361	366	368	371
373	375	378	378	378	379	382	384	389	390
393	394	394	395	396	399	400	403	407	411

The same data set is displayed in two slightly different ways below:

			379		399		
			378		396		
			378		395		
		368	378		394		
	358	366	375	389	394	407	
344	358	361	373	384	393	403	
340	354	360	371	382	390	400	411

In this display, the width of each class interval is 10, as in the stem-and-leaf display above.

		384		
		382	399	
		379	396	
	368	378	395	
	366	378	394	
	361	378	394	411
354	360	375	393	407
344	358	373	390	403
340	358	371	389	400

In this display, the width of each class interval is 15.

WORKING WITH GROUPED DATA

1. An organisation called Auto Rescue recorded the following numbers of calls from motorists each day for roadside service during March 2014:

28	122	217	130	120	86	80	90	120	140
70	40	145	187	113	90	68	174	194	170
100	75	104	97	75	123	100	82	109	120
81									

Set up a tally and frequency table for this set of data values, in intervals of width 40.

2. When geologists go out into the field they make sure they have their rulers and measurement instruments in their bags. They also have their "inbuilt rulers", for example their handspans. A handspan is the distance from the tip of the thumb to the tip of the fifth finger on an outstretched hand. Measure your handspan against the ruler! This frequency table shows the handspans of different Grade 9 learners, in cm.

Handspan of Grade 9 learners (cm)	Frequency
15–18	7
18–21	9
21–24	10
24 and greater	4

(a) How many learner handspans were measured altogether?
(b) How many learner handspans are less than 21 cm wide?
(c) How many handspans are 18 cm or wider?
(d) In which interval would you place a handspan of 18 cm?

22.3 Summarising data

The mean, median, mode and range are single numbers that provide some information about a data set, without listing all the data values.

The **mode** is the value that occurs most frequently. To find the mode, look for the number or category that is listed in the data set most often. Some data sets have more than one mode, and some may have none.

The **median** is the number that separates the set of values into an upper half and a lower half. The median can be found by arranging the values from small to big or big to small. If the data set consists of an even number of items, the median is the sum of the two middle values divided by 2.

The **mean** (average) of a set of numerical data is the sum of the values divided by the number of values in the data set.

Mean = the sum of the values ÷ the number of values.

The **range** is a number that tells us how spread out the data values are. It is the difference between the largest and smallest values.

The mean, median and mode do not work equally well for all sets of data. It depends on the kind of data, and also on whether the data is evenly spread out or not.

ORGANISE, SUMMARISE AND COMPARE SOME DATA

1. A researcher analyses data about the people who are suffering from three different types of the flu virus: A, B and C. The ages of the people in the different groups are:

   Type A: 60, 80, 75, 87, 88, 49, 94, 84, 59, 86, 82, 62, 79, 89 and 78.

   Type B: 27, 39, 43, 29, 36, 70, 56, 25, 54, 36, 66, 45, 33, 46, 14 and 41.

   Type C: 33, 48, 64, 15, 31, 20, 70, 21, 18, 49, 21, 19, 57, 23, 29 and 20.

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For each group:

- Draw a stem-and-leaf plot.
- Calculate the range, mean and median of the ages.
- Look at the shape of the stem-and-leaf displays as well as the summary measures. Discuss the spread of the data in each case, and compare the three different groups.

Work and report on your work.

2. Copy the table and fill in the statistic (mode, mean or median) that would best summarise each data set, and indicate the central tendency of the data:

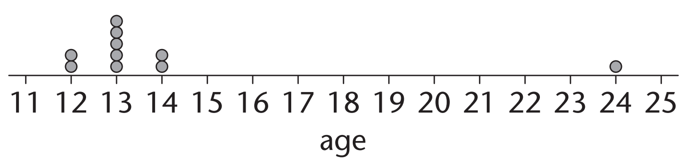
Data set	Best measure of central tendency
The shoe sizes of boys in Grade 9	
An evenly spread set of measurement values, such as the heights of learners in a class	
A set of measurement values with a few very low values and mostly high values	
The number of siblings each person in your class has	
The sizes of properties in a town, where most people live in small apartments or RDP houses, and a few live on large properties	

EXTREME VALUES OR OUTLIERS

An **extreme value** or **outlier** is a data value that lies an abnormal distance from other values in a random sample from a population. Sometimes there are reasons why this data value is so different to the others. It is important to comment on the possible reasons.

When you are summarising data (and also when you analyse data), you need to decide whether or not an outlier makes sense in the context you are looking at.

It is possible that an outlier does not make sense, as it lies too far away and is an unreasonable measurement. Then you need to think about the fact that this data value may be an error. For example:



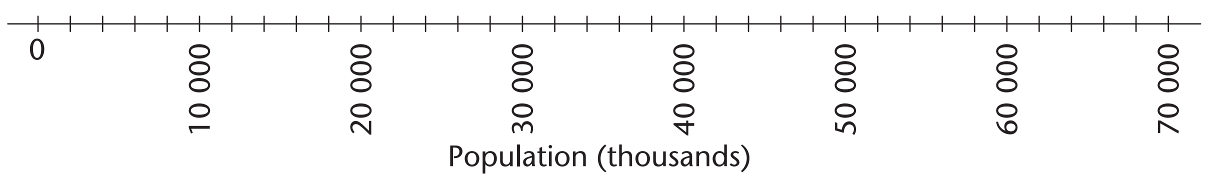
In this case, the value of 24 years old could be an unreasonable value. This depends on the context of the survey.

You will learn more about extreme values and outliers in Chapter 24.

Use this information about 14 countries to answer the questions that follow:

Country	Total population (in 1 000s)	Total annual national income per person (US$)	Percentage of income spent on health
Angola	18 498	4 830	4,6
Botswana	1 950	13 310	10,3
DRC	66 020	280	2,0
Lesotho	2 067	1 970	8,2
Malawi	15 263	810	6,2
Mauritius	1 288	12 580	5,7
Mozambique	22 894	770	5,7
Namibia	2 171	6 250	5,9
Seychelles	84	19 650	4,0
South Africa	50 110	9 790	8,5
Swaziland	1 185	5 000	6,3
Tanzania	43 739	1 260	5,1
Zambia	12 935	1 230	4,8
Zimbabwe	12 523	170	Not available

1. Look at the total population for each country.
   (a) Calculate the mean of the data.
   (b) Copy the number line below and draw a dot plot on the number line to show the data.



   (c) Find the median of the data.
   (d) What is the range of the data?
   (e) Which measure of central tendency do you think represents the data more accurately? Explain.

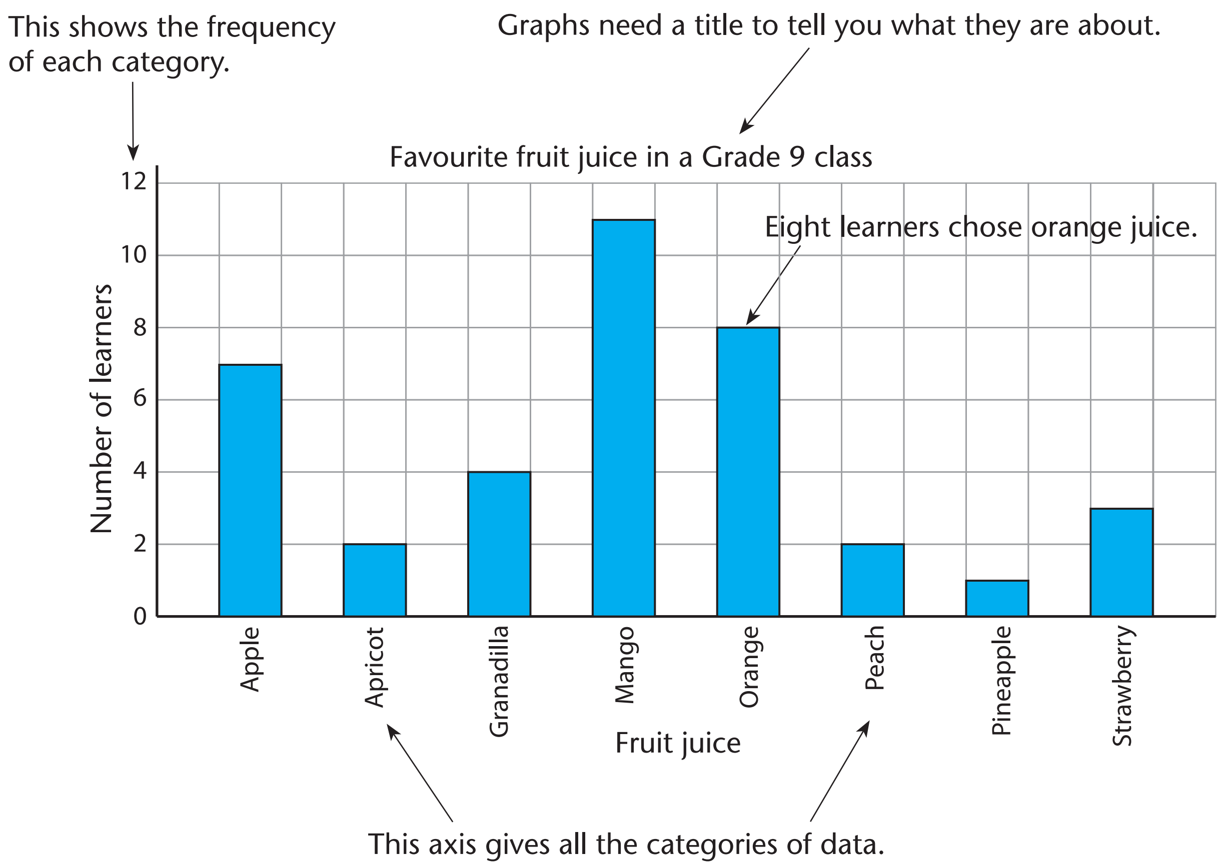
2. Look at the *Total annual national income per person in US dollars* column. Comment on the spread of the data.

CHAPTER 23
Representing data

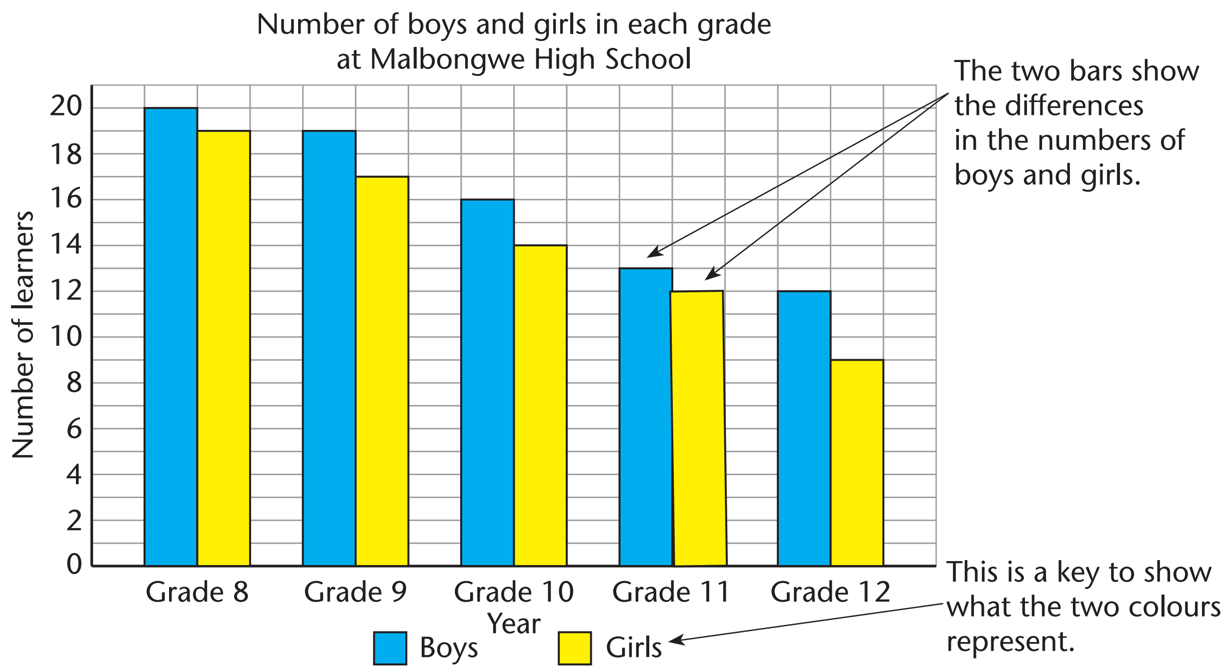
23.1 Bar graphs and double bar graphs

REVISING BAR GRAPHS AND DOUBLE BAR GRAPHS

A **bar graph** shows categories of data along the horizontal axis, and the frequency of each category along the vertical axis. An example is given below.



A **double bar graph** shows two sets of data in the same categories on the same set of axes. This is useful when we need to show two groups within each category.



DRAWING BAR GRAPHS AND DOUBLE BAR GRAPHS

1. Obese (very overweight) people have many health problems. It is a concern all around the world. Health researchers analysed the change over 28 years in the numbers of people who are overweight and obese in different areas of the world. The following table summarises some of the data:

   Percentage of population that is overweight and obese

	1980	**2008**
Sub-Saharan Africa	12%	23%
North Africa and Middle East	33%	58%
Latin America	30%	57%
East Asia (low-income countries)	13%	25%
Europe	45%	58%
North America (high-income countries)	43%	70%

   (a) The table summarises "some" of the data. What would some other important data be? Think of as many things as you can.
   (b) Which data stands out the most for you in the table above? Give your personal opinion.
   (c) On grid paper, plot a double bar graph to compare the data for the areas, and for the two years. Remember to give your graph a key.
   (d) Look carefully at the comparisons that the graph makes. Has your opinion of the most interesting differences changed, now that you see the double bar graph? Explain.

(e) In some countries, the obesity problem has been labelled "Obesity in the face of poverty". Write a short report on the data and your double bar graph to support this argument.

23.2 Histograms

REVISING HISTOGRAMS

A histogram is a graph of the frequencies of data in different **class intervals**, as demonstrated in the example below. Each class interval is used for a range of values. The different class intervals are consecutive and cannot have values that overlap. The data may result from counting or from measurement.

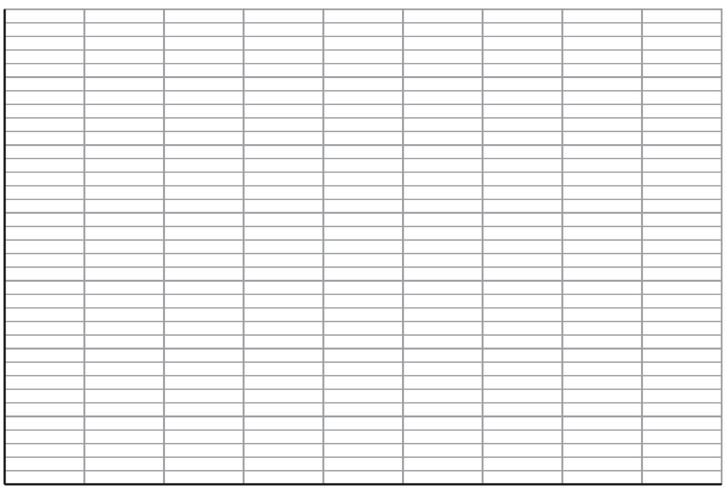
A histogram looks somewhat like a bar graph, but is normally drawn without gaps between the bars.

REPRESENTING DATA IN HISTOGRAMS

1. (a) A fruit farmer wants to know which of his trees are producing good plums and which trees need to be replaced. He collects 100 plums each from two trees and measures their masses. The data below gives the mass of plums from the first tree:

Mass of plums (g)	20–29	30–39	40–49	50–59	60–69
Frequency	6	18	34	30	12

Copy the grid below and use it to represent the data in a histogram.



(b) Now draw another histogram to represent the following data giving the mass of the same type of plums from another tree in the orchard:

Mass of plums (g)	20–29	30–39	40–49	50–59	60–69
Frequency	3	14	26	36	21

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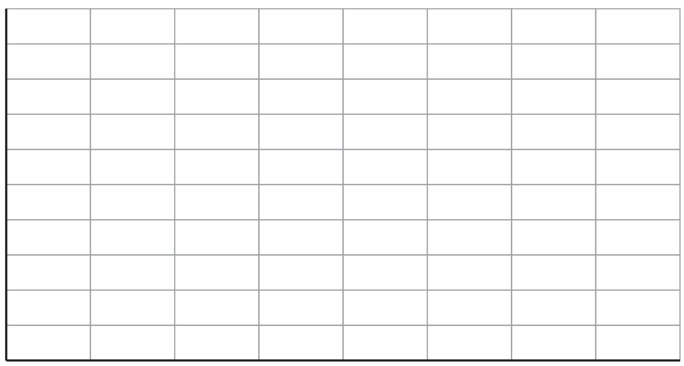
(c) Study the two histograms and then comment on the number of plums produced by the two trees.

2. (a) Use the example below to draw a histogram to represent the data in the table below. Group the data in intervals of 0,5 kg.

Birth weights (kg) of 28 babies at a clinic

3,3	1,34	2,88	2,54	1,87	2,06	2,72
1,89	0,85	1,99	2,43	1,66	2,45	1,62
1,91	1,20	2,45	1,38	0,9	2,65	2,88
1,75	2,11	3,2	1,74	0,6	3,1	1,86



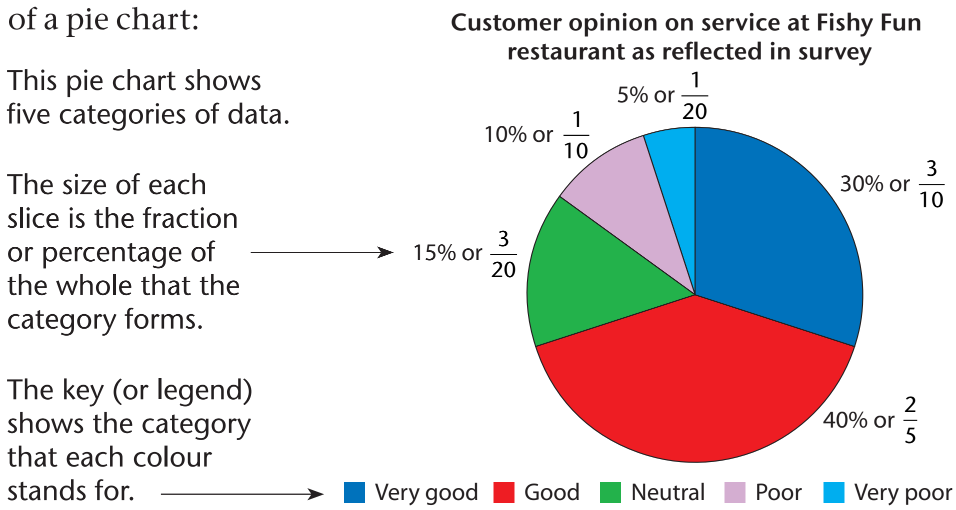
(b) Calculate the mean and median of the data.

(c) Records from the whole country show that the birth weight of babies ranges from 0,5 kg to 4,5 kg, and the mean birth weight is 3,18 kg. Use the graph and the mean and median to write a short report on the data from the clinic.

23.3 Pie charts

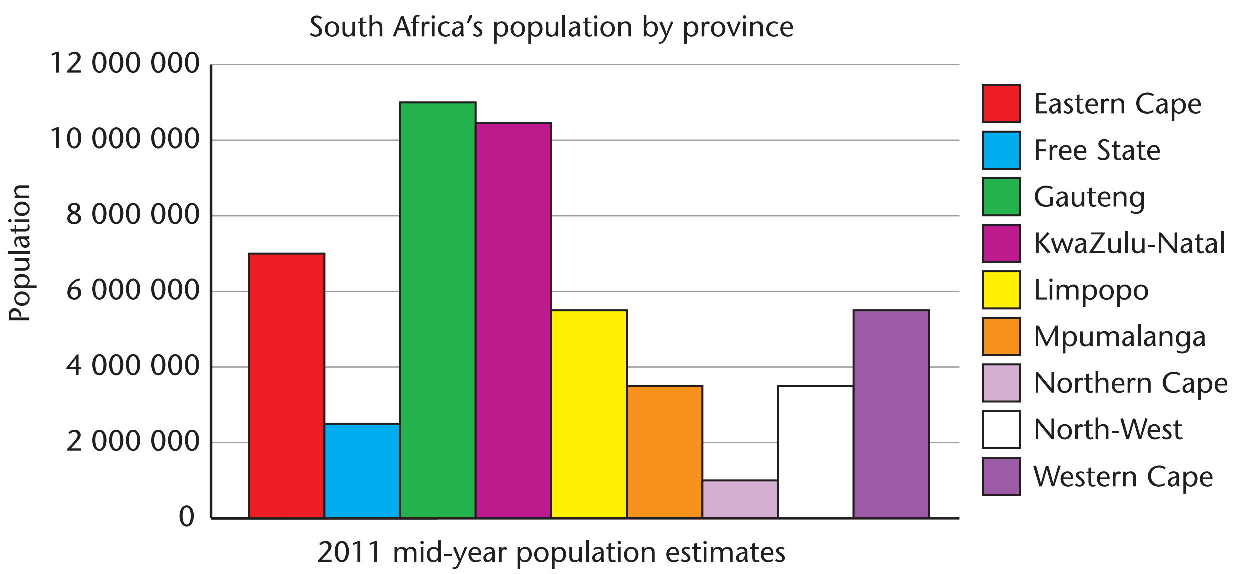
A **pie chart** consists of a circle divided into sectors (slices). Each sector shows one category of data. Bigger categories of data have bigger slices of the circle.

Here is an example of a pie chart:



DRAWING PIE CHARTS

1. The following bar graph shows the population of South Africa by province.



(a) Copy the table and write down the figures in the graph correct to the nearest 500 000.

Province	E Cape	FS	Gau	KZN	Lim	Mpum	NC	NW	WC
Population (× 1 000)									

(b) What is the total of the rounded off numbers?

(c) Work out the percentage of the whole for each province.

Province	E Cape	FS	Gau	KZN	Lim	Mpum	NC	NW	WC
Percentage of total									

(d) Draw a pie chart showing the data in the completed table. (Estimate the sizes of the slices.)

(e) Write a short report explaining the difference in the way the data is represented in the pie chart and the bar graph. Which do you think is a better method to show this data?

23.4 Broken-line graphs

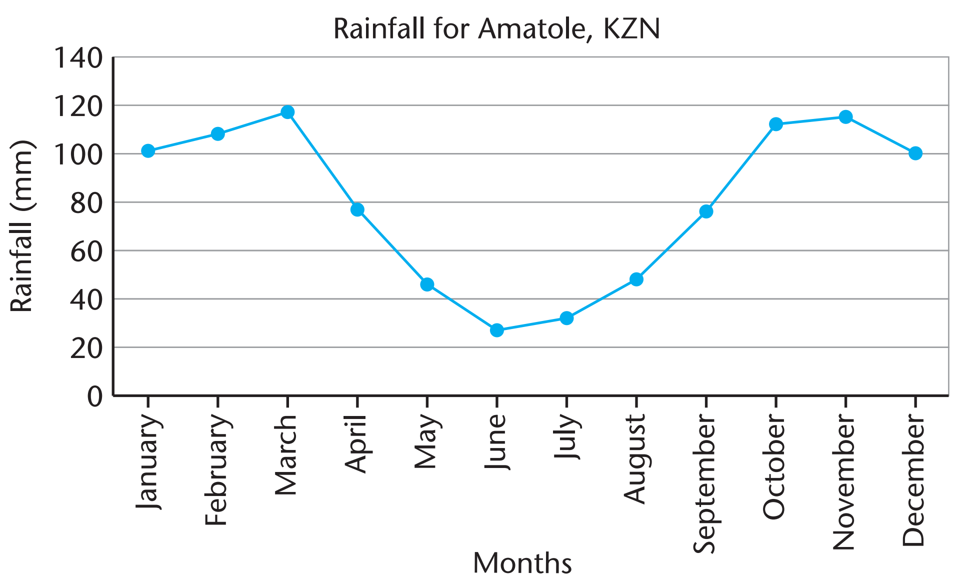
BROKEN-LINE GRAPHS

Broken-line graphs are used to represent data that changes continuously over time. For example, the rainfall for a whole month is captured as one data point, even though the rain is spread out over the month, and it rains on some days and not on others. Broken line graphs are useful to identify and display trends.

Here is some data that can be represented with broken-line graphs:

Rainfall at three locations in South Africa in 2012			
	Amatole, KZN	Mahikeng, NW	Ceres, WC
	Rainfall (mm)	Rainfall (mm)	Rainfall (mm)
January	101	118	27
February	108	90	23
March	117	86	41
April	77	61	60
May	46	14	130
June	27	6	168
July	32	3	152
August	48	7	162
September	76	18	88
October	112	46	60
November	115	75	41
December	100	86	36

Here is a broken line graph for the Amatole rainfall data:



1. During which four months does Amatole have the least rain?
2. During which six months does Amatole have the most rain?
3. During which months would you plan a hike if you were only considering the rainfall patterns?
4. What other factors should you consider when planning a hike in this region?
5. Make a broken-line graph for the Mahikeng rainfall data.
6. Make a broken-line graph for the Ceres rainfall data.
7. Write a few lines on the difference in rainfall patterns between Ceres and Mahikeng.
8. Draw a combined broken-line graph with the information from all three regions on one graph.

23.5 Scatter plots

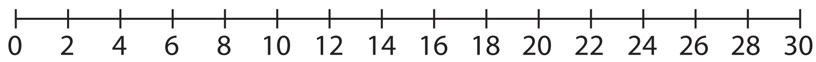
UNDERSTANDING AND CONSTRUCTING SCATTER PLOTS

Scatter plots show how two sets of numerical data are related. Matching pairs of numbers are treated as coordinates and are plotted as a single point. All the points, made up of two data items each, show a scattering across the graph.

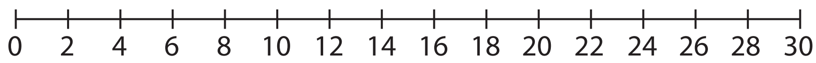
1. This table shows a data set with **two** variables. Study the information in the table.
2. Copy the number lines on the next page and make a dot for each learner's mark for each subject.

Learners	Mathematics marks	Natural Sciences marks
Zinzi	**25**	**26**
John	23	25
Palesa	**22**	**25**
Siza	21	23
Eric	20	23
Chokocha	19	21
Gabriel	17	20
Simon	16	19
Miriam	15	18
Frederik	15	16
Sibusiso	**12**	**15**
Meshack	11	13
Duma	11	12
Samuel	10	12
Lola	10	11
Thandile	9	10
Jabulani	**8**	**10**
Manare	7	9
Marlene	7	7
Mary	5	7

Natural Sciences marks

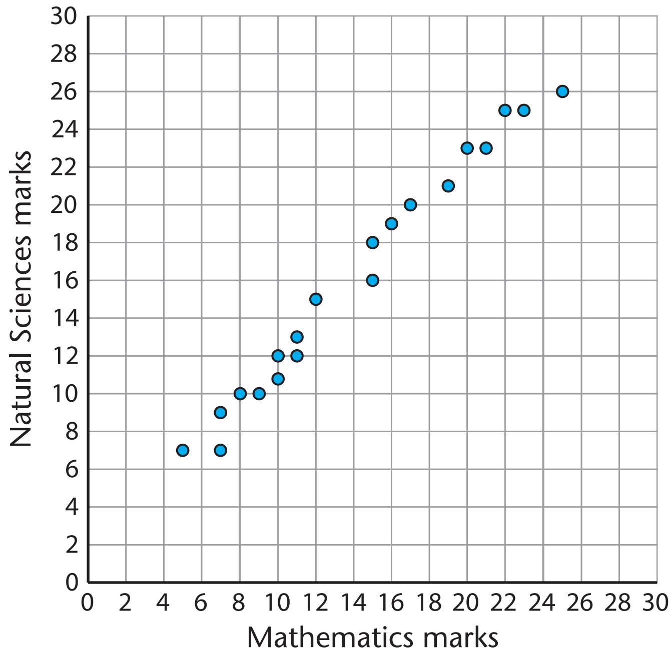


Mathematics marks



3. What if you were to show both sets of marks on the same graph, instead of a separate number line for each set? The graph below shows a scatter plot that represents both sets of data. Each dot represents one learner. Copy the scatter plot.

Correction between Mathematics and Natural Sciences



The scatter plot shows the **relationship** between the Natural Sciences mark and the Mathematics mark.

4. Find the dot for Sibusiso in the data set. He obtained a mark of 12 for the Mathematics test and a mark of 15 for Natural Sciences. Find 12 on the horizontal axis. Follow the vertical line up until you reach a blue dot. Find 15 on the vertical axis. Follow the line horizontally until you reach the same blue dot. This blue dot represents the two marks that belong to Sibusiso. On your scatter plot, circle the blue dot and label it "S".

5. Find the data points for Zinzi, Palesa, Jabulani and Mary. On your scatter plot, circle them and label them Z, P, J and M.

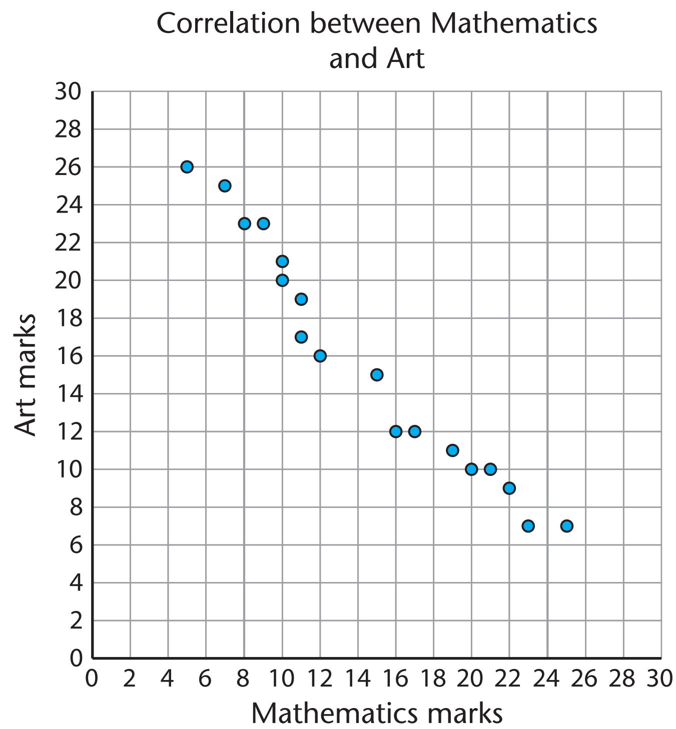
In the example on page 264, a higher Mathematics mark corresponds to a higher Natural Sciences mark. We say there is a **positive correlation** between the Mathematics marks and the Natural Sciences marks.

6. Study this data set and the scatter plot of the data given on the next page. Copy the scatter plot.

Learner	Mathematics marks	Art marks
Zinzi	**25**	**7**
John	23	7
Jabulani	22	9
Siza	21	10
Eric	**20**	**10**
Chokocha	19	11
Gabriel	17	12
Simon	16	12
Miriam	**15**	**15**
Frederik	**15**	**15**
Sibusiso	12	16
Mishack	11	17
Duma	11	19
Samuel	**10**	**20**
Lola	10	21
Thandile	9	23
Palesa	8	23
Manare	7	25
Marlene	7	25
Mary	**5**	**26**

7. Find Eric in the table. Note his marks for Mathematics and Art. Find the dot that represents his marks on the scatter plot. Encircle it and label it E.
8. Find Samuel in the table. Note his marks for Mathematics and Art. Find the dot that represents his marks. Encircle it and label it S.
9. Compare the two sets of marks for Eric and for Samuel. What do you notice about the marks?
10. On your scatter plot, find the data points on the scatter plot for Zinzi, Eric, Miriam, Frederik, Samuel and Mary. Circle the points and label them Z, E, M, F, S and Ma.

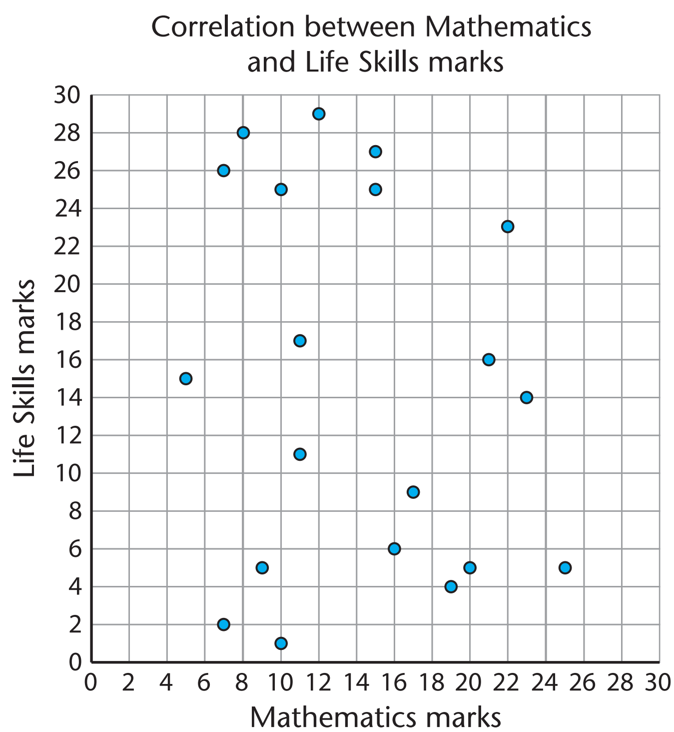
11. What do you notice about the pattern of marks in Mathematics and Art for this data set?



A **negative correlation** is a correlation in which an increase in the value of one piece of data tends to be matched by the decrease in the other set of data. Learners who obtain a high mark for Mathematics appear to obtain a low mark for Art. We say there is a negative correlation between the Mathematics and Art scores for this data set.

A correlation is an assessment of how strongly two sets of data appear to be connected. Two sets of data may be correlated or may show **no correlation**.

Here is the scatter plot for the Mathematics and Life Skills marks of the same group of learners. The table for this data is given on the next page.



12. Study the scatter plot on the previous page and the data table below. Copy the scatter plot.

13. On your scatter plot, find the data points on the scatter plot for Zinzi, Eric, Miriam, Lola and Mary. Circle the points and label them Z, E, M, L and Ma.

14. What do you notice about the pattern of marks in Mathematics and Life Skills for this data set?

Learner	**Mathematics**	**Life Skills**
Zinzi	25	5
John	23	14
Jabulani	22	23
Siza	21	16
Eric	20	5
Chokocha	19	4
Gabriel	17	9
Simon	16	6
Miriam	15	25
Frederik	15	27
Sibusiso	12	29
Meshack	11	17
Duma	11	11
Samuel	10	1
Lola	10	25
Thandile	9	5
Palesa	8	28
Manare	7	26
Marlene	7	2
Mary	5	15

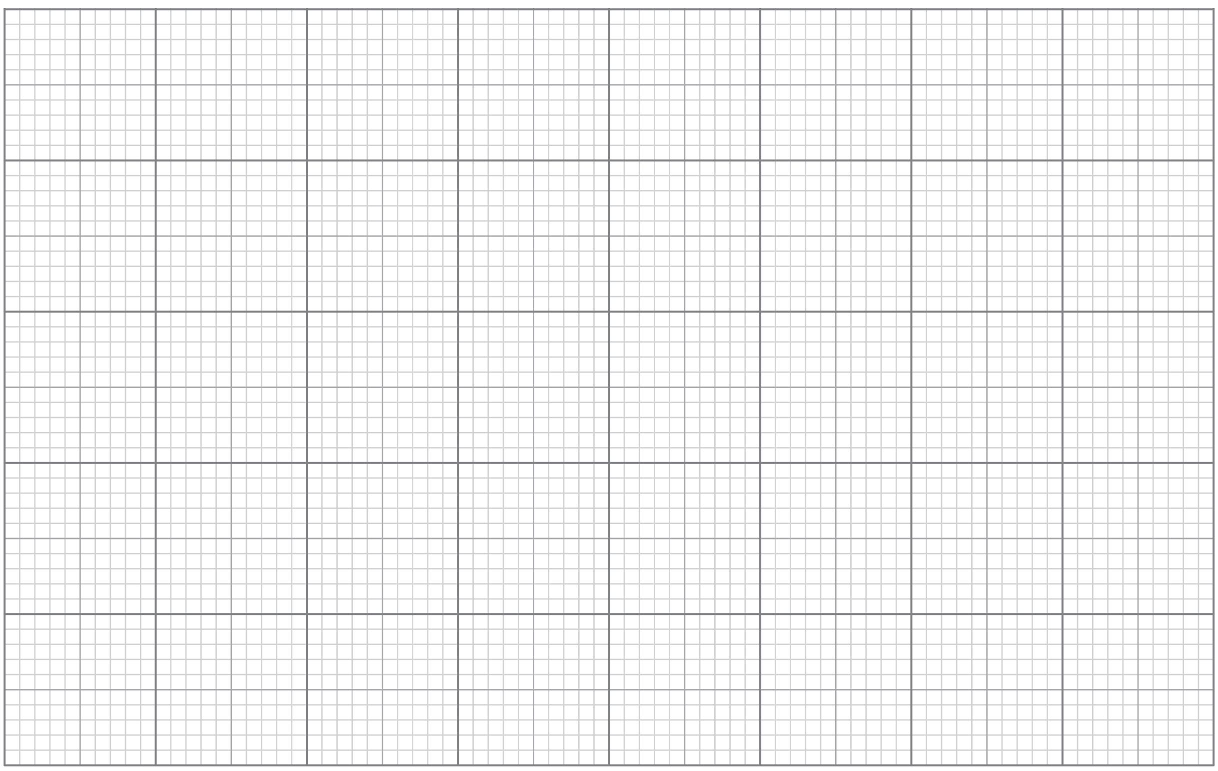
THE RELATIONSHIP BETWEEN ARM SPAN AND HEIGHT

The idea that a person's arm span (the distance from the tip of the middle finger on one hand to the tip of the middle finger on the other hand when the arms are stretched out sideways), is the same as one's height has been explored many times.

A data set for 13 people is given on the next page.

1. Make a scatter plot of this data on a grid like the one below.
   For example, take Cilla's arm span. Find 156 on the horizontal axis. Follow a vertical line up. Then on the vertical axis find 162. Follow a horizontal line across. Where the two points meet, draw a dot.

Person	Arm span	Height
Cilla	156	162
Meshack	159	162
Tony	161	160
Ellen	162	170
Karin	170	170
Sibongile	173	185
Gabriel	177	173
Alpheus	178	178
Mfiki	188	188
Nathi	188	182
Manare	188	192
Khanyi	196	184



2. What would you say about the correlation between the arm span and the height?

Chapter 24
Interpret, analyse and report on data

24.1 Which graph is best?

You have learnt that certain types of graphs are best for displaying certain kinds of information. The type of graph depends mostly on the type of data that needs to be represented. Here is a summary of the advantages of different types of graphs:

Tables show more information than graphs but the patterns are not as easy to see. They do not give a visual impression of particular trends.

Pie charts show a whole divided into parts. They show how the parts relate to each other and how the parts relate to a whole. They do not show the quantities involved.

Bar graphs show the amounts or quantities involved but do not show the relationship as effectively as pie charts. They are useful for showing **quantitative** data. Bar charts allow us to compare the quantities of different categories, for example, the sales of different items.

A **double-bar graph** is used to compare two or more things for each category. For example, we could use a double-bar graph to compare the differences between males and females.

Histograms are used to represent numerical data that is grouped into equal class intervals. Histograms are useful to show the way the data is spread out.

Broken-line graphs show trends or changes in quantities over time.

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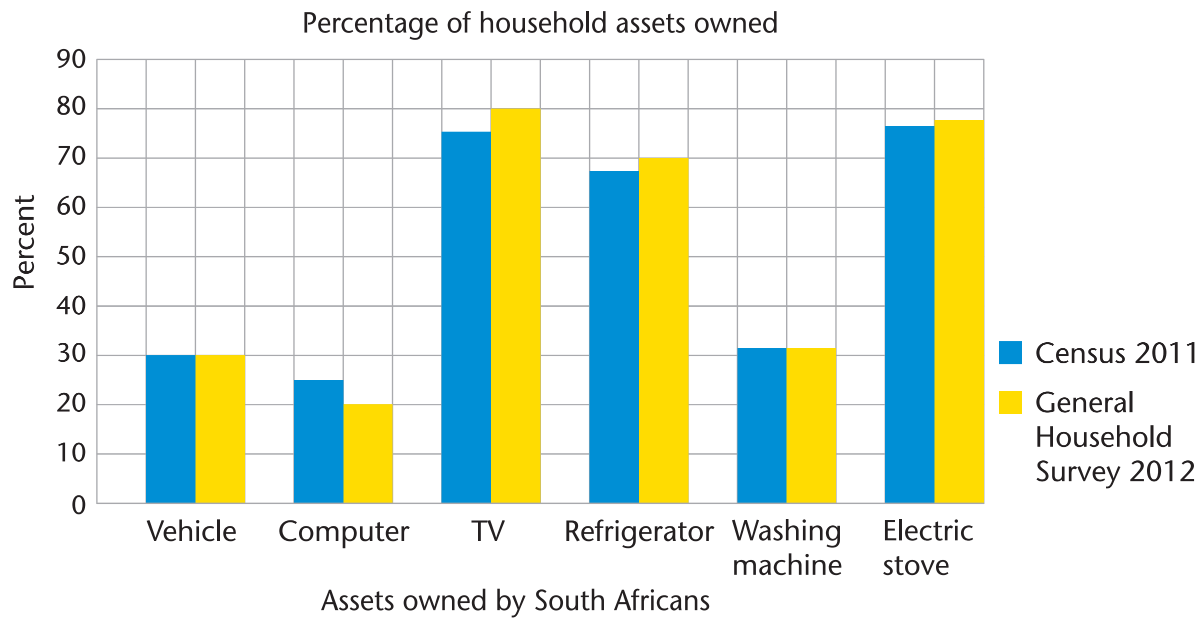
CHOOSE THE BEST REPRESENTATION

1. Which kind of graph is best to represent each of the following? Explain your answers.
   (a) Showing the value of the rand against the US dollar over several years
   (b) Comparing the monthly sales of six different makes of car in 2014 and 2015
   (c) The proportion of people of different age groups in a town
   (d) The quantities of different crops produced on a farm
   (e) The percentages of different goods sold to make up the total sales for a shop
   (f) The change in HIV infection rates over time

2. This graph was published by Statistics South Africa to show the assets owned by South Africans. The blue bar shows the Census 2011 results and the yellow bar shows the General Household Survey 2012 results.



   Give reasons for your answers to the following questions:
   (a) Is it useful to show the differences in the results of Census 2011 and the General Household Survey 2012?
   (b) Is it useful to collect data on assets that people own?
   (c) Is it useful to show that lower percentages of people own certain assets?
   (d) The different coloured bars represent the two different surveys. Draw up a table to show the data in table form. (Read the percentages as accurately as you can from the graph and round off the data to the nearest whole number for the table.)
   (e) Does the table show the data as effectively as the double bar chart? Give your own opinion.

3. The table below shows the employment status of people ages 15–64 years in South Africa. Discuss some ways of representing the data (e.g. graphs). Justify your answers.

	Jul–Sept 2012	Apr–June 2013	Jul–Sep 2013
	Number of people (thousands)		
Population 15–64 years old	33 017	33 352	33 464
Labour force	18 313	18 444	18 638
Employed	**13 645**	**13 720**	**14 028**
Formal sector (non-agricultural)	9 663	9 694	10 008
Informal sector (non-agricultural)	2 197	2 221	2 182
Agriculture	661	712	706
Private households	1 124	1 093	1 132
Unemployed	**4 668**	**4 723**	**4 609**
Not economically active	**14 705**	**14 908**	**14 826**
Discouraged work-seekers	2 170	2 365	2 240
Other (not economically active)	12 535	12 543	12 586
Unemployment rate (%)	25,5	25,6	24,7

(a) The percentages of the employed, unemployed, and not economically active people in July–September 2013

(b) The change in the employment rates over three time periods

(c) The proportions of employed people who work in the formal sector, informal sector, agriculture and private households

(d) The numbers of the employed and unemployed over the three time periods

24.2 The effects of summary statistics on how data is reported

Information articles often use averages to report information. The articles might not use the exact terms for average that you have learnt about: the mean, median and mode. Instead, they may use terms such as "most". However, it is important to be sure about the kind of average to which a report refers, because an average gives us different information.

- Remember that the **mean** is useful for describing a set of measurement values, but can also be used for other numerical data sets. The word "average" usually refers to the "mean" if it is not explained further. The mean is not reliable if a data set is too spread out.
- The **median** is the value in the middle of a data set when it is arranged in order. Half the values in the data set are lower than the median and half of them are higher than the median. The median is often the average used when data values are not uniformly distributed, because the mean is affected by extreme values in the data set, while the median is not. For example, house prices vary widely, so the median would be a better description of the data than the mean. When the median is given in a report, the writer should state that he or she is using the median or middle value.
- The **mode** is the number that occurs most often in a set of data. For example, if we collect data about people's favourite colours, the data set would be a list of colours, and the mode would be the colour that comes up most often. The mode can also be used for numbers. Not all data sets have a mode, because sometimes none of the numbers occurs more than once.

Example: The standard way of reporting house prices in South Africa and internationally is the median house price, which is used by economists in financial reports. The median is regarded as more useful than the mean house price because the sale of a few expensive houses would increase the mean, but would not affect the median.

If a bank gives bonds for eight houses to the value of R100 000, and for two houses to the value of R1 million, then the mean would be R280 000. This does not seem to be an accurate reflection of the value of the houses, because it is distorted by the higher values. The median house price would be R100 000, which is an accurate reflection of the prices.

Remember that the median is the middle point, and half of the values fall below the median, and half above. If the median is lower than the mean, this shows us that there are high values that are distorting the mean.

USING DIFFERENT SUMMARY STATISTICS

1. What kind of average is used in each of these statements?
   (a) The average family has 2,6 children.
   (b) Most families have three children.
   (c) Most people prefer red cars.
   (d) The average height for women is 1,62 m.
   (e) More people shop after work than at any other time during the day.

2. The mean monthly salary of all the staff at company ABC is R8 000 per month, but the median salary is R5 000.
   (a) Explain why the two summary statistics are so different.
   (b) Which summary statistic gives a better idea of the salaries at the company? Give reasons for your answer.

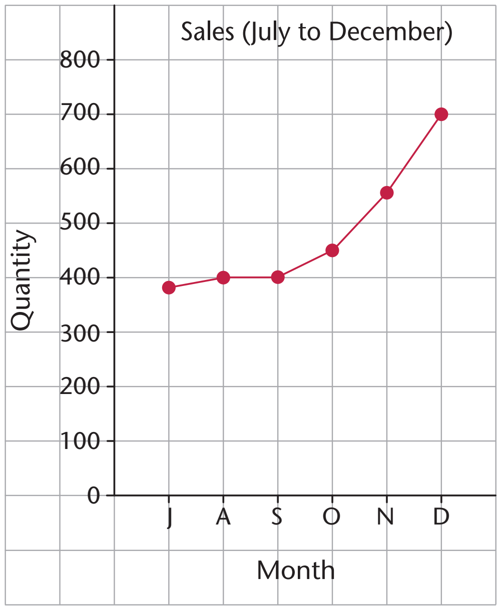
24.3 Misleading graphs

The media (i.e. newspapers, magazines and television), regularly use graphs to show information. Unfortunately, the information is often manipulated to emphasise a particular result. This may be because the writer simply wants to make his or her argument more obvious to the reader.

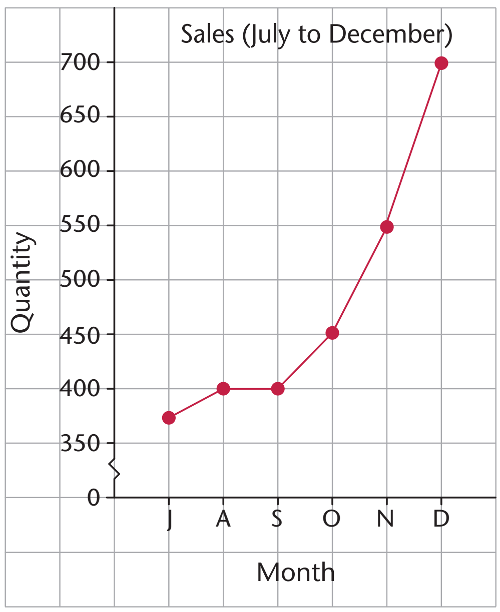
Changing the scale of the axis

If you change the scale of the vertical axis on bar graphs and line graphs, you will change the way the graphs look. For a bar graph, the larger the spaces between the numbers on the vertical axis, the bigger the difference between the bars. The smaller the spaces between the numbers on the axis, the smaller the difference in the height of the bars. The same is true for a line graph which will either have sharp points or be much flatter, depending on how you have changed the scale.

Example: The two broken-line graphs on the next page show the same sales data for a business over a period of six months. Which graph gives the more accurate impression?



Graph A



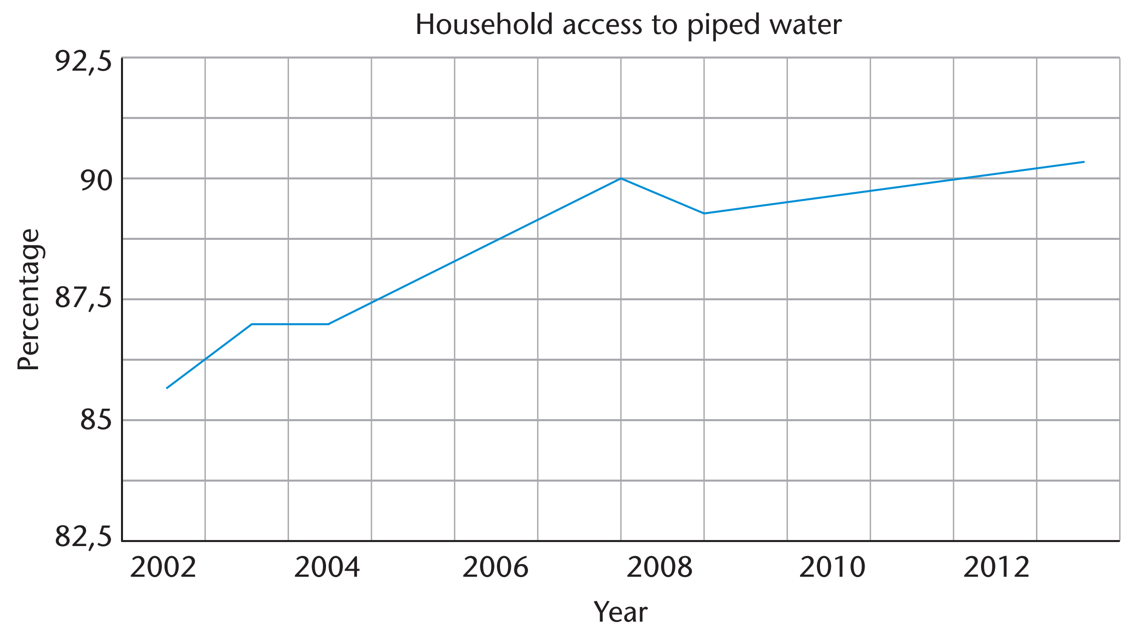
Graph B

Graph B has a different scale on the vertical axis. The vertical axis does not start at 0 and so **two** blocks on the vertical axis represent 100 items instead of only **one** block, as in Graph A. This makes it look as if the sales increased rapidly over the six months.

Note that it is not necessarily wrong to change the scale on the axes or not to start at 0. For example, graphs showing stock exchange fluctuations rarely show the origin on the graph and stockbrokers are taught to interpret the graphs in that form. Sometimes small changes in data values have important effects and in these cases, it may be valid to change the scale to show these.

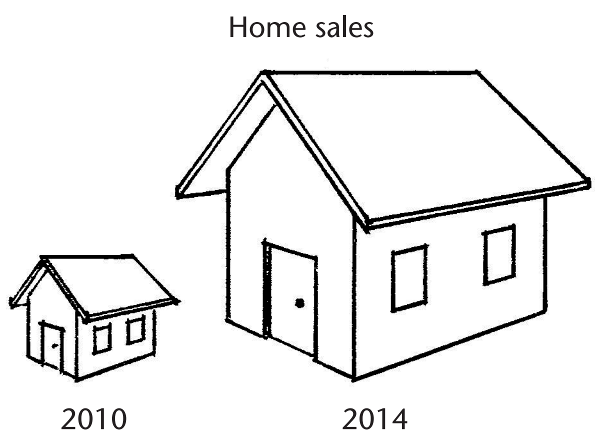
ANALYSING GRAPHS

1. This graph from Statistics South Africa shows the increase in the percentage of households that had access to piped water over a ten-year period.



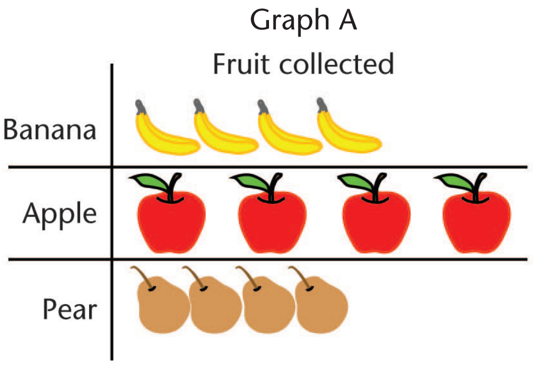
   (a) Comment on the scale used on the vertical axis. Is this a misleading graph?
   (b) How could you redraw the graph so that the differences on the graph are more noticeable?
   (c) How could you draw the graph so that the differences are less noticeable?

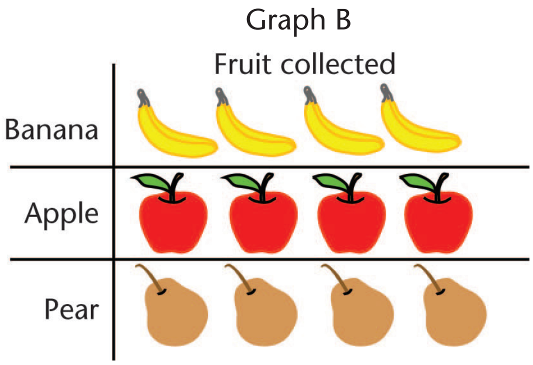
2. In this graph the height of the houses represents the number of sales.



   Do you think that this graph is misleading? Give reason(s) for your answer.

3. Look at the two graphs below:



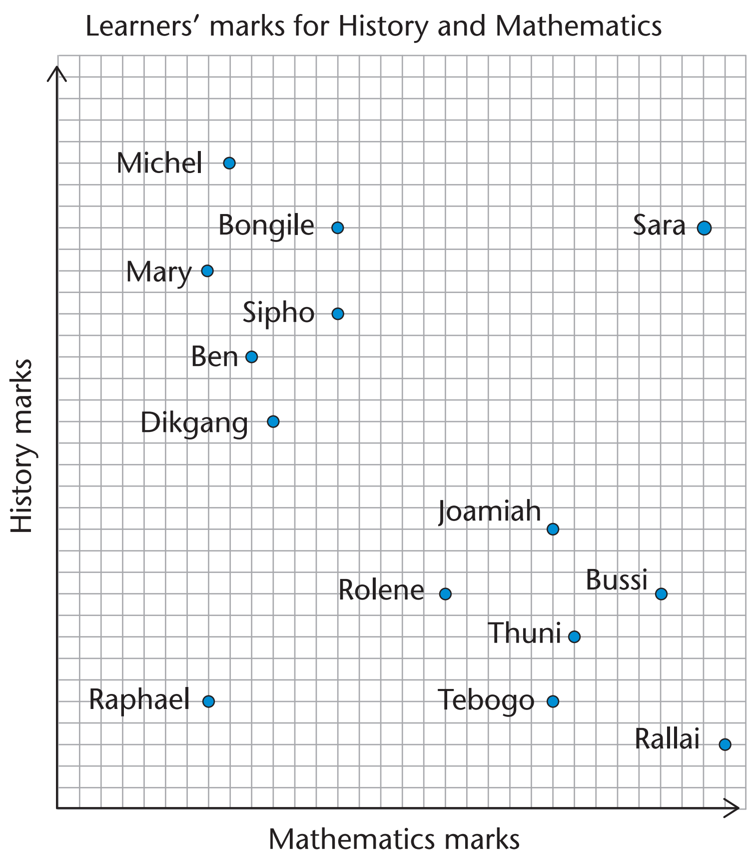


Which graph do you think is drawn correctly? Explain your answer.

24.4 Analysing extreme values and outliers

A data item that is very different from all (or most) of the other items in a data set is called an **outlier**.

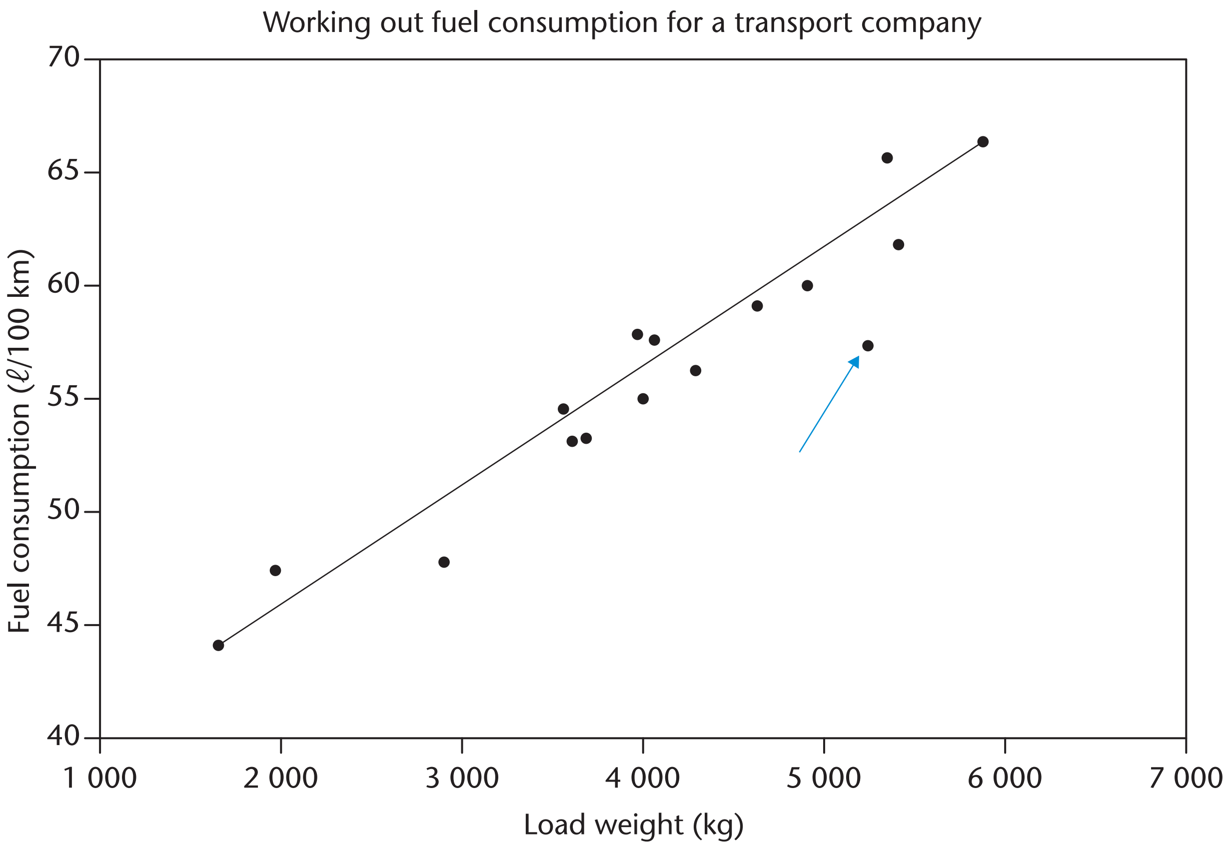
It is sometimes difficult to notice outliers in numerical data. However, outliers often become clearly noticeable when data is displayed with graphs.



1. The scatter plot on the previous page shows the performance of a group of learners in Mathematics and History. Which of the points on the scatter plot can be regarded as outliers? Give reasons for your answer.

Outliers in data sets can be very important. We need to decide if there is a particular reason for the value being so different to the others. Sometimes it gives us important information. In some cases, the data collected for that point could be wrong.

The scatter plot below is for data collected by a transport company.



The company uses just one type of truck. Before each transport job, the company has to specify the price for the job. In order to specify a price before a job, the company needs to estimate how much their costs will be for doing the job. One of the main costs is the cost of fuel, and the main factor influencing the amount of fuel used is the distance. The load weight also plays a role: the greater the load weight, the higher the fuel consumption (litres/100 km).

The table on the next page gives information that was recorded for previous transport jobs. The jobs are numbered from 1 to 16 and for each job the values of the four variables *distance*, *load weight*, *amount of fuel used* and *fuel consumption rate* are given.

2. (a) Which of the four variables are represented on the scatter plot given above?
   (b) What are the values of these two variables for the point indicated by the blue arrow on the scatter plot?

Job number	Distance (km)	Load weight (kg)	Fuel used (ℓ)	Fuel consumption (ℓ/100 km)
1	1 304	5 445	879	67,4
2	1 320	2 954	639	48,4
3	1 151	4 705	698	60,6
4	1 371	4 378	787	57,4
5	325	3 673	176	54,2
6	1 630	5 995	1 113	68,3
7	1 023	5 357	600	58,7
8	620	4 988	382	61,6
9	73	1 992	35	47,9
10	1 071	5 529	680	63,5
11	370	4 140	218	58,9
12	1 423	4 062	843	59,2
13	394	4 068	221	56,1
14	1 536	1 678	682	44,4
15	1 633	3 736	887	54,3
16	435	3 644	241	55,4

3. (a) Consider the scatter plot and the data set. What is the effect of load weight on fuel consumption?
   (b) Is job 7 an exception in this respect? Explain your answer.

4. Further investigations revealed that the driver for jobs 2 and 7 was the same person, and that he was not the driver for any other jobs. What may this indicate?

FIND OUTLIERS

Researchers collected data on the population of some African countries (including the Seychelles), which included the income per person and the percentage of the income spent on health.

Country	Total population (in 1 000s)	Total annual national income per person (US$)	Percentage of income spent on health
Angola	18 498	4 830	4,6
Botswana	1 950	13 310	10,3
DRC	66 020	280	2,0

Country	Total population (in 1 000s)	Total annual national income per person (US$)	Percentage of income spent on health
Lesotho	2 067	1 970	8,2
Malawi	15 263	810	6,2
Mauritius	1 288	12 580	5,7
Mozambique	22 894	770	5,7
Namibia	2 171	6 250	5,9
Seychelles	84	19 650	4,0
South Africa	50 110	9 790	8,5
Swaziland	1 185	5 000	6,3
Tanzania	43 739	1 260	5,1
Zambia	12 935	1 230	4,8

1. What are the three variables in this table?
2. Why do you think it is important to look at income per person in this case, rather than the total income?
3. On graph paper, plot the points for the national income per person and the percentage spent on health care for each country.

4. Write a short report on the data in the table and what the scatter plot shows you about the data. Comment on the general trend and any outliers.

Chapter 25
Probability

25.1 Simple events

REVISION

yellow	green	pink	blue	red	brown	grey	black

1. (a) Suppose the eight coloured buttons above are in a bag and you draw one button from the bag without looking. Can you tell what colour you will draw?
   (b) Suppose you repeatedly draw a button from the bag, note its colour, then put it back. Can you tell in approximately what fraction of all the trials the button will be yellow?

Archie has a theory. Because the eight possible outcomes are equally likely, he believes that if you perform eight trials in a situation like the above you will draw each colour once.

2. If Archie's theory is correct, how many times will each colour be drawn if 40 trials are performed?

3. If Archie's theory is correct, in what fraction of the total number of trials will each colour be drawn?

4. If Archie's theory is correct, how many times will each of the colours be drawn if a total of 40 trials is performed? Copy the table on the following page and write your answers in the second row of the table. Write the predicted relative frequencies in row 3 as fortieths, and in row 4 as two hundredths.

Each time you draw a button from the bag without looking, you perform a **trial**. If you do this and put the button back, and repeat the same actions eight times, you have performed eight trials.

The number of times an event occurs during a set of trials is called the **frequency** of the event.

When the frequency of an event is expressed as a fraction of the total number of trials, it is called the **relative frequency**.

Colour	Yellow	Green	Pink	Blue	Red	Brown	Grey	Black
Frequencies predicted by Archie								
Relative frequencies predicted by Archie expressed in fortieths								
Relative frequencies predicted by Archie expressed in two hundredths								

The relative frequency for each colour that Archie predicted is called the **probability** of drawing that colour. If all the outcomes are equally likely, then:

$$\text{probability of an outcome} = \frac{1}{\text{the total number of equally-likely outcomes}}$$

You will now investigate whether or not Archie's theory is correct.

5. (a) Make eight small cards and write the name of one of the above colours on each card, so that you have cards with the eight colour names. Perform eight trials to check whether or not Archie's theory is correct. Copy the table below and record your results (your tally marks 1 and your frequencies 1) in the relevant row of the table.
   (b) Find out what any four of your classmates found when they did the experiment. Enter their results in your table too (Friend 1, 2, 3 and 4 frequencies).

Table for the results of the experiments

Colour	Yellow	Green	Pink	Blue	Red	Brown	Grey	Black
Your tally marks (1)								
Your frequencies (1)								
Friend 1 frequencies								
Friend 2 frequencies								
Friend 3 frequencies								
Friend 4 frequencies								
Total frequencies for 5 experiments								

6. (a) What was the total number of trials in the five experiments you recorded in the table?
   (b) What is the total of the frequencies for the different colours, in the last row of your table?

7. Is Archie's theory correct?

Bettina has a different theory to Archie's. She believes that if one does many trials with the eight buttons in a bag, each colour will be drawn in **approximately** one-eighth of the cases. In other words, Bettina believes that the relative frequency of each outcome will be close to the probability of that outcome, but may not be equal to it.

8. (a) You and your four classmates performed 40 trials in total. Copy the table below and enter the results in the second row of the table. Also express each frequency as a fraction of 40 and of 200, in fortieths and in two hundredths.

Colour	Yellow	Green	Pink	Blue	Red	Brown	Grey	Black
Actual frequencies obtained in your experiments (40 trials)								
Relative frequencies as fortieths								
Relative frequencies as two hundredths								
Probability as two hundredths								

(b) Do your experiments show that Bettina's theory is correct or not?

Jayden believes that when more trials are performed, the relative frequencies will get closer to the probabilities.

You will now do an investigation to find out whether Jayden's theory is true.

INVESTIGATE WHAT HAPPENS WHEN MORE TRIALS ARE DONE

1. Perform 40 trials by drawing one card at a time from eight small cards with the names of the colours written on them, and enter your results in the second and third rows of a table like the one shown below.

Colour	Yellow	Green	Pink	Blue	Red	Brown	Grey	Black
Tally marks								
Frequencies								
Relative frequencies as fortieths								
Relative frequencies as two hundredths								
Probabilities as two hundredths								

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2. Make a copy of the table on page 282, but leave out the row for tally marks, the row for the relative frequencies as fortieths and the row for the probabilities, on a loose sheet of paper. Exchange it with a classmate. Copy the following Tables 1 and 2 and enter the results of your classmate on Table 1 and 2. Also enter your own results for question 1 on the tables.
3. Get hold of the data reports of three other classmates, and enter these on the tables as well.
4. Add the frequencies of the various colours in the five sets of data for 40 trials each, and calculate the relative frequencies expressed as two hundredths.
5. Is the range of relative frequencies for 200 trials smaller than the ranges for the five different sets of 40 trials each? What does this indicate with respect to Jayden's theory?

When only a small number of trials are done, the actual relative frequencies for different outcomes may differ a lot from the probabilities of the outcomes.

When many trials are done, the actual relative frequencies of the different outcomes are quite close to the probabilities of the outcomes.

Table 1: Frequencies for five sets of 40 trials each

Colour	Yellow	Green	Pink	Blue	Red	Brown	Grey	Black
Frequencies for your own 40 trials in question 1								
Frequencies for 40 trials by classmate 1								
Frequencies for 40 trials by classmate 2								
Frequencies for 40 trials by classmate 3								
Frequencies for 40 trials by classmate 4								
Total frequencies for 200 trials								
Relative frequencies for 200 trials as two hundredths								

Table 2: Relative frequencies for each of the five sets of 40 trials each (expressed as two hundredths)

Colour	Yellow	Green	Pink	Blue	Red	Brown	Grey	Black
Relative frequencies for your own 40 trials								
Relative frequencies for 40 trials by classmate 1								
Relative frequencies for 40 trials by classmate 2								
Relative frequencies for 40 trials by classmate 3								
Relative frequencies for 40 trials by classmate 4								

6. How many different three-digit numbers can be formed with the symbols 3 and 5, if no other symbols are used? You may use one, two or three of the symbols in each number, and you may repeat the same symbol.

25.2 Compound events

TOSSING A COIN AND GIVING BIRTH

1. Simon threw a coin and the outcome was heads. He will now throw the coin again.
   (a) What are the possible outcomes?
   (b) What is the probability of each of the possible outcomes?
   (c) What are the possible outcomes if Simon throws the coin for a third time?
   (d) What is the probability of each of the possible outcomes for the third throw?

What happens when a coin is thrown for a second time has nothing to do with what happened when it was thrown the first time.

The first throw and the second throws are called **independent events**, i.e. what happened on the first throw cannot influence what will happen on the second throw.

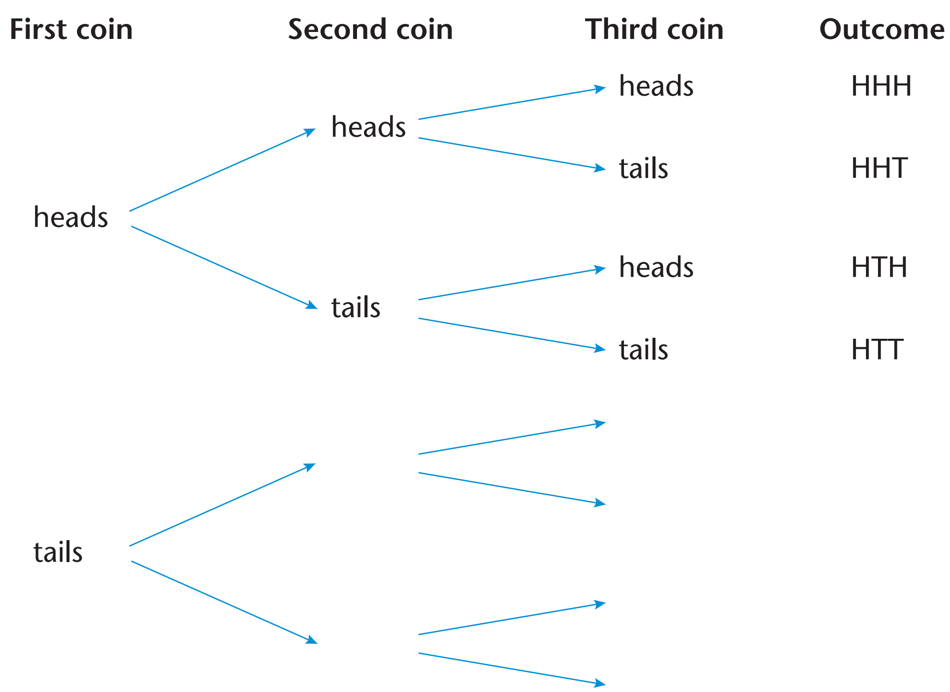
2. (a) If an event has four different equally-likely outcomes, what is the probability of each of the four outcomes?
   (b) Does that mean that if the event is repeated four times, each of the four outcomes will happen once?
   (c) Does your answer in (a) mean that if the event is repeated 100 times, each of the four outcomes will happen 25 times?

3. (a) What are the possible outcomes when two coins are thrown? Copy and complete the **two-way table** below to answer this question. One possible outcome is already given.

	Heads	**Tails**
Heads		H T
Tails		

(b) Do you think these four outcomes are equally likely?
(c) What is the probability of each of the four outcomes?
(d) What is the probability of getting a head and a tail?

4. Let us consider the possible outcomes if three coins are thrown.
Below is a tree diagram that can help you figure out what the different possible outcomes are. Complete the diagram by filling in the missing information.



5. (a) Do you think the eight different outcomes in question 4 are equally likely?
(b) What is the probability of each of the eight outcomes?
(c) What is the probability of throwing two heads and one tail?

6. In question 6 on page 284 you were asked to write down the various numbers that can be formed by using symbols 3 and 5. Think of all the four-letter codes that you can form by using only two letters, P and Q. Any letter can be used more than once in one code. First think about how you will go about finding all the possibilities in a systematic way, and then try to set up a tree diagram to help you.

(a) Draw a tree diagram to help you to solve this problem. List all the outcomes.

(b) If the codes are formed by randomly choosing the letters, what is the probability that the code will consist of the same letter being used four times?

(c) What is the probability that the code will consist of two Ps and two Qs?

When a woman is pregnant, the baby can be a boy or a girl. Suppose we make the assumption that the two possibilities are equally likely, so the probability of a boy is $\frac{1}{2}$ and the probability of a girl is $\frac{1}{2}$.

7. (a) Copy and complete this two-way table to show the possible outcomes of the gender of the two children in a family.

	Boy	**Girl**

(b) List the possible outcomes.

(c) What is the probability that the two children in the family will be of the same gender?

(d) What is the probability that the eldest child will be a boy and that they will then have a girl?

8. A certain woman already has a boy. She now expects a second child. What is the probability of it being a boy again, if we make the assumption that a baby being a boy or a girl are equally likely events?

The assumption that a boy or a girl being born are equally likely events may not actually be true. However, probabilities can only be calculated and used to make predictions if it is assumed that outcomes are equally likely.

9. (a) A woman gets married and plans to have a baby in one year and another baby in the next year. What is the probability that both babies will be girls?

(b) A woman gets married and plans to have a baby in each of the first three years of the marriage. What is the probability that she will have a boy in the first year, and girls in the second and third years?