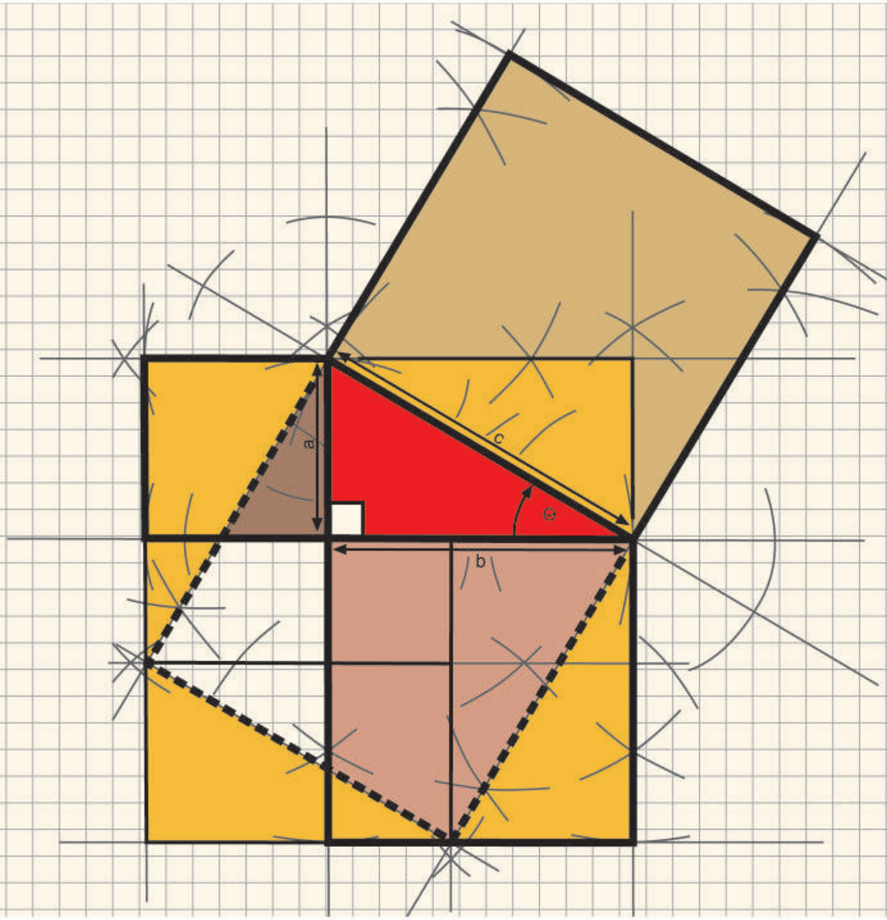
MATHEMATICS

GRADE 9

REVISED EDITION

TEACHER GUIDE



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Mathematics

Grade 9

Teacher Guide

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Mathematics Teacher Guide Grade 9

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Contents

Term 1

Chapter 1: Whole numbers ... 3
Chapter 2: Integers ... 23
Chapter 3: Fractions ... 33
Chapter 4: The decimal notation for fractions ... 47
Chapter 5: Exponents ... 55
Chapter 6: Patterns ... 63
Chapter 7: Functions and relationships ... 73
Chapter 8: Algebraic expressions ... 85
Chapter 9: Equations ... 105

Term 2

Chapter 10: Construction of geometric figures ... 117
Chapter 11: Geometry of 2D shapes ... 135
Chapter 12: Geometry of straight lines ... 153
Chapter 13: Pythagoras' Theorem ... 163
Chapter 14: Area and perimeter of 2D shapes ... 173

Term 3

Chapter 15: Functions ... 189
Chapter 16: Algebraic expressions ... 199
Chapter 17: Equations ... 211
Chapter 18: Graphs ... 223
Chapter 19: Surface area, volume and capacity of 3D objects ... 247
Chapter 20: Transformation geometry ... 257
Chapter 21: Geometry of 3D objects ... 275

Term 4

Chapter 22: Collect, organise and summarise data ... 293
Chapter 23: Representing data ... 303
Chapter 24: Interpret, analyse and report on data ... 319
Chapter 25: Probability ... 331

Term 1

Chapter 1: Whole numbers 3
1.1 Properties of numbers 4
1.2 Calculations with whole numbers 8
1.3 Multiples and factors 14
1.4 Solving problems about ratio, rate and proportion 15
1.5 Solving problems in financial contexts 17

Chapter 2: Integers 23
2.1 Which numbers are smaller than 0? 24
2.2 Adding and subtracting with integers 25
2.3 Multiplying and dividing with integers 27
2.4 Powers, roots and word problems 30

Chapter 3: Fractions 33
3.1 Equivalent fractions 34
3.2 Adding and subtracting fractions 37
3.3 Multiplying and dividing fractions 39
3.4 Equivalent forms 44

Chapter 4: The decimal notation for fractions 47
4.1 Equivalent forms 48
4.2 Calculations with decimals 49
4.3 Solving problems 51
4.4 More problems 52
4.5 Decimals in algebraic expressions and equations 53

Chapter 5: Exponents **55**
5.1 Revision 56
5.2 Integer exponents 58
5.3 Solving simple exponential equations 60
5.4 Scientific notation 61
Chapter 6: Patterns **63**
6.1 Geometric patterns 64
6.2 More patterns 66
6.3 Different kinds of patterns in sequences 68
6.4 Formulae for sequences 70
Chapter 7: Functions and relationships **73**
7.1 Find output numbers for given input numbers 74
7.2 Different ways to represent the same relationship 75
7.3 Different representations of the same relationship 79
Chapter 8: Algebraic expressions **85**
8.1 Algebraic language 86
8.2 Properties of operations 91
8.3 Combining like terms in algebraic expressions 93
8.4 Multiplication of algebraic expressions 95
8.5 Dividing polynomials by integers and monomials 98
8.6 Products and squares of binomials 101
8.7 Substitution into algebraic expressions 103
Chapter 9: Equations **105**
9.1 Solving equations by inspection 106
9.2 Solving equations using additive and multiplicative inverses 106
9.3 Setting up equations 108
9.4 Equations and situations 110
9.5 Solving equations by using the laws of exponents 111

Grade 9 Term 1 Chapter 1 Whole numbers

Learner Book Overview		
Sections in this chapter	**Content**	**Pages in Learner Book**
1.1 Properties of numbers	Natural numbers; whole numbers; integers; rational numbers; irrational numbers	Pages 1 to 4
1.2 Calculations with whole numbers	Estimating, rounding off and compensating; adding, multiplying and subtracting in columns; long division	Pages 5 to 10
1.3 Multiples and factors	Lowest common multiple; highest common factor; prime factorisation	Pages 11 to 12
1.4 Solving problems about ratio, rate and proportion	Ratio and rate; direct proportion; average speed; indirect proportion	Pages 12 to 14
1.5 Solving problems in financial contexts	Discount; profit; loss; hire purchase; simple and compound interest; exchange rate; commission	Pages 14 to 18

CAPS time allocation	4,5 hours
CAPS content specification	Pages 119 to 121

Mathematical background

The **associative and distributive properties** of addition and multiplication and the **distributive property** (distribution of multiplication over addition and subtraction) apply to the sets of natural numbers $\boldsymbol{N} = \{1; 2; 3; 4; ...\}$, whole numbers $\boldsymbol{N_0} = \{0; 1; 2; 3; 4; ...\}$, integers $\boldsymbol{Z} = \{...-5; -4; -3, -2, -1; 0; 1; 2; 3; 4; 5...\}$, rational numbers $\boldsymbol{Q}$ = {numbers of the form $\frac{\text{integer}}{\text{integer}}$} $= \{...-\frac{1}{2}; 1; \frac{5}{2}; ...\}$ and irrational numbers $\boldsymbol{Q'}$ = {numbers that cannot be written in the form $\frac{\text{integer}}{\text{integer}}$} $= \{...\sqrt{2}; \pi...\}$.

We can relate the different number systems to each other by the ideas of **extending** and **closure**:

- The **set of whole numbers** is closed under addition and multiplication. This means that the sum of any two whole numbers is also a whole number, and the product of any two whole numbers is also a whole number. However, the whole numbers are not closed under subtraction, for example 7 – 12 is not a whole number. Extending the whole numbers to the **set of integers** by including the "negative natural numbers" provides closure under subtraction. Although it is not true to say that mathematicians invented (introduced) negative numbers for this purpose. Historically, there were several other motivations for introducing negative numbers. One reason was the desire to make equations solvable. For example, the equation $12 + 4x = 4$ has no solution in the set of whole numbers, but it does have a solution in the set of integers, provided the property (*positive number*) × (*negative number*) *is a negative number* is assigned to the integers.
- Neither the set of whole numbers nor the set of integers is closed under division. If we extend these to the **set of rational numbers**, then we have closure. This is a very practical reason for adding fractions to the numbers we work with. In reality, the division of a whole number of objects into a given number of equal parts often requires fractional parts. However, historical evidence suggests that fractions were first introduced in North Africa in the context of measurement (parts of a unit) rather than equal sharing.
- The idea of a **set of irrational numbers** was conceived as an outcome of several challenges mathematicians experienced in antiquity, including the challenge to find an exact relationship between the radius and circumference of a circle, and the length of the third side of some right-angled triangles with two sides given.

1.1 Properties of numbers

DIFFERENT TYPES OF NUMBERS

The natural numbers

Background information

- The **set of natural numbers** *N* = {1; 2; 3; 4; 5; …} has the following properties:
  - *N* **is closed under addition** because the sum of two or more natural numbers is a natural number.

    Example: 75 + 19 + 6 = 100 (which is a natural number)
  - *N* **is closed under multiplication** because the product of two or more natural numbers is a natural number.

    Example: 25 × 8 = 200 (which is a natural number)
  - *N* contains the **identity element for multiplication (1)** because any natural number multiplied by 1 gives the same number you start with.
- The set of natural numbers *N* **is not closed under subtraction** because the difference between two natural numbers is not always a natural number.

  Example: 5 – 12 does not result in a natural number.
- The set of natural numbers *N* **is not closed under division** because the quotient of two natural numbers is not always a natural number.

  Example: 5 ÷ 12 does not result in a natural number.

Teaching guidelines

Learners investigate whether the set of natural numbers is closed under addition, multiplication, subtraction and division.

Answers

1. (a) Yes, 1.

   (b) No
2. See the answers on LB page 1 alongside.

CHAPTER 1
Whole numbers

1.1 Properties of numbers

DIFFERENT TYPES OF NUMBERS

The natural numbers

The numbers that we use to count are called **natural numbers**:

1 2 3 4 5 6 7 8 9 10 11 12 13 14

Natural numbers have the following properties:
When you add two or more natural numbers, you get a natural number again.
When you multiply two or more natural numbers, you get a natural number again.

Mathematicians describe this by saying: The system of natural numbers is **closed under addition and multiplication**.

However, when a natural number is *subtracted* from another natural number, the answer is not always a natural number again. For example, there is no natural number that provides the answer to 5 – 20.

Similarly, when a natural number is *divided* by another natural number, the answer is not always a natural number again. For example, there is no natural number that provides the answer to 10 ÷ 3.

The system of natural numbers is **not closed under subtraction or division**.

When subtraction or division is done with natural numbers, the answers are not always natural numbers.

1. (a) Is there a smallest natural number, in other words, a natural number that is smaller than all other natural numbers? If so, what is it?

   (b) Is there a largest natural number, in other words, a natural number that is larger than all other natural numbers? If so, what is it?
2. In each of the following cases, say whether the answer is a natural number or not:

   (a) 100 + 400 Yes (b) 100 – 400 No

   (c) 100 × 400 Yes (d) 100 ÷ 400 No

The whole numbers

Background information

- The set of natural numbers combined with 0 is called the **set of whole numbers**.
- The **set of whole numbers** N_0 = {0; 1; 2; 3; 4; 5; ...} has one additional property to those of natural numbers:
    - N_0 contains the **identity element for addition (0)** because 0 added to any whole number gives the same number you started with.
- The set of whole numbers N_0 is **not closed under subtraction** because the difference between two whole numbers is not always a whole number.
- The set of whole numbers N_0 is **not closed under division** because the quotient of two whole numbers is not always a whole number.

Teaching guidelines

Learners discuss the necessity of the symbol 0 and the identity element for addition.

Answers

3. Yes. If you multiply by 1, the number stays the same.
4. (a) 1
   (b) 0

The integers

Background information

- The **set of whole numbers** starts with 0 and extends in one direction: {0; 1; 2; 3; 4; ...}
- The **set of integers** extends in both directions: {...; –4; –3; –2; –1; 0; 1; 2; 3; 4; ...}
- The **set of integers** Z = {...; –4; –3; –2; –1; 0; 1; 2; 3; 4; ...} has these additional properties to those of natural numbers:
    - For each integer there is an **additive inverse** which, when added to the integer, is equal to 0.
      Example: The additive inverse of 20 is –20 because 20 + (–20) = 0.
    - **Z is closed under subtraction** because the difference between two integers is always an integer.
      Examples: 45 – 27 = 18 (which is an integer)
      27 – 45 = –18 (which is an integer)

The whole numbers

Although we do not use 0 for counting, we need it to write numbers. Without 0, we would need a special symbol for 10, all multiples of 10 and some other numbers. For example, all the numbers that belong in the yellow cells below would need a special symbol.

	41	42	43	44	45	46	47	48	49
	51	52	53	54	55	56	57	58	59
	61	62	63	64	65	66	67	68	69
	71	72	73	74	75	76	77	78	79
	81	82	83	84	85	86	87	88	89
	91	92	93	94	95	96	97	98	99
	111	112	113	114	115	116	117	118	119

The natural numbers combined with 0 is called the system of **whole numbers**.

If you are working with natural numbers and you add two numbers, the answer will always be different from any of the two numbers added. For example: 21 + 25 = 46 and 24 + 1 = 25. If you are working with whole numbers, in other words including 0, this is not the case. When 0 is added to a number the answer is just the number you start with: 24 + 0 = 24.

For this reason, 0 is called the **identity element** for addition. In the set of natural numbers there is no identity element for addition.

3. Is there an identity element for multiplication in the whole numbers? Explain your answer.
4. (a) What is the smallest natural number?
   (b) What is the smallest whole number?

The integers

In the set of whole numbers, no answer is available when you subtract a number from a number smaller than itself. For example, there is no whole number that is the answer for 5 – 8. But there is an answer to this subtraction in the system of integers.

For example: 5 – 8 = –3. The number –3 is read as "negative 3" or "minus 3".

Whole numbers start with 0 and extend in one direction:

0 1 2 3 4 5 6 → → → ......

Integers extend in both directions:

...... ← ← ← –5 –4 –3 –2 –1 0 1 2 3 4 5 6 → → → ......

Teaching guidelines

Learners discuss the additional properties of integers.

Answers

5. See the answers on LB page 3 alongside.
6. See the answers on LB page 3 alongside.

The rational numbers

Background information

- To have answers for all possible division questions the set of integers is extended to include fractions and negative fractions of the form $\frac{\text{integer}}{\text{integer}}$.

  This extended set of numbers is called the **set of rational numbers** Q and has these additional properties:
  - Rational numbers are **closed under division** (but division by 0 is not defined).
  - Rational numbers can be expressed in **common fraction notation**, for example, $\frac{12}{5}$.
  - Rational numbers can be expressed in **decimal notation**, for example, 2,4.

Teaching guidelines

Learners discuss the properties of rational numbers.

Answers

7. (a) They will get more than two.
   (b) No
   (c) Two pieces of the chocolate.
   (d) Both are correct.
8. (a) 2,3 and $2\frac{3}{10}$ (b) 4,6 and $4\frac{6}{10}$ or $4\frac{3}{5}$
   (c) 2,3 and $2\frac{3}{10}$ (d) 0,8 and $\frac{8}{10}$ or $\frac{4}{5}$

All whole numbers are also **integers**. The set of whole numbers forms part of the set of integers. For each whole number, there is a negative number that corresponds with it. The negative number –5 corresponds to the whole number 5 and the negative number –120 corresponds to the whole number 120.

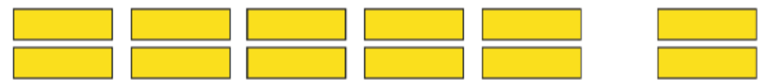
Within the set of integers, the sum of two numbers can be 0.

For example 20 + (–20) = 0 and 135 + (–135) = 0.

20 and –20 are called **additive inverses** of each other.

5. Calculate the following without using a calculator:
   (a) 100 – 165 = –65 (b) 300 – 700 = –400
6. You may use a calculator to calculate the following:
   (a) 123 – 765 = –642 (b) 385 – 723 = –338

The rational numbers



7. Five people share 12 slabs of chocolate equally among them.
   (a) Will each person get more or less than two full slabs of chocolate?
   (b) Can each person get another half of a slab?
   (c) How much more than two full slabs can each person get, if the two remaining slabs are divided as shown here?
   
   (d) Will each person get 2,4 or $2\frac{2}{5}$ slab?

The system of integers does not provide an answer for all possible division questions. For example, as we see above, the answer for 12 ÷ 5 is not an integer.

To have answers for all possible division questions, we have to extend the number system to include fractions and negative fractions, in other words, numbers of the form $\frac{\text{integer}}{\text{integer}}$. This system of numbers is called **rational numbers**. We can represent rational numbers as common fractions or as decimal numbers.

8. Express the answers for each of the following division problems in two ways. Firstly, using the common fraction notation and secondly, using the decimal notation for fractions.
   (a) 23 ÷ 10 (b) 23 ÷ 5
   (c) 230 ÷ 100 (d) 8 ÷ 10

CHAPTER 1: WHOLE NUMBERS 3

Answers

9. The table on LB page 4 alongside clearly shows that the different sets of numbers can be related to each other by the ideas of **extension and closure** (see the diagram below).

Irrational numbers

Background information

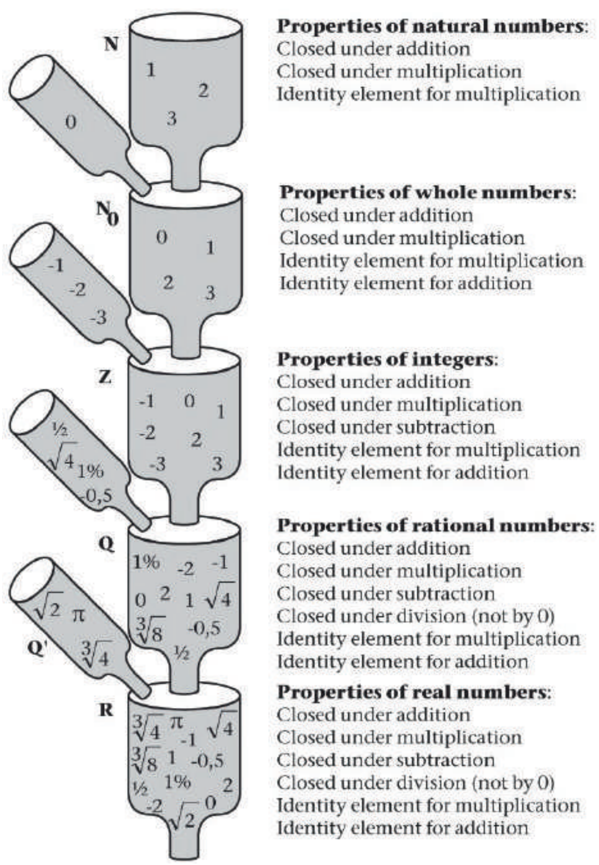
- No rational number, in either the common fraction notation or decimal notation, will produce the solution of the following problem: $(\text{number})^2 = 5$.
- Numbers like these are contained by the **set of irrational numbers** ***Q'***.
- The set of rational numbers ***Q***, together with the set of irrational numbers ***Q'***, is called the **set of real numbers** ***R***.

Teaching guidelines

Learners discuss the concept of irrational numbers.

Mathematical notes

The idea of extension of the various sets of numbers is illustrated by the diagram alongside.



9. Copy the table and answer the statement by writing "yes" or "no" in the appropriate cell.

Statement	Natural numbers	Whole numbers	Integers	Rational numbers
The sum of two numbers is a number of the same kind (closed under addition).	Yes	Yes	Yes	Yes
The sum of two numbers is always bigger than either of the two numbers.	Yes	Yes	No	No
When one number is subtracted from another, the answer is a number of the same kind (closed under subtraction).	No	No	Yes	Yes
When one number is subtracted from another, the answer is always smaller than the first number.	Yes	Yes	No	No
The product of two numbers is a number of the same kind (closed under addition).	Yes	Yes	Yes	Yes
The product of two numbers is always bigger than either of the two numbers.	Yes	Yes	No	No
The quotient of two numbers is a number of the same kind (closed under division).	No	No	No	Yes
The quotient of two numbers is always smaller than the first of the two numbers.	No (e.g. $\frac{2}{1}$)	No (e.g. $\frac{2}{1}$)	No	No

Irrational numbers

Rational numbers do not provide for all situations that may occur in Mathematics. For example, there is no rational number which will produce the answer 2 when it is multiplied by itself.

(number) × (same number) = 2

2 × 2 = 4 and 1 × 1 = 1, so clearly, this number must be between 1 and 2.

But there is no number which can be expressed as a fraction, in either the common fraction or the decimal notation, which will solve this problem. Numbers like these are called **irrational numbers**.

Here are some more examples of irrational numbers:

$\sqrt{5}$ $\sqrt{10}$ $\sqrt{3}$ $\sqrt{7}$ π

Rational and irrational numbers together, are called **real numbers**.

4 MATHEMATICS GRADE 9: TERM 1

1.2 Calculations with whole numbers

ESTIMATING, ROUNDING OFF AND COMPENSATING

Background information

- Estimation is approximation based upon a judgement. However, approximation is not necessarily estimation.
    - **Approximation** is applied to an existing number (actual answer) in order to find a result precise enough for a specific purpose.
    - **Estimation** is the mental skill of making an educated guess of the actual answer of a mathematical problem.
- **Error** is the difference between an estimate and the actual answer.
- **Rounding** is when a number is truncated so as to minimise error.

Teaching guidelines

- **Estimate**: this is the answer to a problem by rounding off the information and guessing a number that can be used to check the actual answer.
- **Approximate**: this is the actual answer rounded off so that people find it easier to understand.
- **Compensate for errors**: helps to minimise the errors.

Answers

1. (a) 500 × R40 = R20 000. No
   (b) 200 × R40 = R8 000. Yes
   (c) 250 × R40 = R10 000. Yes
2. See the answers on LB page 5 alongside.
3. (a) (i) 100 × R200 = R20 000
   (ii) 80 × R240 = R19 200
   (b) (i) 100 × R300 = R30 000
   (ii) 120 × R260 = R26 000 + R5 200 = R31 200
   (c) (i) R5 700 + R3 300 = R9 000
   (ii) R5 670 + R3 280 = R8 950
   (d) (i) R1 200 – R900 = R300
   (ii) R1 230 – R870 = R360
4. (a) Adding it.

1.2 Calculations with whole numbers

Do **not** use a calculator in Section 1.2, unless told to do so.

ESTIMATING, ROUNDING OFF AND COMPENSATING

1. A shop owner wants to buy chickens from a farmer. The farmer wants R38 for each chicken. Answer the following questions without doing written calculations:
   (a) If the shop owner has R10 000 to buy chickens, do you think he can buy more than 500 chickens?
   (b) Do you think he can buy more than 200 chickens?
   (c) Do you think he can buy more than 250 chickens?

What you were trying to do in question 1 is called **estimation**. To estimate, when working with numbers, means to try to get close to an answer without actually doing the calculations. However, you can do other, simpler calculations to estimate.

When the goal is not to get an accurate answer, numbers may be rounded off. For example, the cost of 51 chickens at R38 each may be **approximated** by calculating 50 × 40. This is clearly much easier than calculating 51 × R38.

To approximate something means to try find out more or less how much it is, without measuring or calculating it precisely.

2. (a) How much is 5 × 4? 20
   (b) How much is 5 × 40? 200
   (c) How much is 50 × 40? 2 000

The cost of 51 chickens at R38 each is therefore, approximately R2 000.

This approximation was obtained by rounding both 51 and 38 off to the nearest multiple of 10, and then calculating with the multiples of 10.

3. In each case, estimate the cost by rounding off to calculate the approximate cost, without using a calculator. In each case, make two estimates. First make a rough estimate by rounding the numbers off to the nearest 100 before calculating. Then make a better estimate by rounding the numbers off to the nearest 10 before calculating.
   (a) 83 goats are sold for R243 each
   (b) 121 chairs are sold for R258 each
   (c) R5 673 is added to R3 277
   (d) R874 is subtracted from R1 234

Suppose you have to calculate R823 – R273.
An estimate can be made by rounding the numbers off to the nearest 100:
R800 – R300 = R500.

4. (a) By working with R800 instead of R823, an error was introduced into your answer. How can this error be corrected: by adding R23 to the R500, or by subtracting it from R500?

Answers

4. (b) R500 + R23 = R523 (c) R523 + R27 = R550
5. (a) 800 – 300 = 500 (b) 2 300 – 1 900 = 400
   (c) 800 + 300 = 1 100 (d) 2 300 + 1 900 = 4 200
   (e) 0 + 300 = 300 (f) 3 200 – 1 800 = 1 400
   (g) 8 200 – 2 800 = 5 400 (h) 5 200 – 3 800 = 1 400
6. (a) 800 – 300 = 500; 500 + 12 = 512; 512 – 42 = 470
   (b) 2 300 – 1 900 = 400; 400 + 42 = 442; 442 + 24 = 466
   (c) 800 + 300 = 1 100; 1 100 + 12 = 1 112; 1 112 + 42 = 1 154
   (d) 2 300 + 1 900 = 4 200; 4 200 + 42 = 4 242; 4 242 – 24 = 4 218
   (e) 0 + 300 = 300; 300 + 9 = 309; 309 – 22 = 287
   (f) 3 200 – 1 800 = 1 400; 1 400 + 31 = 1 431; 1 431 + 31 = 1 462
   (g) 8 200 – 2 800 = 5 400; 5 400 + 34 = 5 434; 5 434 + 24 = 5 458
   (h) 5 200 – 3 800 = 1 400; 1 400 + 13 = 1 413; 1 413 + 32 = 1 445

ADDING IN COLUMNS

Background information

For addition and subtraction, the **commutative and associative properties** form the basis of the column methods. Writing the one number below the other one and working in columns is useful because it automatically rearranges the parts of the numbers as is allowed by the commutative and associative properties of addition. For example, in the calculation of 278 + 546, writing one number below the other and working in columns means the same as replacing (200 + 70 + 8) + (500 + 40 + 6) with (200 + 500) + (70 + 40) + (8 + 6) for the purposes of calculation. The transition from (200 + 70 + 8) + (500 + 40 + 6) to (200 + 500) + (70 + 40) + (8 + 6) is a direct application of the associative and commutative properties of addition. You could point this out when learners do the work on LB page 6.

Teaching guidelines

Use the systematic layout on LB page 6 alongside to revise addition in columns.

Answers

1. See the answers on LB page 6 alongside.
2. Step 1: Add 3 000 and 5 000.
   Step 2: Add 700 and 400.
   Step 3: Add 50 and 80.
   Step 4: Add 8 and 6.
3. (a) 7 662 (b) 8 892 (c) 13 393 (d) 20 080

(b) Correct the error to get a better estimate.
(c) Now also correct the error that was made by subtracting R300 instead of R273.

What you did in question 4 is called **compensating for errors**.

5. Estimate each of the following by rounding off the numbers to the nearest 100:

   (a) 812 – 342 (b) 2 342 – 1 876
   (c) 812 + 342 (d) 2 342 + 1 876
   (e) 9 + 278 (f) 3 231 – 1 769
   (g) 8 234 – 2 776 (h) 5 213 – 3 768

6. Find the exact answer for each of the calculations in question 5, by working out the errors caused by rounding, and compensating for them.

ADDING IN COLUMNS

1. (a) Write 8 000 + 1 100 + 130 + 14 as a single number. 9 244
   (b) Write 3 000 + 700 + 50 + 8 as a single number. 3 758
   (c) Write 5 486 in expanded notation, as shown in question 1(b). 5486=5000+400+80+6

You can calculate 3 758 + 5 486 as shown on the left below.

	3 758
	5 486
Step 1	8 000
Step 2	1 100
Step 3	130
Step 4	14
	9 244

You can do this in short, as shown on the right. This is a bit harder on the brain, but it saves paper!

3 758
5 486
9 244

2. Explain how the numbers in each of Steps 1 to 4 are obtained.

It is only possible to use the shorter method if you add the units first, then add the tens, then the hundreds and finally, the thousands. You can then do what you did in question 1(a), without writing the separate terms of the expanded form.

3. Calculate each of the following:

   (a) 3 878 + 3 784 (b) 298 + 8 594
   (c) 10 921 + 2 472 (d) 1 298 + 18 782

4. A farmer buys a truck for R645 840, a tractor for R783 356, a plough for R83 999 and a bakkie for R435 690.

Answers

4. (a) R600 000 + R800 000 + R100 000 + R400 000 = R1 900 000
   (b) R1 948 885
5. (a) R500 000 – R300 000 = R200 000
   (b) R279 323

MULTIPLYING IN COLUMNS

Background information

The **distributive property** allows a product to be broken down into partial products, which can be calculated more easily. For example, 7 × 648, which means 7 × (600 + 40 + 8), can be broken down into 7 × 600 + 7 × 40 + 7 × 8. If learners know the basic multiplication facts (tables), they can easily calculate these partial products and add them up to evaluate 7 × 648. The column format is a convenient way of making the transition from 7 × (600 + 40 + 8) to 7 × 600 + 7 × 40 + 7 × 8.

Teaching guidelines

Use the systematic layout on LB page 7 alongside to revise multiplication in columns.

Answers

1. See the answers on LB page 7 alongside.
2. Step 1: 7 × 9 = 63
   Step 2: 7 × 80 = 560
   Step 3: 7 × 400 = 2 800
   Step 4: 7 × 3 000 = 21 000

(a) Estimate to the nearest R100 000 how much these items will cost altogether.
(b) Use a calculator to calculate the total cost.

5. An investor makes R543 682 in one day on the stock market and then loses R264 359 on the same day.
   (a) Estimate to the nearest R100 000 how much money she has made in total on that day.
   (b) Use a calculator to determine how much money she has made.

MULTIPLYING IN COLUMNS

1. (a) Write 3 489 in expanded notation. 3 489 = 3 000 + 400 + 80 + 9
   (b) Write an expression without brackets that is equivalent to 7 × (3 000 + 400 + 80 + 9). 7 × 3 489

7 × 3 489 may be calculated as shown on the left below.

	3 489
	× 7
Step 1	63
Step 2	560
Step 3	2 800
Step 4	21 000
	24 423

A shorter method is shown on the right.

3 489
× 7
24 423

2. Explain how the numbers in each of Steps 1 to 4 on the above left are obtained.

47 × 3 489 may be calculated as shown on the left below.

	3 489
	× 47
Step 1	63
Step 2	560
Step 3	2 800
Step 4	21 000
Step 5	360
Step 6	3 200
Step 7	16 000
Step 8	120 000
	163 983

A shorter method is shown on the right.

3 489
× 47
24 423
139 560
163 983

Answers

3. Step 5: 40 × 9 = 360
   Step 6: 40 × 80 = 3 200
   Step 7: 40 × 400 = 16 000
   Step 8: 40 × 3 000 = 120 000
4. 40 × 3 000 + 40 × 400 + 40 × 8 + 40 × 9 = 120 000 + 16 000 + 3 200 + 360 = 139 560

SUBTRACTING IN COLUMNS

Background information

There are **various methods** you could use to subtract numbers. For example:

- **adding on**: To find the difference between two numbers, start at the smaller number and add on up to the larger number. This resembles skipping forward from the smaller number to the larger number on a number line: 26 – 14 = 6 + 6 = 12 (start at 14, skip 6 up to 20 and 6 more up to 26).
- **using compensation**: To calculate 26 – 14, use any of these methods:
  - Create an "easier" number **to subtract from** by adding 4 to both numbers:

    26 – 14 = (26 + 4) – (14 + 4) = 30 – 18 = 12
  - Create an "easier" number **to subtract from** by adding 6 to both numbers:

    26 – 14 = (26 + 6) – (14 + 6) = 32 – 20 = 12
- **using expanded form**: Subtracting in columns resembles writing numbers in expanded form and, if necessary, rearranging the first line so that subtraction in each column is possible.

Teaching guidelines

Use the systematic layout on LB page 8 to revise subtraction in columns.

Answers

1. See the answers on LB page 8 alongside.
2. See the notes on LB page 8 alongside.
3. (a) 7 000 + 1 100 + 190 + 14 (b) 2 000 + 1 300 + 120 +11
4. See the answers and notes on LB page 8.
5. (a) 120 – 50 = 70
   (b) 1 300 – 900 = 400
   (c) 7 000 – 3 000 = 4 000
   (d) 4 000 + 400 + 70 + 5 = 4 475

3. Explain how the numbers in each of Steps 5 to 8 on the left on page 7 are obtained.
4. Explain how the number 139 560 that appears in the shorter form on the right on page 7 is obtained.

SUBTRACTING IN COLUMNS

1. Write each of the following as a single number:
   (a) 8 000 + 400 + 30 + 2 = 8 432
   (b) 7 000 + 1 300 + 120 + 12 = 8 432
   (c) 3 000 + 900 + 50 + 7 = 3 957
2. If you worked correctly you should have obtained the same answers for questions 1(a) and 1(b). If this was not the case, redo your work.

The expression 7 000 + 1 300 + 120 + 12 was formed from 8 000 + 400 + 30 + 2 by:

- taking 1 000 away from 8 000 and adding it to the hundreds term to get 1 400
- taking 100 away from 1 400 and adding it to the tens term to get 130
- taking 10 away from 130 and adding it to the units term to get 12.

3. Form an expression like the expression in question 1(b) for each of the following:
   (a) 8 000 + 200 + 100 + 4 (b) 3 000 + 400 + 30 + 1
4. Write expressions like in question 1(b) for the following numbers:
   (a) 7 214 = 6 000 + 1 100 + 100 + 14 (b) 8 103 = 7 000 + 1 000 + 90 + 13

8 432 – 3 957 can be calculated as shown below:

	8 432
	– 3 957
Step 1	5
Step 2	70
Step 3	400
Step 4	4 000
Step 5	4 475

To do the subtraction in each column, you need to think of 8 432 as 8 000 + 400 + 30 + 2; in fact, you have to think of it as 7 000 + 1 300 + 120 + 12.

In Step 1, the 7 of 3 957 is subtracted from 12.

5. (a) How is the 70 in Step 2 obtained?
   (b) How is the 400 in Step 3 obtained?
   (c) How is the 4 000 in Step 4 obtained?
   (d) How is the 4 475 in Step 5 obtained?

Because of the zeros obtained in Steps 2, 3 and 4, the answers need not be written separately as shown above. The work can actually be shown in the short way on the right.

8 432
– 3 957
4 475

8 MATHEMATICS GRADE 9: TERM 1

Answers

6. (a)

9 123
– 3 784
5 339

(b)

8 284
– 3 547
4 737

7. Learners use a calculator to check the answers for questions 6 (a) and (b).

8. (a)

7 243
– 3 182
4 061

(b)

6 221
– 1 888
4 333

9. R87 456 – R44 800 = R42 656
10. R46 964
11. See the answers on LB page 9 alongside.

LONG DIVISION

Background information

- During division:
  - the **dividend** is the number that gets divided
  - the **divisor** is the number that does the division.

4 349 ÷ 35

- **Long division** involves subtraction of multiples of the divisor from the dividend until the remainder, if any, is smaller than the divisor.

Example: Calculate 4 349 ÷ 35.

Use powers of 10, doubling and halving to find multiples of the divisor:

35 × 100 = 3 500

35 × 10 = 350 → 35 × 20 = 700

35 × 1 = 35 → 35 × 2 = 70 → 35 × 4 = 140

Subtract multiples of the divisor from the dividend:

4 349	
– 3 500	(35 × 100 = 3 500)
849	
– 700	(35 × 20 = 700)
149	
– 140	(35 × 4 = 140)
9	(100 + 20 + 4 = 124)

Answer: 4 349 ÷ 35 = 124 remainder 9

6. Calculate each of the following:
   (a) 9 123 – 3 784 (b) 8 284 – 3 547
7. Use a calculator **only** to check your answers. If your answers are wrong, try again.
8. Calculate each of the following:
   (a) 7 243 – 3 182 (b) 6 221 – 1 888

You may use a calculator to do the questions below.

9. Bettina has R87 456 in her savings account. She withdraws R44 800 to buy a car. How much money is left in her savings account?
10. Liesbet starts a savings account by making a deposit of R40 000. Over a period of time she does the following transactions on the savings account:
    - a withdrawal of R4 000
    - a withdrawal of R2 780
    - a deposit of R1 200
    - a deposit of R7 550
    - a withdrawal of R5 230
    - a deposit of R8 990
    - a deposit of R1 234

    How much money does she have in her savings account now?
11. (a) R34 537 – R13 267 R21 270 (b) R135 349 – R78 239 R57 110

LONG DIVISION

Study this method for calculating 13 254 ÷ 56:

	13 254	
200 × 56 = 11 200	11 200	(200 is a rough estimate of the answer for 13 254 ÷ 56)
	2 054	(2 054 remains after 11 200 is taken from 13 254)
30 × 56 = 1 680	1 680	(30 is a rough estimate of the answer for 2 054 ÷ 56)
	374	(374 remains after 1 680 is taken from 2 054)
6 × 56 = 336	336	(6 is an estimate of the answer for 374 ÷ 56)
236 × 56 = 13 216	38	(38 remains)

So, 13 254 ÷ 56 = 236 remainder 38, or 13 254 ÷ 56 = $236\frac{38}{56} = 236\frac{19}{28}$, which can also be written as 236,68 (correct to two decimal figures).

CHAPTER 1: WHOLE NUMBERS 9

Teaching guidelines

Explain the correct layout for long division by working through the layout on LB page 10.

Answers

1. (a) 36 177 ÷ 78
   (b) 78 × 400 = 31 200; 400 is an estimate of the answer to 36 177 ÷ 78.
   (c) 4 977 remains after subtracting 31 200 from 36 177.
   (d) 60 is an approximation of 4 977 ÷ 78. (60 × 78 = 4 680)
2. (a)

```
     524
63|33 030
  31 500
   1 530
   1 260
     270
     252
      18
```

33 030 ÷ 63 = 524 remainder 18

(b)

```
     683
27|18 450
  16 200
   2 250
   2 160
      90
      81
       9
```

18 450 ÷ 27 = 683 remainder 9

3. Learners use a calculator to check the answers in question 2.
4. (a)

```
      265
287|76 287
    57 400
    18 887
    17 220
     1 667
     1 435
       232
```

76 287 ÷ 287 = 265 remainder 232

(b)

```
    1 484
44|65 309
   44 000
   21 309
   17 600
    3 709
    3 520
      189
      176
       13
```

65 309 ÷ 44 = 1 484 remainder 13

5. 1 180,5555 ... ≈ 1 180 street name boards
   1 180 × 72 = 84 960; 85 000 – 84 960 = R40. There will be R40 left.
6. Furniture dealer: R243,48 per desk
   School supply company: R337,78 per desk. The furniture dealer is cheaper.

The work can also be set out as follows:

```
        6
       30
      200
56 | 13 254        or more briefly as        56 | 13 254   (236)
     11 200                                       11 200
      2 054                                        2 054
      1 680                                        1 680
        374                                          374
        336                                          336
         38                                           38
```

1. (a) Mlungisi's work to do a certain calculation is shown on the right. What is the question that Mlungisi tries to answer?
   (b) Where does the number 31 200 in Step 1 come from? How did Mlungisi obtain it, and for what purpose did he calculate it?
   (c) Explain Step 2 in the same way as you explained Step 1.
   (d) Explain Step 3.

```
            463
       78 | 36 177
Step 1      31 200
Step 2       4 977
Step 3       4 680
Step 4         297
Step 5         234
                63
```

2. Calculate each of the following without using a calculator:
   (a) 33 030 ÷ 63 (b) 18 450 ÷ 27
3. Use a calculator to check your answers to question 2. If your answers are wrong, try again. It is important that you learn to do long division correctly.
4. Calculate each of the following:
   (a) 76 287 ÷ 287 (b) 65 309 ÷ 44

Use your calculator to do questions 5 and 6 below.

5. A municipality has budgeted R85 000 for putting up new street name boards. The street name boards cost R72 each. How many new street name boards can be put up, and how much money will be left in the budget?
6. A furniture dealer quoted R840 000 for supplying 3 450 school desks. A school supply company quoted R760 000 for supplying 2 250 of the same desks. Which provider is cheapest, and what do the two providers actually charge for one school desk?

10 MATHEMATICS GRADE 9: TERM 1

1.3 Multiples and factors

LOWEST COMMON MULTIPLE AND PRIME FACTORISATION

Background information

- A **common multiple** is a multiple of two or more numbers.

  Example: 36; 72; 108; ... are common multiples of 12 and 18.
- The **lowest common multiple** (**LCM**) of two or more numbers is the smallest number which is a common multiple of the numbers. It can be found by multiplying all the prime factors of both numbers, without repeating (except where a number is repeated as a factor in one of the numbers).

  Example: $12 = 2 \times 2 \times 3$; $18 = 2 \times 3 \times 3$;
  LCM $= 2 \times 2 \times 3 \times 3 = 36$.
- A **common factor** is a factor of two or more numbers.

  Example: $110 = 2 \times \mathbf{5} \times 11$; $105 = 3 \times \mathbf{5} \times 7$;
  5 is a common factor of 110 and 105.
- The **highest common factor** (**HCF**) of two or more numbers is the biggest number that is a common factor of the numbers. It can be found by multiplying the factors that are common to the two numbers, i.e. in the list of prime factors of both numbers.

  Example: $96 = \mathbf{2} \times \mathbf{2} \times \mathbf{2} \times 2 \times 2 \times \mathbf{3}$; $120 = \mathbf{2} \times \mathbf{2} \times \mathbf{2} \times \mathbf{3} \times 5$;
  HCF $= 2 \times 2 \times 2 \times 3 = 24$

Teaching guidelines

- Revise the concepts listed above.
- Discuss how the LCM is used to add and subtract fractions.

  Example: $\frac{5}{12} + \frac{1}{18} = \frac{15}{36} + \frac{2}{36} = \frac{17}{36}$
- Discuss how the HCF is used to simplify fractions.

  Example: $\frac{110 \div 5}{105 \div 5} = \frac{22}{21}$

Answers

1. (a) 15
   (b) See the tables on LB page 11 alongside.
   (c) 30
2. See the answers on LB page 11 alongside.
3. (a) Yes (b) Yes (c) No

1.3 Multiples and factors

LOWEST COMMON MULTIPLES AND PRIME FACTORISATION

1. Consecutive multiples of 6, starting at 6 itself, are shown in the following table:

6	12	18	24	30	36	42	48	54	60
66	72	78	84	90	96	102	108	114	120
126	132	138	144	150	156	162	168	174	180
186	192	198	204	210	216	222	228	234	240

   (a) The following table also shows multiples of a number. What is the number?

15	30	45	60	75	90	105	120	135	150
165	180	195	210	225	240	255	270	285	300
315	330	345	360	375	390	405	420	435	450
465	480	495	510	525	540	555	570	585	600

   (b) Copy both tables. Draw rough circles around all the numbers that occur in both tables.
   (c) What is the smallest number that occurs in both tables?

90 is a multiple of 6; it is also a multiple of 15.
90 is called a **common multiple** of 6 and 15; it is a multiple of both.
The smallest number that is a multiple of both 6 and 15 is the number 30.
30 is called the **lowest common multiple** or **LCM** of 6 and 15.

2. Calculate, without using a calculator:
   (a) $2 \times 3 \times 5 \times 7 \times 11$ 2 310
   (b) $2 \times 2 \times 5 \times 7 \times 13$ 1 820
   (c) $2 \times 3 \times 3 \times 3 \times 5 \times 13$ 3 510
   (d) $3 \times 5 \times 5 \times 17$ 1 275

Check your answers by using a calculator or by comparing with some classmates.
The number 2 is a factor of each of the numbers 2 310, 1 820 and 3 510.
Another way of saying this is: 2 is a **common factor** of 2 310, 1 820 and 3 510.

3. (a) Is 2×3, in other words, 6, a common factor of 2 310 and 3 510?
   (b) Is $2 \times 3 \times 5$, in other words, 30, a common factor of 2 310 and 3 510?
   (c) Is there any bigger number than 30 that is a common factor of 2 310 and 3 510?

30 is called the **highest common factor** or **HCF** of 2 310 and 3 510.

Answers

4. (a) 1 820 = 2 × 2 × 5 × 7 × 13; 3 510 = 2 × 3 × 3 × 3 × 5 × 13
HCF = 2 × 5 × 13 = 130
LCM = 2 × 2 × 3 × 3 × 3 × 5 × 7 × 13 = 49 140

(b) 2 310 = 2 × 3 × 5 × 7 × 11; 1 275 = 3 × 5 × 5 × 17
HCF = 3 × 5 = 15
LCM = 2 × 3 × 5 × 5 × 7 × 11 × 17 = 196 350

(c) 1 820 = 2 × 2 × 5 × 7 × 13; 3 510 = 2 × 3 × 3 × 3 × 5 × 13; 1 275 = 3 × 5 × 5 × 17
HCF = 5
LCM = 2 × 2 × 3 × 3 × 3 × 5 × 5 × 7 × 13 ×17 = 4 176 900

(d) 2 310 = 2 × 3 × 5 × 7 × 11; 1 275 = 3 × 5 × 5 × 17; 1 820 = 2 × 2 × 5 × 7 × 13
HCF = 5
LCM = 2 × 2 × 3 × 5 × 5 × 7 × 11 × 13 × 17 = 5 105 100

(e) 780 = 2 × 2 × 3 × 5 × 13; 7 700 = 2 × 2 × 5 × 5 × 7 × 11
HCF = 2 × 2 × 5 = 20
LCM = 2 × 2 × 3 × 5 × 5 × 7 × 11 × 13 = 300 300

(f) 360 = 2 × 2 × 2 × 3 × 3 × 5; 1 360 = 2 × 2 × 2 × 2 × 5 × 17
HCF = 2 × 2 × 2 × 5 = 40
LCM = 2 × 2 × 2 × 2 × 3 × 3 × 5 × 17 = 12 240

1.4 Solving problems about ratio, rate and proportion

RATIO AND RATE PROBLEMS

Background information

- The work on **ratio** and **rate** is possibly the most challenging part of the prescribed content on whole numbers. Although we use division to calculate both ratios and rates, the two concepts have quite different meanings.
  - A **ratio** is a relationship between two or more quantities of the same kind.
  - A **rate** is a relationship between two quantities of different kinds.
- Two sets of quantities are in **direct proportion** when a constant multiplier can be used to find an element in one set if the matching element in the other set is given (an increase in one, matches an increase in the other).

  Example: Refer to question 2 on LB page 12 alongside.

In question 2 you can see the list of **prime factors** of the numbers 2 310, 1 820, 3 510 and 1 275.

> The LCM of two numbers can be found by multiplying all the prime factors of both numbers, without repeating (except where a number is repeated as a factor in one of the numbers).
> The HCF of two numbers can be found by multiplying the factors that are common to the two numbers, i.e. in the list of prime factors of both numbers.

4. In each case, find the HCF and LCM of the numbers:

(a) 1 820 and 3 510 (b) 2 310 and 1 275
(c) 1 820 and 3 510 and 1 275 (d) 2 310 and 1 275 and 1 820
(e) 780 and 7 700 (f) 360 and 1 360

1.4 Solving problems about ratio, rate and proportion

RATIO AND RATE PROBLEMS

You **may** use a calculator in this section.

1. Moeneba collects apples in the orchard. She picks about five apples each minute. Approximately how many apples will Moeneba pick in each of the following periods of time?

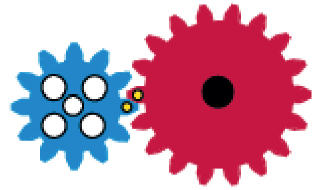
(a) eight minutes 40 apples (b) 11 minutes 55 apples
(c) 15 minutes 75 apples (d) 20 minutes 100 apples

In the situation described in question 1, Moeneba picks apples **at a rate of** about five apples **per minute**.

2. Garth and Kate also collect apples in the orchard, but they both work faster than Moeneba. Garth collects at a rate of about 12 apples per minute, and Kate collects at a rate of about 15 apples per minute. Copy and complete the following table to show approximately how many apples they will each collect in different periods of time:

Period of time in min	1	2	3	8	10	20
Moeneba	5	10	15	40	50	100
Garth	12	24	36	96	120	240
Kate	15	30	45	120	150	300
The three together	32	64	96	256	320	640

Teaching guidelines



- Turning the blue gear with 12 teeth by hand will drive the red gear, which has 18 teeth.
    - For the red gear to make one revolution (so that the yellow dot is back where it started) the blue gear has to make $1\frac{1}{2}$ revolutions. For the red gear to make two revolutions, the blue gear has to make three revolutions. The relationship between the number of blue revolutions and the number of red revolutions is called a **ratio**.
    - Now consider how fast a gear turns. The blue gear may make 30 revolutions in one minute, in which case the red gear will turn at 20 revolutions per minute (rpm). These specifications of turning speed are **rates**. The quantity "30 rpm" states the **amount of one quantity** (in this case, revolutions) **associated with one unit of a different quantity** (in this case, time).
- Use question 2 on LB page 12 to illustrate the concept of direct proportion.

Answers

1. See the answers on LB page 12 on the previous page.
2. See the table on LB page 12 on the previous page.
3. (a) 15 : 12

   (b) Yes, 15:12 simplifies to 5:4 as you divide both sides of the ratio by 3.
4. 5 : 2 : 1 So 500 g flour, 200 g oatmeal and 100 g cocoa powder
5. (a) 90 km in one hour

   (b) No, this is just an average of the total distance travelled. He probably did more in some hours and less in the others.

   (c) 7 × 90 = 630 km

   (d) 900 ÷ 90 = 10 hours
6. For 5(c): distance = average speed × time; For 5(d): time = $\frac{\text{distance}}{\text{average speed}}$
7. (a) Section A: 110 km/h; Section B: 90 km/h; Section C: 70 km/h

   (b) Distance = 440 km + 540 km + 280 km = 1 260 km

   Time = 4 h + 6 h + 4 h = 14 h

   Average speed = 1 260 ÷ 14 = 90 km/h

   (c) If he drives at a similar speed, he will drive at 90 km/h.

   874 km ÷ 90 km/h = 9,711... hr or 9 hours and 43 min

In this situation, the number of apples picked is **directly proportional** to the time taken.

If you filled the table in correctly, you will notice that during any period of time, Kate collected three times as many apples as Moeneba. We can say that during any time interval, the **ratio** between the numbers of apples collected by Moeneba and Kate is **3 to 1**, which can be written as **3: 1**. For any period of time, the ratio between the numbers of apples collected by Garth and Moeneba is 12: 5.

3. (a) What is the ratio between the numbers of apples collected by Kate and Garth during a period of time?

   (b) Would it be correct to also say that the ratio between the numbers of apples collected by Kate and Garth is 5: 4? Explain your answer.
4. To make biscuits of a certain kind, five parts of flour are to be mixed with two parts of oatmeal, and one part of cocoa powder. How much oatmeal and how much cocoa powder must be used if 500 g of flour is used?
5. A motorist covers a distance of 360 km in exactly four hours.

   (a) Approximately how far did the motorist drive in one hour?

   (b) Do you think the motorist covered exactly 90 km in each of the four hours? Explain your answer briefly.

   (c) Approximately how far will the motorist drive in seven hours?

   (d) Approximately how long will the motorist need to travel 900 km?

Some people use these formulae to do calculations like those in question 5:

average speed = $\frac{\textbf{distance}}{\textbf{time}}$, which means distance ÷ time

distance = average speed × time

time = $\frac{\textbf{distance}}{\textbf{average speed}}$, which means distance ÷ average speed

6. For each of questions 5(c) and 5(d), state which formula will produce the correct answer.
7. A motorist completes a journey in three sections, making two long stops to eat and relax between sections. During section A he covers 440 km in four hours. During section B he covers 540 km in six hours. During section C he covers 280 km in four hours.

   (a) Calculate his average speed over each of the three sections.

   (b) Calculate his average speed for the journey as a whole.

   (c) On the next day, the motorist has to travel 874 km. How much time (stops excluded) will he need to do this? Justify your answer with calculations.
8. Different vehicles travel at different average speeds. A large transport truck with a heavy load travels much slower than a passenger car. A small bakkie is also slower than a passenger car. In the table on the following page, some average speeds and the

CHAPTER 1: WHOLE NUMBERS 13

Background information (continued)

- Two sets of quantities are in **indirect (inverse) proportion** when the products of matching elements of the two sets stay constant (an increase in one, matches a decrease in the other).

  Example: Refer to question 8 on LB page 13.

Teaching guidelines (continued)

Use question 8 on LB page 13 to illustrate the concept of indirect (inverse) proportion.

Answers

8. See the table on LB page 14 alongside.
9. (a) It decreases.
   (b) It increases.
   (c) The product speed × time stays constant.

1.5 Solving problems in financial contexts

DISCOUNT, PROFIT AND LOSS

Background information

- **Cost price** is the amount for which a dealer buys an article from a producer or manufacturer.
- **Marked price** is the amount marked on the article and usually includes the dealer's profit (or loss).
- **Selling price** is the price of the article after discount (if any).

Teaching guidelines

Revise the concepts listed above.

Answers

1. (a) R128
   (b) 8 × R128 = R1 024
2. See the answers on LB page 14 alongside.

---

times needed are given for different vehicles that all have to be driven for the same distance of 720 km. Copy and complete the table:

Time in hours	12	9	8	6	5
Average speed in km/h	60	80	90	120	144

9. Look at the table you have just completed.
   (a) What happens to the time needed if the average speed increases?
   (b) What happens to the average speed if the time is reduced?
   (c) What can you say about the product average speed × time, for the numbers in the table?

In the situation above, the average speed is said to be **indirectly proportional** to the time needed for the journey.

1.5 Solving problems in financial contexts

You **may** use a calculator in this section.

DISCOUNT, PROFIT AND LOSS

1. (a) R12 800 is divided equally between 100 people. How much money does each person get?
   (b) How much money do eight of the people together get?

Another word for hundredths is **per cent**.

Instead of $\frac{5}{100}$ we can write 5%. The symbol % means exactly the same as $\frac{}{100}$.
In question 1(a) you calculated $\frac{1}{100}$ or 1% of R12 800, and in question 1(b) you calculated $\frac{8}{100}$ or 8% of R12 800.

> The amount that a dealer pays for an article is called the **cost price**. The price marked on the article is called the **marked price** and the price of the article after the discount is the **selling price**.

2. The marked prices of some articles are given below. A discount of 15% is offered to customers who pay cash. In each case, calculate how much a customer who pays cash will actually pay:
   (a) R850 R722,50 (b) R140 R119
   (c) R32 600 R27 710 (d) R138 R117,30

Lina bought a couch at a sale. It was marked R3 500 but she paid only R2 800.

Background information (continued)

- **Discount** is the amount which is sometimes taken off the marked price of the article and is usually stated as a percentage.
  Example: marked price: R3 500;
  discount: R700; % discount: $\frac{R700}{R3\,500} \times 100\% = 20\%$
- **Profit** is made when something is sold for more than what it originally cost.
  Example: cost price: R400; selling price: R480; profit: R480 – R400 = R80
- **Percentage profit** is the profit expressed as a percentage of the cost price.
  Example: cost price: R400; profit: R80: percentage profit: $\frac{R80}{R400} \times 100\% = 20\%$
- **Loss** is made when something is sold for less than what it originally cost.
  Example: cost price: R699; selling price: R549; loss: R699 – R549 = R150

Teaching guidelines (continued)

Revise the concepts listed above.

Answers

3. (a) R35 (b) 700 ÷ R35 = 20 hundredths (c) The discount was 20%.
4. Percentage profit is calculated as a percentage of the cost price.
   Article A: Discount = R360 – R324 = R36. R36 ÷ R360 × 100 = 10%
   Profit = R324 – R240 = R84. R84 ÷ R240 × 100% = 35%
   Article B: Discount = R700 – R560 = R140. R140 ÷ R700 × 100% = 20%
   Profit = R560 – R540 = R20. R20 ÷ R540 × 100% = 3,7%
   Article C: Discount = R2 000 – R1 700 = R300. R300 ÷ R2 000 × 100% = 15%
   Profit = R1 700 – R1 200 = R500. R500 ÷ R1 200 × 100% = 41,666... ≈ 41,7%
5. (a) Costs: R750 + R3 600 = R4 350
   Income = 40 × R150 = R6 000
   Profit = R6 000 – R4 350 = R1 650
   (b) (25 × R150) + (15 × R100) = R3 750 + R1 500 = R5 250
   Profit amount = R5 250 – R4 350 = R900
   Percentage profit = R900 ÷ R4 350 = 20,68...% ≈ 20,7%
6. She made a loss of R8 per pie. R8 ÷ R60 × 100% = 13,333... % = 13,3% loss.

She was given a discount of R700.

What percentage discount was given to Lina?

This question means:

How many hundredths of the marked price were taken off?

To answer the question we need to know how much $\frac{1}{100}$ (one hundredth) of the marked price is.

3. (a) How much is $\frac{1}{100}$ of R3 500?
   (b) How many hundredths of R3 500 is the same as R700?
   (c) What percentage discount was given to Lisa: 10% or 20%?
4. The cost price, marked price and selling price of some articles are listed below:
   Article A: Cost price = R240; marked price = R360; selling price = R324.
   Article B: Cost price = R540; marked price = R700; selling price = R560.
   Article C: Cost price = R1 200; marked price = R2 000; selling price = R1 700.
   The profit is the difference between the cost price and the selling price.
   For each of the above articles, calculate the percentage discount and profit.
5. Remey decided to work from home and bought herself a sewing machine for R750. She planned to make 40 covers for scatter cushions. The fabric and other items she needed cost her R3 600. Remey planned to sell the covers at R150 each.
   (a) How much profit could Remey make if she sold all 40 covers at this price?
   (b) Remey managed to sell only 25 of the covers and decided to sell the rest at R100 each. Calculate her percentage profit.
6. Zadie bakes and sells pies to earn some extra income. The cost of the ingredients for one chicken pie comes to about R68. She sold the pies for R60 each. Did she make a profit or a loss? Calculate the percentage loss or profit.

HIRE PURCHASE

Sometimes you need an item but do not have enough money to pay the full amount immediately. One option is to buy the item on **hire purchase** (**HP**). You will have to pay a deposit and sign an agreement in which you undertake to pay monthly instalments until you have paid the full amount. Therefore:

HP price = deposit + total of instalments

The difference between the HP price and the cash price is the interest that the dealer charges you for allowing you to pay off the item over a period of time.

1. Sara buys a flat screen television on HP. The cash price is R4 199. She has to pay a deposit of R950 and 12 monthly instalments of R360.

CHAPTER 1: WHOLE NUMBERS 15

HIRE PURCHASE

Background information

- To buy an item on **hire purchase (HP)** means paying a deposit and signing an agreement to pay monthly instalments until you have paid the full amount.
- The **interest on hire purchase** is the difference between the HP price and the cash price and is charged for allowing you to pay off the item over a period of time.

Teaching guidelines

Discuss the concepts listed above.

Answers

1. (a) R950 + 12 × 360 = 950 + R4 320 = R5 270
   (b) R5 270 – R4 199 = R1 071
2. (a) Susie: R178 600; David: R140 300
   (b) R178 600 – R140 300 = R38 300
   (c) Susie: R48 600 ÷ R130 000 × 100% = 37,38...% ≈ 37,4%
   David: R10 300 ÷ R130 000 × 100% = 7,92...% ≈ 7,9%

SIMPLE INTEREST

Background information

Simple interest is calculated at the end of each period and is worked out only on the original amount. No interest is added to the original amount for later interest calculations.

Teaching guidelines

If the amount is invested for part of the year, the time must be written as a fraction of a year.

Answers

1. (a) 5% (b) 15% (c) 10% (d) $\frac{x}{a}$ × 100%
2. (a) 9% × R8 345 = R751,05 (b) R751,05 × 5 = R3 755,25
3. 5% of R3 500 = R175. R875 ÷ R175 = 5 years.
4. See the table on LB page 16 alongside.

(a) Calculate the total HP price.
(b) How much interest does she pay?

2. Susie buys a car on HP. The car costs R130 000. She pays a 10% deposit on the cash price and will have to pay monthly instalments of R4 600 for a period of three years. David buys the same car, but chooses another option where he has to pay a 35% deposit on the cash price and monthly instalments of R3 950 for two years.
   (a) Calculate the HP price for both options.
   (b) Calculate the difference between the total price paid by Susie and by David.
   (c) Calculate the interest that Susie and David have to pay as a percentage of the cash price.

SIMPLE INTEREST

When interest is calculated for a number of years on an amount (i.e. a fixed deposit), without the interest being added to the amount each year for the purpose of later interest calculations, it is referred to as **simple interest**. If the amount is invested for part of a year, the time must be written as a fraction of a year.

Example:

R2 000 invested at 10% per annum simple interest over 2 years:
End of first year: Amount = R2 000 + R200 interest of original amount = R2 200
End of second year: Amount = R2 200 + R200 interest of original amount = R2 400

1. Interest rates are normally expressed as percentages. This makes it easier to compare rates. Express each of the following as a percentage:
   (a) A rate of R5 for every R100
   (b) A rate of R7,50 for every R50
   (c) A rate of R20 for every R200
   (d) A rate of x rands for every a rands
2. Annie deposits R8 345 into a savings account at Bonus Bank. The interest rate is 9% per annum.
   (a) How much interest will she have earned at the end of the first year?
   (b) Annie decides to leave the deposit of R8 345 with the bank for an indefinite period, and to withdraw only the interest at the end of every year. How much interest does she receive over a period of five years?

Per annum means "per year".

3. Maxi invested R3 500 at an interest rate of 5% per annum. Her total interest was R875. For what period did she invest the amount?
4. Money is invested for one year at an interest rate of 8% per annum. Copy and complete the table of equivalent rates:

Sum invested (R)	1 000	2 500	8 000	20 000	90 000	x
Interest earned (R)	80	200	640	1 600	7 200	8% × x or 0,08x

16 MATHEMATICS GRADE 9: TERM 1

Answers

5. R260 × 20% = R52 over 365 days. R52 × $\frac{10}{365}$ = R1,42
6. (a) Final amount = original amount invested plus interest

   Interest = amount invested × 5% = amount invested × 0,05

   Final amount = investment + investment × 0,05 = investment × (1 + 0,05)

   Thuli wanted to multiply that expression by 5 because the money was invested for five years but then the amount invested (P) would also be multiplied by 5.

   (b) (Take P as amount invested and interest = 0,05).

   Final amount = P + interest = P + P.n.i = P(1 + n.i)

   $P = \frac{\text{Final amount}}{1+(0,05)\times 5} = \frac{6\ 250}{(1,25)}$ = 5 000; amount invested is R5 000

COMPOUND INTEREST

Background information

Compound interest is calculated at the end of each period and is worked out on the original amount plus any previous interest already earned.

Teaching guidelines

Use the example on LB page 17 to illustrate the concept of compound interest.

Answers

1. (a) R20 000 + (5% × R20 000) = R20 000 + R1 000 = R21 000

   (b) R21 000 + (5% × R21 000) = R21 000 + R1 050 = R22 050

   (c) R22 050 + (5% × R22 050) = R22 050 + R1 102,50 = R23 152,50
2. (a) R800 + (15% × R800) = R920 (b) R1 058 – R800 = R258
3. Compound interest: R750 + (14% × R750) = R855

   Second year: R855 + (14% × R855) = R974,70

   Simple interest: R750 + 2 × (R750 × 15%) = R975 ∴ Zinzi is correct.

5. Interest on overdue accounts is charged at a rate of 20% per annum. Calculate the interest due on an account that is ten days overdue if the amount owing is R260. (Give your answer to the nearest cent.)
6. A sum of money invested in the bank at 5% per annum, i.e. simple interest, amounted to R6 250 after five years. This final amount includes the interest. Thuli figured out that the final amount is (1 + 0,05 × 5) × amount invested.
   (a) Explain Thuli's thinking.
   (b) Calculate the amount that was invested.

COMPOUND INTEREST

When the interest earned each year is added to the original amount, and the interest for the following year is calculated on this new amount, the result is known as **compound interest**.

Example:

R2 000 is invested at 10% per annum compound interest:

End of first year:	Amount = R2 000 + R200 interest = R2 200
End of second year:	Amount = R2 200 + R220 interest = R2 420
End of third year:	Amount = R2 420 + R242 interest = R2 662

1. An amount of R20 000 is invested at 5% per annum compound interest.
   (a) What is the total value of the investment after one year?
   (b) What is the total value of the investment after two years?
   (c) What is the total value of the investment after three years?
2. Bonus Bank is offering an investment scheme over two years at a compound interest rate of 15% per annum. Mr Pillay wishes to invest R800 in this way.
   (a) How much money will be due to him at the end of the two-year period?
   (b) How much interest will have been earned during the two years?
3. Andrew and Zinzi are arguing about interest on money that they have been given for Christmas. They each received R750. Andrew wants to invest his money in ABC Building Society for two years at a compound interest rate of 14% per annum, while Zinzi claims that she will do better at Bonus Bank, earning 15% simple interest per annum over two years. Who is correct?

CHAPTER 1: WHOLE NUMBERS 17

Answers

4. (a) 12

 (b) $A = P(1 + \frac{r}{100})^n = R12\ 750(1 + 0{,}053)^4 = R15\ 675{,}58$

 (c) R15 675,58 – R12 750 = R2 925,58

5. Formula used to calculate the final amount: A = R5 000(1 + 0,1)3 = R6 655
 R6 655 – R5 000 = R1 655 interest

EXCHANGE RATE AND COMMISSION

Background information

- **Exchange rate** is the value of one currency for the purpose of conversion to another.

 Example: An exchange rate of 0,070 US Dollar to 1 South African rand means:
 - it will cost R1 to buy 7 American cents
 - it will cost 1 ÷ 0,070 = R14,28 to buy 1 US Dollar.
- **Commission on exchange rates** is a fee commonly charged by financial institutions for exchanging one currency to another.
- **Sales commission** is additional compensation to motivate employees to produce more sales and to reward and recognise those who perform most productively.

Teaching guidelines

Discuss the concepts listed above.

Answers

1. (a) £650 × R15,66/£ = R10 179

 R10 179 × 102,5% = R10 433,475 ≈ R10 433,48

 (b) 1 ÷ R15,66 = £0,0638...

2. $25,86 × R9,95/$ = R257,307 ≈ R257,31

3. 3% × R220 000 = R6 600 commission

---

4. Mr Martin invests an amount (P) of R12 750 at 5,3% (r) compound interest over a period (n) of four years. Use the formula: $A = P(1 + \frac{r}{100})^n$ and calculate the final amount (A) that his investment will be worth after four years.
 (a) How many conversion periods will his investment have altogether?
 (b) How much is his investment worth after three years?
 (c) Calculate the total interest that he earns on his initial investment.

5. Calculate the interest generated by an investment (P) of R5 000 at 10% (r) compound interest over a period (n) of three years. A is the final amount. Use the formula: $A = P(1 + \frac{r}{100})^n$ to calculate the interest.

EXCHANGE RATE AND COMMISSION

1. (a) Tim bought £650 at the foreign exchange desk at Gatwick Airport in the UK at a rate of R15,66 per £1. The desk also charged 2,5% commission on the transaction. How much did Tim spend to buy the pounds?
 (b) What was the value of R1 in British pounds on that day?

2. Mandy wants to order a book from the internet. The price of the book is $25,86. What is the price of the book in rands? Say, for example, that the exchange rate is R9,95 for $1.

3. Bongani is a car salesperson. He earns a commission of 3% on the sale of a car with the value of R220 000. Calculate how much commission he earned.

Grade 9 Term 1 Chapter 2 Integers

Learner Book Overview		
Sections in this chapter	**Content**	**Pages in Learner Book**
2.1 Which numbers are smaller than 0?	The need for negative numbers; properties of integers	Page 19
2.2 Adding and subtracting with integers	Addition and subtraction of negative numbers; subtraction of a larger number from a smaller number; addition of a positive and a negative number; subtraction of a negative number; subtraction of a positive number from a negative number; calculations	Pages 20 to 21
2.3 Multiplying and dividing with integers	Multiplication with integers; the commutative and distributive properties; division with integers; mixed calculations with integers	Pages 22 to 25
2.4 Powers, roots and word problems	Positive and negative square roots; cube roots; word problems	Pages 25 to 26

CAPS time allocation	4,5 hours
CAPS content specification	Page 121

Mathematical background

- The set of **integers** Z = {...; –4; –3; –2; –1; 0; 1; 2; 3; 4; ...} is formed by adding the set {...; –4; –3; –2; –1} to the set of whole numbers N_0 = {0; 1; 2; 3; 4; ....}.
- The **focus in this chapter** is on the computational properties of integers and the properties of operations with integers.
- The **computational properties of integers** describe how we do calculations with integers:
  - Subtracting a bigger number from a smaller number results in a negative number, for example, 10 – 20 = (–10) ("negative ten").
  - Any integer and its additive inverse lie the same distance from, and on opposite sides of, 0 on the number line. Therefore, the sum of any integer and its additive inverse is equal to 0, for example, 12 + (–12) = 0 or (–12) + 12 = 0 ("negative twelve plus twelve equals zero").
  - Adding an integer has the same effect as subtracting its additive inverse, for example, 3 + (–10) and 3 – 10 produce the same result.
  - Subtracting an integer has the same effect as adding its additive inverse, for example, 3 – (–10) and 3 + 10 produce the same result.
  - The product of a positive and a negative integer is negative, which means that the quotient of a negative and a positive integer is negative, for example, (–30) ÷ (+10) = –3. Or, the quotient of two negative integers is positive, for example, (–30) ÷ (–10) = +3.
  - The product of two negative integers is positive, which means that the quotient of a positive and a negative integer is negative, for example, (–3) × (–10) = 30.
- The **associative, commutative and distributive properties of operations with integers** may be called the "structural" properties of integers.

2.1 Which numbers are smaller than 0?

WHY PEOPLE DECIDED TO HAVE NEGATIVE NUMBERS

Background information

- **Whole numbers** start at 0 and extend in one direction on a number line.
- **Integers** extend in both directions on a number line.
- For every natural number there is an **additive inverse** which lies the same distance from, and on the other side of, 0 on the number line.
  Example: The additive inverse of 17 is –17 ("negative seventeen").
- The **set of integers** contain all natural numbers and their additive inverses, together with 0.
- When a large number is subtracted from a smaller number the answer may be a **negative number**.
  Example: 5 – 12 = –7 ("five minus twelve = negative seven")

Teaching guidelines

Make sure that learners are always aware of the difference between using the – sign to indicate subtraction and using it to indicate a negative number. To do this, it is useful to get learners into the habit of referring to a negative number using the term "negative" rather than "minus". For example, they should read –6 as "negative 6" and not "minus 6". In the Learner Book, the operation sign "–" is always separated from numbers by a space, so "10 minus 8" is written as, 10 – 8. However, when the – sign is used to indicate a negative number, there is no space between the sign and the number, for example, the number "negative 8" is printed as –8 and not as – 8.

PROPERTIES OF INTEGERS

Teaching guidelines

Make sure that learners understand the content of the table on LB page 19 alongside.

Answers

1. (a) $x = -30$. Property 2
   (b) $x = -30$. Property 1
   (c) $x = -10$. Property 2 and property 3
   (d) $x = -10$. Property 1 and property 3

CHAPTER 2
Integers

2.1 Which numbers are smaller than 0?

WHY PEOPLE DECIDED TO HAVE NEGATIVE NUMBERS

Numbers such as –7 and –500, the additive inverses of whole numbers, are included with all the whole numbers and are called **integers**.
Fractions can be negative too, for example: $-\frac{3}{4}$ and –3,46.

Natural numbers are used for counting and fractions (rational numbers) are used for measuring. Why do we also have negative numbers?

When a larger number is subtracted from a smaller number, the answer may be a negative number: 5 – 12 = –7. This number is called **negative 7**.

One of the most important reasons for inventing negative numbers was to provide solutions for equations like the following:

Equation	Solution	Required property of negative numbers
$17 + x = 10$	$x = -7$ because $17 + (-7) = 17 - 7 = 10$	1. Adding a negative number is just like subtracting the corresponding positive number
$5 - x = 9$	$x = -4$ because $5 - (-4) = 5 + 4 = 9$	2. Subtracting a negative number is just like adding the corresponding positive number
$20 + 3x = 5$	$x = -5$ because $3 \times (-5) = -15$	3. The product of a positive number and a negative number is a negative number

PROPERTIES OF INTEGERS

1. In each case, state what number will make the equation true. Also state which of the properties of integers in the table above, is demonstrated by the equation:
   (a) $20 - x = 50$
   (b) $50 + x = 20$
   (c) $20 - 3x = 50$
   (d) $50 + 3x = 20$

2.2 Adding and subtracting with integers

Background information

- **Number patterns** can be used to illustrate addition and subtraction rules for integers.

Pattern P	Pattern Q	Pattern R	Pattern S
3 + 0 = 3	3 – 0 = 3	6 – 0 = 6	6 + 0 = 6
3 + (–1) = 2	3 – 1 = 2	6 – (–1) = 7	6 + 1 = 7
3 + (–2) = 1	3 – 2 = 1	6 – (–2) = 8	6 + 2 = 8
3 + (–3) = 0	3 – 3 = 0	6 – (–3) = 9	6 + 3 = 9
3 + (–4) = –1	3 – 4 = –1	6 – (–4) = 10	6 + 4 = 10
3 + (–5) = –2	3 – 5 = –2	6 – (–5) = 11	6 + 5 = 11

- Patterns P and Q confirm that adding a negative number has the same effect as subtracting a natural number.
- Patterns R and S confirm that subtracting a negative number has the same effect as adding a natural number.
- Addition and subtraction of negative numbers are done in the **same way** as the addition and subtraction of positive numbers.
  Examples: (–5) + (–3) = –8, just like 5 + 3 = 8
  (–20) – (–7) = –13, just like 20 – 7 = 13.
- The **additive inverse** can be used to subtract a larger number from a smaller number.
  Example: 5 – 9 = 5 – 5 – 4 = 0 – 4 = –4
- **Adding an integer** has the same effect as subtracting its additive inverse.
  Example: 3 + (–10) and 3 – 10 produce the same result.

Teaching guidelines

Work through the examples on LB page 20.

Misconceptions

"Operations may be done on signs."

Example: 7 + (–5) = 7 – 5 because **adding a negative number has the same effect as subtracting its additive inverse**, NOT because "a plus times a minus is a minus". The two properties "subtracting an integer is the same as adding its additive inverse" and "adding an integer is the same as subtracting its additive inverse" have absolutely nothing to do with the property that "a negative number × a positive number = a negative number". Learners should not be allowed to develop the misconception that "operations can be done on signs", for example the idea that "a minus times a minus is a plus".

2.2 Adding and subtracting with integers

Addition and subtraction of negative numbers

Examples: (–5) + (–3) and (–20) – (–7)

(–5) + (–3) can also be written as –5 + (–3) or as –5 + –3

This is done in the same way as the addition and subtraction of positive numbers.

(–5) + (–3) = –8 and –20 – (–7) = –13

This is just like 5 + 3 = 8 and 20 – 7 =13, or R5 + R3 = R8, and R20 – R7 = R13.

Subtraction of a larger number from a smaller number

Examples: 5 – 9 and 29 – 51

Let us first consider the following:

5 + (–5) = 0 10 + (–10) = 0 and 20 + (–20) = 0

If we subtract 5 from 5, we get 0, but then we still have to subtract 4:

5 – 9 = 5 – 5 – 4
= 0 – 4
= –4

We know that –9 = (–4) + (–5)

Suppose the numbers are larger, for example 29 – 51:

29 – 51 = 29 – 29 – 22

–51 = (–29) + (–22)

How much will be left of the 51, after you have subtracted 29 from 29 to get 0?
How can we find out? Is it 51 – 29?

Addition of a positive and a negative number

Examples: 7 + (–5); 37 + (–45) and (–13) + 45

The following statement is true if the unknown number is 5:

20 – (a certain number) = 15

We also need numbers that will make sentences like the following true:

20 + (a certain number) = 15

But to go from 20 to 15 you have to subtract 5.

The number we need to make the sentence *20 + (a certain number) = 15* true, must have the following strange property:

If you **add** this number, it should have the **same effect** as **subtracting 5**.

So, mathematicians agreed that the number called negative 5 will have the property that if you add it to another number, the effect will be the same as subtracting the natural number 5.

This means that mathematicians agreed that 20 + (–5) is equal to 20 – 5.

In other words, instead of adding *negative 5* to a number, you may subtract 5.

Background information (continued)

- The **sum of any integer and its additive inverse** is equal to 0.
  Example: 12 + (–12) = 0 or (–12) + 12 = 0
- **Subtracting a negative number** has the same effect as adding its additive inverse.
  Example: 3 – (–10) and 3 + 10 produce the same result.
- **Subtracting a positive number from a negative number** has the same effect as adding its additive inverse.
  Example: (–7) – 4 and (–7) + (–4) produce the same result.

Teaching guidelines (continued)

Work through the examples on LB page 21 alongside.

Misconceptions

Learners should also be protected against the fallacy that "subtraction of a negative number is the same as multiplying two negative numbers".

Example: 10 – (–4) = 10 + 4 because "**subtracting an integer is the same as adding its additive inverse**", NOT because (–1) × (–4) = 4.

CALCULATIONS WITH INTEGERS

Background information

- Adding an integer has the same effect as subtracting its additive inverse.
  Example: 6 + (–4) = 6 – 4 = 2
- Subtracting an integer has the same effect as adding its additive inverse.
  Example: 6 – (–4) = 6 + 4 = 10

Teaching guidelines

Revise the computational properties for addition and subtraction of integers listed above.

Answers

1. to 6. See the answers on LB page 21 alongside.

Adding a negative number has the same effect as subtracting a corresponding natural number.
For example: 20 + (–15) = 20 – 15 = 5.

Subtraction of a negative number

We have dealt with cases like –20 – (–7) on the previous page.

The following statement is true if the number is 5:

25 + (a certain number) = 30

We also need a number to make this statement true:

25 – (a certain number) = 30

If you subtract this number, it should have the same effect as adding 5.

It was agreed that: *25 – (–5) is equal to 25 + 5.*

Instead of subtracting the negative number, you add the corresponding positive number (the additive inverse):

$$8 - (-3) = 8 + 3$$
$$= 11$$
$$-5 - (-12) = -5 + 12$$
$$= 7$$

We may say that for each "positive" number there is a **corresponding** or **opposite** negative number. Two positive and negative numbers that correspond, for example 3 and (–3), are called **additive inverses**.

Subtraction of a positive number from a negative number

For example: –7 – 4 actually means (–7) – 4.

Instead of subtracting a positive number, you add the corresponding negative number.

For example: –7 – 4 can be seen as (–7) + (–4) = –11.

CALCULATIONS WITH INTEGERS

Calculate each of the following:

1. –7 + 18 = 11
2. 24 – 30 – 7 = –13
3. –15 + (–14) – 9 = –38
4. 35 – (–20) = 55
5. 30 – 47 = –17
6. (–12) – (–17) = 5

2.3 Multiplying and dividing with integers

MULTIPLICATION WITH INTEGERS

Background information

Multiplication of integers is **commutative**.

Teaching guidelines

At the end of this section on LB page 22 alongside, learners should have discovered that multiplication of integers is commutative.

Answers

1. See the answers on LB page 22 alongside.
2. See the answers on LB page 22 alongside.

THE DISTRIBUTIVE PROPERTY

Background information

The **distributive property** allows us to distribute multiplication over addition and/or subtraction.

Examples: 4 × (20 + (–5)) = 4 × 20 + 4 × (–5)

4 × (20 – 5) = 4 × 20 – 4 × 5

Teaching guidelines

At the end of question 3 on LB page 22 alongside, learners should have discovered that multiplication distributes over addition and subtraction with integers.

Answers

1. (a) 20 – 5 = 15 (b) 4 × 15 = 60 (c) 80 – 20 = 60
   (d) –5 – 20 = –25 (e) 4 × (–25) = –100 (f) –20 + (–80) = –100
2. Learners evaluate their answers to question 1 on LB page 22.
3. (a) 5 (b) 4 × 5 = 20 (c) 80 – 60 = 20
   (d) –35 (e) 4 × (–35) = –140 (f) –60 – 80 = –140
   (g) 5 (h) –20 (i) –40 + 20 = –20
4. Adding an integer is the same as subtracting its additive inverse.

2.3 Multiplying and dividing with integers

MULTIPLICATION WITH INTEGERS

1. Calculate each of the following:
   (a) –7 + –7 + –7 + –7 + –7 + –7 + –7 + –7 + –7 + –7 = –70
   (b) –10 + –10 + –10 + –10 + –10 + –10 + –10 = –70
   (c) 10 × (–7) = –70 (d) 7 × (–10) = –70
2. Say whether you agree (✓) or (✗) disagree with each statement:
   (a) 10 × (–7) = 70 ✗ (b) 9 × (–5) = (–9) × 5 ✓
   (c) (–7) × 10 = 7 × (–10) ✓ (d) 9 × (–5) = –45 ✓
   (e) (–7) × 10 = 10 × (–7) ✓ (f) 5 × (–9) = 45 ✗

Multiplication of integers is commutative:
(–20) × 5 = 5 × (–20)

THE DISTRIBUTIVE PROPERTY

1. Calculate each of the following. Note that brackets are used for two purposes in these expressions, i.e. to indicate that certain operations are to be done first, and to show the integers.
   (a) 20 + (–5) (b) 4 × (20 + (–5)) (c) 4 × 20 + 4 × (–5)
   (d) (–5) + (–20) (e) 4 × ((–5) + (–20)) (f) 4 × (–5) + 4 × (–20)
2. If you worked correctly, your answers for question 1 should be 15; 60; 60; –25; –100 and –100. If your answers are different, check to see where you went wrong and correct your work.
3. Calculate each of the following where you can:
   (a) 20 + (–15) (b) 4 × (20 + (–15)) (c) 4 × 20 + 4 × (–15)
   (d) (–15) + (–20) (e) 4 × ((–15) + (–20)) (f) 4 × (–15) + 4 × (–20)
   (g) 10 + (–5) (h) (–4) × (10 + (–5)) (i) (–4) × 10 + ((–4) × (–5))
4. What property of integers is demonstrated in your answers for questions 3(a) and (g)? Explain your answer.

In question 3(i) you had to multiply two negative numbers. What was your guess?
We can calculate (–4) × (10 + (–5)) as in (h). It is (–4) × 5 = –20.

If we want the distributive property to be true for integers, then (–4) × 10 + (–4) × (–5) must be equal to –20.

(–4) × 10 + (–4) × (–5) = –40 + (–4) × (–5)

22 MATHEMATICS GRADE 9: TERM 1

Background information (continued)

Number patterns can be used to investigate multiplication rules for integers.

Pattern P	Pattern Q	Pattern R
5 × 2 = 10	2 × 5 = 10	–5 × 2 = –10
5 × 1 = 5	1 × 5 = 5	–5 × 1 = –5
5 × 0 = 0	0 × 5 = 0	–5 × 0 = 0
5 × (–1) = –5	–1 × 5 = –5	–5 × (–1) = 5
5 × (–2) = –10	–2 × 5 = –10	–5 × (–2) = 10

- The product of two positive numbers is a positive number (see pattern P).
- The product of a positive number and a negative number is a negative number (P).
- The product of a negative number and a positive number is a negative number (Q).
- The product of two negative numbers is a positive number (R).

Teaching guidelines (continued)

Revise the multiplication rules for integers listed above.

Answers

5. (a) 500 – 300 = 200 (b) 20
   (c) 10 × 20 = 200 (d) –80
   (e) –500 – 300 = –800 (f) 10 × (–80) = –800
6. (a) See the underlined expressions on LB page 23 alongside.
   (b) Distributive property
7. (a) Yes
   (b) 10 × 50 + 10 × (–30) = 10 × **(**50 + (–30)**)**
   10 × –50 + 10 × 30 = 10 × (–50 + 30)
8. See the underlined expressions on LB page 23 alongside.
9. 10 × ((–50) – (–30)) = 10 × –20 = –200
   10 × (–50) – (–30) = –500 + 30 = –470
   10 × (–50) – 10 × (–30) = –500 + 300 = –200
10. –10 × 2 = –20
11. Yes, the answers are the same.

Then (–4) × (–5) must be equal to 20.

5. Calculate each of the following:
   (a) 10 × 50 + 10 × (–30) (b) 50 + (–30)
   (c) 10 × **(**50 + (–30)**)** (d) (–50) + (–30)
   (e) 10 × (–50) + 10 × (–30) (f) 10 × **(**(–50) + (–30)**)**

- The product of two positive numbers is a positive number, for example: 5 × 6 = 30.
- The product of a positive number and a negative number is a negative number, for example: 5 × (–6) = –30.
- The product of a negative number and a positive number is a negative number, for example: (–5) × 6 = –30.

6. (a) Write out only the numerical expressions below which you would expect to have the same answers. Do not do the calculations.
   16 × (53 + 68) 53 × (16 + 68) 16 × 53 + 16 × 68 16 × 53 + 68
   (b) What property of operations is demonstrated by the fact that two of the above expressions have the same value?
7. Consider your answers for question 5.
   (a) Does multiplication distribute over addition in the case of integers?
   (b) Illustrate your answer with two examples.
8. Write out only the numerical expression below which you would expect to have the same answers. Do not do the calculations now.
   10 × **(**(–50) – (–30)**)** 10 × (–50) – (–30) 10 × (–50) – 10 × (–30)
9. Do the three sets of calculations given in question 8.
10. Calculate (–10) × **(**5 + (–3)**)**.
11. Now consider the question of whether or not multiplication by a negative number distributes over addition and subtraction of integers. For example, would (–10) × 5 + (–10) × (–3) also have the answer of –20, like (–10) × **(**5 + (–3)**)**?

To make sure that multiplication distributes over addition and subtraction in the system of integers, we have to agree that:

(a negative number) × (a negative number) is a positive number.

For example: (–10) × (–3) = 30.

Teaching guidelines (continued)

Revise the summary of properties of integers on LB page 24 alongside.

Answers

12. (a) 120 (b) –140 (c) 300 + 240 = 540
(d) –30 × –18 = 540 (e) 300 – 240 = 60 (f) –30 × –2 = 60

DIVISION WITH INTEGERS

Background information

Multiplication and division are **inverse operations**. This means that, if two numbers and the value of their product are known, the answers to two division sums are also known.

Example: If 3 × (–5) = –15 then (–15) ÷ 3 = –5 and (–15) ÷ (–5) = 3

- A positive number divided by a positive number is a positive number.
- A positive number divided by a negative number is a negative number.
- A negative number divided by a positive number is a negative number.
- A negative number divided by a negative number is a positive number.

Teaching guidelines

Learners use inverse operations to deduce the division rules for integers.

Answers

1. See the answers on LB page 24 alongside.
2. See the answers on LB page 24 alongside.

MIXED CALCULATIONS WITH INTEGERS

Teaching guidelines

Remind learners to use the conventional order of operations (BODMAS) to solve problems with multiple operations.

Answers

1. (a) 20(–43) = –860 (b) –1 000 + 140 = –860
(c) 20(–57) = –1 140 (d) –1 000 – 140 = –1 140
(e) –20(–57) = 1 140 (f) 1 000 + 140 = 1 140

12. Calculate each of the following:
(a) (–20) × (–6) (b) (–20) × 7
(c) (–30) × (–10) + (–30) × (–8) (d) (–30) × ((–10) + (–8))
(e) (–30) × (–10) – (–30) × (–8) (f) (–30) × ((–10) – (–8))

Here is a summary of the properties of integers that make it possible to do calculations with integers:

- When a number is added to its additive inverse, the result is 0.
For example, (+12) + (–12) = 0.
- Adding an integer has the same effect as subtracting its additive inverse.
For example, 3 + (–10) can be calculated by doing 3 – 10, and the answer is –7.
- Subtracting an integer has the same effect as adding its additive inverse.
For example, 3 – (–10) can be calculated by calculating 3 + 10 is 13.
- The product of a positive and a negative integer is negative.
For example, (–15) × 6 = –90.
- The product of a negative and a negative integer is positive.
For example, (–15) × (–6) = 90.

DIVISION WITH INTEGERS

1. Calculate each of the following:
(a) 5 × (–7) = –35 (b) (–3) × 20 = –60
(c) (–5) × (–10) = 50 (d) (–3) × (–20) = 60

2. Use your answers in question 1 to determine the following:
(a) (–35) ÷ 5 = –7 (b) (–35) ÷ (–7) = 5
(c) (–60) ÷ 20 = –3 (d) (–60) ÷ (–3) = 20
(e) 50 ÷ (–5) = –10 (f) 50 ÷ (–10) = –5
(g) 60 ÷ (–20) = –3 (h) 60 ÷ (–3) = –20

- The quotient of a positive number and a negative number is a negative number.
- The quotient of two negative numbers is a positive number.

MIXED CALCULATIONS WITH INTEGERS

1. Calculate each of the following:
(a) 20(–50 + 7) (b) 20 × (–50) + 20 × 7
(c) 20(–50 + –7) (d) 20 × (–50) + 20 × –7
(e) –20(–50 + –7) (f) –20 × –50 + –20 × –7

24 MATHEMATICS GRADE 9: TERM 1

Answers

2. (a) $40 \times (-4) - 10 \times (-6) - 3 \times (-11) = -160 + 60 + 33 = -67$
   (b) $10 \times 40 + (-10) \times 7 = 400 - 70 = 330$
   (c) $-50(35) + 30(10) - 40(-3) = -1\ 750 + 300 + 120 = -1\ 330$
   (d) $-4 \times (-20) + 7 \times (-30) - 10 \times (-40) = 80 - 210 + 400 = 270$
   (e) $-3 \times (-15) \times (-10) \times (-5) = 2\ 250$

2.4 Powers, roots and word problems

Background information

- A **square** is any number which is a product of two identical factors.
- The **square root of a number** is another number which, when squared, will equal the first number.
  - A square has two square roots, for example 25 has the square roots 5 and –5.
  - The square root sign is used for positive numbers only, so we use $\sqrt{25}$ to indicate 5, and it does not mean –5. We indicate the negative square root by $-\sqrt{25}$.
- A **cube** is any number which is a product of three identical factors.
- The **cube root of a number** is another number which, when cubed, will equal the first number.

Teaching guidelines

Revise the concepts listed above.

Answers

1. See the tables on LB page 25 alongside.
2. (a) $2 - 3 = -1$ (b) $3 - 4 = -1$
   (c) -9 (d) 9
   (e) $16 - 36 + 1 = -19$ (f) $27 - 64 - 8 - 1 = -46$
   (g) $9 - 2 \times 5 = 9 - 10 = -1$ (h) $-(16)(1) = -16$
   (i) $\frac{25}{\sqrt{25}} = 25 \div 5 = 5$ (j) $\frac{-6}{-1-8} = \frac{-6}{-9} = \frac{2}{3}$

2. Calculate each of the following:
   (a) $40 \times (-12 + 8) - 10 \times (2 + -8) - 3 \times (-3 - 8)$
   (b) $(9 + 10 - 9) \times 40 + (25 - 30 - 5) \times 7$
   (c) $-50(40 - 25 + 20) + 30(-10 + 7 + 13) - 40(-16 + 15 - 2)$
   (d) $-4 \times (30 - 50) + 7 \times (40 - 70) - 10 \times (60 - 100)$
   (e) $-3 \times (-14 - 6 + 5) \times (-13 - 7 + 10) \times (20 - 10 - 15)$

2.4 Powers, roots and word problems

Answer all questions in this section **without** using a calculator.

1. Copy and complete the following tables:

(a)

x	1	2	3	4	5	6	7	8	9	10	11	12
x^2	1	4	9	16	25	36	49	64	81	100	121	144
x^3	1	8	27	64	125	216	343	512	729	1 000	1 331	1 728

(b)

x	–1	–2	–3	–4	–5	–6	–7	–8	–9	–10	–11	–12
x^2	1	4	9	16	25	36	49	64	81	100	121	144
x^3	–1	–8	–27	–64	–125	–216	–343	–512	–729	–1 000	–1 331	–1 728

3^2 is 9 and $(-3)^2$ is also 9.
3^3 is 27 and $(-5)^3$ is –125.
Both (–3) and 3 are **square roots** of 9.
3 may be called the **positive square root** of 9, and (–3) may be called the **negative square root** of 9.
3 is called the **cube root** of 27, because $3^3 = 27$.
–5 is called the cube root of –125 because $(-5)^3 = -125$.
10^2 is 100 and $(-10)^2$ is also 100.
Both 10 and (–10) are called **square roots** of 100.

The symbol $\sqrt{\ }$ means that you must take the **positive square root** of the number.

2. Calculate each of the following:
   (a) $\sqrt{4} - \sqrt{9}$ (b) $\sqrt[3]{27} + (-\sqrt[3]{64})$
   (c) $-(3^2)$ (d) $(-3)^2$
   (e) $4^2 - 6^2 + 1^2$ (f) $3^3 - 4^3 - 2^3 - 1^3$
   (g) $\sqrt{81} - \sqrt{4} \times \sqrt[3]{125}$ (h) $-(4^2)(-1)^2$
   (i) $\frac{(-5)^2}{\sqrt{37-12}}$ (j) $\frac{-\sqrt{36}}{-1^3 - 2^3}$

Answers

3. (a) 11 °C – (–2 °C) = 13 °C
   (b) 3 °C + 2 °C = 5 °C
   (c) –75 + 21 = –54 m, so it is 54 m below the surface.
   (d) –37 – 15 = –52 m, so it is 52 m below the surface.

3. Determine the answer to each of the following:
   (a) The overnight temperature in Polokwane drops from 11 °C to –2 °C. By how many degrees has the temperature dropped?
   (b) The temperature in Escourt drops from 2 °C to –1 °C in one hour, and then another two degrees in the next hour. How many degrees in total did the temperature drop over the two hours?
   (c) A submarine is 75 m below the surface of the sea. It then rises by 21 m. How far below the surface is it now?
   (d) A submarine is 37 m below the surface of the sea. It then sinks a further 15 m. How far below the surface is it now?

Learner Book Overview		
Sections in this chapter	**Content**	**Pages in Learner Book**
3.1 Equivalent fractions	Consolidation of understanding of equivalent fractions; conversions between mixed numbers and common fractions	Pages 27 to 30
3.2 Adding and subtracting fractions	Addition and subtraction with fractions using the LCM of the denominators	Pages 30 to 32
3.3 Multiplying and dividing fractions	Multiplication and division with fractions; squares, cubes, square roots and cube roots	Pages 32 to 36
3.4 Equivalent forms	Conversion between fractions in common fraction, decimal and percentage notations	Pages 37 to 38

CAPS time allocation	4,5 hours
CAPS content specification	Page 122

Mathematical background

A **fraction** is a measure of how something is to be divided up or shared out. In this chapter learners find out more about fractions and what they are used for.

- Fractions were invented so that **quantities**, such as the following, can be described accurately:
  - **measures**, for example, "a learner is $1\frac{1}{2}$ m tall"
  - **parts of whole objects**, for example, "quarter of an apple" or "a half-loaf of bread"
  - **parts of collections**, for example, "three-eighths of the learners in a school"
  - **parts of non-physical quantities**, for example, "63 hundredths of the available marks", normally written in percentage notation.
- Fractions can be described using various **notations** such as the following:
  - **common fraction notation**, for example, "three-fifths" of a cake is left over
  - **decimal notation**, for example, "0,6" of a cake is left over
  - **percentage notation**, for example, "60%" of a cake is left over
  - **ratio notation**, for example, the ratio of cake eaten and cake left over is "2 : 3".
- **Equivalent fractions** are fractions that have the same value but are different in form. They enable us to:
  - **convert** a fraction to another notation
  - **reduce** a common fraction by writing it in its simplest form
  - **compare** two or more common fractions by writing them with a common denominator
  - **add and subtract** common fractions by writing them with a common denominator.

3.1 Equivalent fractions

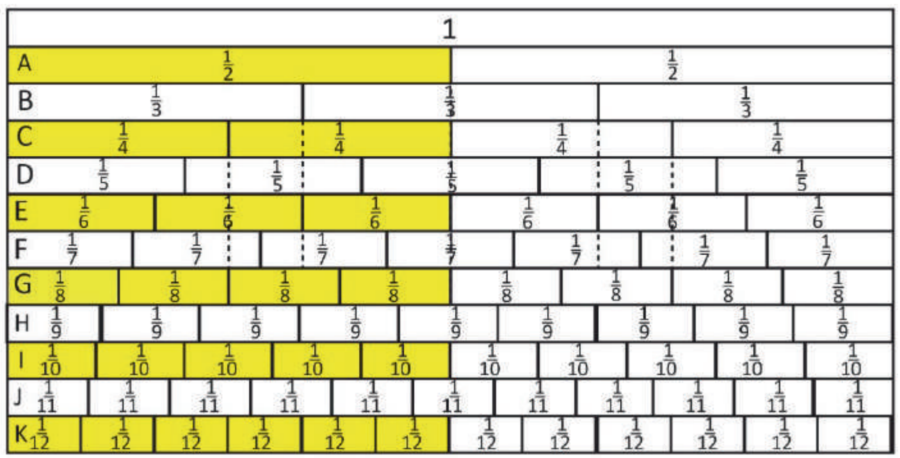
THE SAME NUMBER IN DIFFERENT FORMS

Background information

The activities on LB pages 27 and 28 are intended to provide learners with opportunities to consolidate their understanding of equivalence. While the conventional process to simplify fractions (to multiply or divide the numerator and denominator by the same number) is easy to learn and to imply, knowledge of this process does not guarantee understanding of the meaning of equivalence of fractions, namely that the same number is expressed in different ways. Widely-held misconceptions, such as "$\frac{20}{30} > \frac{2}{3}$", even among university students, indicate that there is a serious need for opportunities to help learners understand what equivalent fractions mean – i.e. that the same number (quantity or ratio) can be expressed in different ways. A fraction wall can be used for this purpose.

Teaching guidelines

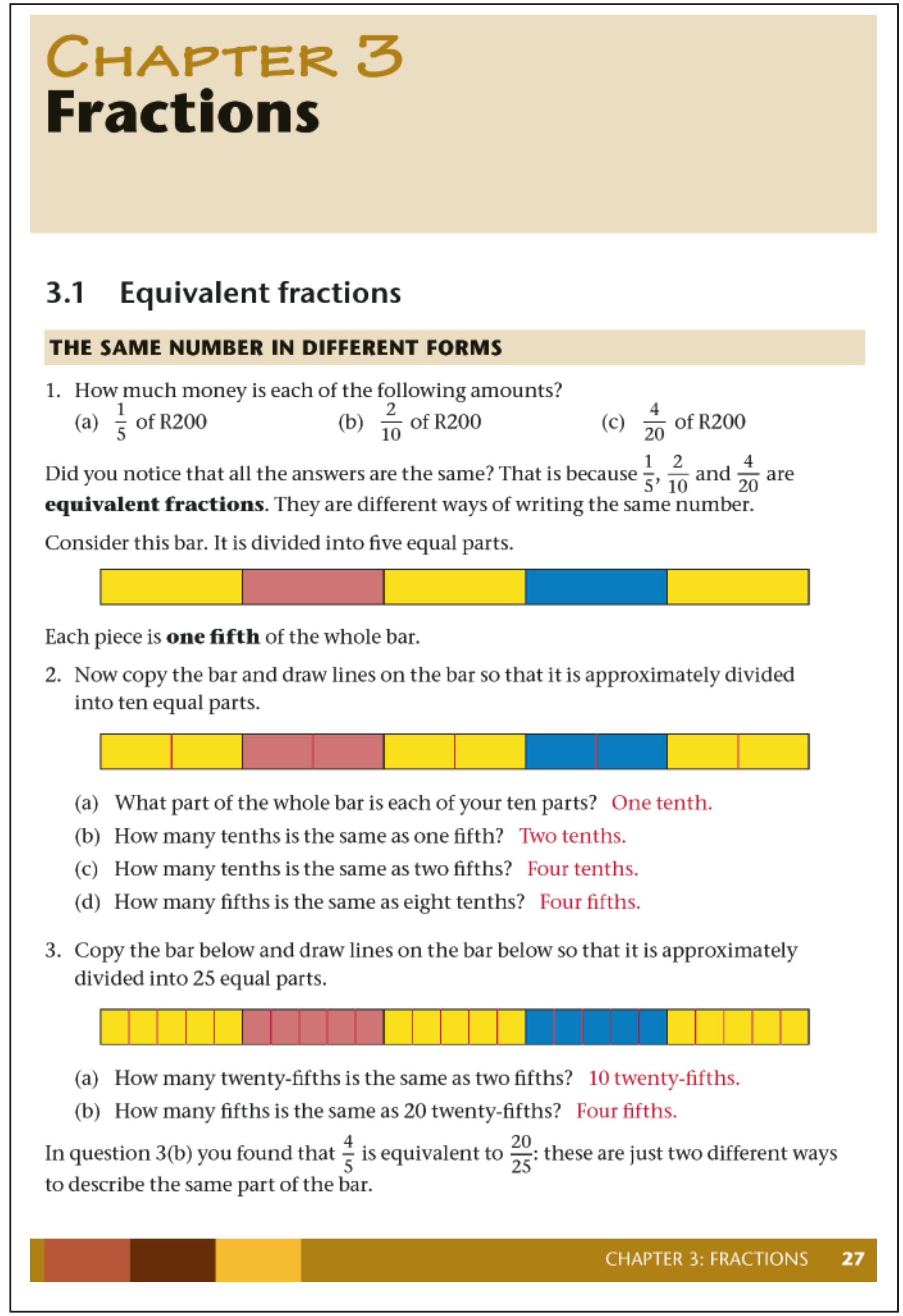
Learners use a fraction wall like the one below to find sets of equivalent fractions.



Example: $\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8} = \frac{5}{10} = \frac{6}{12}$

Answers

1. (a) R40 (b) R40 (c) R40
2. See the answers on LB page 27 alongside.
3. See the answers on LB page 27 alongside.



CHAPTER 3
Fractions

3.1 Equivalent fractions

THE SAME NUMBER IN DIFFERENT FORMS

1. How much money is each of the following amounts?
   (a) $\frac{1}{5}$ of R200 (b) $\frac{2}{10}$ of R200 (c) $\frac{4}{20}$ of R200

Did you notice that all the answers are the same? That is because $\frac{1}{5}$, $\frac{2}{10}$ and $\frac{4}{20}$ are **equivalent fractions**. They are different ways of writing the same number.

Consider this bar. It is divided into five equal parts.

Each piece is **one fifth** of the whole bar.

2. Now copy the bar and draw lines on the bar so that it is approximately divided into ten equal parts.
   (a) What part of the whole bar is each of your ten parts? One tenth.
   (b) How many tenths is the same as one fifth? Two tenths.
   (c) How many tenths is the same as two fifths? Four tenths.
   (d) How many fifths is the same as eight tenths? Four fifths.
3. Copy the bar below and draw lines on the bar below so that it is approximately divided into 25 equal parts.
   (a) How many twenty-fifths is the same as two fifths? 10 twenty-fifths.
   (b) How many fifths is the same as 20 twenty-fifths? Four fifths.

In question 3(b) you found that $\frac{4}{5}$ is equivalent to $\frac{20}{25}$: these are just two different ways to describe the same part of the bar.

CHAPTER 3: FRACTIONS 27

Background information (continued)

It is useful to distinguish the following **three phases** in the development of the concept of equivalent fractions in learners' minds.

- Awareness that the **same part** of a whole, collection, quantity or unit of measurement can be described with different fractions:

  Example: 24 forty-eighths of a slab of chocolate can be described as 12 twenty-fourths, eight sixteenths, six twelfths, four eighths, three sixths, two quarters or one half of the slab

- The ability to **specify equivalent fractions**:

  Example: 24 forty-eighths = 12 twenty-fourths = eight sixteenths = six twelfths = four eighths = three sixths = two quarters = one half

- Producing equivalent fractions with a **formula**:

  Example: Either multiply or divide both the numerator and denominator of a fraction by the same number.

Teaching guidelines (continued)

The common practice to refer to fractions as "a number 'over' a number", for example to refer to $\frac{5}{8}$ as "5 over 8" needs to be discouraged, because it undermines understanding of what fractions really are. It is critical that learners and teachers use language that reflects the true nature of a fraction, namely a number of parts of a certain size, for example, $\frac{5}{8}$ means five parts of a size indicated by the denominator 8, meaning a quantity was divided into eight equal parts.

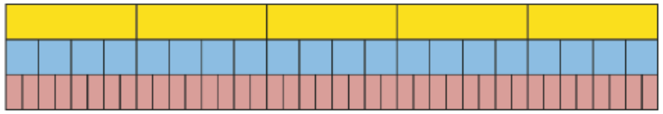
Answers

4. $\frac{2}{10} = \frac{1}{5}$; $\frac{4}{10} = \frac{2}{5}$; $\frac{4}{5} = \frac{8}{10}$; $\frac{10}{25} = \frac{2}{5}$
5. See the answers on LB page 28 alongside.
6. See the answers on LB page 28 alongside.
7. See the table on LB page 28 alongside.

This can be expressed by writing $\frac{4}{5} = \frac{20}{25}$ which means that $\frac{4}{5}$ and $\frac{20}{25}$ are equivalent to each other.

4. Write down all the other pairs of equivalent fractions which you found while doing questions 2 and 3.

The yellow bar is divided into fifths.



5. (a) Into what kind of fraction parts is the blue bar divided? Twentieths.
   (b) Into what kind of fraction parts is the red bar divided? Fortieths.
   (c) If you want to mark the yellow bar in twentieths (like the blue bar), into how many parts do you have to divide each of the fifths? Four parts.
   (d) If you want to mark the yellow bar in fortieths (like the red bar), into how many parts do you have to divide each of the fifths? Eight parts.
   (e) If you want to mark the yellow bar in eightieths, into how many parts do you have to divide each of the fifths? 16 parts.
   (f) If you want to mark the blue bar in eightieths, into how many parts do you have to divide each of the twentieths? Four parts.
6. Suppose this bar is divided into four equal parts, in other words, quarters.

   (a) If the bar is also divided into 20 equal parts, how many of these smaller parts will there be in each quarter? 5 parts (twentieths).
   (b) If each quarter is divided into six equal parts, what part of the whole bar will each small part be? One twenty-fourth.
7. Copy and complete this table of equivalent fractions, as far as you can using whole numbers. All the fractions in each column must be equivalent.

sixteenths	8	4	2	10	14	12
eighths	4	2	1	5	7	6
quarters	2	1				3
twelfths	6	3				9
twentieths	10	5				15

Background information (continued)

- Equivalent fractions look different but have the **same value**.
- Equivalent fractions occupy the **same position** on a number line.
- A common fraction can be **simplified** by dividing the numerator and denominator by the same number.
- The numerator and denominator of a common fraction in its **simplest form** has no common factor but 1.

Teaching guidelines (continued)

Discuss the statements listed above.

Answers

8. There are many equivalent fractions. Some examples are: $\frac{6}{8}; \frac{9}{12}; \frac{12}{16}; \frac{15}{20}; \frac{18}{24}$

9. (a) $\frac{8}{12}$ (b) $\frac{9}{12}$ (c) $\frac{10}{12}$ (d) $\frac{2}{12}$

10. (a) $\frac{2}{5}$ (b) $\frac{1}{4}$ (c) $\frac{1}{5}$ (d) $\frac{1}{5}$ (e) $\frac{1}{4}$ (f) $\frac{1}{11}$

CONVERTING BETWEEN MIXED NUMBERS AND FRACTIONS

Background information

- Mixed numbers consist of **two parts**: a whole number and a fraction: $3\frac{4}{5}$.
- Mixed numbers can be written in **expanded notation**: $3\frac{4}{5} = 3 + \frac{4}{5}$.
- Mixed numbers can be **added or subtracted** by working with the whole number parts and fraction parts separately, for example:

  $3\frac{4}{5} + 13\frac{3}{5} = 16 + \frac{7}{5} = 16 + 1\frac{2}{5} = 17\frac{2}{5}$ or

  $13\frac{3}{5} - 3\frac{4}{5} = 12\frac{8}{5} - 3\frac{4}{5} = 9\frac{4}{5}$ (“borrow” 1 from 13)

- A **proper fraction** has a numerator smaller than its denominator.
- An **improper fraction** has a numerator larger than its denominator.
- **Mixed numbers can be converted** to improper fractions, and vice versa.

Equivalent fractions can be formed by multiplying the numerator and denominator by the same number. For example: $\frac{1}{5} = \frac{4 \times 1}{4 \times 5} = \frac{4}{20}$

8. Write down five different fractions that are equivalent to $\frac{3}{4}$.
9. Express each of the following numbers as twelfths:

   (a) $\frac{2}{3}$ (b) $\frac{3}{4}$ (c) $\frac{5}{6}$ (d) $\frac{1}{6}$

You may divide the numerator and denominator by the same number, instead of multiplying the numerator and denominator by the same number. This gives you a simpler fraction.

The **simplest form** of a fraction has no common factors. For example, you find the simplest form of the fraction $\frac{4}{12}$ is $\frac{1}{3}$ by dividing both the numerator and denominator by the common factor of 4.

10. Convert each of the following fractions to their simplest form:

    (a) $\frac{40}{100}$ (b) $\frac{4}{16}$

    (c) $\frac{5}{25}$ (d) $\frac{6}{30}$

    (e) $\frac{6}{24}$ (f) $\frac{8}{88}$

CONVERTING BETWEEN MIXED NUMBERS AND FRACTIONS

Numbers that have both whole number and fraction parts are called **mixed numbers**.

Examples of mixed numbers: $3\frac{4}{5}$, $2\frac{7}{8}$ and $8\frac{3}{10}$

Mixed numbers can be written in expanded notation, for example:

$3\frac{4}{5}$ means $3 + \frac{4}{5}$ $2\frac{7}{8}$ means $2 + \frac{7}{8}$ $8\frac{3}{10}$ means $8 + \frac{3}{10}$.

To add and subtract mixed numbers, you can work with the whole number parts and the fraction parts separately, for example:

$3\frac{4}{5} + 13\frac{3}{5}$
$= 16\frac{7}{5}$
$= 17\frac{2}{5}$

$13\frac{3}{5} - 3\frac{4}{5}$
$= 12\frac{8}{5} - 3\frac{4}{5}$
$= 9\frac{4}{5}$

(we need to “borrow” a unit from 13, because we cannot subtract $\frac{4}{5}$ from $\frac{3}{5}$)

CHAPTER 3: FRACTIONS 29

Teaching guidelines

Use the examples at the top of LB page 30 alongside to illustrate the conversion of mixed numbers to improper fractions.

Answers

1. (a) $\frac{28}{5}$ (b) $\frac{19}{8}$ (c) $\frac{25}{7}$ (d) $\frac{53}{12}$
2. (a) $6\frac{2}{5}$ (b) $3\frac{1}{8}$ (c) $2\frac{6}{9} = 2\frac{2}{3}$ (d) $1\frac{17}{20}$

3.2 Adding and subtracting fractions

Background information

Common fractions can be added and subtracted if they are expressed with a **common denominator**, which can be either the:

- "product" of the denominators, or
- The LCM (lowest common multiple) of the denominators.

Teaching guidelines

Use the examples at the bottom of LB page 30 alongside and illustrate how to use:

- the product of two denominators to add two fractions
- the LCM of the denominators to add two fractions.

However, this method can be difficult to do with some examples, and it does not work with multiplication and division.

An alternative and preferred method is to convert the mixed number to an **improper fraction**, as shown in the example below:

$$3\frac{4}{5}$$
$$= 3 + \frac{4}{5}$$
$$= \frac{15}{5} + \frac{4}{5}$$
$$= \frac{19}{5}$$

NOTE
You can obtain the numerator of 19 in one step by multiplying the denominator (5) by the whole number (3), and then adding the numerator (4).

So, you can calculate $3\frac{4}{5} + 13\frac{3}{5}$ using this method:

$$3\frac{4}{5} + 13\frac{3}{5}$$
$$= \frac{19}{5} + \frac{68}{5}$$
$$= \frac{87}{5}$$

The answer must be converted to a mixed number again: $\frac{87}{5} = 17\frac{2}{5}$

1. Convert each of the following mixed numbers to improper fractions:
   (a) $5\frac{3}{5}$ (b) $2\frac{3}{8}$ (c) $3\frac{4}{7}$ (d) $4\frac{5}{12}$
2. Convert each of the following improper fractions to mixed numbers:
   (a) $\frac{32}{5}$ (b) $\frac{25}{8}$ (c) $\frac{24}{9}$ (d) $\frac{37}{20}$

3.2 Adding and subtracting fractions

To add or subtract two fractions, they have to be expressed with the *same* denominators first. To achieve that, one or more of the given fractions may have to be replaced with equivalent fractions.

$$\frac{3}{20} + \frac{2}{5}$$
$$= \frac{3}{20} + \frac{2 \times 4}{5 \times 4}$$
$$= \frac{3}{20} + \frac{8}{20}$$
$$= \frac{11}{20}$$

We will refer to this as the LCM method.

$$\frac{5}{12} + \frac{7}{20}$$
$$= \frac{5 \times 20}{12 \times 20} + \frac{7 \times 12}{20 \times 12}$$
$$= \frac{100}{240} + \frac{84}{240}$$
$$= \frac{184}{240}$$
$$= \frac{23}{30}$$

We will later refer to this method of adding or subtracting fractions as Method A.

30 MATHEMATICS GRADE 9: TERM 1

ADDING AND SUBTRACTING FRACTIONS

Teaching guidelines (continued)

Revise how to find the LCM of two numbers by using:

- a **factor tree**, starting with known factors
- a **factor ladder**, starting with the smallest prime factor.

Answers

1. LCM method. Finding the LCM is the quickest method because I know the multiples.

2. (a) $\frac{15}{40}+\frac{16}{40}=\frac{31}{40}$

   (b) $\frac{12}{40}+\frac{35}{40}=\frac{47}{40}=1\frac{7}{40}$

   (c) $3+2+\frac{4}{10}+\frac{3}{10}=5\frac{7}{10}$

   (d) $7+3+\frac{9}{24}+\frac{22}{24}=10\frac{31}{24}=11\frac{7}{24}$

3. (a) $\frac{13}{20}-\frac{8}{20}=\frac{5}{20}=\frac{1}{4}$

   (b) $\frac{7}{12}-\frac{3}{12}=\frac{4}{12}=\frac{1}{3}$

   (c) $5\frac{4}{8}-3\frac{3}{8}=2\frac{1}{8}$

   (d) $4\frac{1}{9}-5\frac{6}{9}=\frac{37}{9}-\frac{51}{9}=-\frac{14}{9}=-1\frac{5}{9}$

4. $\frac{1}{3}=\frac{5}{15}$, and two fifths $=\frac{6}{15}$

   $\frac{15}{15}-\frac{5}{15}-\frac{6}{15}=\frac{4}{15}$ of the pizza.

In the case of $\frac{5}{12}+\frac{7}{20}$, multiplying by 20 and by 12 was a sure way of making equivalent fractions of the same kind, in this case two hundred-and-fortieths. However, the numbers became quite big. Just imagine how big the numbers will become if you use the same method to calculate $\frac{17}{75}+\frac{13}{85}$!

Fortunately, there is a method of keeping the numbers smaller (in many cases) when making equivalent fractions, so that fractions can be added or subtracted. In this method you first calculate the **lowest common multiple** or LCM of the denominators. In the case of $\frac{5}{12}+\frac{7}{20}$, the smaller multiples of the denominators are:

12:	12	24	36	48	60	72	84
20:	20	40	60	80	100	120	140

The smallest number that is a multiple of both 12 and 20 is 60.

Both $\frac{5}{12}$ and $\frac{7}{20}$ can be expressed in terms of sixtieths:

$\frac{5}{12}=\frac{5\times 5}{12\times 5}=\frac{25}{60}$ because to make twelfths into sixtieths you have to divide each twelfth into five equal parts, to get $12 \times 5 = 60$ equal parts, i.e. sixtieths.

Similarly, $\frac{7}{20}=\frac{7\times 3}{20\times 3}=\frac{21}{60}$.

Hence $\frac{5}{12}+\frac{7}{20}=\frac{25}{60}+\frac{21}{60}=\frac{46}{60}=\frac{23}{30}$

We may call this method the LCM method of adding or subtracting fractions.

ADDING AND SUBTRACTING FRACTIONS

1. Which method of adding and subtracting fractions do you think will be the easiest and quickest for you, Method A or the LCM method? Explain.
2. Calculate each of the following:

   (a) $\frac{3}{8}+\frac{2}{5}$ (b) $\frac{3}{10}+\frac{7}{8}$

   (c) $3\frac{2}{5}+2\frac{3}{10}$ (d) $7\frac{3}{8}+3\frac{11}{12}$
3. Calculate each of the following:

   (a) $\frac{13}{20}-\frac{2}{5}$ (b) $\frac{7}{12}-\frac{1}{4}$

   (c) $5\frac{1}{2}-3\frac{3}{8}$ (d) $4\frac{1}{9}-5\frac{2}{3}$
4. Paulo and Sergio buy a pizza. Paulo eats $\frac{1}{3}$ of the pizza and Sergio eats two fifths. How much of the pizza is left over?

Answers

5. (a) $\frac{56}{120} + \frac{55}{120} = \frac{111}{120} = \frac{37}{40}$
LCM method

(b) $\frac{219}{300} - \frac{28}{300} = \frac{191}{300}$
LCM method

(c) $\frac{24}{200} + \frac{65}{200} = \frac{89}{200}$
LCM method

(d) $\frac{45}{80} - \frac{24}{80} = \frac{21}{80}$
LCM method

(e) $\frac{10}{180} + \frac{63}{180} = \frac{73}{180}$
LCM method

(f) $\frac{22}{70} - \frac{15}{70} = \frac{7}{70} = \frac{1}{10}$
LCM method

(g) $\frac{50}{8} = 6\frac{2}{8} = 6\frac{1}{4}$
There was a common denominator already.

3.3 Multiplying and dividing fractions

THINK ABOUT MULTIPLICATION AND DIVISION WITH FRACTIONS

Background information

Multiplication and division with fractions should cover the following calculations:

- Multiply a fraction by a whole number: Use the meaning of the fraction.
  Example: $5 \times \frac{2}{3} = 5 \times 2$ thirds = 10 thirds = $\frac{10}{3}$
- Divide a fraction by a whole number: Use equivalent fractions.
  Example: $\frac{2}{3} \div 5 = \frac{10}{15} \div 5 = \frac{2}{15}$

Teaching guidelines

Use the questions on LB page 32 to introduce the calculations listed above.

Answers

1. (a) $10 \times \frac{5}{8}$ (b) $\frac{5}{8} \div 10$ (c) $\frac{5}{8}$ of R10 (d) $10 \div \frac{5}{8}$
2. (a) Set D (b) Set B (c) Set D (d) Set C

---

5. Calculate each of the following. State whether you use Method A or the LCM method.
   (a) $\frac{7}{15} + \frac{11}{24}$
   (b) $\frac{73}{100} - \frac{7}{75}$
   (c) $\frac{3}{25} + \frac{13}{40}$
   (d) $\frac{9}{16} - \frac{3}{10}$
   (e) $\frac{1}{18} + \frac{7}{20}$
   (f) $\frac{11}{35} - \frac{3}{14}$
   (g) $\frac{5}{8} + \frac{5}{8} + \frac{5}{8} + \frac{5}{8} + \frac{5}{8} + \frac{5}{8} + \frac{5}{8} + \frac{5}{8} + \frac{5}{8} + \frac{5}{8}$

3.3 Multiplying and dividing fractions

THINK ABOUT MULTIPLICATION AND DIVISION WITH FRACTIONS

1. Read the questions below, but do not answer them now. Just describe in each case what calculations you think must be done to find the answer to the question. You can think later about how the calculations may be done.
   (a) Ten people come to a party and each of them must get $\frac{5}{8}$ of a pizza. How many pizzas must be bought to provide for all of them?
   (b) $\frac{5}{8}$ of the cost of a new clinic must be carried by the ten doctors who will work there. What part of the cost of the clinic must be carried by each of the doctors, if they have agreed to share the cost equally?
   (c) If a whole pizza costs R10, how much does $\frac{5}{8}$ of a pizza cost?
   (d) The owner of a spaza shop has ten whole pizzas. How many portions of $\frac{5}{8}$ of a pizza each can he make up from the ten pizzas?
2. Look at the different sets of calculations shown below.
   (a) Which set of calculations is a correct way to find the answer for question 1(a)?
   (b) Which set of calculations is a correct way to find the answer for question 1(b)?
   (c) Which set of calculations is a correct way to find the answer for question 1(c)?
   (d) Which set of calculations is a correct way to find the answer for question 1(d)?

Set A: $\frac{10}{10} \times \frac{5}{8} = \frac{50}{80}$

Set B: $\frac{5}{8} = \frac{50}{80}$. 50 eightieths ÷ 10 = $\frac{5}{80}$

Set C: How many eighths in ten wholes? 80 eighths. How many five-eighths in 80? $80 \div 5 = 16$

Set D: $\frac{5}{8}$ is five eighths. 10 × five eighths = $\frac{50}{8}$

Set E: $\frac{5}{8} \div 10 = \frac{5}{8} \times \frac{10}{1} = \frac{50}{8}$

32 MATHEMATICS GRADE 9: TERM 1

Background information (continued)

- To **multiply a fraction by a whole number**, write the fraction in words, find the answer and, if necessary, convert it to a mixed number.
  Example: $5 \times \frac{2}{3} = 5 \times 2$ thirds $= 10$ thirds $= \frac{10}{3} = 3\frac{1}{3}$
- To **divide a fraction by a whole number**, convert the fraction to an equivalent fraction with a numerator that is a multiple of the divisor.
  Example: $\frac{2}{3} \div 5 = \frac{10}{15} \div 5 = \frac{2}{15}$
- To **find a fraction of a whole number**, find a unit fraction (one part) of the whole number and multiply the answer by the numerator.
  Example: $\frac{7}{12}$ of $36 = 7 \times \left(\frac{1}{12} \text{ of } 36\right) = 7 \times 3 = 21$
- To **find a fraction of a fraction**, simply multiply the numerators as well as the denominators and write the answer in its simplest form.
  Example: $\frac{7}{12}$ of $\frac{36}{50} = 7 \times \left(\frac{1}{12} \text{ of } \frac{36}{50}\right) = 7 \times \frac{3}{50} = \frac{21}{50}$ or $\frac{7}{12} \times \frac{36}{50} = \frac{252}{600} = \frac{21}{50}$

Teaching guidelines (continued)

Use the examples on LB page 33 to illustrate the methods explained above.

Answers

3. (a) 21 fiftieths
   (b) $\frac{252}{600} = \frac{21}{50}$

Multiply a fraction by a whole number

Example:

$8 \times \frac{3}{5} = 8 \times 3$ fifths $= 24$ fifths $= \frac{24}{5} = 4\frac{4}{5}$

Divide a fraction by a whole number

You can divide a fraction by converting it to an equivalent fraction with a numerator that is a multiple of the divisor.

Example:

$\frac{2}{3} \div 5 = \frac{10}{15} \div 5 = 10$ fifteenths $\div 5 = 2$ fifteenths $= \frac{2}{15}$

A fraction of a whole number, and a fraction of a fraction

Examples:

A $\frac{7}{12}$ of R36.

$\frac{1}{12}$ of R36 is the same as R36 ÷ 12 = R3, so $\frac{7}{12}$ of R36 is 7 × R3 = R21.

B $\frac{7}{12}$ of 36 fiftieths.

$\frac{1}{12}$ of 36 fiftieths is the same as 36 fiftieths ÷ 12 = 3 fiftieths,

so $\frac{7}{12}$ of 36 fiftieths is 7 × 3 fiftieths = 21 fiftieths.

$\frac{7}{12} \times \frac{36}{50}$ means $\frac{7}{12}$ of $\frac{36}{50}$, it is the same.

$\frac{1}{12}$ of $\frac{36}{50}$ is the same as $\frac{36}{50} \div 12 = \frac{3}{50}$, so $\frac{7}{12}$ of $\frac{36}{50}$ is $7 \times \frac{3}{50} = \frac{21}{50}$.

3. (a) You calculated $\frac{7}{12} \times \frac{36}{50}$ in the example above. What was the answer?
   (b) Calculate $\frac{7 \times 36}{12 \times 50}$, and simplify your answer.

Example:

$\frac{2}{3} \times \frac{5}{8} = \frac{2}{3}$ of $\frac{15}{24} = \frac{1}{3}$ of $\frac{30}{24} = \frac{10}{24} = \frac{5}{12}$

The same answer is obtained by calculating $\frac{2 \times 5}{3 \times 8}$.

To multiply two fractions, you may simply multiply the numerators and the denominators.

$\frac{2}{3} \times \frac{9}{20} = \frac{2 \times 9}{3 \times 20} = \frac{18}{60} = \frac{3}{10}$

MULTIPLYING AND DIVIDING FRACTIONS

Background information

- Division is the **inverse** of multiplication.
- To find the **inverse of a fraction**, change its numerator to a denominator and its denominator to a numerator.

  Example: The inverse of $\frac{7}{12}$ is $\frac{12}{7}$.
- The **product of a fraction and its inverse** is equal to 1.

  Example: $\frac{7}{12} \times \frac{12}{7} = \frac{84}{84} = 1$
- To **divide by a fraction**, multiply by its inverse.

  Example: $\frac{7}{12} \div \frac{7}{12} = \frac{7}{12} \times \frac{12}{7} = \frac{84}{84} = 1$ (a number divided by itself is equal to 1)

Teaching guidelines

Use the context at the top of LB page 34 to discuss the background information listed above.

Answers

1. (a) $\frac{36}{100} = \frac{9}{25}$ (b) $\frac{36}{400} = \frac{9}{100}$ (c) $\frac{39}{100}$

   (d) $\frac{9}{8} = 1\frac{1}{8}$ (e) $\frac{15}{120} = \frac{1}{8}$ (f) $\frac{9}{400}$

2. (a) $\frac{18}{50} \div \frac{3}{50} = \frac{18}{50} \times \frac{50}{3} = 6$ pans

   (b) $\frac{20}{50} \div \frac{3}{50} = \frac{20}{50} \times \frac{50}{3} = 6\frac{2}{3}$ pans. They can only make whole pans, so they can make 6 pans with $\frac{2}{3}$ of $\frac{3}{50}$ (a pan) left over, i.e. $\frac{6}{150}$ or $\frac{1}{25}$ kg of copper.

   (c) $\frac{2}{5}$ is equivalent to $\frac{20}{50}$, so they can make 6 pans with $\frac{1}{25}$ kg of copper left over.

   (d) $\frac{3}{4} \div \frac{3}{50} = \frac{3}{4} \times \frac{50}{3} = 12{,}5$ pans, so they can make 12 pans with $\frac{3}{100}$ kg of copper left over.

Division by a fraction

When we divide by a fraction, we have a very different situation. Think about this:

If you have 40 pizzas, how many learners can have $\frac{3}{5}$ a pizza each?

To find the number of fifths in 40 pizzas: 40 × 5 = 200 fifths of a pizza.

To find the number of three fifths: 200 ÷ 3 = 66 portions of $\frac{3}{5}$ pizza and two fifths of a pizza is left over.

Since the portion for each learner is three fifths, the two fifths of a pizza that remains is two thirds of a portion.

So, to calculate $40 \div \frac{\mathbf{3}}{\mathbf{5}}$, we multiplied by **5** and divided by **3**, and that gave us 66 and two thirds of a portion.

In fact, we calculated $40 \times \frac{5}{3}$.

> Division is the inverse of multiplication.
> So, to divide by a fraction, you multiply by its inverse.

Example:

$\frac{18}{60} \div \frac{2}{3} = \frac{18}{60} \times \frac{3}{2} = \frac{54}{120} = \frac{9}{20}$

MULTIPLYING AND DIVIDING FRACTIONS

1. Calculate each of the following:

   (a) $\frac{3}{4}$ of $\frac{12}{25}$ (b) $\frac{3}{4} \times \frac{12}{100}$

   (c) $\frac{3}{4}$ of $\frac{13}{25}$ (d) $\frac{3}{4} \times 1\frac{1}{2}$

   (e) $\frac{3}{20} \times \frac{5}{6}$ (f) $\frac{3}{20}$ of $\frac{3}{20}$

2. A small factory manufactures copper pans for cooking. Exactly $\frac{3}{50}$ kg of copper is needed to make one pan.

   (a) How many pans can they make if $\frac{18}{50}$ kg of copper is available?

   (b) How many pans can they make if $\frac{20}{50}$ kg of copper is available?

   (c) How many pans can they make if $\frac{2}{5}$ kg of copper is available?

   (d) How many pans can they make if $\frac{3}{4}$ kg of copper is available?

34 MATHEMATICS GRADE 9: TERM 1

Answers

2. (e) $\frac{144}{50} \div \frac{3}{50} = \frac{144}{50} \times \frac{50}{3} = 48$ pans

   (f) $5 \div \frac{3}{50} = 5 \times \frac{50}{3} = \frac{250}{3} = 83$ pans with $\frac{2}{100}$ kg of copper left over.

3. (a) $\frac{18}{50} \times \frac{50}{3} = 6$ (b) $\frac{18}{50} \div \frac{3}{50} = \frac{18}{50} \times \frac{50}{3} = 6$

   (c) $\frac{144}{50} \times \frac{50}{3} = 48$ (d) $\frac{144}{50} \div \frac{3}{50} = \frac{144}{50} \times \frac{50}{3} = 48$

   (e) $\frac{144}{50} \div \frac{3}{50} = \frac{144}{50} \times \frac{50}{3} = 48$ (f) $\frac{5}{8} \times \frac{50}{3} = \frac{250}{24} = 10\frac{10}{24} = 10\frac{5}{12}$

   (g) $\frac{20}{1} \times \frac{50}{3} = \frac{1\,000}{3} = 333\frac{1}{3}$ (h) $\frac{2}{1} \times \frac{50}{3} = \frac{100}{3} = 33\frac{1}{3}$

   (i) $\frac{1}{1} \times \frac{50}{3} = \frac{50}{3} = 16\frac{2}{3}$ (j) $\frac{1}{2} \times \frac{50}{3} = \frac{50}{6} = 8\frac{1}{3}$

4. (a) $A = l \times b = \frac{29}{8} \times \frac{13}{5} = \frac{377}{40} = 9\frac{17}{40}\ \text{cm}^2$

   (b) $P = 2(l + b) = 2\left(3\frac{5}{8} + 2\frac{3}{5}\right) = 2\left(5 + \frac{25}{40} + \frac{24}{40}\right) = 2\left(5\frac{49}{40}\right) = 2\left(6\frac{9}{40}\right) = 12\frac{9}{20}$ cm

5. $8\frac{1}{6} \div 5\frac{5}{6} = \frac{49}{6} \times \frac{6}{35} = \frac{49}{35} = 1\frac{14}{35} = 1\frac{2}{5}$

6. (a) $\frac{19}{8} \times \frac{29}{5} = \frac{551}{40} = 13\frac{31}{40}$ (b) $\frac{23}{7} \times \frac{31}{12} = \frac{713}{84} = 8\frac{41}{84}$

   (c) $\frac{42}{5} \div \frac{33}{10} = \frac{42}{5} \times \frac{10}{33} = \frac{420}{165} = 2\frac{90}{165} = 2\frac{6}{11}$ (d) $\frac{33}{10} \times \frac{33}{10} = \frac{1\,089}{100} = 10\frac{89}{100}$

   (e) $\frac{21}{8} \div \frac{57}{10} = \frac{21}{8} \times \frac{10}{57} = \frac{210}{456} = \frac{35}{76}$ (f) $\frac{3}{5} \times \frac{5}{3} \times 1\frac{3}{4} = 1\frac{3}{4}$

7. (a) $\frac{2}{3}\left(\frac{15}{20} + \frac{14}{20}\right) = \frac{2}{3}\left(\frac{29}{20}\right) = \frac{29}{30}$ (b) $\frac{2}{3} \times \left(\frac{3}{4} + \frac{7}{10}\right) = \frac{29}{30}$ as in (a)

   (c) $\frac{5}{8}\left(\frac{12}{15} - \frac{5}{15}\right) = \frac{5}{8}\left(\frac{7}{15}\right) = \frac{7}{24}$ (d) $\frac{20}{40} - \frac{5}{24} = \frac{60}{120} - \frac{25}{120} = \frac{35}{120} = \frac{7}{24}$

(e) How many pans can be made if $\frac{144}{50}$ kg of copper is available?

(f) How many pans can be made if 5 kg of copper is available?

3. Calculate each of the following:

   (a) $\frac{18}{50} \div \frac{3}{50}$ (b) $\frac{9}{25} \div \frac{3}{50}$

   (c) $\frac{144}{50} \div \frac{3}{50}$ (d) $2\frac{44}{50} \div \frac{3}{50}$

   (e) $2\frac{22}{25} \div \frac{3}{50}$ (f) $\frac{5}{8} \div \frac{3}{50}$

   (g) $20 \div \frac{3}{50}$ (h) $2 \div \frac{3}{50}$

   (i) $1 \div \frac{3}{50}$ (j) $\frac{1}{2} \div \frac{3}{50}$

4. A rectangle is $3\frac{5}{8}$ cm long and $2\frac{3}{5}$ cm wide.

   (a) What is the area of this rectangle?

   (b) What is the perimeter of this rectangle?

5. A rectangle is $5\frac{5}{6}$ cm long and its area is $8\frac{1}{6}$ cm².

   How wide is this rectangle?

6. Calculate each of the following:

   (a) $2\frac{3}{8}$ of $5\frac{4}{5}$ (b) $3\frac{2}{7} \times 2\frac{7}{12}$

   (c) $8\frac{2}{5} \div 3\frac{3}{10}$ (d) $3\frac{3}{10} \times 3\frac{3}{10}$

   (e) $2\frac{5}{8} \div 5\frac{7}{10}$ (f) $\frac{3}{5} \times 1\frac{2}{3} \times 1\frac{3}{4}$

7. Calculate each of the following:

   (a) $\frac{2}{3}(\frac{3}{4} + \frac{7}{10})$ (b) $\frac{2}{3} \times \frac{3}{4} + \frac{2}{3} \times \frac{7}{10}$

   (c) $\frac{5}{8}(\frac{4}{5} - \frac{1}{3})$ (d) $\frac{5}{8} \times \frac{4}{5} - \frac{5}{8} \times \frac{1}{3}$

8. A piece of land with an area of 40 ha is divided into 30 equal plots. The total price of the land is R45 000. Remember that "ha" is the abbreviation for hectares.

   (a) Jim buys $\frac{2}{5}$ of the land.

Answers

8. (a) (i) $\frac{2}{5} \times \frac{30}{1} = 12$ plots; $\frac{2}{5} \times \frac{45\,000}{1} = \text{R}18\,000$

(ii) $\frac{2}{5} \times \frac{40}{1} = 16$ ha

(b) $\frac{1}{3} \times \frac{30}{1} = 10$ plots; $\frac{1}{3} \times \frac{45\,000}{1} = \text{R}15\,000$

(c) $\frac{15}{15} - \frac{6}{15} - \frac{5}{15} = \frac{4}{15}$

SQUARES, CUBES, SQUARE ROOTS AND CUBE ROOTS

Background information

- The **square of a fraction** is equal to the square of the numerator divided by the square of the denominator.
- The **cube of a common fraction** is the cube of the numerator divided by the cube of the denominator.
- The **square root of a common fraction** is the square root of the numerator divided by the square root of the denominator.
- The **cube root of a common fraction** is the cube root of the numerator divided by the cube root of the denominator.

Teaching guidelines

Use the following examples to revise the concepts listed above.

$$\left(\frac{4}{5}\right)^2 = \frac{4^2}{5^2} = \frac{16}{25} \qquad \left(\frac{4}{5}\right)^3 = \frac{4^3}{5^3} = \frac{64}{125} \qquad \sqrt{\frac{16}{25}} = \frac{\sqrt{16}}{\sqrt{25}} = \frac{4}{5} \qquad \sqrt[3]{\frac{64}{125}} = \frac{\sqrt[3]{64}}{\sqrt[3]{125}} = \frac{4}{5}$$

Answers

1. (a) $\frac{9}{16}$ (b) $\frac{49}{100}$ (c) $\frac{21}{8} \times \frac{21}{8} = \frac{441}{64} = 6\frac{57}{64}$

(d) $\frac{17}{12} \times \frac{17}{12} = \frac{289}{144} = 2\frac{1}{144}$ (e) $\frac{26}{7} \times \frac{26}{7} = \frac{676}{49} = 13\frac{39}{49}$ (f) $\frac{43}{4} \times \frac{43}{4} = \frac{1\,849}{16} = 115\frac{9}{16}$

2. (a) $\frac{5}{7}$ (b) $\frac{6}{11}$ (c) $\frac{8}{5} = 1\frac{3}{5}$ (d) $\sqrt{\frac{144}{49}} = \frac{12}{7} = 1\frac{5}{7}$

3. (a) $\frac{27}{64}$ (b) $\frac{343}{1\,000}$ (c) $\frac{729}{1\,000}$ (d) $\frac{125}{512}$

4. (a) $\frac{3}{10}$ (b) $\frac{5}{6}$ (c) $\frac{10}{6} = 1\frac{4}{6} = 1\frac{2}{3}$ (d) $\sqrt[3]{\frac{125}{8}} = \frac{5}{2} = 2\frac{1}{2}$

(i) How many plots is this and how much should he pay?
(ii) What is the area of the land that Jim buys?

(b) Charlene buys $\frac{1}{3}$ of the land. How many plots is this and how much should she pay?

(c) Bongani buys the rest of the land. Determine the fraction of the land that he buys.

SQUARES, CUBES, SQUARE ROOTS AND CUBE ROOTS

1. Calculate each of the following:

(a) $\frac{3}{4} \times \frac{3}{4}$ (b) $\frac{7}{10} \times \frac{7}{10}$

(c) $2\frac{5}{8} \times 2\frac{5}{8}$ (d) $1\frac{5}{12} \times 1\frac{5}{12}$

(e) $3\frac{5}{7} \times 3\frac{5}{7}$ (f) $10\frac{3}{4} \times 10\frac{3}{4}$

$\frac{9}{16}$ is the square of $\frac{3}{4}$, because $\frac{3}{4} \times \frac{3}{4} = \frac{9}{16}$. $\frac{3}{4}$ is the square root of $\frac{9}{16}$.

2. Find the square root of each of the following numbers:

(a) $\sqrt{\frac{25}{49}}$ (b) $\sqrt{\frac{36}{121}}$

(c) $\sqrt{\frac{64}{25}}$ (d) $\sqrt{2\frac{46}{49}}$

3. Calculate each of the following:

(a) $\frac{3}{4} \times \frac{3}{4} \times \frac{3}{4}$ (b) $\frac{7}{10} \times \frac{7}{10} \times \frac{7}{10}$

(c) $\frac{9}{10} \times \frac{9}{10} \times \frac{9}{10}$ (d) $\frac{5}{8} \times \frac{5}{8} \times \frac{5}{8}$

4. Find the cube root of each of the following numbers:

(a) $\sqrt[3]{\frac{27}{1\,000}}$ (b) $\sqrt[3]{\frac{125}{216}}$

(c) $\sqrt[3]{\frac{1\,000}{216}}$ (d) $\sqrt[3]{15\frac{5}{8}}$

3.4 Equivalent forms

FRACTIONS, DECIMALS AND PERCENTAGE FORMS

Background information

- It is critical that learners do not adhere to the misconception that “common fractions” and “decimal fractions” are different kinds of numbers, but understand that these labels refer to **two different notations for exactly the same numbers**, namely the rational numbers.
- In the **common fraction notation**, accuracy of representing a quantity is achieved by using a denominator, i.e. a fractional unit that facilitates the accurate description of a quantity. For example, if a discount of R600 is given on a purchase of R1 600, the discount can be accurately described as three eighths of the marked price (R600 is three eighths of R1 600).
- The **decimal notation** is limited to tenths, hundredths, thousandths and so on. Hence, to express the same R600 discount in terms of a decimal, it needs to be understood as $\frac{3}{10} + \frac{7}{100} + \frac{5}{1\,000}$, which is written as 0,375 in the decimal notation.
- **Decimals** are fractions as much as common fractions are – they **are just fractions expressed in a different way**.
- **Percentage** is a more limited form (than decimals) of expressing fractions, in the sense that only hundredths are used: 37% means $\frac{37}{100}$, where % means $\frac{}{100}$.

Teaching guidelines

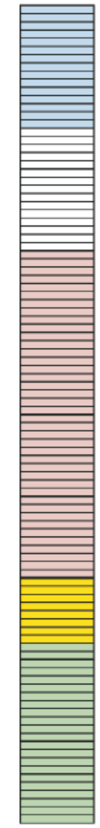
Revise how to convert between the common fraction, decimal and percentage notations.

Answers

1. (a) 100 (b) 10 (c) $\frac{15}{100} = \frac{3}{20}$ (d) $\frac{40}{100} = \frac{4}{10} = \frac{2}{5}$
2. (a) 22% (b) 40%
3. (a) 0,3; 30%; $\frac{3}{10}$ (b) 0,07; 7%; $\frac{7}{100}$ (c) 0,37; 37%; $\frac{37}{100}$
   (d) 0,7; 70%; $\frac{7}{10}$ (e) 0,4; 40%; $\frac{2}{5}$ (f) 0,35; 35%; $\frac{7}{20}$

3.4 Equivalent forms

FRACTIONS, DECIMALS AND PERCENTAGE FORMS



1. The rectangle on the right is divided into small parts.
   (a) How many of these small parts are there in the rectangle?
   (b) How many of these small parts are there in one tenth of the rectangle?
   (c) What fraction of the rectangle is blue?
   (d) What fraction of the rectangle is pink?

Instead of “six hundredths” we may say “6 per cent” or, in short, “6%”. It means the same thing.

15 per cent of the rectangle on the right is blue.

2. (a) What percentage of the rectangle is green?
   (b) What percentage of the rectangle is pink?

0,37 and 37% and $\frac{37}{100}$ are different ways of writing the same value (**37 hundredths**).

3. Express each of the following in three ways, namely as a decimal, a percentage and a fraction (in simplest form):
   (a) three tenths (b) seven hundredths
   (c) 37 hundredths (d) seven tenths
   (e) two fifths (f) seven twentieths

CHAPTER 3: FRACTIONS 37

Answers

4. See the completed table on LB page 38 alongside.
5. (a) 25%
   (b) $\frac{3}{4}$
   (c) $(1 - 0{,}18) \div 2 = 0{,}41 = \frac{41}{100}$

4. Copy the table and fill in the missing values.

Decimal	Percentage	Common fraction (simplest form)
0,2	20%	$\frac{1}{5}$
0,4	40%	$\frac{2}{5}$
0,375	37,5%	$\frac{3}{8}$
0,05	5%	$\frac{1}{20}$

5. (a) Jannie eats a quarter of a watermelon. What percentage of the watermelon is this?
   (b) Sibu drinks 75% of the milk in a bottle. What fraction of the milk in the bottle has he drunk?
   (c) Jem used 0,18 of the paint in a tin. If he uses half of the remaining amount the next time he paints, what fraction (in simplest form) is left over?

The decimal notation for fractions

Learner Book Overview		
Sections in this chapter	**Content**	**Pages in Learner Book**
4.1 Equivalent forms	Conversions of fractions in common fraction, decimal and percentage notation	Pages 39 to 40
4.2 Calculations with decimals	Addition, subtraction, multiplication and division with decimal fractions	Pages 40 to 42
4.3 Solving problems	Simple problems	Pages 42 to 43
4.4 More problems	Problems in contexts	Pages 43 to 44
4.5 Decimals in algebraic expressions and equations	Decimals in algebra	Page 44

CAPS time allocation	4,5 hours
CAPS content specification	Page 123

Mathematical background

It is critical that learners are empowered to understand that "common fractions", "decimals" and "percentages" are **different ways to represent exactly the same numbers**.

- To express a number as a **common fraction**, only one kind of fractional unit is used. In the common fraction representation, precision is achieved by selecting a **fractional unit with which the given quantity can be measured accurately**.

  Example: A distance may be expressed as five eighths of a kilometre, which can also be written as $\frac{5}{8}$ km. Precision is achieved by dividing a kilometre into eight fractional parts and selecting five of those parts. The same quantity can be expressed as a common fraction in different ways using different fractional parts like sixteenths $\left(\frac{10}{16}\right)$, fortieths $\left(\frac{25}{40}\right)$ or thousandths $\left(\frac{625}{1\,000}\right)$.

- To express the same quantity as a **percentage**, only **hundredths** of the measuring unit is used.

  Example: Five eighths is $62\frac{1}{2}$ hundredths, which can be written as 62,5%.

- To express the same quantity as a **decimal**, a combination of **tenths, hundredths, thousandths,** etc. of the measuring unit is used.

  Example: In decimals, five eighths is expressed as six tenths + two hundredths + five thousandths.

To **add and subtract numbers** expressed as common fractions, conversion to equivalent fractions ("a common denominator") is often necessary. There is no such need when numbers are expressed as decimals. However, as in the case of addition and subtraction with whole numbers, the usual "column methods" are based on decomposition ("expanded notation") and regrouping (using the commutative and associative properties), and can only be logically understood in terms of decomposition and regrouping.

4.1 Equivalent forms

COMMON FRACTIONS, DECIMAL FRACTIONS AND PERCENTAGES

Background information

- To **write a decimal as a common fraction**, write it with a denominator that is a power of 10 and simplify if possible.

  Example: $0{,}64 = \frac{64}{100} = \frac{16}{25} \times \frac{4}{4} = \frac{16}{25}$

- To **write a common fraction as a decimal fraction**, convert it to a common fraction with a power of 10 as a denominator.

  Example: $\frac{3}{8} = \frac{3}{8} \times \frac{125}{125} = \frac{375}{1\,000} = 0{,}375$

Teaching guidelines

Learners discuss how to write:

- a common fraction in decimal notation: divide the numerator by the denominator
- a common fraction in percentage notation: divide the numerator by the denominator and multiply by 100
- a decimal fraction in common fraction notation: write as tenths, hundredths or thousandths and simplify
- a decimal fraction in percentage notation: multiply by 100
- a percentage in common fraction notation: write as hundredths and simplify
- a percentage in decimal notation: divide by 100.

Answers

1. (a) $\frac{56}{100} = \frac{14}{25} \times \frac{4}{4} = \frac{14}{25}$

   (b) $\frac{387}{100}$ or $3\frac{87}{100}$

   (c) $\frac{19}{10}$ or $1\frac{9}{10}$

   (d) $\frac{5\,205}{1\,000} = \frac{1\,041}{200} \times \frac{5}{5} = \frac{1\,041}{200}$ or $5\frac{41}{200}$

CHAPTER 4

The decimal notation for fractions

4.1 Equivalent forms

Decimal fractions and common fractions are simply different ways of expressing the same number. They are different **notations** showing the same value.

To write a decimal fraction as a common fraction: Write the decimal with a denominator that is a power of ten (10, 100, 1 000, etc.) and then simplify it if possible.

For example: $0{,}35 = \frac{35}{100} = \frac{7}{20} \times \frac{5}{5} = \frac{7}{20}$

To write a common fraction as a decimal fraction: Change the common fraction to an equivalent fraction with a power of ten as a denominator.

For example: $\frac{3}{4} = \frac{3}{4} \times \frac{25}{25} = \frac{75}{100} = 0{,}75$

If you are permitted to use your calculator, simply type in 3 ÷ 4 and the answer will be given as 0,75. On some calculators you will need to press an additional button to convert the exact fraction to a decimal.

Notation means a set of symbols that are used to show a special thing.

COMMON FRACTIONS, DECIMAL FRACTIONS AND PERCENTAGES

Do *not* use a calculator in this exercise.

1. Write the following decimal fractions as common fractions in their simplest form:

   (a) 0,56 (b) 3,87
   (c) 1,9 (d) 5,205

Answers

2. (a) $\frac{9}{20} \times \frac{5}{5} = \frac{45}{100} = 0{,}45$ (b) $\frac{7}{5} \times \frac{20}{20} = \frac{140}{100} = 1{,}4$

   (c) $\frac{24}{25} \times \frac{4}{4} = \frac{96}{100} = 0{,}96$ (d) $\frac{2 \times 8 + 3}{8} \times \frac{125}{125} = \frac{2\,375}{1\,000} = 2{,}375$

3. (a) $\frac{70}{100} = \frac{7}{10}$ (b) $\frac{5}{100} = \frac{1}{20}$ (c) $\frac{125}{1\,000} = \frac{1}{8}$

4. (a) $\frac{6}{10} \times \frac{10}{10} = \frac{60}{100} = 60\%$ (b) $\frac{43}{100} = 43\%$

   (c) $\frac{8}{100} = 8\%$ (d) $\frac{265}{1\,000} = \frac{26{,}5}{100} = 26{,}5\%$

   (e) $\frac{5}{1\,000} = \frac{0{,}5}{100} = 0{,}5\%$

5. (a) $\frac{7}{10} \times \frac{10}{10} = \frac{70}{100} = 70\%$ (b) $\frac{3}{4} \times \frac{25}{25} = \frac{75}{100} = 75\%$

   (c) $\frac{33}{50} \times \frac{2}{2} = \frac{66}{100} = 66\%$ (d) 100%

   (e) $\frac{2}{25} \times \frac{4}{4} = \frac{8}{100} = 8\%$ (f) $\frac{29}{50} \times \frac{2}{2} = \frac{58}{100} = 58\%$

6. (a) $\frac{40}{50} = \frac{4}{5}$

   (b) Devi: 80%; Jane: $\frac{60}{80} = \frac{15}{20} \times \frac{5}{5} = \frac{75}{100} = 75\%$

   (c) Devi performed better.

7. (a) $\frac{4}{12} = \frac{1}{3}$

   (b) $\frac{8}{12} = \frac{2}{3} = 66{,}6...\%$

4.2 Calculations with decimals

CALCULATIONS WITH DECIMALS

Background information

- To **add and subtract decimals**:
  - add tenths to tenths; subtract tenths from tenths
  - add hundredths to hundredths; subtract hundredths from hundredths
  - add thousandths to thousandths; subtract thousandths from thousandths.

---

2. Write the following common fractions as decimal fractions:
   (a) $\frac{9}{20}$ (b) $\frac{7}{5}$
   (c) $\frac{24}{25}$ (d) $2\frac{3}{8}$
3. Write the following percentages as common fractions in their simplest form:
   (a) 70% (b) 5% (c) 12,5%
4. Write the following decimal fractions as percentages:
   (a) 0,6 (b) 0,43 (c) 0,08
   (d) 0,265 (e) 0,005
5. Write the following common fractions as percentages:
   (a) $\frac{7}{10}$ (b) $\frac{3}{4}$ (c) $\frac{33}{50}$
   (d) $\frac{60}{60}$ (e) $\frac{2}{25}$ (f) $\frac{29}{50}$
6. Jane and Devi are in different schools. At Jane's school the year mark for Mathematics was out of 80, and Jane got 60 out of 80. At Devi's school the year mark was out of 50 and Devi got 40 out of 50.
   (a) What fraction of the total marks, in its simplest form, did Devi obtain at her school?
   (b) What percentage did Devi and Jane get for Mathematics?
   (c) Who performed better, Jane or Devi?
7. During a basketball game, Lebo tried to score 12 times. Only four of her attempts were successful.
   (a) What fraction of her attempts was successful?
   (b) What percentage of her attempts was not successful?

4.2 Calculations with decimals

When you **add** and **subtract** decimal fractions:
- Add tenths to tenths.
- Subtract tenths from tenths.
- Add hundredths to hundredths.
- Subtract hundreds from hundredths.

And so on!

40 MATHEMATICS GRADE 9: TERM 1

- To **multiply decimals**:
  - write the decimals as common fractions
  - multiply the numerators as well as the denominators
  - divide the product of the numerators by the product of the denominators.
- To **divide decimal fractions**:
  - convert the dividend and divisor to whole numbers by multiplying both by the same power of 10
  - divide the new dividend by the new divisor.

Teaching guidelines

Revise how to add, subtract, multiply and divide with decimals.

Answers

1. (a) 3,30 + 4,83 = 8,13
   (b) (0,6 + 4,4) + 18,3 = 5 + 18,3 = 23,3
   (c) 16,90 – 1,23 = 15,67
   (d) 8,4 – 0,6 = 7,8
   (e) 9,43 – 4,75 = 4,68
   (f) 1,21 + 2,50 – 2,07 = 3,71 – 2,07 = 1,64
2. (a) 2,0 (b) 0,3
   (c) 0,032 (d) 0,0030 (or 0,003)
   (e) 0,214 (f) 0,00032
3. (a) 2,4 (b) $\frac{12}{0,3} \times \frac{10}{10} = 120 \div 3 = 40$
   (c) $\frac{0,15}{0,5} \times \frac{10}{10} = 1,5 \div 5 = 0,3$ (d) 10 000 ÷ 2 = 5 000
   (e) 300 ÷ 6 = 50 (f) 2,4 ÷ 8 = 0,3
4. (a) See the circled answer on LB page 41 alongside.
   (b) See the circled answer on LB page 41 alongside.

When you **multiply** decimal fractions, you change the decimals to whole numbers, do the calculation and lastly, change them back to decimal fractions.

Example: To calculate 13,1 × 1,01, you first calculate 131 × 101 (which equals 13 231). Then, since you have multiplied the 13,1 by 10, and the 1,01 by 100 in order to turn them into whole numbers, you need to divide this answer by 10 × 100 (i.e. 1 000). Therefore, the final answer is 13,231.

When you **divide** decimal fractions, you can use equivalent fractions to help you.

Example: $21,7 \div 0,7 = \frac{21,7}{0,7} = \frac{21,7}{0,7} \times \frac{10}{10} = \frac{217}{7} = 31$

Notice how you multiply both the numerator and denominator of the fraction by the same number (in this case, 10). Always multiply by the *smallest* power of ten that will convert both values to whole numbers.

CALCULATIONS WITH DECIMALS

Do *not* use a calculator in this exercise. Ensure that you show all steps of your working.

1. Calculate the value of each of the following:
   (a) 3,3 + 4,83 (b) 0,6 + 18,3 + 4,4
   (c) 9,3 + 7,6 – 1,23 (d) (16,0 – 7,6) – 0,6
   (e) 9,43 – (3,61 + 1,14) (f) 1,21 + 2,5 – (2,3 – 0,23)
2. Calculate the value of each of the following:
   (a) 4 × 0,5 (b) 15 × 0,02 (c) 0,8 × 0,04
   (d) 0,02 × 0,15 (e) 1,07 × 0,2 (f) 0,016 × 0,02
3. Calculate the value of each of the following:
   (a) 7,2 ÷ 3 (b) 12 ÷ 0,3 (c) 0,15 ÷ 0,5
   (d) 10 ÷ 0,002 (e) 0,3 ÷ 0,006 (f) 0,024 ÷ 0,08
4. Write down the value that is equal to or closest to the answer to each calculation:
   (a) 3 × 0,5 (b) 4,4 ÷ 0,2
   A: 6 A: 8,8
   B: 1,5 B: 2,2
   C: 0,15 C: 22

Answers

4. (c) See the circled answer on LB page 42 alongside.
5. See the spider diagram on LB page 42 alongside.
6. (a) 0,01 (b) 0,0009 (c) 6,25
   (d) 0,2 (e) 0,4 (f) 0,7
   (g) 0,008 (h) 0,064 (i) 0,000027
   (j) 0,4 (k) 0,5 (l) 0,6
7. (a) $5{,}0 \div 10 = 0{,}5$
   (b) $4{,}2 - 6{,}0 = -1{,}8$
   (c) $\frac{12{,}75}{0{,}05} \times \frac{100}{100} = 1\,275 \div 5 = 255$
   (d) $420 \div 21 + 0{,}135 = 20 + 0{,}135 = 20{,}135$

4.3 Solving problems

ALL KINDS OF PROBLEMS

Background information

Operations with decimals should include examples such as the following:

- multiplying a decimal by a whole number
- multiplying a decimal by a decimal
- dividing a whole number by a decimal
- dividing a decimal by a decimal.

Teaching guidelines

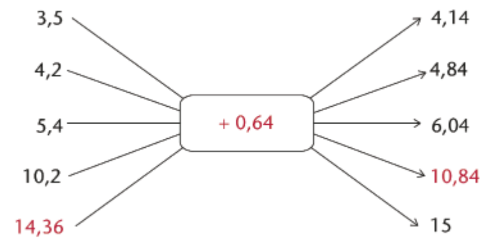
Create simple examples like those listed above.

Answers

1. The value is the same, because the first number is divided by ten and the second number is multiplied by ten to get the right-hand side figures. We know that multiplying by ten and dividing by ten is the same as multiplying by one.
2. (a) 10,8 (b) 108,0 (c) 1,08
   (d) 1,08 (e) 0,00108 (f) 1,08

(c) $56 \times 1{,}675$
   A: more than 56
   B: more than 84
   C: more than 112

5. Copy the diagram. Determine the operator and the unknown numbers and fill them in:



6. Calculate each of the following:
   (a) $(0{,}1)^2$ (b) $(0{,}03)^2$ (c) $(2{,}5)^2$
   (d) $\sqrt{0{,}04}$ (e) $\sqrt{0{,}16}$ (f) $\sqrt{0{,}49}$
   (g) $(0{,}2)^3$ (h) $(0{,}4)^3$ (i) $(0{,}03)^3$
   (j) $\sqrt[3]{0{,}064}$ (k) $\sqrt[3]{0{,}125}$ (l) $\sqrt[3]{0{,}216}$
7. Calculate each of the following:
   (a) $2{,}5 \times 2 \div 10$ (b) $4{,}2 - 5 \times 1{,}2$
   (c) $\frac{5{,}4 + 7{,}35}{0{,}05}$ (d) $4{,}2 \div 0{,}21 + 0{,}45 \times 0{,}3$

4.3 Solving problems

ALL KINDS OF PROBLEMS

Do *not* use a calculator in this exercise. Ensure that you show all steps of working.

1. Is $6{,}54 \times 0{,}81 = 0{,}654 \times 8{,}1$? Explain your answer.
2. You are given that $45 \times 24 = 1\,080$. Use this result to determine:
   (a) $4{,}5 \times 2{,}4$ (b) $4{,}5 \times 24$ (c) $4{,}5 \times 0{,}24$
   (d) $0{,}045 \times 24$ (e) $0{,}045 \times 0{,}024$ (f) $0{,}045 \times 24$

42 MATHEMATICS GRADE 9: TERM 1

Answers

3. (a) 28 (b) 0,8 (c) 100
4. (a) Yes, he is correct. He multiplied the divisor by 100 in step 1.
   He must multiply the dividend by 100 too.
   He is doing the same as this calculation:
   $\frac{6,5}{0,02} \times \frac{100}{100} = 650 \div 2 = 325$
   (b) (i) $4,8 \div 3 = 1,6$; $1,6 \times 10 = 16$
   (ii) $21 \div 3 = 7$; $7 \times 1\,000 = 7\,000$
5. See the answers on LB page 43 alongside.

4.4 More problems

MORE PROBLEMS AND CALCULATIONS

Background information

Calculations with decimals feature in a variety of real-life contexts, for example:

- recording distances and times in sports like athletics
- interest rates charged by banks and financial institutions
- perimeter and area of 2D figures
- prices of products and change to be paid out
- fuel, water and electricity consumption, etc.

Teaching guidelines

Discuss examples where decimals feature in real-life situations.

Answers

1. See the answers on LB page 43 alongside.
2. $0,890 - 0,581 = 0,309$
3. (a) $2 \times (12,34 + 31,67) = 88,02$ cm
   (b) $12,34 \times 31,67\ \text{cm}^2 = 390,81\ \text{cm}^2$
4. (a) R5,95 + R3,25 + R4,60 = R13,80
   (b) R20 – R13,80 = R6,20
5. $11,25 \times 4 \div 3$ (0,75 is the same as three quarters) = 15 ℓ

---

3. Without actually dividing, choose which answer in brackets is the correct answer, or the closest to the correct answer.
   (a) $14 \div 0,5$ (7; 28; 70) (b) $0,58 \div 0,7$ (8; 80; 0,8)
   (c) $2,1 \div 0,023$ (10; 100; 5)
4. (a) John is asked to calculate $6,5 \div 0,02$. He does the following:
   Step 1: $6,5 \div 2 = 3,25$
   Step 2: $3,25 \times 100 = 325$
   Is he correct? Why?
   (b) Use John's method in part (a) to calculate:
   (i) $4,8 \div 0,3$ (ii) $21 \div 0,003$
5. Given: $0,174 \div 0,3 = 0,58$. Using this fact, write down the answers for the following without doing any further calculations:
   (a) $0,3 \times 0,58$ = 0,174 (b) $1,74 \div 3$ = 0,58
   (c) $17,4 \div 30$ = 0,58 (d) $174 \div 300$ = 0,58
   (e) $0,0174 \div 0,03$ = 0,58 (f) $0,3 \times 5,8$ = 1,74

4.4 More problems

MORE PROBLEMS AND CALCULATIONS

You *may* use a calculator for this exercise.

1. Calculate the following, rounding off all answers correct to two decimal places:
   (a) $8,567 + 3,0456$ = 11,61 (b) $2,781 - 6,0049$ = –3,22
   (c) $1,234 \times 4,056$ = 5,01 (d) $\frac{5,678 + 3,245}{1,294 - 0,994}$ = 29,74
2. What is the difference between 0,890 and 0,581?
3. If a rectangle is 12,34 cm wide and 31,67 cm long:
   (a) What is the perimeter of the rectangle?
   (b) What is the area of the rectangle? Round off your answer to two decimal places.
4. Alison buys a cooldrink for R5,95, a chocolate for R3,25 and a packet of chips for R4,60. She pays with a R20 note.
   (a) How much did she spend?
   (b) How much change did she get?
5. A tractor uses 11,25 ℓ of fuel in 0,75 hours. How many litres does it use in one hour?

Answers

6. (a) 32,65 ÷ 5,83 + 5,326 = 10,92634305 kℓ (or 5,6 + 5,326 = 10,926 kℓ)
   (b) Amount @ R1,42/kWh: (R417,59 – 100 × R1,13) ÷ 1,42 = 214,5 kWh
   Electricity used: 214,5 + 100 + 10 = 324,5 kWh
7. 25 ÷ 1,35 = 18,519, therefore 18 dresses can be made.
   Material left over: 25 – 1,35 × 18 = 0,7 m
8. 28,6 × 0,679 = 19,4194 = 19,42 kg
9. 332,523 – 321,573 = 10,95 kℓ = 10,95 × 1 000 = 10 950 ℓ

4.5 Decimals in algebraic expressions and equations

DECIMALS IN ALGEBRA

Teaching guidelines

Restrict examples to the following:

- addition and subtraction of simple algebraic expressions
- multiplication and division of simple algebraic expressions
- algebraic expressions that contain squares, cubes, square roots and cube roots
- equations that require only one operation to find the answer.

Answers

1. (a) $0,3x^{18}$ (b) $-3,2x^3$
   (c) $24x^3y^6$ (d) $11,75x^2 - 6x^2 = 5,75x^2$
   (e) $\frac{2,2x}{4,4x} = \frac{1}{2}$ (f) $0,2x^4 + 0,4x^4 = 0,6x^4$
   (g) $3,1x^2 - 41,7$ (h) $\frac{1,6y}{-2,4x} = \frac{0,2y}{-0,3x} = -\frac{2y}{3x}$
2. (a) $\frac{0,5x^9}{0,02x^3} \times \frac{100}{100} = \frac{50x^9}{2x^3} = 25x^6$ (b) $\frac{-1,35}{x^2}$
   (c) $\frac{36x}{15y^3} \times \frac{50y}{6x} = \frac{1\,800xy}{90xy^3} = \frac{20}{y^2}$ (d) $\frac{95x^2}{12x} \times \frac{4y^8}{5x} = \frac{19xy^6}{3} = 6\frac{xy^6}{3}$
3. (a) $x = 0,31 - 0,24 = 0,07$ (b) $x = 7,23 - 5,61 = 1,62$
   (c) $x = 9,87 + 3,14 = 13,01$ (d) $x = 4,21 - 2,74 = 1,47$
   (e) $x = 0,48 \div 0,96 = 0,5$ (f) $x = 1,5 \times 0,03 = 0,045$

6. Mrs Ruka received her municipal bill.
   (a) Her water consumption charge for one month is R32,65. The first 5,326 kℓ are free, then she pays R5,83 per kilolitre for every kilolitre thereafter.
   How much water did the Ruka household use?
   (b) The electricity charge for Mrs Ruka for the same month was R417,59. The first 10 kWh are free. For the next 100 kWh the charge is R1,13 per kWh, and thereafter for each kWh the charge is R1,42.
   How much electricity did the Ruka household use?
7. A roll of material is 25 m long. To make one dress, you need 1,35 m of material. How many dresses can be made out of a roll of material and how much material is left over?
8. If one litre of petrol weighs 0,679 kg, what will 28,6 ℓ of petrol weigh?
9. The reading on a water meter at the beginning of the month is 321,573 kℓ. At the end of the month the reading is 332,523 kℓ. How much water (in ℓ) was used during this month?

4.5 Decimals in algebraic expressions and equations

DECIMALS IN ALGEBRA

1. Simplify the following:
   (a) $\sqrt{0,09x^{36}}$ (b) $7,2x^3 - 10,4x^3$
   (c) $(2,4x^2y^3)(10y^3x)$ (d) $11,75x^2 - 1,2x \times 5x$
   (e) $\frac{3,4x - 1,2x}{1,1x \times 4}$ (f) $\sqrt[3]{0,008x^{12}} + \sqrt{0,16x^8}$
   (g) $3x^2 + 0,1x^2 - 45,6 + 3,9$ (h) $\frac{0,4y + 1,2y}{0,6x - 3x}$
2. Simplify the following:
   (a) $\frac{0,5x^9}{0,02x^3}$ (b) $\frac{0,325}{x^2} - \frac{1,675}{x^2}$
   (c) $\frac{3,6x}{1,5y^3} \times \frac{5y}{0,6x}$ (d) $\frac{9,5x^2}{1,2y^2} \div \frac{0,05x}{0,04y^8}$
3. Solve the following equations:
   (a) $0,24 + x = 0,31$ (b) $x + 5,61 = 7,23$
   (c) $x - 3,14 = 9,87$ (d) $4,21 - x = 2,74$
   (e) $0,96x = 0,48$ (f) $x \div 0,03 = 1,5$

WORKSHEET

Answers

1. See the table on LB page 45 alongside.
2. (a) 1,86
   (b) 0,085
   (c) 20
   (d) 4,2 – 0,48 + 7,37 = 11,09
   (e) 0,0144
   (f) $\frac{3 \times 0{,}02}{0{,}3} = \frac{3 \times 2}{3} = 2$
3. (a) 6,84
   (b) 0,00684
   (c) 360
4. (a) $4{,}95x - 1{,}2 - 3{,}65x - 3{,}1 = 1{,}3x - 4{,}3$
   (b) $\frac{2\,750x^{50}}{5x^{25}} = 550x^{25}$
5. 100 – 2 × 6,98 – 3 × 6,48 – 5 × 7,95 = R26,85

WORKSHEET

You are *not* permitted to use a calculator in this exercise, *except* for question 5. Ensure that you show all steps of working, where relevant.

1. Copy and complete the following table:

Percentage	Common fraction	Decimal fraction
2,5%	$\frac{1}{40}$	0,025
6%	$\frac{15}{250}$	0,06
0,9%	$\frac{9}{1\,000}$	0,009

2. Calculate each of the following:
   (a) 6,78 – 4,92 (b) 1,7 × 0,05 (c) 7,2 ÷ 0,36
   (d) 4,2 – 0,4 × 1,2 + 7,37 (e) $(0{,}12)^2$ (f) $\frac{3\sqrt{0{,}04}}{\sqrt[3]{0{,}027}}$
3. 36 × 19 = 684. Use this result to determine:
   (a) 3,6 × 1,9 (b) 0,036 × 0,19 (c) 68,4 ÷ 0,19
4. Simplify:
   (a) $(4{,}95x - 1{,}2) - (3{,}65x + 3{,}1)$ (b) $\frac{2{,}75x^{50}}{0{,}005x^{25}}$
5. Mulalo went to the shop and purchased two tubes of toothpaste for R6,98 each; three cans of cooldrink for R6,48 each, and five tins of baked beans for R7,95 each. If he pays with a R100 note, how much change should he get?

Grade 9 Term 1 Chapter 5 Exponents

Learner Book Overview		
Sections in this chapter	**Content**	**Pages in Learner Book**
5.1 Revision	The exponential form; exponent; base; order of operations; laws of exponents	Pages 46 to 48
5.2 Integer exponents	Negative exponents	Pages 48 to 49
5.3 Solving simple exponential equations	Exponential equations	Pages 50 to 51
5.4 Scientific notation	Very small and very large numbers; calculations using scientific notation	Pages 51 to 52

CAPS time allocation	5 hours
CAPS content specification	Page 124

Mathematical background

- The **exponential notation** is a shorthand notation for **repeated multiplication with the same number**. The exponent, for example the "5" in 2^5, indicates the number of occurrences of the factor 2 in the product $2 \times 2 \times 2 \times 2 \times 2$.
- The **multiplication notation** is a shorthand notation for **repeated addition with the same number**, for example, 2×5 is shorthand for $5 + 5$. Multiplication of whole numbers is commutative, therefore $2 \times 5 = 5 \times 2$, for which the shorthand is $2 + 2 + 2 + 2 + 2$.
- This chapter starts with revision of work done on exponents in Grade 8, where the focus was on **positive exponents**. In the context of positive exponents, the emphasis was on **multiplication**. Learners may need to be reminded that they can only **add the exponents** if the variables that are multiplied have the **same base**.
- **Negative exponents** are introduced in Section 5.2. The meaning given to negative exponents is motivated by emphasising that the properties of exponents are maintained.
- Learners will also learn how to **solve simple exponential equations** such as $2^x = 8$. The big question when solving exponential equations of this nature is: *To what power must the base be raised to make the statement true?* This question is introduced in the context of a table of values where learners simply read the solution from the table. The aim is for them to engage with the idea of what it means to solve an exponential equation before they manipulate equations in order to answer the very same question.
- In Grade 8 **scientific notation** was limited to big numbers where the power of 10 was **positive**. In Grade 9, learners have to learn about representing small numbers in scientific notation where the power of 10 is **negative**. They also do **calculations** using their knowledge of scientific notation.
- The idea of **inverse processes** is important in the work on exponents, for example, raising to a power and finding a root are inverse processes.

5.1 Revision

THE EXPONENTIAL FORM OF A NUMBER

Teaching guidelines

Revise the concepts of **exponential notation** (shorthand for repeated multiplication of the same factor), **base** (the repeated factor) and **exponent** (number of repeated factors), as well as the **laws of exponents** listed in the table on LB page 46.

Misconceptions

Interpreting, for example 2^3, as 2 × 3 instead of 2 × 2 × 2, can be addressed by evaluating expressions like 2 × 2 × 2 and 2 + 2 + 2 and writing these in shorter forms. It is critical that learners understand that the:

- **exponential notation** is a short form to indicate **repeated multiplication** of the same number
- **multiplication notation** is a short form to indicate **repeated addition** of the same number.

Answers

1.	(a) 2^5	(b) s^4	(c) $(-6)^3$
	(d) 2^4s^4	(e) $3^3 \times 7^2$	(f) $500 \times (1{,}02)^2$
2.	(a) 3^4; 9^2	(b) 5^3	(c) 10^3; $2^3 \times 5^3$
	(d) 2^6; 4^3; 8^2	(e) 6^3; $2^3 \times 3^3$	(f) 2^{10}; 4^5

ORDER OF OPERATIONS

Teaching guidelines

Point out that the conventional order of operations (BODMAS) must still be applied to simplify expressions with multiple operations.

Misconceptions

Some learners may think that "exponents distribute", for example: $(2 + 3)^2 = 2^2 + 3^2$. To help learners who make this mistake, ask them to calculate 5^2 as well as $2^2 + 3^2$.

Answers

1. Nathaniel. He calculated $7^2 = 7 \times 7 = 49$ and 49 – 4 = 45.
   Bathabile has mistaken 7^2 for 7 × 2.

CHAPTER 5
Exponents

5.1 Revision

Remember that exponents are a shorthand way of writing repeated multiplication of the same number by itself. For example: $5 \times 5 \times 5 = 5^3$. The **exponent**, which is 3 in this example, stands for however many times the value is being multiplied. The number that is being multiplied, which is 5 in this example, is called the **base**.

If there are mixed operations, then the powers should be calculated before multiplication and division. For example: $5^2 \times 3^2 = 25 \times 9$.

You learnt these laws for working with exponents in previous grades:

Law	Example
$a^m \times a^n = a^{m+n}$	$3^2 \times 3^3 = 3^{2+3} = 3^5$
$a^m \div a^n = a^{m-n}$	$5^4 \div 5^2 = 5^{4-2} = 5^2$
$(a^m)^n = a^{m \times n}$	$(2^3)^2 = 2^{2 \times 3} = 2^6$
$(a \times t)^n = a^n \times t^n$	$(3 \times 4)^2 = 3^2 \times 4^2$
$a^0 = 1$	$32^0 = 1$

THE EXPONENTIAL FORM OF A NUMBER

1. Write the following in exponential notation:

(a) 2 × 2 × 2 × 2 × 2	(b) $s \times s \times s \times s$	(c) (–6) × (–6) × (–6)
(d) $2 \times 2 \times 2 \times 2 \times s \times s \times s \times s$	(e) 3 × 3 × 3 × 7 × 7	(f) 500 × (1,02) × (1,02)

2. Write each of the numbers in exponential notation in some different ways, if possible:

(a) 81	(b) 125	(c) 1 000
(d) 64	(e) 216	(f) 1 024

ORDER OF OPERATIONS

1. Calculate the value of $7^2 - 4$.
   Bathabile did the calculation like this: $7^2 - 4 = 14 - 4 = 10$
   Nathaniel did the calculation differently: $7^2 - 4 = 49 - 4 = 45$
   Which learner did the calculation correctly? Give reasons for your answer.

Answers

2. $5 + 3 \times 2^2 - 10 = 5 + 3 \times 4 - 10$ Calculate: $2^2 = 4$.
$= 5 + 12 - 10$ Order of operations: first multiply $3 \times 4 = 12$.
$= 7$ Do the operations from left to right.

3. There are two operations involved, multiplication and subtraction. A whole number exponent tells us that there are repeated factors.
So first calculate the numbers in exponential form: $2^6 - 6^2 = 64 - 36$.
Then do the subtraction: $64 - 36 = 28$.

4. $(4 + 1)^2 + 8 \times \sqrt[3]{64}$ Add the numbers inside the brackets first and calculate the cube root of 64.
$= (5)^2 + 8 \times 4$ Calculate the square of 5 and the product of 8 and 4.
$= 25 + 32$
$= 57$ Calculate the sum of 25 and 32.

LAWS OF EXPONENTS

Teaching guidelines

Illustrate the following laws for working with exponents:

- The product of two powers with the same base: $a^m \times a^n = a^{m+n}$
- The quotient of two powers with the same base: $a^m \div a^n = a^{m-n}$
- A power of a power: $(a^m)^n = a^{m \times n}$
- A power of a product: $(a \times t)^n = a^n \times t^n$
- A power of zero: $a^0 = 1$, remember $\frac{2^3}{2^3} = \frac{8}{8} = 1$

Answers

1. (a) $2^{2+4} = 2^6$ (b) $3^{4-2} = 3^2$ (c) $1 + 3^4$
(d) $2^{3 \times 2} = 2^6$ (e) $2^2 \times 5^2$ (f) $(2^2)^3 \times 7^3 = 2^6 \times 7^3$
2. See the first table on LB page 47 alongside.
3. Yes, each expression simplifies to y^5, because:
$y \times y^4 = y^{1+4} = y^5$; and $y^2 \times y^3 = y^{2+3} = y^5$
4. See the second table on LB page 47 alongside.
5. (a) Yes, each expression simplifies to y^2, because:
$y^4 \div y^2 = y^{4-2} = y^2$; and $y^3 \div y^1 = y^{3-1} = y^2$
(b) $y^4 \div y^2 = y^2$. So the value of $y^4 \div y^2$ when $y = 15$ is $y^2 = (15)^2 = 225$

2. Calculate: $5 + 3 \times 2^2 - 10$, with explanations.
3. Explain how to calculate $2^6 - 6^2$.
4. Explain how to calculate $(4 + 1)^2 + 8 \times \sqrt[3]{64}$.

LAWS OF EXPONENTS

1. Use the laws of exponents to simplify the following (leave answer in exponential form):
(a) $2^2 \times 2^4$ (b) $3^4 \div 3^2$ (c) $3^0 + 3^4$
(d) $(2^3)^2$ (e) $(2 \times 5)^2$ (f) $(2^2 \times 7)^3$
2. Copy and complete the table. Substitute the given number for y. The first column has been done as an example.

	y	2	3	4	5
(a)	$y \times y^4$	2×2^4 $= 2^{1+4}$ $= 2^5$ $= 32$	3×3^4 $= 3^5$ $= 243$	4×4^4 $= 4^5$ $= 1\,024$	5×5^4 $= 5^5$ $= 3\,125$
(b)	$y^2 \times y^3$	$2^2 \times 2^3$ $= 2^{2+3}$ $= 4 \times 8$ $= 32$	$3^2 \times 3^3$ $= 9 \times 27$ $= 243$	$4^2 \times 4^3$ $= 16 \times 64$ $= 1\,024$	$5^2 \times 5^3$ $= 25 \times 125$ $= 3\,125$
(c)	y^5	$2^5 = 32$	$3^5 = 243$	$4^5 = 1\,024$	$5^5 = 3\,125$

3. Are the expressions $y \times y^4$, $y^2 \times y^3$ and y^5 equivalent? Explain.
4. Copy and complete the table. Substitute the given number for y.

	y	2	3	4	5
(a)	$y^4 \div y^2$	$2^4 \div 2^2$ $= 16 \div 4$ $= 4$	$3^4 \div 3^2$ $= 81 \div 9$ $= 9$	$4^4 \div 4^2$ $= 256 \div 16$ $= 16$	$5^4 \div 5^2$ $= 625 \div 25$ $= 25$
(b)	$y^3 \div y^1$	$2^3 \div 2^1$ $= 8 \div 2$ $= 4$	$3^3 \div 3^1$ $= 27 \div 3$ $= 9$	$4^3 \div 4^1$ $= 64 \div 4$ $= 16$	$5^3 \div 5^1$ $= 125 \div 5$ $= 25$
(c)	y^2	$2^2 = 4$	$3^2 = 9$	$4^2 = 16$	$5^2 = 25$

5. (a) From the table, is $y^4 \div y^2 = y^3 \div y^1 = y^2$? Explain.
(b) Evaluate $y^3 \div y^1$ for $y = 15$.

Misconceptions

Learners may multiply bases and add or multiply exponents, for example:
$3^4 5^2 = 15^6$ or $3^4 3^2 = 9^6$ or 9^8.
Learners who initially learnt about the exponential notation without understanding what it represents may be especially prone to making this error. When the exponential notation is introduced, too much emphasis is often placed on the terminology *base*, *power* and *exponent* while the meaning, *a short way of indicating repeated multiplication*, is not entrenched.

Answers

6. See the table on LB page 48 alongside.
7. (a) No. The two expressions have different values for the same values of x.
   (b) $(2 \times 5)^x$ and $2^x \times 5^x$ are equal, because they have the same values for the same values of x.
8. (a) The solution is incorrect. Wilson should add the exponents instead of multiplying them. The correct solution is $b^3 \times b^8 = b^{3+8} = b^{11}$.
   (b) The solution is incorrect. Wilson should have raised both factors to the power of 2. He raised only x to the power of 2. The correct thinking in order to determine the solution is $(5x)^2 = 5^2 \times x^2 = 25x^2$.
   (c) The solution is correct.

5.2 Integer exponents

NEGATIVE EXPONENTS

Background information

- **Positive exponents** are used to indicate the number of repetitions of the base. Example: $5 \times 5 \times 5 \times 5 = 5^4$.
- **Negative exponents** are used to indicate the number of repetitions of the multiplicative inverse of the base. Example: $\frac{1}{5} \times \frac{1}{5} \times \frac{1}{5} \times \frac{1}{5} \times \frac{1}{5} = 5^{-4}$ or $(\frac{1}{5})^4 = 5^{-4}$.
- Negative exponents have the **same properties** as positive exponents.

Teaching guidelines

Use the quotient of two powers with the same base to explain negative exponents, for example: $2^3 \div 2^4 = \frac{(2 \times 2 \times 2)}{(2 \times 2 \times 2 \times 2)} = \frac{1}{2}$ has the same value as $2^3 \div 2^4 = 2^{3-4} = 2^{-1}$.

6. Copy and complete the following table:

	x	2	3	4	5
(a)	2×5^x	2×5^2 $= 2 \times 25$ $= 50$	2×5^3 $= 2 \times 125$ $= 250$	2×5^4 $= 2 \times 625$ $= 1\,250$	2×5^5 $= 2 \times 3\,125$ $= 6\,250$
(b)	$(2 \times 5)^x$	$(2 \times 5)^2$ $= 10^2$ $= 100$	$(2 \times 5)^3$ $= 10^3$ $= 1\,000$	$(2 \times 5)^4$ $= 10^4$ $= 10\,000$	$(2 \times 5)^5$ $= 10^5$ $= 100\,000$
(c)	$2^x \times 5^x$	$2^2 \times 5^2$ $= 4 \times 25$ $= 100$	$2^3 \times 5^3$ $= 8 \times 125$ $= 1\,000$	$2^4 \times 5^4$ $= 16 \times 625$ $= 10\,000$	$2^5 \times 5^5$ $= 32 \times 3\,125$ $= 100\,000$

7. (a) From the table above, is $2 \times 5^x = (2 \times 5)^x$? Explain.
   (b) Which expressions in question 6 are equivalent? Explain.
8. Below is a calculation that Wilson did as homework. Mark each problem correct or incorrect and explain the mistakes.
   (a) $b^3 \times b^8 = b^{24}$
   (b) $(5x)^2 = 5x^2$
   (c) $(-6a) \times (-6a) \times (-6a) = (-6a)^3$

5.2 Integer exponents

5^4 means $5 \times 5 \times 5 \times 5$. The exponent 4 indicates the number of appearances of the repeated factor.

What may a negative exponent mean, however? For example, what may 5^{-4} mean?

Mathematicians have decided to use negative exponents to indicate repetition of the multiplicative inverse of the base, for example 5^{-4} is used to indicate $\frac{1}{5} \times \frac{1}{5} \times \frac{1}{5} \times \frac{1}{5}$ or $(\frac{1}{5})^4$, and x^{-3} is used to indicate $(\frac{1}{x})^3$, which is $\frac{1}{x} \times \frac{1}{x} \times \frac{1}{x}$.

This decision was not taken blindly – mathematicians were well aware that it makes good sense to use negative exponents in this meaning. One major advantage is that the negative exponents, when used in this meaning, have the same properties as positive exponents, for example:

$2^{-3} \times 2^{-4} = 2^{(-3)+(-4)} = 2^{-7}$ because $2^{-3} \times 2^{-4}$ means $(\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}) \times (\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2})$ which is $(\frac{1}{2})^7$.

$2^{-3} \times 2^4 = 2^{(-3)+4} = 2^1$ because $2^{-3} \times 2^4$ means $(\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}) \times (2 \times 2 \times 2 \times 2)$ which is 2.

Answers

1. (a) $\left(\frac{1}{5}\right)^6$ and 5^{-6} (b) $\left(\frac{1}{3}\right)^4$ and 3^{-4}

2. (a) True, $\frac{1}{10}\times\frac{1}{10}\times\frac{1}{10}=\frac{1}{1\,000}=0{,}001$

   (b) False, $3^{-5}9^2=\frac{3\times3\times3\times3}{3\times3\times3\times3\times3}=\frac{1}{3}$

   (c) False, $5^4\times2^{-6}\times5^{-6}\times2^6=5^{-2}$

   (d) True, $\frac{1}{5}$ is the multiplicative inverse of 5.

3. (a) $10^{-3}\times2^4\times10^4=10\times2^4=160$ (b) $5^4=625$

4. (a) See the table on LB page 49 alongside.

   (b) $10^{-3}=\frac{1}{10^3}=\frac{1}{1\,000}=0{,}001$

5. (a) $\frac{1}{(3^3)^2}=\frac{1}{(27)^2}=\frac{1}{729}$ or $3^{3\times-2}=3^{-6}=\frac{1}{3^6}=\frac{1}{729}$

   (b) $16\times\frac{1}{16}=1$ or $4^{2-2}=4^0=1$

   (c) $\frac{1}{25}\times\frac{1}{5}=\frac{1}{125}$ or $5^{-2-1}=5^{-3}=\frac{1}{5^3}=\frac{1}{125}$

6. (a) $\frac{1}{2^3}=\frac{1}{8}$ (b) $3^2\times\frac{1}{3^2}=1$ (c) $(5)^{-2}=\frac{1}{5^2}=\frac{1}{25}$

   (d) $\frac{1}{3^2}\times\frac{1}{2^3}=\frac{1}{9}\times\frac{1}{8}=\frac{1}{72}$ (e) $\frac{1}{2^3}+\frac{1}{3^3}=\frac{1}{8}+\frac{1}{27}=\frac{35}{216}$ (f) $\frac{1}{10^3}=\frac{1}{1\,000}$

   (g) $8+\frac{1}{2^3}=8+\frac{1}{8}=8\frac{1}{8}$ (h) $3^{-1\times-1}=3^1=3$ (i) $2^{-3\times2}=2^{-6}=\frac{1}{2^6}=\frac{1}{64}$

7. (a) False. $6^{-1}=\frac{1}{6}$

   (b) False. $3x^{-2}=\frac{3}{x^2}$

   (c) True

   (d) True. $(ab)^{-2}=\frac{1}{(ab)^2}=\frac{1}{(a^2b^2)}$

   (e) True. $\left(\frac{2}{3}\right)^{-2}=\frac{2^{-2}}{3^{-2}}=\frac{1}{2^2}\div\frac{1}{3^2}=\frac{1}{4}\times\frac{9}{1}=\frac{9}{4}=\left(\frac{3}{2}\right)^2$

   (f) True. $\left(\frac{1}{3}\right)^{-1}=(3^{-1})^{-1}=3^{-1\times-1}=3^1=3$

NEGATIVE EXPONENTS

1. Express each of the following in the exponential notation in two ways: with positive exponents and with negative exponents:

   (a) $\frac{1}{5}\times\frac{1}{5}\times\frac{1}{5}\times\frac{1}{5}\times\frac{1}{5}\times\frac{1}{5}$ (b) $\frac{1}{3}\times\frac{1}{3}\times\frac{1}{3}\times\frac{1}{3}$

2. In each case, check whether the statement is true or false. If it is false, write a correct statement. If it is true, give reasons why you say so.

   (a) $10^{-3}=0{,}001$ (b) $3^{-5}\times9^2=3$

   (c) $25^2\times10^{-6}\times2^6=5$ (d) $\left(\frac{1}{5}\right)^{-4}=5^4$

3. Calculate each of the following, without using a calculator:

   (a) $10^{-3}\times20^4$ (b) $\left(\frac{1}{5}\right)^{-4}$

4. (a) Use a scientific calculator to determine the decimal values of the given powers.

   Example: To find 3^{-1} on your calculator, use the key sequence: 3 y^x 1 ± =

Power	2^{-1}	5^{-1}	$(-2)^{-1}$	$(0{,}3)^{-1}$	0^{-1}	10^{-1}	10^{-2}
Decimal value	0,5	0,2	–0,5	$3{,}\dot{3}$	undef.	0,1	0,01

   (b) Explain the meaning of 10^{-3}.

5. Determine the value of each of the following in two ways:

   A. By using the definition of powers (for example, $5^2\times5^0=25\times1=25$).
   B. By using the properties of exponents (for example, $5^2\times5^0=5^{2+0}=5^2=25$).

   (a) $(3^3)^{-2}$ (b) $4^2\times4^{-2}$ (c) $5^{-2}\times5^{-1}$

6. Calculate the value of each of the following. Express your answers as common fractions.

   (a) 2^{-3} (b) $3^2\times3^{-2}$ (c) $(2+3)^{-2}$
   (d) $3^{-2}\times2^{-3}$ (e) $2^{-3}+3^{-3}$ (f) 10^{-3}
   (g) 2^3+2^{-3} (h) $(3^{-1})^{-1}$ (i) $(2^{-3})^2$

7. Which of the following are true? Correct any false statement.

   (a) $6^{-1}=-6$ (b) $3x^{-2}=\frac{1}{3x^2}$ (c) $3^{-1}x^{-2}=\frac{1}{3x^2}$

   (d) $(ab)^{-2}=\frac{1}{a^2b^2}$ (e) $\left(\frac{2}{3}\right)^{-2}=\left(\frac{3}{2}\right)^2$ (f) $\left(\frac{1}{3}\right)^{-1}=3$

5.3 Solving simple exponential equations

SOLVING EXPONENTIAL EQUATIONS

Background information

- An **exponential equation** is an equation in which the variable is in the exponent. Example: $4^x = 64$.
- Exponential equations are solved by rewriting both sides of the equation as **powers with the same base**. Example: If $a^m = a^n$ then $m = n$.
- To solve an exponential equation, ask the following question: *To what power must the base be raised to make the statement true?*

Teaching guidelines

- Discuss the background information provided above.
- Use the examples on LB page 50 alongside to explain how to solve exponential equations.

Notes on question 1

- The question "*To what power must the base be raised to make the statement true?*" is introduced in the context of a table of values where learners simply read the solution from the table.
- The aim is for learners to engage with the idea of what it means to solve an exponential equation before they proceed to manipulate equations in order to answer the very same question.

Answers

1. (a) $x = 5$ (b) $x = 4$ (c) $x = 5$

(d) $x = 3$ (e) $x = 4$ (f) $x = 2$

(h) $5^{x+1} = 5^2$
$x + 1 = 2$
$x = 1$

(i) $3^{x+2} = 3^3$
$x + 2 = 3$
$x = 1$

(j) $2^{x-1} = 2^3$
$x - 1 = 3$
$x = 4$

5.3 Solving simple exponential equations

An exponential equation is an equation in which the variable is in the exponent. So, when you solve exponential equations, you are solving questions of the form; *"To what power must the base be raised for the statement to be true?"*

To solve this kind of equation, remember that:

If $a^m = a^n$, then $m = n$.

In other words, if the base is the same on either side of the equation, then the exponents are the same.

Example:

$3^x = 243$

$3^x = 3^5$ (rewrite using the same base)

$x = 5$ (since the bases are the same, we equate the exponents)

Some exponential equations are slightly more complex:

Examples: $3^{x+3} = 243$

$3^{x+3} = 3^5$ (rewrite using the same base)

$x + 3 = 5$ (equate the exponents)

$x = 2$

Check: LHS $3^{2+3} = 3^5 = 243$

$3^{x+3} = 1$ (remember $1 = 3^0$)

$3^{x+3} = 3^0$ (rewrite using the same base)

$x + 3 = 0$ (equate the exponents)

$x = -3$

Remember that the exponent can also be negative. However, you follow the same method to solve these kinds of equations.

Example: $2^x = \frac{1}{32}$

$2^x = 2^{-5}$ (rewrite using the same base)

$x = -5$ (equate the exponents)

SOLVING EXPONENTIAL EQUATIONS

1. Use the following table to answer questions that follow:

x	2	3	4	5
2^x	4	8	16	32
3^x	9	27	81	243
5^x	25	125	625	3 125

Find the value of x:

(a) $2^x = 32$ (b) $3^x = 81$ (c) $5^x = 3\,125$

(d) $2^x = 8$ (e) $5^x = 625$ (f) $3^x = 9$

(g) $5^{x+1} = 25$ (h) $3^{x+2} = 27$ (i) $2^{x-1} = 8$

Answers

2. (a) $4^x = 4^{-3}$
$x = -3$

(b) $6^{2x} = 6^4$
$2x = 4$
$x = 2$

(c) $2^{x-1} = 2^{-3}$
$x - 1 = -3$
$x = -2$

(d) $3^{x+2} = 3^{-6}$
$x + 2 = -6$
$x = -8$

(e) $5^{x+1} = 5^6$
$x + 1 = 6$
$x = 5$

(f) $2^{x+3} = 2^{-2}$
$x + 3 = -2$
$x = -5$

(g) $4^{x+3} = 4^{-4}$
$x + 3 = -4$
$x = -7$

(h) $3^{2-x} = 3^4$
$2 - x = 4$
$-x = 2$
$x = -2$

(i) $5^{3x} = 5^{-3}$
$3x = -3$
$x = -1$

5.4 Scientific notation

WRITING VERY SMALL AND VERY LARGE NUMBERS

Background information

- In **scientific notation**, a number is expressed as a product of:
  - a number from 1 to 9
  - a power of 10 with an exponent equal to an integer.
- **Numbers that are too big or too small** to be written clearly in decimal form are written in scientific notation, for example:
  - The average distance of 150 000 000 km from the sun to the Earth is written in scientific notation as $1{,}5 \times 10^8$ km.
  - The diameter of 0,000000053 mm of a hydrogen atom is written in scientific notation as $5{,}3 \times 10^{-8}$ mm.

Teaching guidelines

- To write the decimal number 324 000 000 as $3{,}24 \times 10^8$ in scientific notation, the decimal comma is moved eight places to the left. Since the number is divided by 100 000 000 or 10^8 it means that 3,24 must be multiplied by 10^8 to keep the same value.
- To write the decimal number 0,00000065 as $6{,}5 \times 10^{-7}$ in scientific notation, the decimal comma is moved seven places to the right. Since the number is multiplied by 10 000 000 or 10^7 it means that 6,5 must be divided by 10^7, i.e. multiplied by 10^{-7}, to keep the same value.

2. Solve these exponential equations. You may use your calculator if necessary.

(a) $4^x = \frac{1}{64}$ (b) $6^{2x} = 1\,296$ (c) $2^{x-1} = \frac{1}{8}$

(d) $3^{x+2} = \frac{1}{729}$ (e) $5^{x+1} = 15\,625$ (f) $2^{x+3} = \frac{1}{4}$

(g) $4^{x+3} = \frac{1}{256}$ (h) $3^{2-x} = 81$ (i) $5^{3x} = \frac{1}{125}$

5.4 Scientific notation

Scientific notation is a way of writing numbers that are too big or too small to be written clearly in decimal form. The diameter of a hydrogen atom, for example, is a very small number. It is 0,000000053 mm. The distance from the sun to the earth is, on average, 150 000 000 km.

In scientific notation, the diameter of the hydrogen molecule is written as $5{,}3 \times 10^{-8}$ and the distance from the sun to the earth is written as $1{,}5 \times 10^8$. It is easier to compare and to calculate numbers like these, as it is very cumbersome to count the zeros when you work with these numbers.

Look at more examples below:

Decimal notation	Scientific notation
6 130 000	$6{,}13 \times 10^6$
0,00001234	$1{,}234 \times 10^{-5}$

A number written in scientific notation is written as the product of two numbers, in the form $\pm a \times 10^n$. Here, *a* is a decimal number between 1 and 10, and *n* is an integer.

Any number can be written in scientific notation, for example:

$40 = 4{,}0 \times 10$

$2 = 2 \times 10^0$

The decimal number 324 000 000 is written as $3{,}24 \times 10^8$ in scientific notation, because the decimal comma is moved eight places to the left to form 3,24.

The decimal number 0,00000065 written in scientific notation is $6{,}5 \times 10^{-7}$, because the decimal point is moved seven places to the right to form the number 6,5.

Answers

1. (a) $1{,}3456 \times 10^{2}$ (b) $5{,}678 \times 10^{-7}$
   (c) $8{,}765 \times 10^{8}$ (d) $3{,}21 \times 10^{-11}$
   (e) $6{,}789 \times 10^{-3}$ (f) $8{,}91 \times 10^{13}$
   (g) 1×10^{-3} (h) 1×10^{2}
2. (a) 1 234 000 (b) 0,5
   (c) 450 000 (d) 0,00000000006543
3. Because 34 is a number greater than 10. In scientific notation it would be $3{,}4 \times 10^{4}$.
4. (a) No. $9{,}03 \times 10^{-4}$ (b) No. 1×10^{4} (c) Yes.
   (d) Yes. (e) No. 1×10^{1} (f) No. 6×10^{7}

CALCULATIONS USING SCIENTIFIC NOTATION

Teaching guidelines

Use the examples on LB page 52 alongside to explain how to use scientific notation to:

- multiply two very large numbers
- add numbers in scientific notation if their powers of 10 differ.

Answers

1. (a) $1{,}35 \times 10^{5} \times 2{,}46 \times 10^{8}$
   $= 3{,}321 \times 10^{13}$
   (b) $9{,}87654 \times 10^{5} \times 1{,}23456 \times 10^{5}$
   $= 12{,}1931812224 \times 10^{10}$
   $= 1{,}21931812224 \times 10^{11}$
   (c) $6{,}5 \times 10^{-5} \times 2{,}16 \times 10^{-4}$
   $= 6{,}5 \times 2{,}16 \times 10^{-5} \times 10^{-4}$
   $= 14{,}04 \times 10^{-9}$
   $= 1{,}404 \times 10^{-8}$
   (d) $6{,}39 \times 10^{-7} \times 5{,}87 \times 10^{-5}$
   $= 6{,}39 \times 5{,}87 \times 10^{-7} \times 10^{-5}$
   $= 37{,}5093 \times 10^{-12}$
   $= 3{,}75093 \times 10^{-11}$
2. (a) $7{,}16 \times 10^{5} + 0{,}023 \times 10^{5}$
   $= 7{,}183 \times 10^{5}$
   (b) $0{,}23 \times 10^{-3} + 6{,}5 \times 10^{-3}$
   $= 6{,}73 \times 10^{-3}$
   (c) $4{,}31 \times 10^{7} + 0{,}157 \times 10^{7}$
   $= 4{,}467 \times 10^{7}$
   (d) $6{,}13 \times 10^{-10} + 389 \times 10^{-10}$
   $= 395{,}13 \times 10^{-10}$
   $= 3{,}9513 \times 10^{-8}$

WRITING VERY SMALL AND VERY LARGE NUMBERS

1. Express the following numbers in scientific notation:
   (a) 134,56 (b) 0,0000005678
   (c) 876 500 000 (d) 0,0000000000321
   (e) 0,006789 (f) 89 100 000 000 000
   (g) 0,001 (h) 100
2. Express the following numbers in ordinary decimal notation:
   (a) $1{,}234 \times 10^{6}$ (b) 5×10^{-1}
   (c) $4{,}5 \times 10^{5}$ (d) $6{,}543 \times 10^{-11}$
3. Why do we say that 34×10^{3} is not written in scientific notation? Rewrite it in scientific notation.
4. Is each of these numbers written in scientific notation? If not, rewrite it so that it is in scientific notation.
   (a) $90{,}3 \times 10^{-5}$ (b) 100×10^{2} (c) $1{,}36 \times 10^{5}$
   (d) $2{,}01 \times 10^{-2}$ (e) $0{,}01 \times 10^{3}$ (f) $0{,}6 \times 10^{8}$

CALCULATIONS USING SCIENTIFIC NOTATION

Example: $123\,000 \times 4\,560\,000$
$= 1{,}23 \times 10^{5} \times 4{,}56 \times 10^{6}$ (write in scientific notation)
$= 1{,}23 \times 4{,}56 \times 10^{5} \times 10^{6}$ (multiplication is commutative)
$= 5{,}6088 \times 10^{11}$ (Use a calculator to multiply the decimals, but add the powers mentally.)

1. Use scientific notation to calculate each of the following. Give the answer in scientific notation.
   (a) 135 000 × 246 000 000 (b) 987 654 × 123 456
   (c) 0,000065 × 0,000216 (d) 0,000000639 × 0,0000587

Example: $5 \times 10^{3} + 4 \times 10^{4}$
$= 0{,}5 \times 10^{4} + 4 \times 10^{4}$ (Form like terms)
$= 4{,}5 \times 10^{4}$ (Combine like terms)

2. Calculate the following. Leave the answer in scientific notation.
   (a) $7{,}16 \times 10^{5} + 2{,}3 \times 10^{3}$ (b) $2{,}3 \times 10^{-4} + 6{,}5 \times 10^{-3}$
   (c) $4{,}31 \times 10^{7} + 1{,}57 \times 10^{6}$ (d) $6{,}13 \times 10^{-10} + 3{,}89 \times 10^{-8}$

52 MATHEMATICS GRADE 9: TERM 1

Learner Book Overview		
Sections in this chapter	**Content**	**Pages in Learner Book**
6.1 Geometric patterns	Investigating and extending geometric patterns that generate sequences where there is a constant difference between consecutive terms; working with geometric patterns that generate sequences that do not differ by a constant value or a constant ratio	Pages 53 to 55
6.2 More patterns	Drawing, investigating and extending patterns that have a constant difference between consecutive terms and sequences that do not have a constant value or a constant ratio	Pages 55 to 57
6.3 Different kinds of patterns in sequences	Sequences formed by adding (or subtracting) repeatedly (constant difference); multiplying or dividing repeatedly (constant ratio); neither a constant difference nor a constant ratio	Pages 57 to 59
6.4 Formulae for sequences	Describing relationships between the value of a term and the term number; making a rule	Pages 59 to 61

CAPS time allocation	4,5 hours
CAPS content specification	Pages 126 to 129

Mathematical background

Patterns can be given in geometric and numeric form. There are two ways of identifying the pattern or relationship that describes a sequence of numbers.

1. Given a sequence of numbers, learners have to identify a pattern or relationship between consecutive terms. Such patterns could be generated when:
   - a constant value is added (or subtracted) from each successive term, for example: 3; 6; 9; 12; ... (add 3; a constant difference between terms)
   - successive terms are multiplied or divided by the same number, for example: 1; 3; 9; 27; ... (multiply by 3; a constant ratio between terms)
   - the amount added increases from term to term, for example: 1; 3; 6; 10; 15; ... (start by adding 2 and add one more each time), or 1; 4; 9; 16; 25; ... (add increasing odd numbers). There is not a constant difference or a constant ratio between consecutive terms.
2. Given a sequence of numbers, learners have to identify a pattern or relationship between the term and its position in the sequence. We can describe a sequence by the relationship between a term and its position, n, in the sequence:
   - for example, 3; 6; 9; 12; ... constant difference is 3; which is 3×1; 3×2; 3×3; 3×4; ... and therefor has the rule: $3n$
   - for example, 1; 3; 9; 27; ... constant ratio is 3; which is 3^0; 3^1; 3^2; 3^3; ... and therefor has the rule:3^{n-1}.

   When there is no constant difference or ratio, writing a rule is not as straightforward. Some rules are easy such as 1; 4; 9; 16; 25; ... rule: n^2 and 2; 5; 10; 17; ... = 1 + 1; 4 + 1; 9 + 1; 16 + 1; ... rule $n^2 + 1$, while the description of other rules is more complicated.

   Working with sequences in this way focuses on the functional relationship between a term and its value. This can be used to find values of the independent variable (the term number) that corresponds with a given dependent variable of function value. For example, we have to find what term number 626 will be in the sequence 2; 5; 10; 17; .... We know the term is obtained by squaring the position number of the term and adding 1, then the position number can be obtained by subtracting 1 and then finding the square root of that answer. Hence, 626 will be the twenty-fifth term in the sequence since: $626 - 1 = 625$ and $\sqrt{625} = 25$. This amounts to solving the equation $n^2 + 1 = 626$.

6.1 Geometric patterns

INVESTIGATING AND EXTENDING

Teaching guidelines

Learners identify the pattern that exists between the consecutive terms of a sequence.

The sequences of numbers generated by the geometric patterns can be continued by adding a constant value to each term to get the next term, so there is a constant difference between the terms, except in questions 4 and 7, where there is a different pattern to the sequences.

Notes on the questions

Focusing on the functional relationship between the term **position** and the term **value** is useful for analysing and extending sequences that do not have constant differences or ratios between terms. The CAPS (page 126) specifies sequences "not limited to sequences involving a constant difference or ratio". Questions 4 and 7 in this section deal with sequences like these (the white tiles in question 4, and the black tiles in question 7).

Answers

1. (a) There are 1, 2, 3 and 4 yellow tiles in arrangements 1, 2, 3 and 4 respectively.
   (b) There are 8, 10, 12 and 14 blue tiles in arrangements 1, 2, 3 and 4 respectively.
   (c) Five yellow tiles and 16 blue tiles
   (d) See the answer on LB page 53 alongside.
   (e) 58 blue tiles
   (f) 206
   (g) *Possible answers*:

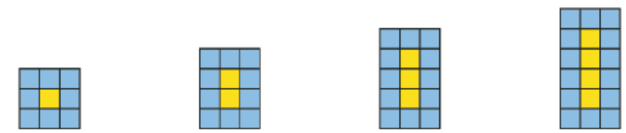
   I noticed that the top and bottom rows in each tile are identical and remain constant. There are always three blue tiles in the top row and three blue tiles in the bottom row. On the left and right of each yellow tile there is always one blue tile. So, if there are 100 yellow tiles there will be 100 blue tiles on the left and 100 blue tiles on the right, and three blue tiles at the top, and three blue tiles at the bottom, that is 100 + 100 + 3 + 3 = 206 blue tiles.

   Or: blue tiles = 2 × yellow tiles + 6 = 2 × 100 + 6 = 206 tiles

CHAPTER 6
Patterns

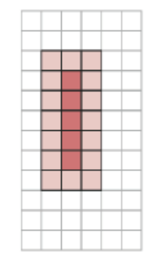
6.1 Geometric patterns

INVESTIGATING AND EXTENDING



Arrangement 1 Arrangement 2 Arrangement 3 Arrangement 4

1. Blue and yellow square tiles are combined to form the above arrangements.
   (a) How many yellow tiles are there in each arrangement?
   (b) How many blue tiles are there in each arrangement?
   (c) If more arrangements are made in the same way, how many blue tiles and how many yellow tiles will there be in arrangement 5? Check your answer by drawing the arrangement onto grid paper.

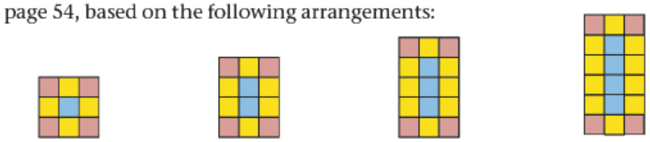


   (d) Copy and complete the following table:

Number of yellow tiles	1	2	3	4	5	8
Number of blue tiles	8	10	12	14	16	22

   (e) How many blue tiles will there be in a similar arrangement with 26 yellow tiles?
   (f) How many blue tiles will there be in a similar arrangement with 100 yellow tiles?
   (g) Describe how you thought to produce your answer for (f)?

2. (a) In these arrangements there are red tiles too. Copy and complete the table on page 54, based on the following arrangements:



Arrangement 1 Arrangement 2 Arrangement 3 Arrangement 4

Answers

2. (a) See completed table on LB page 54 alongside.
   (b) There are four red tiles in each arrangement.
   (c) It varies, depending on how many blue tiles there are. There is one above and one below and then one on either side of every blue tile, so: 2 × number of blue tiles + 2.
3. A variable

Teaching guidelines

The pattern that exists between the sequence of numbers that describes the white squares in question 4 (continued in question 5) does not involve a constant difference. Learners have to recognise that the pattern between the sequence of grey squares in question 4 increases with a constant value of four added to consecutive terms, while the pattern that describes the sequence of white squares is that increasing odd numbers are added to get consecutive terms, or that it consists of the squares of consecutive whole numbers.

The same argument applies to question 7 where the pattern that describes the sequence of the black tiles involves the squares of whole numbers added to four.

Answers

4. (a) See the answers on LB page 54 alongside.
   (b) The number of black squares
   (c) The number of grey squares and the number of white squares are both variables.
5. (a) See the answers on LB page 54 alongside.
   (b) 60, by counting in fours, for example, from 40 (arrangement number 10): 40; 44; 48; 52; 56; 60. Or, by calculation: 15 × 4 = 60.
   (c) 4. It is constant.

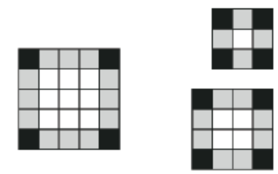
Number of blue tiles	1	2	3	4	5	6	7
Number of yellow tiles	4	6	8	10	12	14	16
Number of red tiles	4	4	4	4	4	4	4

(b) How many red tiles are there in each arrangement?
(c) How many yellow tiles are there in each arrangement?

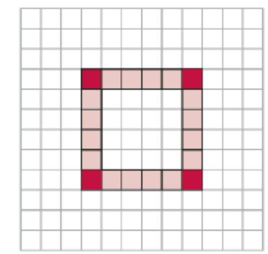
The number of red tiles in arrangements like those in question 2, is **constant.** It is always four, no matter how many blue and yellow tiles there are.

The number of blue tiles is different for different arrangements. We can say the number of blue tiles **varies**. We can also say the number of blue tiles is a **variable**.

3. Is the number of yellow tiles in the above arrangements a constant or is it a variable?



4. Look at the arrangements on the right. They consist of black squares, grey squares and white squares.
   (a) Draw another arrangement of the same kind, but with a different length, on grid paper.
   (b) Describe what is constant in these arrangements.
   (c) What are the variables in these arrangements?



The smallest arrangement above may be called arrangement 1, the next bigger one may be called arrangement 2, and so on.

5. (a) Copy and complete the table for arrangements like those in question 4.

Arrangement number	1	2	3	4	5	6	7	10	20
Number of black squares	4	4	4	4	4	4	4	4	4
Number of grey squares	4	8	12	16	20	24	28	40	80
Number of white squares	1	4	9	16	25	36	49	100	400

(b) How many grey squares do you think there will be in arrangement 15? Explain your answer.
(c) How many black squares do you think there will be in arrangement 15? Explain your answer.

Answers

5. (d) $15^2 = 225$ Or, by adding increasing odd number steps, for example: $100 + 21 = 121$, $121 + 23 = 144$, etc. (Note: the last answer is quite sophisticated.)
6. The grey squares: ...; 28; 32; 36; 40; 44. The white squares: ...; 64; 81; 100; 121; 144
7. (a) See the answers on LB page 55. The number of black tiles on the corners remains the same: 4. The number of black tiles in the centre, increase by one row and one column in consecutive terms, starting at one tile. The sequence is: 1 + 4; 4 + 4; 9 + 4; 16 + 4; 25 + 4; ...
   (b) 20; it is the fourth arrangement, so there are $4 \times 4 + 4 = 20$
   (c) 29; 40; 53; 68

DO SOMETHING MORE

Teaching guidelines

In this instance learners have to solve equations. The number of grey squares increase by a constant value of 4, starting at 4, so if there are 20 grey squares it means we are looking at the arrangement number 5 ($20 \div 4$).

The number of white squares is given by adding increasing odd numbers, so the sequence would be 1: $1 + 3 = 4$; $1 + 3 + 5 = 9$; $1 + 3 + 5 + 7 = 16$; $1 + 3 + 5 + 7 + 9 = 25$; or 5^2. The number of white squares is 256, so it is the sixteenth term as the square root of 256 = 16. The number of grey squares is 4 times 16 = 64. The same procedure is followed to find the other answers.

6.2 More patterns

DRAWING AND INVESTIGATING

Teaching guidelines

The questions in this section deal with two kinds of patterns: adding a constant value to consecutive terms and sequences that do not involve constant differences, but increasing differences between consecutive terms, for example question 2.

If the successive terms of the sequence generated by the arrangements of dots in question 2 are added, it leads to another sequence, the squares of whole numbers from 2.

Answers

1. (a) See the answers on LB page 55 alongside.
   (b) Yes, the number of black squares is constant. It is 1 in all the arrangements.
   (c) Yes, the number of grey squares is a variable: 4, 8, 12, 16, 20.

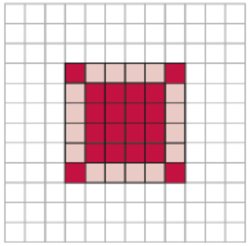
(d) How many white squares do you think there will be in arrangement 15? Explain your answer.

The numbers of grey squares in the different arrangements in question 4 form a pattern: 4; 8; 12; 16; 20; 24; . . . , and so on.

The numbers of white squares in the different arrangements also form a pattern: 1; 4; 9; 16; 25; 36; 49; . . . , and so on.

6. What are the next five numbers in each of the above patterns?

7. (a) On grid paper, draw the next arrangement that follows the same pattern:



(b) How many black tiles are there in the arrangement you have drawn?
(c) How many black tiles will there be in each of the next four arrangements?

DO SOMETHING MORE

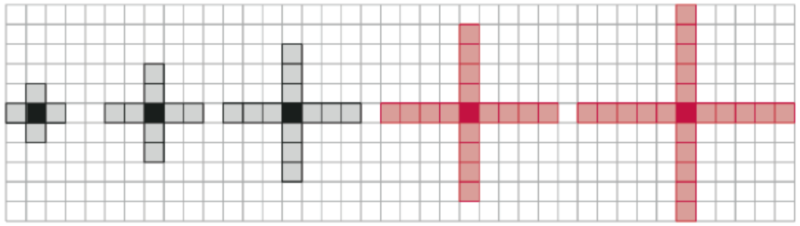
Consider the arrangements in question 4 on page 54 again. If there are 20 grey tiles in such an arrangement, how many white tiles are there? Copy and complete the table, entering your answer in the table.

Number of grey squares	20	36	52	64	60	100
Number of white squares	25	81	169	256	225	625

6.2 More patterns

DRAWING AND INVESTIGATING

1. (a) On grid paper, make two more arrangements of black and grey squares so that a pattern is formed.



CHAPTER 6: PATTERNS 55

Answers

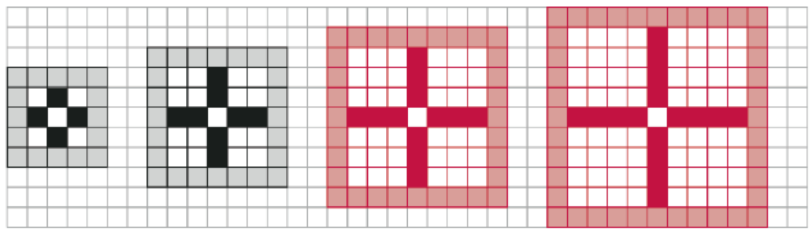
2. (a) See the answers on LB page 56 alongside.
   (b) 21 in the sixth arrangement, 28 in the seventh.
   The pattern is 1, 3, 6, 10, 15, 21, 28 ...
   Or: The number added increases by 1 each time.
   (c) 4
   (d) 9
   (e) 16
   (f) 25
   (g) 4, 9, 16, 25. These are the squares of the whole numbers from 2.
3. (a) See the answers on LB page 56 alongside.
   (b) The numbers of white squares, grey squares and black squares.
   (c) 12
   (d) 20 black squares, because the pattern is 4, 8, 12, 16, 20; add 4 to consecutive terms, starting with 4.
   101 white squares, because the pattern is 5, 17, 37, 65, 101 which is:
   4 + 1; 16 + 1; 36 + 1; 64 + 1, and this can be written as
   $1^2 \times 4 + 1$; $2^2 \times 4 + 1$; $3^2 \times 4 + 1$; $4^2 \times 4 + 1$
   48 grey squares, because the pattern is 16, 24, 32, 40, 48; add 8 to consecutive terms starting with 16.
4. (a) Learners' own work.
   (b) Learners' own work.

(b) Is there a constant in your pattern? If yes, what is its value?
(c) Is there a variable in your pattern? If yes, give the values of the variable.

2. (a) Make three more arrangements with dots to form the sequence 1; 3; 6; 10; 15 . . .

(b) How many dots will there be in the sixth and seventh arrangements? Explain how you got your answer.
(c) How many dots are there in arrangements 1 and 2 together?
(d) How many dots are there in arrangements 2 and 3 together?
(e) How many dots are there in arrangements 3 and 4 together?
(f) How many dots are there in arrangements 4 and 5 together?
(g) Describe the pattern in your answers for (c), (d), (e) and (f).

3. (a) On grid paper, draw two more arrangements to make a pattern.



(b) What are the variables in your pattern?
(c) The number of black squares is a variable in these arrangements. The value of this variable is four in the first arrangement and eight in the second arrangement. What is the value of this variable in the third arrangement?
(d) What are the values of each of the variables in the fifth arrangement in your pattern? Explain your answers.

4. (a) Now, on grid paper, make a pattern of your own.

6.3 Different kinds of patterns in sequences

DO THE SAME THING REPEATEDLY

Teaching guidelines

In the previous sections the pattern to continue a sequence was found by adding or subtracting a constant value to successive terms. In other words, there is a constant difference between successive terms. For example: 3, 7, 11, 15, ... Starting with 3, 4 is added to each successive term.

A pattern can also be identified between terms in a sequence when consecutive terms are multiplied or divided by the same value. In other words, there is a constant ratio between successive terms. For example: 3; 9; 27; 81; ... Starting with 3, each successive term is multiplied by 3. We see this when we find the ratio between successive terms: $\frac{9}{3} = 3$; $\frac{27}{9} = 3$.

Some sequences are formed in other ways, for example 1, 4, 8, 13, 19, ... where the pattern is neither a constant difference nor a constant ratio between successive terms, but adding more to each successive term to generate the sequence.

Learners generate sequences by repeating the same action to each successive term.

Note that it is possible for two different sequences to have the same first three terms, i.e. the sequence 5, 10, 20, 40, ... and another sequence 5, 10, 20, 35, ... So, if only the first three terms of a sequence are given, it is important to note that it is possible that there could be more than one interpretation of this sequence.

Answers

1. (a) See the completed sequences on LB page 57 alongside.
   (b) Sequence A is formed by adding 4 each time.
   Sequence B is formed by multiplying by 2 each time.
   Sequence C is formed by adding consecutive odd numbers: 5, then 7, then 9, ...
2. 5 13 21 29 37 45 53 61 69 77 85 93 101

(b) Copy this table and use it to describe the variables in your pattern, and their values:

Arrangement number	1	2	3	4	5	6

6.3 Different kinds of patterns in sequences

DO THE SAME THING REPEATEDLY

1. (a) Write the next three numbers in each of the sequences below.

   Sequence A: 5 9 13 17 21 25 29 33

   Sequence B: 5 10 20 40 80 160 320 640

   Sequence C: 5 10 17 26 37 50 65 82

   (b) Describe the differences in the ways in which the three sequences are formed.

2. You will now make a sequence with the first term 5. Write 5 on the left on the line below. Then add 8 to the first term (5) to form the second term of your sequence. Write the second term next to the first term (5) in the line below. Now add 8 to the second term to form the third term. Continue like this to form ten more terms.

The numbers in a sequence are also called the **terms** of the sequence.

A sequence can be formed by repeatedly adding or subtracting the same number. In this case the **difference** between consecutive terms in a sequence is **constant**.

A sequence can be formed by repeatedly multiplying or dividing. In this case the **ratio** between consecutive terms is **constant**.

A sequence can also be formed in such a way that neither the difference nor the ratio between consecutive terms is constant.

To write more terms of sequence A in question 1(a), you **added** 4 **repeatedly**.

To write more terms of sequence B in question 1(a), you **multiplied** by 2 **repeatedly**.

To write more terms of sequence C in question 1(a) you did not add the same number each time, nor did you multiply by the same number.

3. Write the next three terms of each sequence. In each case also describe what the pattern is, for example "there is a constant difference of –5 between consecutive terms".

Answers

3. (a) See the completed sequences on LB page 58 alongside.
There is a constant difference of –8 between consecutive terms.
(b) See the completed sequences on LB page 58 alongside.
The terms differ by the odd numbers starting from 3: 3; 5; 7, etc.
The first term is 1 squared, the second term is 2 squared, etc.
(c) See the completed sequences on LB page 58 alongside.
The terms differ by the sequence 6; 10; 14; 18, etc.
The first term is 2 × (one squared), the second term is 2 × (2 squared), etc.
(d) See the completed sequences on LB page 58 alongside.
The terms differ by the odd numbers 3; 5; 7, etc.
The first term is 1 squared plus 2, the second term is 2 squared plus 2, etc.
(e) See the completed sequences on LB page 58 alongside.
There is a constant ratio of $\frac{1}{2}$ between consecutive terms.
Each term is halved to form the next term.
(f) See the completed sequences on LB page 58 alongside.
The terms differ by consecutive natural numbers 1; 2; 3, etc.

4. (a) 1 2 4 8 16 32 64 128
(b) 256 224 192 160 128 96 64 32
(c) 256 128 64 32 16 8 4 2

5. (a) to (f) See the answers on LB pages 58 alongside and LB page 59 on following page.

(a) 16; 8; 0; –8; –16; –24; –32
(b) 1; 4; 9; 16; 25; 36; 49
(c) 2; 8; 18; 32; 50; 72; 98
(d) 3; 6; 11; 18; 27; 38; 51
(e) 640; 320; 160; 80; 40; 20
(f) 1; 2; 4; 7; 11; 16; 22; 29

4. In each case, follow the instruction to make a sequence with eight terms.
(a) Start with 1 and multiply by 2 repeatedly.
(b) Start with 256 and subtract 32 repeatedly.
(c) Start with 256 and divide by 2 repeatedly.

The sequence that you made in question 2 can be represented with a table like the one shown below:

Term number	1	2	3	4	5	6	7	8	9	10
Term value	5	13	21	29	37	45	53	61	69	77

5. In each case make a sequence by following the instructions. Copy the tables and write the term numbers and the term values in the tables.
(a) Term 1 = 10. Add 15 repeatedly.

Term number	1	2	3	4	5	6	7	8
Term value	10	25	40	55	70	85	100	115

(b) Term 1 = 10. Term value = 15 × term number – 5.

Term number	1	2	3	4	5	6	7	8
Term value	10	25	40	55	70	85	100	115

(c) Term 1 = 10. Multiply by 2 repeatedly.

Term number	1	2	3	4	5	6
Term value	10	20	40	80	160	320

(d) Term 1 = 20. Term value = $10 \times 2^{\text{term number}}$

Term number	1	2	3	4	5	6
Term value	20	40	80	160	320	640

Answers

6. One way is to work from one term to the next. The other way is to do a calculation with the term number (the position of the term in the sequence).

6.4 Formulae for sequences

MAKE TWO FORMULAE FOR THE SAME SEQUENCE

Teaching guidelines

In the previous sections the pattern to continue a sequence was found by adding a constant value to successive terms or by multiplying by a constant value. Allow learners to distinguish between these two different ways by referring them to question 5 on the previous page of the LB.

A pattern can also be identified between a term and its position in a sequence. For example, in the sequence 1, 4, 9, 16, 25, ... the nth term will be n^2 as the pattern that describes the sequence is the position of the term squared.

Another example is 1, 4, 7, 10, ... where the pattern can be identified as: multiply the position of the term by 3 and subtract 2, or $3n - 2$.

Learners will learn to find the relationship between the term number and the term value in terms of a calculation or rule that describes the pattern of the sequence.

Answers

1. (a) Sample answer: 3, 8, 13, 18, 23, 28, 33, 38, 43, 48, ...
   (b) See the answers on LB page 59 alongside.
   (c) They both have gaps of five between consecutive terms.

(e) Term 1 = 10. Term value = $10 \times 2^{\text{term number} - 1}$

Term number	1	2	3	4	5	6
Term value	10	20	40	80	160	320

(f) Term 4 = 30. Add 5 repeatedly.

Term number	1	2	3	4	5	6	7	8
Term value	15	20	25	30	35	40	45	50

6. Instructions for forming a sequence are given in two different ways in question 5. How would you describe the two different ways for giving instructions to form a sequence?

6.4 Formulae for sequences

The formula for a number sequence can be written in two different ways:

- A description of the **relationship between consecutive terms**: In other words, the calculations that you do to a term to produce the next term, as in questions 5(a), (c) and (f) on the previous page. The first (or another) term must be given. This kind of formula has two parts: the first term and the relationship between terms.
- A description of the **relationship between the value of the term and its position in the sequence**: This relationship describes the calculations that can be done **on the term number** to produce the **term value**, as in question 5(b), (d) and (e) on the previous page.

MAKE TWO FORMULAE FOR THE SAME SEQUENCE

1. Choose any whole number smaller than 10 as the first term of a sequence.
   (a) Copy the table. Use your chosen first term to form a sequence by adding 5 repeatedly.
   (b) Multiply each term number below by 5 to form a sequence:

Term number	1	2	3	4	5	6	7	8
Term value	5	10	15	20	25	30	35	40

   (c) What is similar about the two sequences you have formed?

Answers

1. (d) See the answers on LB page 60 alongside.
   (e) For the sample answer in (a): subtract 2 from each term.
   (f) See the answers on LB page 60 alongside.
2. (a) Multiply the term number by 3 and add 5 to get the term value.
   Term value = (3 × term number) + 5
   (b) Multiply the term number by 3 and add 9 to get the term value.
   Term value = (3 × term number) + 9
   (c) Multiply the term number by 3 and subtract 1 to get the term value.
   Term value = (3 × term number) – 1
3. (a) Start with 15, then add 10 to get the next term.
   (b) Start with 2, then add 5 to get to the next term.

(d) Now fill in your own sequence in the same table:

Term number	1	2	3	4	5	6	7	8
Term value in (b)	5	10	15	20	25	30	35	40
Term value of your own sequence in (a)	3	8	13	18	23	28	33	38

(e) What must you add to or subtract from each term value in (b) to get the same sequence as the one you made in (a)?

(f) Copy and fill in the following to write a formula for each sequence:
For the sequence in (b): Term value = ...5 ×... (term number)
For the sequence in (a): Term value = ...5 ×... (term number) – 2

2. Now you are going to repeat what you did in question 1, with a different set of sequences. In this sequence, the term number is multiplied by 3 to get the term value.

Term number	1	2	3	4	5	6	7	8
Term value	3	6	9	12	15	18	21	24

Now make a formula describing the relationship of the **term value** to the **term number** for each of these sequences:
(a) The sequence that starts with 8 and is formed by adding 3 repeatedly.
(b) The sequence that starts with 12 and is formed by adding 3 repeatedly.
(c) The sequence that starts with 2 and is formed by adding 3 repeatedly.

3. Copy the tables. Write the first eight terms of each of the following sequences and in each case, describe how each term can be calculated from the previous term.

(a) Term value = 10 × term number + 5

Term number	1	2	3	4	5	6	7	8
Term value	15	25	35	45	55	65	75	85

(b) Term value = 5 × term number – 3

Term number	1	2	3	4	5	6	7	8
Term value	2	7	12	17	22	27	32	37

Teaching guidelines

Learners generate a sequence using two different descriptions: one where they add a constant value to consecutive terms or multiply consecutive terms by a constant value; the other where they apply a rule to the term number.

In each situation in question 4(a) to (c) learners have to apply the two methods of describing the pattern of a sequence.

They may see that if there is a constant difference (which could be negative) between terms, we multiply the term number by that constant difference and then adjust the rule by adding or subtracting a value to get the terms.

For example, a sequence of numbers is 4, 6, 8, 10, 12, ... the common difference is 2. Test the rule: 2 × term number = 2, check for term number 1, 2 and 3 and see that the rule works.

Learners must be careful when working with sequences where the pattern is given by a constant ratio, for example in 1, 3, 9, 27, 81, ... the common ratio is 3, but if we multiply the term number by 3 we get term 1 = 3 and term 2 = 9, and so on, but if we divide each answer by 3 we get the correct value, so the rule to get a term is $\frac{3^{\text{term number}}}{3}$. For example, for term 2 it is $\frac{3^2}{3}$ which we can simplify to 3^{2-1}, or if the term number is indicated by n, the rule is 3^{n-1}. This treatment applies to (c).

Answers

4. (a) to (c) See the answers on LB page 61 alongside.

4. For each sequence, write a formula to obtain each term from the previous term. Try to write a formula which relates each term to its position in the sequence. Check both your formulae by applying them, and write the results in a table like the one below.

(a) 7 11 15 19 23 27 31 35 39 43

A. Relationship between consecutive terms: Start with 7, then add 4 to each term to get the next term.

B. Relationship between term value and its position in sequence: Multiply the term number by 4 and add 3.

Term number	1	2	3	4	5
Term value using A	7	11	15	19	23
Term value using B	7	11	15	19	23

(b) 60 57 54 51 48 45 42 39 36

A. Relationship between consecutive terms: Subtract 3 from each term to get the next term.

B. Relationship between term value and its position in sequence: Multiply the term number by –3 and add 63.

Term number	1	2	3	4	5
Term value using A	60	57	54	51	48
Term value using B	60	57	54	51	48

(c) 1 2 4 8 16 32 64 128

A. Relationship between consecutive terms: Multiply each term by 2 to get the next term.

B. Relationship between term value and its position in sequence: The term value is 2 to the power (term number – 1).

Term number	1	2	3	4	5
Term value using A	1	2	4	8	16
Term value using B	1	2	4	8	16

Learner Book Overview		
Sections in this chapter	**Content**	**Pages in Learner Book**
7.1 Find output numbers for given input numbers	Working with different sets of input numbers; concept of domain	Pages 62 to 63
7.2 Different ways to represent the same relationship	Ways of representing the same relationship: words, flow diagram, formula, table of values, graph	Pages 63 to 65
7.3 Different representations of the same relationship	Practise in representing relationships in the different ways	Pages 66

CAPS time allocation	4 hours
CAPS content specification	Page 129

Mathematical background

A **relationship** exists between two quantities where the one quantity, the input value (independent variable), is substituted into a formula to give another value, the output value (dependent variable). A **function** is a special relationship where each input has a single output.

A numeric pattern has a set of input values (independent variable), a formula and a set of output values (dependent variable) and is therefore an example of a **functional relationship**. The values of a function (values of the dependent variable) for consecutive integer values of the independent variable (input numbers), in fact for any set of evenly spaced input numbers, always form a sequence.

Output values for given input values can be calculated by using a formula. If the formula is $y = 2x - 3$ and the input values are 5 and 10, the corresponding output values can be calculated by substitution: $y = 2 \times 5 - 3 = 7$ and $y = 2 \times 10 - 3 = 17$.

If the output value 13 is given, it can be substituted in the formula to produce the equation $13 = 2x - 3$, and solving this equation gives an input value of $x = 8$.

In a description of a function, the formula as well as the domain of input values should be given, for example, the input values could be the whole numbers, or the integers. If the function describes a real-life situation, care should be taken to ensure that the domain is correct. For example, if the input is the number of days and the output is the cost to hire a car, it is clear that the input numbers are whole numbers.

A function can be represented in the following different, equivalent ways:

- in a flow diagram: input value $\longrightarrow$ +5 $\longrightarrow$ × 3 $\longrightarrow$ output value
- verbally: "add five to the input number and then multiply by three to get the corresponding output number"
- $y = 3(x + 5)$
- a table of values of the two variables (this does not describe how the output numbers can be calculated, it simply lists the corresponding values)

x	1	2	3	4	5	6	7	8	9
y	18	21	24	27	30	33	36	39	42

- a graph.

7.1 Find output numbers for given input numbers

TWO DIFFERENT SETS OF INPUT NUMBERS

Teaching guidelines

Learners are given experiences that will make them aware of different possible sets of input values for the same rule, thereby developing the concept of the domain of a function or relationship.

Remind learners that we represent the input numbers by the symbol x, so that they will not be confused when they do question 2.

Encourage learners to make sure that they know which numbers belong to the different sets, for example, the natural numbers smaller than 1 000 (1; 2; 3; ...; 998; 999).

Answers

1. (a) Learners' own answer, for example: 50 – (5 × 4) = 50 – 20 = 30
   (b) Learners' own answer, for example: 50 – (5 × 50) = 50 – 250 = –200
   (c) Yes, because choosing even the smallest number, 20, will mean that you need to subtract 100 from 50.
2. (a) 45; 40; 35; 30; 25; 20; 15; 10; 5
   (b) –50; –100; –150; –200; –250; –300; –350; –400
3. (a) See the answers on LB page 62 alongside.
   (b) See the answers on LB page 62 alongside.
4. (a) 5; 9; 13; 17; 21; ... Note that the set of input numbers is continued, so we use three dots to show this.

CHAPTER 7

Functions and relationships

7.1 Find output numbers for given input numbers

TWO DIFFERENT SETS OF INPUT NUMBERS

In this activity you will do some calculations with:

- Set A: the natural numbers smaller than 10, i.e. 1, 2, 3, 4, 5, 6, 7, 8 and 9.
- Set B: multiples of 10 that are bigger than 10 but smaller than 100, i.e. the numbers 20, 30, 40, 50, 60, 70, 80 and 90.

1. You are going to choose a number, multiply it by 5, and subtract the answer from 50.
   (a) Choose any number from set A and do the above calculations.
   (b) Choose any number from set B and do the above calculations.
   (c) If you choose any other number from set B, do you think the answer will also be a negative number?
2. (a) Write down all the different output numbers that will be obtained when the calculations $50 - 5x$ are performed on the different numbers in set A.
   (b) Write down the output numbers that will be obtained when the formula $50 - 5x$ is applied to set B.

Output numbers are numbers that you obtain when you apply the rule to the input numbers.

3. (a) Copy and complete the following table for set A:

Input numbers	1	2	3	4	5	6	7	8	9
Values of $50 - 5x$	45	40	35	30	25	20	15	10	5

   (b) Copy and complete the following table for set B:

Input numbers	20	30	40	50	60	70	80	90
Values of $50 - 5x$	–50	–100	–150	–200	–250	–300	–350	–400

4. In this question your set of input numbers will be the even numbers: 2; 4; 6; 8; 10; ...
   (a) What will all the output numbers be if the rule $2n + 1$ is applied to the set of even numbers? Write a list.

62 MATHEMATICS GRADE 9: TERM 1

Answers

4. (b) 3; 7; 11; 15; 19; ...
   (c) 9; 13; 17; 21; 25; ...
   (d) 7; 13; 19; 25; 31; ...
5. (a) The output numbers will all be negative.
   (b) The output numbers will all be mixed numbers or decimals between 10 and 11, for example, $10\frac{1}{5}$; $10\frac{2}{5}$; 10,7 and so on.
   (c) They will all be whole numbers (not equal to 0), because the denominators are all factors of 30.

7.2 Different ways to represent the same relationship

Teaching guidelines

Remind learners of the work they did on sequences in Chapter 6. One way to describe the pattern of the sequence was to describe the relationship between the term number and the term with a rule or formula, for example the sequence 15; 25; 35; 45; ... could be described by the rule $10n + 5$ where n is the term number.

In the new context we could call the term number the input variable and the term value the output variable. This is an example of a function. There is only one output value for every input value.

Discuss the fact that there are different ways to represent a function. Each representation stresses one or two aspects of the function or relationship it represents. The discussion from LB page 64 onwards explains this.

One representation that can be added to the others is a verbal description. A relationship can also be described in words.

Misconceptions

Learners are often not clear about what a function is. It is the set of input numbers and the corresponding set of output numbers. The different ways of representing a function (or a relationship) are exactly that: ways of representing the ordered set of numbers.

(b) What will the output numbers be if the rule $2n - 1$ is applied?
(c) What will the output numbers be if the rule $2n + 5$ is applied?
(d) What will the output numbers be if the rule $3n + 1$ is applied?

5. (a) What kind of output numbers will be obtained by applying the rule $x - 1\ 000$ to natural numbers smaller than 1 000?
   (b) What kind of output numbers will be obtained by applying the rule $\frac{x}{10} + 10$ to natural numbers smaller than 10?
   (c) If you use the rule $30x + 2$, and use input numbers that are positive fractions with denominators 2, 3 and 5, what kind of output numbers will you obtain?

7.2 Different ways to represent the same relationship

Consider the work that you did in Section 6.4 of Chapter 6. In each question, there were two variable quantities.

A quantity that changes is called a **variable quantity** or just a **variable**.

If one variable quantity is influenced by another, we say there is a **relationship** between the two variables. You can sometimes work out which number is linked to a specific value of the other variable.

An algebraic expression, such as $10x + 5$, describes what calculations must be done to find the output number that corresponds to a given input number.

The output number can also be called the output value, or the value of the expression, which is $10x + 5$ in this case.

For any input number, application of the rule $10x + 5$ produces only one output number, and it is very clear what that number is. For instance, if the formula is applied to $x = 3$, the output number is 35.

A relationship between two variables in which there is only one output number for each input number, is called a **function**.

Functions can be represented in:

- a table that shows some values of the two variables as it clearly shows which value of the output variable corresponds to each particular value of the input variable
- a flow diagram, which shows what calculations are to be done to calculate the output number that corresponds to a given input variable

Teaching guidelines

- The first representation we look at is a **flow diagram**. Learners should be familiar with flow diagrams. Learners can gain the following information from a flow diagram:
  - every input value is connected to one output value (the flow diagram shows which input and output values are connected)
  - the calculations that are performed on the input values are clearly shown.

  Only some values can be shown in the flow diagram, for example, if the input values are the integers, only a few of the numbers can be shown.
- Another representation is an expression that can be written as a **formula**. This can be written:
  - in words, for example: an output value equals five times the input value + 10. (we can call the output values the "function values" (indicated by the symbol y) and indicate the input values by the symbol x)
  - as an algebraic formula, for example function value = $5x + 10$ or $y = 5x + 10$.

  This representation shows only the calculations that have to be done.
- The third representation is a **table**.
  - In a table we can see which input and output values go together.
  - We cannot usually see what the formula of the function is and we cannot see all the values in a table.
- The fourth representation is a **graph**. We draw a graph by plotting the pairs of values that we can see in a table.
  - The input values are plotted against the horizontal axis and the output values against the vertical axis. The point where the lines in line with the input value and output value meet represents the input-output pair.

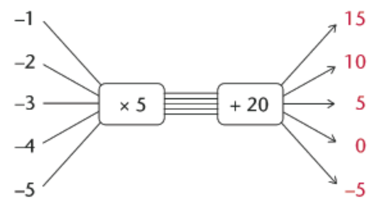
Answers

1. See the answers on LB page 64 alongside.
2. See the answers on LB page 64 alongside.
3. See graph of the function discussed in questions 1 and 2 on TG page 77.

- a formula, which also describes what calculations are to be done to calculate the output number that corresponds to a given input variable
- a graph.

Examples of these four ways of describing a function are given on the next two pages.

1. Copy and complete the following flow diagram:



A completed flow diagram shows two kinds of information:
- It shows what calculations are done to produce the output numbers.
- It shows which output number is connected to which input number.

The flow diagram that you completed shows:
- that each input number is multiplied by 5, then 20 is added, to produce the output numbers
- which output numbers correspond to which input numbers.

The calculations that need to be done can also be described with an expression. The expression $5x + 20$ describes the calculations that you did in question 1. You can also write this as a formula:

- A **verbal formula**:
  output number = 5 × input number + 20
- An **algebraic formula**:
  output number = $5x + 20$

The output numbers of a function are also called **function values**. Hence the formula can also be written as *function value* = $5x + 20$.

2. Copy and complete this table for the function described by $5x + 20$:

Input numbers	–1	–2	–3	–4	–5
Function values	15	10	5	0	–5

3. Draw a graph of this function discussed in question 1 and 2 on graph paper.

64 MATHEMATICS GRADE 9: TERM 1

Answers

3. See the answer to question 3 alongside.

Teaching guidelines

Show learners how to draw the horizontal (x-axis) and vertical (y-axis) axes and to number the axes with evenly spaced values, starting with 0 at the origin (where the two lines cross).

Help learners to read the coordinate pairs from the table and plot them on the graph.

At this stage, all learners need to know how to plot the points. They do not need to know all the features of the graphs.

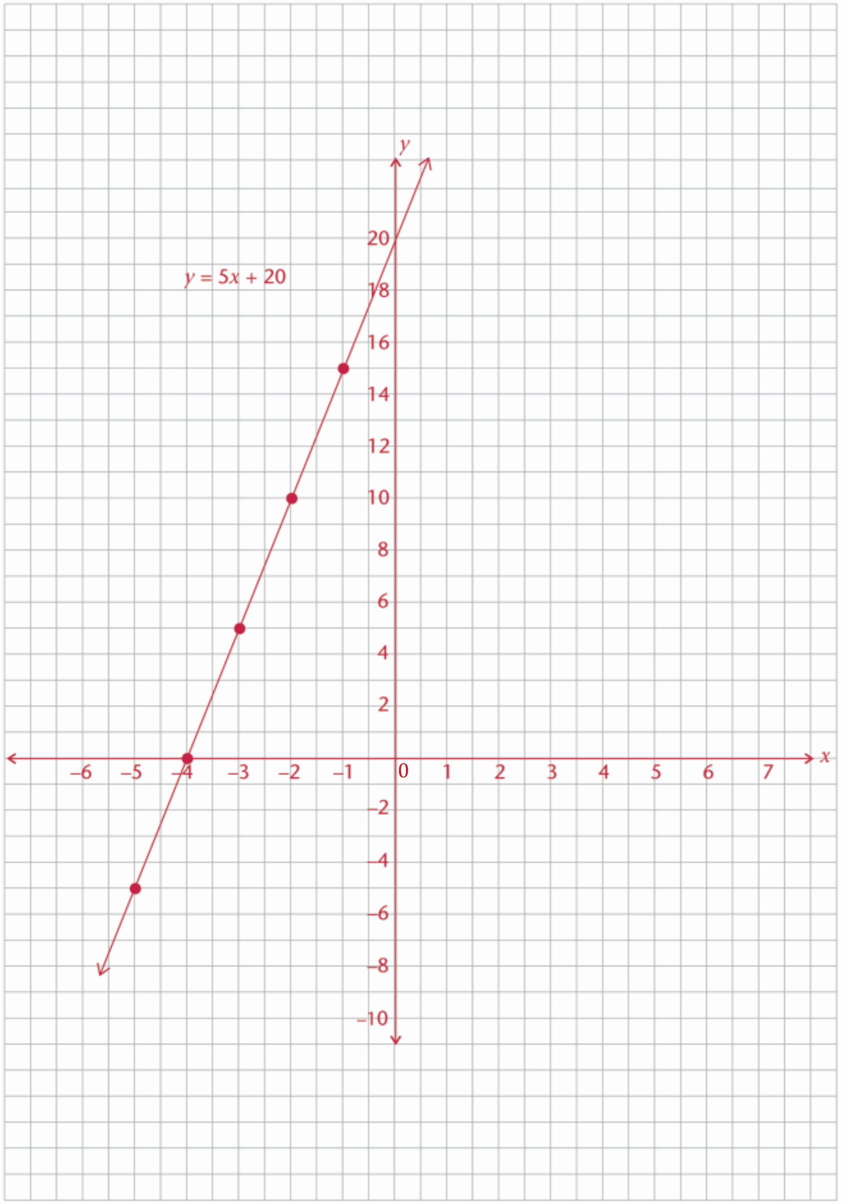
Make special mention of the position of the points we get when $x = 0$ and when $y = 0$; these are the points at which the graph cuts the axes.

Misconceptions

Learners find it difficult to plot points when they are not familiar with this work. To plot (–3; 5) for example, they will make a dot at $x = -3$ and another dot at $y = 5$. They typically plot the values on the axes.

Show them how to plot a point by drawing a line through $x = -3$, parallel to the y-axis and another line through $y = 5$, parallel to the x-axis. The point where the two lines cross represents the coordinate pair.

Answer: to question 3 on LB page 64



Answers

4. See the completed graph on LB page 65 alongside.

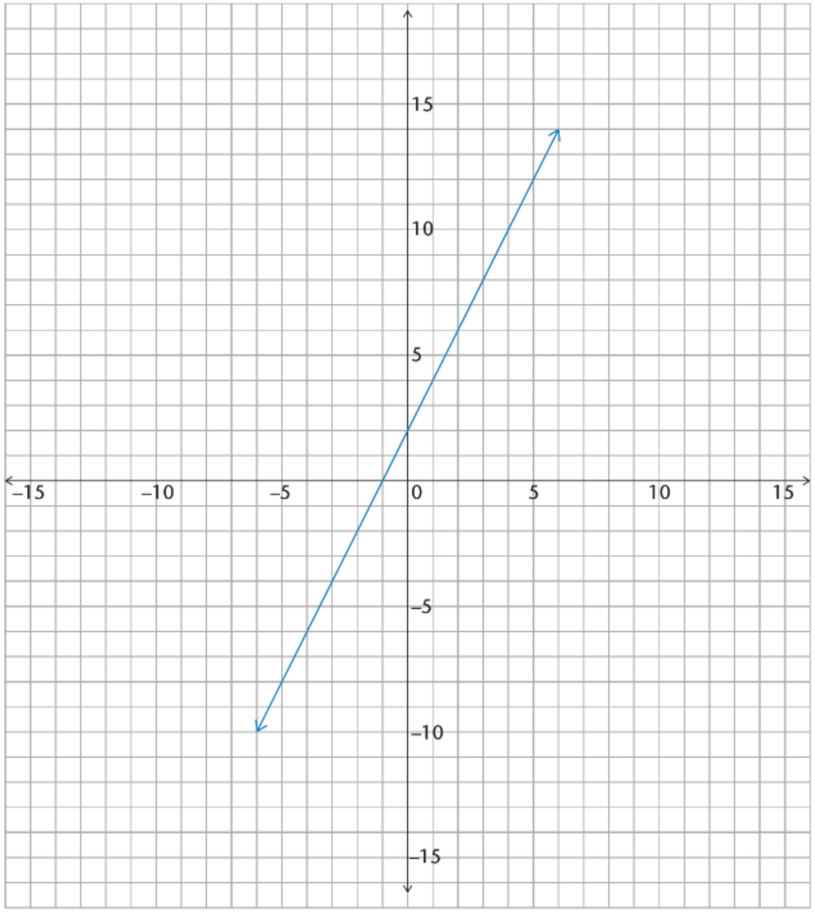
Teaching guidelines

Learners can gain experience in finding function and input values from graphs. Let them choose points where the graph crosses grid lines, for example where $x = -5$. They choose a point and find the corresponding x-value and y-value.

Help them to read off the answer correctly from the axes.

4. A graph of a certain function is given below. Copy and complete the table for this function:

Input numbers	–5	–2	0	1	2
Function values	–8	–2	2	4	6



7.3 Different representations of the same relationship

Teaching guidelines

As shown on the following pages, learners can:

- complete the flow diagrams
- draw up the tables of values
- draw the graph to represent the given expressions from questions 1 to 7.

Teach learners to write the formula of the function on the graph next to the line, in other words, to name the graph.

They should also label the axes. Vertical axis: y and horizontal axis: x.

Notes on Answers to Questions

For question 4, discuss with learners why the graph is slanted in the opposite direction to the previous graphs. (The function values become smaller as the input values become bigger.)

With question 6, let learners note that as the input values increase, the function values also increase.

With question 7, let learners note that the function values decrease as the input values increase.

Answers

Please see answers to questions 1 to 7 on the following TG pages 80 to 83. Suggest to learners to use input values {–5; –2; 0; 2; 5} for the flow diagrams.

7.3 Different representations of the same relationship

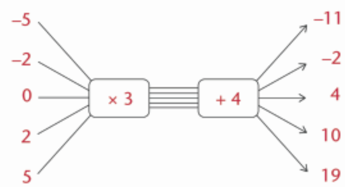
On separate pages, represent each of the following functions with the following:

(a) a flow diagram
(b) a table of values for the set of integers from –5 to 5
(c) a graph

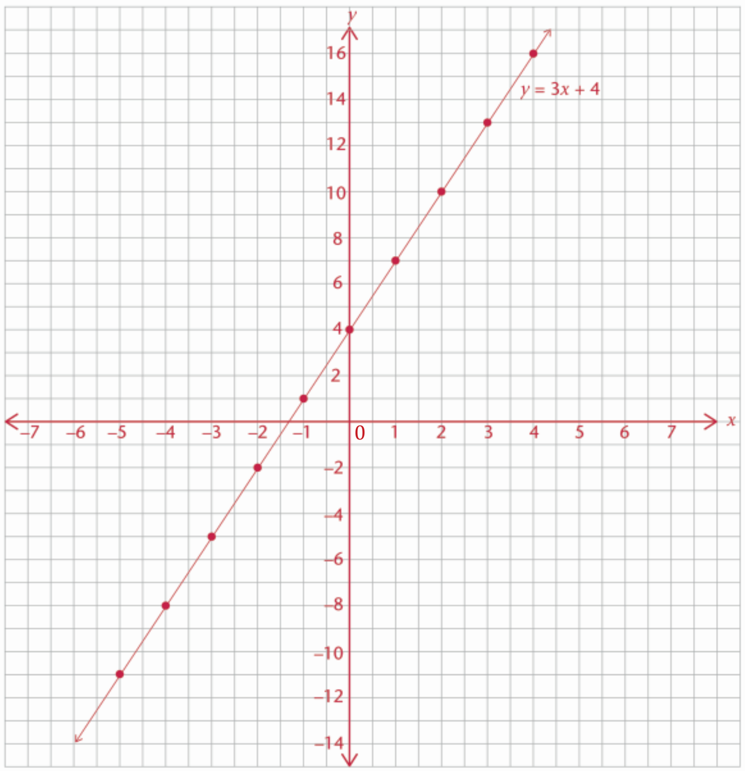
1. The relationship described by the expression $3x + 4$.
2. The relationship described by the expression $2x - 5$.
3. The relationship described by the expression $\frac{1}{2}x + 2$.
4. The relationship described by the expression $-3x + 4$.
5. The relationship described by the expression $2{,}5x + 1{,}5$.
6. The relationship described by the expression $0{,}2x + 1{,}4$.
7. The relationship described by the expression $-2x - 4$.

66 MATHEMATICS GRADE 9: TERM 1

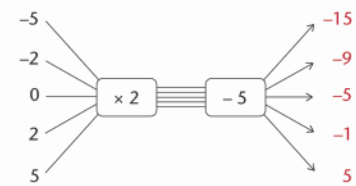
Answer: to question 1 on LB page 66.



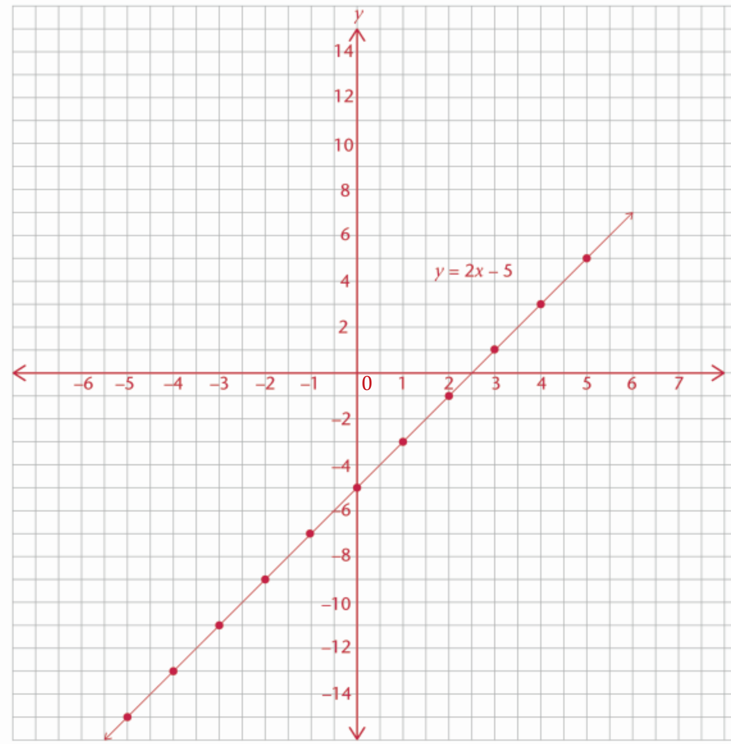
x	–5	–4	–3	–2	–1	0	1	2	3	4	5
$3x + 4$	–11	–8	–5	–2	1	4	7	10	13	16	19



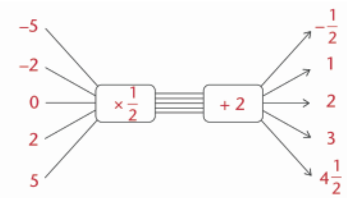
Answer: to question 2 on LB page 66.



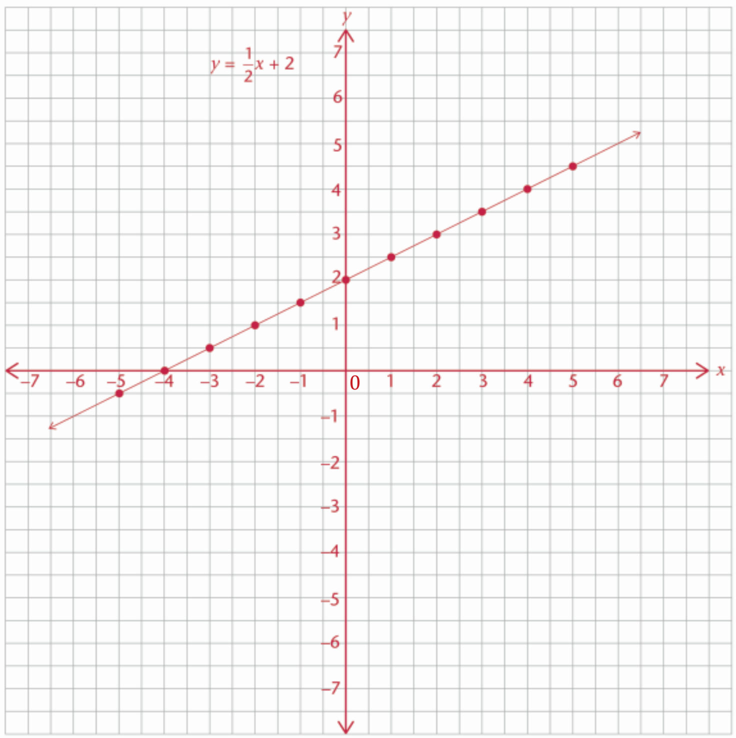
x	–5	–4	–3	–2	–1	0	1	2	3	4	5
$2x - 5$	–15	–13	–11	–9	–7	–5	–3	–1	1	3	5



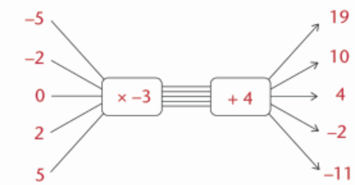
Answer: to question 3 on LB page 66.



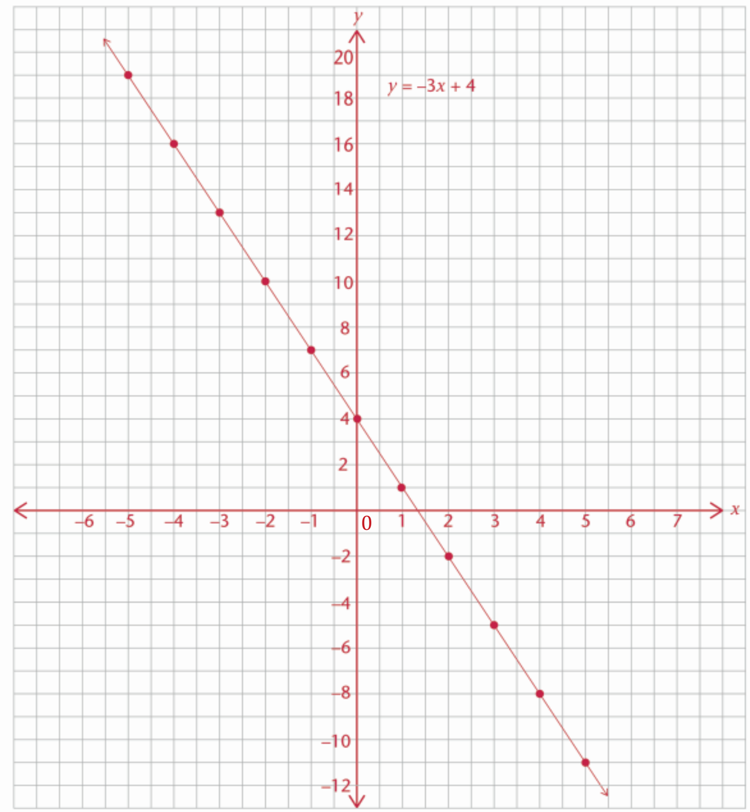
x	–5	–4	–3	–2	–1	0	1	2	3	4	5
$\frac{1}{2}x + 2$	$-\frac{1}{2}$	0	$\frac{1}{2}$	1	$1\frac{1}{2}$	2	$2\frac{1}{2}$	3	$3\frac{1}{2}$	4	$4\frac{1}{2}$



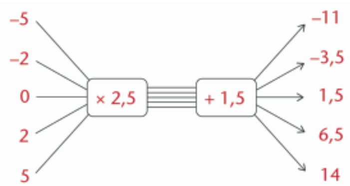
Answer: to question 4 on LB page 66.



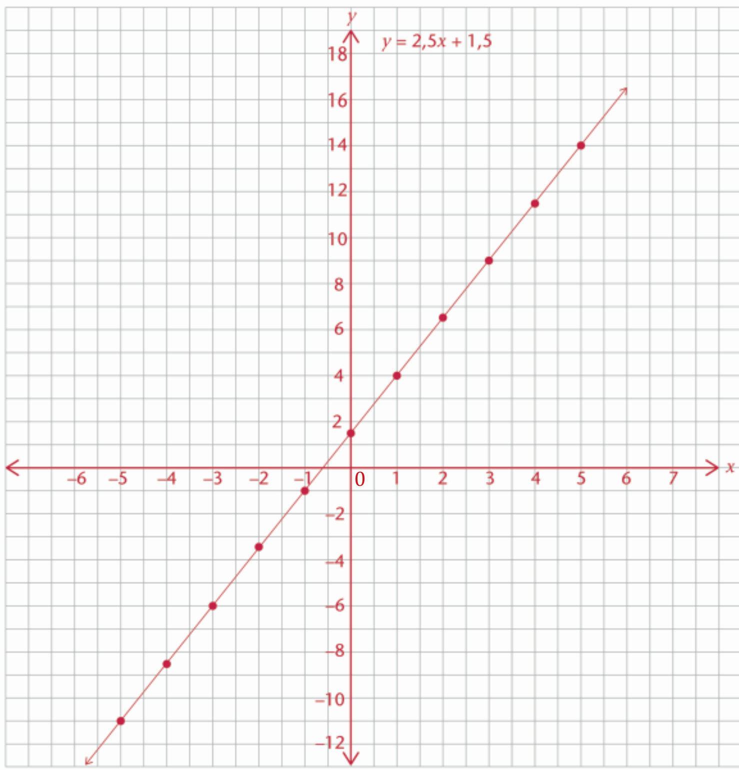
x	–5	–4	–3	–2	–1	0	1	2	3	4	5
$-3x + 4$	19	16	13	10	7	4	1	–2	–5	–8	–11



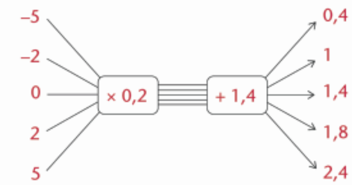
Answer: to question 5 on LB page 66.



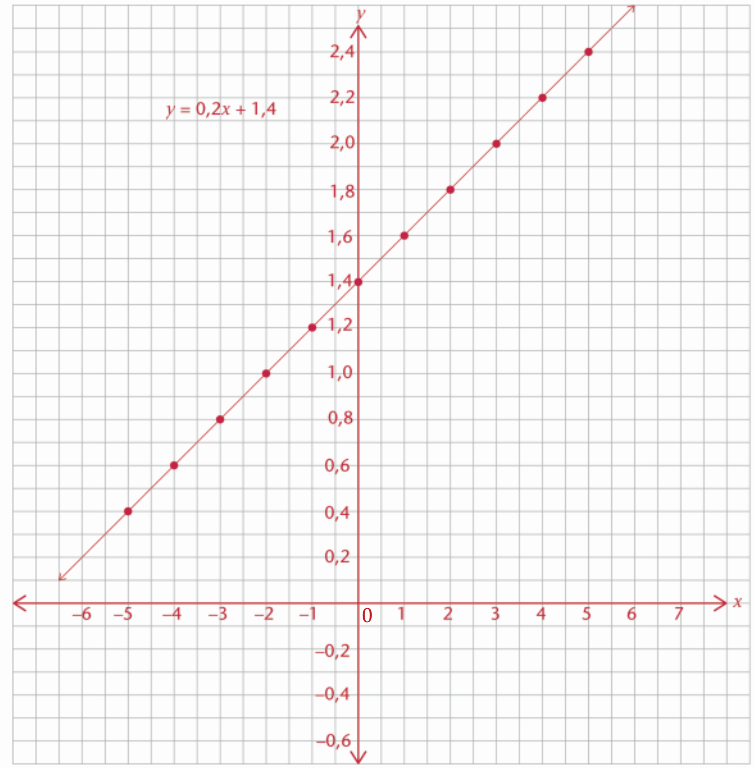
x	–5	–4	–3	–2	–1	0	1	2	3	4	5
$2,5x + 1,5$	–11	–8,5	–6	–3,5	–1	1,5	4	6,5	9	11,5	14



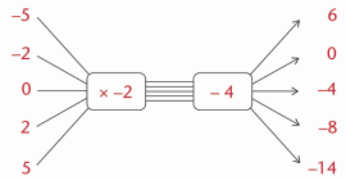
Answer: to question 6 on LB page 66.



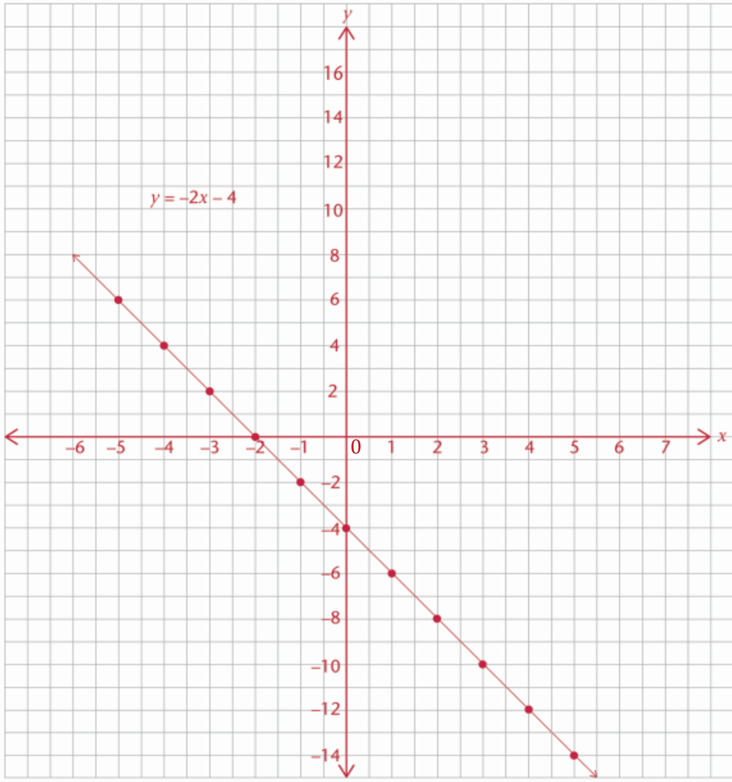
x	–5	–4	–3	–2	–1	0	1	2	3	4	5
$0,2x + 1,4$	0,4	0,6	0,8	1	1,2	1,4	1,6	1,8	2,0	2,2	2,4



Answer: to question 7 on LB page 66.



x	–5	–4	–3	–2	–1	0	1	2	3	4	5
$-2x - 4$	6	4	2	0	–2	–4	–6	–8	–10	–12	–14



Grade 9 Term 1 Chapter 8 Algebraic expressions

Learner Book Overview		
Sections in this chapter	**Content**	**Pages in Learner Book**
8.1 Algebraic language	The meaning of the words used in algebra: algebraic expression, polynomial, variable, coefficient, constants, equivalent expressions; conventions used in algebra, order of operations	Pages 67 to 72
8.2 Properties of operations	Distributive, commutative and associative properties	Pages 72 to 74
8.3 Combining like terms in algebraic expressions	Using the properties of operations to rearrange and add like terms	Pages 74 to 76
8.4 Multiplication of algebraic expressions	Using the distributive property to multiply a polynomial by a monomial; finding squares and square roots of monomials	Pages 76 to 79
8.5 Dividing polynomials by integers and monomials	Using the right-distributive property of division to divide polynomials by monomials	Pages 79 to 82
8.6 Products and squares of binomials	Extending the knowledge of multiplying a binomial by a monomial to multiplying by a binomial; finding the square of a binomial	Pages 82 to 83
8.7 Substitution into algebraic expressions	Finding values of equivalent expressions thereby finding reasons to simplify expressions for easier calculation	Page 84

CAPS time allocation	4,5 hours
CAPS content specification	Pages 130 to 131

Mathematical background

The manipulation of algebraic expressions to get equivalent expressions is done by understanding and using the conventions and the properties of operations such as the order of operations; using brackets to indicate that a particular calculation has to be done before others; and the distributive, commutative and associative properties (refer to TG pages 92 and 94).

Expressions are simplified for easier calculation, for example:

- by rearranging terms in order to combine like terms
- by multiplying polynomials by monomials and binomials
- by finding the square of a binomial
- by dividing polynomials by integers and monomials.

In the process it may be necessary to find squares or square roots of monomials.

8.1 Algebraic language

WORDS, DIAGRAMS AND EXPRESSIONS

Background information

Algebra at school level mainly involves describing and analysing relationships between variables. An algebraic expression, for example $3x + 5$, is a description of calculations performed on values of a variable (represented by the symbol x) to produce the values of another variable. For example, the algebraic expression $3x + 5$ "tells us" that:

- if $x = 6$, the value of the other variable is $3 \times \boxed{6} + 5$ which is 23
- if $x = 4$, the value of the other variable is $3 \times \boxed{4} + 5$ which is 17
- if $x = 2{,}6$, the value of the other variable is $3 \times \boxed{2{,}6} + 5$ which is 12,8
- if $x = -2{,}6$, the value of the other variable is $3 \times \boxed{-2{,}6} + 5$ which is $-2{,}8$
- and the same for infinitely many other values of x, in fact for all numbers.

We could symbolise this information more completely by representing the dependent variable (the numbers at the ends of the above sentences) with a letter, for example, by writing $y = 3x + 5$. The algebraic expression $3x + 5$ and the formula $y = 3x + 5$ represent exactly the same mathematical situation.

Teaching guidelines

Understanding algebraic expressions as descriptions of sequences of calculations is critical for learners to understand what they actually do mathematically when they perform algebraic manipulations.

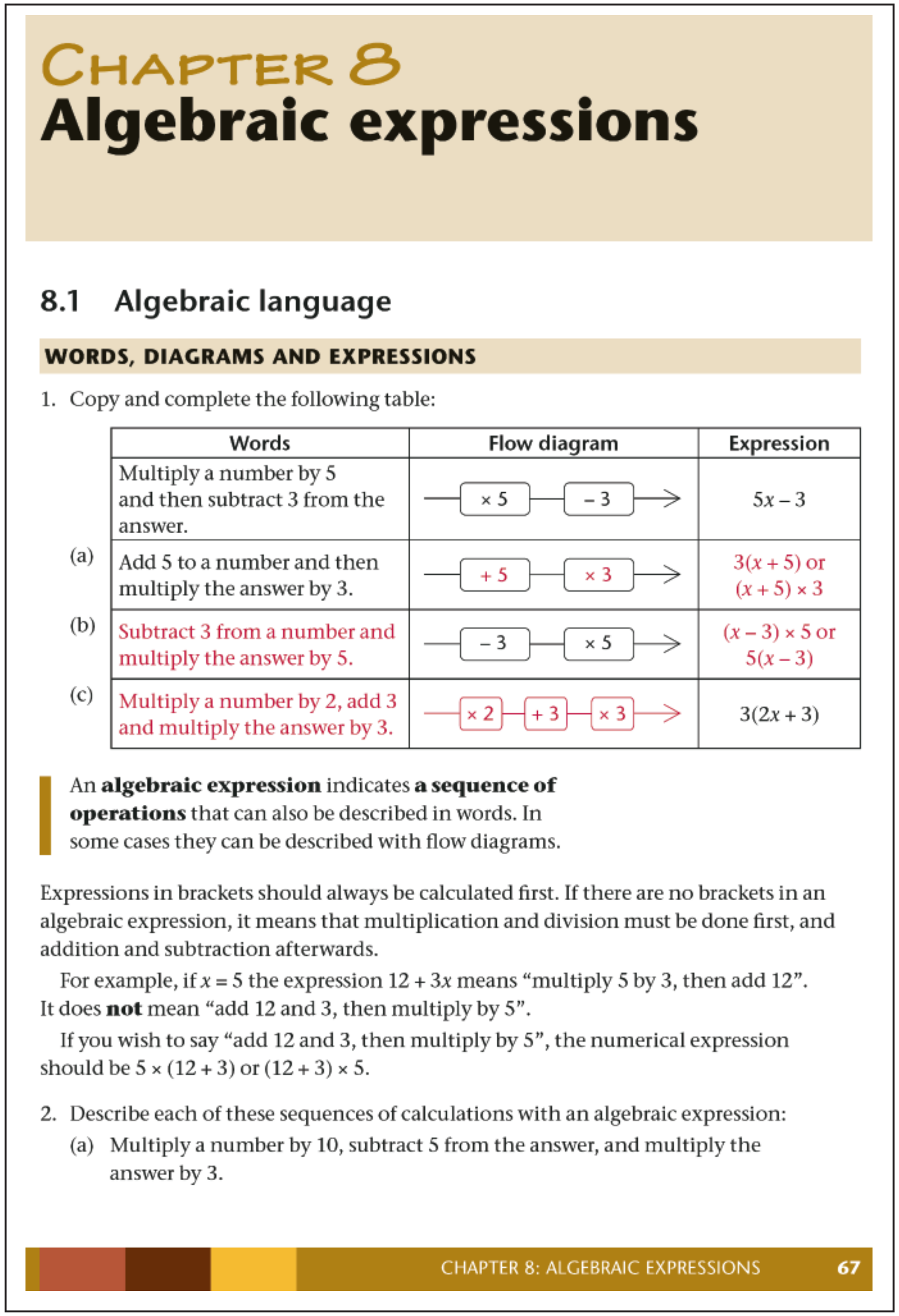
Explain the order of the operations and how to use brackets to change the meaning of the calculation instruction.

Misconceptions

Learners might not know how to use brackets to write an expression so that it is clear which calculation should be done first.

Answers

1. See the answers on LB page 67 alongside.
2. (a) $(10x - 5) \times 3$ or $3(10x - 5)$



CHAPTER 8
Algebraic expressions

8.1 Algebraic language

WORDS, DIAGRAMS AND EXPRESSIONS

1. Copy and complete the following table:

	Words	Flow diagram	Expression
	Multiply a number by 5 and then subtract 3 from the answer.	× 5 → − 3 →	$5x - 3$
(a)	Add 5 to a number and then multiply the answer by 3.	+ 5 → × 3 →	$3(x + 5)$ or $(x + 5) \times 3$
(b)	Subtract 3 from a number and multiply the answer by 5.	− 3 → × 5 →	$(x - 3) \times 5$ or $5(x - 3)$
(c)	Multiply a number by 2, add 3 and multiply the answer by 3.	× 2 → + 3 → × 3 →	$3(2x + 3)$

An **algebraic expression** indicates **a sequence of operations** that can also be described in words. In some cases they can be described with flow diagrams.

Expressions in brackets should always be calculated first. If there are no brackets in an algebraic expression, it means that multiplication and division must be done first, and addition and subtraction afterwards.

For example, if $x = 5$ the expression $12 + 3x$ means "multiply 5 by 3, then add 12". It does **not** mean "add 12 and 3, then multiply by 5".

If you wish to say "add 12 and 3, then multiply by 5", the numerical expression should be $5 \times (12 + 3)$ or $(12 + 3) \times 5$.

2. Describe each of these sequences of calculations with an algebraic expression:
   (a) Multiply a number by 10, subtract 5 from the answer, and multiply the answer by 3.

CHAPTER 8: ALGEBRAIC EXPRESSIONS 67

Answers

2. (b) $(x - 5) \times 10 \times 3$ or $(10(x - 5)) \times 3$ or $3(10(x - 5))$
3. (a) $200 - 5 \times 10 = 200 - 50 = 150$ (b) $195x = 195 \times 10 = 1\,950$
   (c) $5 \times 10 + 40 = 50 + 40 = 90$ (d) $5 \times (10 + 40) = 5 \times 50 = 250$
   (e) $40 + 5 \times 10 = 40 + 50 = 90$ (f) $5 \times 10 + 5 \times 40 = 50 + 200 = 250$

SOME WORDS WE USE IN ALGEBRA

Teaching guidelines

Learners need to understand and learn the language we use in Mathematics. The words they need to know at this stage are discussed on LB page 68.

It is always useful to use the correct terminology in discussions with learners about any mathematical content. For example, refer to "polynomials", "coefficient of x in the first term", "the constant", "variables", and so on.

Misconceptions

Learners are sometimes confused between the use of the words "coefficient" and "constant". They use the latter when they mean the former.

Answers

1. See the answers on LB page 68 alongside.
2. (a) 1
   (b) 1
   (c) Yes. It will be $-3x^0$, which is simply -3.

EQUIVALENT ALGEBRAIC EXPRESSIONS

Teaching guidelines

The mathematical meaning of manipulating an expression is to replace a given expression. For example, replacing $3x^2 + 13x + 7 + 2x^2 - 8x - 12$ with a different expression that describes an alternative way of calculating the output numbers, such as the expression $5x^2 + 5x - 5$, or the expression $5(x^2 + x - 1)$. Such a replacement with an equivalent expression may be useful. In this particular case it is much less work to evaluate $3x^2 + 13x + 7 + 2x^2 - 8x - 12$ for $x = 8$ by substituting into the expression $5(x^2 + x - 1)$ rather than into the original expression, even if you use a calculator.

Learners need to experience the equivalence of expressions and the possible advantages of using one expression rather than another. Consolidate these experiences by having class discussions after the learners have completed these questions.

(b) Subtract 5 from a number, multiply the answer by 10, and multiply this answer by 3.

3. Evaluate each of these expressions for $x = 10$:
   (a) $200 - 5x$ (b) $(200 - 5)x$
   (c) $5x + 40$ (d) $5(x + 40)$
   (e) $40 + 5x$ (f) $5x + 5 \times 40$

SOME WORDS WE USE IN ALGEBRA

- An expression with one term only, like $3x^2$, is a **monomial**.
- An expression which is a sum of two terms, like $5x + 4$, is called a **binomial**.
- An expression which is a sum of three terms, like $3x^3 + 2x + 9$, is called a **trinomial**.

The symbol x is often used to represent the **variable** in an algebraic expression, but other letter symbols may also be used.

In the monomial $3x^2$, the 3 is called the **coefficient** of x^2.

In the binomial $5x + 4$, and the trinomial $3x^2 + 2x + 9$, the numbers 4 and 9 are called **constants**.

1. Copy and complete the table, using the completed first row as an example:

Expression	Type of expression	Symbol used to represent the variable	Constant	Coefficient of ...
$x^2 + 6x + 10$	Trinomial	x	10	the second term is: 6
$6s^3 + s^2 + 5$	Trinomial	s	5	s^2 is: 1
$\frac{k}{3} + 12$	Binomial	k	12	the first term is: $\frac{1}{3}$
$4p^{10}$	Monomial	p	0	p^{10} is: 4

2. Consider the polynomial pattern starting with $7x^5 + 5x^4 + 3x^3 + x^2 + ...$
   (a) What is the coefficient of the fourth term?
   (b) What is the exponent value of the fifth term?
   (c) Do you think the sixth term will be a constant? Why?

EQUIVALENT ALGEBRAIC EXPRESSIONS

1. Copy and complete the table on page 69 by doing the necessary calculations. Calculate the numerical value of the expressions for the various values of x.

68 MATHEMATICS GRADE 9: TERM 1

Answers

1. See the answers on LB page 69 alongside.
2. $2x - 3$ and $\frac{(3x+2)(2x-3)}{3x+2}$ and $\frac{6x^2 - 5x - 6}{3x+2}$

   $3x + 2 + 2x - 3$ and $2x - 3 + 3x + 2$ and $5x - 1$

   $(3x + 2)(2x - 3)$ and $3x(2x - 3) + 2(2x - 3)$ and $6x^2 - 5x - 6$
3. See the answers on LB page 69 alongside.
4. See the answers on LB page 69 alongside.
5. (a) Yes, they have the same numerical value for any given value of x. If you simplify the longer expression by combining the like terms, it gives you the shorter expression.

	x	–2	–1	0	1	2
(a)	$3x + 2$	–4	–1	2	5	8
(b)	$2x - 3$	–7	–5	–3	–1	1
(c)	$3x + 2 + 2x - 3$	–11	–6	–1	4	9
(d)	$2x - 3 + 3x + 2$	–11	–6	–1	4	9
(e)	$5x - 1$	–11	–6	–1	4	9
(f)	$(3x + 2)(2x - 3)$	28	5	–6	–5	8
(g)	$3x(2x - 3) + 2(2x - 3)$	28	5	–6	–5	8
(h)	$6x^2 - 5x - 6$	28	5	–6	–5	8
(i)	$\frac{(3x+2)(2x-3)}{3x+2}$	–7	–5	–3	–1	1
(j)	$\frac{6x^2-5x-6}{3x+2}$	–7	–5	–3	–1	1

2. Although they may look different, make a list of all the algebraic expressions above which have the same numerical value for the same value of x.

Equivalent expressions are algebraic expressions that have different sequences of operations, but have the same numerical value for any given value of x.

It is often convenient not to work with a given expression, but to **replace** it with an equivalent expression.

3. Copy and complete the following table:

x	2	3	5	10	–5	–10
$12x - 7 + 3x + 10 - 5x$	23	33	53	103	–47	–97

4. Copy and complete the following table:

x	2	3	5	10	–5	–10
$10x + 3$	23	33	53	103	–47	–97

5. (a) Is $10x + 3$ equivalent to $12x - 7 + 3x + 10 - 5x$? Explain your answer.

Answers

5. (b) First simplify the expression by combining the like terms, then substitute for x.

CONVENTIONS FOR WRITING ALGEBRAIC EXPRESSIONS

Teaching guidelines

Discuss the conventions for writing algebraic expressions and recognition of coefficients, exponents and different types of polynomials as covered in this section. For example, explain to learners that writing $3x$ is the correct, conventional way of writing an expression even though $3 \times x$ or $x3$ give the same instruction.

In the last part of this section, we revise the conventions about brackets and order of operations because this is often a stumbling block for many learners in the higher grades.

Make sure learners understand that brackets are used to show which calculations should be done first. Brackets are usually introduced when certain additions or calculations have to be done before others, for example $3(x + y) + 2(x + 3y)$.

Misconceptions

Learners get confused when they are working with algebraic expressions that do not follow the order of operations.

Answers

1. (a) $4x + xy - 3y$
   (b) $70 - 7x + 50x + 30$
2. 60; 60 and 40
3. (a) 13
   (b) 13
   (c) $6(5) - 3(10) + 5(2) = 10$ or $10(5) - 3(10) + 5(2) - 4(5) = 10$
   (d) $14(5) - 5(2) = 60$ or $10(5) - 3(10) - 5(2) + 4(5) + 3(10) = 60$

(b) Suppose you need to know how much $12x - 7 + 3x + 10 - 5x$ is for $x = 37$ and $x = -43$. What do you think is the easiest way to find out?

CONVENTIONS FOR WRITING ALGEBRAIC EXPRESSIONS

Here are some things that mathematicians have agreed upon, and it makes mathematical work much easier if all people stick to these agreements.

A **convention** is something that people have agreed to do in the same way.

The multiplication sign is often omitted in algebraic expressions: We normally write $4x$ instead of $4 \times x$, and $4(x - 5)$ instead of $4 \times (x - 5)$.

It is a convention to write a known number first in a product, i.e. we write $3 \times x$ rather than $x \times 3$, and we write $3x$ but **not** $x3$.

1. Rewrite each of the following in the normal way of writing algebraic expressions:
   (a) $x \times 4 + x \times y - y \times 3$
   (b) $7 \times (10 - x) + (5 \times x + 3)10$

People all over the world have agreed that, in expressions that do not contain brackets, addition and subtraction should be performed as they appear from left to right.

According to this convention, $x - y + z$ means that you first have to subtract y from x, then add z. For example if $x = 10$, $y = 5$ and $z = 3$, $x - y + z$ is $10 - 5 + 3$ and it means $10 - 5 = 5$, then $5 + 3 = 8$. It does not mean $5 + 3 = 8$, then $10 - 8 = 2$.

2. Calculate $50 - 20 + 30$ and $50 + 30 - 20$ and $50 - 30 + 20$.

3. Evaluate each of the following expressions for $x = 10$, $y = 5$ and $z = 2$:
   (a) $x + y - z$
   (b) $x - z + y$
   (c) $10y - 3x + 5z - 4y$
   (d) $10y - 3x - 5z + 4y + 3x$

People have also agreed that, in expressions that do not contain brackets, we should do multiplication (and division) **before** addition and subtraction.

Hence, $5 + 3 \times 4$ should be understood as "multiply 4 by 3, then add the answer to 5"; not as "add 5 and 3 then multiply the answer by 4".

Also, $3 \times 4 + 5$ should be understood as "multiply 4 by 3, then add 5 to the answer"; not as "add 4 and 5 then multiply the answer by 3".

Teaching guidelines

Use the information on LB page 71 alongside about using brackets in calculations to help learners understand when and how to insert brackets.

Brackets are used if there are calculations that have to be done first, for example, if the intention is that 4 be subtracted from 20 and the answer be multiplied by 2, it is **incorrect** to write 20 – 4 × 2 because according to the convention, multiplication should be done first. Therefore, learners should learn that a bracket dictates that the operations inside it should be done first. The correct instruction would be: 2(20 – 4).

Explain the correct way to write an expression with brackets, namely 2(20 – 4) and not (20 – 4) × 2 or (20 – 4)2.

Answers

4. See the answers on LB page 71 alongside.
5. 4 × 3 + 5

   5 + 4 × 3
6. See the answers on LB page 71 alongside.
7. See the answers on LB page 71 alongside.

4. Do each of the following calculations:
   (a) Multiply 4 by 3, then add 5 to the answer. 17
   (b) Add 4 and 5, then multiply the answer by 3. 27
   (c) Multiply 4 by 3, then add the answer to 5. 17
   (d) Add 5 and 3, then multiply the answer by 4. 32
5. Rewrite the instructions in 4(a) and 4(c) without using words.
6. Calculate each of the following:
   (a) 10 × 5 + 30 80 (b) 30 + 10 × 5 80
   (c) 10 × 5 – 30 20 (d) 30 – 10 × 5 –20
7. (a) Add 4 and 5, then subtract the answer from 20. 11
   (b) Subtract 4 from 20 and then add 5. 21
   (c) Add 4 and 5, then multiply the answer by 3. 27
   (d) Multiply 3 by 5 and then add the answer to 4. 19

If we want to specify the calculations in 7(a) and 7(c) without using words, we will face challenges.

We cannot write 20 – 4 + 5 for "*add 4 and 5 then subtract the answer from 20*", because that would mean "*subtract 4 from 20, then add 5*". We need a way to indicate, without using words, that we want the addition to be performed before the subtraction in this case.

Similarly, we cannot write 4 + 5 × 3 for "*add 4 and 5 then multiply the answer by 3*", because that would mean "*multiply 3 by 5 and then add the answer to 4*". We need a way to indicate, without using words, that we want the addition to be performed *before* the multiplication in this case.

Mathematicians have agreed to use brackets to address the above challenges. The following convention is used all over the world:

> Whenever there are brackets in an expression, the calculations within the brackets should be performed first.

Hence, 20 – (4 + 5) means "*add 4 and 5 then subtract the answer from 20*", but 20 – 4 + 5 means "*subtract 4 from 20, then add 5*".

(4 + 5) × 3 or 3 × (4 + 5) means "*add 4 and 5 then multiply the answer by 3*", but 4 + 5 × 3 means "*multiply 3 by 5, then add the answer to 4*".

10 + 2(5 + 9) means "*add 5 and 9, multiply the answer by 2, then add this answer to 10*":

5 + 9 = 14 14 × 2 = 28 28 + 10 = 38

CHAPTER 8: ALGEBRAIC EXPRESSIONS 71

Answers

8. (a) 120 (b) 120
   (c) 80 (d) 20
   (e) 3(6) + 2 = 20 (f) 10(12) + 3(10) = 150
   (g) 250 – 10(20) + 35 = 85 (h) 40(10) = 400
   (i) 240(20) + 35 = 4 835 (j) 20 + 20(10) = 220
   (k) 200 + 100(40) = 4 200 (l) 300(30) + 5 = 9 005
9. (a) 52 (b) 70
   (c) 20 (d) 70 (like (b))
   (e) 48 (f) 30
   (g) 0 (h) 40
   (i) 13 (j) 7
   (k) 3 (l) 3 (like (k))
   (m) 13 (like (i)) (n) 7 (like(j))

8.2 Properties of operations

Teaching guidelines

Learners are discovering that in algebraic language they need to know the properties that go with the operations on numbers and how to use them. These properties help us to simplify expressions and make them easier to work with.

The distributive property over addition means that if a term is multiplied by terms in brackets, we need to "distribute" the multiplication over all the terms inside the bracket, for example: $3x(2 + y) = 6x + 3xy$. Even though order of operations says that we must add the terms inside the parenthesis first, with the distributive property we can simplify the expression by multiplying every term inside the brackets by the multiplier.

(Teaching guidelines are continued alongside LB page 73 and on the following page.)

Answers

1. See the answers on LB page 72 alongside.

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8. Calculate each of the following:
   (a) 100 + 50 – 30 (b) 100 + (50 – 30)
   (c) 100 – 50 + 30 (d) 100 – (50 + 30)
   (e) 3(10 – 4) + 2 (f) 10(5 + 7) + 3(18 – 8)
   (g) 250 – 10 × (18 + 2) + 35 (h) (20 + 20) × (20 – 10)
   (i) (250 – 10) × (18 + 2) + 35 (j) 20 + 20 × (20 – 10)
   (k) 200 + (100 × 2(15 + 5)) (l) (200 + 100) × 2 × 15 + 5

In algebra, we normally write $3(x + 2y)$ instead of $(x + 2y) \times 3$, and we write $3(x - 2y)$ instead of $(x - 2y) \times 3$. Do not let this conventional way of writing in algebra confuse you. The expression $3(x + 2y)$ does not mean that multiplication by 3 is the first thing you should do when you evaluate the expression for certain values of x and y. The first thing you should do is to add the values of x and y. That is what the brackets tell you!

However, performing the instructions $3(x + 2y)$ is not the only way in which you can find out how much $3(x + 2y)$ is for any given values of x and y. Instead of working out $3(x + 2y)$, you may work out $3x + 6y$. In this case you will multiply each term before you add them together.

9. Evaluate each of the following expressions for $x = 10$, $y = 5$ and $z = 2$:
   (a) $xy + z$ (b) $x(y + z)$
   (c) $x + yz$ (d) $xy + xz$
   (e) $xy - z$ (f) $x(y - z)$
   (g) $x - yz$ (h) $xy - yz$
   (i) $x + (y - z)$ (j) $x - (y - z)$
   (k) $x - (y + z)$ (l) $x - y - z$
   (m) $x + y - z$ (n) $x - y + z$

8.2 Properties of operations

1. Calculate each of the following:
   (a) 5(3 + 4) 35 (b) 5 × 3 + 5 × 4 35
   (c) 6 × 3 + (4 + 6) 28 (d) (6 + 4) + 3 × 6 28
   (e) 3 × (4 × 5) 60 (f) (3 × 4) × 5 60

You should have noticed that for each row the results are the same. This is because operations with numbers have certain properties, namely the **distributive**, **commutative** and **associative** properties.

Answers

2. (a) 10 (b) 10
   (c) 350 (d) 350
   (e) 45 (f) 45
3. Yes. $5(3 - 2) = 5(3) - 5(2)$
   $2(6 - 4) = 2(6) - 2(4)$
4. No. $10 - 3 \neq 3 - 10$
5. $16 + 14 = 30$ and $33 + 17 = 50$ and $30 + 50 = 80$

Teaching guidelines and mathematical background (continued)

The commutative property applies to addition and multiplication and it says that we can add or multiply numbers in any order without changing the result, for example: for addition $7x + 9 = 9 + 7x$, and for multiplication $2 \times 5 \times 3x = 3x \times 2 \times 5 = 5 \times 3x \times 2$.

The associative property means that we can group numbers in a sum or a product in any way we want and still get the same answer. This is because we can only add or multiply two numbers at a time. So if there are more numbers in the expression, we can decide which two to add or multiply first. For example, in the case of addition $(2x + 5x) + 9x = 2x + (5x + 9x)$ where we work out the brackets first; and in the case of multiplication $(2x \times x) \times 5y = 2x \times (x \times 5y)$.

These properties often lie at the heart of the manipulations and "laws" we use. For example, when we manipulate $(a.b)^3$ to $a^3.b^3$ we can do it because of the definition of a power ($a^n = a.a.a....$ for n factors) and the commutative property: $(a.b)^3 = (a.b) \times (a.b) \times (a.b) = a \times a \times b \times b \times a \times b = a \times a \times b \times a \times b \times b = a.a.a \times b.b.b = a^3.b^3$. The shaded parts show where the commutative property was applied.

The long, detailed explanation of why the rule $(a.b)^3 = a^3.b^3$ is true, shows that using the properties of the operations reduces the number of calculations.

The **distributive** property is used each time you multiply a number in parts. For example:

The number thirty-four is actually $30 + 4$. You may calculate 5×34 by calculating 5×30 and 5×4, and then adding the two answers:

$5 \times 34 = 5 \times 30 + 5 \times 4$

The word "distribute" means to spread out. The distributive property may be described as follows:
$a(b + c) = ab + ac$
where a, b and c can be any numbers.
We may say: "multiplication distributes over addition".

2. Calculate each of the following:
   (a) $5(x - y)$ for $x = 10$ and $y = 8$ (b) $5x - 5y$ for $x = 10$ and $y = 8$
   (c) $5(x - y)$ for $x = 100$ and $y = 30$ (d) $5x - 5y$ for $x = 100$ and $y = 30$
   (e) $5(x - y + z)$ for $x = 10$, $y = 3$ and $z = 2$ (f) $5x - 5y + 5z$ for $x = 10$, $y = 3$ and $z = 2$
3. We say "multiplication distributes over addition". Does multiplication also distribute over subtraction? Give examples to support your answer.

For any values of x and y:
- $x + y$ and $y + x$ give the same answers, and
- xy and yx give the same answers.

This is called the **commutative property** of addition and multiplication.

4. We say "addition is commutative" and "multiplication is commutative".
   Is subtraction also commutative? Demonstrate your answer with an example.

The **associative property** allows you to arrange three or more numbers in any sequence when adding or multiplying. For any values of x, y and z, the following expressions all have the same answer:

$x + y + z$ $y + x + z$ $z + y + x$

5. Calculate $16 + 33 + 14 + 17$ in the easiest possible way.

The associative property of multiplication allows you to simplify something like the following:

$abc + bca + cba$

Because the order of multiplication does not change the result we can rewrite this expression as: $abc + abc + abc$.

This then can be simplified by adding like terms to be $3abc$. You will be able to use these properties throughout this chapter and when you do algebraic manipulations.

When you form an expression that is equivalent to a given expression, you say that you *manipulate* the expression.

Answers

6. (a) $17 \times (43 + 57)$ (43 + 57 make a convenient 100)
   (b) $(5 + 12)(8 \times 4 \times 7) - 9 \times 5 \times 8 \times 4 = ((5 + 12)(7) - 9(5))(8 \times 4)$
   (c) $(43 + 57) \times 17$ (43 + 57 make a convenient 100)
   (d) $100x$ (100 multiplies conveniently)
7. (a), (c) and (d) – distributive (reversed)
   (b) – associative, distributive (reversed)

8.3 Combining like terms in algebraic expressions

REARRANGE TERMS, THEN COMBINE LIKE TERMS

Teaching guidelines

Show learners that the commutative property (discussed in Section 8.2) allows us to rearrange terms. We do this so that we can manipulate the expressions to form simpler expressions when circumstances require.

At this stage learners should be familiar with the concept of like terms. Work through the examples on LB page 74 to remind learners why they can add like terms.

Work through an example using all the properties previously discussed to show learners how everything we do is justified by some or other property of numbers or operations. See LB page 75 for more information on the mathematical background relating to this topic.

Misconceptions

Learners remove brackets incorrectly when they apply the distributive property. Discuss the values in the tables given in question 1 to help learners see the correct way of applying the distributive property.

Answers

1. (a) See the answers on LB page 74 alongside.
   The two expressions are not equivalent. The table below shows the answers for each input number.
   (b) See the answers on LB page 74 alongside.
   The two expressions are equivalent. The table below shows the answers for each input number.

6. Replace each of the following expressions with a simpler expression that will give the same answer. **Do not do any calculations now**. In each case, state why your replacement will be easier to do.
   (a) $17 \times 43 + 17 \times 57$
   (b) $7 \times 5 \times 8 \times 4 + 12 \times 8 \times 4 \times 7 - 9 \times 4 \times 5 \times 8$
   (c) $43 \times 17 + 57 \times 17$
   (d) $43x + 57x$ (for $x = 213$ or any other value)
7. Which properties of operations did you use in each part of question 6?

8.3 Combining like terms in algebraic expressions

REARRANGE TERMS, THEN COMBINE LIKE TERMS

To check whether two expressions are possibly equivalent, you can evaluate both expressions for several different values of the variable.

1. In each case below, copy the tables, then predict whether the two expressions are equivalent. Check by evaluating both for $x = 1$, $x = 10$, $x = 2$ and $x = -2$ in the tables.
   (a) $x(x + 3)$ and $x^2 + 3$

	1	10	2	–2
$x(x + 3)$	4	130	10	–2
$x^2 + 3$	4	103	7	7

   (b) $x(x + 3)$ and $x^2 + 3x$

	1	10	2	–2
$x(x + 3)$	4	130	10	–2
$x^2 + 3x$	4	130	10	–2

Some expressions can be simplified by rearranging the terms and combining "like terms".

In the expression $5x^2 + 13x + 7 + 2x^2 - 8x - 12$, the terms $5x^2$ and $2x^2$ are like terms.

Two or more like terms can be combined to form a single term.

$5x^2 + 2x^2$ can be replaced by $7x^2$ because for any value of x, for example $x = 2$ or $x = 10$, calculating $5x^2 + 2x^2$ and $7x^2$ will produce the same output value. Try it!

Answers

2. See the completed table at the top of LB page 75 alongside.
3. (a) See the answers in the table on LB page 75 alongside.
   (b) The answers are the same.
   (c) Both have the same terms; they are just arranged differently.
   (d) $5x^2 + 5x - 5$
   (e) It depends on the learners' answers for (d). If they answered $5x^2 + 5x - 5$, the values should be 545, 25 and 145.
   (f) Help learners who do not understand the question yet.
   A learner who justifies equivalence by pointing out that the answers obtained in (e) are the same as those in (a) clearly knows what equivalence means.
4. (a) $8x^2 + 6x + 12$ (b) $9x^2 + 2x + 3$
   (c) $10x^2 - 2x + 1$ (d) $-3x^2 - 3x - 8$
   (e) $-5x^2 - 4x + 2$ (f) $2y^2$

Mathematical background (continued)

The example below shows how we can use the properties to justify a simplification.

Simplify $4x - 6y + 9x$.

Step 1: apply the commutative property	$4x + 9x - 6y$
Step 2: apply the associative property	$(4x + 9x) - 6y$
Step 3: apply the distributive property	$x(4 + 9) - 6y$
Step 4: simplify within the bracket	$x(13) - 6y$
Step 5: apply the commutative property	$13x - 6y$

Normally we will simply add $4x$ and $9x$ to get $13x$. The explanation above shows exactly why we may do it.

2. Copy and complete the following table:

x	10	2	5	1
$5x^2 + 2x^2$	700	28	175	7
$7x^2$	700	28	175	7
$13x - 8x$	50	10	25	5
$5x$	50	10	25	5

It is difficult to see the like terms in a long expression like $3x^2 + 13x + 7 + 2x^2 - 8x - 12$. Fortunately, you can rearrange the terms in an expression so that the like terms are next to each other.

3. (a) Copy the table and complete the second and third rows of the table. You will complete the next two rows when you do question 3(g).

x	10	2	5	1
$3x^2 + 13x + 7 + 2x^2 - 8x - 12$	545	25	145	5
$3x^2 + 2x^2 + 13x - 8x + 7 - 12$	545	25	145	5
$5x^2 + 5x - 5$	545	25	145	5
$5(x^2 + x - 1)$	545	25	145	5

   (b) What do you observe?
   (c) How does the one expression in the above table differ from the other one?
   (d) Combine like terms in $3x^2 + 2x^2 + 13x - 8x + 7 - 12$ to make a shorter equivalent expression.
   (e) Evaluate your shorter expression for $x = 10$, $x = 2$ and $x = 5$.
   (f) Is your shorter expression equivalent to $3x^2 + 13x + 7 + 2x^2 - 8x - 12$? Explain how you know whether it is or is not.
   (g) Evaluate $5x^2 + 5x - 5$ and $5(x^2 + x - 1)$ for $x = 10$, $x = 2$, $x = 5$ and $x = 1$, and write your answers in the last two rows of the table.

4. Simplify each expression:
   (a) $(3x^2 + 5x + 8) + (5x^2 + x + 4)$ (b) $(7x^2 + 3x + 5) + (2x^2 - x - 2)$
   (c) $(6x^2 - 7x - 4) + (4x^2 + 5x + 5)$ (d) $(2x^2 - 5x - 9) - (5x^2 - 2x - 1)$
   (e) $(-2x^2 + 5x - 3) + (-3x^2 - 9x + 5)$ (f) $(y^2 + y + 1) + (y^2 - y - 1)$

Answers

5. See the completed table on LB page 76 alongside.

   It is of little value if learners can simplify expressions, but do not use this capacity to simplify the work required by questions like these. The two expressions simplify to $10x + 10$ and $x + 20$ respectively, which can be calculated mentally with ease.

6. (a) $9r^2 - 5r - 17$ (b) $-5r^2 + 11r + 7$
   (c) $-10x + 7xy + 8y$ (d) $14x + 3xy - 2y$

7. (a) $2x^3 - 2x^2 - 2x + 10x^3 + 15x^2 - 25x - 3x^3 - 6x^2 - 3x = 9x^3 + 7x^2 - 30x$
   $x = 3$: $9(27) + 7(9) - 90 = 216$ $x = -2$: $9(-8) + 7(4) - 30(-2) = 16$
   $x = 5$: $9(125) + 7(25) - 30(5) = 1\,150$ $x = -3$: $9(-27) + 7(9) - 30(-3) = -90$
   (b) $x^2 - 10x + 20$
   $x = 3$: $(9) - 10(3) + 20 = -1$ $x = -2$: $(4) - 10(-2) + 20 = 44$
   $x = 5$: $(25) - 10(5) + 20 = -5$ $x = -3$: $(9) - 10(-3) + 20 = 59$

8. (a) $3x^4 - x^2 - 2x$ (b) $3x^4 - x^2 + 2x$
   (c) $3x^4 + x^2 - 2x$ (d) $x - y - z + t$

9. (a) $2y^2 + y^2 - 3y = 3y^2 - 3y$ (b) $3x^2 + x^2 + 5x = 4x^2 + 5x$
   (c) $6x^2 - 3x^2 - x^4 = -x^4 + 3x^2$ (d) $2t^2 - 3t^2 + 5t^3 = 5t^3 - t^2$
   (e) $6x^2 - 4x^2 + 3x - 5x = 2x^2 - 2x$ (f) $2r^2 + 3r^2 - 5r - 7r + 7 - 8 = 5r^2 - 12r - 1$
   (g) $5x^2 + 2x^2 + 5x + 6x = 7x^2 + 11x$ (h) $2x^2 + 5x^2 - 6x + 10x = 7x^2 + 4x$

10. (a) $3x^2 + x^2 + 3x = 4x^2 + 3x$ (b) $5x + 7x - 2x^2 = -2x^2 + 12x$
    (c) $6r^2 - 2r^2 + 10r = 4r^2 + 10r$ (d) $2a^2 + 6a + 5a^2 - 10a = 7a^2 - 4a$
    (e) $6y^2 + 6y - 3y^2 - 6y = 3y^2$ (f) $8x^2 - 12x - 3x^2 - 6x = 5x^2 - 18x$
    (g) $2x^3 - 10x^2 - 3x^3 + 2x = -x^3 - 10x^2 + 2x$
    (h) $x^2 - x + 2x^2 + 3x - 6x^2 - 2x = -3x^2$

8.4 Multiplication of algebraic expressions

MULTIPLY POLYNOMIALS BY MONOMIALS

Teaching guidelines

The distributive property is used when learners expand expressions by multiplying a polynomial in a bracket by a monomial. This section develops the expansion of expressions thoroughly. Let learners work through it in detail.

5. Copy and complete the table. (Hint: Save yourself some work by simplifying first!)

x	2,5	3,7	6,4	12,9	35	−4,7	−0,04
$(3x + 6{,}5) + (7x + 3{,}5)$	35	47	74	139	360	−37	9,6
$(13x - 6) + (26 - 12x)$	22,5	23,7	26,4	32,9	55	15,3	19,96

6. Simplify:
   (a) $(2r^2 + 3r - 5) + (7r^2 - 8r - 12)$ (b) $(2r^2 + 3r - 5) - (7r^2 - 8r - 12)$
   (c) $(2x + 5xy + 3y) - (12x - 2xy - 5y)$ (d) $(2x + 5xy + 3y) + (12x - 2xy - 5y)$

7. Evaluate the following expressions for $x = 3$, $x = -2$, $x = 5$ and $x = -3$:
   (a) $2x(x^2 - x - 1) + 5x(2x^2 + 3x - 5) - 3x(x^2 + 2x + 1)$
   (b) $(3x^2 - 5x + 7) - (7x^2 + 3x - 5) + (5x^2 - 2x + 8)$

8. Write equivalent expressions without brackets:
   (a) $3x^4 - (x^2 + 2x)$ (b) $3x^4 - (x^2 - 2x)$
   (c) $3x^4 + (x^2 - 2x)$ (d) $x - (y + z - t)$

9. Write equivalent expressions without brackets, rearrange so that like terms are grouped together, and then combine the like terms:
   (a) $2y^2 + (y^2 - 3y)$ (b) $3x^2 + (5x + x^2)$
   (c) $6x^2 - (x^4 + 3x^2)$ (d) $2t^2 - (3t^2 - 5t^3)$
   (e) $6x^2 + 3x - (4x^2 + 5x)$ (f) $2r^2 - 5r + 7 + (3r^2 - 7r - 8)$
   (g) $5(x^2 + x) + 2(x^2 + 3x)$ (h) $2x(x - 3) + 5x(x + 2)$

10. Write equivalent expressions without brackets and simplify these expressions as far as possible.

    Example: $5r^2 - 2r(r + 5) = 5r^2 - 2r^2 - 10r$
    $= 3r^2 - 10r$

    (a) $3x^2 + x(x + 3)$ (b) $5x + x(7 - 2x)$
    (c) $6r^2 - 2r(r - 5)$ (d) $2a(a + 3) + 5a(a - 2)$
    (e) $6y(y + 1) - 3y(y + 2)$ (f) $4x(2x - 3) - 3x(x + 2)$
    (g) $2x^2(x - 5) - x(3x^2 - 2)$ (h) $x(x - 1) + x(2x + 3) - 2x(3x + 1)$

8.4 Multiplication of algebraic expressions

MULTIPLY POLYNOMIALS BY MONOMIALS

1. (a) Calculate 3×38 and 3×62, and add the two answers.
   (b) Add 38 and 62, then multiply the answer by 3.

Answers

1. (a) Learners can use any method, for example:

   $3 \times 38 = (3 \times 30) + (3 \times 8) = 90 + 24 = 114$

   $3 \times 62 = (3 \times 60) + (3 \times 2) = 180 + 6 = 186$

   $114 + 186 = 300$

   (b) $38 + 62 = (30 + 60) + (8 + 2) = 90 + 10 = 100$

   $100 \times 3 = 300$

2. See the answers on LB page 77 alongside.
3. (a) Sample answer: $12 = x$; $2 = y$ $\quad x + y = 14 \quad 3(x + y) = 3 \times 14 = 42$

   (b) Sample answer: $3 \times x + 3 \times y = (3 \times 12) + (3 \times 2) = 36 + 6 = 42$
4. See the completed table on LB page 77 alongside.

(c) If you do not get the same answer for (a) and (b), you have made a mistake. Rework until you get it right.

The fact that if you work correctly, you get the same answer in questions 1(a) and (b), is a demonstration of the **distributive property**.

The distributive property may be described as follows:
$a(b + c) = ab + ac$ and
$a(b - c) = ab - ac$,
where a, b and c can be any numbers.

What you saw in question 1 was that:
$3 \times 100 = 3 \times 38 + 3 \times 62$.
This can also be expressed by writing $3(38 + 62) = 3 \times 38 + 3 \times 62$.

2. (a) Calculate 10×56. $(10 \times 50) + (10 \times 6) = 500 + 60 = 560$

   (b) Calculate $10 \times 16 + 10 \times 40$. $(10 \times 10) + (10 \times 6) + (10 \times 40) = 100 + 60 + 400 = 560$

3. (a) Write down any two numbers smaller than 100. Let us call them x and y. Add your two numbers and multiply the answer by 3.
   (b) Calculate $3 \times x$ and $3 \times y$, and add the two answers.
   (c) If you do not get the same answers for (a) and (b), you have made a mistake somewhere. Correct your work.

4. Copy and complete the following table:

x	12	50	5
y	4	30	10
$5x - 5y$	$(5 \times 12) - (5 \times 4)$ $= 60 - 20$ $= 40$	$(5 \times 50) - (5 \times 30)$ $= 250 - 150$ $= 100$	$(5 \times 5) - (5 \times 10)$ $= 25 - 50$ $= -25$
$5(x - y)$	$(12 - 4) \times 5$ $= 8 \times 5$ $= 40$	$(50 - 30) \times 5$ $= 20 \times 5$ $= 100$	$(5 - 10) \times 5$ $= -5 \times 5$ $= -25$
$5x + 5y$	$(5 \times 12) + (5 \times 4)$ $= 60 + 20$ $= 80$	$(5 \times 50) + (5 \times 30)$ $= 250 + 150$ $= 400$	$(5 \times 5) + (5 \times 10)$ $= 25 + 50$ $= 75$
$5(x + y)$	$(12 + 4) \times 5$ $= 16 \times 5$ $= 80$	$(50 + 30) \times 5$ $= 80 \times 5$ $= 400$	$(5 + 10) \times 5$ $= 15 \times 5$ $= 75$

Performing the instructions $5(x + y)$ is not the only way in which you can find out how much $5(x + y)$ is for any given values of x and y. Instead of doing $5(x + y)$, you may do $5x + 5y$. In this case you will multiply first, and again, before you add.

CHAPTER 8: ALGEBRAIC EXPRESSIONS 77

Answers

5. (a) $8(x + y) = 8(10 + 20) = 8 \times 30 = 240$
   (b) $8(x + y) = (8 \times 10) + (8 \times 20) = 80 + 160 = 240$
6. $20(x - y) = 20(5 - 3) = 20 \times 2 = 40$; $20(x - y) = (20 \times 5) - (20 \times 3) = 100 - 60 = 40$
7. (a) $ab + ac$ (b) $ab + ac + ad$
   (c) $x^2 + x$ (d) $x^3 + x^2 + x$
   (e) $x^4 + x^3 + x^2 + x$ (f) $x^4 - x^3 + 3x^2$
   (g) $6x^4 + 4x^2$ (h) $6x^5 + 12x^4 - 15x^3$
   (i) $-2x^7 + 4x^6 + 8x^5 - 10x^4$ (j) $a^5b - a^4b + a^3b + a^2b$
   (k) $3x^4y^4 + x^3y^5 - x^2y^4$ (l) $-2x^4 + 2xy^3$
   (m) $6a^4b + 4a^4b^3 + 8a^2b^3$ (n) $6a^4b^2 - 2ab^2$
8. (a) $5x - 10 + 3x + 12$
   $= 8(5) + 2 = 42$
   (b) $x^2 + 4x - 4x - 16$
   $= 5^2 - 16 = 9$
   (c) $x^2 - 4x + 4x - 16$
   $= 5^2 - 16 = 9$
   (d) $x^3 + 3x^2 + 9x - 3x^2 - 9x - 27$
   $= 5^3 - 27 = 98$
   (e) $x^3 - 3x^2 + 9x + 3x^2 - 9x + 27$
   $= 5^3 + 27 = 152$
   (f) $x^4 - 3x^3 + 4x^2 - x^4 - 4x^3 - 2x^2 - 3x$
   $= -7(5^3) + 2(5^2) - 3(5) = -840$
9. (a) $x^3 + 2x2y + xy^2 + x^2y + 2xy^2 + y^3 = x^3 + 3x^2y + 3xy^2 + y^3$
   (b) $x^4y - 2x^3y^2 + x^2y^3 - 2x^3y^2 + 3x^2y^3 + xy^4 = x^4y - 4x^3y^2 + 4x^2y^3 + xy^4$
   (c) $ab^4c^3 - a^2b^2c^2 + a^2b^2c^4 + ab^3c^6$
   (d) $p^3q^3 + p^3q + p^2q^2 + p^2q - pq^3$

SQUARES AND CUBES AND ROOTS OF MONOMIALS

Teaching guidelines

Learners often omit to apply the exponent to the coefficient when they work with brackets, for example, for $(3x)^2$ they write as $3x^2$ instead of $9x^2$. Doing exercises should help them avoid making this mistake.

Answers

1. (a) $9x^2 \therefore 36; 225; 900$ (b) $9x^2 \therefore 36; 225; 900$
   (c) $4x^2 \therefore 16; 100; 400$ (d) $4x^2 \therefore 16; 100; 400$
   (e) $8x^3 \therefore 64; 1\,000; 8\,000$ (f) $8x^3 \therefore 64; 1\,000; 8\,000$
   (g) $25x^2 \therefore 100; 625; 2\,500$ (h) $9x^2 \therefore 36; 225; 900$
2. (a) $25x^2$ (b) $125x^3$ (c) $400x^2$
   (d) $1\,000x^3$ (e) $81x^2$ (f) $343x^3$

5. (a) For $x = 10$ and $y = 20$, evaluate $8(x + y)$ by first adding 10 and 20, and then multiplying by 8.
   (b) Now evaluate $8(x + y)$ by doing $8x + 8y$; in other words, first calculate 8×10 and 8×20.
6. In question 5 you evaluated $8(x + y)$ in two different ways for the given values of x and y. Now also evaluate $20(x - y)$ in two different ways, for $x = 5$ and $y = 3$.
7. Use the distributive property in each of the following cases to make a different expression that is equivalent to the given expression:
   (a) $a(b + c)$ (b) $a(b + c + d)$
   (c) $x(x + 1)$ (d) $x(x^2 + x + 1)$
   (e) $x(x^3 + x^2 + x + 1)$ (f) $x^2(x^2 - x + 3)$
   (g) $2x^2(3x^2 + 2)$ (h) $3x^3(2x^2 + 4x - 5)$
   (i) $-2x^4(x^3 - 2x^2 - 4x + 5)$ (j) $a^2b(a^3 - a^2 + a + 1)$
   (k) $x^2y^3(3x^2y + xy^2 - y)$ (l) $-2x(x^3 - y^3)$
   (m) $2a^2b(3a^2 + 2a^2b^2 + 4b^2)$ (n) $2ab^2(3a^3 - 1)$

What you do in this question is sometimes called "multiplication of a polynomial by a monomial". One may also say that in each case you **expand** the expression, or you write an equivalent expression in **expanded form**.

8. Expand the parts of each expression and simplify. Then evaluate the expression for $x = 5$.
   (a) $5(x - 2) + 3(x + 4)$ (b) $x(x + 4) - 4(x + 4)$
   (c) $x(x - 4) + 4(x - 4)$ (d) $x(x^2 + 3x + 9) - 3(x^2 + 3x + 9)$
   (e) $x(x^2 - 3x + 9) + 3(x^2 - 3x + 9)$ (f) $x^2(x^2 - 3x + 4) - x(x^3 + 4x^2 + 2x + 3)$
9. Write in expanded form:
   (a) $x(x^2 + 2xy + y^2) + y(x^2 + 2xy + y^2)$
   (b) $x^2y(x^2 - 2xy + y^2) - xy^2(2x^2 - 3xy - y^2)$
   (c) $ab^2c(b^2c^2 - ac) + b^2c^4(a^2 + abc^2)$
   (d) $p^2q(pq^2 + p + q) + pq(p - q^2)$

SQUARES AND CUBES AND ROOTS OF MONOMIALS

1. Evaluate each of the following expressions for $x = 2$, $x = 5$ and $x = 10$:
   (a) $(3x)^2$ (b) $9x^2$
   (c) $(2x)^2$ (d) $4x^2$
   (e) $(2x)^3$ (f) $8x^3$
   (g) $(2x + 3x)^2$ (h) $(10x - 7x)^2$
2. In each case, write an equivalent monomial without brackets:
   (a) $(5x)^2$ (b) $(5x)^3$
   (c) $(20x)^2$ (d) $(10x)^3$
   (e) $(2x + 7x)^2$ (f) $(20x - 13x)^3$

The square root of $16x^2$ is $4x$, because $(4x)^2 = 16x^2$.

Teaching guidelines

Finding the square roots of algebraic expressions works in the same way as finding the square root of numbers without algebraic extensions. At this stage, learners should only have to find the square root of expressions such as $25x^2$ or $(5x)^2$.

Remind learners that we cannot find the square root of a sum or a difference of terms. We first have to add (or subtract) the terms, for example question 3(*f*).

Work through a few examples to help learners understand.

Answers

3. (a) $7x$ (b) $3x$ (c) $20x$
   (d) $10x$ (e) $5x$ (f) $5x$
   (g) $5x$ (h) $5x$
4. (a) $7x$ (b) $3x$ (c) $20x$
   (d) $10x$ (e) $5x$ (f) $5x$

8.5 Dividing polynomials by integers and monomials

Teaching guidelines

Dividing by a number or an expression can also be seen as multiplication by its reciprocal. For example, dividing by 2 can be seen as multiplying by $\frac{1}{2}$. Therefore, if there is a bracket divided by an expression, the distributive property applies, but the division sign should be on the right-hand side of the bracket.

Answers

1. See the completed table on LB page 79 alongside.
   The expressions are equivalent. The top expression simplifies to the bottom expression, so all values for x yield the same results.
2. (a) R240 ÷ 20 = R12 each
   (b) Yes
   (c) No
3. (a) LHS = 12. RHS = 12. True
   (b) LHS = 12. RHS = 20 + 30. False
   (c) LHS = 12. RHS = 12. True

3. Write down the square root of each of the following expressions:
   (a) $\sqrt{(7x)^2}$ (b) $\sqrt{9x^2}$
   (c) $\sqrt{(20x)^2}$ (d) $\sqrt{100x^2}$
   (e) $\sqrt{(20x - 15x)^2}$ (f) $\sqrt{16x^2 + 9x^2}$
   (g) $\sqrt{(21x - 16x)^2}$ (h) $\sqrt{(5x)^2}$

The cube root of $64x^3$ is $4x$, because $(4x)^3 = 64x^3$.

4. Write down the cube root of each of the following expressions:
   (a) $\sqrt[3]{(7x)^3}$ (b) $\sqrt[3]{27x^3}$
   (c) $\sqrt[3]{(20x)^3}$ (d) $\sqrt[3]{1\,000x^3}$
   (e) $\sqrt[3]{(20x - 15x)^3}$ (f) $\sqrt[3]{125x^3}$

8.5 Dividing polynomials by integers and monomials

1. Copy and complete the following table:

x	20	10	5	–5	–10	–20
$(100x - 5x^2) \div 5x$	0	10	15	25	30	40
$20 - x$	0	10	15	25	30	40

   Can you explain your observations?

2. (a) R240 prize money must be shared equally between 20 netball players. How much should each one get?
   (b) Mpho decided to do the calculations below. Do not do Mpho's calculations, but think about this: Will Mpho get the same answer that you got for question (a)?
   (140 ÷ 20) + (100 ÷ 20) Yes.
   (c) Gert decided to do the calculations below. Without doing the calculations, say whether or not Gert will get the same answer that you got for question (a).
   (240 ÷ 12) + (240 ÷ 8) No.

3. Do the necessary calculations to find out whether the following statements are true or false:
   (a) (140 + 100) ÷ 20 = (140 ÷ 20) + (100 ÷ 20)
   (b) 240 ÷ (12 + 8) = (240 ÷ 12) + (240 ÷ 8)
   (c) (300 – 60) ÷ 20 = (300 ÷ 20) – (60 ÷ 20)

CHAPTER 8: ALGEBRAIC EXPRESSIONS 79

Teaching guidelines

The distributive property for division applies to terms that are added in the brackets and terms that are subtracted, refer to the LB page alongside where this is described.

Answers

4. (a) $2x^2 + x$; 10; 210 (b) $2x^2 + x$; 10; 210
   (c) $2x^2 + x$; 10; 210 (d) $2x + 1$; 5; 21
   (e) $2x + 1$; 5; 21 (f) $2x + 1$; 5; 21
5. (a) $2x^2 + 4x - 3$
   (b) $(10x^2 + 20x - 15) \div 5 = 2x^2 + 4x - 3$
6. (a) $x + y$ (b) $x + 2y$
   (c) $5y + 4$ (d) $7x - 1$
   (e) $28x^2 - 7x + 1$ (f) $3x + 2$
   (g) $10x - 8$
7. (a) $\frac{9x}{y} + 1$ (b) $24 - 15b + 8b^2$
   (c) $3a + 1$ (d) $13 - 17b$
   (e) $3a + 5a^2$ (f) $39a + 13 + b$

Teaching guidelines

Introduce the division notation. This notation may help some learners to see clearly what the division instruction looks like.

Inform learners that when they use this notation they must make sure that they draw the "division line" underneath the whole expression; from the coefficient of the first term to the end of the last term. Often learners are untidy in their writing and write a division like this $\frac{10x^2}{5} + 20x - 15$ instead of this $\frac{10x^2 + 20x - 15}{5}$.

Misconceptions

When a division is given in the form discussed above, learners will typically divide only $10x^2$ by 5. Using the distributive property of division may clear up this misconception.

Division is **right-distributive** over addition and subtraction, for example:
$(2 + 3) \div 5 = (2 \div 5) + (3 \div 5)$.

The division symbol is to the right of the brackets; it is not left-distributive, for example:
$10 \div (2 + 4) \neq (10 \div 2) + (10 \div 4)$.

For example: $(200 + 40) \div 20 = (200 \div 20) + (40 \div 20) = 10 + 2 = 12$, and
$(500 + 200 - 300) \div 50 = (500 \div 50) + (200 \div 50) - (300 \div 50)$

4. Evaluate each expression for $x = 2$ and $x = 10$:
   (a) $(10x^2 + 5x) \div 5$ $= 2x^2 + x$ (b) $(10x^2 \div 5) + (5x \div 5)$ $= 2x^2 + x$
   (c) $2x^2 + x$ (d) $(10x^2 + 5x) \div 5x$ $= 2x + 1$
   (e) $(10x^2 \div 5x) + (5x \div 5x)$ $= 2x + 1$ (f) $2x + 1$

The distributive property of division can be expressed in the following way:
$(x + y) \div z = (x \div z) + (y \div z)$
$(x - y) \div z = (x \div z) - (y \div z)$

5. (a) Do not do any calculations. Which of the following expressions do you *think* will have the same value as $(10x^2 + 20x - 15) \div 5$, for $x = 10$ as well as $x = 2$?
   $2x^2 + 20x - 15$ $10x^2 + 20x - 3$ $2x^2 + 4x - 3$ $2x^2 + 4x - 3$
   (b) Do the necessary calculations to check your answer.
6. Simplify:
   (a) $(2x + 2y) \div 2$ (b) $(4x + 8y) \div 4$
   (c) $(20xy + 16x) \div 4x$ (d) $(42x - 6) \div 6$
   (e) $(28x^4 - 7x^3 + x^2) \div x^2$ (f) $(24x^2 + 16x) \div 8x$
   (g) $(30x^2 - 24x) \div 3x$
7. Simplify:
   (a) $(9x^2 + xy) \div xy$ (b) $(48a - 30ab + 16ab^2) \div 2a$
   (c) $(3a^3 + a^2) \div a^2$ (d) $(13a - 17ab) \div a$
   (e) $(3a^2 + 5a^3) \div a$ (f) $(39a^2b + 13ab + ab^2) \div ab$

The instruction $72 \div 6$ may also be written as $\frac{72}{6}$.

This notation, which looks just like the common fraction notation, is often used to indicate division.

Hence, instead of $(10x^2 + 20x - 15) \div 5$, we may write $\frac{10x^2 + 20x - 15}{5}$.

Since $(10x^2 + 20x - 15) \div 5$ is equivalent to $(10x^2 \div 5) + (20x \div 5) - (15 \div 5)$,
$\frac{10x^2 + 20x - 15}{5}$ is equivalent to $\frac{10x^2}{5} + \frac{20x}{5} - \frac{15}{5}$.

Answers

8. (a) $4x - 3$ (b) $4x^2 - 3$
   (c) $4x^2 - 3x$ (d) $4x - 3$
   (e) $8x^2 - 6x$ (f) $2x^2 - 1\frac{1}{2}$

9. See the answers on LB page 81 alongside.

10. (a) LHS simplifies to RHS using distributive property. Division by 0 is not allowed.
    (b) LHS simplifies to RHS using distributive property. Division by 0 is not allowed.

11. See the completed table on LB page 81 alongside.

12. See the answers on LB page 81 alongside.

8. Find a simpler equivalent expression for each of the following expressions (clearly, these expressions do not make sense if $x = 0$):

(a) $\frac{16x^2 - 12x}{4x}$ (b) $\frac{16x^3 - 12x}{4x}$

(c) $\frac{16x^3 - 12x^2}{4x}$ (d) $\frac{16x^3 - 12x^2}{4x^2}$

(e) $\frac{16x^3 - 12x^2}{2x}$ (f) $\frac{16x^3 - 12x}{8x}$

9. In each case check if the statement is true for $x = 10$, $x = 100$, $x = 5$, $x = 1$ and $x = -2$.

(a) $\frac{x^2}{x} = x$ True (b) $\frac{x^3}{x} = x^2$ True (c) $\frac{x^3}{x^2} = x$ True

(d) $\frac{5x^3}{x} = 5x^2$ True (e) $\frac{5x^3}{x} = 5^3$ False, RHS should be $5x^2$, which is 5^3 only if $x = 5$

(f) $\frac{5x}{x^2} = \frac{5}{x}$ True

10. Explain why the equations below are true:

(a) $\frac{100x - 5x^2}{5x} = 20 - x$ for all values of x, except $x = 0$.

(b) $\frac{15x^2 - 10x}{5x}$ is equivalent to $3x - 2$, excluding $x = 0$.

11. Copy and complete the following table:

x	1,5	2,8	–3,1	0,72
$\frac{3x + 12}{3}$	5,5	6,8	0,9	4,72
$\frac{18x^2 + 6}{6}$	7,75	24,52	29,83	2,5552
$\frac{5x^2 + 7x}{x}$	14,5	21	–8,5	10,6

(Hint: Simplify the expressions first to save yourself some work!)

12. Simplify each expression to the equivalent form requiring the fewest operations:

(a) $\frac{3a + a^2}{a}$ $3 + a$ (b) $\frac{x^3 + 2x^2 - x}{x}$ $x^2 + 2x - 1$ (c) $\frac{2a + 12ab}{2a}$ $1 + 6b$

(d) $\frac{12x^2 + 10x}{2x}$ $6x + 5$ (e) $\frac{21ab - 14a^2}{7a}$ $3b - 2a$ (f) $\frac{15a^2b + 30ab^2}{5ab}$ $3a + 6b$

(g) $\frac{7x^3 + 21x^2}{7x^2}$ $x + 3$ (h) $\frac{3x^2 + 9x}{3x}$ $x + 3$

Answers

13. (a) $x + 5 = 20$ (b) $5 - 3x = 2$
$x = 15$ $x = 1$

14. See the completed table on LB page 82 alongside.

15. (a) $\frac{15x^2 + 12x + 30x^2 + 18x}{5x}$ (b) $2x - 4 + 8 - 6x = -4x + 4$
$\frac{45x^2 + 30x}{5x} = 9x + 6$

8.6 Products and squares of binomials

Teaching guidelines

Learners have learnt to apply the distributive property for multiplying a binomial by a monomial, for example, $a(b + c) = ab + ac$. Extend this expansion to a multiplication of a binomial by a binomial by changing a to a binomial, for example $(x + y)$, as follows:

$a(b + c) = ab + ac$

$(x + y)(b + c) = (x + y)b + (x + y)c$ Then apply the distributive property again.

$(x + y)(b + c) = xb + yb + xc + yc$

The expansion leads to four terms. The expressions on either side of the equal sign are equivalent.

Now we can investigate special cases of the products of two binomials, for example:

1. the algebraic parts of the terms in the two brackets are the same but the coefficients differ, for example:
$(3x + 2)(x + 4) = (3x + 2)x + (3x + 2)4 = 3x^2 + 2x + 12x + 8 = 3x^2 + 14x + 8$
2. the two brackets are exactly the same: $(x + y)(x + y) = x^2 + 2xy + y^2$, the product can be written as a square of the binomial: $(x + y)^2$.

Answers

1. Replace x with various values in both expressions and check whether the outputs are the same.
2. See the answers in the table on LB page 83 on the following page.

13. Solve the equations:
(a) $\frac{3x^2 + 15x}{3x} = 20$ (b) $\frac{30x - 18x^2}{6x} = 2$

14. Copy and complete the following table:

	x	1,1	1,2	1,3	1,4	1,5
(a)	$\frac{x^3 + 2x^2 - x}{x}$	2,41	2,84	3,29	3,76	4,25
(b)	$\frac{7x^3 + 21x^2}{7x^2}$	4,1	4,2	4,3	4,4	4,5
(c)	$\frac{50x^2 + 5x}{5x}$	12	13	14	15	16

15. Simplify the following expressions:
(a) $\frac{3x(5x + 4) + 6x(5x + 3)}{5x}$ (b) $\frac{14x^2 - 28x}{7x} + \frac{24x - 18x^2}{3x}$

8.6 Products and squares of binomials

How can we obtain the expanded form of $(x + 2)(x + 3)$?

In order to expand $(x + 2)(x + 3)$, you can first keep $(x + 2)$ as it is, and apply the distributive property:

$(\mathbf{x + 2})(x + 3)$
$= (\mathbf{x + 2})x + (\mathbf{x + 2})3$
$= x^2 + 2x + 3x + 6$
$= x^2 + 5x + 6$

1. Describe how can you check if $(x + 2)(x + 3)$ is actually equivalent to $x^2 + 5x + 6$.

To expand $(x - y)(x + 3y)$ it can be written as $(x - y)x + (x - y)3y$, and the two parts can then be expanded.

$(x - y)(x + 3y)$
$= (x - y)x + (x - y)3y$
$= x^2 - xy + 3xy - 3y^2$
$= x^2 + 2xy - 3y^2$

2. Do some calculations to check whether $(x - y)(x + 3y)$ and $x^2 + 2xy - 3y^2$ are equivalent. Write the results of your calculations in a table like the one on page 83.

Answers

3. (a) $x(x + 4) + 3(x + 4)$
$= x^2 + 4x + 3x + 12$
$= x^2 + 7x + 12$

(b) $x(4 - x) + 3(4 - x)$
$= 4x - x^2 + 12 - 3x$
$= -x^2 + x + 12$

(c) $x(x - 5) + 3(x - 5)$
$= x^2 - 5x + 3x - 15$
$= x^2 - 2x - 15$

(d) $2x^2(3x - 4) + 1(3x - 4)$
$= 6x^3 - 8x^2 + 3x - 4$

(e) $x(x + 2y) + y(x + 2y)$
$= x^2 + 2xy + xy + 2y^2$
$= x^2 + 3xy + 2y^2$

(f) $a(2a + 3b) - b(2a + 3b)$
$= 2a^2 + 3ab - 2ab - 3b^2$
$= 2a^2 + ab - 3b^2$

(g) $k^2(k^2 + 2m) + m(k^2 + 2m)$
$= k^4 + 2k^2m + k^2m + 2m^2$
$= k^4 + 3k^2m + 2m^2$

(h) $2x(2x - 3) + 3(2x - 3)$
$= 4x^2 - 6x + 6x - 9$
$= 4x^2 - 9$

(i) $5x(5x - 2) + 2(5x - 2)$
$= 25x^2 - 10x + 10x - 4$
$= 25x^2 - 4$

(j) $ax(ax + by) - by(ax + by)$
$= a^2x^2 + abxy - abxy - b^2y^2$
$= a^2x^2 - b^2y^2$

4. (a) $a^2 + 2ab + b^2$ (b) $a^2 - 2ab + b^2$
(c) $x^2 + 2xy + y^2$ (d) $x^2 - 2xy + y^2$
(e) $4a^2 + 12ab + 9b^2$ (f) $4a^2 - 12ab + 9b^2$
(g) $25x^2 + 20xy + 4y^2$ (h) $25x^2 - 20xy + 4y^2$
(i) $a^2x^2 + 2abx + b^2$ (j) $a^2x^2 - 2abx + b^2$

5. (a) $m^2 + 2mn + n$ (b) $m^2 - 2mn + n^2$
(c) $9x^2 + 12xy + 4y^2$ (d) $9x^2 - 12xy + 4y^2$

6. (a) $a^2x^2 + 2abx + b^2$ (b) $a^2x^2 - 2abx + b^2$
(c) $4s^2 + 20s + 25$ (d) $4s^2 - 20s + 25$
(e) $a^2x^2 + 2abxy + b^2y^2$ (f) $a^2x^2 - 2abxy + b^2y^2$
(g) $4s^2 + 20rs + 25r^2$ (h) $4s^2 - 20rs + 25r^2$

7. (a) $24x^2 + 16x + 18x + 12 + 24x^2 + 15x + 16x + 10 = 48x^2 + 65x + 22$
(b) $24x^2 + 16x + 18x + 12 - 24x^2 - 15x - 16x - 10 = 3x + 2$

x	-1	0	0	1	2
y	0	1	2	-1	0
$(x - y)(x + 3y)$	1	-3	-12	-4	4
$x^2 + 2xy - 3y^2$	1	-3	-12	-4	4

3. Expand each of these expressions:
(a) $(x + 3)(x + 4)$ (b) $(x + 3)(4 - x)$
(c) $(x + 3)(x - 5)$ (d) $(2x^2 + 1)(3x - 4)$
(e) $(x + y)(x + 2y)$ (f) $(a - b)(2a + 3b)$
(g) $(k^2 + m)(k^2 + 2m)$ (h) $(2x + 3)(2x - 3)$
(i) $(5x + 2)(5x - 2)$ (j) $(ax - by)(ax + by)$

4. Expand each of these expressions:
(a) $(a + b)(a + b)$ (b) $(a - b)(a - b)$
(c) $(x + y)(x + y)$ (d) $(x - y)(x - y)$
(e) $(2a + 3b)(2a + 3b)$ (f) $(2a - 3b)(2a - 3b)$
(g) $(5x + 2y)(5x + 2y)$ (h) $(5x - 2y)(5x - 2y)$
(i) $(ax + b)(ax + b)$ (j) $(ax - b)(ax - b)$

5. Can you guess the answer to each of the following questions without working it out as you did in question 3? Try them out and then check your answers.
Expand the following expressions:
(a) $(m + n)(m + n)$ (b) $(m - n)(m - n)$
(c) $(3x + 2y)(3x + 2y)$ (d) $(3x - 2y)(3x - 2y)$

All the expressions in questions 4 and 5 are **squares of binomials**, for example $(ax + b)^2$ and $(ax - b)^2$.

6. Expand:
(a) $(ax + b)^2$ (b) $(ax - b)^2$
(c) $(2s + 5)^2$ (d) $(2s - 5)^2$
(e) $(ax + by)^2$ (f) $(ax - by)^2$
(g) $(2s + 5r)^2$ (h) $(2s - 5r)^2$

7. Expand and simplify:
(a) $(4x + 3)(6x + 4) + (3x + 2)(8x + 5)$
(b) $(4x + 3)(6x + 4) - (3x + 2)(8x + 5)$

8.7 Substitution into algebraic expressions

Teaching guidelines

Discuss the fact that the expressions that are simplified, equivalent versions of other expressions, are easier to work with, for example 2(b) and 2(f).

Let learners discuss their answers and find the simplified version of the expressions in question 3, for example.

Answers

1. Learners' own work
2. See the completed table on LB page 84 alongside.
3. See the completed table on LB page 84 alongside.
4. All the answers for question 3 have a constant increase every time x increases by 1. This is because all the expressions simplify to expressions where x is simply multiplied and added to, as all the higher powers of x fall away.
5. See the completed table on LB page 84 alongside.

8.7 Substitution into algebraic expressions

1. In question 2 you have to find the values of different expressions, for some given values of x. Look carefully at the different expressions in the table. Do you think some of them may be equivalent?

   Simplify the longer expression to check whether you end up with the shorter expression.

2. Copy and complete the following table:

	x	13	–13	2,5	10
(a)	$(2x + 3)(3x - 5)$	986	1 012	20	575
(b)	$10x^2 + 5x - 7 + 3x^2 - 4x - 3$	2 200	2 174	73,75	1 300
(c)	$3(10x^2 - 5x + 2) - 5x(6x - 4)$	71	–59	18,5	56
(d)	$13x^2 + x - 10$	2 200	2 174	73,75	1 300
(e)	$6x^2 - x - 15$	986	1 012	20	575
(f)	$5x + 6$	71	–59	18,5	56

3. Copy and complete the following table:

	x	1	2	3	4
(a)	$(2x + 3)(5x - 3) + (10x + 9)(1 - x)$	10	20	30	40
(b)	$\frac{9x^2 + 30x}{3x}$	13	16	19	22
(c)	$3x(10x - 5) - 5x(6x - 4)$	5	10	15	20
(d)	$5x(4x + 3) - 2x(7 + 13x) + 2x(3x + 2)$	5	10	15	20

4. Describe any patterns that you observe in your answers for question 3.

5. Copy and complete the following table:

	x	1,5	2,5	3,5	4,5
(a)	$(2x + 3)(5x - 3) + (10x + 9)(1 - x)$	15	25	35	45
(b)	$\frac{9x^2 + 30x}{3x}$	14,5	17,5	20,5	23,5
(c)	$3x(10x - 5) - 5x(6x - 4)$	7,5	12,5	17,5	22,5
(d)	$5x(4x + 3) - 2x(7 + 13x) + 2x(3x + 2)$	7,5	12,5	17,5	22,5

Grade 9 Term 1 Chapter 9 Equations

Learner Book Overview		
Sections in this chapter	**Content**	**Pages in Learner Book**
9.1 Solving equations by inspection	Using a table of given values with values of different expressions to find equivalent equations	Page 85
9.2 Solving equations using additive and multiplicative inverses	Understanding that inverse operations undo the original operations; using flow diagrams; solving equations using inverse operations	Pages 85 to 87
9.3 Setting up equations	Constructing equations by writing sets of equivalent equations, starting with a value as a solution and building up an equation; solving equations using the equivalent equations; recognising the ordered pairs of number patterns as solutions to equations	Pages 87 to 89
9.4 Equations and situations	Describing real situations in algebraic terms by making an equation to describe the situation; solving the equations to find solutions to the problem	Pages 89 to 90
9.5 Solving equations by using the laws of exponents	Finding an unknown if it is an exponent in an exponential equation; finding an unknown if it is the base in an exponential equation	Pages 90 to 91

CAPS time allocation	4 hours
CAPS content specification	Page 133

Mathematical background

Solving an equation means finding the value of the unknown that makes the equation true.

We can solve equations by inspection, for example, by finding the solution to $3x - 4 = 17$ from a table of values.

If we use a table of values for different expressions, we can find (by inspection) the value of x for which, for example, $3x - 2$ and $x + 4$ have the same value, in other words, for which $3x - 2 = x + 4$.

The idea of equivalent equations (different equations with the same solution) helps us understand how we use inverse operations (and later factoring) to solve equations. For example: Start with a simple solution and build up an equation by performing the same operation on both sides of the equation.

$x = 4$ multiply both sides by the same number

$3x = 12$ add or subtract the same number from both sides

$3x + 5 = 17$

To solve the equation, the inverse operations will be performed on the equation – i.e. subtract five from both sides and then divide by three on both sides.

Setting up equations to describe situations is easier if we first analyse the situation and consider how we would calculate what we want to know, and then try to describe the calculations algebraically. This is the process of creating a mathematical model in the form of an equation or equations by which we describe the situation, and then solve the equations, test the solutions against the situation and improve it if need be.

9.1 Solving equations by inspection

Teaching guidelines

Solving equations by inspection requires the learners to consciously ask the question, "For what value of x will the equation be true?" It is a valuable exercise at the beginning of this chapter to ensure that learners know what solving equations is really about. You could set the following as a diagnostic task:

(a) Solve the equation $20x - 12 + 4(5 - 3x) = 64$

(b) How much is $20x - 12 + 4(5 - 3x)$ for $x = 7$?

(c) Are you sure you worked correctly in question (a)? If not, do it again and check whether you now get the correct answer.

Learners who solve the equation incorrectly in (a) and then find that $20x - 12 + 4(5 - 3x)$ is 64 for $x = 7$ when they do (b), should realise that they have made a mistake in (a) and try to correct it when they do (c). If they do not, it suggests they may not realise that to solve the equation (question (a)) means to find the value of x.

Some learners may even solve the equation correctly in (a) but then laboriously evaluate $20x - 12 + 4(5 - 3x)$ for $x = 7$ in (b), and make mistakes. It may be interesting to hear how such learners respond to the question, "Why do you do these calculations, don't you already know the value of the expression if $x = 7$?"

In this case learners can ask, "For what value of x will the left-hand side of the equation be equal to the right-hand side?"

Answers

1. (a) $x = 0$ (b) $x = 1$ (c) $x = 1$ (d) $x = 2$ (e) $x = 3$ (f) $x = 2$
2. (b) and (c) have the same solution, $x = 1$; (d) and (f) have the same solution, $x = 2$.

9.2 Solving equations using additive and multiplicative inverses

Teaching guidelines

Follow the development given in the LB on pages 85 to 87 for learners to understand what is meant by using the inverse operations.

Learners should see that an inverse operation undoes the original operation. When we solve equations, we want to get back to the number or value we started with when the equation was set up. Flow diagrams help to make this clear.

Answers

1. (a) $x = 3$ from $10 - 7$ (b) $x = 5$ from $13 - 3$ then dividing the answer by 2

CHAPTER 9
Equations

9.1 Solving equations by inspection

1. Six equations are listed in the table below. Use the table to find out for which of the given values of x will be true that the left-hand side of the equation is equal to the right-hand side.

"Searching" for the solution of an equation by using tables is called **solution by inspection**.

x	–3	–2	–1	0	1	2	3	4
$2x + 3$	–3	–1	1	3	5	7	9	11
$x + 4$	1	2	3	4	5	6	7	8
$9 - x$	12	11	10	9	8	7	6	5
$3x - 2$	–11	–8	–5	–2	1	4	7	10
$10x - 7$	–37	–27	–17	–7	3	13	23	33
$5x + 3$	–12	–7	–2	3	8	13	18	23
$10 - 3x$	19	16	13	10	7	4	1	–2

(a) $2x + 3 = 5x + 3$ (b) $5x + 3 = 9 - x$ (c) $2x + 3 = x + 4$
(d) $10x - 7 = 5x + 3$ (e) $3x - 2 = x + 4$ (f) $9 - x = 2x + 3$

Two equations can have the same solution. For example, $5x = 10$ and $x + 2 = 4$ have the same solution; $x = 2$ is the solution for both equations.

Two equations are called **equivalent** if they have the same solution.

2. Which of the equations in question 1 have the same solutions? Explain.

9.2 Solving equations using additive and multiplicative inverses

1. In each case find the value of x:

(a) x → [+ 7] → 10 $x = 3$

(b) x → [× 2] → [+ 3] → 13 $x = 5$

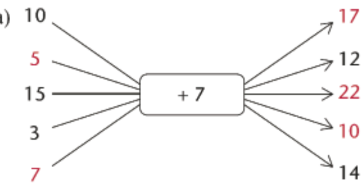
CHAPTER 9: EQUATIONS 85

Answers

2. See the answers on LB page 86 alongside.
3. See the answers on LB page 86 alongside.
4. Multiply a number by 2 and add 3 to the answer: $2x + 3$
5. See the answers on LB page 86 alongside.
6. The input numbers in question 2(b) are the output numbers in question 5 and the output numbers in question 2(b) are the input numbers in question 5.
7. (a) You get the number you started with.
   (b) You get the number you started with.

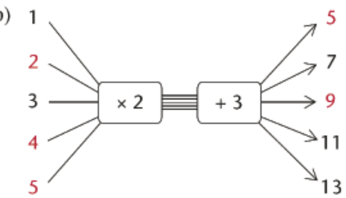
2. Copy and complete the flow diagrams. Fill in all the missing numbers.

(a)



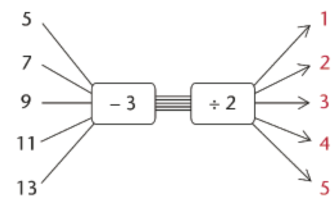
To find the second input number you may say to yourself, *"After I added 7, I had 12. What did I have before I added 7?"*

(b)



To find the input number that corresponds to 13, you may ask yourself, *"What did I have before I added 3?"* and then, *"What did I have before I multiplied by 2?"*

3. Use your answers for question 2 to check your answers for question 1.
4. Describe the instructions in flow diagram 2(b) in words, and also with a symbolic expression.
5. Copy and complete the following flow diagram:



This flow diagram is called the **inverse** of the flow diagram in question 2(b).

6. Compare the input numbers and the output numbers of the flow diagrams in question 2(b) and question 5. What do you notice?
7. (a) Add 5 to any number and then subtract 5 from your answer. What do you get?
   (b) Multiply any number by 10 and then divide the answer by 10. What do you get?

If you add a number and then subtract the same number, you are back where you started. This is why addition and subtraction are called **inverse operations**.

If you multiply by a number and then divide by the same number, you are back where you started. This is why multiplication and division are called **inverse operations**.

Answers

8. (a) $2x = 18$
   $x = 9$
   (b) $3x = 21$
   $x = 7$
   (c) $5x = 55$
   $x = 11$
   (d) $\frac{1}{3}x = 8$
   $x = 24$
   (e) $10x + 30 = 88$
   $10x = 58$
   $x = 5{,}8$
   (f) $2x - 26 = 14$
   $2x = 40$
   $x = 20$

9.3 Setting up equations

CONSTRUCTING EQUATIONS

Teaching guidelines

Constructing equations may help learners to understand what is achieved when an equation is solved. They start with a solution, as demonstrated on LB page 87 and build up the equation with each step. As each new operation is performed on both sides of the equation, the original solution holds, which means the set of equations have the same solution and are therefore equivalent.

The idea of equivalent equations (different equations with the same solution) helps learners understand how we use inverse operations and factoring to solve equations.

Answers

1. $x = 5$
2. Learners' answers will differ. Below is a sample answer:

	$x = 3$
Subtract 5 on both sides	$x - 5 = -2$
Multiply both sides by –5	$-5x + 25 = 10$

3. See Bongile's completed working on LB page 87 alongside.
4. (a) 15
   (b) $x = 5$

The expression $5x - 3$ says *"multiply by 5 then subtract 3"*. This instruction can also be given with a flow diagram: → [× 5] → [– 3] →

The equation $5x - 3 = 47$ can also be written as a flow diagram:

→ [× 5] → [– 3] → 47

8. Solve the equations below. You may do this by using the inverse operations. You may write a flow diagram to help you to see the operations.
   (a) $2x + 5 = 23$
   (b) $3x - 5 = 16$
   (c) $5x - 60 = -5$
   (d) $\frac{1}{3}x + 11 = 19$
   (e) $10(x + 3) = 88$
   (f) $2(x - 13) = 14$

9.3 Setting up equations

CONSTRUCTING EQUATIONS

You can easily make an equation that has 5 as the solution. Here is an example:

Start by writing the solution	$x = 5$
Add 3 to both sides	$x + 3 = 8$
Multiply both sides by 5	$5x + 15 = 40$

1. What is the solution of the equation $5x + 15 = 40$?
2. Make your own equation with the solution $x = 3$.
3. Bongile worked like this to make the equation $2(x + 8) = 30$, but he rubbed out part of his work:

Start by writing the solution	$x = 7$
Add 8 to both sides	$x + 8 = 15$
Multiply both sides by 2	$2(x + 8) = 30$

   Copy and complete Bongile's writing to solve the equation $2(x + 8) = 30$.

4. This is how Bongile made a more difficult equation:

Start by writing the solution	$x = 5$
Multiply by 3 on both sides	$3x = 15$
Subtract 9 from both sides	$3x - 9 = 6$
Add $2x$ to both sides	$5x - 9 = 2x + 6$

   (a) What was on the right-hand side before Bongile subtracted 9?
   (b) What is the solution of $5x - 9 = 2x + 6$?

Answers

5. See the steps taken by Bongile on LB page 88 alongside.

SOLVING EQUATIONS

Teaching guidelines

Learners should apply the inverse operations to the equation to isolate the unknown. Flow diagrams can help them decide what should be done first (that which was done last in the build-up of the equation).

Answers

1. (a) $5x = 21 - 2x$
   $7x = 21$
   $x = 3$

   (b) $2x = -13$
   $x = -6\frac{1}{2}$

   (c) $-x = x - 6$
   $-2x = -6$
   $x = 3$

   (d) $12x + 6 = 0$
   $12x = -6$
   $x = -\frac{1}{2}$

2. (a) $4 - 8x = 12 - 7x$
   $4 - x = 12$
   $-x = 8$
   $x = -8$

   (b) $8 - 24x = 20x + 30$
   $-24x = 20x + 22$
   $-44x = 22$
   $x = -\frac{1}{2}$

   (c) $4x - 10 = 7$
   $4x = 17$
   $x = \frac{17}{4}$
   $x = 4\frac{1}{4}$

   (d) $1{,}6x = 3{,}5x - 3{,}8$
   $-1{,}9x = -3{,}8$
   $x = \frac{(-3{,}8)}{(-1{,}9)}$
   $x = 2$

NUMBER PATTERNS AND EQUATIONS

Teaching guidelines

Learners make a table of ordered pairs by substituting into an expression. This means that for each value of y, an equation like $7n - 34 = y$ has a different solution, the value of n.

Answers

1. (a) Rule E produces the number pattern in the second row of the table.
   (b) $5n + 3 = 143$, hence $5n = 140$, hence $n = 28$.
   (c) Term value = $5n + 3$, so term 28 = $28 \times 5 + 3 = 140 + 3 = 143$.

5. Bongile started with a solution and he ended up with an equation. Write down the steps that Bongile took to make the equation, and solve the equation:

	$x = 4$
Multiply by 8	$8x = 32$
Add 3	$8x + 3 = 35$
Subtract $5x$	$3x + 3 = 35 - 5x$

SOLVING EQUATIONS

To make an equation, you can apply the same operation on both sides.

To solve an equation, you can apply the inverse operation on both sides.

	$x = 4$	
Multiply by 8	$8x = 32$	Divide by 8
Add 3	$8x + 3 = 35$	Subtract 3
Subtract $5x$	$3x + 3 = 35 - 5x$	Add $5x$

Use any appropriate method to solve the following equations:

1. (a) $5x + 3 = 24 - 2x$ (b) $2x + 4 = -9$
   (c) $3 - x = x - 3$ (d) $6(2x + 1) = 0$
2. (a) $4(1 - 2x) = 12 - 7x$ (b) $8(1 - 3x) = 5(4x + 6)$
   (c) $7x - 10 = 3x + 7$ (d) $1{,}6x + 7 = 3{,}5x + 3{,}2$

NUMBER PATTERNS AND EQUATIONS

1. (a) Which of the following rules will produce the number pattern given in the second row of the table below?
   A. Term value = $8n$ where n is the term number
   B. Term value = $6n - 1$ where n is the term number
   C. Term value = $6n + 2$ where n is the term number
   D. Term value = $10n - 2$ where n is the term number
   E. Term value = $5n + 3$ where n is the term number

Term number	1	2	3	4	5	6	7	8	9
Term value	8	13	18	23	28	33	38	43	48

   (b) The sixth term of the sequence has the value 33. Which term will have the value 143? You may set up and solve an equation to find out.
   (c) Apply rule E to your answer, to check if your answer is correct.

88 MATHEMATICS GRADE 9: TERM 1

Answers

2. (a) The rule is: Term value = $3n + 2$.
   (b) $3n + 2 = 221$, hence $3n = 219$, hence $n = 73$.
3. (a) See the completed table on LB page 89 alongside.
   (b) $4n + 11 = 7n - 34$, hence $3n = 45$, hence $n = 15$ (not seen in this table).

9.4 Equation and situations

Teaching guidelines

A big challenge in Mathematics education all over the world is for learners to describe real situations in algebraic terms, i.e. to make "mathematical models" such as equations of real situations. This translation from reality to mathematics may be easier if learners first consider how they would calculate what they need to know, and then try to to describe the calculations algebraically. This is the aim of questions 1 and 2 on LB page 89.

Answers

1. (a) R80 × 10 + R400 = R800 + R400 = R1 200
   (b) R80 × 15 + R400 = R1 200 + R400 = R1 600
2. C. Total cost = 80 × number of days + 400
3. Different methods can be used.
   For example, R2 800 – R400 = R2 400, then R2 400 ÷ R80 = 30, or
   Let x = number of days. Then $80x + 400 = 2\,800$
   so $80x = 2\,400$
   so $x = 2\,400 \div 80$
   $= 30$ days

2. (a) Write the rule that will produce the number pattern in the second row of this table. You may have to experiment to find out what the rule is.

Term number	1	2	3	4	5	6	7	8	9
Term value	5	8	11	14	17	20	23	26	29

   (b) Which term will have the value 221?

3. The rule for number pattern A is $4n + 11$, and the rule for pattern B is $7n - 34$.
   (a) Copy and complete the following table for the two patterns:

Term number	1	2	3	4	5	6	7	8	9
Pattern A	15	19	23	27	31	35	39	43	47
Pattern B	–27	–20	–13	–6	1	8	15	22	29

   (b) For which value of n are the terms of the two patterns equal?

9.4 Equation and situations

1. Consider this situation:
   To rent a room in a certain building, you have to pay a deposit of R400 and then R80 per day.
   (a) How much money do you need to rent the room for ten days?
   (b) How much money do you need to rent the room for 15 days?
2. Which of the following best describes the method that you used to do question 1(a) and (b)?
   A. Total cost = R400 + R80
   B. Total cost = 400(number of days + 80)
   C. Total cost = 80 × number of days + 400
   D. Total cost = (80 + 400) × number of days
3. For how many days can you rent the room described in question 1, if you have R2 800 to pay?

If you want to know for how many days you can rent the room if you have R720, you can set up an equation and solve it.

Example: You know the total cost is R720 and you know that you can work out the total cost like this:

Total cost = $80x + 400$, where x is the number of days.
So, $80x + 400 = 720$ and x = four days.

In each of the cases on page 90 (given in questions 4 to 7), find the unknown number by setting up an equation and solving it.

Answers

4. (a) Let x = number of days.

Then $120x + 300 = 1\,740$, so $120x = 1\,440$, so $x = 1\,440 \div 120 = 12$ days.

(b) $120 \times 10 + 300 = R1\,500$

$120 \times 11 + 300 = R1\,620$

$120 \times 12 + 300 = R1\,740$

(c) $120x + 300 = 3\,300$

so $120x = 3\,000$

so $x = 3\,000 \div 120 = 25$ days

(d) $120x + 300 = 3\,000$

so $120x = 2\,700$

so $x = 2\,700 \div 120 = 22{,}5$. Answer: 22 days

5. Let the number be x.

Then $5x + 12 = 9x - 16$, and so $x = 7$.

6. Work out how many days you have rented the car for if you had to pay R2 910. $x = 10$ days

7. Let the number of days be x.

Then $70x + 320 = 150x$, and $x = 4$.

9.5 Solving equations by using the laws of exponents

Teaching guidelines

The two sides of the equation have the same value. Therefore, the side that does not contain the unknown should be changed to the exponential form with the same base as the other side, for example, $5^x = 125$.

The right-hand side can be written as a product of its prime factors: 5^3. Therefore, $5^x = 5^3$. Since the bases are the same and the powers are the same (by the = sign), the exponents have to be the same. Therefore, $5^x = 125$

$5^x = 5^3$, and we can write: $x = 3$

4. To rent a certain room, you have to pay a deposit of R300 and then R120 per day.
   (a) For how many days can you rent the room if you can pay a total of R1 740? (If you experience trouble in setting up the equation, it may help you to decide first how you will work out what it will cost to rent the room for six days.)
   (b) What will it cost to rent the room for ten days, 11 days and 12 days?
   (c) For how many days can you rent the room if you have R3 300 available?
   (d) For how many days can you rent the room if you have R3 000 available?

5. Ben and Thabo decide to do some calculations with a certain number. Ben multiplies the number by 5 and adds 12. Thabo gets the same answer as Ben when he multiplies the number by 9 and subtracts 16. What is the number they worked with?

6. The cost of renting a certain car for a period of x days can be calculated with the following formula:

   Rental cost in rands = $260x + 310$

   What information about renting this car will you get, if you solve the equation $260x + 310 = 2\,910$?

7. Sarah paid a deposit of R320 for a stall at a market, and she also pays R70 per day rental for the stall. She sells fruit and vegetables at the stall, and finds that she makes about R150 profit each day. After how many days will she have earned as much as she has paid for the stall, in total?

9.5 Solving equations by using the laws of exponents

You may need to look back at Chapter 5 to remember the laws of exponents.

One kind of exponential equation that you deal with in Grade 9 has one or more terms with a base that is raised to a power containing a variable.

Example: $2^x = 16$

When we need to find the unknown value, we are asking the question: *"To what power must the base be raised for the statement to be true?"*

Example: $2^x = 16$ Make sure that the terms with x are on their own on one side.

$2^x = 2^4$ Write the known term in the same base as the term with the exponent.

$x = 4$ Equate the exponents.

In the example above, we can equate the exponents because the two numbers are equal only when they are raised to the same power.

Answers

1. (a) $5^{x-1} = 5^3$
   $x - 1 = 3$
   $x = 4$
   (b) $2^{x+3} = 2^3$
   $x + 3 = 3$
   $x = 0$
   (c) $10^x = 10^4$
   $x = 4$
   (d) $4^{x+2} = 4^3$
   $x + 2 = 3$
   $x = 1$
   (e) $7^{x+1} = 7^0$
   $x + 1 = 0$
   $x = -1$
   (f) x can be any number except 0.
2. (a) $7^x = 7^{-2}$
   $x = -2$
   (b) $10^x = 10^{-3}$
   $x = -3$
   (c) $6^x = 6^{-3}$
   $x = -3$
   (d) $10^{x-1} = 10^{-3}$
   $x - 1 = -3$
   $x = -2$
   (e) $4^{-x} = 4^{-2}$
   $-x = -2$
   $x = 2$
   (f) $x = -3$

SOLVING EQUATIONS WITH A VARIABLE IN THE BASE

Teaching guidelines

The two sides of the equation have the same value. Therefore, the side that does not contain the unknown should be changed to an exponential form with the same exponent as the other side.

Since the powers are the same (by the = sign) and the exponents are the same, it follows that the bases have to be the same. For example, $x^3 = 2^3$, the $x = 2$.

Answers

1. See the completed table on LB page 91 alongside.
   (a) $x = 4$ (b) $x = 2$ (c) $x = 4$
   (d) $x = 2$ (e) $x = 2$ (f) $x = 5$
2. (a) $x = 6$ $(6^3 = 216)$ (b) $x = 18$ $(18^2 = 324)$
   (c) $x = 10$ $(10^4 = 10\,000)$ (d) $x = 3$ $(8^3 = 512)$
   (e) $x = 2$ $(18^2 = 324)$ (f) $x = 3$ $(6^3 = 216)$

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1. Solve for x:
   (a) $5^{x-1} = 125$ (b) $2^{x+3} = 8$
   (c) $10^x = 10\,000$ (d) $4^{x+2} = 64$
   (e) $7^{x+1} = 1$ (f) $x^0 = 1$

Example: Solve for x: $3^x = \frac{1}{27}$

$3^x = 3^{-3}$ (Rewrite $\frac{1}{27}$ as a number to base 3.)

$x = -3$ (Equate the exponents.)

2. Solve for x:
   (a) $7^x = \frac{1}{49}$ (b) $10^x = 0{,}001$ (c) $6^x = \frac{1}{216}$
   (d) $10^{x-1} = 0{,}001$ (e) $4^{-x} = \frac{1}{16}$ (f) $7^x = 7^{-3}$

In another kind of equation involving exponents, the variable is in the base.

> When we need to find the unknown value, we ask the question: *"Which number must be raised to the given power for the statement to be true?"*

For these equations, you should remember what you know about the powers of numbers such as 2, 3, 4, 5 and 10.

SOLVING EQUATIONS WITH A VARIABLE IN THE BASE

1. Copy and complete the table below and answer the questions that follow:

	x	2	3	4	5
(a)	x^3	$2^3 = 8$	$3^3 = 27$	$4^3 = 64$	$5^3 = 125$
(b)	x^5	$2^5 = 32$	$3^5 = 243$	$4^5 = 1\,024$	$5^5 = 3\,125$
(c)	x^4	$2^4 = 16$	$3^4 = 81$	$4^4 = 256$	$5^4 = 625$

For what value of x is:
(a) $x^3 = 64$ (b) $x^5 = 32$ (c) $x^4 = 256$
(d) $x^3 = 8$ (e) $x^4 = 16$ (f) $x^5 = 3\,125$

2. Solve for x and give a reason:
   (a) $x^3 = 216$ (b) $x^2 = 324$
   (c) $x^4 = 10\,000$ (d) $8^x = 512$
   (e) $18^x = 324$ (f) $6^x = 216$

WORKSHEET

Answers

1. $5x + 3 - x = 11$
   $4x = 8$
   $x = 2$

2. (a) $3x - 6 = 4x + 4$
   $-x - 6 = 4$
   $-x = 10$
   $x = -10$

   (b) $5x + 10 = -6 + 3x$
   $2x + 10 = -6$
   $2x = -16$
   $x = -8$

   (c) $0{,}8x = -24$
   $8x = -240$
   $x = -30$

   (d) no solution
   impossibility

   (e) $2{,}5x = 0{,}5x + 5$
   $2x = 5$
   $x = 2{,}5$

   (f) $7x - 14 = 14 - 7x$
   $14x = 28$
   $x = 2$

   (g) $2x - 3 = 10$
   $2x = 13$
   $x = \frac{13}{2}$
   $x = 6\frac{1}{2}$

   (h) $2x - 9 - 3x = 5x + 9$
   $-x - 9 = 5x + 9$
   $-6x = 18$
   $x = -3$

WORKSHEET

1. Ahmed multiplied a number by 5, added 3 to the answer, and then subtracted the number he started with. The answer was 11. What number did he start with?

2. Use any appropriate method to solve the equations:

   (a) $3(x - 2) = 4(x + 1)$
   (b) $5(x + 2) = -3(2 - x)$
   (c) $1{,}5x = 0{,}7x - 24$
   (d) $5(x + 3) = 5x + 12$
   (e) $2{,}5x = 0{,}5(x + 10)$
   (f) $7(x - 2) = 7(2 - x)$
   (g) $\frac{1}{2}(2x - 3) = 5$
   (h) $2x - 3(3 + x) = 5x + 9$

EQUATIONS

Term 2

Chapter 10: Construction of geometric figures **117**
10.1 Constructing perpendicular lines 118
10.2 Bisecting angles 121
10.3 Constructing special angles without a protractor 123
10.4 Angle bisectors in triangles 125
10.5 Interior and exterior angles in triangles 126
10.6 Constructing congruent triangles 128
10.7 Diagonals of quadrilaterals 130
10.8 Angles in polygons 132
Chapter 11: Geometry of 2D shapes **135**
11.1 Revision: Classification of triangles 136
11.2 Finding unknown angles in triangles 138
11.3 Quadrilaterals 139
11.4 Congruent triangles 143
11.5 Similar triangles 146
11.6 Extension questions 152
Chapter 12: Geometry of straight lines **153**
12.1 Angle relationships 154
12.2 Identify and name angles 160
12.3 Solving problems 161

Chapter 13: Pythagoras' Theorem 163
13.1 Investigating the sides of a right-angled triangle 164
13.2 Checking for right-angled triangles 165
13.3 Finding missing sides 167
13.4 More practice using Pythagoras' Theorem 170
Chapter 14: Area and perimeter of 2D shapes 173
14.1 Area and perimeter of squares and rectangles 174
14.2 Area and perimeter of composite figures 175
14.3 Area and perimeter of circles 177
14.4 Converting between units 178
14.5 Area of other quadrilaterals 179
14.6 Doubling dimensions of a 2D shape 183

Grade 9 Term 2 Chapter 10 Construction of geometric figures

Learner Book Overview		
Sections in this chapter	**Content**	**Pages in Learner Book**
10.1 Constructing perpendicular lines	Using circles to construct a line, perpendicular to another line, from a point outside the line and from a point on the line	Pages 93 to 96
10.2 Bisecting angles	Using circles to understand how to bisect angles	Pages 96 to 97
10.3 Constructing special angles without a protractor	Constructing angles of 30°, 45°, 60° as special angles and using construction to find multiples of these angles	Pages 98 to 99
10.4 Angle bisectors in triangles	Constructing angle bisectors in a triangle; finding that they intersect at one point	Page 100
10.5 Interior and exterior angles in triangles	Identifying interior and exterior angles of a triangle; investigating their properties	Pages 101 to 103
10.6 Constructing congruent triangles	Finding the minimum conditions for congruency; constructing congruent triangles	Pages 103 to 105
10.7 Diagonals of quadrilaterals	Drawing diagonals of quadrilaterals and investigating their properties	Pages 105 to 107
10.8 Angles in polygons	Drawing diagonals to investigate the sum of the angles in polygons	Page 107 to 108

CAPS time allocation	9 hours
CAPS content specification	Page 134

Mathematical background

In any two circles that overlap, intersecting at two points, is a line segment with a perpendicular bisector. We can use this property of circles to construct perpendicular lines (TG page 119).

If the centre of a circle lies on the edge of another circle with the same radius, equilateral triangles with angles of 60° are formed. The line that joins the points of intersection of the two equal circles bisects any angle with its vertex on the line of intersection, and its arms going through the centres of the circles. We can use this property of circles to bisect any angle with compasses (TG pages 121 and 123).

The bisectors of the angles in triangles intersect at a point (TG page 125).
An exterior angle of a triangle equals the sum of the opposite interior angles (TG page127).

The minimum conditions for two triangles to be congruent are: three sides are equal; two sides and the angle between them are equal; two angles and a side in the corresponding position are equal; a right angle, the hypotenuse and a right-angle side are equal (TG page 128).
The diagonals of any parallelogram (which includes squares, rectangles and rhombi) bisect each other; those of a rectangle and a square are also the same length; those of a square, a rhombus and a kite are perpendicular (TG page 130).
The sum of the interior angles of a polygon with n sides is given by $(n - 2)180°$ (TG page 132).

10.1 Constructing perpendicular lines

REVISING PERPENDICULAR LINES

Teaching guidelines

Learners should be quite clear that perpendicular lines are lines that cut each other at a 90° angle. We can also say that perpendicular lines cut each other "at right angles".

You can use question 2(a) alongside to point out to learners that the lines may seem to be perpendicular, but we can only be sure that they are if we measure them or if the markings on the drawing indicate that they are perpendicular.

Answers

1. Two lines are perpendicular when the angle of intersection is a right angle.
2. (a) The angles are 103°, 77°, 103° and 77°. The lines are not perpendicular.
   (b) The angles are all equal to 90°, so the lines are perpendicular.

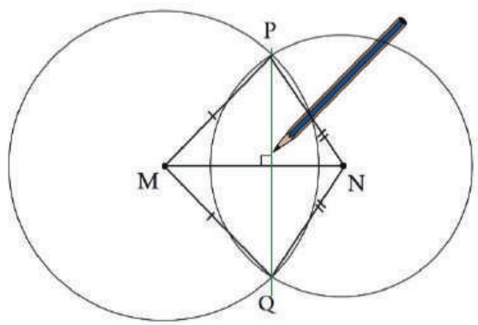
LINES THAT FORM WHEN CIRCLES INTERSECT

Teaching guidelines

Learners worked with these constructions in Grade 8.

Mathematical background

The lines are perpendicular because
PM = QM and PN = QN, so PMQN is a kite.
The diagonals of a kite are perpendicular.



Answers

1. (a) Learners' own work
   (b) Learners' own work
   (c) Learners' own work

CHAPTER 10

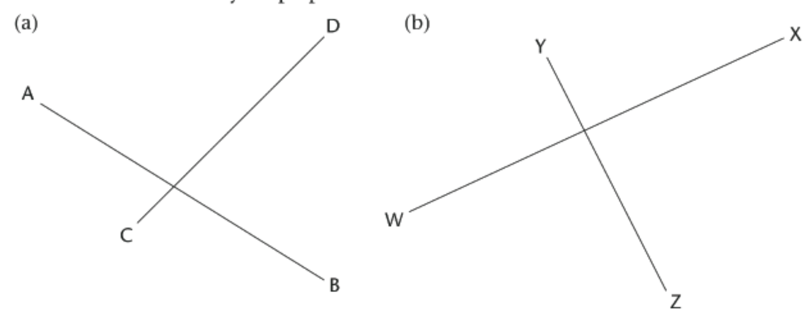
Construction of geometric figures

10.1 Constructing perpendicular lines

REVISING PERPENDICULAR LINES

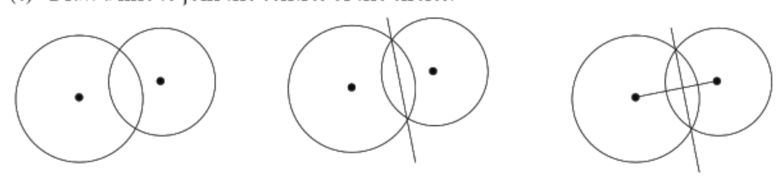
In Grade 8, you learnt about **perpendicular lines**.

1. What does it mean if we say that two lines are perpendicular?
2. Use your protractor to measure the angles between the following pairs of lines. Then state whether they are perpendicular or not.



LINES THAT FORM WHEN CIRCLES INTERSECT

1. Do the following:
   (a) Use a compass to draw two overlapping circles of different sizes.
   (b) Draw a line through the points where the circles intersect (overlap).
   (c) Draw a line to join the centres of the circles.



Step (a) *Step (b)* *Step (c)*

Answers

1. (d) 90°

   (e) If all the angles at the point of intersection are equal, then they are all right angles, and the lines are perpendicular to each other.

2. Learners' own work

3. The two lines are perpendicular to each other.

USING CIRCLES TO CONSTRUCT PERPENDICULAR LINES

Teaching guidelines

Case 1: Learners use the construction in the previous section to construct a line perpendicular to a given line from a point that is not on the line. The two circles should go through the point that is not on the line.

Start with the line on which the perpendicular line has to be drawn. Mark off the points M and N, if they are given or arbitrarily if they have not been given.

Follow the instructions step by step.

(d) Use your protractor to measure the angles between the intersecting lines.

(e) What can you say about the intersecting lines?

2. Repeat questions 1(a) to (e) with circles that are the same size.

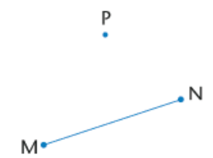
3. What conclusion can you make about a line drawn between the intersection points of two overlapping circles and a line through their centres?

USING CIRCLES TO CONSTRUCT PERPENDICULAR LINES

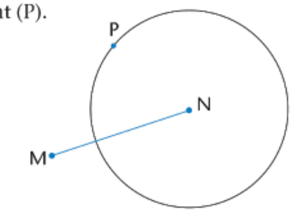
Case 1: A perpendicular through a point that is not on the line segment

Copy the steps below:

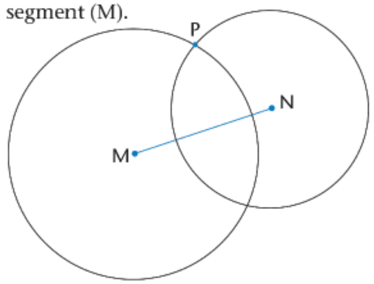
You are given line segment MN with point P at a distance from it. You must construct a line that is perpendicular to MN, so that the perpendicular passes through point P.



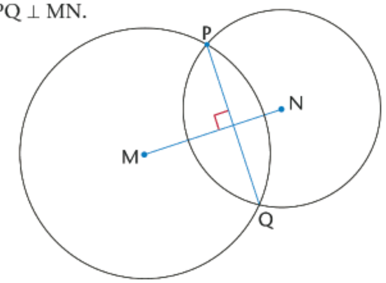
Step 1
Use your compass to draw a circle whose centre is the one end point of the line segment (N) and passes through the point (P).



Step 2
Repeat step 1, but make the centre of your circle the other end point of the line segment (M).



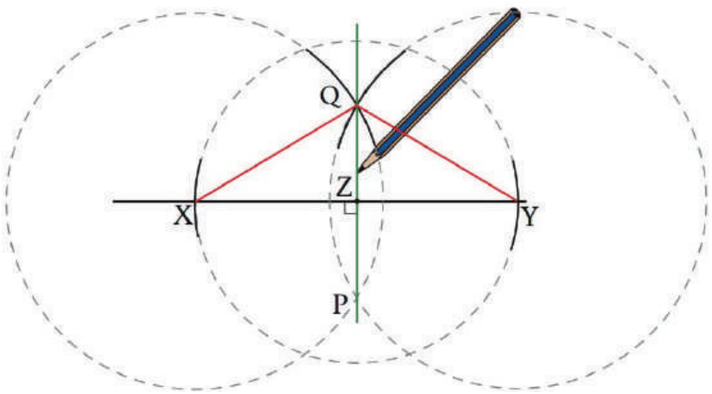
Step 3
Join the points where the circles intersect: PQ ⊥ MN.



94 MATHEMATICS GRADE 9: TERM 2

Teaching guidelines

Case 2: The first circle sets the points on the line from where the second set of circles should be drawn. The midpoints of these circles are the same distance from the point where the line should be drawn and they have the same radius, therefore joining the points where they intersect will give a perpendicular line through the given point.



PRACTISE USING CIRCLES TO CONSTRUCT PERPENDICULAR LINES

Teaching guidelines

The constructions explained above should be used in the following constructions. In question 1, the point is not on the line and in question 2 it is on the line.

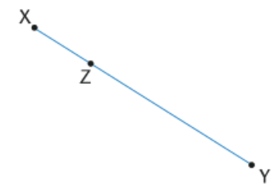
Answers

1. See the construction on LB page 95 alongside.

Case 2: A perpendicular at a point that is on the line segment

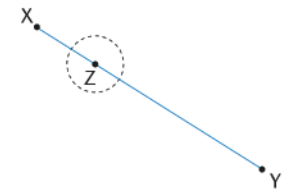
Copy the steps below:

You are given line segment XY with point Z on it. You must construct a perpendicular line passing through Z.



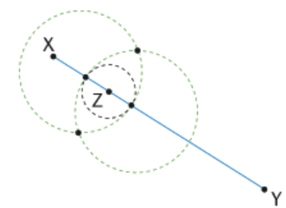
Step 1

Use your compass to draw a circle whose centre is Z. Make its radius smaller than ZX. Note the two points where the circle intersects XY.



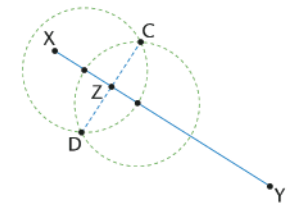
Step 2

Set your compass wider than it was for the circle with centre Z. Draw two circles of the same size whose centres are at the two points where the first (black) circle intersects XY. The two (green) circles will overlap.



Step 3

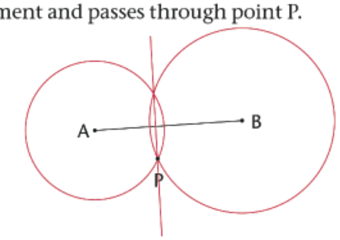
Join the intersection points of the two overlapping circles. Mark these points C and D: CD ⊥ XY and passes through point Z.



PRACTISE USING CIRCLES TO CONSTRUCT PERPENDICULAR LINES

In each of the following two cases, copy the line segment, and draw a line that is perpendicular to the segment and passes through point P.

1.



Answers

2. In this case, point P corresponds to the midpoint of the first circle drawn to determine the midpoints of the two equal-sized interlocking circles.
   To find the midpoints, centre the compass at P and construct a circle that cuts segment CD twice.

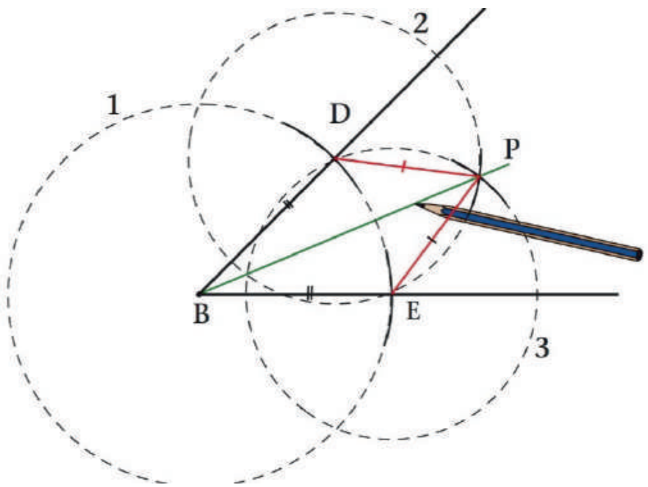
10.2 Bisecting angles

USING CIRCLES TO BISECT ANGLES

Teaching guidelines

The circle numbered 1 is drawn with B as the centre.

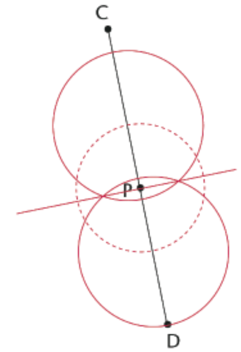
Where circle 1 cuts the arms of the angle at D and E, we draw circle 2 (at D) and circle 3 (at E), both of which have the same radius. Where circle 2 and 3 intersect at P, join P and B to construct the bisector of the angle.



Answers

See the completion of the construction on LB page 96 alongside and LB page 97 on the following page.

2.

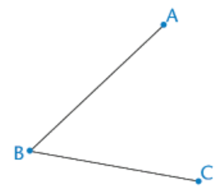
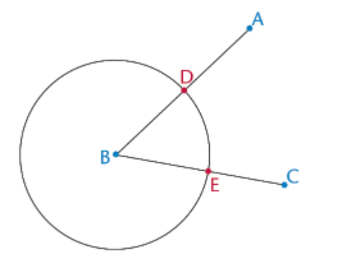


10.2 Bisecting angles

USING CIRCLES TO BISECT ANGLES

Work through the following example of using intersecting circles to **bisect** an angle. Do the following steps yourself.

To bisect something means "to cut in half".

You are given AB̂C. You must bisect the angle.	**Step 1** Draw a circle with centre B to mark off an equal length on both arms of the angle. Label the points of intersection D and E: DB = BE.
	

96 MATHEMATICS GRADE 9: TERM 2

PRACTISE BISECTING ANGLES

Teaching guidelines

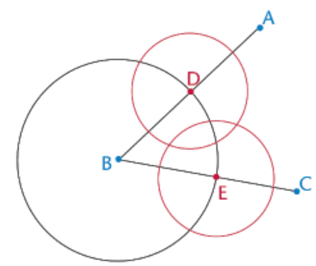
Let learners explain which point on the system of three interlocking circles corresponds with the vertices of the angles. Let them explain how they will find the centres of the interlocking circles.

Answers

See the constructions on LB page 97 alongside.

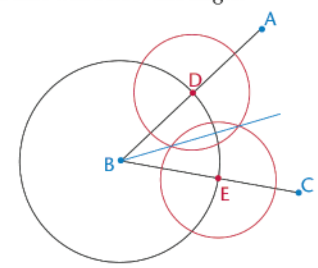
Step 2

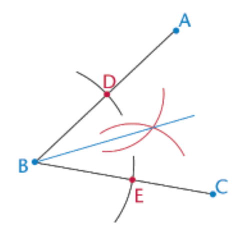
Draw two equal circles with centres at D and at E. Make sure the circles overlap.



Step 3

Draw a line from B through the points where the two equal circles intersect. This line will bisect the angle.





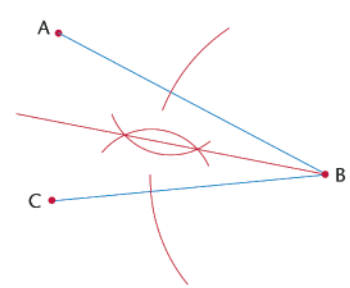
Same construction as in step 3 above

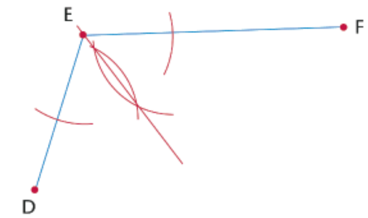
Can you explain why the method above works to bisect an angle?

Can you also see that we need not draw full circles, but can merely use parts of circles (arcs) to do the above construction?

PRACTISE BISECTING ANGLES

Copy the following angles and then bisect them without using a protractor:





10.3 Constructing special angles without a protractor

CONSTRUCTING A 45° ANGLE

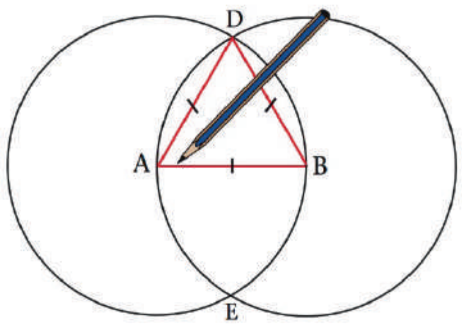
Teaching guidelines

To construct a 45° angle, learners construct a right angle and bisect the right angle, using the system of intersecting circles as they did before.

CONSTRUCTING 60° AND 30° ANGLES

Teaching guidelines

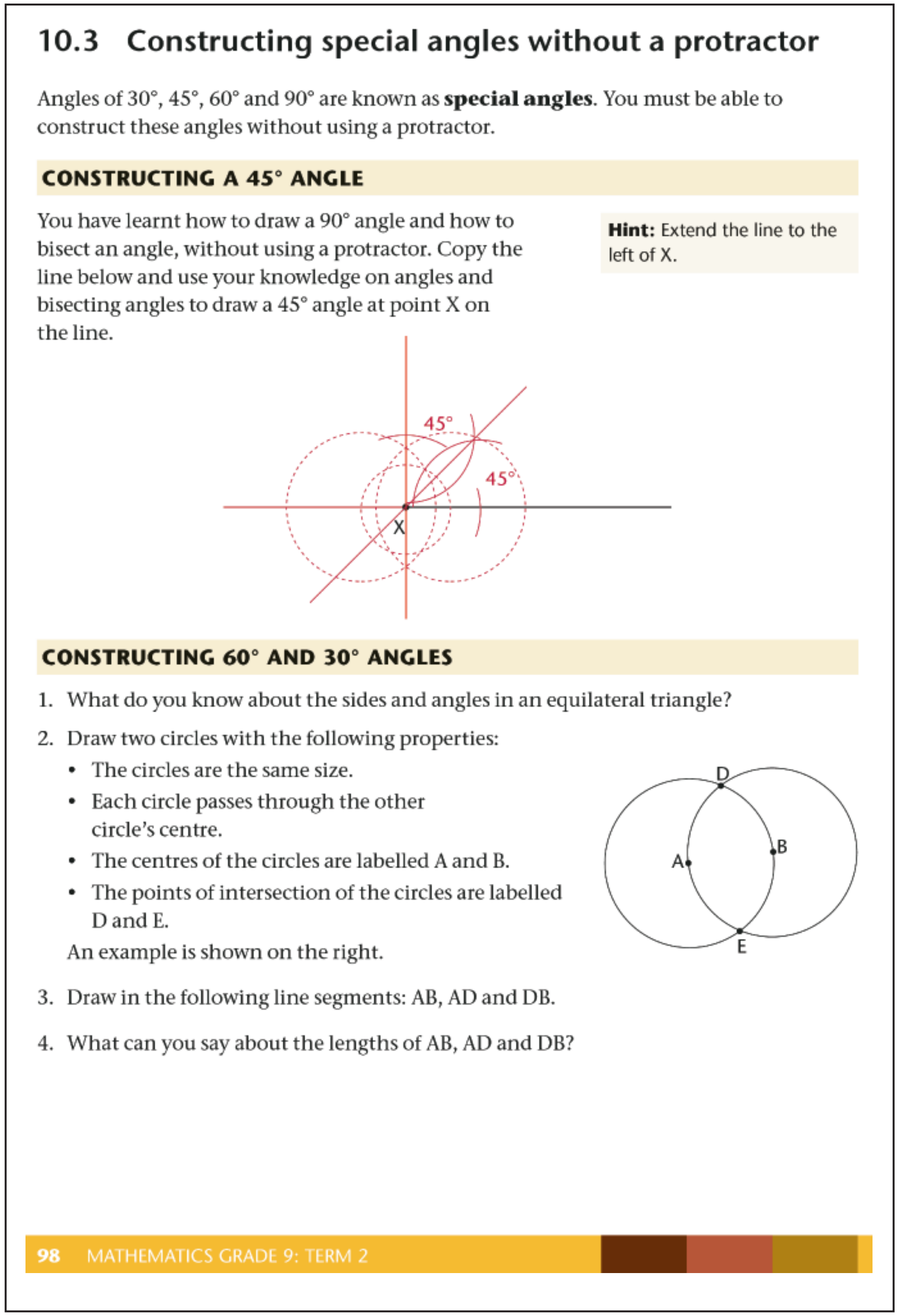
The radii of circles A and B are equal, therefore, the triangle that is formed is equilateral and results in a 60° angle. Remind learners that AB is the length of the radius of both circle A and circle B.



Once learners have the 60° angle they use the constructions explained in the sections above to construct the 30° angle.

Answers

1. The sides are equal and all angles are equal to 60°.
2. Learners' own work
3. Learners' own work
4. All are equal, since they are radii of circles that are the same size.



10.3 Constructing special angles without a protractor

Angles of 30°, 45°, 60° and 90° are known as **special angles**. You must be able to construct these angles without using a protractor.

CONSTRUCTING A 45° ANGLE

You have learnt how to draw a 90° angle and how to bisect an angle, without using a protractor. Copy the line below and use your knowledge on angles and bisecting angles to draw a 45° angle at point X on the line.

Hint: Extend the line to the left of X.

CONSTRUCTING 60° AND 30° ANGLES

1. What do you know about the sides and angles in an equilateral triangle?
2. Draw two circles with the following properties:
   - The circles are the same size.
   - Each circle passes through the other circle's centre.
   - The centres of the circles are labelled A and B.
   - The points of intersection of the circles are labelled D and E.

   An example is shown on the right.
3. Draw in the following line segments: AB, AD and DB.
4. What can you say about the lengths of AB, AD and DB?

98 MATHEMATICS GRADE 9: TERM 2

Answers

5. It is an equilateral triangle.
6. Each angle is equal to 60°.
7. Learners' own work
8. Learner's own work. See the answer on LB page 99 alongside.

CONSTRUCTING THE MULTIPLES OF SPECIAL ANGLES

Teaching guidelines

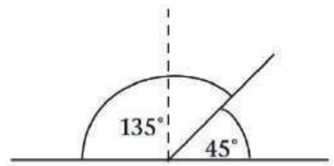
Learners can construct a right angle. From that they can construct a 45° angle by bisecting the right angle. Now they can construct any multiple of 45° and 90°.

Learners know how to construct an angle of 60° and they can bisect it to construct a 30° angle. From these constructions, and using the fact that a straight line is 180° and a revolution is 360°, they can construct any multiple of 30° and 60°.

Answers

1. See the answers on LB page 99 alongside.
2. (a) Draw a circle and draw its diameter. At the centre, construct an angle of 60° using another circle with the same radius. The supplementary angle of the 60° angle is 120°.

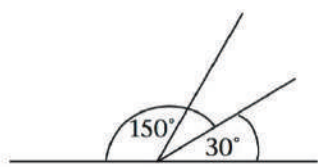
   (b) Draw a line and mark a point on it. Construct a right angle and bisect it to get 45°. The supplementary angle is 135°.



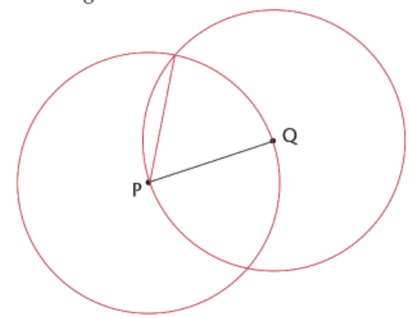
   (c) Construct an angle of 90°. The outer angle is the 270° angle.

   (d) Construct two 60° angles adjacent to each other making a 120° angle. The outer angle is the 240° angle.

   (e) Draw a line and mark a point on it. Construct an angle of 60°, bisect it to get 30°. The supplementary angle to 30° is the 150° angle.



5. What kind of triangle is ABD?
6. Therefore, what do you know about $\hat{A}$, $\hat{B}$ and $\hat{D}$?
7. Use your knowledge of bisecting angles to create an angle of 30° on the construction you made in question 2.
8. Copy the line segment below and use what you have learnt to construct an angle of 60° at point P on the line segment.



CONSTRUCTING THE MULTIPLES OF SPECIAL ANGLES

1. Copy and complete the table below. The first one has been done for you.

Angle	Multiples below 360°	Angle	Multiples below 360°
30°	30°; 60°; 90°; 120°; 150°; 180°; 210°; 240°; 270°; 300°; 330°	45°	45°, 90°, 135°, 180°, 225°, 270°, 315°
60°	60°; 120°; 180°; 240°; 300°	90°	90°; 180°; 270°

2. Construct the following angles without using a protractor. You will need to do more than one construction to create each angle.

   (a) 120° (b) 135° (c) 270° (d) 240° (e) 150°

10.4 Angle bisectors in triangles

Teaching guidelines

The questions in this section give learners ample opportunity to practise the constructions they learnt in the previous sections. Any problems they still have can be addressed now.

Background information

The point at which the angle bisectors intersect is the centre of the inscribed circle. If a perpendicular line is drawn from that point to any of the sides of the triangle, a circle with that distance as radius can be drawn. It will touch each of the three sides inside the triangle.

Answers

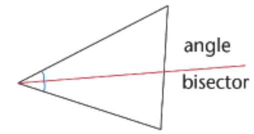
1. (a) See the constructions on LB page 100 alongside.
   (b) See the constructions on LB page 100 alongside.
   (c) The bisectors all intersect at the same point.
2. (a) See the constructions on LB page 100 alongside.
   (b) The bisectors all intersect at the same point.
3. Learners' own work

Extension

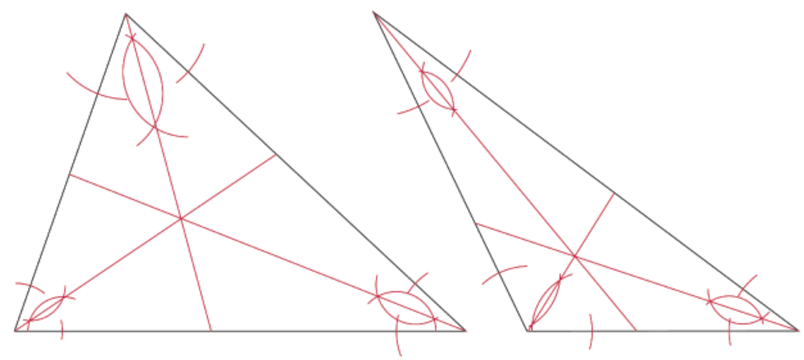
Let learners construct the perpendicular line from the point of intersection to any side of the triangle. With the point of intersection as centre and length of the perpendicular line as radius, let them draw a circle. It should touch each of the sides.

10.4 Angle bisectors in triangles

You learnt how to bisect an angle in Section 10.2.
Now you will investigate the angle bisectors in a triangle.
An **angle bisector** is a line that cuts an angle in half.



1. (a) Copy the acute triangle below. Bisect each of the angles of the acute triangle.
   (b) Extend each of the bisectors to the opposite side of the triangle.
   (c) What do you notice?
2. (a) Copy the obtuse angle below. Do the same with the obtuse triangle.
   (b) What do you notice?



3. Compare your triangles with those of two classmates. You should have the same results.

You should have found that the three **angle bisectors** of a triangle **intersect at one point**. This point is the same distance away from each side of the triangle.

10.5 Interior and exterior angles in triangles

WHAT ARE INTERIOR AND EXTERIOR ANGLES?

Teaching guidelines

Use drawings to help define the concepts “interior angle” and “exterior angle”.

IDENTIFYING EXTERIOR ANGLES AND INTERIOR OPPOSITE ANGLES

Teaching guidelines

In question 1(c), make sure that learners do not name angle 10 as an exterior angle to the triangle that contains angles 13, 14 **and** 15.

Answers

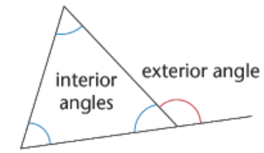
1. See the answers on LB page 101 alongside.

10.5 Interior and exterior angles in triangles

WHAT ARE INTERIOR AND EXTERIOR ANGLES?

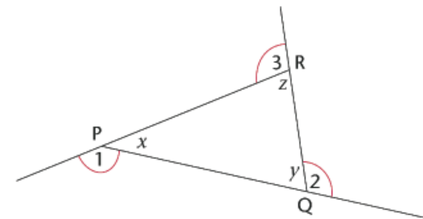
An **interior angle** is an angle that lies between two sides of a triangle. It is inside the triangle. A triangle has three interior angles.

An **exterior angle** is an angle between a side of a triangle and another side that is extended. It is outside the triangle.



Look at ΔPQR. Its three sides are extended to create three exterior angles.

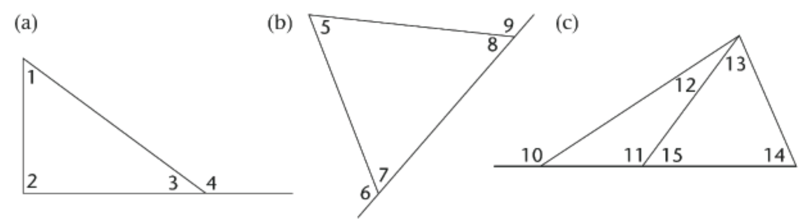
Each exterior angle has one interior adjacent angle (next to it) and two **interior opposite angles**, as described in the following table:



Exterior angle	Interior adjacent angle	Interior opposite angles
1	x	z and y
2	y	x and z
3	z	x and y

IDENTIFYING EXTERIOR ANGLES AND INTERIOR OPPOSITE ANGLES

1. Copy the following table and name each exterior angle and its two interior opposite angles below.



Ext. ∠	4	9	6	10	11
Int. opp. ∠s	1 and 2	5 and 7	5 and 8	11 and 12 12 & 13 and 14	13 and 14

Answers

2. (a) $\angle ABC$, $\angle BCA$ and $\angle CAB$. Learners could also label the angles with a number to indicate $\angle A_1$; $\angle B_1$; $\angle C_1$
   (b) $\angle MBC$, $\angle LBA$, $\angle BAG$, $\angle NAC$, $\angle ACK$, $\angle HCB$
   (c) It does not comply with the definition: an exterior is an angle between a side of a triangle and another side that is extended.
   (d) $\angle GAN$ and $\angle HCK$

INVESTIGATING THE EXTERIOR AND INTERIOR ANGLES IN A TRIANGLE

Teaching guidelines

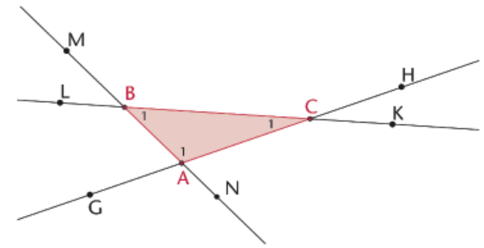
Learners should know that the sum of the angles in a triangle is 180°. So by reasoning they could see that:

	$68° + (45° + 67°) = 180°$	(sum of angles of a triangle)
and	$68° + 112° = 180°$	(angles on a line)
so	$45° + 67° = 112°$	(an exterior angle equals the sum of the opposite interior angles)

Answers

1. to 3. See the answers on LB page 102 alongside.
4. They are equal.

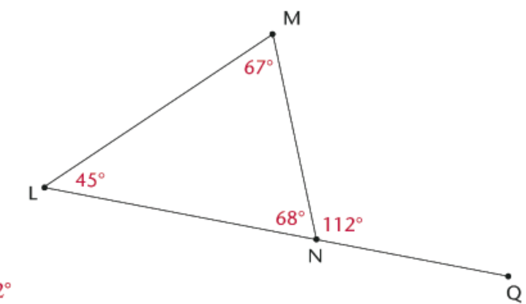
2. ΔABC below has each side extended in both directions to create six exterior angles.



(a) Write down the names of the interior angles of the triangle.
(b) Since a triangle has three sides that can be extended in both directions, there are two exterior angles at each vertex. Write down the names of all the exterior angles of the triangle.
(c) Explain why $M\hat{B}L$ is not an exterior angle of ΔABC.
(d) Write down two other angles that are neither interior nor exterior.

INVESTIGATING THE EXTERIOR AND INTERIOR ANGLES IN A TRIANGLE

1. Consider ΔLMN. Write down the name of the exterior angle. $M\hat{N}Q$
2. Use a protractor to measure the interior angles and the exterior angle. Copy the drawing and write the measurements on the drawing.



3. Use your findings in question 2 to complete the following sum:

   $L\hat{M}N + M\hat{L}N = 67° + 45° = 112°$

4. What is the relationship between the exterior angle of a triangle and the sum of the interior opposite angles?

The **exterior angle** of a triangle is equal to the sum of the interior opposite angles.

Answers

5. (a) $a = 85°$ (ext. ∠ of △)
   (b) $b = 127° - 23° = 104°$ (ext. ∠ of △)
   $c = 127°$ (vertically opposite angles are equal)
   (c) $d + f = 180° - 78° = 102°$ (sum of ∠s of △)
   $d = f$ (angles opposite equal sides)
   Therefore $d = 51° = f$
   $e = 78° + 51° = 129°$ (ext. ∠ of △)

10.6 Constructing congruent triangles

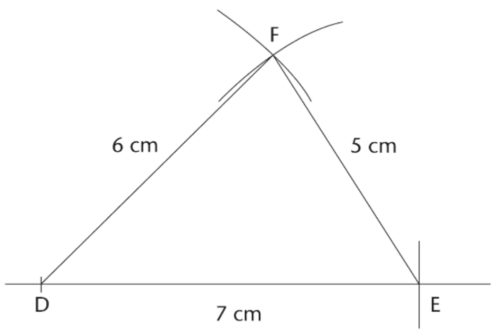
MINIMUM CONDITIONS FOR CONGRUENCY

Teaching guidelines

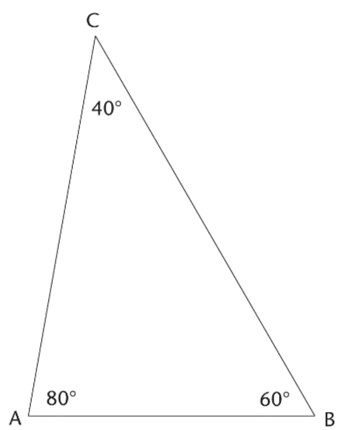
Remind learners what we mean by congruent triangles.

Answers

1. (a) Learners should get congruent triangles.

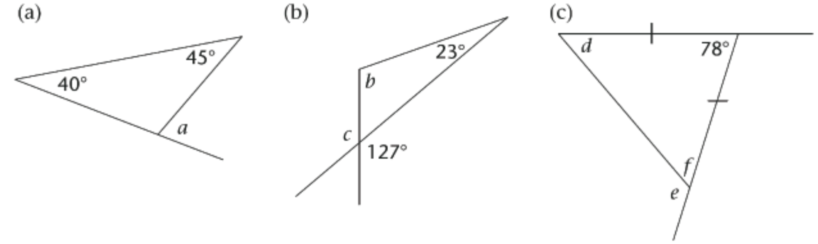


   (b) No sides are given, so learners should get similar shapes, but they won't necessarily be congruent.



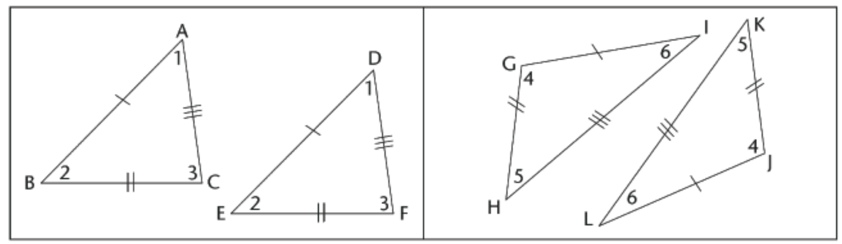
5. Work out the sizes of angles *a* to *f* below, without using a protractor. Give reasons for the statements you make as you work out the answers.

(a) (b) (c)



10.6 Constructing congruent triangles

Two triangles are **congruent** if they have exactly the **same shape** and **size**, i.e. they are able to fit exactly on top of each other. This means that all three corresponding sides and three corresponding angles are equal, as shown in the following two pairs:



ΔABC ≡ ΔDEF and ΔGHI ≡ ΔJKL. In each pair, the corresponding sides and angles are equal.

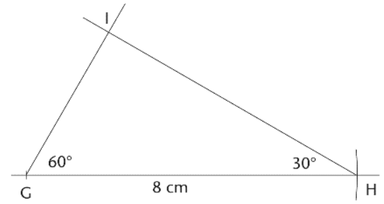
MINIMUM CONDITIONS FOR CONGRUENCY

To determine if two triangles are congruent, we need a certain number of measurements, but not all of these. Let's investigate which measurements give us only one possible triangle.

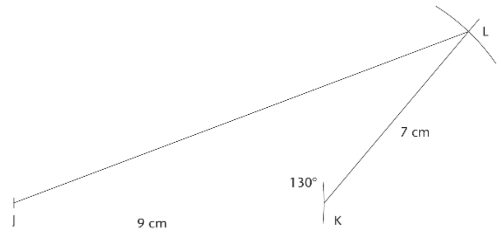
1. Use a ruler, compass and protractor to construct the following triangles. Each time minimum measurements are given.
   (a) Given three sides: side, side, side (SSS):
   ΔDEF with DE = 7 cm, DF = 6 cm and EF = 5 cm.
   (b) Given three angles: angle, angle, angle (AAA):
   ΔABC with $\hat{A} = 80°$, $\hat{B} = 60°$ and $\hat{C} = 40°$.

Answers

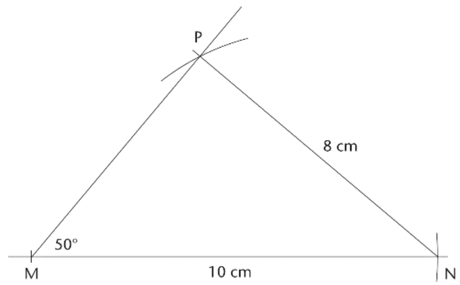
1. (c) Learners should get congruent triangles.



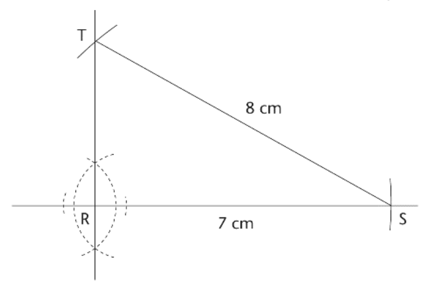
(d) Learners should get congruent triangles.



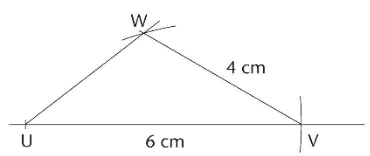
(e) The triangles will not be congruent.



(f) The triangles should be congruent.



(g) The triangles will not be congruent.



(c) Given one side and two angles: side, angle, angle (SAA):
ΔGHI with GH = 8 cm, $\hat{G}$ = 60° and $\hat{H}$ = 30°.

(d) Given two sides and an included angle: side, angle, side (SAS):
ΔJKL with JK = 9 cm, $\hat{K}$ = 130° and KL = 7 cm.

(e) Given two sides and an angle that is not included: side, side, angle (SSA):
ΔMNP with MN = 10 cm, $\hat{M}$ = 50° and PN = 8 cm.

(f) Given a right angle, the hypotenuse and a side (RHS):
ΔTRS with TR ⊥ RS, RS = 7 cm and TS = 8 cm.

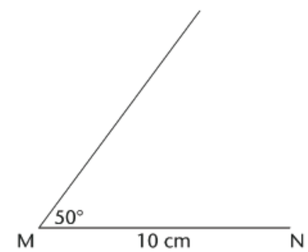
(g) Triangle UVW with UV = 6 cm and VW = 4 cm.

2. Compare your triangles with those of three classmates. Which of your triangles are congruent to theirs? Which are not congruent?

3. Go back to ΔMNP (question 1e). Did you find that you can draw two different triangles that both meet the given measurements? One of the triangles will be obtuse and the other acute. Follow the construction steps below to see why this is so.

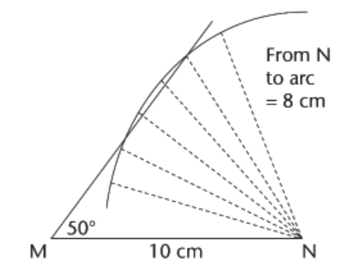
Step 1

Construct MN = 10 cm and the 50° angle at M, even though you do not know the length of the unknown side (MP).



Step 2

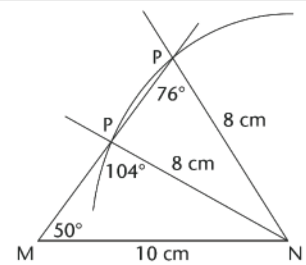
$\hat{N}$ is unknown, but NP = 8 cm. Construct an arc 8 cm from N. Every point on the arc is 8 cm from N.



Step 3

Point P must be 8 cm from N and fall on the unknown side of the triangle. The arc intersects the third side at two points, so P can be at either point.

So two triangles are possible, each meeting the conditions given, i.e. MN = 10 cm, NP = 8 cm and $\hat{M}$ = 50°.



104 MATHEMATICS GRADE 9: TERM 2

2. Triangles (a), (c), (d) and (f) are congruent.
   Triangles (b), (e) and (g) are not congruent.
3. Learners' own work
4. See the answers on LB page 105 alongside.

10.7 Diagonals of quadrilaterals

DRAWING DIAGONALS

Teaching guidelines

Point out that a diagonal is a line drawn in a quadrilateral from a vertex to the opposite vertex.

Answers

2. See the answers on LB page 105 alongside.
3. Four
4. Four
5. Two

4. Copy and complete the table. Write down whether or not we can construct a congruent triangle when the following conditions are given.

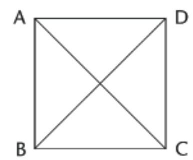
Conditions	Congruent?
Three sides (SSS)	Yes
Two sides (SS)	No
Three angles (AAA)	No
Two angles and a side (AAS)	Yes
Two sides and an angle not between the sides (SSA)	No
Two sides and an angle between the sides (SAS)	Yes
Right-angled with the hypotenuse and a side (RHS)	Yes

10.7 Diagonals of quadrilaterals

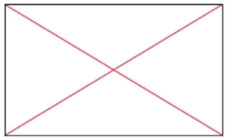
DRAWING DIAGONALS

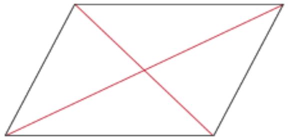
A **diagonal** is a straight line inside a figure that joins two vertices of the figure, where the vertices are not next to each other.

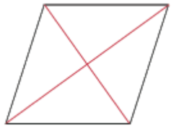
1. Look at the quadrilaterals below. The two diagonals of the square have been drawn in: AC and BD.
2. Copy the quadrilaterals below and draw in the diagonals.

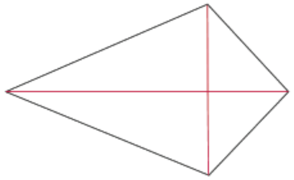


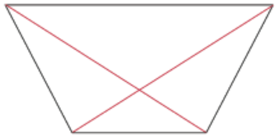
Square


Rectangle


Parallelogram


Rhombus


Kite


Trapezium

3. How many sides does a quadrilateral have?
4. How many angles does a quadrilateral have?
5. How many diagonals does a quadrilateral have?

DIAGONALS OF A RHOMBUS

Teaching guidelines

Remind learners that a rhombus has all four sides equal. Therefore, they can construct a rhombus using two equal circles (radii form the sides of the rhombus). From the previous work they can deduce that the diagonals are perpendicular and that they bisect each other.

Learners can also use congruent triangles to show that the diagonals bisect each other.

Answers

1. See the answer on LB page 106 alongside.
2. The lines are the diagonals of the rhombus.
3. Yes
4. ...intersect at 90° and will bisect each other.

DIAGONALS OF A KITE

Teaching guidelines

The construction is the same as for bisecting the angle D, therefore DE bisects the line joining the points of intersection of the circles. The long diagonal bisects the short diagonal.

Learners can also use congruent triangles to show that the long diagonal of the kite bisects the short diagonal.

Answers

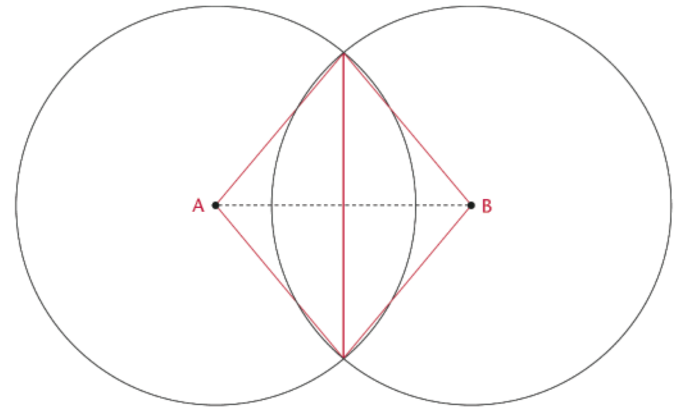
1. to 3. See the answers on LB page 106 alongside.

DIAGONALS OF A RHOMBUS

Below are two overlapping circles with centres A and B. The circles are the same size.

1. Construct a rhombus inside the circles by joining the centre of each circle with the intersection points of the circles. Join AB.
2. Copy the circles and construct the perpendicular bisector of AB. (Go back to Section 10.1 if you need help.) What do you find?

A **perpendicular bisector** is a line that cuts another line in half at a right angle (90°).

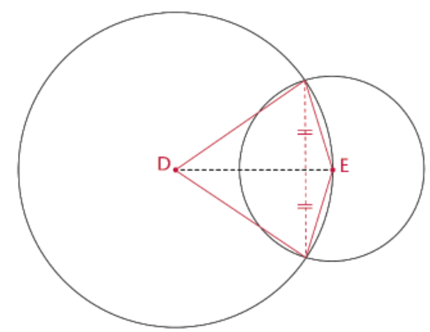


3. Do the diagonals bisect each other?
4. Copy and complete the sentence: The diagonals of a rhombus will always . . .

DIAGONALS OF A KITE

Below are two overlapping circles with centres D and E. The circles are different sizes.

1. Copy the circles and construct a kite by joining the centre points of the circles to the intersection points of the circles.
2. Draw in the diagonals of the kite.
3. Mark all lines that are of the same length.



106 MATHEMATICS GRADE 9: TERM 2

Answers

4. Yes
5. No. Only one of the diagonals is bisected.
6. The diagonals of a rhombus both bisect each other.

DIAGONALS OF PARALLELOGRAMS, RECTANGLES AND SQUARES

Teaching guidelines

Learners can use the properties of these quadrilaterals (for example, opposite sides are parallel and equal) that they have learnt before to draw the quadrilaterals.

Answers

1. to 3. Learners' own work

4. See the completed table on LB page 107 alongside.

10.8 Angles in polygons

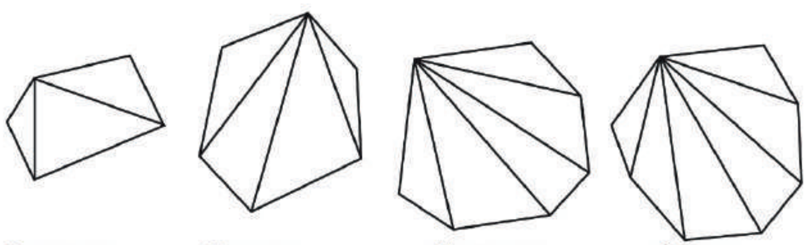
USING DIAGONALS TO INVESTIGATE THE SUM OF THE ANGLES IN POLYGONS

Teaching guidelines

If learners complete the table correctly, they should see the pattern. The number of triangles in a polygon is two less than the number of sides. The sum of the interior angles of a triangle is 180°.

The number of triangles in a polygon with n sides is $n - 2$. Therefore, the sum of the angles of a polygon with n sides is given by $(n - 2)180°$.

Learners may get confused when they have to draw the diagonals. Show them that when you draw the diagonals from the same vertex to every other vertex, you are sure to find the least number of triangles.



Pentagon 3 triangles | Hexagon 4 triangles | Heptagon 5 triangles | Octagon 6 triangles

Answers

1. See the answers on LB page 107 alongside and LB page 108 on the next page.

4. Are the diagonals of the kite perpendicular?
5. Do the diagonals of the kite bisect each other?
6. What is the difference between the diagonals of a rhombus and those of a kite?

DIAGONALS OF PARALLELOGRAMS, RECTANGLES AND SQUARES

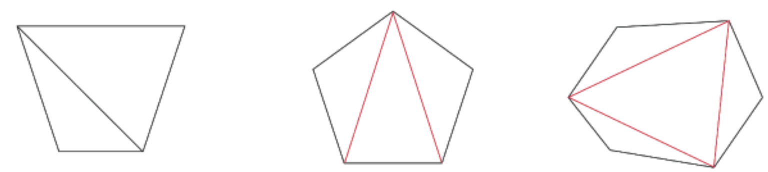
1. Draw a parallelogram, rectangle and square onto grid paper.
2. Draw in the diagonals of the quadrilaterals.
3. Indicate on each shape all the lengths in the diagonals that are equal. (Use a ruler.)
4. Use the information you have found to copy and complete the table below. Fill in "yes" or "no".

Quadrilateral	Diagonals equal	Diagonals bisect	Diagonals meet at 90°
Parallelogram	No	Yes	No
Rectangle	Yes	Yes	No
Square	Yes	Yes	Yes

10.8 Angles in polygons

USING DIAGONALS TO INVESTIGATE THE SUM OF THE ANGLES IN POLYGONS

1. We can divide a quadrilateral into two triangles by drawing in one diagonal.
   (a) Copy the polygons below and draw in diagonals to divide each of the polygons into as few triangles as possible.
   (b) Write down the number of triangles in each polygon.

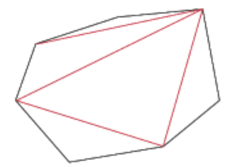


Quadrilateral | *Pentagon* | *Hexagon*

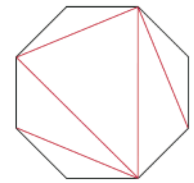
No. of Δs	2	3	4
Sum of ∠s	2 × 180° = 360°	3 × 180° = 540°	4 × 180° = 720°

Answers

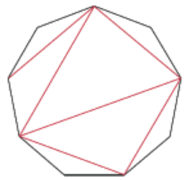
2. Beware that learners may want to draw triangles that share the centre of the polygon as a vertex. In that case, they have to subtract 360° (the central angle) from the sum of the angles of all the triangles.



Heptagon



Octagon



Nonagon

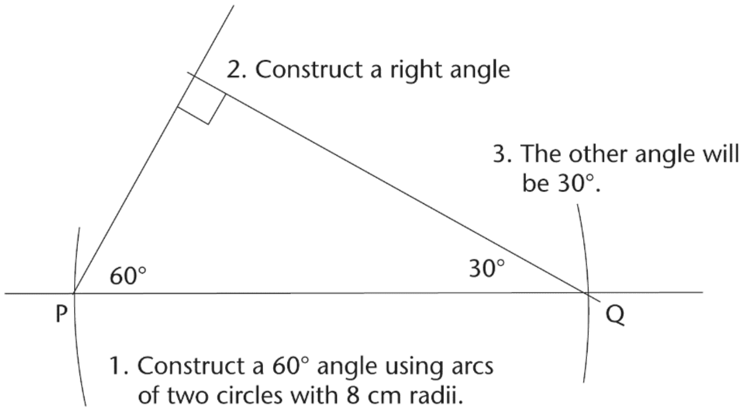
No. of Δs	5	6	7
Sum of ∠s	900°	1 080°	1 260°

2. The sum of the angles of one triangle = 180°. A quadrilateral is made up of two triangles, so the sum of the angles in a quadrilateral = 2 × 180° = 360°. Work out the sum of the interior angles of each of the other polygons above.

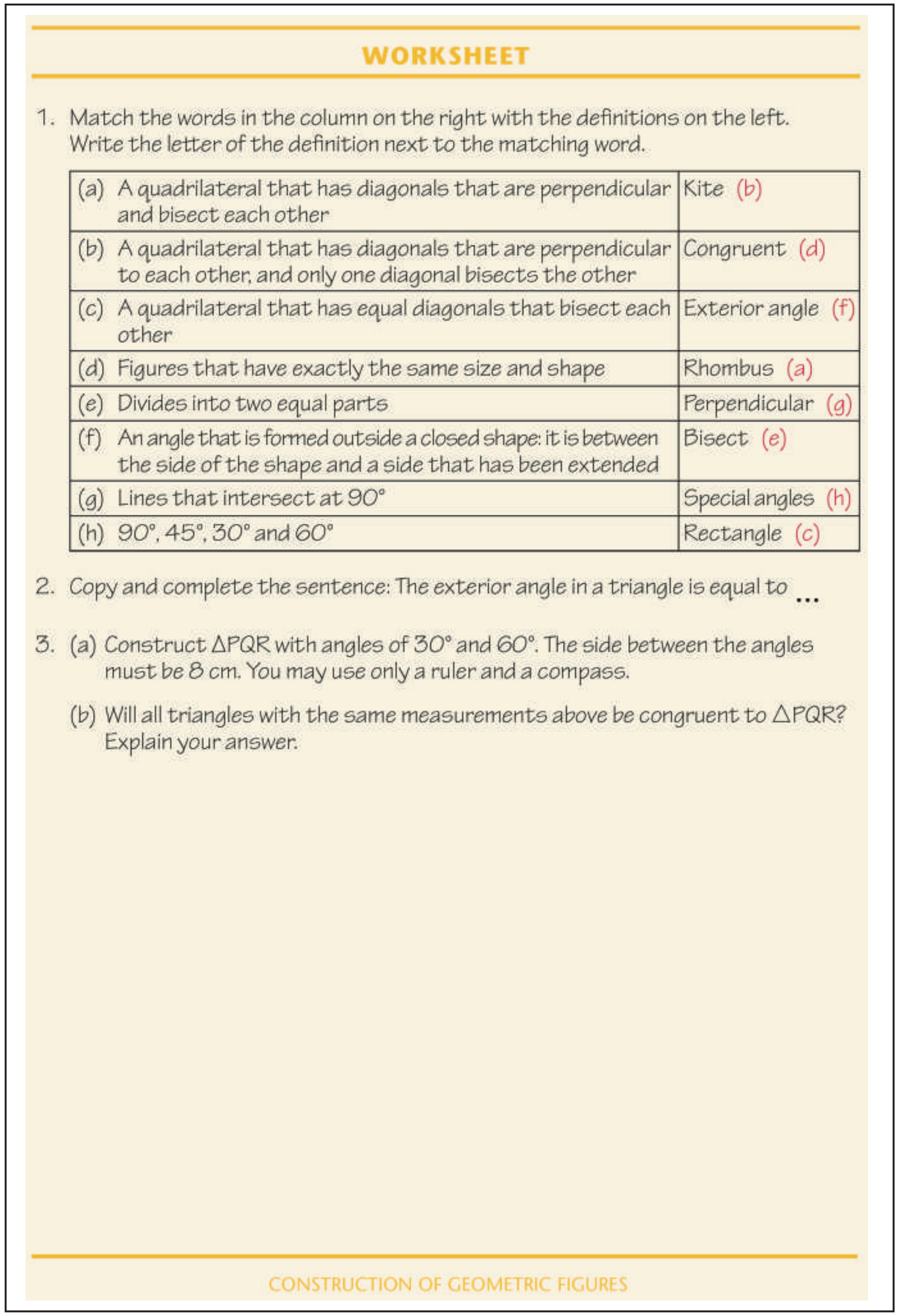
WORKSHEET

Answers

1. See the answers on page 109 of the LB alongside.
2. ...the sum of the opposite two interior angles.
3. (a)



(b) Yes, all sides and angles of such triangles will be equal. The triangles may be in different positions. This is because the third angle = 90°.



WORKSHEET

1. Match the words in the column on the right with the definitions on the left. Write the letter of the definition next to the matching word.

(a) A quadrilateral that has diagonals that are perpendicular and bisect each other	Kite (b)
(b) A quadrilateral that has diagonals that are perpendicular to each other, and only one diagonal bisects the other	Congruent (d)
(c) A quadrilateral that has equal diagonals that bisect each other	Exterior angle (f)
(d) Figures that have exactly the same size and shape	Rhombus (a)
(e) Divides into two equal parts	Perpendicular (g)
(f) An angle that is formed outside a closed shape: it is between the side of the shape and a side that has been extended	Bisect (e)
(g) Lines that intersect at 90°	Special angles (h)
(h) 90°, 45°, 30° and 60°	Rectangle (c)

2. Copy and complete the sentence: The exterior angle in a triangle is equal to ...

3. (a) Construct ΔPQR with angles of 30° and 60°. The side between the angles must be 8 cm. You may use only a ruler and a compass.

   (b) Will all triangles with the same measurements above be congruent to ΔPQR? Explain your answer.

CONSTRUCTION OF GEOMETRIC FIGURES

Geometry of 2D shapes

Learner Book Overview		
Sections in this chapter	**Content**	**Pages in Learner Book**
11.1 Revision: Classification of triangles	Angles of triangles; sides of triangles	Pages 110 to 111
11.2 Finding unknown angles in triangles	Unknown sides and angles of triangles	Pages 112 to 113
11.3 Quadrilaterals	Properties of quadrilaterals; unknown sides and angles of quadrilaterals	Pages 113 to 116
11.4 Congruent triangles	Definition and notation of congruent triangles; minimum conditions for congruent triangles; problem solving	Pages 116 to 120
11.5 Similar triangles	Properties of similar triangles; application of properties of similar triangles; minimum conditions for similarity; problem solving	Pages 120 to 125
11.6 Extension questions	Multi-step problem solving	Page 126

CAPS time allocation	9 hours
CAPS content specification	Page 136

Mathematical background

- In this chapter, learners work with 2D shapes that they have already encountered in previous years. They now focus on the **properties of different types of triangles and quadrilaterals**. Although learners should be able to identify and classify triangles and quadrilaterals, it is best to revise the basic properties of each type of figure. They need a solid understanding of the properties of these figures, as they need to start using these properties to work out information which may be unknown.
- Learners have established the **difference between congruent and similar figures** in previous grades. In this grade, their study becomes more formalised, as they now consolidate their understanding of the minimum conditions for congruent and similar triangles, which they started investigating in the previous chapter.
- Once learners have a good grounding in the properties of triangles and quadrilaterals, as well as in congruency and similarity, they should be able to **solve geometric problems**. Make sure that learners provide reasons for their statements to justify their reasoning. (Note: this will take practice and you may need to model this method for learners until they know what is expected.)

11.1 Revision: Classification of triangles

Background information

- **Triangles** are 2D figures enclosed by three straight sides which form three interior angles. In any triangle:
  - the **sum of the interior angles** is 180°
  - any **exterior angle** is equal to the sum of the opposite interior angles.
- Triangles are **classified** according to the **sizes of their interior angles**, or the **lengths of their sides**, or both. A triangle with:
  - **three acute interior angles** is an acute-angled triangle.
  - **one right interior angle** is a right-angle triangle.
  - **one obtuse interior angle** is an obtuse-angled triangle.
  - **three sides equal** is an equilateral triangle.
  - **two sides equal** is an isosceles triangle.
  - **no sides equal** is a scalene triangle.
- An **equilateral triangle** has:
  - all three sides equal in length
  - all three interior angles equal to 60° each.
- An **isosceles triangle** has two sides equal.
  - The two interior angles opposite the equal sides are equal.
- A **right-angled triangle** has one right interior angle.
  - The **hypotenuse** is the longest side of a triangle and lies opposite the right angle.
  - The area of the square on the hypotenuse is equal to the sum of the areas of the squares on the other two sides (**Theorem of Pythagoras**).

Teaching guidelines

Learners can only solve geometric problems on triangles if they know all the properties of triangles listed above.

Answers

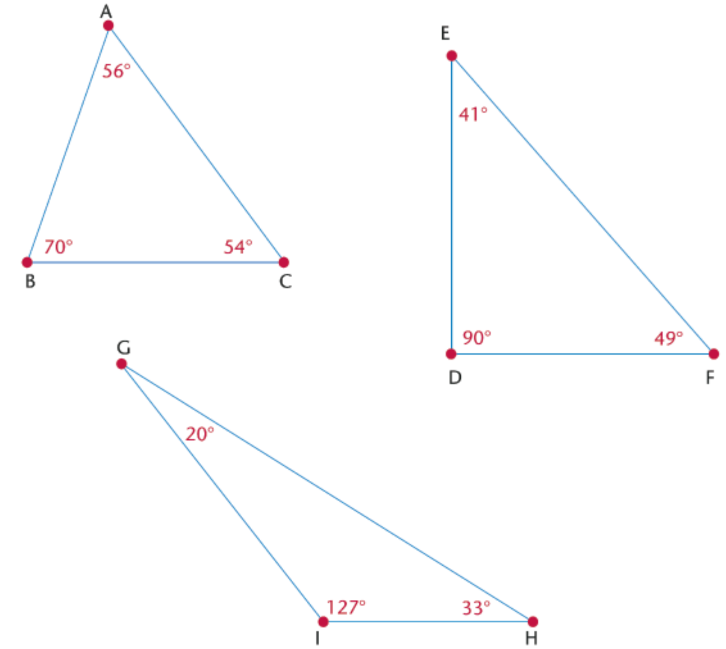
1. See the answers on LB page 110 alongside.
2. See the answers on LB page 110 alongside and LB page 111 on the following page.

CHAPTER 11

Geometry of 2D shapes

11.1 Revision: Classification of triangles

1. Use a protractor to measure the interior angles of each of the following triangles. Write down the sizes of the angles.



2. Classify the triangles in question 1 according to their angle properties. Copy and complete the following statements by choosing from the following types of triangles: **acute-angled**, **obtuse-angled** and **right-angled**.
   (a) ΔABC is an acute angled triangle, because all interior angles are smaller than 90°.

Answers

3. See the answers on LB page 111 alongside.
4. See the table on LB page 111 alongside.

(b) ΔEDF is a right-angled triangle, because it has a 90° angle.

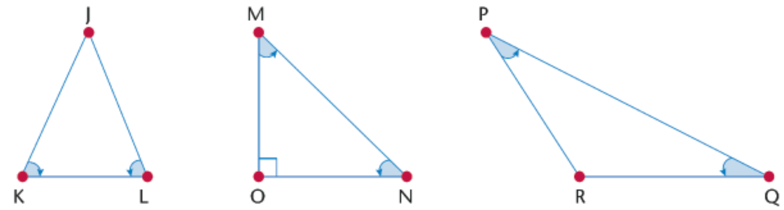
(c) ΔGHI is an obtuse-angled triangle, because one of the interior angles is larger than 90°.

3. The marked angles in each triangle below are equal. Copy and complete the following statements and classify the triangles according to angle and side properties.

(a) Δ JKL is an acute isosceles triangle, because $\hat{K} = \hat{L}$ and $\hat{J} < 90°$.

(b) Δ MON is a right-angled isosceles triangle, because $\hat{M} = \hat{N}$ and $\hat{O} = 90°$.

(c) Δ PRQ is an obtuse isosceles triangle, because $\hat{P} = \hat{Q}$ and $\hat{R} > 90°$.



4. Copy the table below. Say for what kind of triangle each statement is true. If it is true for all triangles, then write "All triangles".

	Statement	True for:
(a)	Two sides of the triangle are equal.	Isosceles
(b)	One angle of the triangle is obtuse.	Obtuse-angled
(c)	Two angles of the triangle are equal.	Isosceles
(d)	All three angles of the triangle are equal to 60°.	Equilateral
(e)	The size of an exterior angle is equal to the sum of the opposite interior angles.	All triangles
(f)	The longest side of the triangle is opposite the biggest angle.	All triangles, except equilateral triangles
(g)	The sum of the two shorter sides of the triangle is bigger than the length of the longest side.	All triangles, except equilateral triangles
(h)	The square of the length of one side is equal to the sum of the squares of the other sides.	Right-angled
(i)	The square of the length of one side is bigger than the sum of the squares of the other sides.	Obtuse-angled
(j)	The sum of the interior angles of the triangle is 180°.	All triangles

11.2 Finding unknown angles in triangles

FINDING UNKNOWN LENGTHS AND ANGLES

Background information

- The **sum of the interior angles** of a triangle is 180°.
- In an **equilateral triangle** each interior angle is equal to 60°.
- In an **isosceles triangle** the interior angles can be found when the size of:
  - one base angle is given, or
  - the angle between the equal sides is given.
- In a **scalene triangle** the interior angles can be found when:
  - the sizes of two interior angles are given, or
  - the proportional sizes of the interior angles are given.

Teaching guidelines

Discuss the layout of the solution provided on LB page 112. Point out that a reason should be provided for every statement that is made.

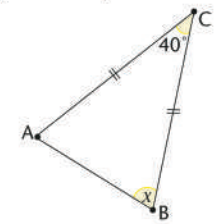
Answers

1. In ΔDEF: $a = 49°$ (sum ∠s △)
   In ΔJIG: $b = 105°$ (sum ∠s △)
   $c = 75°$ (ext. ∠s △ or ∠s on a str. line = 180°)
2. In ΔSTU: $d = 76°$ (isos. △, ∠s opp equal sides)
   $e = 28°$ (sum ∠s △)
   $f = 284°$ (∠s around a point)
   In ΔMON: $g = 51°$ (ext. ∠s △)
   $h = 51°$ (isos. △, ∠s opp equal sides or ∠s on str. line or sum ∠s △)
   $z = 102°$ (∠s on str. line or ext. ∠s △)
   MO = 8 m (MN = MO)

11.2 Finding unknown angles in triangles

When you have to determine the size of an unknown angle or length of a shape in geometry, you must give a reason for each statement you make. Complete the example below.

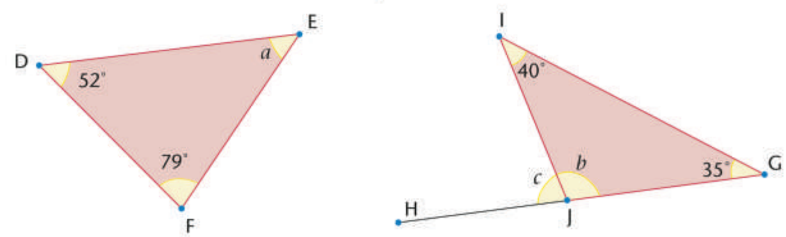
In ΔABC, AC = BC and $\hat{C} = 40°$. Find the size of $\hat{B}$ (shown in the diagram as x).



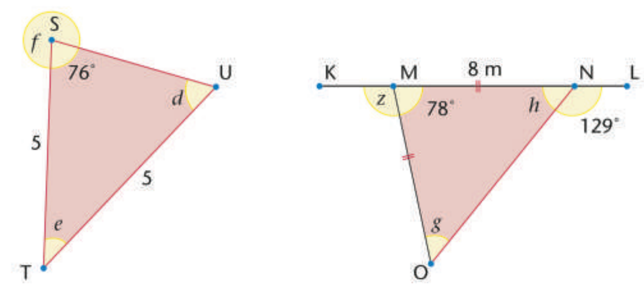
Statement	Reason
AC = BC	Given
$\therefore \hat{A} = \hat{B}$	Isos Δ angles opposite equal sides
$180° = 40° + x + x$	Sum ∠s Δ
$180° - 40° = 2x$	
$\therefore x = 70°$	

FINDING UNKNOWN LENGTHS AND ANGLES

1. Calculate the sizes of the unknown angles.



2. Determine the sizes of the unknown angles and the length of MO.



112 MATHEMATICS GRADE 9: TERM 2

Answers

3. In ΔPQR: $P\hat{R}Q = 360° - 344° = 16°$ (∠s around a point)

$2y = 180° - (26° + 16°)$ (sum ∠s △)

$y = 69°$

In ΔABC: $6x = 180°$ (sum ∠s △)

$x = 30°$

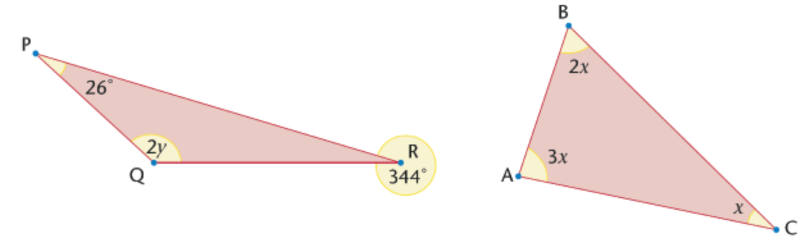
11.3 Quadrilaterals

PROPERTIES OF QUADRILATERALS

Background information

- **Quadrilaterals** are 2D figures enclosed by four straight sides which form four interior angles. In any quadrilateral the **sum of the interior angles** is 360°.
- **Squares** have the following properties:
  - opposite sides are parallel
  - all four sides are equal
  - each interior angle is equal to 90°
  - diagonals are perpendicular
  - diagonals bisect each other
  - diagonals are equal.
- **Rhombi** have the following properties:
  - opposite sides are parallel
  - all four sides are equal
  - opposite angles are equal
  - diagonals are perpendicular
  - diagonals bisect each other.
- **Rectangles** have the following properties:
  - opposite sides are parallel
  - opposite sides are equal
  - each interior angle is equal to 90°
  - diagonals bisect each other
  - diagonals are equal.

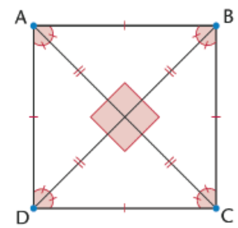
3. Calculate the sizes of y and x.



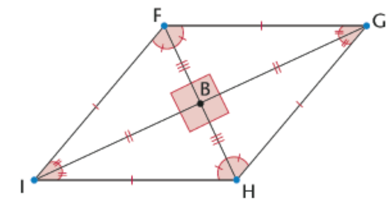
11.3 Quadrilaterals

PROPERTIES OF QUADRILATERALS

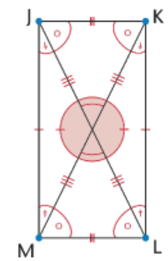
1. Name the following quadrilaterals. Copy the quadrilaterals and mark equal angles and equal sides in each figure. Use your ruler and protractor to measure angle sizes and lengths where necessary.



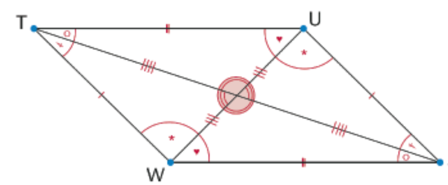
Square



Rhombus



Rectangle



Parallelogram

CHAPTER 11: GEOMETRY OF 2D SHAPES 113

- **Parallelograms** have the following properties:
  - opposite sides are parallel
  - opposite sides are equal
  - opposite angles are equal
  - diagonals bisect each other.
- **Trapeziums** have one pair of opposite sides parallel.
- **Kites** have the following properties:
  - two pairs of adjacent sides are equal
  - diagonals are perpendicular
  - one diagonal bisects the other diagonal.

Teaching guidelines

To solve geometric problems on quadrilaterals, learners must know all the properties of quadrilaterals listed above.

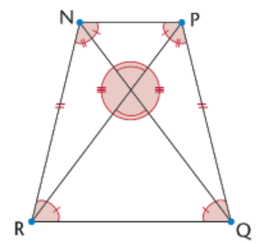
Note on question 2

The completed table on LB page 114 can be used to find those properties which will confirm statements such as the following:

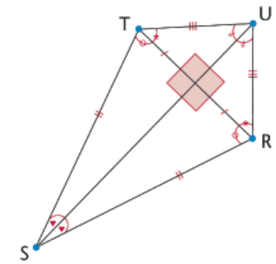
- A rectangle is a special kind of parallelogram.
- A rhombus is a special kind of parallelogram.
- A square is a special kind of rectangle.
- A square is a special kind of rhombus.

Answers

1. See the answers on LB pages 113 on the previous page and LB page 114 alongside.
2. See the table on LB pages 114 alongside and LB page 115 on the following page.



Trapezium



Kite

2. Copy and complete the following table:

Properties	True for the following quadrilaterals					
	Square	Rhombus	Rectangle	Parallelogram	Kite	Trapezium
At least one pair of opposite angles is equal.	yes	yes	yes	yes	yes	no
Both pairs of opposite angles are equal.	yes	yes	yes	yes	no	no
At least one pair of adjacent angles is equal.	yes	no	yes	no	no	no
All four angles are equal.	yes	no	yes	no	no	no
Any two opposite sides are equal.	yes	yes	yes	yes	no	no
Two adjacent sides are equal and the other two adjacent sides are also equal.	yes	yes	no	no	yes	no
All four sides are equal.	yes	yes	no	no	no	no
At least one pair of opposite sides is parallel.	yes	yes	yes	yes	no	yes
Any two opposite sides are parallel.	yes	yes	yes	yes	no	no
The two diagonals are perpendicular.	yes	yes	no	no	yes	no
At least one diagonal bisects the other one.	yes	yes	yes	yes	yes	no

Answers

3. (a) No. A square and a rhombus share the property that all four sides are equal. A square has right angles at the vertices, but a rhombus does not. The only angle property they share is that the opposite angles are equal. A square has equal diagonals and a rhombus does not.
   (b) Yes. The properties of a rhombus are shared by a square (see question 2).
   (c) See the answers on LB page 115 alongside.
4. (a) No. A rectangle does not have equal sides, diagonals that intersect at 90°, or diagonals that bisect the angles at the vertices.
   (b) Yes. A square has all the properties of a rectangle as well as additional properties.
   (c) See the answers on LB page 115 alongside.
5. (a) Yes. A rectangle has all the properties of a parallelogram, and additional properties.
   (b) No. A parallelogram does not have to have equal diagonals, or angles of 90°.
   (c) See the answers on LB page 115 alongside.
6. Yes. A rhombus has all the properties of a parallelogram as well as additional properties.
7. A kite does not have the following properties, which are needed for it to be a parallelogram: opposite sides equal, both pairs of opposite sides parallel and both diagonals bisect each other.
8. A trapezium does not have the following properties, which are needed for it to be a parallelogram: equal opposite sides, both pairs of opposite sides parallel and the diagonals do not bisect each other.

The two diagonals bisect each other.	yes	yes	yes	yes	no	no
The two diagonals are equal.	yes	no	yes	no	no	no
At least one diagonal bisects a pair of opposite angles.	yes	yes	no	no	yes	no
Both diagonals bisect a pair of opposite angles.	yes	yes	no	no	no	no
The sum of the interior angles is 360°.	yes	yes	yes	yes	yes	yes

3. Look at the properties of a square and a rhombus.
   (a) Are all the properties of a square also the properties of a rhombus? Explain.
   (b) Are all the properties of a rhombus also the properties of a square? Explain.
   (c) Which statement is true? Write down the statement.

   A square is a special kind of rhombus. True

   A rhombus is a special kind of square. False

4. Look at the properties of rectangles and squares.
   (a) Are all the properties of a square also the properties of a rectangle? Explain.
   (b) Are all the properties of a rectangle also the properties of a square? Explain.
   (c) Which statement is true? Write down the statement.

   A square is a special kind of rectangle. True

   A rectangle is a special kind of square. False

5. Look at the properties of parallelograms and rectangles.
   (a) Are all the properties of a parallelogram also the properties of a rectangle? Explain.
   (b) Are all the properties of a rectangle also the properties of a parallelogram? Explain.
   (c) Which statement is true? Write down the statement.

   A rectangle is a special parallelogram. True

   A parallelogram is a special rectangle. False

6. Look at the properties of a rhombus and a parallelogram. Is a rhombus a special kind of parallelogram? Explain.
7. Compare the properties of a kite and a parallelogram. Why is a kite not a special kind of parallelogram?
8. Compare the properties of a trapezium and a parallelogram. Why is a trapezium not a special kind of parallelogram?

UNKNOWN SIDES AND ANGLES IN QUADRILATERALS

Background information

- The **sum of the interior angles** of a quadrilateral is 360°.
- In a **parallelogram, rectangle, rhombus and square** the opposite angles are equal.
- In a **parallelogram and rhombus** all interior angles can be found if the size of one interior angle is given because pairs of co-interior angles are supplementary.
- In a **rectangle and square** each interior angle is equal to 90°.
- In a **trapezium** two pairs of co-interior angles are supplementary.
- In a **rhombus and square** all sides are equal in length.
- In some quadrilaterals the **Theorem of Pythagoras** can be applied to find the length of some line segments:
  - In **rectangles and squares** all interior angles are right angles.
  - In **rhombi, squares and kites** the diagonals are perpendicular to each other.

Teaching guidelines

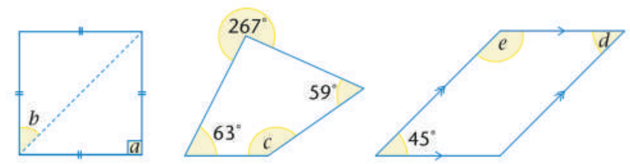
- Determine the minimum information that will enable learners to find the sizes of all interior angles of different types of quadrilaterals.
- Remind learners that they need to provide a reason for every statement that they make.

Answers

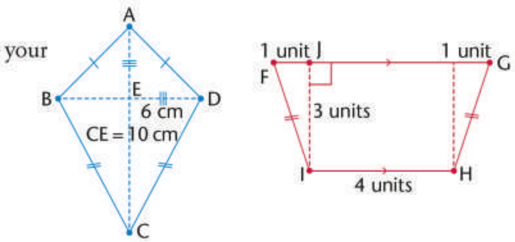
1. a = 90° (angle of square)
   b = 45° (diagonals of square bisect ∠s)
   c = 145° (∠s round a point, int. ∠s quad = 360°)
   d = 45° (opp. ∠s parallelogram)
   e = 135° (opp. ∠s and int. ∠s quad = 360°)
2. In ABCD: AD = $\sqrt{72}$ cm (Pythagoras, diagonals of a kite)
   CD = $\sqrt{136}$ cm (Pythagoras)
   Perimeter of ABCD = 40,29 cm
   In FGHI: FI = $\sqrt{10}$ units (Pythagoras)
   Perimeter of FIHG = 16,32 units

UNKNOWN SIDES AND ANGLES IN QUADRILATERALS

1. Determine the sizes of angles a to e in the quadrilaterals below. Give reasons for your answers.



2. Calculate the perimeters of the quadrilaterals on the right. Give your answers to two decimal places.

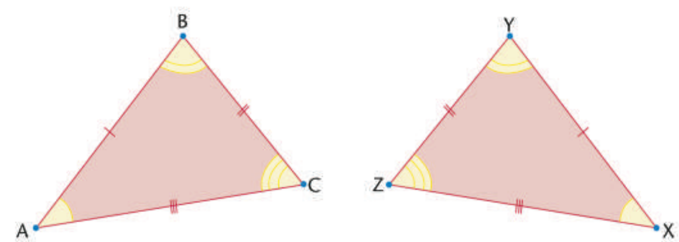


11.4 Congruent triangles

DEFINITION AND NOTATION OF CONGRUENT TRIANGLES

If two triangles are congruent, then they have exactly the same size and shape. In other words, if you cut out one of the triangles and place it on the other, they will match exactly.

If you know that two triangles are congruent, then each side in the one triangle will be equal to each corresponding side in the second triangle. Also, each angle in the one triangle will be equal to each corresponding angle in the second triangle.



11.4 Congruent triangles

DEFINITION AND NOTATION OF CONGRUENT TRIANGLES

Background information

- **Congruent triangles** have the same shape and size.
- The **symbol** ≡ means "is congruent to".
- If $\Delta ABC \equiv \Delta XYZ$:
  - the **corresponding angles are equal**: $\hat{A} = \hat{X}$, $\hat{B} = \hat{Y}$ and $\hat{C} = \hat{Z}$, and
  - the **corresponding sides are equal**: AB = XY, BC = YZ and CA = ZX.

Teaching guidelines

Discuss the definition and notation of congruent triangles. Point out that the letters of the corresponding vertices between the two triangles must appear in the same position.

Answers

1. See the answer on LB page 117 alongside.
2. See the answer on LB page 117 alongside.

MINIMUM CONDITIONS FOR CONGRUENT TRIANGLES

Background information

There are four conditions for congruency:

- **SSS**: all corresponding sides are equal
- **SAS**: two corresponding sides and the angle between the two sides are equal
- **AAS**: two corresponding angles and any corresponding side are equal
- **RHS**: both triangles are right-angled and have equal hypotenuses and one other side equal.

Teaching guidelines

Revise the condition for congruent triangles.

Answers

1. See the answer on LB page 117 alongside.
2. See the answer on LB page 117 alongside.

In the triangles on the previous page, you can see that $\Delta ABC \equiv \Delta XYZ$.

Congruency symbol

≡ means "is congruent to".

The order in which you write the letters when stating that two triangles are congruent is very important. The letters of the corresponding vertices between the two triangles must appear in the same position in the notation. For example, the notation for the triangles on the previous page should be: $\Delta ABC \equiv \Delta XYZ$, because it indicates that $\hat{A} = \hat{X}$, $\hat{B} = \hat{Y}$, $\hat{C} = \hat{Z}$, AB = XY, BC = YZ and AC = XZ.

It is incorrect to write $\Delta ABC \equiv \Delta ZYX$. Although the letters refer to the same triangles, this notation indicates that $\hat{A} = \hat{Z}$, $\hat{C} = \hat{X}$, AB = ZY and BC = YX, and these statements are not true.

Write down the equal angles and sides according to the following notations:

1. $\Delta KLM \equiv \Delta PQR$: $\hat{K} = \hat{P}$, $\hat{L} = \hat{Q}$, $\hat{M} = \hat{R}$, KL = PQ, LM = QR, KM = PR
2. $\Delta FGH \equiv \Delta CST$: $\hat{F} = \hat{C}$, $\hat{G} = \hat{S}$, $\hat{H} = \hat{T}$, FG = CS, GH = ST, FH = CT

MINIMUM CONDITIONS FOR CONGRUENT TRIANGLES

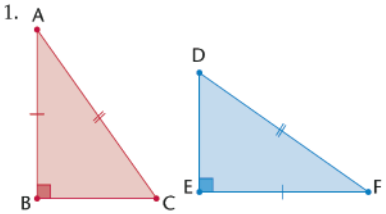
Earlier in this chapter, you investigated the minimum conditions that must be satisfied in order to establish that two triangles are congruent.

The conditions for congruency consist of:

- SSS (all corresponding sides are equal)
- SAS (two corresponding sides and the angle between the two sides are equal)
- AAS (two corresponding angles and any corresponding side are equal)
- RHS (both triangles have a 90° angle and have equal hypotenuses and one other side equal).

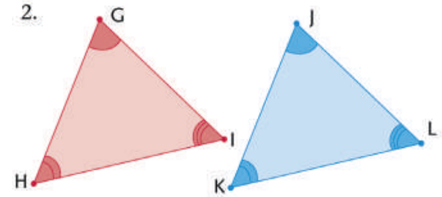
Decide whether or not the triangles in each pair below are congruent. For each congruent pair, write the notation correctly and give a reason for congruency.

1.



$\Delta ABC \equiv \Delta FED$ (RHS)

2.



Not necessarily congruent

CHAPTER 11: GEOMETRY OF 2D SHAPES 117

Note on question 3

Start with the pair of vertices of the obtuse angles, then the pair of vertices of the given acute angles and then the pair of vertices of the unknown angles.

Note on question 5

Start with the pair of vertices opposite 3 cm, then the pair of vertices opposite 2 cm and then the pair of vertices opposite 4 cm.

Note on question 6

Start with the pair of vertices opposite the given equal sides, then the pair of vertices opposite the common side and then the pair of vertices opposite the unknown side.

Answers

3. to 6. See the answers on LB page 118 alongside.

PROVING THAT TRIANGLES ARE CONGRUENT

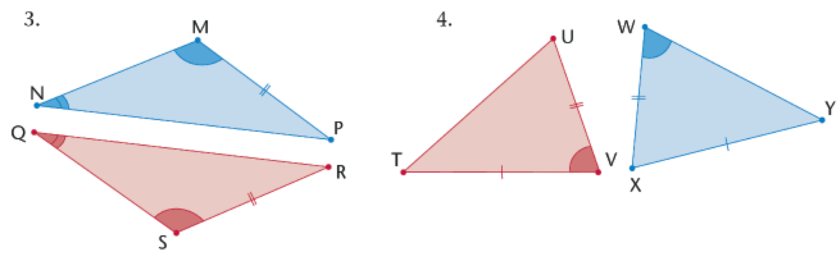
Background information

A **proof for congruency** must contain the following:

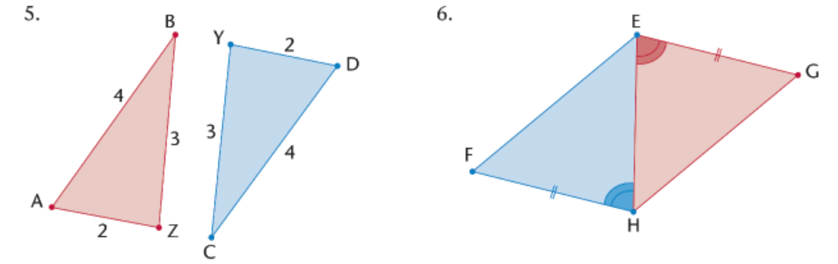
- You must give **three statements** to prove any two triangles congruent.
- You must provide a **reason** for each statement.
- Finally, you must give the **reason for congruence**.

Teaching guidelines

Use the example on LB pages 118 and 119 to explain the layout of a proof for congruency.



ΔMNP ≡ ΔSQR (AAS) Not necessarily congruent



ΔABZ ≡ ΔDCY (SSS) ΔEFH ≡ ΔHGE (SAS)

PROVING THAT TRIANGLES ARE CONGRUENT

You can use what you know about the minimum conditions for congruency to prove that two triangles are congruent.

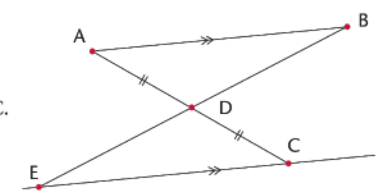
When giving a proof for congruency, remember the following:

- Each statement you make needs a reason.
- You must give three statements to prove any two triangles congruent.
- Give the reason for congruency.

Example:

In the sketch on the right: AB || EC and AD = DC.

Prove that the triangles are congruent.



118 MATHEMATICS GRADE 9: TERM 2

Mathematical notes

- It is good practice, but not compulsory, to do the following:
  - At the **start of the proof**, write down the names of the two triangles in the correct order.
  - **During the proof**, write down the three statements in the same order as the reason for congruency.
  - At the **end of the proof**, write down the reason for congruency.

Example: Refer to the example on LB pages 118 and 119.

In ΔABD and ΔCED:

1)	$A\hat{D}B = C\hat{D}E$	(vert. opp. ∠s)
2)	$B\hat{A}D = E\hat{C}D$	(alt. ∠s; AB || EC)
3)	AD = CD	(given)
∴	ΔABD ≡ ΔCED	(AAS)

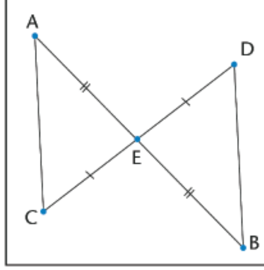
Answers

1. to 2. See the answers on LB page 119 alongside.

Solution:

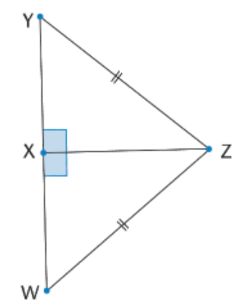
Statement	Reason
In ΔABD and ΔCED:	
1) AD = DC	Given
2) $A\hat{D}B = C\hat{D}E$	Vert. opp. ∠s
3) $B\hat{A}D = E\hat{C}D$	Alt. ∠s (AB || EC)
∴ ΔABD ≡ ΔCED	AAS

1. Copy the table with the sketch, and prove that ΔACE ≡ ΔBDE.



Statement	Reason
1) $A\hat{E}C = B\hat{E}D$	Vert. opp. ∠s
2) AE = BE	Given
3) CE = DE	Given
∴ ΔACE ≡ ΔBDE	SAS

2. Copy the table with the sketch, and prove that ΔWXZ ≡ ΔYXZ.



Statement	Reason
1) $W\hat{X}Z = Y\hat{X}Z = 90°$	Given
2) XZ is common	
3) WZ = YZ	Given
∴ ΔWXZ ≡ ΔYXZ	RHS

CHAPTER 11: GEOMETRY OF 2D SHAPES **119**

Note on question 3

Once the two triangles are proven congruent it can be deduced that all corresponding pairs of sides are equal, therefore QR = SP.

Note on question 4

Once the two triangles are proven congruent it can be deduced that all corresponding pairs of angles are equal, therefore $Q\hat{P}M = 41°$.

Answers

3. to 4. See the answers on LB page 120 alongside.

11.5 Similar triangles

PROPERTIES OF SIMILAR TRIANGLES

Background information

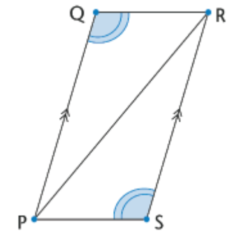
- **Similar triangles** have the same shape, but may vary in size.
- The **symbol** ||| means "is similar to".
- If ΔABC ||| ΔXYZ:
  - the **corresponding angles are equal**: $\hat{A} = \hat{X}$, $\hat{B} = \hat{Y}$ and $\hat{C} = \hat{Z}$

  OR
  - the **corresponding sides are in proportion**: AB : XY = BC : YZ = CA : ZX

Teaching guidelines

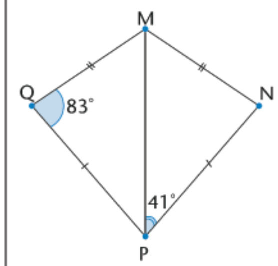
- Discuss the definition and notation of similar triangles. Point out that the letters of the corresponding vertices between the two triangles must appear in the same position.
- By the end of question 3 on LB page 121 learners should be able to:
  - formulate the properties for similar triangles
  - explain the notation for similar triangles.

3. Copy the table with the sketch, and prove that QR = SP. (Hint: First prove that the triangles are congruent.)



Statement	Reason
1) $P\hat{Q}R = R\hat{S}P$	Given
2) PR is common	
3) $Q\hat{P}R = P\hat{R}S$	Alt. ∠s (QP ‖ RS)
∴ ΔPQR ≡ ΔRSP	AAS
∴ QR = SP	

4. Copy the table with the sketch, and prove that the triangles below are congruent. Then find the size of $Q\hat{M}P$.

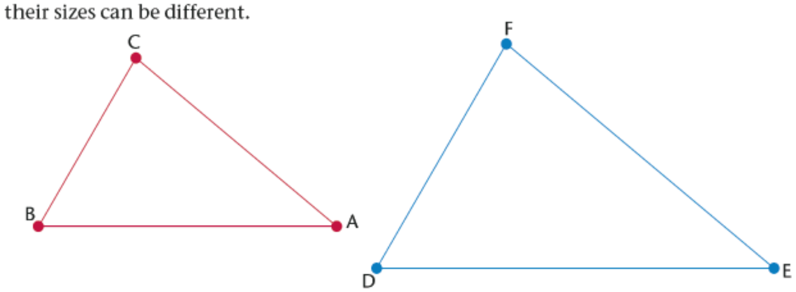


Statement	Reason
1) QM = NM	Given
2) MP is common	
3) QP = NP	Given
∴ ΔPQM ≡ ΔPNM	SSS
∴ $Q\hat{P}M = 41°$	$Q\hat{P}M = M\hat{P}N$ (congruency)
$Q\hat{M}P = 180° - (83° + 41°)$	Sum ∠s Δ
$= 56°$	

11.5 Similar triangles

PROPERTIES OF SIMILAR TRIANGLES

ΔBAC and ΔDEF below are similar to each other. Similar figures have the same shape, but their sizes can be different.



120 MATHEMATICS GRADE 9: TERM 2

Answers

1. (a) See the first table on LB page 121 alongside.
   (b) The corresponding angles in each triangle are equal.
2. (a) See the second table on LB page 121 alongside.
   (b) The corresponding sides are in proportion. In this case, △DEF's side lengths are $1\frac{1}{3}$ times as long as those of △BAC.
3. For similar triangles, the order of the letters in the notation indicates which angles and sides correspond. The equal angles must be in the same position for both triangles. Writing "△BAC" and "△DEF" correctly indicates that $\hat{B} = \hat{D}$, $\hat{A} = \hat{E}$ and $\hat{C} = \hat{F}$.

1. (a) Use a protractor to measure the angles in each triangle on the previous page. Then copy and complete the following table:

Angle	Angle	What do you notice?
$\hat{B} = 60°$	$\hat{D} = 60°$	$\hat{B} = \hat{D}$
$\hat{A} = 40°$	$\hat{E} = 40°$	$\hat{A} = \hat{E}$
$\hat{C} = 80°$	$\hat{F} = 80°$	$\hat{C} = \hat{F}$

   (b) What can you say about the sizes of the angles in similar triangles?

2. (a) Use a ruler to measure the lengths of the sides in each triangle in question 1. Then copy and complete the following table:

Length (cm)	Length (cm)	Ratio
BA = 6 cm	DE = 8 cm	BA : DE = 6 : 8 = $1 : 1\frac{1}{3}$
BC = 3,9 cm	DF = 5,2 cm	BC : DF = 3,9 : 5,2 = $1 : 1\frac{1}{3}$
CA = 5,25 cm	FE = 7 cm	CA : FE = 5,25 : 7 = $1 : 1\frac{1}{3}$

   (b) What can you say about the relationship between the sides in similar triangles?

Ratio reminder
You read 2 : 1 as "two to one".

3. The following notation shows that the triangles are similar: ΔBAC ||| ΔDEF. Why do you think we write the first triangle as ΔBAC and not as ΔABC?

The properties of similar triangles:
- The corresponding angles are equal.
- The corresponding sides are in proportion.

Notation for similar triangles:
If ΔXYZ is similar to ΔPQR, then we write: ΔXYZ ||| ΔPQR.

As for the notation of congruent figures, the order of the letters in the notation of similar triangles indicates which angles and sides are equal.
For ΔXYZ ||| ΔPQR:

Angles: $\hat{X} = \hat{P}$, $\hat{Y} = \hat{Q}$ and $\hat{Z} = \hat{R}$

Sides: XY : PQ = XZ : PR = YZ : QR

If the triangles' vertices were written in a different order, then the statements above would not be true.

When proving that triangles are similar, you either need to show that the corresponding angles are equal, or you must show that the sides are in proportion.

WORKING WITH PROPERTIES OF SIMILAR TRIANGLES

Teaching guidelines

Learners determine whether the corresponding sides of each pair of triangles are in proportion.

Notes on question 1 (a)

- For the two triangles, first find the ratio of the pair of longest sides $\frac{EF}{LN}$, then the ratio of the pair of shortest sides $\frac{EG}{LM}$ and then the ratio of the pair of remaining sides $\frac{GF}{MN}$. If the three ratios are equal, the triangles are similar.
- Write down the triangles by starting with the vertices opposite the longest sides, then the vertices opposite the shortest sides and then the vertices opposite the remaining sides.

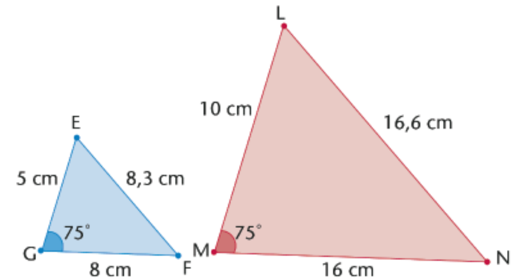
Answers

1. See the answers on LB page 122 alongside.

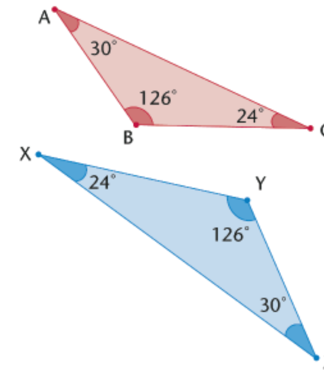
WORKING WITH PROPERTIES OF SIMILAR TRIANGLES

1. Decide if the following triangles are similar to each other:

(a)



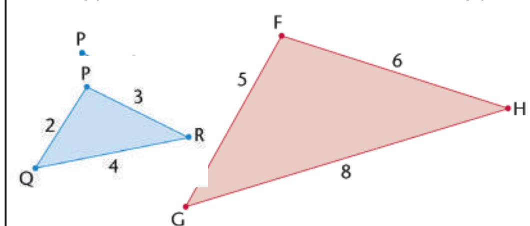
(b)



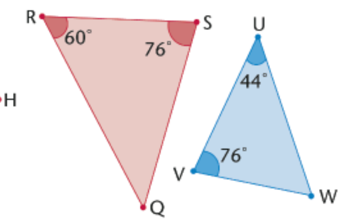
ΔEFG ||| ΔLMN (sides in proportion)

ΔABC ||| ΔZYX (AAA)

(c)



(d)



Not similar (sides not in proportion)

ΔRST ||| ΔWVU (AAA)

2. Do the following task:
   - Use a ruler and protractor to construct the triangles described in (a) to (d) on the next page.
   - Use your knowledge of similarity to draw the second triangle in each question.
   - Indicate the sizes of the corresponding sides and angles on the second triangle.

122 MATHEMATICS GRADE 9: TERM 2

Answers

2. (a) Learners construct ΔABC similar to a given ΔEFG.
   (b) Learners construct ΔPQR similar to a given ΔMNO.
   (c) Learners construct ΔVWX similar to a given ΔRST.
   (d) Learners construct ΔXYZ similar to a given ΔKLM.

INVESTIGATION: MINIMUM CONDITIONS FOR SIMILARITY

Background information

Triangles are **similar** when:

- their **corresponding angles are equal**
  OR
- their **corresponding sides are in proportion**.

Teaching guidelines

Learners determine whether some given conditions are minimum conditions for similar triangles.

Answers

1. See the answers on LB page 123 alongside.

(a) In ΔEFG, $\hat{G} = 75°$, EG = 4 cm and GF = 5 cm.
ΔABC is an enlargement of ΔEFG, with its sides three times longer.

(b) In ΔMNO, $\hat{M} = 45°$, $\hat{N} = 30°$ and MN = 5 cm.
ΔPQR is similar to ΔMNO. The sides of ΔMNO to ΔPQR are in proportion 1 : 3.

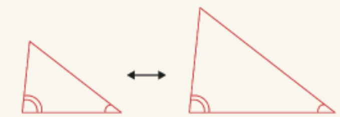
(c) ΔRST is an isosceles triangle. $\hat{R} = 40°$, RS is 10 cm and RS = RT.
ΔVWX is similar to ΔRST. The sides of ΔRST to ΔVWX are in proportion $1 : \frac{1}{2}$.

(d) ΔKLM is right-angled at $\hat{L}$, LM is 7 cm and the hypotenuse is 12 cm.
ΔXYZ is similar to ΔKLM, so that the sides are a third of the length of ΔKLM.

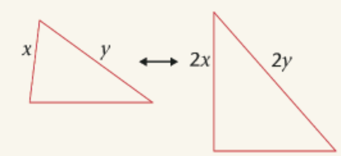
INVESTIGATION: MINIMUM CONDITIONS FOR SIMILARITY

Which of the following are minimum conditions for similar triangles?

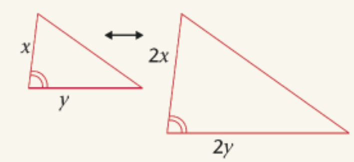
(a) Two angles in one triangle are equal to two angles in another triangle. Yes



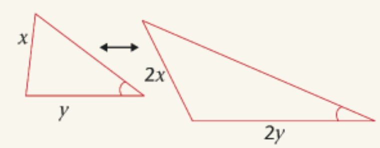
(b) Two sides of one triangle are in the same proportion as two sides in another triangle. No



(c) Two sides of one triangle are in the same proportion as two sides in another triangle, and the angle between the two sides is equal to the angle between the corresponding sides. Yes



(d) Two sides of one triangle are in the same proportion as two sides in another triangle, and one angle not between the two sides is equal to the corresponding angle in the other triangle. No (this only works if the side opposite the angle given is the longer of the two sides.)



SOLVING PROBLEMS WITH SIMILAR TRIANGLES

Background information

A **proof for similarity** must contain the following:

- You must give **three statements** to prove any two triangles similar: EITHER that the three **corresponding angles** are equal or that the three **corresponding sides are in the same proportion**.
- Each statement must be accompanied by a **reason**.
- Finally, you must give the **reason for similarity**.

Teaching guidelines

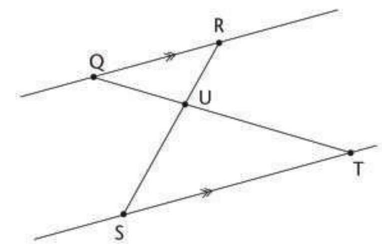
Use the table on LB page 124 to explain the layout of a proof for similarity.

Answers

1. See the table on LB page 124 alongside.
2. (a) Yes. Angles are equal (same proof as in question 1).
   (b) $\triangle$ABE ||| $\triangle$DCE; $\triangle$FGJ ||| $\triangle$HIJ
   (c) Yes. If there are parallel lines, the pairs of alternate angles will always be equal. So, you have similar triangles.

SOLVING PROBLEMS WITH SIMILAR TRIANGLES

1. Line segment QR is parallel to line segment ST.

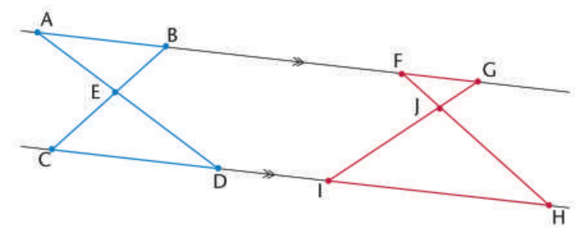


Parallel lines never meet. Two lines are parallel to each other if the distance between them is the same along the whole length of the lines.

Copy and complete the following proof that ΔQRU ||| ΔTSU:

Statement	Reason	
$R\hat{Q}T = Q\hat{T}S$	Alt. $\angle$s	(QR || ST)
$Q\hat{R}S = R\hat{S}T$	Alt. $\angle$s	(QR || ST)
$Q\hat{U}R = T\hat{U}S$	Vert. opp. $\angle$s	
$\therefore$ ΔQRU ||| ΔTSU	Equal $\angle$s	(or AAA)

2. The following intersecting line segments form triangle pairs between parallel lines.



(a) Are the triangles in each pair similar? Explain.
(b) Write down pairs of similar triangles.
(c) Are triangles like these always similar? Explain how you can be sure without measuring every possible triangle pair.

Mathematical notes

- It is good practice, but not compulsory, to do the following:
  - At the **start of the proof**, write down the names of the two triangles in the correct order.
  - **During the proof**, write down the three statements in the same order as the reason for similarity: either, three pairs of equal angles or, three pairs of proportional sides.
  - At the **end of the proof**, write down the reason for similarity.

Example: Refer to question 4 on LB page 125.

In ΔADE and ΔABC:

1) $\hat{A} = \hat{A}$ (common)
2) $A\hat{D}E = A\hat{B}C$ (corr. ∠s; DE || BC)
3) $A\hat{E}D = A\hat{C}B$ (corr. ∠s; DE || BC)

∴ ΔABD ||| ΔCED (AAA)

Answers

3. See the answer on LB page 125 alongside.
4. See the answer on LB page 125 alongside.
5. In △STW and △UVW:
   1) $\hat{W}$ is common
   2) $S\hat{T}W = U\hat{V}W = 90°$ (context of problem)
   3) $T\hat{S}W = V\hat{U}W$ (sum ∠s △)

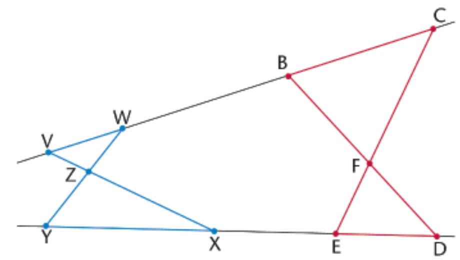
   ∴ △STW ||| △UVW (AAA)

   ∴ proportion = 5,1 : 1,7 = 1 : 3

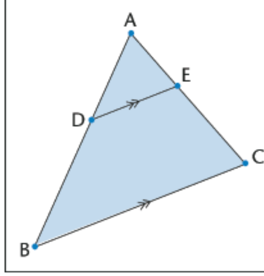
   ∴ ST = 3 × UV = 3 × 1 = 3 m
6. See the answer on LB page 125 alongside.

3. The intersecting lines on the right form triangle pairs between the line segments that are not parallel. Are these triangle pairs similar? Explain why or why not.

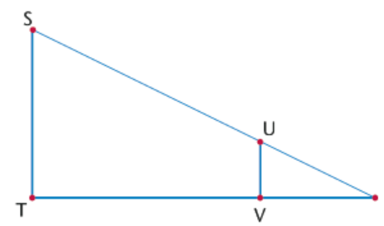
They are not similar. The corresponding angles in each triangle are not equal and the sides are not in proportion.



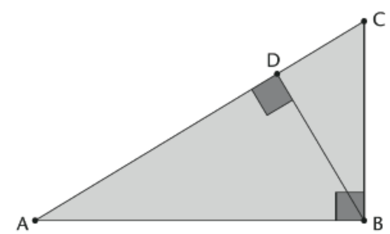
4. Consider the triangles below. DE || BC. Copy the table with the sketch, and prove that ΔABC ||| ΔADE.

	Statement	Reason
	$\hat{A}$ is common $A\hat{B}C = A\hat{D}E$ $A\hat{C}B = A\hat{E}D$ ∴ ΔABC ||| ΔADE	 Corr. ∠s (DE || BC) Corr. ∠s (DE || BC) OR Sum ∠sΔ AAA

5. In the diagram on the right, ST is a telephone pole and UV is a vertical stick. The stick is 1 m high and it casts a shadow of 1,7 m (VW). The telephone pole casts a shadow of 5,1 m (TW). Use similar triangles to calculate the height of the telephone pole.



6. How many similar triangles are there in the diagram below? Explain your answer.



In ΔABC and ΔADB:
$\hat{A}$ is shared
$\hat{B} = \hat{D} = 90°$ (given) ∴ ΔABC ||| ΔADB
In ΔABC and ΔBDC:
$\hat{C}$ is shared
$\hat{B} = \hat{D} = 90°$ (given) ∴ ΔABC ||| ΔBDC
∴ ΔABC ||| ΔADB ||| ΔBDC (AAA).

CHAPTER 11: GEOMETRY OF 2D SHAPES 125

11.6 Extension questions

Teaching guidelines

Learners solve multi-step problems on congruency and similarity.

Answers

1. (a) $\hat{B} = 60°$ (ABC is equilateral, given)
   $BA = BC$ (ABC is equilateral, given)
   $\therefore BD = BE$ (D and E are midpoints)
   $B\hat{D}E = B\hat{E}D$ (isos. $\triangle$)
   $\therefore B\hat{D}E = B\hat{E}D = 60°$ (sum $\angle$s $\triangle$)
   $\therefore \triangle BDE$ is equilateral

   (b) In $\triangle BDE$ and $\triangle ADF$:
   1) $D\hat{B}E = F\hat{A}D = 60°$ ($\triangle ABC$ is equilateral)
   2) $BD = AD$ (D is midpoint, given)
   3) $BE = AF$ ($BC = AC$ equilateral ΔABC; $BE = EC = CF = AF$ midpoints)

   $\therefore \triangle BDE \equiv \triangle ADF$ (SAS)

   Similarly, $\triangle BDE \equiv \triangle CFE$ hence $\triangle ADF \equiv \triangle CFE$.

   $DE = EF = AF$ ($\triangle BDE$ is equilateral, proved in 1(a))

   $\therefore \triangle BDE \equiv \triangle DEF$

   $\therefore \triangle BDE \equiv \triangle DEF \equiv \triangle ADF \equiv \triangle CFE$

   (c) $\triangle ABC \,|||\, \triangle BDE \,|||\, \triangle DEF \,|||\, \triangle ADF \,|||\, \triangle CFE$ (all equilateral)

   (d) For $\triangle ABC \,|||\, \triangle BDE$, the sides are in the proportion 2 : 1.

   (e) $B\hat{A}C = 60°$ ($\triangle ABC$ is equilateral)
   $B\hat{D}E = 60°$ ($\triangle BDE$ is equilateral)
   $DE \neq AC$ ($B\hat{A}C = B\hat{D}E$, Corr. $\angle$s)

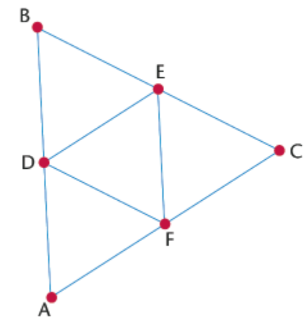
   (f) Yes. Similar reason as 1(e).

2. (a) The radii are equal, therefore $FJ = FL$, $FI = FK$ and $FG = FH$. Therefore, the sides of each triangle are in proportion. $\hat{F}$ is shared in all three triangles. Therefore, the triangles are similar (SAS, or sides in proportion and a shared angle).

   (b) Each triangle shares $\hat{U}$. Each triangle has a 90° angle. Therefore, they are similar (AAA).

11.6 Extension questions

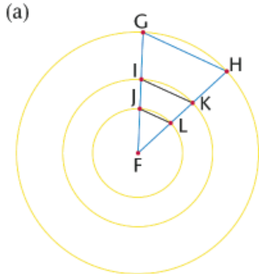
1. ΔABC on the right is equilateral. D is the midpoint of AB, E is the midpoint of BC and F is the midpoint of AC.



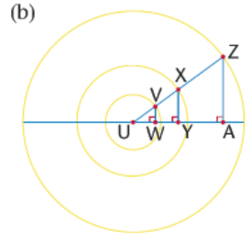
   (a) Prove that ΔBDE is an equilateral triangle.
   (b) Find all the congruent triangles. Give a proof for each.
   (c) Name as many similar triangles as you can. Explain how you know they are similar.
   (d) What is the proportion of the corresponding sides of the similar triangles?
   (e) Prove that DE is parallel to AC.
   (f) Is DF parallel to BC? Is EF parallel to BA? Explain.

2. Consider the similar triangles drawn below using concentric circles. Explain why the triangles are similar in each diagram.

(a)



(b)



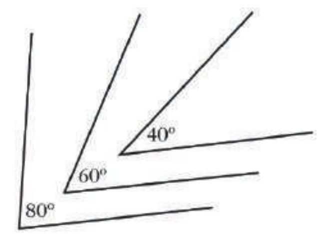
Grade 9 Term 2 Chapter 12 Geometry of straight lines

Learner Book Overview		
Sections in this chapter	**Content**	**Pages in Learner Book**
12.1 Angle relationships	Defining supplementary angles; angles formed by perpendicular lines; vertically opposite angles; corresponding, alternate and co-interior angles; angles formed by parallel lines	Pages 127 to 133
12.2 Identify and name angles	Identifying conditions for corresponding angles and alternate angles to be equal and co-interior angles to be supplementary	Pages 133 to 134
12.3 Solving problems	Using the knowledge gained about angles and lines to solve problems relating to angles that are equal to given angles, giving reasons for the answers	Pages 134 to 135

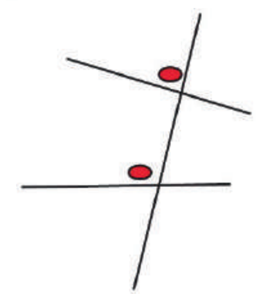
CAPS time allocation	9 hours
CAPS content specification	Page 137

Mathematical background

When straight lines meet or intersect, angles are formed. Some of these concepts are:



- Supplementary angles are angles that add up to 180°. The word "supplementary" gives information about the numerical value of angles. Supplementary angles can share a vertex (be part of the same figure). Any number of angles may be described by the phrase "supplementary angles", and the angles need not be elements of the same figure. For example, angles of 40°, 60° and 80° are supplementary, but they need not be part of the same figure.
- Perpendicular lines form right angles.
- Vertically opposite angles share a vertex, but not sides. The word "vertically" gives information about the spatial arrangement of the angles as well as implicit information about their sizes – i.e. they are equal.
- Corresponding, alternate and co-interior angles are formed when two lines are cut by a transversal. The names only indicate how the angles are positioned in relation to each other in an arrangement of straight lines.
  Angles can only be corresponding, alternate and co-interior if they are elements of the same figure, and specifically if they have one common side.
  If the lines that are cut by the transversal are parallel, corresponding and alternate angles are equal and co-interior angles are supplementary.



12.1 Angle relationships

Background information

Learners need to acquire the correct terminology, phrases and language usage to communicate effectively about geometry. The role of language in learning is not just to communicate with other people. Language also provides the framework in which personal knowledge is formed, and if learners do not have a command of the necessary language, they cannot learn geometry effectively.

Teaching guidelines

The words "supplementary angles" describe something about the sizes of the angles, i.e. the sum of the angles is 180°. Note that any number of angles may be described by the words "supplementary angles", and that the angles need not be elements of the same figure. Three angles of 40°, 60° and 80° are supplementary angles irrespective of where these angles occur. But only two of these angles are not supplementary: for example, angles of 40° and 60° are not supplementary because they do not add up to 180°.

Answers

1. (a) smaller
   (b) bigger
2. (a) 105°
   (b) $C\hat{M}A + C\hat{M}B = A\hat{M}B = 180°$

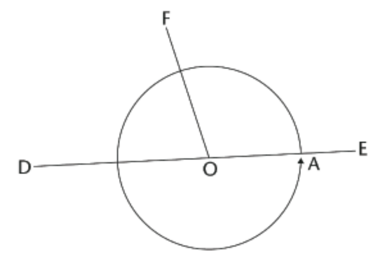
CHAPTER 12
Geometry of straight lines

12.1 Angle relationships

Remember that 360° is one full revolution.

If you look at something and then turn all the way around so that you are looking at it again, you have turned through an angle of 360°. If you turn only halfway around so that you look at something that was right behind your back, you have turned through an angle of 180°.

1. Answer the questions about the figure below.

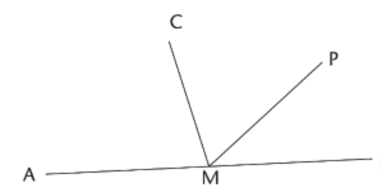


   (a) Is angle FOD in the figure smaller or bigger than a right angle?
   (b) Is angle FOE in the above figure smaller or bigger than a right angle?

In the figure above, $F\hat{O}D + F\hat{O}E$ = half of a revolution = 180°.

> The sum of the angles on a straight line is 180°.
> When the sum of angles is 180°, the angles are called **supplementary**.

2. $C\hat{M}A$ in the figure on the right is 75°.
   AMB is a straight line.
   (a) How big is $C\hat{M}B$?
   (b) Why do you say so?



CHAPTER 12: GEOMETRY OF STRAIGHT LINES 127

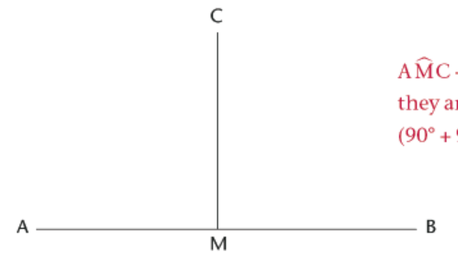
Teaching guidelines

Remind learners what that when two lines are perpendicular, they form right angles.

Answers

3. (a) $105° - 40° = 65°$
   (b) $C\hat{M}P + P\hat{M}B = 105° = C\hat{M}B$
4. (a) 90°
   (b) $A\hat{M}C + B\hat{M}C$ form straight line $A\hat{M}B$. If they are equal, each angle must be 90°. $(90° + 90° = 180°)$
5. (a) Yes
   (b) Yes, by using the angle definition of a straight line.
   $C\hat{M}A + C\hat{M}B = 180°$ and $B\hat{M}D + C\hat{M}B = 180°$
   If $C\hat{M}B$ is subtracted on both sides, it follows that $C\hat{M}A = B\hat{M}D$.
   (c) 180°
   They are angles on straight line CD.
   (d) 180°
   They are angles on straight line AB.

3. $P\hat{M}B$ in the figure in question 2 is 40°.
   (a) How big is $C\hat{M}P$?
   (b) Explain your reasoning.
4. In the figure below, AMB is a straight line and $A\hat{M}C$ and $B\hat{M}C$ are equal angles.
   (a) How big are these angles?
   (b) How do you know this?

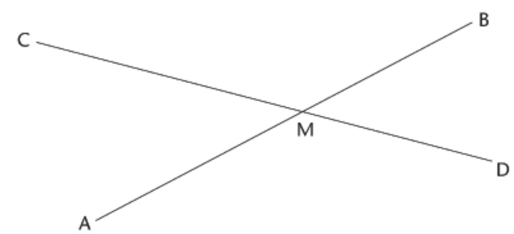


$A\hat{M}C + B\hat{M}C$ form straight line AMB. If they are equal, each angle must be 90°. $(90° + 90° = 180°)$

When one line forms two equal angles where it meets another line, the two lines are said to be **perpendicular**.

Because the two equal angles are angles on a straight line, their sum is 180°, hence each angle is 90°.

5. In the figure below, lines AB and CD intersect at point M.

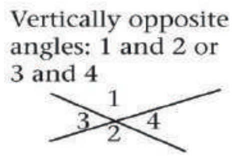


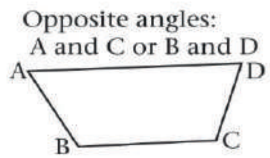
   (a) Does it look as if $C\hat{M}A$ and $B\hat{M}D$ are equal?
   (b) Can you explain why they are equal?
   (c) What does $C\hat{M}A + D\hat{M}A$ equal? Why do you say so?
   (d) What is $C\hat{M}A + C\hat{M}B$? Why do you say so?

In this chapter, you are required to give good reasons for every statement you make.

Teaching guidelines

Make sure that learners understand the difference between angles that are vertically opposite in that they share a vertex but not a side, and angles that are opposite each other in a shape. Opposite angles do not share a common vertex.





Answers

5. (e) Yes
   (f) $C\widehat{M}A$
6. Each side of the equal equation has one angle that is the same size, so the other two angles have to be equal to each other.
7. (a) 125°
   (b) It is vertically opposite $B\widehat{M}C$.

LINES AND ANGLES

Background information

Although the phrase "corresponding angles" sounds similar to supplementary angles, it has an entirely different kind of meaning. The term "corresponding" does not contain any information about the sizes of the angles; it only indicates the position of the angles in relation to each other in an arrangement of straight lines. Angles can only be corresponding if they are elements of the same figure, and specifically if they have one common arm. Corresponding angles are not necessarily equal – they are only equal if the non-shared arms are parallel.

Teaching guidelines

Remind learners what a transversal is.

Talk about "corresponding", "alternate" and "co-interior" angles, and explain that these words only say something about the positions of the angles.

Discuss the conditions under which the corresponding angles and alternate angles will be equal (if the lines are parallel), and when co-interior angles will be supplementary.

Misconceptions

Learners think that angles are equal simply because they are corresponding or alternate.

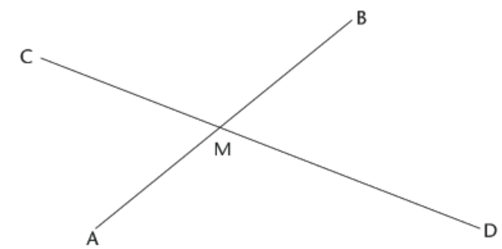
(e) Is it true that $C\widehat{M}A + D\widehat{M}A = C\widehat{M}A + C\widehat{M}B$?

(f) Which angle occurs on both sides of the equation in (e)?

6. Look carefully at your answers to questions 5(c) to (e). Now try to explain your observation in question 5(a).

7. In the figure below, AB and CD intersect at M. Four angles are formed. Angle CMB and angle AMD are called **vertically opposite** angles. Angle CMA and angle BMD are also **vertically opposite**.

When two straight lines intersect, the vertically opposite angles are equal.

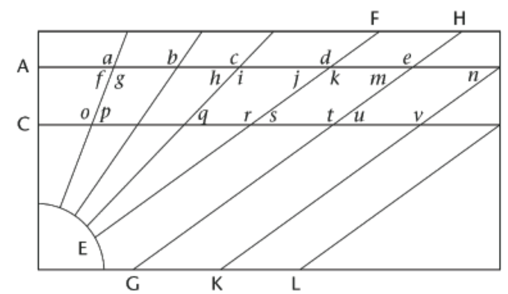


(a) If angle BMC = 125°, how big is angle AMD?

(b) Why do you say so?

LINES AND ANGLES

A line that intersects other lines is called a **transversal**.



In the above pattern, AB is parallel to CD and EF || GH || KB || LD.

CHAPTER 12: GEOMETRY OF STRAIGHT LINES 129

Answers

1. No, some of the angles look bigger or smaller than others.
2. Only $d = e$
3. (g, i, k); (o, r, t, v); (u, s, q, p)
4. Angles on the same side of the transversal and either all angles left of the other lines, or angles on the right-hand side of the other lines.
5. Yes
6. Yes, they are equal and they are angles on lines AB and CD, which are parallel to each other.
7. m and u; h and q; f and p
8. They are on alternate sides of the transversal and between the other lines.
9. Corresponding angles are also equal when AB is parallel to CD.
10. g and p; j and r; m and t; v and n

ANGLES FORMED BY PARALLEL LINES

Teaching guidelines

We have already established that corresponding angles are only equal if the transversal that forms them cuts parallel lines.

Discuss with learners that this also provides a way to show that two lines are parallel. If we can prove that corresponding angles are equal, we know the lines cut by the transversal are parallel.

Background information

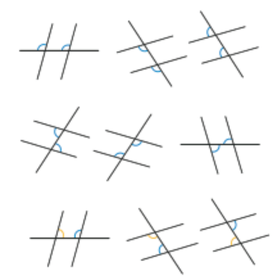
By convention, the definition of parallel lines is that if corresponding angles are equal when two lines are cut by a transversal, the lines are parallel.

1. Angles a, b, c, d and e are **corresponding angles**. Do the corresponding angles appear to be equal?
2. Investigate whether or not the corresponding angles are equal by using tracing paper. Trace the angle you want to compare to other angles and place it on top of the other angle to find out if they are equal. What do you notice?
3. Angles f, h, j, m and n are also corresponding angles. Identify all the other groups of corresponding angles in the pattern.
4. Describe the position of corresponding angles that are formed when a transversal intersects other lines.
5. The following are pairs of **alternate angles**: g and o; j and s; and k and r. Do these angles appear to be equal?
6. Investigate whether or not the alternate angles are equal by using tracing paper. Trace the angle you want to compare and place it on top of the other angle to find out if they are equal. What do you notice?
7. Identify two more pairs of alternate angles.
8. Clearly describe the relative position of alternate angles that are formed when a transversal intersects other lines.
9. Did you notice anything about some of the pairs of corresponding angles when you did the investigation in question 6? Describe your finding.
10. Angles f and o, i and q and k and s are all pairs of **co-interior angles**. Identify three more pairs of co-interior angles in the pattern.

The angles in the same relative position at each intersection where a straight line crosses two others are called **corresponding angles**.

Angles on different sides of a transversal and between two other lines are called **alternate angles**.

Angles on the same side of the transversal and between two other lines are called **co-interior angles**.



ANGLES FORMED BY PARALLEL LINES

Corresponding angles

The lines AB and CD shown on the following page never meet. Lines that never meet and are at a fixed distance from one another are called parallel lines. We write AB || CD.

130 MATHEMATICS GRADE 9: TERM 2

Answers

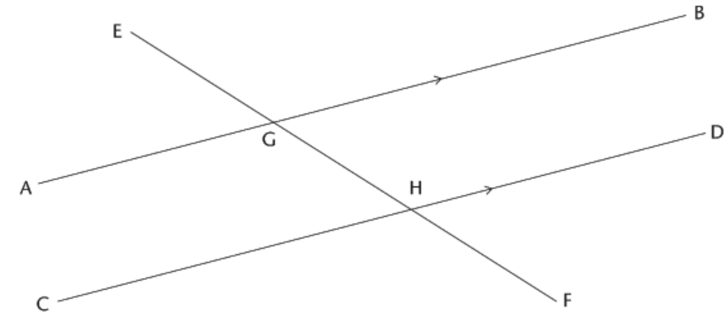
1. (a) Yes
   (b) They are equal.
2. Yes. These angles are corresponding angles and equal because AB||CD.

Teaching guidelines

Learners should remember that alternate angles are only equal if the transversal that forms them cuts parallel lines.

Discuss with learners that this also provides a way to show that two lines are parallel. If we can prove that alternate angles are equal, we know that the lines cut by the transversal are parallel.

Parallel lines have the same direction, i.e. they form **equal corresponding angles** with any line that intersects them.



The line EF cuts AB at G and CD at H.

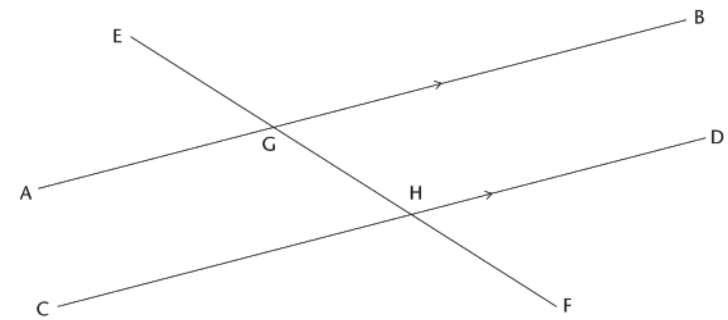
EF is a transversal that cuts parallel lines AB and CD.

1. (a) Look carefully at the angles EGA and EHC in the above figure. They are called corresponding angles. Do they appear to be equal?
   (b) Measure the two angles to check if they are equal. What do you notice?
2. Suppose $E\hat{G}A$ and $E\hat{H}C$ are really equal. Would $E\hat{G}B$ and $E\hat{H}D$ then also be equal? Give reasons to support your answer.

When two parallel lines are cut by a transversal, the corresponding angles are equal.

Alternate angles

The angles $B\hat{G}F$ and $C\hat{H}E$ below are called alternate angles. They are on opposite sides of the transversal.



Answers

3. Yes, they are on different sides of transversal EF and between lines AB and CD.
4. They are equal if AB||CD.
5. ∠GHD = corresponding ∠EGB but ∠EGB = vertically opposite ∠AGH, so ∠AGH= alternate ∠GHD. In general, an angle is vertically opposite to the alternate angle of its equal corresponding angle.
6. Yes. They are in the same positions if the lines are crossed by a transversal and they are equal if the lines are parallel.
7. (a) They are supplementary (their sum is 180°) angles, because AB is a straight line.
   (b) They are supplementary (their sum is 180°) angles, because CD is a straight line.
   (c) Yes, because the sums on either side of the equal sign are both equal to 180°.
   (d) Yes, because $C\hat{H}G$ is vertically opposite and equal to $D\hat{H}F$, which is in turn corresponding to $B\hat{G}H$, because AB and CD are parallel (see question 6).
8. Corresponding angles are equal if parallel lines are intersected by a transversal. Alternate angles will also be equal, because their vertically opposite angles are equal.

Teaching guidelines

Learners should remember that co-interior angles only add up to 180° if the transversal that forms them cuts parallel lines.

Discuss with learners that this also provides a way to show that two lines are parallel. If we can prove that co-interior angles are supplementary, we know the lines cut by the transversal are parallel.

---

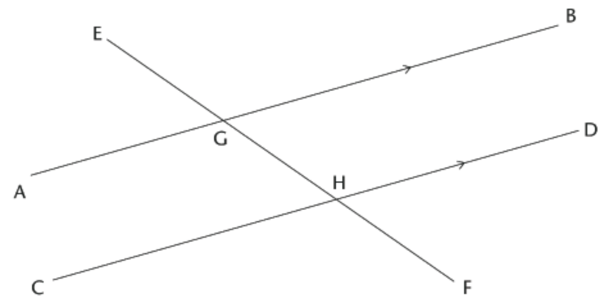
3. Do you think angles AGF and DHE should also be called alternate angles?
4. Do you think alternate angles are equal? Investigate by using the tracing paper like you did previously, or measure the angles accurately with your protractor. What do you notice?

When parallel lines are cut by a transversal, the alternate angles are equal.

5. Try to explain why alternate angles are equal when the lines that are cut by a transversal are parallel, keeping in mind that corresponding angles are equal.

By answering the following questions, you should be able to see how you can explain why alternate angles are equal when parallel lines are cut by a transversal.

6. Are angles $B\hat{G}H$ and $D\hat{H}F$ in the figure corresponding angles? What do you know about corresponding angles?



7. (a) What can you say about $B\hat{G}H + A\hat{G}H$? Give a reason.
   (b) What can you say about $D\hat{H}G + C\hat{H}G$? Give a reason.
   (c) Is it true that $B\hat{G}H + A\hat{G}H = D\hat{H}G + C\hat{H}G$? Explain.
   (d) Will the equation in (c) still be true if you replace angle $B\hat{G}H$ on the left-hand side with angle $C\hat{H}G$?
8. Look carefully at your work in question 7 and write an explanation why alternate angles are equal, when two parallel lines are cut by a transversal.

Co-interior angles

The angles $A\hat{G}H$ and $C\hat{H}G$ in the figure on the following page are called co-interior angles. They are on the same side of the transversal.

The prefix "co-" means together. The word "co-interior" means on the same side.

Answers

9. (a) They are supplementary (their sum = 180°) because CD is a straight line.
   (b) They are supplementary (their sum = 180°) because AB is a straight line.
   (c) They are alternate angles and they are equal, because AB||CD.
   (d) They are also supplementary (their sum = 180°). This is because we have already shown that:

   $C\hat{H}G$ is equal to $B\hat{G}H$ (alt. ∠s AB||CD)

   $B\hat{G}H$ is supplementary to $A\hat{G}H$ (∠s on a str. line).

12.2 Identify and name angles

Teaching guidelines

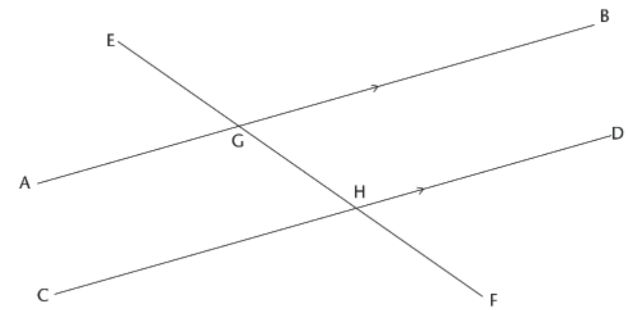
Learners should be able to identify and name pairs of corresponding angles and alternate angles, and determine whether they are equal. They should also be able to identify and name co-interior angles, and determine whether or not they are supplementary. The questions in this section help learners to realise that they need to make sure that lines are parallel to establish which angles are equal or supplementary.

They can only be sure that lines are parallel if they are marked as parallel or if they are told that one of the conditions apply, for example if they are shown that corresponding angles are equal.

Answers

1. (a) Not necessarily, as we don't know whether AB||CD.
   (b) There are many possibilities: $R\hat{P}B$ and $B\hat{P}Q$ (on str. line RF); angles $B\hat{P}Q$ and $G\hat{P}Q$ (on str. line AB); $A\hat{G}E$ and $E\hat{G}P$ (on str. line AB); angles $A\hat{G}E$ and $A\hat{G}H$ (on str. line EF).

   Co-interior angle pairs $A\hat{G}H$ and $C\hat{H}G$, $P\hat{G}H$ and $G\hat{H}Q$, $G\hat{P}Q$ and $P\hat{Q}H$, $B\hat{P}Q$ and $P\hat{Q}D$ will be supplementary only if AB||CD. Remember that supplementary angles are angles that add up to 180°, they do not have to be adjacent angles.



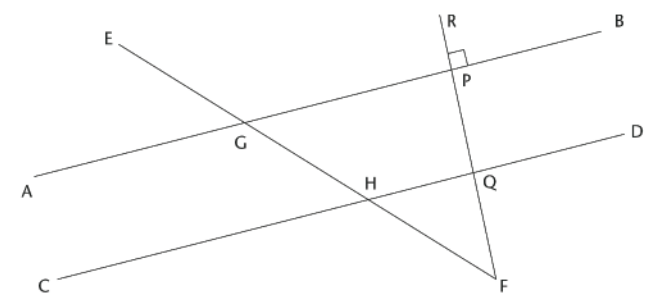
9. (a) What do you know about $C\hat{H}G$ + $D\hat{H}G$? Explain.
   (b) What do you know about $B\hat{G}H$ + $A\hat{G}H$? Explain.
   (c) What do you know about $B\hat{G}H$ + $C\hat{H}G$? Explain.
   (d) What conclusion can you draw about $A\hat{G}H$ + $C\hat{H}G$? Give detailed reasons for your conclusion.

When two parallel lines are cut by a transversal, the sum of two co-interior angles is 180°.

Another way of saying this is to say that the two co-interior angles are **supplementary**.

12.2 Identify and name angles

1. In the figure below, the line RF is perpendicular to AB.



   (a) Is RF also perpendicular to CD? Justify your answer.
   (b) Name four pairs of supplementary angles in the figure. In each case, say how you know that the angles are supplementary.

Answers

1. (c) Co-interior angles $A\hat{G}H$ and $C\hat{H}G$, $P\hat{G}H$ and $G\hat{H}Q$, $G\hat{P}Q$ and $P\hat{Q}H$, $B\hat{P}Q$ and $P\hat{Q}D$
   (d) $E\hat{G}P$ and $G\hat{H}Q$; PGH and $Q\hat{H}F$; $B\hat{P}Q$ and $D\hat{Q}F$; $R\hat{P}B$ and $P\hat{Q}D$ (there are four other possibilities as well)
   (e) $A\hat{G}H$ and $Q\hat{H}G$; $P\hat{G}H$ and $C\hat{H}G$; $G\hat{P}Q$ and $P\hat{Q}D$; $B\hat{P}Q$ and $H\hat{Q}P$
   (Note that learners can also list the alternate exterior angles in the figure.)
2. (a) Yes, AB||CD, therefore RF will be perpendicular to CD because $H\hat{Q}P$ and $G\hat{P}R$ are corresponding angles and thus $H\hat{Q}P$ will also be 90°.
   (b) Because it is stated that AB||CD, the co-interior angles will also be supplementary: $P\hat{G}H$ and $G\hat{H}Q$; $G\hat{P}Q$ and $H\hat{Q}P$; $B\hat{P}Q$ and $D\hat{Q}P$; $A\hat{G}H$ and $C\hat{H}G$
   Other angles that are supplementary are adjacent angles on a straight line:
   $C\hat{H}F$ and $F\hat{H}Q$ on line CD; $H\hat{Q}P$ and $H\hat{Q}F$ on line RF; etc.
   (c) $G\hat{H}C = E\hat{G}A = x$ (corr. ∠s; AB||CD); $P\hat{G}H = E\hat{G}A = x$ (vert. opp. ∠s);
   $Q\hat{H}F = G\hat{H}C = x$ (vert. opp. ∠s);
   $E\hat{G}P = 180° - E\hat{G}A$ (suppl. adj. ∠s or ∠s on str. line) $= 180° - x$;
   $A\hat{G}H = E\hat{G}P = 180° - x$ (vert. opp. ∠s); $G\hat{H}Q = E\hat{G}P = 180° - x$ (corr. ∠s; AB||CD) and $C\hat{H}F = G\hat{H}Q = 180° - x$ (vert. opp. ∠s); $Q\hat{F}H = 90° - x$ ($H\hat{Q}F = 90°$; sum of ∠s of △).
   (Other valid reasons may be given in various places, such as alternate angles that are equal etc.)

12.3 Solving problems

Teaching guidelines

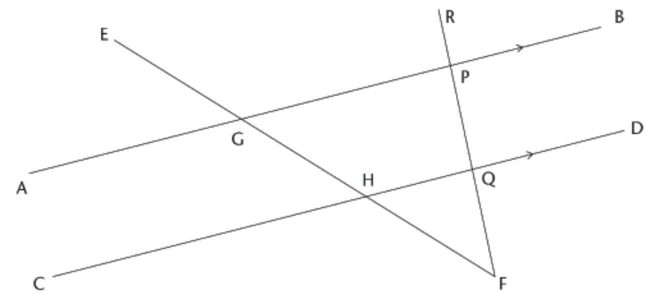
Impress on learners that if two angles are equal because they are corresponding, this will always be the case because the lines that are cut by the transversal, are parallel and that fact should be part of the answer. So they should never write:
∠A = ∠D (corr. ∠s), but ∠A = ∠D (corr. ∠s, AB||DE).

Answers

1. (a) $C\hat{H}F = G\hat{H}D$ (vert. opp. ∠s); $D\hat{L}K = G\hat{H}D$ (corr. ∠s, EF||IJ);
   $A\hat{G}H = G\hat{H}D$ (alt. ∠s, AB||CD); $E\hat{G}K = G\hat{H}D$ (corr. ∠s, AB||CD);
   $H\hat{L}J = G\hat{H}D$ (alt. ∠s, EF||IJ)
   (b) $A\hat{K}J = A\hat{G}H$ (corr. ∠s, EF||IJ); $E\hat{G}K = A\hat{G}H$ (vert. opp. ∠s);
   $I\hat{K}M = E\hat{G}K$ (corr. ∠s, EF||IJ) and $E\hat{G}K = A\hat{G}H$ (proven above);
   $C\hat{H}F = A\hat{G}H$ (corr. ∠s, AB||CD); $G\hat{H}L = A\hat{G}H$ (alt. ∠s, AB||CD)

(c) Name four pairs of co-interior angles in the figure.
(d) Name four pairs of corresponding angles in the figure.
(e) Name four pairs of alternate angles in the figure.

2. Now you are given that AB and CD in the figure below are parallel.

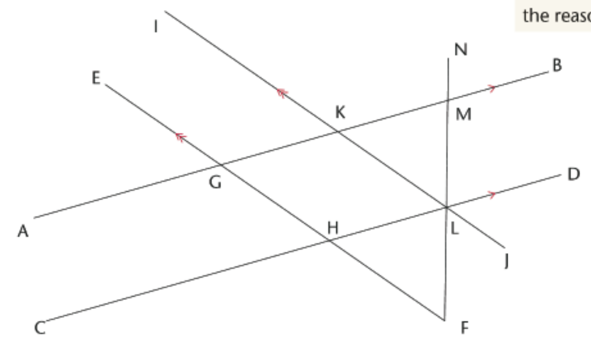


(a) If it is also given that RF is perpendicular to AB, will RF also be perpendicular to CD? Justify your answer.
(b) Name all pairs of supplementary angles in the figure. In each case, say how you know that the angles are supplementary.
(c) Suppose $E\hat{G}A = x$. Give the size of as many angles in the figure as you can, in terms of x. Each time give a reason for your answer.

12.3 Solving problems

When you solve problems in geometry you can use a short-hand way to write your reasons. For example, if two angles are equal because they are corresponding angles, then you can write (corr ∠s, AB || CD) as the reason.

1. Line segments AB and CD in the figure below are parallel. EF and IJ are also parallel. Copy the figure and mark these facts on the figure, and then answer the questions.



134 MATHEMATICS GRADE 9: TERM 2

2. Learners should be able to find all 23 of the angles from the information given:

$K\hat{M}L = 80°$ (vert. opp. ∠)

$N\hat{M}K = 100°$ and $B\hat{M}L = 100°$ (∠s on str. lines AB and NF)

$M\hat{L}D = 80°$ (AB||CD, corr. ∠s equal)

$M\hat{L}H = 100°$ (∠s on str. line CD) $D\hat{L}K = 60°$ (vert.opp.to $H\hat{L}K$)

$H\hat{L}F = 80°$ (vert. opp. to $M\hat{L}D$) $D\hat{L}F = 100°$ (vert. opp. to $M\hat{L}H$)

$M\hat{L}K = 40°$ (vert. opp. to $J\hat{L}F$) $H\hat{L}K = 100° - 40° = 60°$

$I\hat{K}G = 60°$ (corr. ∠ to $H\hat{L}K$) $L\hat{K}M = 60°$ (vert. opp. to $I\hat{K}G$)

$G\hat{K}L = 120°$ (∠s on a str. line IJ) $I\hat{K}B = 120°$ (vert. opp. $G\hat{K}L$)

$E\hat{G}K = 120°$ (corr. ∠ to IKB) $A\hat{G}H = 120°$ (vert. opp. EGK)

$E\hat{G}A = 60°$ (∠s on str. line AB) $K\hat{G}H = 60°$ (∠s on str. line AB)

$G\hat{H}L = 120°$ (corr. ∠ with $E\hat{G}K$) $G\hat{H}C = 60°$ (∠s on str. line CD)

$C\hat{H}F = 120°$ (vert. opp. $G\hat{H}L$) $F\hat{H}L = 60° = G\hat{H}C$ (vert. opp. ∠s)

∴ $H\hat{F}L = 180° - (H\hat{L}F + F\hat{H}L)$ (sum of ∠s of triangle)

∴ $H\hat{F}L = 180° - (80° + 60°) = 40°$

3. (a) Challenge learners to find the sizes of all the angles in the figure. In most instances, there are also other valid reasons.

$\hat{N}_1 = 50°$ (vert. opp. $R\hat{N}D$); $\hat{N}_2 = 130°$ (∠s on str. line); $\hat{N}_4 = 130°$ (∠s on str. line); $\hat{P}_1 = 50°$ (corr. to $\hat{N}_1$, AB||CD); $S\hat{P}B = 130°$ (∠s on str. line); $\hat{P}_4 = 50°$ (vert. opp. $\hat{P}_1$); $O\hat{P}N = 130°$ (vert. opp. $S\hat{P}B$); $\hat{S}_1 = 50°$ (corr. to $\hat{P}_1$, EF||AB); $\hat{S}_2 = 130°$ (∠s on str. line); $\hat{S}_3 = 50°$ (vert. opp. $\hat{S}_1$); $\hat{S}_4 = 130°$ (∠s on str. line); $\hat{H}_2 = 130°$ (co-int. ∠ with $\hat{S}_1$, GH//JR); $\hat{H}_1 = 50°$ (∠s on str. line); $\hat{H}_4 = 130°$ (vert. opp. $G\hat{H}S$); $\hat{H}_3 = 50°$ (vert. opp. $\hat{H}_1$); $O_2 = 130°$ (corr. to $\hat{H}_2$, EF||AB); $\hat{O}_1 = 50°$ (∠s on str. line); $\hat{O}_4 = 130°$ (vert. opp. $\hat{O}_2$); $\hat{O}_3 = 50°$ (∠s on str. line); $\hat{M}_1 = 50°$ (co-int. with $\hat{O}_4$, AB||CD); $\hat{M}_4 = 50°$ (vert. opp. $\hat{M}_1$); $N\hat{M}O = 130°$ (∠s on str. line); $C\hat{M}H = 130°$ (vert. opp. $N\hat{M}O$); $\hat{Q}_2 = \hat{P}_3 = P\hat{M}N = 60°$ (corr. ∠s, EF||AB and AB||CD); $\hat{Q}_1 = 120°$ (∠s on str. line), $Q\hat{P}O = P\hat{M}C = \hat{Q}_1 = 120°$ (corr. ∠s, EF||AB and AB||CD); $\hat{Q}_4 = 60°$ (vert. opp. $\hat{Q}_2$); $\hat{Q}_3 = 120°$ (vert. opp. $\hat{Q}_1$); $\hat{P}_6 = 60°$ (corr. to $\hat{Q}_4$, SQ || PO); $B\hat{P}M = 120°$ (∠s on str. line); $\hat{P}_2 = Q\hat{P}O - \hat{P}_1 = 120° - 50° = 70°$; $\hat{M}_2 = \hat{P}_2 = 70°$ (corr. ∠s, JR||GH); $Q\hat{P}N = 110°$ (∠s on str. line); $\hat{M}_5 = \hat{M}_2 = 70°$ (vert. opp. ∠s); $\hat{M}_6 = \hat{M}_3 = 60°$ (vert. opp. ∠s); $\hat{P}_6 = 60°$ (vert. opp. $\hat{P}_3$); $\hat{P}_5 = 70°$ (vert. opp. P_2)

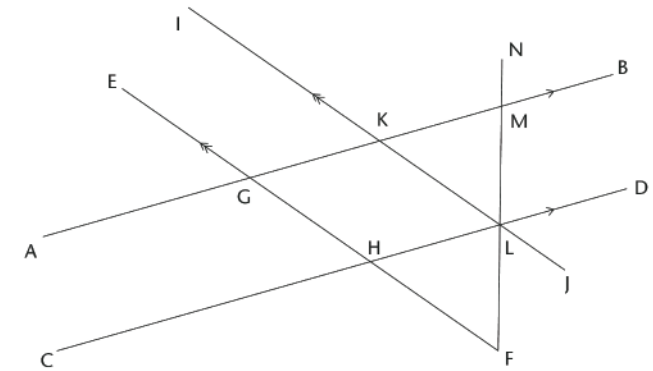
(b) Yes, corresponding angles $J\hat{S}H$ and $M\hat{N}P$ are equal.

Also, if AB||CD and AB||EF, then it follows that EF||CD.

(a) Name five angles in the figure that are equal to $G\hat{H}D$. Give a reason for each of your answers.

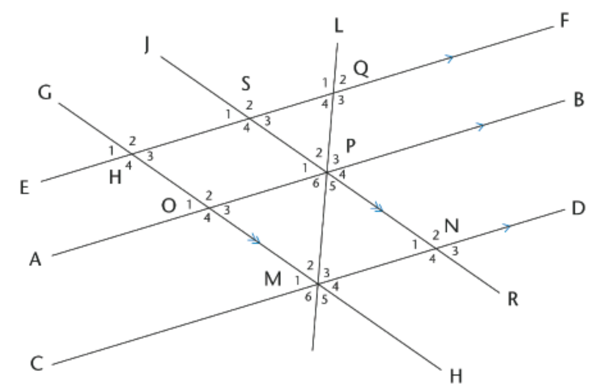
(b) Name all the angles in the figure that are equal to $A\hat{G}H$. Give a reason for each of your answers.

2. AB and CD in the figure below are parallel. EF and IJ are also parallel. $N\hat{M}B = 80°$ and $J\hat{L}F = 40°$.



Find the sizes of as many angles in the figure as you can, giving reasons.

3. In the figure below, AB || CD; EF || AB; JR || GH. You are also given that $P\hat{M}N = 60°$, $R\hat{N}D = 50°$.



(a) Find the sizes of as many angles in the figure as you can, giving reasons.

(b) Are EF and CD parallel? Give reasons for your answers.

Learner Book Overview		
Sections in this chapter	**Content**	**Pages in Learner Book**
13.1 Investigating the sides of a right-angled triangle	Definition of a theorem; the hypotenuse of a right-angled triangle; Theorem of Pythagoras	Pages 136 to 137
13.2 Checking for right-angled triangles	Definition of the converse of a theorem; the converse of the Theorem of Pythagoras	Pages 137 to 138
13.3 Finding missing sides	Surds; finding the length of the hypotenuse; finding the length of a right-angle side; Pythagorean triplets	Pages 139 to 142
13.4 More practice using Pythagoras' Theorem	Multi-step solutions; classification of triangles using the converse of the Theorem of Pythagoras	Pages 142 to 143

CAPS time allocation	5 hours
CAPS content specification	Page 138

Mathematical background

- **Pythagoras' Theorem** states that, in a right-angled triangle, a square formed on the hypotenuse will have the same area as the sum of the areas of the two squares formed on the other two sides of the triangle.
  - Pythagoras' Theorem **provides information about physical situations**.

    Example: If the lengths of two sides of a right-angled triangle are known, the length of the other side can be calculated by using the theorem.
  - Pythagoras' Theorem **provides a meaningful context for finding square roots**.

    Example: The length of the hypotenuse of a right-angled triangle with right-angle sides 3 m and 4 m is the square root of 25 m^2, which is 5 m.
  - Pythagoras' Theorem **provides a context for the idea of irrational numbers**, i.e. numbers that cannot be expressed either in the common fraction notation or as finite or recurring decimals.

    Example: The length of the hypotenuse of a right-angled triangle with right-angle sides of 1 m each, is $\sqrt{1^2} + \sqrt{1^2} = \sqrt{2}$ m.
- The **electronic calculator** is an enormously useful tool. However, it has certain disadvantages from a teaching viewpoint and these should be properly managed in the classroom.

  Example: A calculator will produce a finite decimal as an answer for $\sqrt{2}$, like 1,414213562. For the square of this number, the calculator will produce 2, creating the impression that 1,414213562 squared is exactly 2, i.e. that $\sqrt{2}$ can be expressed as a finite decimal. If learners are left with this impression, their understanding of irrational numbers will be seriously undermined. One way to help them to understand that the calculator answers for $\sqrt{2}$ and 1,414213562 are inexact is to ask them what the last digit of the answer will be if 1,414213562 × 1,414213562 is calculated by hand. Clearly, the answer will be of the form 1, ..., 4, with 4 as the last digit, and hence cannot be equal to 2,00000000000000000000... which is 2. Spending some time in class on this will reinforce learners' understanding of irrational numbers.

13.1 Investigating the sides of a right-angled triangle

INVESTIGATING SQUARES ON THE SIDES OF RIGHT-ANGLED TRIANGLES

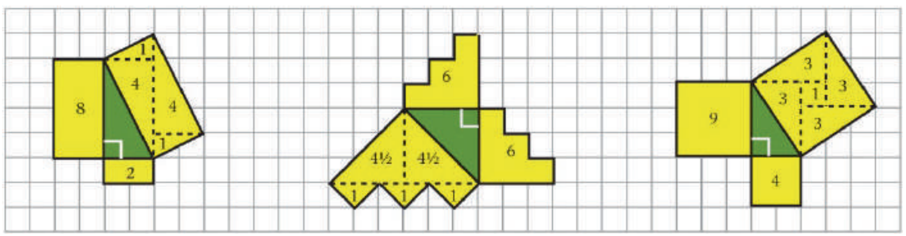
Background information

A **theorem** is a rule or statement that has been proved through reasoning. Pythagoras' Theorem is a rule that applies to only **right-angled triangles**.

- A right-angled triangle has **one interior angle of 90°**.
- The **longest side** of a right-angled triangle always lies opposite the right angle and is called the **hypotenuse**.

Similar figures have the same shape but may differ in size.

- Similar figures can be formed with **corresponding sides** on the sides of a right-angled triangle.
- If similar figures are drawn on the sides of a right-angled triangle, the area of the figure drawn on the hypotenuse is equal to the sum of the areas of the two figures drawn on the other sides.



- The easiest similar figure to form on the sides of a right-angled triangle is a **square**.

Pythagoras' Theorem states that, in a right-angled triangle, a square formed on the hypotenuse will have the same area as the sum of the areas of the two squares formed on the other two sides of the triangle.

Teaching guidelines

At the end of question 3 on LB page 137, learners should be able to interpret and formulate Pythagoras' Theorem.

Answers

1. (a) See the answers on LB page 136 alongside.
   (b) 9 + 16 = 25 square units
   (c) Area of square B + Area of square C = Area of square A

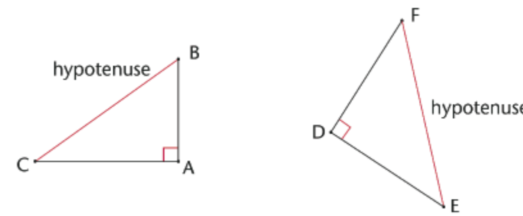
CHAPTER 13
Pythagoras' Theorem

13.1 Investigating the sides of a right-angled triangle

A **theorem** is a rule or a statement that has been proved through reasoning. **Pythagoras' Theorem** is a rule that applies only to **right-angled triangles**. The theorem is named after the Greek mathematician, Pythagoras.

A right-angled triangle has one 90° angle. The longest side of the right-angled triangle is called the **hypotenuse**.

Pythagoras (569–475 BC)
Pythagoras was an influential mathematician. Like many Greek mathematicians of 2 500 years ago, he was also a philosopher and a scientist. He formulated the best-known theorem, today known as Pythagoras' Theorem.
However, the theorem had already been in use 1 000 years earlier, by the Chinese and the Babylonians.

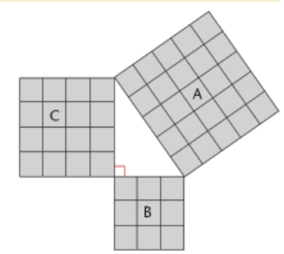


The hypotenuse is the side opposite the 90° angle in a right-angled triangle. It is always the longest side.

How to say it:
"high - *pot* - eh - news"

INVESTIGATING SQUARES ON THE SIDES OF RIGHT-ANGLED TRIANGLES

1. The figure shows a right-angled triangle with squares on each of the sides.
   (a) Write down the areas of the following:
   Square A: 5 × 5 = 25 square units
   Square B: 3 × 3 = 9 square units
   Square C: 4 × 4 = 16 square units
   (b) Add the area of square B and the area of square C.
   (c) What do you notice about the areas?



Answers

2. (a) See the answer on LB page 137 alongside.
   (b) See the answers on LB page 137 alongside.
   (c) The sum of the areas of the two squares formed on the sides of the right-angled triangle is equal to the area of the square formed on the hypotenuse.
   Yes, it is similar to the answer in 1(c).
3. (a) $15 \text{ cm} \times 15 \text{ cm} = 225 \text{ cm}^2$
   $8 \text{ cm} \times 8 \text{ cm} = 64 \text{ cm}^2$
   Area of square along hypotenuse $= 225 \text{ cm}^2 + 64 \text{ cm}^2 = 289 \text{ cm}^2$
   (b) Length of hypotenuse $= \sqrt{289} \text{ cm} = 17 \text{ cm}$

13.2 Checking for right-angled triangles

Background information

- A **theorem** is a statement which has been proved to be true.
- A **converse** is a statement that swaps around what is given in a theorem and what must be determined.
- **Pythagoras' Theorem** states the following: If any triangle is a right-angled triangle, then the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the other two sides.

  Example: If a triangle is right-angled, the sides will have the following relationship: $(\text{Hypotenuse})^2 = (\text{Side } 1)^2 + (\text{Side } 2)^2$
- The **converse of Pythagoras' Theorem** states the following: If the sum of the squares of the lengths of two sides of a triangle is equal to the square of the length of the longest side, then the triangle is a right-angled triangle.

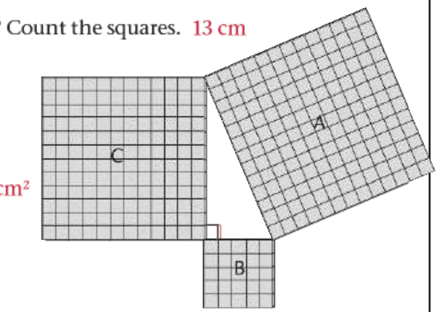
  Example: If the sides have the relationship: $(\text{Longest side})^2 = (\text{Side } 1)^2 + (\text{Side } 2)^2$, then the triangle is right-angled.

Teaching guidelines

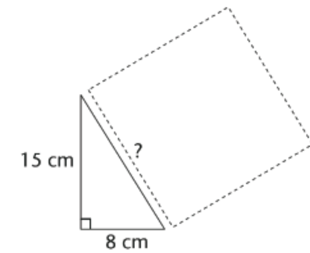
Discuss the difference between a theorem and its converse.

---

2. The figure below is similar to the one in question 1. The lengths of the sides of the right-angled triangle are 5 cm and 12 cm.
   (a) What is the length of the hypotenuse? Count the squares. 13 cm
   (b) Use the squares to find the following:
   Area of A: $13 \times 13 = 169 \text{ cm}^2$
   Area of B: $5 \times 5 = 25 \text{ cm}^2$
   Area of C: $12 \times 12 = 144 \text{ cm}^2$
   Area of B + Area of C: $25 + 144 = 169 \text{ cm}^2$
   (c) What do you notice about the areas?
   Is it similar to your answer in 1(c)?

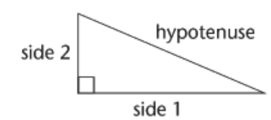


3. A right-angled triangle has side lengths of 8 cm and 15 cm. Use your findings in the previous questions to answer the following questions:
   (a) What is the area of the square drawn along the hypotenuse?
   (b) What is the length of the triangle's hypotenuse?



In the previous activity, you should have discovered Pythagoras' Theorem for right-angled triangles.

> Pythagoras' Theorem says:
> In a right-angled triangle, a square formed on the hypotenuse will have the same area as the sum of the area of the two squares formed on the other sides of the triangle. Therefore:
> $(\text{Hypotenuse})^2 = (\text{Side } 1)^2 + (\text{Side } 2)^2$



13.2 Checking for right-angled triangles

Pythagoras' Theorem applies in two ways:
- If a triangle is right-angled, the sides will have the following relationship: $(\text{Hypotenuse})^2 = (\text{Side } 1)^2 + (\text{Side } 2)^2$.
- If the sides have the relationship: $(\text{Longest side})^2 = (\text{Side } 1)^2 + (\text{Side } 2)^2$, then the triangle is a right-angled triangle.

So, we can test if any triangle is right-angled without using a protractor.

ARE THESE RIGHT-ANGLED TRIANGLES?

Teaching guidelines

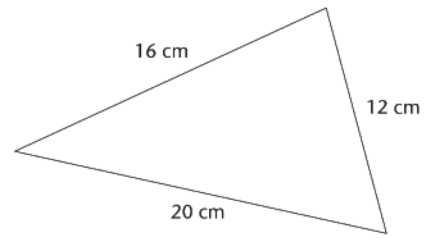
- Use the example on LB page 138 to illustrate how the converse of Pythagoras' Theorem is used to determine whether a triangle is right-angled or not.
- At the end of question 3 on LB page 138, learners should be able to:
  - o determine whether a triangle with three given sides is right-angled or not
  - o state the converse of Pythagoras' Theorem.

Answers

1. (a) $(\text{Longest side})^2 = 29^2 = 841\text{ mm}^2$
   $(\text{Side }1)^2 + (\text{Side }2)^2 = 20^2 + 21^2$
   $400 + 441 = 841\text{ mm}^2$ ∴ The triangle is right-angled.
   (b) See the marked angle on LB page 138 alongside.
2. (a) $4^2 + 4^2 = 16 + 16 = 32$; $6^2 = 36$; not right-angled
   (b) $4^2 + 12^2 = 16 + 144 = 160$; $15^2 = 225$; not right-angled
   (c) $5^2 + 3^2 = 25 + 9 = 34$; $7^2 = 49$; not right-angled
3. (a) $7^2 = 49$; $9^2 = 81$; $12^2 = 144$; $49 + 81 = 130$; not right-angled
   (b) $7^2 = 49$; $12^2 = 144$; $14^2 = 196$; $49 + 144 = 193$; not right-angled
   (c) $16^2 = 256$; $8^2 = 64$; $10^2 = 100$; $100 + 64 = 164$; not right-angled
   (d) $6^2 = 36$; $8^2 = 64$; $10^2 = 100$; $36 + 64 = 100$; right-angled
   (e) $8^2 = 64$; $15^2 = 225$; $17^2 = 289$; $64 + 225 = 289$; right-angled
   (f) $16^2 = 256$; $21^2 = 441$; $25^2 = 625$; $256 + 441 = 697$; not right-angled

Example:

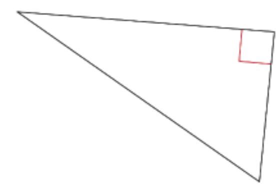
Is a triangle with sides 12 cm, 16 cm and 20 cm right-angled?



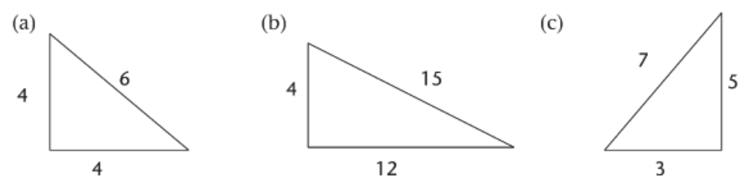
$(\text{Longest side})^2 = 20^2 = 400\text{ cm}^2$
$(\text{Side }1)^2 + (\text{Side }2)^2 = 12^2 + 16^2 = 144 + 256 = 400\text{ cm}^2$
$(\text{Longest side})^2 = (\text{Side }1)^2 + (\text{Side }2)^2$
∴ The triangle is right-angled.

ARE THESE RIGHT-ANGLED TRIANGLES?

1. This triangle's side lengths are 29 mm, 20 mm and 21 mm.
   (a) Prove that it is a right-angled triangle.
   (b) Copy the triangle and mark the right angle in the diagram.



2. Use Pythagoras' Theorem to determine whether these triangles are right-angled. All values are in the same units.



3. Determine whether the following side lengths would form right-angled triangles. All values are in the same units.
   (a) 7, 9 and 12 (b) 7, 12 and 14 (c) 16, 8 and 10
   (d) 6, 8 and 10 (e) 8, 15 and 17 (f) 16, 21 and 25

138 MATHEMATICS GRADE 9: TERM 2

13.3 Finding missing sides

FINDING THE MISSING HYPOTENUSE

Background information

- Pythagoras' Theorem enables us to find the **length of the hypotenuse** of a right-angled triangle if the lengths of the other two sides are given.
- A **surd** is the square root of a whole number, or a simple fraction which produces an irrational number.

  Examples: $\sqrt{7}$ is a surd but $\sqrt{9}$ is not a surd because $\sqrt{9} = 3$

  $\sqrt{\frac{3}{5}}$ is a surd but $\sqrt{\frac{9}{16}}$ is not a surd because $\sqrt{\frac{9}{16}} = \frac{3}{4}$

Teaching guidelines

Use the examples on LB page 139 to explain:

- how to find the missing hypotenuse of a right-angled triangle if the other two sides are given
- when the length of the hypotenuse is a surd.

Misconceptions

- A calculator will produce a finite decimal as an answer for $\sqrt{2}$, like 1,414213562. For the square of this number, the calculator will produce 2, creating the impression that 1,414213562 squared is exactly 2, i.e. that $\sqrt{2}$ can be expressed as a finite decimal. If learners are left with this impression, their understanding of irrational numbers will be seriously undermined.
- One way to help learners to understand that the calculator answers for $\sqrt{2}$ and 1,414213562 are inexact is to ask them what the last digit of the answer will be if 1,414213562 × 1,414213562 is, calculated by hand. Clearly, the answer will be of the form 1, ..., 4, with 4 as the last digit, and hence cannot be equal to 2,00000000000000000000... which is 2. Spending some time in class on this will provide learners with a better understanding of the idea of irrational numbers.

Mathematical note

Remember, when taking the square root, the length is always positive.

13.3 Finding missing sides

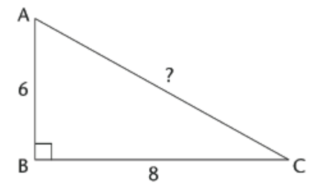
You can use the Pythagoras' Theorem to find the lengths of missing sides if you know that a triangle is right-angled.

FINDING THE MISSING HYPOTENUSE

Example: Calculate the length of the hypotenuse if the lengths of the other two sides are six units and eight units.

ΔABC is right-angled, so:

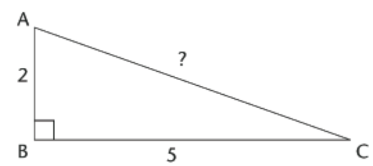
$AC^2 = AB^2 + BC^2$
$= (6^2 + 8^2)$ units^2
$= 36 + 64$ units^2
$= 100$ units^2
$AC = \sqrt{100}$ units
$= 10$ units



Sometimes the square root of a number is not a whole number or a simple fraction. In these cases, you can leave the answer under the square root sign. This form of the number is called a **surd**.

Surd form
You pronounce *surd* so that it rhymes with *word*.
$\sqrt{5}$ is an example of a number in surd form.
$\sqrt{9}$ is not a surd because you can simplify it:
$\sqrt{9} = 3$

Example: Calculate the length of the hypotenuse of ΔABC if $\hat{B} = 90°$, AB = two units and BC = five units. Leave your answer in surd form, where applicable. Remember when taking the square root that length is always positive.



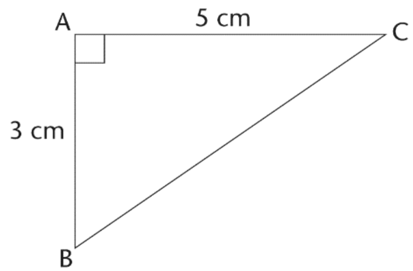
$AC^2 = AB^2 + BC^2$
$= 2^2 + 5^2$ units^2
$= 4 + 25$ units^2
$= 29$ units^2
$AC = \sqrt{29}$ units

1. Find the length of the hypotenuse in each of the triangles shown on the following page. Leave the answers in surd form where applicable.

CHAPTER 13: PYTHAGORAS' THEOREM 139

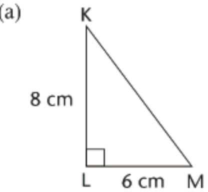
Answers

1. (a) $KM^2 = 8^2 + 6^2 = 64 + 36 = 100\text{ cm}^2$; $KM = \sqrt{100}\text{ cm} = 10\text{ cm}$
   (b) $NO^2 = 55^2 + 48^2 = 3\,025 + 2\,304 = 5\,329\text{ m}^2$; $NO = \sqrt{5\,329}\text{ m} = 73\text{ m}$
   (c) $SR^2 = 1^2 + 1^2 = 1 + 1 = 2\text{ m}^2$; $SR = \sqrt{2}\text{ m}$
   (d) $XZ^2 = 7^2 + 12^2 = 49 + 144 = 193\text{ units}^2$; $XZ = \sqrt{193}\text{ units}$
2. See the answer on LB page 140 alongside.
3. $BC^2 = AB^2 + AC^2 = 3^2 + 5^2 = 9 + 25 = 34\text{ cm}^2$; $BC = \sqrt{34}\text{ cm}$

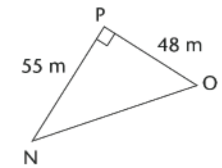


4. (a) $HC^2 = HG^2 + CG^2 = 16^2 + 8^2 = 256 + 64 = 320\text{ cm}^2$; $HC = \sqrt{320}\text{ cm}$
   $AH^2 = DH^2 + AD^2 = 8^2 + 10^2 = 64 + 100 = 164\text{ cm}^2$; $AH = \sqrt{164}\text{ cm}$
   $AC^2 = AB^2 + BC^2 = 16^2 + 10^2 = 256 + 100 = 356\text{ cm}^2$; $AC = \sqrt{356}\text{ cm}$
   (b) $320\text{ cm}^2 + 164\text{ cm}^2 = 484\text{ cm}^2$
   No, ΔACH is not a right-angled triangle.

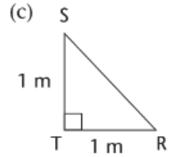
(a)



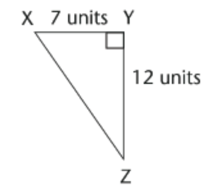
(b)



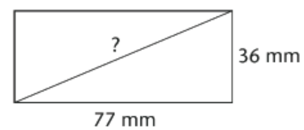
(c)



(d)



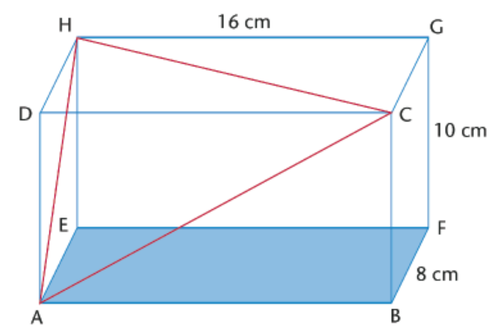
2. A rectangle has sides with lengths of 36 mm and 77 mm. Find the length of the rectangle's diagonal.



$77^2 + 36^2 = 5\,929 + 1\,296$
$= 7\,225\text{ mm}^2$
Diagonal $= \sqrt{7\,225}\text{ mm} = 85\text{ mm}$

3. ΔABC has $\hat{A} = 90°$, AB = 3 cm and AC = 5 cm. Make a rough sketch of the triangle, and then calculate the length of BC.

4. A rectangular prism is made of glass. It has a length of 16 cm, a height of 10 cm and a breadth of 8 cm. ABCD and EFGH are two of its faces. ΔACH has been drawn inside the prism. Is ΔACH right-angled? Answer the questions to find out.



140 MATHEMATICS GRADE 9: TERM 2

FINDING ANY MISSING SIDE IN A RIGHT-ANGLED TRIANGLE

Background information

Pythagoras' Theorem enables us to find the **length of the missing side** of a right-angled triangle if the lengths of the hypotenuse and another side are given.

Teaching guidelines

Use the examples on LB page 141 to explain:

- how to find a missing side if the hypotenuse and another side of a right-angled triangle are given
- when the length of the missing side is a surd.

Answers

1. (a) $x^2 = 25^2 - 24^2 = 625 - 576 = 49;\ x = \sqrt{49} = 7$

   (b) $y^2 = 20^2 - 15^2 = 400 - 225 = 175;\ y = \sqrt{175}$

   (c) $z^2 = 26^2 - 24^2 = 676 - 576 = 100;\ z = \sqrt{100} = 10$

2. (a) $AC^2 = 18^2 - 12^2 = 324 - 144 = 180\text{ cm}^2$

   $\therefore AC = \sqrt{180}\text{ cm} \approx 13{,}42\text{ cm}$

   (b) $EF^2 = 58^2 - 41^2 = 3\,364 - 1\,681 = 1\,683\text{ cm}^2$

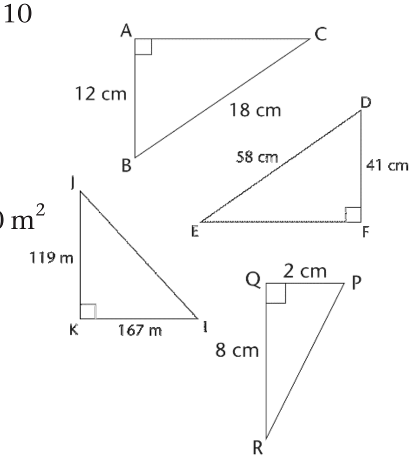
   $\therefore EF = \sqrt{1683}\text{ cm} \approx 41{,}02\text{ cm}$

   (c) $JL^2 = 119^2 + 167^2 = 14\,161 + 27\,889 = 42\,050\text{ m}^2$

   $\therefore JL = \sqrt{42\,050}\text{ m} \approx 205{,}06\text{ m}$

   (d) $PR^2 = 2^2 + 8^2 = 4 + 64 = 68\text{ cm}^2$

   $\therefore PR = \sqrt{68}\text{ cm} \approx 8{,}25\text{ cm}$



3. (a) height against wall = h

   $h^2 = 5^2 - 1^2 = 25 - 1 = 24\text{ m}^2$

   $h = \sqrt{24}\text{ m} \approx 4{,}90\text{ m}$

   The ladder reaches 4,9 m up the wall.

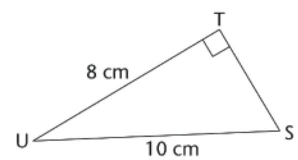
   (b) $5^2 = 4{,}5^2 + d^2;\ 25 = 20{,}25 + d^2;\ 4{,}75\text{ m}^2 = d^2;\ d = \sqrt{(4{,}75)} \approx 2{,}18\text{ m}$

(a) Calculate the length of the sides of ΔACH. Note that all three sides of the triangles are diagonals of rectangles. AC is in rectangle ABCD, AH is in ADHE and HC is in HDCG.

(b) Is ΔACH right-angled? Explain your answer.

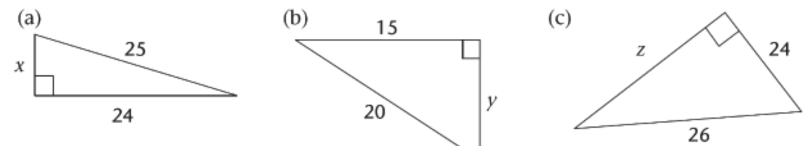
FINDING ANY MISSING SIDE IN A RIGHT-ANGLED TRIANGLE

Example: Find the length of TS in the triangle below.

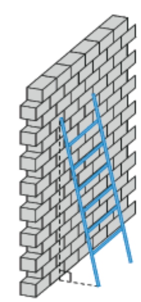


$$\begin{aligned} US^2 &= TU^2 + TS^2 \\ 10^2 &= 8^2 + TS^2 \\ 100 &= 64 + TS^2 \\ 36 &= TS^2 \\ \sqrt{36} &= TS \\ \therefore TS &= 6\text{ cm} \end{aligned}$$

1. In the right-angled triangles below, calculate the length of the sides that have not been given. Leave your answers in surd form where applicable.



2. Calculate the length of the third side of each of the following right-angled triangles. First draw a rough sketch of each of the triangles before you do any calculations. Round off your answers to two decimal places.
   (a) ΔABC has AB = 12 cm, BC = 18 cm and $\hat{A} = 90°$. Calculate AC.
   (b) ΔDEF has $\hat{F} = 90°$, DE = 58 cm and DF = 41 cm. Calculate EF.
   (c) ΔJKL has $\hat{K} = 90°$, JK = 119 m and KL = 167 m. Calculate JL.
   (d) ΔPQR has PQ = 2 cm, QR = 8 cm and $\hat{Q} = 90°$. Calculate PR.



3. (a) A ladder with a length of 5 m is placed at an angle against a wall. The bottom of the ladder is 1 m away from the wall. How far up the wall will the ladder reach? Round off to two decimal places.
   (b) If the ladder reaches a height of 4,5 m against the wall, how far away from the wall was it placed? Round off to two decimal places.

PYTHAGOREAN TRIPLES

Background information

Pythagorean triples are sets of three numbers that that can be used as the sides of a right-angled triangle because they match the requirements of Pythagoras' Theorem.

Examples: 3; 4; 5 because $3^2 + 4^2 = 5^2$ 5; 12; 13 because $5^2 + 12^2 = 13^2$
7; 24; 25 because $7^2 + 24^2 = 25^2$ 8; 15; 17 because $8^2 + 15^2 = 17^2$
9; 40; 41 because $9^2 + 40^2 = 41^2$ 11; 60; 61 because $11^2 + 60^2 = 61^2$
12; 35; 37 because $12^2 + 35^2 = 37^2$ 13; 84; 85 because $13^2 + 84^2 = 85^2$

Teaching guidelines

Learners find as many Pythagorean triples as they can without using multiples of another one. Here are a few more examples:

15; 112; 113	16; 63; 65	17; 144; 145	19; 180; 181	20; 21; 29
21; 220; 221	23; 264; 265	24; 143; 145	25; 312; 313	27; 364; 365

13.4 More practice using Pythagoras' Theorem

Background information

Pythagoras' Theorem can be used to calculate lengths such as the following:

- the length of a **diagonal of a rectangle** if its dimensions are known
- the length of the **sides of a square** if the length of any diagonal is known
- the length of the sides of a rhombus if the length of both diagonals are known.

Teaching guidelines

Learners use Pythagoras' Theorem to solve multi-step problems.

Answers

1. See the answers on LB page 142 alongside.
2. (a) $6 \text{ cm} \times 4{,}5 \text{ cm} = 27 \text{ cm}^2$
   (b) $KM^2 = 6^2 + 4{,}5^2 = 36 + 20{,}25 = 56{,}25$
   $KM = \sqrt{(56{,}25)} = 7{,}5 \text{ cm}$
   Perimeter: $6 + 4{,}5 + 7{,}5 = 18 \text{ cm}$

PYTHAGOREAN TRIPLES

Sets of **whole numbers** that can be used as the sides of a right-angled triangle are known as **Pythagorean triples**, for example:

3-4-5   5-12-13   7-24-25   16-30-34   20-21-29

You extend these triples by finding multiples of them. For examples, triples from the 3-4-5 set include the following:

3-4-5   6-8-10   9-12-15   12-16-20

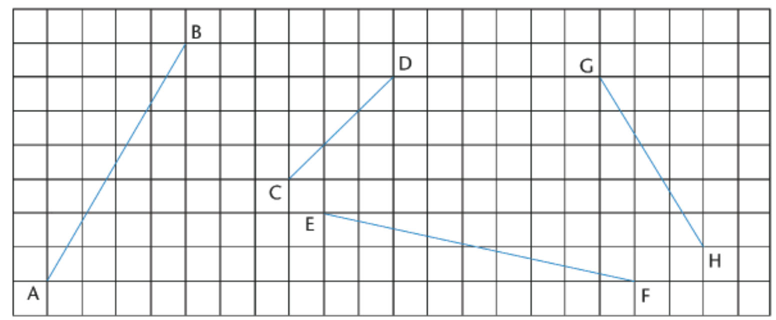
There are many old writings that record Pythagorean triples. For example, from 1900 to 1600 BC, the Babylonians had already calculated very large Pythagorean triples, such as:

1 679-2 400-2 929.

How many Pythagorean triples can you find? What is the largest one you can find that is not a multiple of another one?

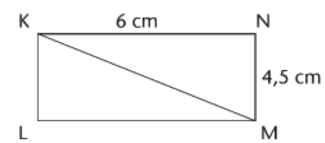
13.4 More practice using Pythagoras' Theorem

1. Four lines have been drawn on the grid below. Each square is one unit long. Calculate the lengths of the lines: AB, CD, EF and GH. Do the calculations and write the answers in surd form.



$AB = \sqrt{65}$ units $CD = \sqrt{18}$ units $EF = \sqrt{85}$ units $GH = \sqrt{34}$ units

2. (a) Calculate the area of rectangle KLMN.
   (b) Calculate the perimeter of ΔKLM.



3. ABCD is a rectangle with AB = 4 cm, BC = 7 cm and CQ = 1,5 cm. Round off your answers to two decimal places if the answers are not whole numbers.

Answers

3. (a) QD = 4 – 1,5 = 2,5 cm

   (b) $PQ^2 = 4{,}2^2 + 1{,}5^2 = 17{,}64 + 2{,}25 = 19{,}89\ cm^2$; $PQ = \sqrt{(19{,}89)} \approx 4{,}46$ cm

   (c) $AQ^2 = 7^2 + 2{,}5^2 = 49 + 6{,}25 = 55{,}25\ cm^2$; $AQ = \sqrt{(55{,}25)}$ cm ≈ 7,43 cm

   $A = \frac{1}{2} b \times h = \frac{(7 \times 2{,}5)}{2} = 8{,}75\ cm^2$

4. (a) $\frac{1}{2} b \times h = \frac{9 \times 12}{2} = 54\ mm^2$

   (b) $MN^2 = 9^2 + 12^2 = 81 + 144 = 225\ mm^2$; $MN = \sqrt{225} = 15$ mm

   Perimeter = 2(15 + 9 + 30) = 2 × 54 = 108 mm

PYTHAGORAS' THEOREM AND OTHER TYPES OF TRIANGLES

Background information

The converse of Pythagoras' Theorem enables us to determine whether a triangle is:

- an **acute-angled** triangle
- a **right-angled** triangle, or
- an **obtuse-angled** triangle.

Teaching guidelines

Learners discuss how to use the converse of Pythagoras' Theorem to classify a triangle if the lengths of all its sides are known.

Misconceptions

Ensure that learners do not develop the common misconception that the Theorem of Pythagoras applies to all triangles. In this regard it may help to engage learners in working on questions such as the following:

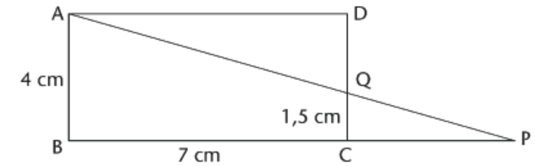
ΔABC has $\hat{B} = 90°$ with AB = 8 cm and AC = 17 cm. D is a point on BC so that BD = 8 cm.

(a) Calculate the length of AD. (Answer: AD = 11,3 cm)

(b) Calculate the length of DC. (Answer: DC = BC – BD = 15 – 8 = 7 cm)

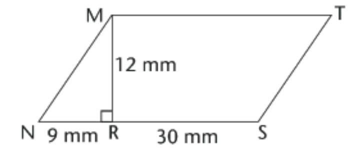
Answers

See the table on LB page 143 alongside.



(a) What is the length of QD?
(b) If CP = 4,2 cm, calculate the length of PQ.
(c) Calculate the length of AQ and the area of ΔAQD.

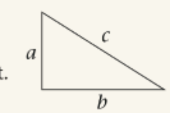
4. MNST is a parallelogram. NR = 9 mm and MR = 12 mm.
   (a) Calculate the area of ΔMNR.
   (b) Calculate the perimeter of MNST.



PYTHAGORAS' THEOREM AND OTHER TYPES OF TRIANGLES

Pythagoras' Theorem works only for right-angled triangles. But we can also use it to find out whether other triangles are acute or obtuse.

- If the square of the longest side is *less* than the sum of the squares of the two shorter sides, the *biggest angle is acute*.
  For example, in a 6-8-9 triangle: $6^2 + 8^2 = 100$ and $9^2 = 81$.
  81 is less than 100 ∴ the 6-8-9 triangle is acute.
- If the square of the longest side is *more* than the sum of the squares of the two shorter sides, the *biggest angle is obtuse*.
  For example, in a 6-8-11 triangle: $6^2 + 8^2 = 100$ and $11^2 = 121$.
  121 is more than 100 ∴ the 6-8-11 triangle is obtuse.



Copy and complete the following table. It is based on the triangle on the right. Decide whether each triangle described is right-angled, acute or obtuse.

a	b	c	$a^2 + b^2$	c^2	Fill in =, > or <	Type of triangle
3	5	6	$3^2 + 5^2 = 9 + 25 = 34$	$6^2 = 36$	$a^2 + b^2 < c^2$	Acute
2	4	6	$2^2 + 4^2 = 4 + 16 = 20$	$6^2 = 36$	$a^2 + b^2$ ...<... c^2	Acute
5	7	9	$5^2 + 7^2 = 25 + 49 = 74$	$9^2 = 81$	$a^2 + b^2$ ...<... c^2	Acute
12	5	13	$12^2 + 5^2 = 144 + 25 = 169$	$13^2 = 169$	$a^2 + b^2$ ...=... c^2	Right-angled
12	16	20	$12^2 + 16^2 = 144 + 256 = 400$	$20^2 = 400$	$a^2 + b^2 = c^2$	Right-angled
7	9	11	$7^2 + 9^2 = 49 + 81 = 130$	$11^2 = 121$	$a^2 + b^2$ ...>... c^2	Obtuse
8	12	13	$8^2 + 12^2 = 64 + 144 = 208$	$13^2 = 169$	$a^2 + b^2$ ...>... c^2	Obtuse

WORKSHEET

Answers

1. Sample answer: In a right-angled triangle, a square formed on the hypotenuse will have the same area as the sum of the area of the two squares formed on the other sides of the triangle.
2. (a) $DF^2 = 4^2 + 6^2 = 16 + 36 = 52\text{ cm}^2$

   $DF = \sqrt{52}\text{ cm} = 4 \times 13\text{ cm} = 2\sqrt{13}\text{ cm}$

   (b) $QR^2 = 13^2 - 12^2 = 169 - 144 = 25\text{ m}^2$

   $QR = \sqrt{25}\text{ cm} = 5\text{ cm}$
3. (a) $AD^2 = 225 - 144 = 81\text{ m}^2$

   $AD = 9\text{ m}$

   $\text{Perimeter} = 2(9 + 15) = 48\text{ m}$

   (b) $A = b \times h$

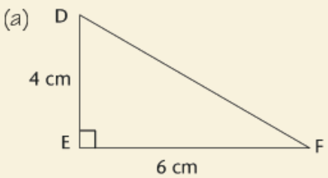
   $\ldots = 9 \times 12$

   $\ldots = 108\text{ m}^2$

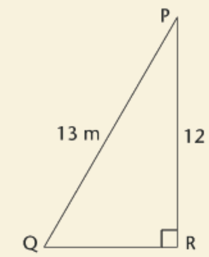
WORKSHEET

1. Write down Pythagoras' Theorem in the way that you best understand it.
2. Calculate the lengths of the missing sides in the following triangles. Leave the answers in surd form if necessary.

(a)



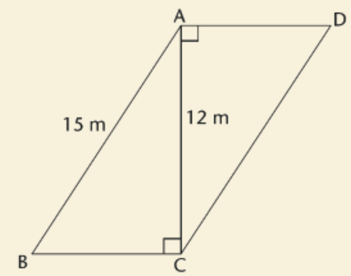
(b)



3. ABCD is a parallelogram.

   (a) Calculate the perimeter of ABCD.

   (b) Calculate the area of ABCD.



PYTHAGORAS' THEOREM

Grade 9 Term 2 Chapter 14 Area and perimeter of 2D shapes

Learner Book Overview		
Sections in this chapter	**Content**	**Pages in Learner Book**
14.1 Area and perimeter of squares and rectangles	Revising calculations of area and perimeter of squares and rectangles	Pages 145 to 146
14.2 Area and perimeter of composite figures	Breaking up shapes to make calculations easier	Pages 146 to 147
14.3 Area and perimeter of circles	Calculating area and perimeter of circles	Pages 148 to 149
14.4 Converting between units	Converting between units of length and area; mm to cm, mm^2 to cm^2, etc.	Page 149
14.5 Area of other quadrilaterals	Deriving formulae for parallelograms, rhombi, kites and trapeziums; calculating areas of composite shapes	Pages 150 to 153
14.6 Doubling dimensions of a 2D shape	Investigating the effect on the perimeter and area if the dimensions of a shape are doubled	Pages 154 to 155

CAPS time allocation	5 hours
CAPS content specification	Pages 139 to 140

Mathematical background

The following formulae are used to calculate the **perimeter of 2D shapes**:	square rectangle the circumference of a circle	$P = 4s$ $P = 2(l + b)$ or $P = 2l + 2b$ $c = 2\pi r$ or πd
The following formulae are used to calculate the **area of 2D shapes**:	square rectangle triangle circle	$A = l^2$ $A = l \times b$ $A = \frac{1}{2}(b \times h)$ $A = \pi r^2$
The areas of more complex polygons can be worked out by breaking them up into known shapes such as squares, rectangles and triangles. The following formulae are used to calculate for the **area of rhombi, kites, parallelograms and trapeziums**:	rhombus kite parallelogram trapezium	$A = l \times h$ $A = \frac{1}{2}(\text{diagonal}_1 \times \text{diagonal}_2)$ $A = \text{base} \times h$ $A = \frac{1}{2}(\text{sum of parallel sides}) \times h$

If the dimensions of a shape are doubled, the perimeter of the new shape is doubled and the area is four times larger than the original shape.

14.1 Area and perimeter of squares and rectangles

REVISING CONCEPTS

Teaching guidelines

Learners should understand the difference between the perimeter and the area of a figure. The questions in this section serve to refresh their memories.

You could write a formula on the board without giving an explanation to remind learners about the formulae they used in Grade 8 to calculate area and perimeter. For example:

$P = 4s$
$P = 2(l + b)$ or $P = 2l + 2b$
$A = l^2$
$A = l \times b$

Answers

1. See the answers on LB page 145 alongside and LB page 146 on the following page.

CHAPTER 14

Area and perimeter of 2D shapes

14.1 Area and perimeter of squares and rectangles

REVISING CONCEPTS

1. Each block in figures A to F below measures 1 cm × 1 cm. What is the perimeter and area of each of the figures?
   Copy and complete the table below.

The **perimeter** (P) of a shape is the distance along the sides of the shape. The **area** (A) of a figure is the size of the flat surface enclosed by the figure.

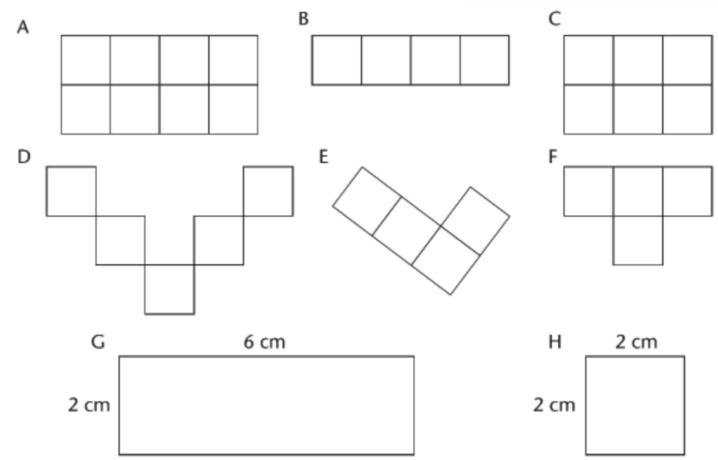


Figure	Perimeter	Area	Number of 1 cm × 1 cm squares
A	12 cm	8 cm²	8
B	10 cm	4 cm²	4
C	10 cm	6 cm²	6
D	20 cm	5 cm²	5

Answers

2. (a) 20. There are four rows of five squares each, so that is 4 × 5 = 20 squares. (Six squares are hidden.)
   (b) 20 cm^2
3. Sipho's area: l × b = 4 m × 10 m = 40 m^2
   Theunis's area: l × b = 5 m × 8 m = 40 m^2
   The areas they paint are the same size, so they should be paid the same.
4. Area = l^2 = l × l = 12 mm × 12 mm = 144 mm^2
5. Area = length × breadth
   72 cm^2 = 8 cm × breadth
   Breadth = 72 cm^2 ÷ 8 cm = 9 cm

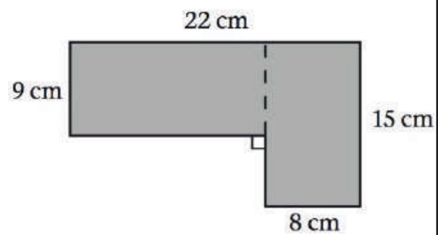
14.2 Area and perimeter of composite figures

BREAKING UP FIGURES AND PUTTING THEM BACK TOGETHER AGAIN

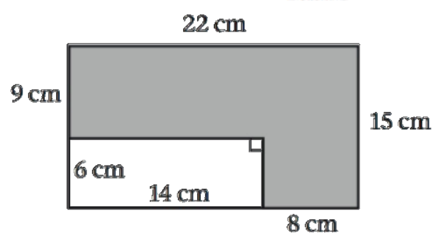
Teaching guidelines

There are two ways for learners to calculate the areas of shapes like these:

1. They can divide the shape into two rectangles, as in question 1 on LB page 146. They calculate the areas of the two rectangles separately and then add the two answers.



2. They can draw a rectangle into which the shape fits perfectly, calculate the area of the whole rectangle and then subtract the additional piece (which is a smaller rectangle). For example, the added piece has measurements of length = 22 – 8 = 14 cm; breadth = 15 – 9 = 6 cm.



   Area of grey shape = 22 × 15 – 14 × 6 = 246 cm^2

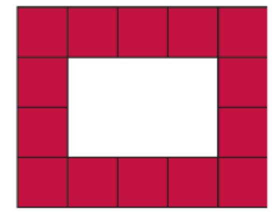
Point out to learners that if they divide the shape according to the first method above, that the division line does not form part of the perimeter of the shape.

The second method explained above, gives a quick way to find the perimeter and shows that for rectangles like these, the formula for calculation of the perimeter is as before.

Figure	Perimeter	Area	Number of 1 cm × 1 cm squares
E	10 cm	4 cm^2	4
F	10 cm	4 cm^2	4
G	16 cm	12 cm^2	12
H	8 cm	4 cm^2	4

2. Consider the rectangle below on the right-hand side. It is formed by tessellating identical squares that are 1 cm by 1 cm each. The white part has squares that are hidden.
   (a) Write down, without counting, the total number of squares that form this rectangle, including those that are hidden. Explain your reasoning.
   (b) What is the area of the rectangle, including the white part?

To **tessellate** means to cover a surface with identical shapes in such a way that there are no gaps or overlaps. Another word for tessellating is **tiling**.



Area of a rectangle = length × breadth
= *l* × *b*
Area of a square = *l* × *l*
= l^2

Both length (*l*) and breadth (*b*) are expressed in the same unit.

3. Sipho and Theunis each paint a wall to earn some money during the school holidays. Sipho paints a wall 4 m high and 10 m long. Theunis's wall is 5 m high and 8 m long. Who should be paid more? Explain.
4. What is the area of a square with a length of 12 mm?
5. The area of a rectangle is 72 cm^2 and its length is 8 cm. What is its breadth?

14.2 Area and perimeter of composite figures

BREAKING UP FIGURES AND PUTTING THEM BACK TOGETHER AGAIN

1. The diagram on the left on the following page shows the floor plan of a room.
   We can calculate the area of the room by dividing the floor into two rectangles, as shown in the diagram on the right on the following page.

Answers

1. (a) From question (a): wooden floor area = 126 m^2; carpeted area = 120 m^2

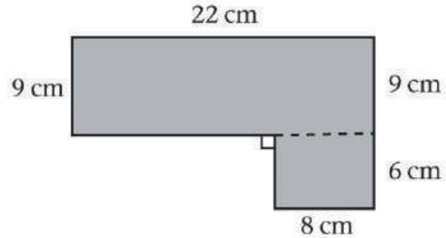
   (b) Area of the room = Area of top rectangle + Area of bottom rectangle

   $= (l \times b) + (l \times b)$

   $= (8 \times 6) + (22 \times 9)$

   $= 48 + 198$

   $= 246\ m^2$



2. Area of figure A: $1 \times 4 + (3 - 1) \times 1$
   $= 4 + 2 = 6\ cm^2$

   or: $1 \times (4 - 1) + 3 \times 1 = 3 + 3$
   $= 6\ cm^2$

   Area of figure B: $0{,}5 \times 5 + 3{,}5 \times 3$
   $= 2{,}5 + 10{,}5 = 13\ cm^2$

3. The first three are equivalent expressions and correct, while the last one is not correct as it calculates only half of the perimeter.

4. $P = 2(1 + 4) + 2(2 + 1) - 2(1)$

   $= 10 + 6 - 2$

   $= 14$ cm

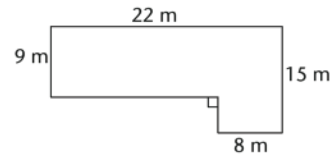
   Note: The two rectangles share a 1 cm side, which does not form part of the perimeter.

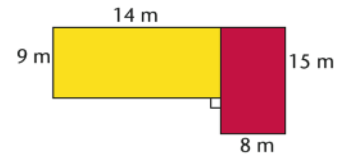
5. $P = 2l + 2b$

   $28 = 2l + 2(6)$

   $2l = 16$

   $l = 8$ cm



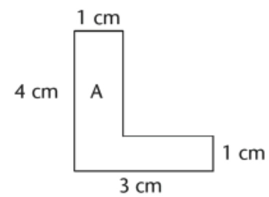


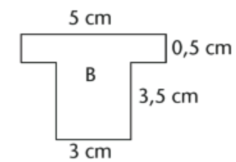
Area of the room = Area of yellow rectangle + Area of red rectangle
$= (l \times b) + (l \times b)$
$= (14 \times 9) + (15 \times 8)$
$= 126 + 120$
$= 246\ m^2$

(a) The yellow part of the room has a wooden floor and the red part is carpeted. What is the area of the wooden floor? What is the area of the carpeted floor?

(b) Calculate the area of the room dividing the floor into two other shapes. Draw a sketch.

2. Calculate the area of the figures below.





3. Which of the following rules can be used to calculate the perimeter (P) of a rectangle? Explain.

   - Perimeter = $2 \times (l + b)$ ✓
   - Perimeter = $l + b + l + b$ ✓
   - Perimeter = $2l + 2b$ ✓
   - Perimeter = $l + b$

l and *b* refer to the length and the breadth of a rectangle.

The following are equivalent expressions for perimeter:
$P = 2l + 2b$ and $P = 2(l + b)$ and $P = l + b + l + b$

4. Check with two classmates that the rule or rules you have chosen above are correct; then apply it to calculate the perimeter of figure A. Think carefully!

5. The perimeter of a rectangle is 28 cm and its breadth is 6 cm. What is its length?

14.3 Area and perimeter of circles

REVISING CONCEPTS FROM PREVIOUS GRADES

Teaching guidelines

Revise these concepts by asking questions such as: "What do we call the perimeter of a circle?" "What do we mean by the diameter of a circle?", and so on.

Write the formulae for calculating the circumference and the area of a circle on the board.

$c = 2\pi r$ or πd
$A = \pi r^2$

Remind learners that the diameter of a circle is twice the radius: $d = 2r$.

CIRCLE CALCULATIONS

Teaching guidelines

Learners apply their knowledge of circles when answering these questions.

Take care that learners use the correct value for the radius, for example if the diameter is given. Note that units are only included in the final answers.

Answers

1. (a) $P = 2\pi r$
$= 2(3{,}14)5$
$= 31{,}4$ m
$A = \pi r^2$
$= 3{,}14(5^2)$
$= 78{,}5$ m^2

(b) $P = 2\pi r$
$= 2(3{,}14)9$ $[r = \frac{1}{2}d = 9$ mm$]$
$= 56{,}52$ mm
$A = \pi r^2$
$= 3{,}14(9^2)$
$= 254{,}34$ mm^2

2. (a) $c = 2\pi r$
$53 = 2(3{,}14)r$
$53 = 6{,}28r$
$r \approx 8{,}44$ cm

(b) $c = 2\pi r$
$206 = 2(3{,}14)r$
$206 = 6{,}28r$
$r \approx 32{,}8$ mm

3. A: $A = \pi r^2$
$= 3{,}14(15^2)$
$= 3{,}14 \times 225$
$= 706{,}5$ cm^2

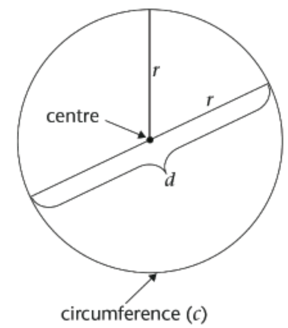
B: $A = \pi r^2$
$= 0{,}5(3{,}14)(21{,}5^2)$ $[r = \frac{1}{2}d = 21{,}5]$
$= 0{,}5 \times 3{,}14 \times 462{,}25$
$\approx 725{,}73$ cm^2

14.3 Area and perimeter of circles

REVISING CONCEPTS FROM PREVIOUS GRADES

The perimeter of a circle is called the **circumference** of a circle. You will remember the following about circles from previous grades:

- The distance across the circle through its centre is called the **diameter** (d) of the circle.
- The distance from the centre of the circle to any point on the circumference is called the **radius** (r).
- The circumference (c) of a circle divided by its diameter is equal to the irrational value we call **pi** (π). To simplify calculations, we often use the approximate values: $\pi \approx 3{,}14$ or $\frac{22}{7}$.



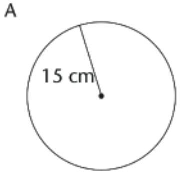
The following are important formulae to remember:
- $d = 2r$ and $r = \frac{1}{2}d$
- Circumference of a circle $(c) = 2\pi r$
- Area of a circle $(A) = \pi r^2$

CIRCLE CALCULATIONS

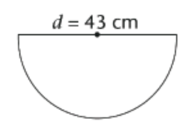
In the following calculations, use $\pi = 3{,}14$ and round off your answers to two decimal places. If you take a square root, remember that length is always positive.

1. Calculate the perimeter and area of the following circles:
(a) A circle with a radius of 5 m
(b) A circle with a diameter of 18 mm
2. Calculate the radius of a circle with:
(a) a circumference of 53 cm
(b) a circumference of 206 mm
3. Work out the area of the following shapes:

A



B



Answers

4. (a) $A = \pi r^2$
   $r^2 = 200 \div 3{,}14$
   $r^2 \approx 63{,}69$
   $r \approx 7{,}98$ m
   [length is always positive]
   $d = 2r = 15{,}96$ m

   (b) $A = \pi r^2$
   $r^2 = 1\,000 \div 3{,}14$
   $r^2 \approx 318{,}47$
   $r \approx 17{,}85$ m
   $d = 2r = 35{,}7$ m

5. See the answers on LB page 149 alongside.

14.4 Converting between units

CONVERTING BETWEEN UNITS USED FOR PERIMETER AND AREA

Teaching guidelines

To convert from a larger unit to a smaller unit for perimeter, we multiply by the conversion factor and from a smaller unit to a larger unit, we divide by the conversion factor. For example, to change centimetres (the larger unit) to millimetres (the smaller unit), multiply by 10, etc.

Pay particular attention to conversion of square units.

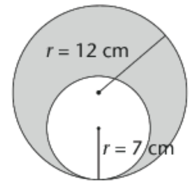
Misconceptions

Learners become confused about the relative size of units and, for example, divide centimetres by 10 to get millimetres. Encourage them to ask the question, "If I change this unit, for example, from centimetres to metres, will the number be more or less than it is?" In this case the answer is less, so learners divide centimetres by 100 to get metres.

Answers

2. (a) $650(0{,}1)(0{,}1)\text{ cm}^2 = 6{,}5\text{ cm}^2$
   (b) $1\,200(0{,}1)(0{,}1)\text{ cm}^2 = 12\text{ cm}^2$
   (c) $18(100)(100)\text{ cm}^2 = 180\,000\text{ cm}^2$
   (d) $0{,}045(100)(100)\text{ cm}^2 = 450\text{ cm}^2$
   (e) $93(0{,}1)(0{,}1)\text{ cm}^2 = 0{,}93\text{ cm}^2$
   (f) $177(100)(100)\text{ cm}^2 = 1\,770\,000\text{ cm}^2$
3. (a) $93(0{,}001)(0{,}001)\text{ cm}^2 = 0{,}000093\text{ m}^2$
   (b) $0{,}017(1\,000)(1\,000)\text{ cm}^2 = 17\,000\text{ m}^2$

4. Calculate the radius and diameter of a circle with:
   (a) an area of 200 m² (b) an area of 1 000 m²
5. Calculate the area of the shaded part.



Area of larger circle:
$A = \pi r^2 = 3{,}14 \times 12^2$
$= 3{,}14 \times 144$
$= 452{,}16\text{ cm}^2$

Area of smaller circle:
$A = \pi r^2 = 3{,}14 \times 7^2$
$= 3{,}14 \times 49$
$= 153{,}86\text{ cm}^2$

Area of shaded part: $452{,}16 - 153{,}86 = 298{,}3\text{ cm}^2$

14.4 Converting between units

CONVERTING BETWEEN UNITS USED FOR PERIMETER AND AREA

Always make sure that you use the correct units in your calculations. Practise the conversions below.

Remember:

1 cm = 10 mm	1 mm = 0,1 cm
1 m = 100 cm	1 cm = 0,01 m
1 km = 1 000 m	1 m = 0,001 km

1. Copy and complete the following conversions:
   (a) 34 cm = 340 mm (b) 501 m = 0,501 km
   (c) 226 m = 22 600 cm (d) 0,58 km = 580 m
   (e) 1,9 cm = 19 mm (f) 73 mm = 7,3 cm
   (g) 924 mm = 0,924 m (h) 32,23 km = 32 230 m

Remember, to convert between square units, you can use method shown below:

To convert cm² to m²:
$1\text{ cm}^2 = 1\text{ cm} \times 1\text{ cm}$
$= 0{,}01\text{ m} \times 0{,}01\text{ m}$
$= 0{,}0001\text{ m}^2$

Example

Convert 50 cm² to m²
$1\text{ cm}^2 = 0{,}0001\text{ m}^2$
$\therefore 50\text{ cm}^2 = 50 \times 0{,}0001\text{ m}^2$
$= 0{,}005\text{ m}^2$

2. Convert to cm²:
   (a) 650 mm² (b) 1 200 mm²
   (c) 18 m² (d) 0,045 m²
   (e) 93 mm² (f) 177 m²
3. (a) Convert 93 mm² to m². (b) Convert 0,017 km² to m².

14.5 Area of other quadrilaterals

PARALLELOGRAMS

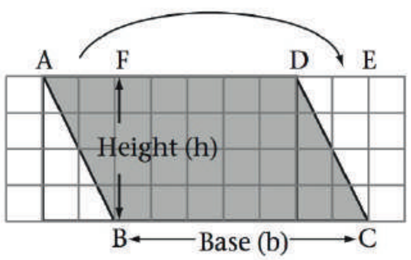
Teaching guidelines

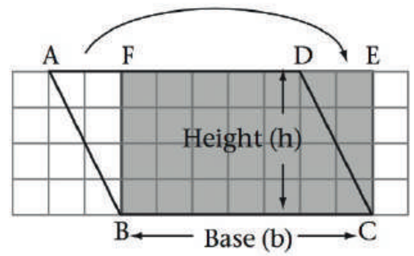
You can let learners draw a parallelogram on squared paper and change it into a rectangle on the same base and between the same parallel lines. The length of a rectangle can also be called the base and the breadth can be called the height.

The opposite sides of a parallelogram are parallel and equal. An altitude (or height) of a parallelogram is a line drawn perpendicularly between two opposite, parallel sides.

Any parallelogram, for example ABCD, can be changed into a rectangle if we cut it along the line FB (an altitude) and move the triangle we cut off to the opposite side so that the original side AB lies along the side CD. The result is rectangle FBCE, which has the same base and the same height as the parallelogram. The areas of the two shapes are equal.

The area of a rectangle is given by $A = l \times b$ or A = base × height = $b \times h$. Therefore, the formula for the area of any parallelogram is the same: A = base × height.

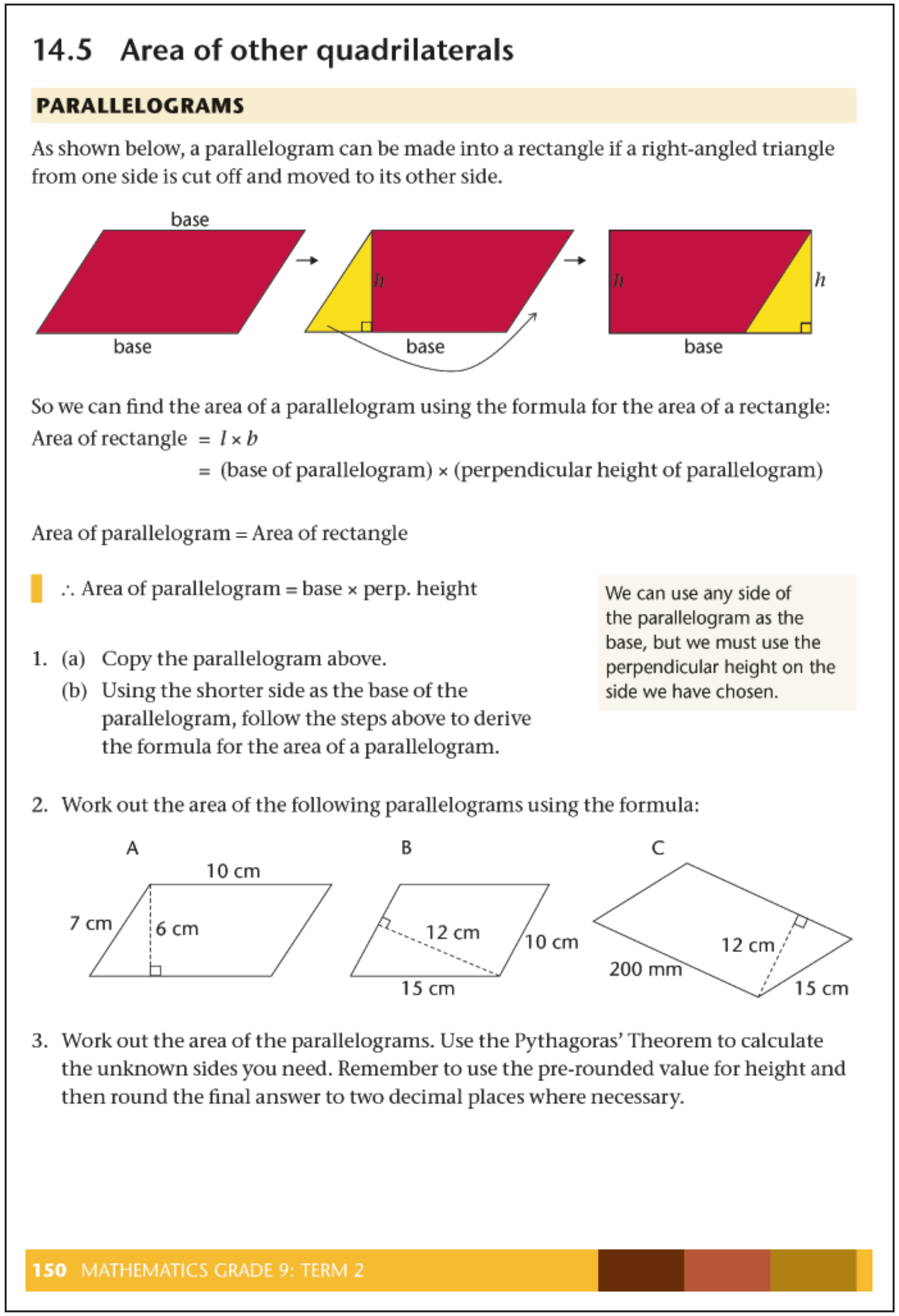




Answers

1. Learners' own work
2. Area of parallelogram = base × perpendicular height

   Area of A = 10 × 6 = 60 cm^2 Area of B = 10 × 12 = 120 cm^2 Area of C = 20 × 12 = 240 cm^2



14.5 Area of other quadrilaterals

PARALLELOGRAMS

As shown below, a parallelogram can be made into a rectangle if a right-angled triangle from one side is cut off and moved to its other side.

So we can find the area of a parallelogram using the formula for the area of a rectangle:

Area of rectangle = $l \times b$

= (base of parallelogram) × (perpendicular height of parallelogram)

Area of parallelogram = Area of rectangle

∴ Area of parallelogram = base × perp. height

We can use any side of the parallelogram as the base, but we must use the perpendicular height on the side we have chosen.

1. (a) Copy the parallelogram above.
   (b) Using the shorter side as the base of the parallelogram, follow the steps above to derive the formula for the area of a parallelogram.
2. Work out the area of the following parallelograms using the formula:

3. Work out the area of the parallelograms. Use the Pythagoras' Theorem to calculate the unknown sides you need. Remember to use the pre-rounded value for height and then round the final answer to two decimal places where necessary.

150 MATHEMATICS GRADE 9: TERM 2

Answers

3. A: $\text{height}^2 + 3^2 = 5^2$ [Pythagoras]
$\text{height}^2 + 9 = 25$
$\text{height}^2 = 25 - 9$
$\text{height}^2 = 16$
$\text{height} = 4$
$\text{Area} = (8 + 3) \times 4$
$= 44\ \text{cm}^2$

B: $\text{height}^2 = 8^2 - 2^2$
$\text{height}^2 = 64 - 4$
$\text{height} = \sqrt{60}$ cm (keep this value in your calculator)
$\text{Area} = (5 + 2) \times \text{height}$
$= 7 \times \text{height}$
$\approx 54{,}22\ \text{cm}^2$

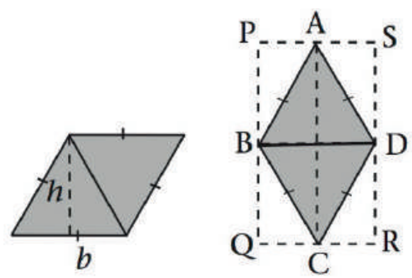
C: $\text{Area} = \text{base} \times \text{height}$
$= 12 \times 8$
$= 96\ \text{m}^2$

D: $\text{height}^2 + (15 - 10)^2 = 8^2$ [Pythagoras]
$\text{height}^2 + 25 = 64$
$\text{height}^2 = 39$
$\text{height} \approx 6{,}24$
$\text{Area} = \text{base} \times \text{height}$
$= 15 \times 6{,}24$
$\approx 93{,}67\ \text{cm}^2$

RHOMBI

Teaching guidelines

The area of a rhombus can be found by working with the base and the height (as we did for the parallelogram), or by working with the diagonals. Look at the second drawing on the right of ABCD in PQRS, as follows:



$\text{Area ABCD} = \frac{1}{2}\text{Area of PQRS} = \frac{1}{2}\text{PQ} \times \text{QR} = \frac{1}{2}\text{AC} \times \text{BD}$ and (AC = PQ; BD = QR)

$= \frac{1}{2} \times$ product of the diagonals.

The method used will depend on the information given.

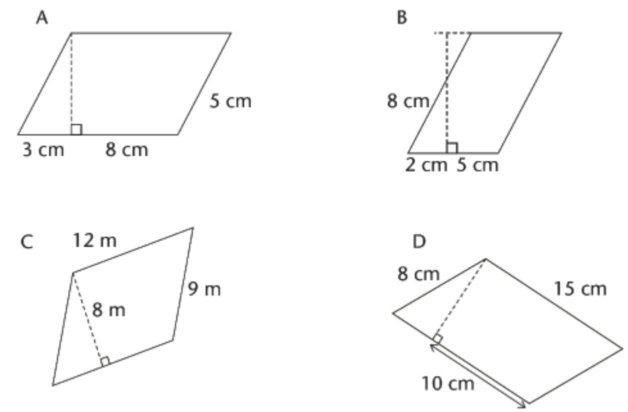
Answers

2. A: $\text{Area} = \text{length} \times \text{height}$
$= 10 \times 8$
$= 80\ \text{m}^2$

B: $\text{Area} = \text{length} \times \text{height}$
$= 15 \times 11$
$= 165\ \text{cm}^2$

C: $\text{height}^2 + 2^2 = 7^2$ [Pythagoras]
$\text{height}^2 = 49 - 4$
$\text{height} \approx 6{,}71$
$\text{Area} = 7 \times \text{height}$
$= 46{,}96\ \text{cm}^2$

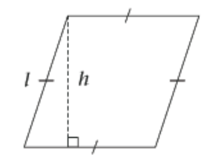
D: $\text{height}^2 + (9 - 6)^2 = 9^2$ [Pythagoras]
$\text{height}^2 = 81 - 9 = 72$
$\text{height} \approx 8{,}49$
$\text{Area} = 9 \times \text{height} = 9 \times 8{,}49$
$= 76{,}41\ \text{cm}^2$



RHOMBI

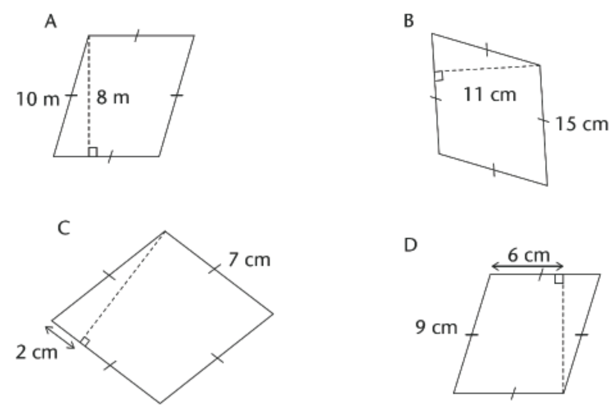
A rhombus is a parallelogram with all its sides equal.

In the same way we derived the formula for the area of a parallelogram, we can show the following:



Area of a rhombus = length × perp. height

1. Show how to derive the formula for the area of a rhombus.
2. Calculate the areas of the following rhombi. Round off answers to two decimal places where necessary.

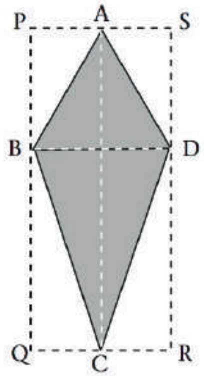


CHAPTER 14: AREA AND PERIMETER OF 2D SHAPES 151

KITES

Teaching guidelines

The diagonals of a kite bisect each other at right angles. If we draw a rectangle around the kite, the diagonals of the kite divide the rectangle into four smaller rectangles, each of which has a diagonal which is a side of the kite. Notice that the area of the kite is half of the area of the rectangle.



Area of kite ABCD:

$= \frac{1}{2}$ Area PQRS

$= \frac{1}{2}$ PQ $\times$ QR and PQ = AC; QR = BD

$= \frac{1}{2}$ AC $\times$ BD

$= \frac{1}{2} \times$ the product of the diagonals

In words: The area equals half of the diagonal product.

Answers

1. (a) Area $= \frac{1}{2}(\text{diagonal}_1 \times \text{diagonal}_2)$
$= \frac{1}{2}(150 \times 200)$
$= 15\ 000\ \text{mm}^2$
$= 0{,}015\ \text{m}^2$

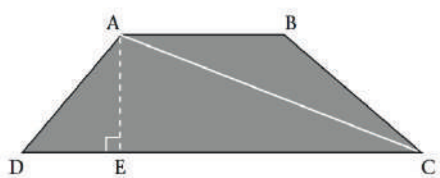
(b) Area $= \frac{1}{2}(\text{diagonal}_1 \times \text{diagonal}_2)$
$= \frac{1}{2}(25 \times 40)$
$= 500\ \text{cm}^2$
$= 0{,}05\ \text{m}^2$

2. See the answer on LB page 152 alongside.

TRAPEZIUMS

Teaching guidelines

We can divide the trapezium ABCD into two triangles with the same perpendicular height AE, namely ΔABC and Δ ADC. Then:



Area of trapezium ABCD:

= Area ΔABC + Area ΔADC

$= \frac{1}{2}$ AB × AE + $\frac{1}{2}$ DC × AE

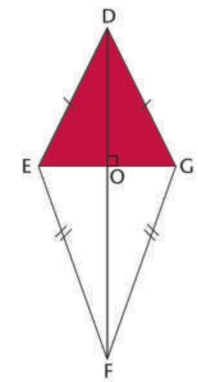
$= \frac{1}{2}$ AE(AB + DC)

In words: The area equals half of the sum of the parallel sides multiplied by the height.

Remember: The area of a rectangle is given by length × breadth.

KITES

To calculate the area of a kite, you use one of its properties, namely that the diagonals of a kite are perpendicular.



Area of kite DEFG = Area of ΔDEG + Area of ΔEFG

$= \frac{1}{2}(b \times h) + \frac{1}{2}(b \times h)$

$= \frac{1}{2}(\text{EG} \times \text{OD}) + \frac{1}{2}(\text{EG} \times \text{OF})$

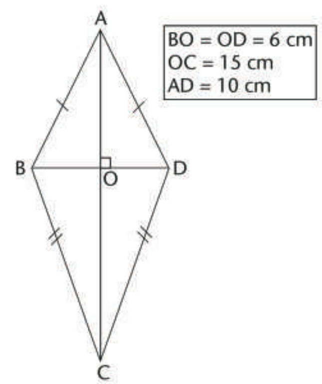
$= \frac{1}{2}\text{EG}(\text{OD} + \text{OF})$

$= \frac{1}{2}\text{EG} \times \text{DF}$

Notice that EG and DF are the diagonals of the kite.

$\therefore$ Area of a kite $= \frac{1}{2}(\text{diagonal}_1 \times \text{diagonal}_2)$

1. Calculate the area of kites with the following diagonals. Give your answers in m^2.
   (a) 150 mm and 200 mm (b) 25 cm and 40 cm
2. Calculate the area of the kite.



$10^2 = 6^2 + \text{OA}^2$ [Pythagoras]

$\text{OA}^2 = 100 - 36$

$\text{OA}^2 = 64$

OA = 8 cm

Area of kite $= \frac{1}{2}(\text{diagonal}_1 \times \text{diagonal}_2)$

$= \frac{1}{2}((6 + 6) \times (15 + 8))$

$= \frac{1}{2}(12 \times 23)$

$= 138\ \text{cm}^2$

TRAPEZIUMS

A trapezium has two parallel sides. If we tessellate (tile) two trapeziums, as shown in the diagram on the following page, we form a parallelogram. (The yellow trapezium is the same size as the blue one. The base of the parallelogram is equal to the sum of the parallel sides of the trapezium.)

152 MATHEMATICS GRADE 9: TERM 2

Answers

1. A: Area $= \frac{1}{2} \times 19(20 + 10)$
$= \frac{1}{2} \times 19 \times 30$
$= 285 \text{ mm}^2$

B: Area $= \frac{1}{2} \times 52(65 + 105)$
$= \frac{1}{2} \times 52 \times 170$
$= 4\,420 \text{ cm}^2$

AREAS OF COMPOSITE SHAPES

Teaching guidelines

Remind learners that "height" always refers to "perpendicular height", and make sure that they know how to work out the perpendicular heights of the various polygons.

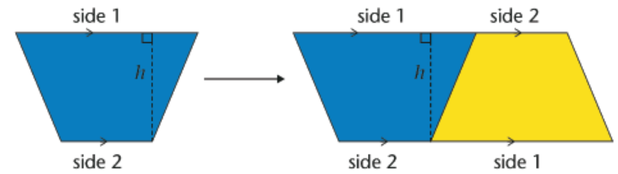
Answers

1. (a) $EC^2 = 5^2 - 4^2$ [Pythagoras]
$EC^2 = 25 - 16 = 9$
$EC = 3 \text{ cm}$
Total area $= l \times h + \frac{1}{2}(b \times h)$
$= 12 \times 4 + \frac{1}{2}(3 \times 4)$
$= 48 + 6 = 54 \text{ cm}^2$

(b) Total area $= \frac{1}{2}(12 + 10) \times h + l \times h$
$= \frac{1}{2}(22 \times 6) + 7 \times 6$
$= \frac{1}{2} \times 132 + 42$
$= 66 + 42$
$= 108 \text{ m}^2$

(c) $OM = PO = 5$ cm [kite]
$ON^2 = 13^2 - 5^2$ [Pythagoras]
$= 169 - 25 = 144$
$ON = 12$ cm
$OL^2 = 20^2 - 5^2$ [Pythagoras]
$= 400 - 25 = 375$
$OL \approx 19{,}36$ cm
Area $= \frac{1}{2}(OL + 12)(5 + 5)$
$= 156{,}8 \text{ cm}^2$

(d) $FO^2 = 10^2 - 6^2$ [Pythagoras]
$= 100 - 36 = 64$
$FO = 8$
$HO^2 = 8^2 - 6^2$ [Pythagoras]
$= 64 - 36 = 28$
$HO \approx 5{,}29$
Area $= 18 \times 8 + \frac{1}{2}(18 + 9)HO$
$= 215{,}42 \text{ m}^2$



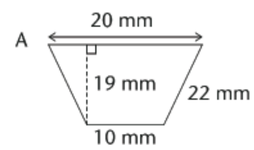
We can use the formula for the area of a parallelogram to work out the formula for the area of a trapezium as follows:

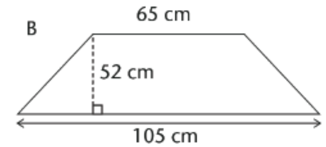
Area of parallelogram = base × height
= (side 1 + side 2) × height

Area of trapezium = $\frac{1}{2}$ area of parallelogram
= $\frac{1}{2}$(side 1 + side 2) × height

∴ Area of a trapezium = $\frac{1}{2}$(sum of parallel sides) × perp. height

Calculate the area of the following trapeziums:

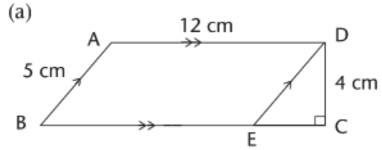




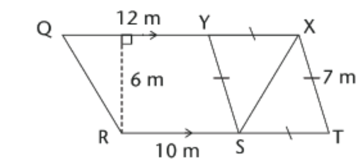
AREAS OF COMPOSITE SHAPES

Calculate the areas of the following 2D shapes. Round off your answers to two decimal places where necessary.

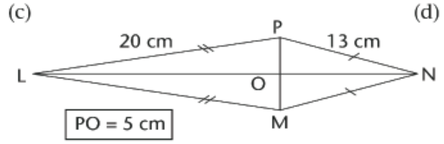
(a)



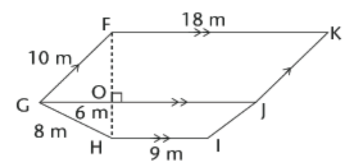
(b)



(c)



(d)



14.6 Doubling dimensions of a 2D shape

Teaching guidelines

You can show learners how doubling dimensions influences the perimeter and the area. For example:

- A rectangle: length l and breadth b; area = lb. Now double the dimensions to $2l$ and $2b$, then the area becomes $A = 2l \times 2b = 4lb$ which is four times the original area.
- A kite: let the diagonals be a and b. The area $= \frac{1}{2}ab$. Double the dimensions, then the area becomes $A = \frac{1}{2} \times 2a \times 2b = \frac{1}{2} \times 4 \times ab = 4 \times \frac{1}{2}ab$; four times the original area.

Answers

1. See the answers on LB pages 154 alongside and LB page 155 on the following page.
2. None; the shapes in column 3 are similar to those in column 1.
3. See the answers on LB page 154 alongside.

14.6 Doubling dimensions of a 2D shape

Remember that a 2D shape has two dimensions, namely length and breadth. You have used length and breadth in different forms, to work out the perimeters and areas of shapes, for example:

- length and breadth for rectangles and squares
- bases and perpendicular heights for triangles, rhombi and parallelograms
- two diagonals for kites.

But how does doubling one or both of the dimensions of a figure affect the figure's perimeter and area?

"Doubling" means to multiply by 2.

The four sets of figures on the next page are drawn on a grid of squares. Each row shows an original figure; the figure with one of its dimensions doubled, and the figure with both of its dimensions doubled. Each square has a side of one unit.

1. Work out the perimeter and area of each shape. Round off your answers to two decimal places where necessary.
2. Which figure in each set is congruent to the original figure?
3. Copy the table below and fill in the perimeter (P) and area (A) of each figure:

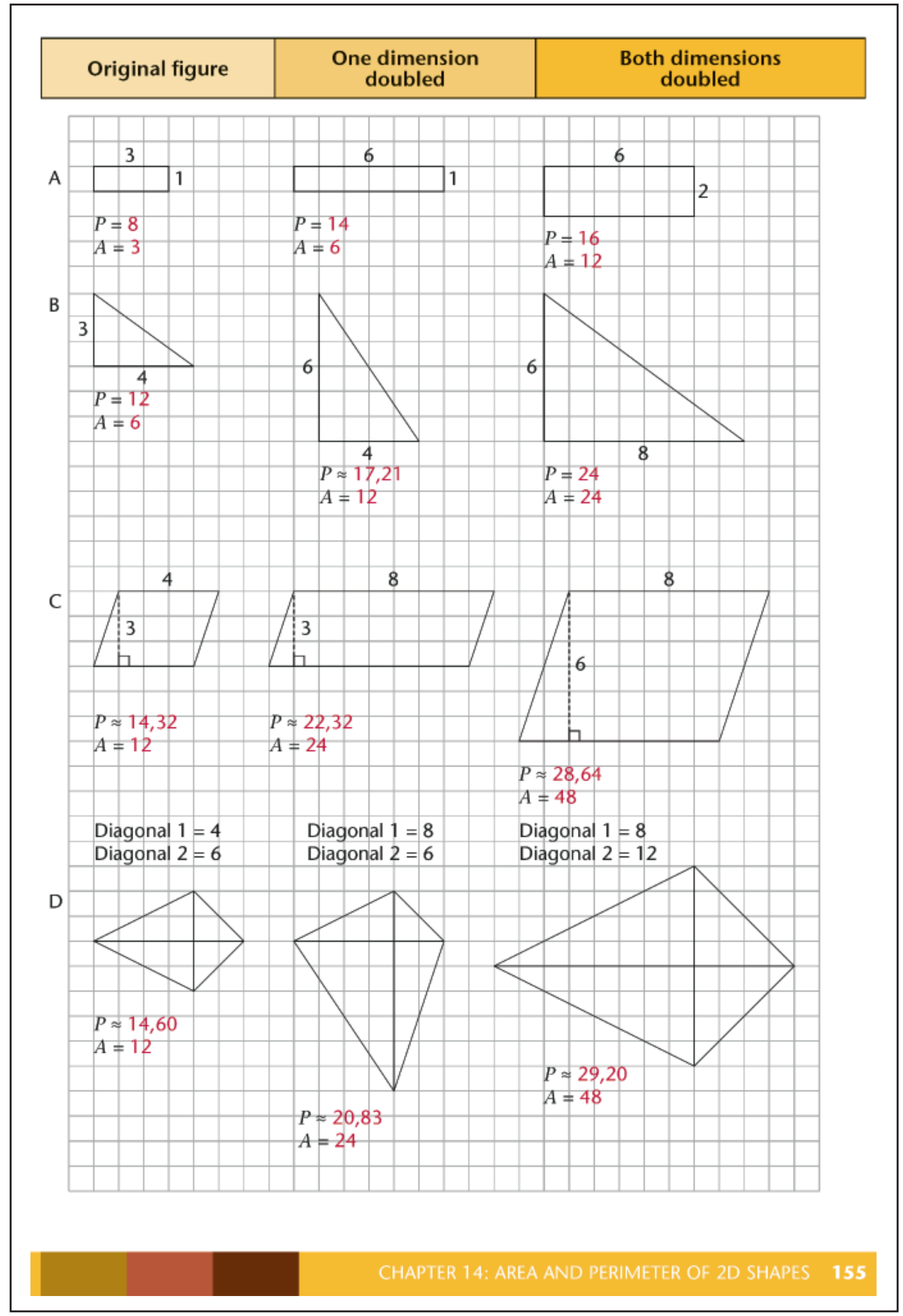
Figure	Original figure	Figure with both dimensions doubled
A	P = 8 A = 3	P = 16 A = 12
B	P = 12 A = 6	P = 24 A = 24
C	P = ≈ 14,32 A = 12	P = ≈ 28,64 A = 48
D	P = ≈ 14,60 A = 12	P = ≈ 29,20 A = 48

4. Look at the completed table. What patterns do you notice? Choose one:
   - When both dimensions of a shape are doubled, its **perimeter is doubled** and its **area is doubled**.
   - When both dimensions of a shape are doubled, its **perimeter is doubled** and its area is **four times bigger**.

Answer

4. See the answers on LB page 155 alongside.

   The pattern in the table on LB page 155 alongside shows that if both dimensions of a shape are doubled, its perimeter is doubled and its area is four times bigger.



WORKSHEET

Answers

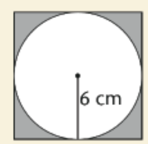
1. See the answers on LB page 156 alongside.
2. (a) $d = 2r = 12$ cm
   $P = 4(12) = 48$ cm
   Area $= l^2 - \pi r^2$
   $= 12^2 - \pi 6^2$
   $= 144 - 113{,}04$
   $= 30{,}96 \text{ cm}^2$
   (b) JO = OL = 3 m; JM = LM = 7 m; JK = KL = 5 m [kite]
   $OM^2 = 7^2 - 3^2 = 49 - 9$ [Pythagoras]
   $OM \approx 6{,}32$
   $OK^2 = 5^2 - 3^2 = 25 - 9 = 16$ [Pythagoras]
   $OK = 4$
   $A = \frac{1}{2}(\text{diagonal}_1 \times \text{diagonal}_2)$
   $= \frac{1}{2}(3 + 3)(4 + OM)$
   $\approx 30{,}97 \text{ m}^2$

WORKSHEET

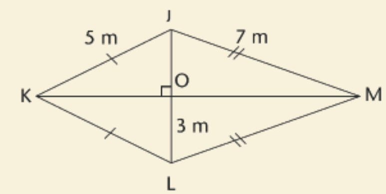
1. Write down the formulae for the following:

Perimeter of a square	$P = 4l$
Perimeter of a rectangle	$P = 2(l + b)$
Area of a square	$A = l^2$
Area of a rectangle	$A = l \times b$
Area of a triangle	$A = \frac{1}{2}(b \times h)$
Area of a rhombus	A = length × perp. height
Area of a kite	$A = \frac{1}{2}(\text{diagonal}_1 \times \text{diagonal}_2)$
Area of a parallelogram	$A = b \times$ perp. h
Area of a trapezium	$A = \frac{1}{2}$(sum of parallel sides) × height
Diameter of a circle	$d = 2r$
Circumference of a circle	$c = 2\pi r$ or πd
Area of a circle	$A = \pi r^2$

2. (a) Calculate the perimeter of the square and the area of the shaded parts of the square:



(b) Calculate the area of the kite:



AREA AND PERIMETER OF 2D SHAPES

Term 3

Chapter 15: Functions **189**
15.1 From formulas to words, tables and graphs 190
15.2 Tables and graphs 192
Chapter 16: Algebraic expressions **199**
16.1 Introduction 200
16.2 Factors of expressions of the form $ab + ac$ 201
16.3 Factors of expressions of the form $x^2 + (b + c)x + bc$ 203
16.4 Factors of expressions of the form $a^2 - b^2$ 205
16.5 Simplification of algebraic fractions 207
Chapter 17: Equations **211**
17.1 Introduction 212
17.2 Solving by factorisation (Part 1) 214
17.3 Solving by factorisation (Part 2) 216
17.4 Solving by factorisation (Part 3) 217
17.5 Set up equations to solve problems 218
17.6 Equations and ordered pairs 220
Chapter 18: Graphs **223**
18.1 Global graphs 224
18.2 Changes at different rates 232
18.3 Draw graphs from tables of ordered pairs 234
18.4 Gradient 236
18.5 Finding the formula for a graph 240
18.6 x- and y-intercepts 244
18.7 Graphs of non-linear functions 245

Chapter 19: Surface area, volume and capacity of 3D objects 247
19.1 Surface area 248
19.2 Volume 250
19.3 Capacity 253
19.4 Doubling dimensions and the effect on volume 254
Chapter 20: Transformation geometry 257
20.1 Points on a coordinate system 258
20.2 Reflection (flip) 259
20.3 Translation (slide) 262
20.4 Enlargement (expansion) and reduction (shrinking) 266
Chapter 21: Geometry of 3D objects 275
21.1 Classifying 3D objects 276
21.2 Nets and models of prisms and pyramids 278
21.3 Platonic solids 279
21.4 Euler's formula 283
21.5 Cylinders 284
21.6 Spheres 287

Grade 9 Term 3 Chapter 15 Functions

Learner Book Overview		
Sections in this chapter	**Content**	**Pages in Learner Book**
15.1 From formulas to words, tables and graphs	Translating mathematical instructions for a calculation from words to flow diagrams and formulae and vice versa; completing tables using formulae	Pages 157 to 159
15.2 Tables and graphs	Completing tables to produce ordered pairs of input and output numbers; plotting points on graphs to represent the ordered pairs; changing the scales on the axes and comparing the resulting graphs; matching a formula to a table of values; investigating pattern differences	Pages 159 to 164

CAPS time allocation	5 hours
CAPS content specification	Page 141

Mathematical background

A function is a relationship where each input has a single output.

A relationship exists between two quantities where the one quantity, the input value (independent variable), is substituted into a formula to give another value, the output value (dependent variable).

A function can be represented in the following different, but equivalent ways:

- in a flow diagram: input value ⟶ × 3 ⟶ + 5 ⟶ output value
- verbally: for example, multiply the input number by 3 and then add 5 to get the corresponding output number
- $y = 3x + 5$
- a table of values that shows the input and corresponding output values as pairs of numbers
- a graph on which the ordered pairs of numbers are plotted as points.

We can switch from one representation to another.

The scales on one or both of the axes of a graph can change, which will change the appearance of the graph, but not the information in the graph.

The graphs of relationships show clearly what the patterns are that the output values show. Some patterns show:

- increases in the output values as the input values increase, for example in $y = 10 + x$
- decreases in the output values as the input values increase, for example in $y = 10 - x$
- both decreases and increases in the output values as the input values increase, for example $y = x^2$.

15.1 From formulae to words, tables and graphs

THE SAME INSTRUCTIONS IN WORDS AND IN SYMBOLS

Teaching guidelines

Revise the substitution of input numbers in formulae to get the corresponding output numbers. You can use a table to organise the answers.

Learners have worked with translations between the different representations of functions before. Concentrate on the change from formulae to words and the translation from words to formulae.

Learners will recognise equivalent formulae, namely those that give the same output values for the same input values.

Answers

1. (a) 5; 8; 11; 35 (b) 15; 18; 21; 45 (c) 0; 8; 16; 80
   (d) 5; 8; 17; 305 (e) 0; 8; 22; 350 (f) 0; 18; 42; 450
2. (a) Multiply the input number by 3 and add 5 to get the output number.
   (b) Add 5 to the input number and multiply the answer by 3.
   (c) Multiply the input number by 3, multiply the input number by 5 and add the two answers.
   (d) Square the input number, multiply the answer by 3 and add 5 to that answer.
   (e) Square the input number and multiply the answer by 3. Also, multiply the input number by 5 and add to the previous answer.
   (f) Add 5 to the input number and multiply the answer by the input number. Multiply this answer by 3.
3. See the answers on LB page 157 alongside.
4. See the answers on LB page 158 on the following page.

CHAPTER 15 Functions

15.1 From formulae to words, tables and graphs

THE SAME INSTRUCTIONS IN WORDS AND IN SYMBOLS

1. Each formula below indicates a relationship between two sets of numbers that may be called the *input numbers* and the *output numbers*. For each formula, calculate the output numbers that correspond to the input numbers 0, 1, 2 and 10.

   (a) $y = 3x + 5$ (b) $y = 3(x + 5)$ (c) $y = 3x + 5x$
   (d) $y = 3x^2 + 5$ (e) $y = 3x^2 + 5x$ (f) $y = 3x(x + 5)$

2. The information provided in the formula $y = 5x^2 - 3x$ can also be represented in words, for example: *To get the output number, you have to subtract three times the input number from five times the square of the input number.*

   Represent each of the formulae in question 1 in words:

   (a) $y = 3x + 5$ (b) $y = 3(x + 5)$ (c) $y = 3x + 5x$
   (d) $y = 3x^2 + 5$ (e) $y = 3x^2 + 5x$ (f) $y = 3x(x + 5)$

3. For each set of instructions, write a formula that provides the same information.

   (a) *multiply the input number by 10, then subtract 3 to get the output number* $y = 10x - 3$
   (b) *subtract 3 from the square of the input number, then multiply by 10 to get the output number* $y = 10(x^2 - 3)$
   (c) *multiply the square of the input number by 10, then add 5 times the input number to get the output number* $y = 10x^2 + 5x$
   (d) *subtract 7 times the square of the input number from 100, then multiply by 3 to get the output number* $y = 3(100 - 7x^2)$
   (e) *add 4 to the input number, then subtract the answer from 50 to get the output number* $y = 50 - (x + 4)$
   (f) *multiply the input number by 3, then subtract the answer from 15 to get the output number* $y = 15 - 3x$

4. Copy and complete the table on the following page to check your answers for question 3. First apply the verbal instructions for the input numbers 1, 5 and 10 in each case. Then choose another input number and do the same thing. Next, use the formula you have written to calculate the output numbers. Do corrections where there are differences.

Answers

5. (a) Formula: $y = 3x + 17$

   To get the output number, multiply the input number by 3 and add 17.

   (b) Formula: $y = 3(x + 5) + 2$

   To get the output number, add 5 to the input number, multiply by 3 and then add 2.

   (c) Formula: $y = 3(x - 2) + 23$

   To get output number, subtract 2 from the input number, multiply by 3 and add 23.

   (d) Formula: $y = 5(2x + 3) + 4$

   To get the output number, multiply the input number by 2, then add 3, then multiply that answer by 5 and add 4.

   (e) Formula: $y = 5[2(x + 3) + 4]$

   To get the output number, add 3 to the input number, then multiply by 2, then add 4 to that answer and multiply the last answer by 5.

   (f) Formula: $y = 10x + 19$

   To get the output number, multiply the input number by 10 and add 19.

   (g) Formula: $y = 10(x + 5)$

   To get the output number, add 5 to the input number and then multiply by 10.

6. (a) See the answers on LB page 158 alongside.

   (b) Learners' own work

		1	5	10	
(a)	verbal description	7	47	97	
	formula	7	47	97	
(b)	verbal description	–20	220	970	
	formula	–20	220	970	
(c)	verbal description	15	275	1 050	
	formula	15	275	1 050	
(d)	verbal description	279	–225	–1 800	
	formula	279	–225	–1 800	
(e)	verbal description	45	41	36	
	formula	45	41	36	
(f)	verbal description	12	0	–15	
	formula	12	0	–15	

5. In certain cases, flow diagrams can be used to provide instructions on how output numbers can be calculated. For each flow diagram below, represent the information in a formula and also in words:

   (a) → × 3 → + 17 → $y = 3x + 17$

   (b) → + 5 → × 3 → + 2 → $y = 3(x + 5) + 2$

   (c) → – 2 → × 3 → + 23 → $y = 3(x - 2) + 23$

   (d) → × 2 → + 3 → × 5 → + 4 → $y = 5(2x + 3) + 4$

   (e) → + 3 → × 2 → + 4 → × 5 → $y = 5[2(x + 3) + 4]$

   (f) → × 10 → + 19 → $y = 10x + 19$

   (g) → + 5 → × 10 → $y = 10(x + 5)$

6. (a) Copy and complete the following table:

x	0	1	2	3
y according to your formula for 5(a)	17	20	23	26
y according to your formula for 5(b)	17	20	23	26
y according to your formula for 5(c)	17	20	23	26

Answers

7. See the answers on LB page 159 alongside.
8. The formula for (b) is $y = 3(x + 5) + 2 = 3x + 15 + 2 = 3x + 17$, as in (a).
   The formula for (c) is $y = 3(x - 2) + 23 = 3x - 6 + 23 = 3x + 17$, as in (a).

15.2 Tables and graphs

Teaching guidelines

Learners often find it difficult to plot ordered pairs because they do not understand how to read off points on a graph. Looking at the graph on LB page 159 alongside, any point on the blue line is six units away from the origin to the right in the horizontal direction. In the same way, any point on the red line is nine units upward from the origin in the vertical direction. The ordered pair (6; 9) is represented by the point where the blue line and the red line cross.

If learners can read off the coordinates of the other given points correctly, you can assume that they understand how to plot points.

Hand out graph paper to learners to plot the ordered pairs in questions 7, 8 and 9 on LB page 161.

Misconceptions

When learners start working on graph paper to plot points, they forget that it is an ordered pair of which the first number is plotted along the horizontal axis and the second number along the vertical axis. They easily switch the numbers in their efforts to plot them. Or they plot the one number on the x-axis and the other on the y-axis, so that they have two points, one on each axis.

Notes on the questions

The set of yellow and blue squares at the top of LB page 160 on the following page are used to make the table below it. The independent variable is the blue squares and the dependent variable is the yellow squares.

Answers

1. See the answers on LB page 159 alongside.
2. (a) (8; 13)
   (b) (2; 1)

(b) If your output numbers for 5(a), 5(b) and 5(c) are not the same, you have made a mistake somewhere. If this is the case, find your mistake and correct it.

7. (a) Copy and complete the following table:

x	–3	–2	–1	0
y according to your formula for 5(d)	–11	–1	9	19
y according to your formula for 5(e)	20	30	40	50
y according to your formula for 5(f)	–11	–1	9	19
y according to your formula for 5(g)	20	30	40	50

   (b) If your output numbers for 5(d) and 5(f) are not the same, you have made a mistake somewhere. If this is the case, find your mistake and correct it.
   (c) If your output numbers for 5(e) and 5(g) are not the same, you have made a mistake somewhere. If this is the case, find your mistake and correct it.

8. Explain why the output numbers in 5(a), (b) and (c) are the same.

15.2 Tables and graphs

1. Copy and complete the table to show some of the input and output numbers of the relationship described by the formula $y = 2x - 3$.

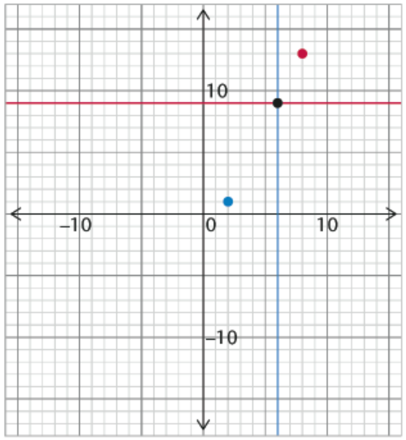
Input numbers	–5	0	2	4	6	8
Output numbers	–13	–3	1	5	9	13

The vertical blue line on this graph represents the input number 6.

The heavy horizontal red line represents the output number 9.

The black point where the blue and red lines intersect indicates that the input number 6 is associated with the output number 9.

We also say the black point represents the **ordered number pair** (6; 9).



2. (a) Which ordered number pair does the red point on the graph represent?
   (b) Which ordered number pair does the blue point on the graph represent?

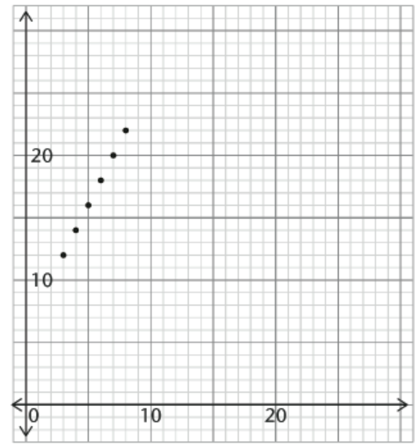
Answers

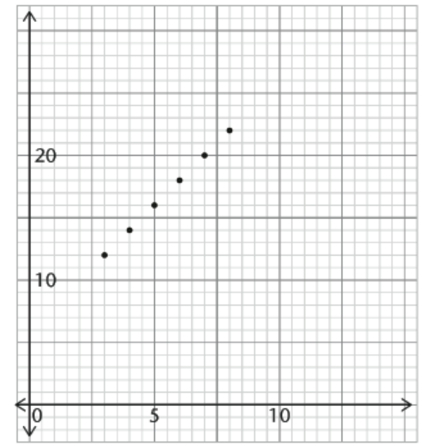
3. The same data, because the output numbers associated with each input number are the same. For example, the output number associated with 3 is 12 on both graphs. This is true of all the points on the graphs.
4. The scales are different on the axes that represent the input numbers.
5. 30
6. No – there is a different relationship between input and output numbers, because the output numbers associated with the input number 5 are different.

A relationship between two variables can be represented by a table that shows the values of the independent and dependent variables (input and output numbers):

Values of the independent variable	3	4	5	6	7	8
Values of the dependent variable	12	14	16	18	20	22

The same information can also be shown on a graph:





3. Do the two graphs show the same relationship, or different relationships between two variables?
4. How do the two graphs differ?
5. Use one of the graphs to find out how many yellow squares there will be, in an arrangement like those at the top of the page, with 12 blue squares.
6. Does the table below represent the same relationship as the table at the top of the page? Explain your answer.

Values of the independent variable	0	5	10	15	20	25
Values of the dependent variable	8	18	28	38	48	58

160 MATHEMATICS GRADE 9: TERM 3

Teaching guidelines

The table in question 7 gives the ordered pairs that will plot a parabola. Learners do not know this yet; they will only plot the points. Learners may be able to see what the shape of the graph will be if the points are joined.

The ordered pairs generated in question 8 will be plotted in a straight line with a positive gradient.

The ordered pairs generated in question 9 will be plotted in a straight line with a negative gradient. The point (0; 15) belongs to $y = 15 + x$ and to $y = 15 - x$.

Answers

7. (a) See the answers on LB page 161 alongside.
   (b) See the answers on LB page 161 alongside. The coordinates of the points have been written next to the point on the answer sheet.

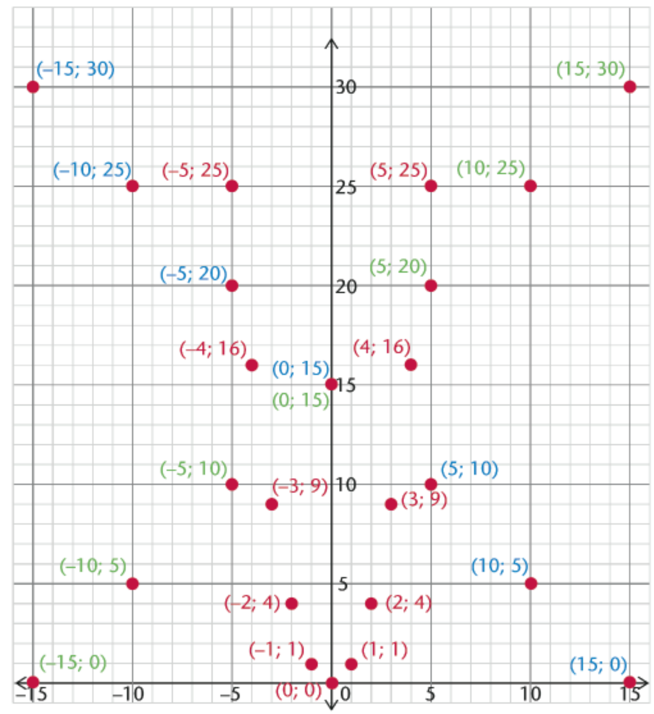
Let learners use different colours for the answers to questions 7, 8 and 9.

8. The points have been plotted on the graph on LB page 161 alongside.
9. The points have been plotted on the graph on LB page 161 alongside.

7. (a) Copy and complete the following table for the relationship described by $y = x^2$:

x	–5	–4	–3	–2	–1	0	1	2	3	4	5
y	25	16	9	4	1	0	1	4	9	16	25

(b) On a graph sheet, copy the axis as below and represent the ordered number pairs in the table.



8. Copy and complete the table for the relationship $y = 15 + x$. Represent the ordered number pairs on the graph sheet you used in question 7(b) above.

x	–15	–10	–5	0	5	10	15
$15 + x$	0	5	10	15	20	25	30

9. Copy and complete the table for the relationship $y = 15 - x$. Represent the ordered number pairs on the graph sheet you used in question 7(b) above.

x	–15	–10	–5	0	5	10	15
$15 - x$	30	25	20	15	10	5	0

CHAPTER 15: FUNCTIONS 161

Answers

10. (a) For $y = x^2$ the rate at which the output values increase and decrease is not constant, but for $y = 15 + x$, the output values increase at a constant rate.
    (b) The graph of $y = x^2$ is a curve and the graph of $y = 15 + x$ is a straight line.
11. (a) For $y = 15 + x$ the output values increase by 5, for $y = 15 - x$ they decrease by 5. In both cases the input values increase by 5.
    (b) Both are straight lines but their directions differ: $y = 15 + x$ goes upwards from left to right as the input values increase, and $y = 15 - x$ goes downwards from left to right as the input values increase.
12. See the answers on LB page 162 alongside.
13. The output numbers in (a) increase by double the amount in (b). The graph of (a) is steeper than the one in (b).
14. (a) See the completed tables on LB page 163 on the following page.

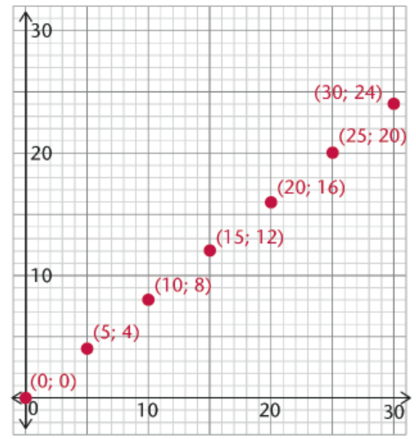
10. (a) The output values for $y = x^2$ and $y = 15 + x$ show patterns. Describe, in words, how the patterns differ. Use the words *increase* and *decrease* in your description.
    (b) Describe, in words, how the graphs of $y = x^2$ and $y = 15 + x$ differ.
11. (a) Describe, in words, how the patterns in the output values for $y = 15 + x$ and $y = 15 - x$ differ. Use the words *increase* and *decrease* in your description.
    (b) Describe, in words, how the graphs of $y = 15 + x$ and $y = 15 - x$ differ.
12. Copy and complete each of the following tables by extending the pattern in the output numbers. Also represent the relationship on graph sheets as the two shown below.

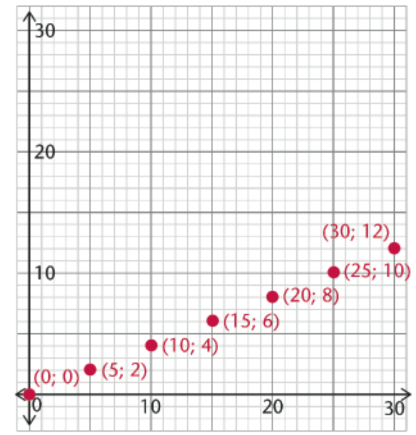
(a)

Input numbers	0	5	10	15	20	25	30
Output numbers	0	4	8	12	16	20	24

(b)

Input numbers	0	5	10	15	20	25	30
Output numbers	0	2	4	6	8	10	12





13. How do the patterns in 12(a) and (b) differ, and how do the graphs differ?
14. Each table on the following page shows some values for a relationship represented by one of these rules:

$y = -2x + 3$	$y = 2x - 5$	$y = -3x + 5$	$y = -3(x + 2)$
$y = 3x + 2$	$y = 5(x - 2)$	$y = 2x + 3$	$y = 2x + 5$
$y = -3x + 6$	$y = 5x + 10$	$y = 5x - 10$	$y = -x + 3$

(a) Copy and complete the tables by extending the patterns in the output values.
(b) For each table, describe what you did to produce more output values. Also write down the rule (formula) that corresponds to the table.

Answers

14. (b) Sample answers are given for the descriptions to produce more output values.

A: $y = 3x + 2$; add three to each consecutive answer, or multiply the input number by three and add two

B: $y = -2x + 3$; subtract two from each consecutive answer, or multiply the input number by minus two and add three

C: $y = 5x - 10$; add five to each consecutive answer, or multiply the input number by five and subtract ten

D: $y = 2x - 5$; add two to each consecutive answer, or multiply the input number by two and subtract five

E: $y = -3x + 6$; subtract three from each consecutive answer, or multiply the input number by minus three and add six

F: $y = -x + 3$; subtract one from each consecutive answer, or multiply the input number by minus one and add three

G: $y = 2x + 3$; add two to each consecutive answer, or multiply the input number by two and add three

AN INVESTIGATION: PATTERNS IN DIFFERENCES

Teaching guidelines

Learners could use the first row of the table in question 1, $y = x^2$ to find the other values. For example, to find $z = x^2 + 1^2$ learners need to find $z = y + 1$, so simply add 1 to each value of the first row, and $w = y + 4$, in other words the first row + 4, and for $s = y + 9$, add 9 to every value in the first row.

Learners should look for the patterns when they work in question 2. For example, $p = (x + 1)^2$ can be found from $y = x^2$ by moving each value one column to the left.

Answers

1. See the answers on LB page 163 alongside.
2. See the answers on LB page 163 alongside.

A.

x	0	1	2	3	4	5	6	7
y	2	5	8	11	14	17	20	23

B.

x	0	1	2	3	4	5	6	7
y	3	1	–1	–3	–5	–7	–9	–11

C.

x	0	1	2	3	4	5	6	7
y	–10	–5	0	5	10	15	20	25

D.

x	0	1	2	3	4	5	6	7
y	–5	–3	–1	1	3	5	7	9

E.

x	0	1	2	3	4	5	6	7
y	6	3	0	–3	–6	–9	–12	–15

F.

x	0	1	2	3	4	5	6	7
y	3	2	1	0	–1	–2	–3	–4

G.

x	0	1	2	3	4	5	6	7
y	3	5	7	9	11	13	15	17

AN INVESTIGATION: PATTERNS IN DIFFERENCES

1. Copy and complete the tables for $y = x^2$, $z = x^2 + 1^2$, $w = x^2 + 2^2$ and $s = x^2 + 3^2$:

x	1	2	3	4	5	6	7	8	9	10
y	1	4	9	16	25	36	49	64	81	100
z	2	5	10	17	26	37	50	65	82	101
w	5	8	13	20	29	40	53	68	85	104
s	10	13	18	25	34	45	58	73	90	109

2. Copy and complete the tables for $y = x^2$, $p = (x + 1)^2$, $q = (x + 2)^2$ and $r = (x + 3)^2$:

(a)

x	1	2	3	4	5	6	7	8	9	10
p	4	9	16	25	36	49	64	81	100	121
y	1	4	9	16	25	36	49	64	81	100
$p - y$	3	5	7	9	11	13	15	17	19	21

(b)

x	1	2	3	4	5	6	7	8	9	10
q	9	16	25	36	49	64	81	100	121	144
y	1	4	9	16	25	36	49	64	81	100
$q - y$	8	12	16	20	24	28	32	36	40	44

(c)

x	1	2	3	4	5	6	7	8	9	10
r	16	25	36	49	64	81	100	121	144	169
y	1	4	9	16	25	36	49	64	81	100
$r - y$	15	21	27	33	39	45	51	57	63	69

CHAPTER 15: FUNCTIONS 163

Answers

3. The output values for the relationships z; w and s have already been calculated in question 1, and the values for p, q and r have been done in question 2.
4. (a) See the answers on LB page 164 alongside.
   (b) See the answers on LB page 164 alongside.
5. (a) See the answers on LB page 164 alongside.
   (b) 2, 4 and 6 respectively
   (c) Learners should be allowed to make their own conjectures.
   (d) See the answers on LB page 164 alongside.
6. (a) The expression $2x + 1$ has the same values as $(x + 1)^2 - x^2$ for all values of x. This can be explained by expanding the right-hand side:
   $(x + 1)^2 - x^2 = x^2 + 2x + 1 - x^2 = 2x + 1$ and similarly for the other cases.
   (b) $8x + 16$ and $10x + 25$

3. In each of the following cases, you should have different output values for the two relationships. If your output values are the same, find your mistakes and correct your work.
   (a) $z = x^2 + 1^2$ and $p = (x + 1)^2$
   (b) $w = x^2 + 2^2$ and $q = (x + 2)^2$
   (c) $s = x^2 + 3^2$ and $r = (x + 3)^2$
4. Copy and complete the tables, for $y = x^2$, $p = (x + 1)^2$, $q = (x + 2)^2$ and $r = (x + 3)^2$:

(a)

x	1	2	3	4	5	6	7	8	9	10
$p - y$	3	5	7	9	11	13	15	17	19	21
$q - y$	8	12	16	20	24	28	32	36	40	44
$r - y$	15	21	27	33	39	45	51	57	63	69

(b)

x	10	11	12	13	14	15	16	17
$p - y$	21	23	25	27	29	31	33	35
$q - y$	44	48	52	56	60	64	68	72
$r - y$	69	75	81	87	93	99	105	111

5. (a) Copy and complete the following table:

x	1	2	3	4	5	6	7	8	9	10
$2x + 1$	3	5	7	9	11	13	15	17	19	21
$4x + 4$	8	12	16	20	24	28	32	36	40	44
$6x + 9$	15	21	27	33	39	45	51	57	63	69

   (b) What are the constant differences in the sequences of values of $2x + 1$, $4x + 4$ and $6x + 9$, for $x = 1;\ 2;\ 3;\ 4 \ldots\ldots$?
   (c) Do you have an idea whether or not the corresponding sequence for $12x + 36$ will also have a constant difference and what the constant difference may be?
   (d) There are certain patterns in the coefficients and constant terms in the expressions in the table above. Copy the table below and continue the patterns to make some more similar expressions for your expressions.

x	1	2	3	4	5	6	7	8	9	10
$8x + 16$	24	32	40	48	56	64	72	80	88	96
$10x + 25$	35	45	55	65	75	85	95	105	115	125

6. (a) If your answers for the tables in 4(a) and 5(a) are correct, they will be the same. Try to explain why they are the same.
   (b) What expressions, similar to those in question 5(a), may have the same values as $(x + 4)^2 - x^2$ and $(x + 5)^2 - x^2$, respectively?

164 MATHEMATICS GRADE 9: TERM 3

Learner Book Overview		
Sections in this chapter	**Content**	**Pages in Learner Book**
16.1 Introduction	Writing a polynomial as a product of its factors; expanding a product; an identity formed by an expression and its factorised form	Pages 165 to 166
16.2 Factors of expressions of the form $ab + ac$	Finding the greatest common factor in an expression	Pages 166 to 168
16.3 Factors of expressions of the form $x^2 + (b + c)x + bc$	Terminology relating to quadratic trinomials; finding the factors of trinomials using the factors of the last term and the middle term, also using grouping of terms to factorise expressions	Pages 168 to 170
16.4 Factors of expressions of the form $a^2 - b^2$	Finding the factors of a difference of two squares $a^2 - b^2$ as $(a - b)(a + b)$; using this knowledge to simplify calculations with areas	Pages 170 to 172
16.5 Simplification of algebraic fractions	Simplifying algebraic fractions using factorisation; division by 0 is undefined; further practice in factorising expressions and simplifying fractions	Pages 172 to 175

CAPS time allocation	9 hours
CAPS content specification	Pages 142 to 143

Mathematical background

Expanding factors of an algebraic expression creates an identity where a single term, consisting of factors, is manipulated to a polynomial, for example: $(x + 3)(x + 4) = x^2 + 7x + 12$. It is an identity because the two expressions have the same value for all values of x. Changing from more than one term to a single term is called factorisation. Factorising expressions can be done according to the following structure:

1. Find and remove any common factors from all the terms. Once that has been done, inspect the bracket.
2. Look for a difference between two squares and factorise it. If there is not a difference between two squares, move to the next step.
3. If the content of the bracket is a quadratic trinomial, factorise it.
4. If none of the above steps apply, try grouping terms that have a common factor in order to create a common factor with which to start from step 1 again.

For example: $3x^2 - 6xy - 9y^2 = 3(x^2 - 2xy - 3y^2) = 3(x - 3y)(x + y)$ (steps 1 and 3)
or: $ax - 3by + bx - 3ay = ax + bx - 3ay - 3by = x(a + b) - 3y(a + b) = (a + b)(x - 3y)$ (steps 4 and 1)

To simplify algebraic fractions, we must factorise the denominator and the numerator and use the fact that the quotient of two equal factors = 1, keeping in mind values of the variable that could make the denominator 0, as division by 0 is not defined.

For example: $\frac{(x^2 - 4)}{x - 2} = \frac{(x - 2)(x + 2)}{x - 2} = x + 2$; $x \neq 2$, because we know that $\frac{x - 2}{x - 2} = 1$.

16.1 Introduction

MANIPULATING EXPRESSIONS

Teaching guidelines

In Chapter 8 we used the distributive property to multiply a polynomial by a monomial and a binomial by a binomial. That was called expansion of a product. In this chapter, we will do the reverse – i.e. we factorise an expression to end up with a product of factors.

We factorise expressions in order to make calculations easier. Whether factorising will make a calculation easier or not depends on what we have to do in the calculation.

The instruction to simplify can mean different things in different situations. Sometimes we have to expand an expression in order to simplify it and at other times we need to factorise an expression. For example:

1. Add $(x + 3)(x + 2)$ and $2x^2 - x + 1$. We cannot add unless we expand the first expression:
   $x^2 + 5x + 6 + 2x^2 - x + 1 = 3x^2 + 4x + 7$.
2. Simplify $\frac{x^2 - 9}{x + 3}$. We cannot simplify unless we first factorise the numerator:
   $\frac{x^2 - 9}{x + 3} = \frac{(x - 3)(x + 3)}{x + 3} = x - 3$.

An identity makes it possible for us to manipulate expressions for easier calculation without changing the value of the expression.

Misconceptions

Learners simplify fractions incorrectly, for example, in $\frac{x^2 - 9}{x + 3}$ they will divide x^2 by x and 9 by 3, in other words, divide terms by terms instead of dividing factors by factors.

Answers

1. Learners who do not simplify but get the correct answers are not wrong, but should be made aware that they did unnecessary work.
   (a) $x + 5 = 12 + 5 = 17$ (b) $x - 3 = 12 - 3 = 9$
   (c) $x = 12$ (d) $x + 5 = 12 + 5 = 17$
2. (a) $(x + 3)(x - 3) = x^2 - 9$ (b) $(x + 3)(x - 2) = x^2 + x - 6$
   (c) $(x + 3)(x + 1) = x^2 + 4x + 3$ (d) $x(x + 3) = x^2 + 3x$
3. See the answers on LB page 165 alongside.
4. (a) $x - 3$ (b) $x - 2$ (c) $x + 3$ (d) 1

CHAPTER 16
Algebraic expressions

16.1 Introduction

MANIPULATING EXPRESSIONS

The process of writing a polynomial as a product is called **factorisation**. This is the inverse of expansion.

$$x^2 + 5x + 6 \underset{\text{expansion}}{\overset{\text{factorisation}}{\rightleftharpoons}} (x + 2)(x + 3).$$

A numerical or algebraic expression that requires multiplication as a last step, is called a **product**. For example, $12(37 + 63)$, $2x(x - 5)$ and xyz are called products. A product is a monomial.

Each part of a product is called a **factor** of the expression. If $c = ab$, then a and b are factors of c. $x + 2$ and $x + 3$ are the factors of $(x + 2)(x + 3)$. Since $x^2 + 5x + 6 = (x + 2)(x + 3)$, $x + 2$ and $x + 3$ are the factors of $x^2 + 5x + 6$.

1. Calculate the value of each of the following expressions for $x = 12$:
   (a) $\frac{(x+2)(x+5)}{x+2}$ (b) $\frac{(x-3)(x-4)}{x-4}$
   (c) $\frac{x(2x+1)}{2x+1}$ (d) $\frac{(x+5)(x-5)}{x-5}$
2. Check if the following statements are identities by expanding the expressions on the right.

   A statement like $2(x + 3) = 2x + 6$, which is true for all values of x you can think of, is called an **identity**.

   (a) $x^2 - 9 = (x + 3)(x - 3)$ (b) $x^2 + x - 6 = (x + 3)(x - 2)$
   (c) $x^2 + 4x + 3 = (x + 3)(x + 1)$ (d) $x^2 + 3x = x(x + 3)$
3. Write down the factors of each of the following expressions:
   (a) $x^2 + x - 6$ $= (x + 3)(x - 2)$ (b) $x^2 + 3x$ $= x(x + 3)$
   (c) $x^2 + 4x + 3$ $= (x + 3)(x + 1)$ (d) $x^2 - 9$ $= (x + 3)(x - 3)$
4. Simplify the following quotients (algebraic fractions):
   (a) $\frac{x^2 - 9}{x + 3}$ (b) $\frac{x^2 + x - 6}{x + 3}$ (c) $\frac{x^2 + x - 6}{x - 2}$ (d) $\frac{x^2 + 4x + 3}{(x + 3)(x + 1)}$

Answers

5. (a) $x - 3$
   (b) Learners should be sure. Let them try both expressions to make sure.

16.2 Factors of expressions of the form $ab + ac$

THE GREATEST COMMON FACTOR

Teaching guidelines

Removing a common factor is the first step in factorising an expression.

Make sure that learners can factorise expressions where a negative sign is involved, for example:

- $2(a - b) - y(a - b) = (a - b)(2 - y)$
- $(a - b)x + (b - a)2y = (a - b)x - (a - b)2y = (a - b)(x - 2y)$ with the switch from $b - a$ to $-(a - b)$

Misconceptions

Learners tend to make mistakes when they factorise something like $6x^2 + 9x + 3$, they write $3(2x^2 + 3x)$ instead of $3(2x^2 + 3x + 1)$. A strategy to correct this is to let learners decide what the common factor is and then to divide each term by the common factor to see what will remain in the term from which they removed the common factor (i.e. take the common factor 3 then divide by 3 to see what remains: $\frac{6x^2}{3} + \frac{9x}{3} + \frac{3}{3} = 2x^2 + 3x + 1$).

Answers

1. (a) Yes, $4 \times 5 = 20$ (b) Yes, $6 \times 5 = 30$
   (c) Yes, $10 \times 5 = 50$ (d) Yes, $2 \times 5 = 10$
2. (a) Yes, $a \times b = ab$ (b) Yes, $a \times c = ac$
   (c) Yes, $a \times (b + c) = ab + ac$ (d) $b + c$
   (e) $\frac{ab + ac}{a} = b + c$ except for $a = 0$
3. See the answers on LB page 166 alongside.

5. (a) Suppose you have to find the value of the expression for $x = 15$.
   Which expression will be the least amount of work: $\frac{x^2 - 9}{x + 3}$ or $x - 3$?
   (b) Are you sure that you will get the same answers for the two expressions?

In the following sections you will learn how to factorise certain types of expressions. The following identities are useful for the purposes of factorisation:

$a(b + c) = ab + ac$ $(x + a)(x + b) = x^2 + (a + b)x + ab$ $(a + b)(a - b) = a^2 - b^2$

16.2 Factors of expressions of the form $ab + ac$

THE GREATEST COMMON FACTOR

1. (a) Is 5 a factor of 20? (b) Is 5 a factor of 30?
   (c) Is 5 a factor of 30 + 20? (d) Is 5 a factor of 30 – 20?
2. (a) Is a a factor of ab? (b) Is a a factor of ac?
   (c) Is a a factor of $ab + ac$? (d) Find another factor of $ab + ac$.
   (e) Now try and simplify: $\frac{ab + ac}{a}$.

Suppose you have to factorise $4x^3 + 2x^2 - 6x$: It is clear that $2x$ is a factor of every term, hence it is a factor of $4x^3 + 2x^2 - 6x$.

By division we get $\frac{4x^3 + 2x^2 - 6x}{2x} = 2x^2 + x - 3$. Hence $4x^3 + 2x^2 - 6x = 2x(2x^2 + x - 3)$.

It is always a good idea to check factorisation by expanding the answer and making sure that the result is equal to the original expression.

3. Copy and complete the following table:

(a) For each expression, find:	$3x + 6y$	$4a^3 + 2a$	$5x - 2x^2$	$ax^2 - bx^3$	$12a^2b + 18ab^2$
the factors of the first term	3; x	2; 2; a^3	5; x	a; x; x	2^2; 3; a^2; b
the factors of the second term	2; 3; y	2; a	2; x; x	b; x; x; x	2; 3^2; a; b^2
the greatest common factor of the two terms	3	$2a$	x	x^2	$6ab$
(b) Write the expression in factor form	$3(x + 2y)$	$2a(2a^2 + 1)$	$x(5 - 2x)$	$x^2(a - bx)$	$6ab(2a + 3b)$

4. Study the example and then factorise the expressions that follow.

$(a - b)x + (b - a)y = (a - b)x - (a - b)y$
$= (a - b)(x - y)$

Note that:
$b - a = -a + b = -(a - b)$

Answers

4. (a) $(a - b)x + (a - b)$
$= (a - b)(x + 1)$
(b) $(a - b)x - (a - b)$
$= (a - b)(x - 1)$
(c) $(a + b)(a + b) - (a + b)$
$= (a + b)(a + b - 1)$
(d) $(a + b)x - (a + b)$
$= (a + b)(x - 1)$
(e) $3x(2x - 3) + (2x - 3)$
$= (2x - 3)(3x + 1)$
(f) $y(y - 4) + 3(y - 4)$
$= (y - 4)(y + 3)$

SOMETHING IN BETWEEN

Background information

This is a method to factorise a quadratic trinomial. If we expand $x^2 + (b + c)x + bc$, we get $x^2 + bx + cx + bc$.

We can factorise this expression by grouping. We get: $x(x + b) + c(x + b) = (x + b)(x + c)$. So the coefficient of the middle term is the sum of the last terms in each bracket, while their product is the last term of the expanded form.

We can use this strategy when we want to factorise an expression such as $x^2 + 16x + 63$. We find the factors of 63 (the last term) that add up to 16 (the coefficient of the middle term).

Teaching guidelines

Point out that we find the sets of factors of the last term in a trinomial and choose the factors of which the sum gives the coefficient of the term in x (or the middle term).

Answers

1. See the answers on page LB 167 alongside.
2. (a) 3 and 10 (b) –2 and –15
(c) –5 and –6 (d) 5 and 6
3. (a) –15 and 2 (b) –10 and –3
(c) –2 and 15 (d) 5 and –6
(e) –5 and 6
4. (a) 3 and 12 (b) 20 and 2 (c) 18 and 2
(d) 20 and –2 (e) –18 and –2 (f) –20 and 2
5. (a) value is 0 for $x = 2$, $x^2 + 3x - 10$ (b) value is 28 for $x = 2$, $x^2 + 7x + 10$
(c) value is 0 for $x = 2$, $x^2 - 7x + 10$ (d) value is –12 for $x = 2$, $x^2 - 3x - 10$

(a) $(a - b)x + a - b$
(b) $(a - b)x - a + b$
(c) $(a + b)^2 - (a + b)$
(d) $(a + b)x - a - b$
(e) $3x(2x - 3) - (3 - 2x)$
(f) $(y^2 - 4y) + (3y - 12)$

SOMETHING IN BETWEEN

1. By copying and completing the tables below you will learn something that will help you to find the factors of expressions of the form $x^2 + (b + c)x + bc$, for example: $x^2 + 17x + 30$.

b	1	2	3	5	–1	–2	–3	–5
c	30	15	10	6	–30	–15	–10	–6
$b + c$	31	17	13	11	–31	–17	–13	–11
bc	30	30	30	30	30	30	30	30

b	–1	–2	–3	–5	1	2	3	5
c	30	15	10	6	–30	–15	–10	–6
$b + c$	29	13	7	1	–29	–13	–7	–1
bc	–30	–30	–30	–30	–30	–30	–30	–30

2. For each case below, find two numbers x and y so that their product xy is 30 and their sum $x + y$ is the given number:
(a) $xy = 30$ and $x + y = 13$
(b) $xy = 30$ and $x + y = -17$
(c) $xy = 30$ and $x + y = -11$
(d) $xy = 30$ and $x + y = 11$
3. Find x and y in each case:
(a) $xy = -30$ and $x + y = -13$
(b) $xy = 30$ and $x + y = -13$
(c) $xy = -30$ and $x + y = 13$
(d) $xy = -30$ and $x + y = -1$
(e) $xy = -30$ and $x + y = 1$
4. Find x and y in each case:
(a) $xy = 36$ and $x + y = 15$
(b) $xy = 40$ and $x + y = 22$
(c) $xy = 36$ and $x + y = 20$
(d) $xy = -40$ and $x + y = 18$
(e) $xy = 36$ and $x + y = -20$
(f) $xy = -40$ and $x + y = -18$
5. Evaluate each expression for $x = 2$. Also expand each expression.
(a) $(x + 5)(x - 2)$
(b) $(x + 5)(x + 2)$
(c) $(x - 5)(x - 2)$
(d) $(x - 5)(x + 2)$
6. Evaluate each polynomial you formed in question 5 for $x = 2$. Compare the answers with the values of the corresponding product expressions in question 1. In cases where the values differ, you have made a mistake somewhere. Sort out any mistakes before you continue with question 7.

Teaching guidelines

Show learners that the expansion can happen as follows:

$(x + 3)(x+ 8) = x^2 + 3x + 8x + 3 \times 8 = x^2 + (3 + 8)x + 3 \times 8.$

In reverse this makes it possible and easy to factorise an expression.

Misconceptions

When learners start using short methods too soon, they usually leave out the middle term when they expand a bracket, for example: $(x + 3)(x+ 8) = x^2 + 24$. Be on the lookout for this mistake.

Answers

7. See the answers on LB page 168 alongside.

16.3 Factors of expressions of the form $x^2 + (b + c)x + bc$

TRY TO FIND THE FACTORS

Teaching guidelines

Introduce the terminology learners have to use, for example: quadratic trinomial; what the different terms are called and how to recognise them as the middle term and the last term.

Show learners that factorising an expanded expression is the inverse of the expansion of the factors of the expression. See page LB 168 alongside.

Question 2 illustrates the effect that negative signs have on the answers. Let learners work through the exercise so that they can become aware of the influence of the negative sign.

Answers

1. (a) 6 (b) 9
   (c) 6 (d) $(x + 3)$
   (e) $x^2 - x - 6 = (x - 3)(x + 2)$

7. Expand each product:
   (a) $(x + 3)(x + 8)$ $= x^2 + 11x + 24$
   (b) $(x + 2)(x + 12)$ $= x^2 + 14x + 24$
   (c) $(x + 4)(x + 6)$ $= x^2 + 10x + 24$
   (d) $(x + 1)(x + 24)$ $= x^2 + 25x + 24$
   (e) $(x + 3)(x - 8)$ $= x^2 - 5x - 24$
   (f) $(x + 2)(x - 12)$ $= x^2 - 10x - 24$
   (g) $(x + 4)(x - 6)$ $= x^2 - 2x - 24$
   (h) $(x + 1)(x - 24)$ $= x^2 - 23x - 24$

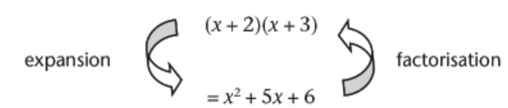
16.3 Factors of expressions of the form $x^2 + (b + c)x + bc$

The expanded form of a product of two linear binomials, such as $(x + 3)(x + 8)$, or $(x + 3)(x - 8)$, is a **quadratic trinomial**, such as $x^2 + 11x + 24$ or $x^2 - 5x - 24$ with:

- a term in x^2
- a term in x that is called the **middle term**, which is $+11x$ in $x^2 + 11x + 24$ and $-5x$ in $x^2 - 5x - 24$
- a constant term also called the **last term**, which is $+24$ in $x^2 + 11x + 24$, and -24 in $x^2 - 5x - 24$.

To factorise an expression, such as $x^2 + 5x + 6$, means to reverse the process of expansion. This means that we have to find out which binomials will produce the trinomial when the product of the binomials is expanded, for example:

$x^2 + 5x + 6 = (\mathbf{?} + \mathbf{?})(\mathbf{?} + \mathbf{?})$



TRY TO FIND THE FACTORS

1. Write the following out and fill in the missing parts of the factors in each case:
   (a) $(x + 3)(x$ $+ 6$ $) = x^2 + 9x + 18$
   (b) $(x + 2)(x$ $+ 9$ $) = x^2 + 11x + 18$
   (c) $(x + 3)(x -$ 6 $) = x^2 + 9x - 18$
   (d) (x + 3)$(x + 2) = x^2 + 5x + 6$
   (e) $x^2 - x - 6$

Answers

2. See the answers on LB page 169 alongside.
3. (a) $(x + 2)(x + 6)$ (b) $(x - 2)(x - 6)$

PRACTICE MAKES PERFECT

Teaching guidelines

Learners can practise the method learnt in the sections above to factorise these expressions.

Answers

1. (a) $(a + 7)(a + 2)$ (b) $(x - 3)(x + 6)$
   (c) $(x - 17)(x - 1)$ (d) $(y + 15)(y + 2)$
   (e) $(y - 15)(y + 2)$ (f) $(y + 10)(y - 3)$
   (g) $(x + 5)(x - 3)$ (h) $(m + 7)(m - 3)$
   (i) $(x - 3)(x - 3) = (x - 3)^2$ (j) $(b + 7)(b + 8)$
   (k) $(a - 9)(a + 7)$ (l) $(a + 5b)(a - 6b)$
   (m) $(x - 8y)(x + 3y)$ (n) $(x - 8)(x - 5)$

Teaching guidelines

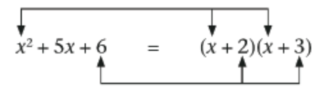
Grouping in order to create common factors is one of the methods of factorising. This method is usually used if:

- there is not a common factor that appears in each term in the expression
- the expression is not a quadratic trinomial
- the expression is not a difference between two squares.

This method can also be used to break up a trinomial into four terms by splitting the middle term, grouping and then removing the common factor from each group (see question 3).

2. Expand each product:
   (a) $(x + p)(x + q)$ $x^2 + px + qx + pq$ or $x^2 + (p + q)x + pq$
   (b) $(x + p)(x - q)$ $x^2 + px - qx - pq$ or $x^2 + (p - q)x - pq$
   (c) $(x - p)(x + q)$ $x^2 - px + qx - pq$ or $x^2 - (p - q)x - pq$
   (d) $(x - p)(x - q)$ $x^2 - px - qx + pq$ or $x^2 - (p + q)x + pq$

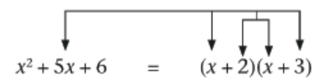
The product of the first terms of the factors must be equal to the x^2 term of the trinomial.



Meaning: $x \times x = x^2$

Meaning: $2 \times 3 = 6$

The product of the last terms of the factors must be equal to the last term (the constant term) of the trinomial. The sum of the inner and outer products must be equal to the term in x (the middle term) of the trinomial.



Meaning: $2x + 3x = (2 + 3)x = 5x$

$(x + a)(x + b) = x \times x + ax + bx + a \times b = x^2 + (a + b)x + ab$

3. Try to factorise the following trinomials:
   (a) $x^2 + 8x + 12$ (b) $x^2 - 8x + 12$

PRACTICE MAKES PERFECT

1. Factorise the following trinomials. (Remember to check your answer by expanding the factors to test if you do get the original expression.)
   (a) $a^2 + 9a + 14$ (b) $x^2 + 3x - 18$
   (c) $x^2 - 18x + 17$ (d) $y^2 + 17y + 30$
   (e) $y^2 - 13y - 30$ (f) $y^2 + 7y - 30$
   (g) $x^2 + 2x - 15$ (h) $m^2 + 4m - 21$
   (i) $x^2 - 6x + 9$ (j) $b^2 + 15b + 56$
   (k) $a^2 - 2a - 63$ (l) $a^2 - ab - 30b^2$
   (m) $x^2 - 5xy - 24y^2$ (n) $x^2 - 13x + 40$

An alternative method

2. Study the example and then factorise the expressions on page 170.

Answers

2. (a) $p(x + y) + q(x + y)$
$= (x + y)(p + q)$

(b) $9x^2(x - 3) + (x - 3)$
$= (x - 3)(9x^2 + 1)$

(c) $4(a + b) + 3p(a + b)$
$= (a + b)(4 + 3p)$

(d) $a^3(a + 1) + 3(a + 1)$
$= (a + 1)(a^3 + 3)$

(e) $x(y + 1) + (y + 1)$
$= (y + 1)(x + 1)$

(f) $a(c - d) - b(c - d)$
$= (c - d)(a - b)$

3. (a) $(x + 3)(x + 4)$ (b) $(x - 4)(x - 3)$

16.4 Factors of expressions of the form $a^2 - b^2$

PRELIMINARY WORK

Teaching guidelines

Learners develop the process of factorising the difference between two squares by completing the table of values. The table shows the expression of the difference between two squares (the expanded expression); the factors separately; and the product of its factors.

The numerical values should help learners to see the connections between the factors and the expanded expression: $a^2 - b^2 = (a + b)(a - b)$.

Answers

1. See the answers on LB page 170 alongside.

Example: Factorise $ac + bc + bd + ad$

$ac + bc + bd + ad = (ac + bc) + (bd + ad)$ (Order and group terms with common factors)
$= c(a + b) + d(b + a)$ (Take out the common factor)
$= (a + b)(c + d)$ (Write expression as a product)

(a) $px + py + qx + qy$
(b) $9x^3 - 27x^2 + x - 3$
(c) $4a + 4b + 3ap + 3bp$
(d) $a^4 + a^3 + 3a + 3$
(e) $xy + x + y + 1$
(f) $ac - ad - bc + bd$

Another method

Example 1:	Example 2:	Action:
$x^2 + 4x + 3$	$x^2 + 3x - 4$	
$= x^2 + x + 3x + 3$	$= x^2 - x + 4x - 4$	(Re-write middle term as sum of two terms)
$= (x^2 + x) + (3x + 3)$	$= (x^2 - x) + (4x - 4)$	(Group)
$= x(x + \mathbf{1}) + 3(x + \mathbf{1})$	$= x(x - \mathbf{1}) + 4(x - \mathbf{1})$	(Take out the GCF of each group)
$= (x + \mathbf{1})(x + 3)$	$= (x - \mathbf{1})(x + 4)$	(Write it as a product)

3. Factorise:
(a) $x^2 + 7x + 12$
(b) $x^2 - 7x + 12$

The challenge is to re-write the middle term as the sum of two terms in a way that you are able to take out the common factor.

16.4 Factors of expressions of the form $a^2 - b^2$

PRELIMINARY WORK

1. Copy and complete the following table and see if you can notice a pattern (rule) whereby you can predict the answers to the first column's calculations without squaring it:

(a)	$3^2 - 2^2$	$3 + 2$	$3 - 2$	$(3 + 2)(3 - 2)$
	5	5	1	5
(b)	$4^2 - 3^2$	$4 + 3$	$4 - 3$	$(4 + 3)(4 - 3)$
	7	7	1	7
(c)	$6^2 - 4^2$	$6 + 4$	$6 - 4$	$(6 + 4)(6 - 4)$
	20	10	2	20
(d)	$9^2 - 3^2$	$9 + 3$	$9 - 3$	$(9 + 3)(9 - 3)$
	72	12	6	72

Answers

2. The answers in column 1 are the same as the answers in column 4:
   $a^2 - b^2 = (a + b)(a - b)$
3. (a) $(17 + 13)(17 - 13) = 30 \times 4 = 120$
   (b) $(54 + 46)(54 - 46) = 100 \times 8 = 800$
   (c) $(28 + 22)(28 - 22) = 50 \times 6 = 300$
4. $a^2 - b^2 = (a + b)(a - b)$
5. The signs need to differ as it is actually:
   $(a + b)(a - b) = a^2 + ab - ab - b^2 = a^2 + 0ab - b^2 = a^2 - b^2$

FACTORISING DIFFERENCE BETWEEN TWO SQUARES' EXPRESSIONS

Teaching guidelines

When we factorise we should always keep the following in mind:

- remove any common factors first
- factorise completely
- 1 is a square, for example $16x^2 - 1 = (4x + 1)(4x - 1)$.

Answers

1. (a) $(2a + b)(2a - b)$ (b) $(m + 3n)(m - 3n)$
   (c) $(5x + 6y)(5x - 6y)$ (d) $(11x + 12y)(11x - 12y)$
   (e) $(4p + 7q)(4p - 7q)$ (f) $(8a + 5bc)(8a - 5bc)$
   (g) $(x + 2)(x - 2)$ (h) $(4x + 6y)(4x - 6y)$
2. (a) $(x^2 + 1)(x + 1)(x - 1)$ (b) $(4a^2 + 9)(2a + 3)(2a - 3)$
   (c) $(1 + abc)(1 - abc)$ (d) $(5x^5 + 7y^4)(5x^5 - 7y^4)$
   (e) $2(x^2 - 9) = 2(x + 3)(x - 3)$ (f) $2(100 - b^2) = 2(10 + b)(10 - b)$
   (g) $3x(y^2 - 16a^2) = 3x(y + 4a)(y - 4a)$ (h) $5(a^4 - 4b^2) = 5(a^2 + 2b)(a^2 - 2b)$

2. Do you notice a pattern (rule) whereby you can predict the answers to such calculations?
3. Now predict the answers to each of the following without squaring. Check your answers where necessary. Does the rule that you discovered in question 2 also hold for the following cases?
   (a) $17^2 - 13^2$ (b) $54^2 - 46^2$ (c) $28^2 - 22^2$
4. Formulate your rule in symbols:
   $a^2 - b^2 = (a + b)(a - b)$
5. Can you explain why factors of $a^2 - b^2$ have this form?

Stated differently: If p and q are perfect squares, also "algebraic squares", then:

$$p - q = (\sqrt{p} + \sqrt{q})(\sqrt{p} - \sqrt{q})$$

$$9x^4 - 4y^2 = (\sqrt{9x^4} + \sqrt{4y^2})(\sqrt{9x^4} - \sqrt{4y^2})$$

$$= (3x^2 + 2y)(3x^2 - 2y)$$

(Note the operations within the brackets differ.)

An expression of the form $a^2 - b^2$ is called the **difference between two squares**.
To factorise a difference between squares, we use the identity: $a^2 - b^2 = (a + b)(a - b)$ where a and b represent numbers or algebraic expressions.

FACTORISING DIFFERENCE BETWEEN TWO SQUARES EXPRESSIONS

1. Use the skills you learnt in the previous exercises to factorise the following:
   (a) $4a^2 - b^2$ (b) $m^2 - 9n^2$
   (c) $25x^2 - 36y^2$ (d) $121x^2 - 144y^2$
   (e) $16p^2 - 49q^2$ (f) $64a^2 - 25b^2c^2$
   (g) $x^2 - 4$ (h) $16x^2 - 36y^2$

Always factorise completely.
Always take out the greatest common factor if there is one.
One is a perfect square: $1 = 1^2$ and $1^m = 1$.
The exponential law: $a^m.a^n = a^{m+n}$.

2. Factorise:
   (a) $x^4 - 1$ (b) $16a^4 - 81$
   (c) $1 - a^2b^2c^2$ (d) $25x^{10} - 49y^8$
   (e) $2x^2 - 18$ (f) $200 - 2b^2$
   (g) $3xy^2 - 48xa^2$ (h) $5a^4 - 20b^2$

IN EACH CASE CALCULATE THE AREA OF THE SHADED PART

Teaching guidelines

Both drawings consist of a square within a square, so the area of the smaller square must be subtracted from the larger square. This produces a difference between two squares.

It is easier to add and subtract than to calculate squares and then subtract.

Answers

(a) $25^2 - 9^2 = (5 + 3)(5 - 3) = 8 \times 2 = 16$ square units

(b) $36^2 - 4^2 = (6 + 2)(6 - 2) = 8 \times 4 = 32$ square units

16.5 Simplification of algebraic fractions

WORKING WITH ALGEBRAIC FRACTIONS

Teaching guidelines

If we manipulate the expressions by factorising, we reduce the number of calculations and thereby the possibility of making mistakes.

We can simplify the expression $\frac{(x+2)(x+3)}{x+2}$ to $x + 3$ for example, because $\frac{(x+2)}{x+2} = 1$.

Misconceptions

Avoid using the term "cancel" as many learners interpret the term incorrectly. Some will say that in $\frac{(x+2)(x+3)}{x+2}$, "$x + 2$ falls away" or even that it is 0.

Furthermore, learners are often confused about the meaning of "cancel" and see $x - x$ as cancelling with an answer of 0, and then $x \div x$ as cancelling with an answer of 0 as well.

Answers

1. Madodo's solution is simple and does not require many calculations.

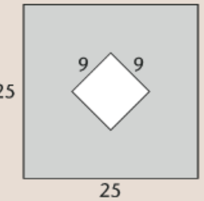
2. (a) $\frac{(x+2)(x+3)}{x+2}$
$= x + 3$
$= 23 + 3$
$= 26$

(b) $\frac{(x+2)(x+3)}{x+2} = 19$
$x + 3 = 19$
$x = 16$

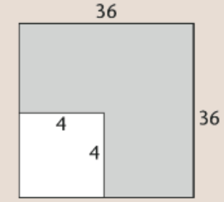
FACTORISATION CAN MAKE CALCULATION EASY

In each case, calculate the area of the shaded part. Use the shortest possible method.

(a)



(b)



THIS IS HOW FACTORISATION CAN MAKE CALCULATION EASY!

16.5 Simplification of algebraic fractions

WORKING WITH ALGEBRAIC FRACTIONS

Liza and Madodo have to determine the value of $\frac{x^2 - 2x - 3}{x - 3}$ for $x = 4{,}6$.

Liza's solution:	Madoda's solution:
$\frac{x^2 - 2x - 3}{x - 3}$ $= \frac{(4{,}6)^2 - 2(4{,}6) - 3}{4{,}6 - 3}$ (Substitute $x = 4{,}6$) $= \frac{21{,}16 - 9{,}2 - 3}{4{,}6 - 3}$ $= \frac{8{,}96}{1{,}6}$ $= 5{,}6$	$\frac{x^2 - 2x - 3}{x - 3}$ $= \frac{(x-3)(x+1)}{x-3}$ (Factorise the numerator) $= x + 1$ (Simplify the expression) $= 4{,}6 + 1$ (Substitute $x = 4{,}6$) $= 5{,}6$

1. Which solution do you prefer? Why?

It is useful to manipulate quotient expressions, such as $\frac{x^2 + 5x + 6}{x + 2}$, into simpler but equivalent sum expressions, like $x + 3$ in this case. It makes substitution and the solving of equations easier.

2. Solve the following problems:
(a) Evaluate $\frac{x^2 + 5x + 6}{x + 2}$ if $x = 23$.
(b) Solve $\frac{x^2 + 5x + 6}{x + 2} = 19$.

3. Determine the value of each of the expressions on page 173 if $x = 36$. See if you can use the shortest possible method.

Answers

3. (a) $\frac{(x+3)(x-3)}{x+3}$
$= x - 3$
$= 36 - 3 = 33$

(b) $\frac{(x+3)(x-2)}{x+3}$
$= x - 2$
$= 36 - 2 = 34$

HOW IS IT POSSIBLE THAT 2 = 1?

Answers

$a = b$, so in step 3 we already have $0 = 0$, as well as in step 4. In step 5 an attempt is made to divide by 0, which will not work as division by 0 is undefined.

DIVIDING BY ZERO CANNOT BE DONE

Teaching guidelines

Learners must be convinced that division by 0 is not possible as it is not defined. Let learners experience that there may be some values of the variable for which an expression is undefined. These values are called "excluded values".

Answers

1. See the answers on LB page 173 alongside.
2. It is undefined.
3. There is no such value.
4. "Math Error". Division by zero is not possible as noted in question 3.

DEFINING THE UNDEFINED

Teaching guidelines

Let learners work through the exercise and correct the statements where needed.

Answers

1. (a) It is not true for all values of x because if $x = 0$ then $\frac{x}{x}$ is undefined, corrected statement: $\frac{x}{x} = 1$ if $x \neq 0$.
(b) It is not true for all values of x because if $x = 0$ then $\frac{x^3}{x^2}$ is undefined, corrected statement: $\frac{x^3}{x^2} = x$ if $x \neq 0$.
(c) It is not true for all values of x because if $x = 3$ then $\frac{x-3}{x-3}$ is undefined, corrected statement: $\frac{x-3}{x-3} = 1$ if $x \neq 3$.

(a) $\frac{x^2-9}{x+3}$ (b) $\frac{x^2+x-6}{x+3}$

HOW IS IT POSSIBLE THAT 2 = 1?

What went wrong in the following argument?

Let:		$a = b$	(If: $b \neq 0$)
$\times a$:	⇔	$a^2 = ab$	
$-b^2$:	⇔	$a^2 - b^2 = ab - b^2$	
Factorise:	⇔	$(a+b)(a-b) = b(a-b)$	
$\div (a-b)$:	⇔	$a + b = b$	
But $a = b$:	⇔	$b + b = b$	
Add terms:	⇔	$2b = b$	
$\div b$:	⇔	$2 = 1$	

Explain what went wrong and why it is wrong?

DIVIDING BY ZERO CANNOT BE DONE

1. Copy and complete the following table by evaluating the value of the expression $\frac{x+2}{x-2}$ for the x-values given in the top row.

x	-2	0	2	4
$\frac{x+2}{x-2}$	$= \frac{-2+2}{-2-2}$ $= 0$	$= \frac{0+2}{0-2}$ $= -1$	$= \frac{2+2}{2-2}$ = Undefined	$= \frac{4+2}{4-2}$ $= 3$

2. If $x = 2$, then $\frac{x+2}{x-2}$ will have the value $\frac{4}{0}$. What is the value of $\frac{4}{0}$?
3. One way to determine the value of $\frac{4}{0}$, is to set it as $\frac{4}{0} = a$. Then $4 = 0 \times a$. Which values of a will make this statement true?
4. What is the result of the calculation of $4 \div 0$ on your calculator? Can you explain the message on your calculator?

Division by 0 is not possible. The algebraic fraction $\frac{x+2}{x-2}$ cannot have a value if the denominator $(x-2)$ is equal to 0. We may say the expression $\frac{x+2}{x-2}$ is **undefined** for $x - 2 = 0$, i.e. for $x = 2$. We also say $x = 2$ is an **excluded value** of x for $\frac{x+2}{x-2}$.

DEFINING THE UNDEFINED

1. Are the following statements true? If not, correct the statement.
(a) $\frac{x}{x} = 1$ for all values of x.

(d) If $x(x + 1) = 0$, $x = 0$ or -1. The given statement is not true for all values of x because if $x = 0$ or -1 then $\frac{x^2 + x}{x(x + 1)}$ is undefined, corrected statement: $\frac{x^2 + x}{x(x + 1)} = 1$ if $x \neq 0$ or -1.

2. See the answers on LB page 174 alongside.

SIMPLIFYING ALGEBRAIC FRACTIONS

Teaching guidelines

Remind learners that when we divide, we make use of the property of numbers that when a number is divided by itself the answer is 1.

We can only divide if we have factors in both the numerator and the denominator. Therefore, we have to factorise the numerator and, if needed, the denominator as well. For example: $\frac{x^2 - x - 6}{2x - 6} = \frac{(x + 2)(x - 3)}{2(x - 3)} = \frac{(x + 2)}{2}$, where $\frac{x - 3}{x - 3} = 1$.

Misconceptions

Learners do not factorise and try to simplify by dividing terms by terms.

Answers

1. (a) $\frac{y(3x + y)}{3x + y} = y$ (If $3x + y \neq 0$) (b) $\frac{ab(a + b)}{a + b} = ab$ (If $a + b \neq 0$)
   (c) $\frac{3xy(x - 2xy)}{3xy} = x - 2xy$ (If $3xy \neq 0$) (d) $\frac{5x^2(2x^2 + 3x)}{5x^2} = 2x^2 + 3x$ (If $5x^2 \neq 0 \therefore x \neq 0$)
2. (a) $\frac{(x + 2)(x + 3)}{x + 2} = x + 3$ (If $x \neq -2$) (b) $\frac{(x - 2)(x + 4)}{x - 2} = x + 4$ (If $x \neq 2$)
   (c) $\frac{(x + 5)(x - 10)}{x + 5} = x - 10$ (If $x \neq -5$) (d) $\frac{(x - 15)(x - 1)}{x - 15} = x - 1$ (If $x \neq 15$)
3. (a) $\frac{(x - 2)(x + 2)}{x - 2} = x + 2$ (If $x \neq 2$) (b) $\frac{(2x - 1)(2x + 1)}{2x + 1} = 2x - 1$ (If $x \neq -\frac{1}{2}$)

FACTORISATION CAN REDUCE CALCULATIONS

(a) Area $= \pi 22^2 - \pi 18^2 = \pi(22^2 - 18^2) = 3{,}142(22 - 18)(22 + 18)$
$= 3{,}142 \times 4 \times 40 = 502{,}72$ square units

(b) $a^2 = 61^2 - 11^2 = (61 - 11)(61 + 11) = 50 \times 72 = 2 \times 25 \times 2 \times 36 = 2^2 \times 5^2 \times 6^2$;
$a = \sqrt{2^2 \times 5^2 \times 6^2} = 2 \times 5 \times 6 = 60$ OR $a^2 = 50 \times 72 = 3\,600 = 36 \times 100$;
$a = \sqrt{3\,600} = 60$

(b) $\frac{x^3}{x^2} = x$ for all values of x.

(c) $\frac{x - 3}{x - 3} = 1$ for all values of x.

(d) $\frac{x^2 + x}{x(x + 1)} = 1$ for all values of x.

2. For which values of the variables will each expression be undefined?
   (a) $\frac{7(y + 5)}{y + 2}$ Undefined for $y + 2 = 0$, i.e. for $y = -2$
   (b) $\frac{3x + 2}{x + 4}$ Undefined for $x + 4 = 0$, i.e. for $x = -4$
   (c) $\frac{2x + 1}{x^2 - 1}$ Undefined for $x^2 - 1 = 0$, i.e. for $x = 1$ and -1
   (d) $\frac{2x^2 - 1}{(x - 2)(x + 3)}$ Undefined for $x = 2$ and $x = -3$

SIMPLIFYING ALGEBRAIC FRACTIONS

To simplify an algebraic fraction that contains a polynomial as a numerator or denominator, the polynomial should be factorised first.

To prevent division by zero, the excluded values must be stated.

1. Simplify each of the following algebraic fractions by factorising the numerator and then using the property $\frac{ax}{a} = x$ if $a \neq 0$. Give the excluded values.
   (a) $\frac{3xy + y^2}{3x + y}$ (b) $\frac{a^2b + ab^2}{a + b}$
   (c) $\frac{3x^2y - 6x^2y^2}{3xy}$ (d) $\frac{10x^4 + 15x^3}{5x^2}$
2. Simplify each of the following algebraic fractions by factorising the numerator and then using the property $\frac{ax}{a} = x$ if $a \neq 0$. (See if you can factorise the trinomials.)
   (a) $\frac{x^2 + 5x + 6}{x + 2}$ (b) $\frac{x^2 + 2x - 8}{x - 2}$
   (c) $\frac{x^2 - 5x - 50}{x + 5}$ (d) $\frac{x^2 - 16x + 15}{x - 15}$
3. Simplify each of the following algebraic fractions by factorising the numerator and then using the property $\frac{ax}{a} = x$ if $a \neq 0$:
   (a) $\frac{x^2 - 4}{x - 2}$ (b) $\frac{4x^2 - 1}{2x + 1}$

174 MATHEMATICS GRADE 9: TERM 3

MORE PRACTICE

Teaching guidelines

Learners need to understand the structure for the process of factorising:

- first look for common factors; if there are none, or if they have been removed, then
- check whether the expression is a difference between two squares, or if not
- check whether the expression is a quadratic trinomial, or if not
- check whether you can group in order to make a common factor and start from the first point again.

Answers

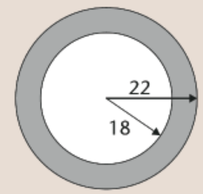
1. (a) $2(2a+3b)$ (b) $(x+7)(x+1)$
   (c) $(c+3)(c-3)$ (d) $(y-5)(y-3)$
   (e) $b(-3a+1)$ or $-b(3a-1)$ (f) $(b-1)(-3a+1)$
   (g) $dg(fg+d-f^2)$ (h) $(x+4)(x+2)$
   (i) $(a+3)(a+2)$ (j) $(x-10)(x+2)$
   (k) $x^3y^3(x^2-y^2)=x^3y^3(x+y)(x-y)$ (l) $xy(x^2-y^2)=xy(x+y)(x-y)$
   (m) $(2-y)(2-y)$ (n) $3(a^2+6a-7)=3(a+7)(a-1)$
   (o) $6(a^2-9)=6(a+3)(a-3)$ (p) $-(a^2+11a+30)=-(a+5)(a+6)$
   (q) $2(a^2+5a-36)=2(a+9)(a-4)$ (r) $5x(x^2-3x-40)=5x(x-8)(x+5)$
   (s) $(x+2+y)(x+2-y)$ (t) $(x+y+a)(x+y-a)$
   (u) $(a-1)^2-b^2=(a-1+b)(a-1-b)$ (v) $1-(a-b)^2=(1+a-b)(1-a+b)$
   (w) $(a-b)x-(a-b)y=(a-b)(x-y)$ (x) $a(2x-y)-(2x-y)=(2x-y)(a-1)$
   (y) $2x^2y^2(y^8-4x^8)$
   $=2x^2y^2(y^4+2x^4)(y^4-2x^4)$
   (z) $(a+b)[(a+b)^2-4]$
   $=(a+b)(a+b+2)(a+b-2)$
   (aa) $(a+b)(a+b)-(a+b)$
   $=(a+b)(a+b-1)$
   (ab) $(x+y)(a-b)-(x+y)(b-a)$
   $=(x+y)[a-b-(b-a)]=(x+y)(2a-2b)$
   $=2(x+y)(a-b)$

2. (a) $\frac{(4+3x)(4-3x)}{4+3x}=4-3x$ (If $x\neq-\frac{4}{3}$) (b) $\frac{(5x+6)(5x-6)}{x(5x+6)}=\frac{5x-6}{x}$ (If $x\neq-\frac{6}{5}$)
   (c) $\frac{x(x+6)(x-5)}{x+6}=x^2-5x$ (If $x\neq-6$) (d) $\frac{(2x+3)(x+1)}{2x+3}=x+1$ (If $x\neq-\frac{3}{2}$)
   (e) $\frac{b(a+c)}{abc}=\frac{a+c}{ac}$ (If $b\neq0$) (f) $\frac{p(a+b)}{a+b}=p$ (If $a+b\neq0$)

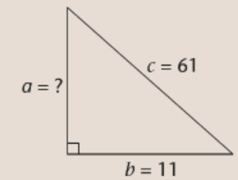
FACTORISATION CAN REDUCE CALCULATIONS

In each case, use the shortest possible method to get to your answer.

(a) Calculate the shaded area.
(Area = πr^2 and use $\pi = 3.142$)



(b) Calculate the length of side a.
(Pythagoras: $c^2=a^2+b^2$)



THIS IS HOW FACTORISATION CAN SAVE YOU TIME!

MORE PRACTICE

1. Factorise the following expressions completely:
   (a) $4a+6b$ (b) x^2+8x+7
   (c) c^2-9 (d) $y^2-8y+15$
   (e) $-3ab+b$ (f) $-3a(b-1)+(b-1)$
   (g) $dfg^2+d^2g-df^2g$ (h) x^2+6x+8
   (i) a^2+5a+6 (j) $x^2-8x-20$
   (k) $x^5y^3-x^3y^5$ (l) x^3y-xy^3
   (m) $4-4y+y^2$ (n) $3a^2+18a-21$
   (o) $6a^2-54$ (p) $-a^2-11a-30$
   (q) $2a^2+10a-72$ (r) $5x^3-15x^2-200x$
   (s) $(x+2)^2-y^2$ (t) $(x+y)^2-a^2$
   (u) $(a^2-2a+1)-b^2$ (v) $1-(a^2-2ab+b^2)$
   (w) $(a-b)x+(b-a)y$ (x) $a(2x-y)+(y-2x)$
   (y) $2x^2y^{10}-8x^{10}y^2$ (z) $(a+b)^3-4(a+b)$
   (aa) $(a+b)^2-a-b$ (ab) $(x+y)(a-b)+(-x-y)(b-a)$
2. Simplify each of the following algebraic fractions as far as possible:
   (a) $\frac{16-9x^2}{4+3x}$ (b) $\frac{25x^2-36}{5x^2+6x}$
   (c) $\frac{x^3+x^2-30x}{x+6}$ (d) $\frac{2x^2+5x+3}{2x+3}$
   (e) $\frac{ab+bc}{abc}$ (f) $\frac{pa+pb}{a+b}$

CHAPTER 16: ALGEBRAIC EXPRESSIONS 175

Learner Book Overview		
Sections in this chapter	**Content**	**Pages in Learner Book**
17.1 Introduction	Finding solutions by inspection; defining an equation, an identity and an impossibility; understanding how inverse operations are used to solve equations by creating a set of equivalent equations	Pages 176 to 178
17.2 Solving by factorisation (Part 1)	Defining the zero-product property; solving equations by removing common factors; understanding factorisation as the inverse of expansion of products	Pages 178 to 180
17.3 Solving by factorisation (Part 2)	Solving equations by factorising trinomials; solving equations by factorising the difference between two squares	Pages 180 to 181
17.4 Solving by factorisation (Part 3)	Solving equations by using properties of exponents	Pages 181 to 182
17.5 Set up equations to solve problems	Understanding how the modelling process works	Pages 182 to 184
17.6 Equations and ordered pairs	Generating input and output numbers and writing them as a set of ordered pairs; solving the equation obtained by equating the formulae of two functions to find the ordered pair that satisfies both	Pages 184 to 186

CAPS time allocation	9 hours
CAPS content specification	Page 144

Mathematical background

Solving an equation means to find the value of the unknown that makes the equation true.

The number of different solutions to an equation is not more than the degree of the equation, where the degree is the highest power of the unknown.

The **zero product property** means that if a product equals 0, at least one of the factors (or all of them) are 0.

To solve a **quadratic trinomial** we need to have a factorised quadratic expression equal to zero. We then use the zero product property to solve linear equations, for example $x^2 + 5x + 6 = 0$; $(x + 2)(x + 3) = 0$, then $x + 2 = 0$, so $x = -2$, or $x + 3 = 0$, so $x = -3$.

A **quadratic equation** can also be a difference between two squares. The procedure is the same as for a trinomial. For example:

$$4x^2 - 25 = 0;\ (2x + 5)(2x - 5) = 0;\ \text{so } 2x + 5 = 0 \text{ or } 2x - 5 = 0;\ x = -\frac{5}{2} \text{ or } \frac{5}{2}.$$

Setting up equations to describe situations is called **mathematical modelling**. A situation is described in mathematical terms. The mathematical problem is solved and the solution is interpreted and tested to see if it fits the original situation.

A **function** is a set of ordered pairs of numbers. The input value is the first element and the output value is the second element of the ordered pair. When we set the formulae of two functions equal, we can solve the equation and find the ordered pair that belongs to both functions.

17.1 Introduction

SOLUTION BY INSPECTION

Teaching guidelines

An equation is a mathematical statement with two expressions, one on the left-hand side (LHS) and one on the right-hand (RHS) side of an equal sign, for example $7x - 4 = 4x + 11$.

Some statements are true for only certain values of the unknown (refer to top of LB page 177). They are called equations. There are various ways to solve an equation.

Some statements, called identities, are true for all the values of the unknown. When learners try to solve an identity they usually end up with something like $0 = 0$.

For some statements there are no values for which they are true. They are called impossible.

We can use a table of values to solve an equation by inspection. Point out that we test a potential answer by substituting it first into the LHS and then into the RHS. If the two answers are equal, the value is a solution to the equation.

Answers

1. See the answers on LB page 176 alongside.
2. See the answers on LB page 176 alongside.

CHAPTER 17
Equations

17.1 Introduction

SOLUTION BY INSPECTION

1. Copy and complete the following table. Substitute the given x-values into the equation until you find the value that makes the equation true.

You can read the solutions of an equation from a table.

	Equation	LHS if $x = 4$	Is LHS = RHS ?	LHS if $x = 5$	Is LHS = RHS ?	LHS if $x = 6$	Is LHS = RHS ?	Correct solution
(a)	$3x - 4 = 11$	8	No	11	Yes	14	No	$x = 5$
(b)	$2x + 7 = 19$	15	No	17	No	19	Yes	$x = 6$
(c)	$13 - 5x = -7$	–7	Yes	–12	No	–17	No	$x = 4$

(LHS = left-hand side and RHS = right-hand side)

2. In the following table, you are given equations with their solutions. Copy the table and insert **+** or **–** or **=** signs between each term to make the equations true for the solution given.

The "searching" for the solution of an equation is referred to as solving the equation by **inspection**.

	Equation	Solution	
(a)	$2x + 7 = 15$	$x = 4$	$2x + 7 = 15$
(b)	$3 - 2x = 11$	$x = -4$	$3 - 2x = 11$
(c)	$-x + 7 = 3$	$x = 4$	$-x + 7 = 3$
(d)	$28 - 5x = 3$	$x = 5$	$28 - 5x = 3$

Statements like $21 - x = 2x + 3$ and $(x - 3)(x - 5) = 0$, which are true for only some values of x, are called **equations**.

SOLVING EQUATIONS THROUGH INVERSE OPERATIONS

Teaching guidelines

A second method of solving equations is by using inverse operations.

Learners have already worked with equivalent equations. Remind learners that a set of equations is equivalent if every equation in the set has the same solution. When we build up a set of equations, we create a set of equivalent equations.

Using inverse operations to solve an equation in turn also creates a set of equivalent equations until we reach the solution, for example, $x = c$.

Explain what we mean by the degree of an equation. It is the highest power of the unknown that appears in the equation, for example $x^3 + 5 = 13$ is an equation of the third degree, while $x^2 + 5x + 6 = 0$ is an expression of the second degree, and $2x + 9 = 14$ is an equation of the first degree.

Even if an expression is written as a product of factors, as in $(x + 2)(x - 1)(x + 1)$, it is an expression of the third degree, because the highest power of x we will get will be x^3.

Misconceptions

Learners do not understand that the inverse operation has to be performed on both sides of the equal side and either do it only on one side or do opposite operations on the two sides, for example subtract 2 on the LHS and add 2 on the RHS.

Answers

1. (a) 0 (b) 0 (c) 0 (d) 0
2. Using the inverse operation for addition or subtraction produces zero as the answer.
3. (a) 1 (b) 1 (c) 1 (d) 1
4. Using the inverse operation for multiplication or division produces 1 as the answer.

A statement like $2(x + 3) = 2x + 6$, which is true for all values of x you can think of, is called an **identity**.

A statement like $2(x + 3) = 2x + 3$, where there are no values of x for which it is true, is called an **impossibility**.

SOLVING EQUATIONS THROUGH INVERSE OPERATIONS

In this section you are going to explore a different way of solving equations.

1. Complete the following calculations:
   (a) $3 - 3$ (b) $-9\ 765 + 9\ 765$
   (c) $-a + a$ (d) $13a - 13a$
2. What do you notice?
3. Complete the following calculations:
   (a) $3 \div 3$ (b) $3 \times \frac{1}{3}$
   (c) $\frac{1}{x} \times x$ (d) $\frac{x}{3} \times \frac{3}{x}$
4. What do you notice?

We can start with a solution as an equation and then apply some operations to it to turn it into an equivalent but more complicated equation.

Two equations are **equivalent** if they have the same solution.

Building an equation

Action on both sides	Equivalent equations
Solution (1)	$x = 3$
$\times 3$	$3x = 9$
$+ 2$	$3x + 2 = 11$
Solution (2)	$x = -9$
$\times 2$	$2x = -18$
$+ 6$	$2x + 6 = -12$
$+ x$	$3x + 6 = x - 12$
factorise	$3(x + 2) = x - 12$

Solving an equation

Action on both sides	Equivalent equations
Equation (1)	$3x + 2 = 11$
$- 2$	$3x = 9$
$\div 3$	$x = 3$
Equation (2)	$3(x + 2) = x - 12$
remove brackets	$3x + 6 = x - 12$
$- x$	$2x + 6 = -12$
$- 6$	$2x = -18$
$\div 2$	$x = -9$

Answers

5. (a) $2x + 8 + 9 = 15$
$2x + 17 = 15$
$2x = -2$
$x = -1$

(b) $5x - 10 = 14 - 7x$
$12x = 24$
$x = 2$

(c) $2x - (2 \times 3) = (12 \times 3)$
$2x - 6 = 36$
$2x = 42$
$x = 21$

(d) $3(3y - 3) + 3(5) = 2(5y)$
$9y - 9 + 15 = 10y$
$-y = -6$
$y = 6$

17.2 Solving by factorisation (Part 1)

DEVELOPING A STRATEGY: MULTIPLYING BY ZERO

Teaching guidelines

The principle by which it is possible to solve a quadratic equation is that if a product of two numbers equals 0, then one or both of the numbers must be 0. This is the zero-product property.

Some quadratic expressions are given as a product of two factors equal to 0, which means we can use the zero-product property.

For example, $(x - 3)(x + 2) = 0$
Use the zero-product property: $(x - 3) = 0$ or $(x + 2) = 0$
Solve the two simple equations: $x = 3$ or $x = -2$

Or $5x(x - 7) = 0$
$x = 0$ or $x - 7 = 0$
$x = 7$

Misconceptions

The learners sometimes incorrectly apply the zero-product property to general equations like $a \times b = 6$ by arguing that if $a \times b = 6$ then the only solutions are $a = 6$ or $b = 6$. They do not consider other possibilities.

Answers

1. One or both the factors must be zero. $x = 0$ OR $y = 0$; or $x = 0$ AND $y = 0$.

Building an equation

Action on both sides	Equivalent equations
Solution (3)	$x = 1$
$\times -1$	$-x = -1$
$+3$	$-x + 3 = 2$
$+2x$	$+x + 3 = 2 + 2x$
$\div 2$	$\frac{(x+3)}{2} = 1 + x$

Solving an equation

Action on both sides	Equivalent equations
Equation (3)	$\frac{(x+3)}{2} = 1 + x$
$\times 2$	$x + 3 = 2 + 2x$
$-2x$	$-x + 3 = 2$
-3	$-x = -1$
$\div -1$	$x = 1$

Try making up your own equations and then solving them. Did you get the "solution" with which you started?

When you solve an equation, you actually reverse the making of the equation.

5. Solve for x:
   (a) $2(x + 4) + 9 = 15$
   (b) $5(x - 2) = 7(2 - x)$
   (c) $\frac{2x}{3} - 2 = 12$
   (d) $\frac{3y-3}{2} + \frac{5}{2} = \frac{5y}{3}$

Up to now you have only dealt with equations of the **first degree**. That means they contained only *first powers* of the unknown (x), for example $3x - 2 = 5x + 7$. In the following sections you will solve equations of the **second degree**, where the expressions contain second powers. This is an equation of the second degree:

$x^2 + 1 = x + 13$.

When the expression part of the equation is written as the product of a monomial and a binomial (e.g. $x(x - 2) = 0$); or the product of two binomials (e.g. $(x - 2)(x + 3) = 0$), the result is also an equation of the second degree.

17.2 Solving by factorisation (Part 1)

DEVELOPING A STRATEGY: MULTIPLYING BY ZERO

1. Can you find two numbers x and y so that if you multiply them the answer is 0, i.e. $xy = 0$?

Each part of a product is called a **factor** of the expression.
If $c = ab$, then a and b are factors of c.
If $x^2 + 5x + 6 = (x + 2)(x + 3)$, then $x + 2$ and $x + 3$ are factors of $x^2 + 5x + 6$.

Answers

2. See the answers on LB page 179 alongside.

TAKING OUT THE HIGHEST COMMON FACTOR

Teaching guidelines

In the previous chapter learners practised removing a common factor.

Remind learners how to look for a common factor and to remove the highest possible common factor from all the terms. This is a way to simplify an expression which makes it possible to solve equations of the type $x^2 - 4x = 0$.

Factorise (common factor x): $x(x - 4) = 0$
Use the zero-product property: $x = 0$ or $x - 4 = 0$, so $x = 4$

Misconceptions

Be aware that if learners have to solve for x in $x^2 = 5x$, some learners will divide by x on either side or get $x = 5$. Show learners that this is not allowed, because one of the values of $x = 0$ (that they have eliminated), and division by 0 is undefined. They have to change the equation to get it equal to 0.

So
$$x^2 = 5x$$
$$x^2 - 5x = 0$$
$$x(x - 5) = 0 \quad \text{Factorise (common factor)}$$
$$x = 0 \text{ or } x = 5 \quad \text{Zero-product property}$$

Answers (to questions at the top of LB page 180 on the following page)

1. $x^2 + 3x = 0$
$x(x + 3) = 0$
$x = 0$ or $x + 3 = 0$
$x = 0$ or $x = -3$

2. $3x^2 - 6x = 0$
$3x(x - 2) = 0$
$3x = 0$ or $x - 2 = 0$
$x = 0$ or $x = 2$

3. $6x + 3x = -12x^2$
$12x^2 + 9x = 0$
$3x(4x + 3) = 0$
$3x = 0$ or $4x + 3 = 0$
$x = 0$ or $x = -\frac{3}{4}$

4. $x = 2x - x^2$
$x^2 - x = 0$
$x(x - 1) = 0$
$x = 0$ or $x - 1 = 0$
$x = 0$ or $x = 1$

2. Copy and complete the following table:

	Equation	Factors	Product	First possible solution	Second possible solution
Example	$x(x-2)=0$	x and $(x-2)$	0	$x=0$	$x-2=0$ $x=2$
(a)	$x(x+5)=0$	x and $(x+5)$	0	$x=0$	$x+5=0$ $x=-5$
(b)	$2x(3x-12)=0$	$2x$ and $(3x-12)$	0	$2x=0$ $x=0$	$3x-12=0$ $x=4$
(c)	$0=(x+2)(x-2)$	$(x+2)$ and $(x-2)$	0	$x+2=0$ $x=-2$	$x-2=0$ $x=2$

You can rewrite an equation so that it is in the form *expression* = 0; for example you can write $x^2 - 2x = 3x + 6$ as $x^2 - 5x - 6 = 0$.

You can factorise $x^2 - 5x - 6$ and then use the zero-product property to solve the equation, as shown below:
$x^2 - 5x - 6 = 0$
$(x - 6)(x + 1) = 0$
$x = 6$ or $x = -1$

Zero-product property
If: $a \times b = 0$
Then: $a = 0$ or
$b = 0$ or
$a = 0$ and $b = 0$

In a later section you will solve equations like the above example. You have to write the equation in the form, *expression* = 0, then factorise the left-hand side and then use the zero-product property.

TAKING OUT THE HIGHEST COMMON FACTOR

The process of writing a sum expression (polynomial) as a product (monomial) is called **factorisation**. This is the inverse of **expansion**.

Look at the expression $2x^2 - 6x$.

$2x$ is a factor of both terms, therefore it is a factor of $2x^2 - 6x$.

By division we get $\frac{2x^2 - 6x}{2x} = x - 3$.

Hence $2x^2 - 6x = 2x(x - 3)$.

It is unnecessary to write out the division step of this method. After finding the common factor, we write down the product form directly:
$2x^2 - 6x = 2x(x - 3)$

CHAPTER 17: EQUATIONS 179

17.3 Solving by factorisation (Part 2)

SOLVING BY FACTORISING TRINOMIALS

Teaching guidelines

The fact that we can usually factorise a quadratic expression to get a product of two factors means we can use the zero-product property. But this means that the RHS (or the LHS) of the equation must be 0.

There may be some manipulation needed to get the equation in to that form.

For example,	$(x - 3)(x + 2) = 6$
Expand the brackets on the LHS:	$x^2 - x - 6 = 6$
Add – 6 to both sides:	$x^2 - x - 12 = 0$
Factorise:	$(x - 4)(x + 3) = 0$
Now use the zero product property:	$(x - 4) = 0$ or $(x + 3) = 0$
Solve the two simple equations:	$x = 4$ or $x = -3$

Misconceptions

Learners forget to make sure that the quadratic expression equals 0 and simply ignore the number on the RHS.

Answers

1. $x^2 + 9x + 14 = 0$
   $(x + 7)(x + 2) = 0$
   $x + 7 = 0$ or $x + 2 = 0$
   $x = -7$ or $x = -2$
2. $x^2 + 3x - 18 = 0$
   $(x - 3)(x + 6) = 0$
   $x - 3 = 0$ or $x + 6 = 0$
   $x = 3$ or $x = -6$
3. $x^2 - 18x + 17 = 0$
   $(x - 17)(x - 1) = 0$
   $x - 17 = 0$ or $x - 1 = 0$
   $x = 17$ or $x = 1$
4. $x^2 - 11x + 30 = 0$
   $(x - 6)(x - 5) = 0$
   $x - 6 = 0$ or $x - 5 = 0$
   $x = 6$ or $x = 5$
5. $x^2 - 13x - 30 = 0$
   $(x + 2)(x - 15) = 0$
   $x + 2 = 0$ or $x - 15 = 0$
   $x = -2$ or $x = 15$
6. $x^2 + 7x - 30 = 0$
   $(x - 3)(x + 10) = 0$
   $x - 3 = 0$ or $x + 10 = 0$
   $x = 3$ or $x = -10$

SOLVING BY FACTORISING THE DIFFERENCE BETWEEN TWO SQUARES

Teaching guidelines

The pattern for factorising the difference of two squares is $a^2 - b^2 = (a - b)(a + b)$.

Determine the values of x which will make the following statements true:

1. $x^2 = -3x$
2. $x^2 + 2x^2 = 6x$
3. $\frac{6x}{3} + x = -4x^2$
4. $x = x(2 - x)$

17.3 Solving by factorisation (Part 2)

SOLVING BY FACTORISING TRINOMIALS

The product of the first terms of the factors must be equal to the x^2 term of the trinomial. The product of the last terms of the factors must be equal to the last term (the constant term) of the trinomial.

$x^2 + 5x + 6 = (x + 2)(x + 3)$ Meaning: $x.x = x^2$

Meaning: $2.3 = 6$

The sum of the inner and outer products must be equal to the x term of the trinomial.

$x^2 + 5x + 6 = (x + 2)(x + 3)$ Meaning: $(2 + 3)x = 5x$

The factors are of the form: $(x.x) + (a + b)x + (a.b) = (x + a)(x + b)$.

Determine the values of x which will make the following statements true.

Remember to write the equation in the form *expression* = 0 so that you can use the zero-product property.

1. $x^2 + 9x = -14$
2. $x^2 + 3x = 18$
3. $x^2 - 18x = -17$
4. $x^2 + 30 = 11x$
5. $x^2 = 13x + 30$
6. $x^2 + 7x = 30$

SOLVING BY FACTORISING THE DIFFERENCE BETWEEN TWO SQUARES

Remember from the previous chapter:
If p and q are perfect squares, also "algebraic squares", then:

$p - q = (\sqrt{p} + \sqrt{q})(\sqrt{p} - \sqrt{q})$

$9x^4 - 4y^2 = (\sqrt{9x^4} + \sqrt{4y^2})(\sqrt{9x^4} - \sqrt{4y^2})$
$= (3x^2 + 2y)(3x^2 - 2y)$

An expression of the form $a^2 - b^2$ is called the **difference between two squares**.
To factorise a difference between squares, we use the identity: $a^2 - b^2 = (a + b)(a - b)$ where a and b represent numbers or algebraic expressions.

Determine the values of the unknown (x or a or n, etc.) which will make the statements on the next page true.

Answers

1. $x^2 - 4 = 0$
$(x+2)(x-2) = 0$
$x+2=0$ or $x-2=0$
$x=-2$ or $x=2$

2. $x^2 - 16 = 0$
$(x+4)(x-4) = 0$
$x+4=0$ or $x-4=0$
$x=-4$ or $x=4$

3. $4a^2 - 9 = 0$
$(2a+3)(2a-3) = 0$
$2a+3=0$ or $2a-3=0$
$a=-\frac{3}{2}$ or $a=\frac{3}{2}$

4. $81 - 9n^2 = 0$
$(9+3n)(9-3n) = 0$
$9+3n=0$ or $9-3n=0$
$n=-3$ or $n=3$

5. $25x^2 - 36 = 0$
$(5x+6)(5x-6) = 0$
$5x+6=0$ or $5x-6=0$
$x=-\frac{6}{5}$ or $x=\frac{6}{5}$

6. $121x^2 - 144 = 0$
$(11x+12)(11x-12) = 0$
$11x+12=0$ or $11x-12=0$
$x=-\frac{12}{11}$ or $x=\frac{12}{11}$

7. $16p^2 - 49 = 0$
$(4p+7)(4p-7) = 0$
$4p+7=0$ or $4p-7=0$
$p=-\frac{7}{4}$ or $p=\frac{7}{4}$

8. $64a^2 - 25 = 0$
$(8a+5)(8a-5) = 0$
$8a+5=0$ or $8a-5=0$
$a=-\frac{5}{8}$ or $a=\frac{5}{8}$

17.4 Solving by factorisation (Part 3)

SOLVING BY USING PROPERTIES OF EXPONENTS

Teaching guidelines

Both sides of the equation have to be written as powers of the same base, to which end the number not written as a power is written as a product of its prime factors.

Answers

1. (a) 2^7 (b) 3^5 (c) 5^3 (d) 7^4
2. (a) $x = 7$ (b) $x = 5$ (c) $x = 3$ (d) $x = 4$
3. (a) $2^x = 2^7$, $x = 7$ (b) $3^x = 3^5$, $x = 5$ (c) $5^x = 5^3$, $x = 3$ (d) $7^x = 7^4$, $x = 4$
(e) $2^x = 16$, $2^x = 2^4$, $x = 4$ (f) $3^x = \frac{1}{9} = 3^{-2}$, $3^x = 3^{-2}$, $x = -2$

Remember to write the equation in the form *expression* = 0 so that you can use the zero-product property.

1. $x^2 = 4$
2. $x^2 = 16$
3. $4a^2 = 9$
4. $81 = 9n^2$
5. $25x^2 = 36$
6. $121x^2 = 144$
7. $16p^2 = 49$
8. $64a^2 = 25$

17.4 Solving by factorisation (Part 3)

SOLVING BY USING PROPERTIES OF EXPONENTS

1. Write the following numbers as the product of their prime factors:
(a) 128 (b) 243
(c) 125 (d) 2 401

All numbers can be written as the product of their prime factors:
16 = 4 × 4 = 2 × 2 × 2 × 2 = $\mathbf{2^4}$
Factorise the number until all the factors are prime numbers.

2. Determine the values of x which will make the following statements true:
(a) $2^x = 2^7$ (b) $3^x = 3^5$
(c) $5^x = 5^3$ (d) $7^x = 7^4$

If the base of the LHS is the same as the base of the RHS, then the exponent on the LHS must be equal to the exponent on the RHS.
If $a^x = a^y$, then $x = y$.

3. Determine the values of x which will make the following statements true:
(a) $2^x = 128$ (b) $3^x = 243$
(c) $5^x = 125$ (d) $7^x = 2\,401$
(e) $2^x + 9 = 25$ (f) $27(3^x) = 3$

In the equation $2^x = 16$, the letter symbol (x) is the exponent. Equations with the letter symbol as an exponent are referred to as **exponential equations**.

MIXED EXERCISES FOR MORE PRACTICE

Determine the values of the unknown (x or m or b, etc.) which will make the following statements true:

1. $\frac{6x}{3} + x = -4x^2$
2. $x = x(2 - x)$
3. $x^2 + 2x = 15$
4. $m^2 + 4m = 21$
5. $x^2 + 3 = 4x$
6. $b^2 - 16b = -15$

MIXED EXERCISES FOR MORE PRACTICE

Teaching guidelines

Remind learners of the following when complete the questions: removing a common factor; factorising a quadratic equation; using the zero product property; and the properties of exponents.

Answers (to questions starting from LB page 181 on the previous page)

1. $6x + 3x = -12x^2$
   $12x^2 + 9x = 0$
   $3x(4x + 3) = 0$
   $3x = 0$ or $4x + 3 = 0$
   $x = 0$ or $x = -\frac{3}{4}$

2. $x = 2x - x^2$
   $x^2 - x = 0$
   $x(x - 1) = 0$
   $x = 0$ or $x - 1 = 0$
   $x = 0$ or $x = 1$

3. $x^2 + 2x - 15 = 0$
   $(x - 3)(x + 5) = 0$
   $x - 3 = 0$ or $x + 5 = 0$
   $x = 3$ or $x = -5$

4. $m^2 + 4m - 21 = 0$
   $(m + 7)(m - 3) = 0$
   $m + 7 = 0$ or $m - 3 = 0$
   $m = -7$ or $m = 3$

5. $x^2 - 4x + 3 = 0$
   $(x - 3)(x - 1) = 0$
   $x - 3 = 0$ or $x - 1 = 0$
   $x = 3$ or $x = 1$

6. $b^2 - 16b + 15 = 0$
   $(b - 15)(b - 1) = 0$
   $b - 15 = 0$ or $b - 1 = 0$
   $b = 15$ or $b = 1$

7. $a^2 - 1 = 0$
   $(a + 1)(a - 1) = 0$
   $a + 1 = 0$ or $a - 1 = 0$
   $a = -1$ or $a = 1$

8. $25x^2 - 7^2 = 0$
   $(5x + 7)(5x - 7) = 0$
   $5x + 7 = 0$ or $5x - 7 = 0$
   $x = -\frac{7}{5}$ or $x = \frac{7}{5}$

9. $2^x = 16$
   $2^x = 2^4$
   $x = 4$

10. $3^x = \frac{1}{27}$
    $3^x = 3^{-3}$
    $x = -3$

17.5 Set up equations to solve problems

THE MATHEMATICAL MODELLING PROCESS

Teaching guidelines

Work through the notes on LB page 182 and explain the steps in the process. Explain that we work with numbers, so we solve problems that can be quantified. We describe the quantities we want to calculate in terms of unknowns, for example x.

7. $1 = a^2$
8. $25x^2 = 49$
9. $2^x - 25 = -9$
10. $81(3^x) = 3$

17.5 Set up equations to solve problems

THE MATHEMATICAL MODELLING PROCESS

Consider this problem involving a practical situation:
Printing shop A charges 45c per page and R12 for binding a book.
Printing shop B charges 35c per page and R15 for binding a book.
For a book with the same amount of pages, will the two shops charge the same?

You can write an equation to describe the problem.

Let the number of pages for which the work costs the same be x. Then:

$45x + 1\,200 = 35x + 1\,500$.

> The equation represents a mathematical problem that can be solved without necessarily keeping the practical situation in mind. It is called a **mathematical model** of the practical situation.

Now solve the equation.

$45x + 1\,200 = 35x + 1\,500$
$45x - 35x = 1\,500 - 1\,200$
$10x = 300$
$x = 30$

> We describe this as **analysing** the mathematical model to produce a **mathematical solution**.

We may now ask what the solution to the mathematical problem ("$x = 30$") means in terms of the practical situation. When the equation was set up above, the symbol x was used as a placeholder for the number of pages in a book for which the two shops would charge the same. So, what does the solution tell you?

> The mathematical solution may be interpreted to establish what it means in the practical situation.

Now check to see if the two shops will charge the same for a book with 30 pages.
At shop A, 30 pages will cost 30 × 45c = 1 350c = R13,50. Binding is R12; total cost is R25,50.
At shop B, 30 pages will cost 30 × 35c = 1 050c = R10,50. Binding is R15; total cost is R25,50.

The solution to the mathematical problem is also a solution to the practical problem.

> The mathematical solution should be tested in the practical situation, because mistakes may have been made.

PRACTISE YOUR MODELLING SKILLS

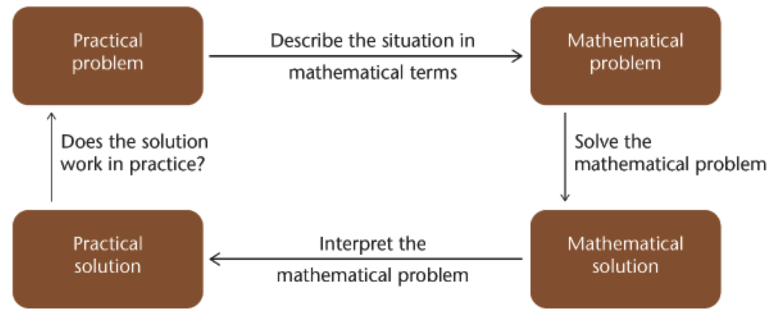
Teaching guidelines

Learners should decide what the unknown quantity is and consign a variable to it, for example: let the age be x, or let the number of days be x.

Remind learners to always make an interpretation.

Answers

1. See the answer on LB page 183 alongside.
2. See the answer on LB page 183 alongside.



When people work like this, we say they do **mathematical modelling**.

PRACTISE YOUR MODELLING SKILLS

For each situation in questions 1 to 3, the mathematical model is outlined and some clues are provided. Fill in the missing information.

1. Louis is six years older than Karin and Karin is four years older than Heidi. The sum of their ages is 53 years. How old is Heidi?

Model:	Let x be:	Heidi's age
	Then:	Karin's age will be $x + 4$
	And:	Louis' age will be $(x + 4) + 6 = x + 10$
	Hence:	$x + (x + 4) + (x + 10) = 53$
Analysis:	$x + (x + 4) + (x + 10) = 53$	
	$3x + 14 = 53$ $3x = 39$ $x = 13$	
Interpretation:	So Heidi is: 13 years old	

2. The sum of two numbers is 15. Three times the smaller number is 5 more than the larger number. Calculate the two numbers. (**Hint:** Let the smaller number be x.)

Model:	Let x be:	The smaller number
	Then:	$3x - 5$ is the larger number
	Hence:	$x + (3x - 5) = 15$
Analysis:	$x + (3x - 5) = 15$, hence $4x = 20$ and $x = 5$	
Interpretation:	So the smaller number is: 5	
	And the larger number is: $3x - 5 = 3(5) - 5 = 10$	

CHAPTER 17: EQUATIONS 183

Answers

3. See the answers on LB page 184 alongside.
4. (a) Cost = 500 + 30t
   Cost = 500 + 30(10) if t = 10
   Cost = 800
   Firm A will charge R800 for a job that takes ten days.
   (b) Cost = 260 + 48t
   596 = 260 + 48t if Cost = 596
   336 = 48t
   7 = t
   If Firm B charges R596, then the job will take them seven days.
   (c) If the jobs cost the same:
   Cost (A) = Cost (B)
   Hence: 260 + 48t = 500 + 30t
   18t = 240
   $t = 13\frac{1}{3}$
   This job will take $13\frac{1}{3}$ days to complete.

17.6 Equations and ordered pairs

WHEN UNKNOWNS BECOME VARIABLES

Teaching guidelines

If the value of the expression (output number) is also expressed as a variable, y, it means we could choose values for y, for example 27, and solve the equation 27 = 5x + 2.

For each value of x, we can generate a value for y. The sets of values are called ordered pairs meaning there is a first value, the input value, and a second value, the output value.

Answers

1. Two letter symbols: x and y.
2. No, there are two variables; you need the value of the one to get the other value.
3. See the answers on LB page 184 alongside.

3. The sum of three consecutive even numbers is 108. What are the numbers?
   Hint: Consecutive numbers are numbers that follow on from each other.
   We define an even number as a number of the form 2n where n is a counting number.
   Model: Let the first number be: 2n
   Then: The next two numbers will be: 2n + 2 and 2n + 4
   Hence: 2n + (2n + 2) + (2n + 4) = 108
   Analysis: 2n + (2n + 2) + (2n + 4) = 108,
   hence 6n = 102 n = 17
   Interpretation: So the first number is: 2n → 2(17) = 34
   The second number is: 2n + 2 → 2(17) + 2 = 36
   And the third number is: 2n + 4 → 2(17) + 4 = 38

4. Firm A calculates the cost of a job using the formula: Cost = 500 + 30t, where t is the number of days it takes to complete the job.
   Firm B calculates the cost of the same job using the formula: Cost = 260 + 48t, where t is the number of days needed to complete the job.
   (a) What would Firm A charge for a job that takes ten days?
   (b) How long would Firm B take to complete a job for which their charge is R596?
   (c) Here is a specific job for which firms charge the same and take the same time to complete. How long does this job take?

17.6 Equations and ordered pairs

WHEN UNKNOWNS BECOME VARIABLES

In the previous sections we dealt with equations which had fixed or limited solutions. They only had one letter symbol, which in this case acted as a placeholder for the value/s which will make the statement true.

Study the equation: $y = 5x + 2$.

1. How many letter symbols does the equation have? (List them.)
2. Is it possible to solve this "equation"?
3. Copy and complete the following table:

x	12	10	20	5	6	–5	–10
5x + 2	62	52	102	27	32	–23	–48

FUNCTIONS AS SETS OF ORDERED PAIRS

Teaching guidelines

We already know that a function can be represented in different forms.

A function is the set of all the ordered pairs that satisfy the description of the function. In the case of a relationship that is a function, there is only one unique y value for every x value.

The ordered pairs represent all the input values and their corresponding output values given either in a table

x	1	2	3	4	...
y	2	4	6	8	...

or as a set of brackets, for example: (1; 2), (2; 4), (3; 6), ... and so on.

Answers

1. (a) See the answers on LB page 185 alongside and learners' own answers.
   (b) See the answers on LB page 185 alongside and learners' own answers.
   (c) See the answers on LB page 185 alongside and learners' own answers.

FUNCTIONS AS SETS OF ORDERED PAIRS

A specific input number, for example 10, and the output number associated with it (52 in the case of the function described by $y = 5x + 2$), is called an **ordered pair**. Ordered pairs can be represented in a table like the one you completed in question 3 on the previous page.

Ordered pairs can also be written in brackets: (input number; output number).

For example, the ordered pairs you entered into the table in question 3 can be written as: (12; 62), (10; 52), (20; 102), (5; 27), (6; 32), (–5; –23), (–10; –48)

In the function indicated by $y = 5x + 2$, the letter symbol in the formula (x) represents the **input** or **independent** variable while the other letter symbol (y) represents the **output** or **dependent** variable.

If there is precisely one value of y for each value of x, we say that y is a **function** of x.

1. Copy and complete each table by writing the ordered pairs in brackets below the table, in the table, as shown in the example. Then choose two more input numbers and write down two additional ordered pairs that belong to each given function.

   For the function with the rule $y = 4x + 5$:

x	–2	0	1	2	5	10	20
y	–3	5	9	13	25	45	85

(–2; –3), (0; 5), (1; 9), (2; 13), (5; 25), and (10; 45) and (20; 85)

(a) For the function with the rule $y = x^2 + 9$:

x	5	3	0	–3	–5
y	34	18	9	18	34

(5;34), (3; 18), (0; 9), (–3; 18), (–5; 34), and (. . ; . . .), and (. . ; . . .)

(b) For the function with the rule $y = 3x – 2$:

x	5	1	0	–3	–5
y	13	1	–2	–11	–17

(5; 13), (1; 1), (0; –2), (–3; –11), (–5; –17), and (. . . ; . . .) and (. . ; . . .)

(c) For the function with the rule $y = 5x – 4$:

x	–5	–3	1	2	5
y	–29	–19	1	6	21

(–5; –29), (–3; –19), (1; 1), (2; 6), (5; 21), and (. . . ; . . .) and (. . ; . . .)

Teaching guidelines

Explain that in order to answer question 3, learners can assume that the y values are the same, and therefore, the expressions that gave these y values will also be equal. Point out that we need to solve the equation we get by setting the expressions equal. For example, the ordered pair that belong to $y = 3x - 2$ and to $y = 5x - 4$ will be calculated as follows:

$$5x - 4 = 3x - 2$$
$$2x = 2$$
$$x = 1, \text{ therefore } y = 3(1) - 2 = 1$$

So the ordered pair is (1; 1).

Answers

1. (d) See the answers on LB page 186 alongside and learners' own answers.
   (e) See the answers on LB page 186 alongside and learners' own answers.
   (f) See the answers on LB page 186 alongside and learners' own answers.
2. (a) (1; 1)
   (b) (2; 6)
3. If $5x + 7 = 3x + 25$, then $2x = 18$ and $x = 9$.
   For $x = 9$, $5x + 7 = 3x + 25 = 52$. The pair (9; 52) belongs to both functions.

(d) For the function with the rule $y = 12 - 3x$:

x	1	2	3	4	5
y	9	6	3	0	–3

(1; 9), (2; 6), (3; 3), (4; 0), (5; –3), and (. . . ; . . .) and (. . ; . . .)

(e) For the function with the rule $y = x^2 + 2$:

x	–12	–7	–2	3	10
y	146	51	6	11	102

(–12; 146), (–7; 51), (–2; 6), (3; 11), (10; 102), and (. . . ; . . .) and (. . ; . . .)

(f) For the function with the rule $y = 2^x + 2$:

x	0	1	2	3	4
y	3	4	6	10	18

(0; 3), (1; 4), (2; 6), (3; 10), (4; 18) and (. . . ; . . .) and (. . ; . . .)

2. (a) Which ordered pair belongs to both $y = 3x - 2$ and $y = 5x - 4$?
   (b) Which ordered pair belongs to both $y = 12 - 3x$ and $y = 5x - 4$?
3. Which ordered pair belongs to both $y = 5x + 7$ and $y = 3x + 25$?

186 MATHEMATICS GRADE 9: TERM 3

Grade 9 Term 3 Chapter 18 Graphs

Learner Book Overview		
Sections in this chapter	**Content**	**Pages in Learner Book**
18.1 Global graphs	Discussing discrete and continuous variables; how increase and decrease is shown on graphs	Pages 187 to 194
18.2 Changes at different rates	Discussing how some graphs decrease or increase slower than others, depending on the rate of change	Pages 195 to 197
18.3 Draw graphs from tables of ordered pairs	Making tables to generate ordered pairs to plot on graphs; drawing graphs of functions with constant differences	Pages 197 to 199
18.4 Gradient	Developing the concept of gradient; determining the gradient	Pages 199 to 203
18.5 Finding the formula for a graph	Using tables to find formulae; the gradient and y-intercept; finding the equation of a straight line from given points and from sketches of graphs	Pages 203 to 207
18.6 x- and y-intercepts	Coordinates of intercepts with axes; equations of vertical and horizontal lines	Pages 207 to 208
18.7 Graphs of non-linear functions	Identifying non-linear relationships by making tables of values and comparing with given plotted points	Pages 208 to 209

CAPS time allocation	12 hours
CAPS content specification	Page 145

Mathematical background

When discrete data is plotted the points are not joined, but sometimes joining the points can give greater clarity to the situation.

The rate of change of a function is shown on the graph by the shape of the graph. A line indicates a constant rate of change.

We can make tables of ordered pairs from formulae and plot the ordered pairs on graph paper to represent the function or relationship graphically.

Graphs of linear functions have constant differences between the function values. These differences give the gradient of the graph, which is a measure of the steepness with which the graph increases or decreases, as well as a measure of the rate of change of the graph.

The gradient is calculated between any two points on the line and is given by $m = \frac{y_2 - y_1}{x_2 - x_1}$ where the two points are $(x_1; y_1)$ and $(x_2; y_2)$.

The equation of a line is given by $y = mx + c$, where c represents the y-intercept of the line.

Lines parallel to the y-axis have equation $x = k$ and lines parallel to the x-axis have equations $y = p$.

Lines with a positive value of m are increasing and lines with a negative value of m are decreasing.

18.1 Global graphs

DISCRETE AND CONTINUOUS VARIABLES

Teaching guidelines

Let learners explain what they understand by continuous variables and discrete variables.

Make sure learners understand that continuous variables are usually those variables that can be measured while discrete variables can be counted.

Point out that when discrete data is plotted the points are not joined, for example in the graphs of the number of workers against the number of jars filled. The number of workers can only be shown by isolated points, as we cannot have half a person. We don't usually join the points by a line, but in some instances it is useful to draw a line between the points to make a trend clear.

Answers

1. (a) 12 workers
   (b) 30 jars
   (c) He would need five workers, because one cannot have four and a $\frac{1}{2}$ people.
   (d) The number of workers employed and the number of jars filled in a week.
2. Yes we can use the graph: 9 workers and 12 workers respectively.

Chapter 18 Graphs

18.1 Global graphs

DISCRETE AND CONTINUOUS VARIABLES

Sibongile collects honey on his farm and puts it in large jars to sell. His business is doing so well that he can no longer do all the work himself. He needs to get some help. Sibongile knows that one person can normally fill two jars in three days. He sets up this table to help him determine how many full-time workers he should employ to fill different numbers of jars in a five-day week.

Jars per week	$3\frac{1}{3}$	$6\frac{2}{3}$	10	$13\frac{1}{3}$	$16\frac{2}{3}$	20	$23\frac{1}{3}$
Workers	1	2	3	4	5	6	7

1. (a) If Sibongile needs to produce 40 jars a week, how many workers does he need?
   (b) How many jars can nine workers fill in a week?
   (c) How many workers does Sibongile need to produce 15 jars per week?
   (d) What are the two variables in the above situation?

In a situation like the one above, one can have any number of jars, as well as fractions of a jar. One can have a whole number of jars (for example, four jars) or a fractional quantity of jars (for example, $6\frac{2}{3}$ or 4,45 jars). The other variable in the above situation, the number of full-time employees, is different. Only whole numbers of people are possible.

Quantities like the quantity of jars of honey, which can include any fraction, are sometimes called "continuous quantities" or "continuous variables". Quantities that can be counted, like a number of people or a number of motor cars or rivers or towns, are sometimes called "discrete quantities" or "discrete variables".

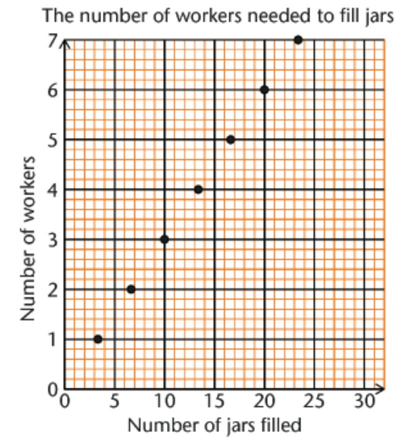
When a graph of a discrete variable is drawn, it does not normally make sense to join the dots with a line, but for some purposes it may be useful.

2. Can you use the second graph on the next page to find out how many workers are needed to fill 30 jars in a week, and how many to fill 40 jars? Check your answers by doing calculations.

Answers

3. See the answer on LB page 188 alongside.
4. (a) discrete
   (b) continuous
   (c) continuous

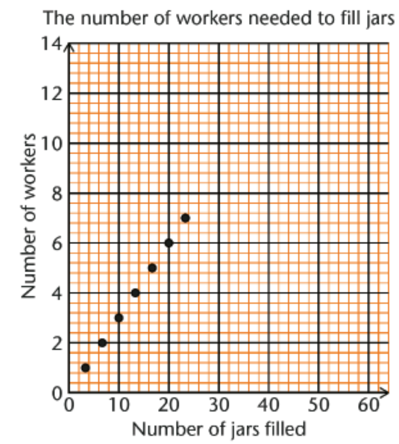
Here is a graph with the information in Sibongile's table:



Here is another graph with the same information:

3. In what way are these two graphs different?

   The width of one small square does not represent the same number of workers and jars in the lower graph and the upper graph. The scales of the two graphs are different.

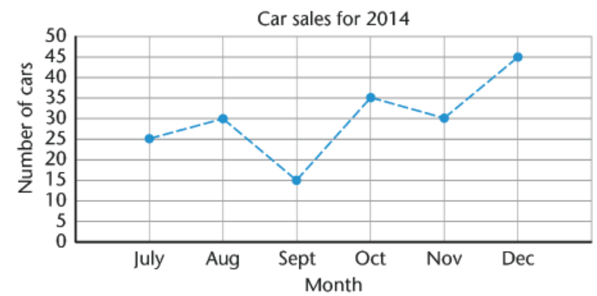


4. In each case, say whether the variables are "discrete" or "continuous".
   (a) You order pizzas for a class party and you need one pizza for every three learners.
   (b) Your height measured at different stages as you grew up.
   (c) The speed the car is travelling as you drive to town.

Answers

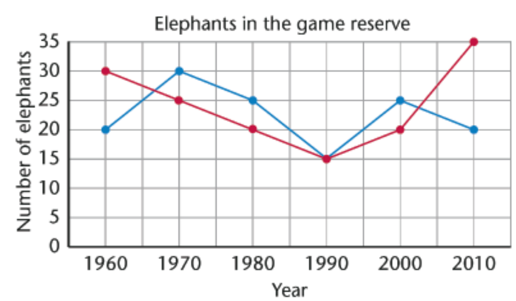
5. (a) It is discrete. Cars and months are both natural numbers.
   (b) 30
   (c) maximum: December; minimum: September
   (d) Five cars
   (e) During September and during November
   (f) Yes. The trend is that the car sales increased.
6. (a) It decreased
   (b) Between 1960 and 1970; 1990 and 2000

5. The following line graph shows the number of cars that a company sold between July and December of 2014:



   (a) Is the data shown in the graph discrete or continuous? Explain your answer.
   (b) How many cars were sold in August?
   (c) During which months were the maximum and minimum number of cars sold?
   (d) How many more cars were sold in November than in July?
   (e) During which months did the car sales decrease?
   (f) Would you say that the car sales generally improved over the six months? Explain your answer.

6. The graph below shows the population of elephants at a game reserve in South Africa between 1960 and 2010. Study the graph and answer the questions that follow.



   (a) Did the elephant population increase or decrease between 1970 and 1990?
   (b) Between which years did the elephant population increase?

Answers

6. (c) 1970
   (d) It is discrete. The number of elephants and years are natural numbers.
   (e) 20 elephants
   (f) See the graph on LB page 189 on the previous page.
   (g) No. They only had more elephants before about 1967 and after about 2002. The rest of the time they had fewer.

SHOWING INCREASE AND DECREASE ON GRAPHS

Teaching guidelines

Explain that the independent variable is always shown on the horizontal axis and the dependent variable on the vertical axis. Time is usually the independent variable and is shown on the horizontal axis.

Explain what it means if a graph increases (or decreases). Draw a system of axes on the board to help learners understand. We look at what happens to the values of the dependent variable as the independent variable increases. On the graph, the independent variable increases from left to right.

- If the dependent value becomes smaller as the independent value becomes bigger, the graph will be decreasing.
- If the dependent value becomes bigger as the independent value becomes bigger, the graph will be increasing.

Misconceptions

Learners often cannot relate the concepts "increase" and "decrease" to graphs correctly, even though they know the meaning of the words.

Answers

1. (a) The temperatures shown on the graph would be in summer rather than in winter in the Free State.
   (b) The highest temperature was 23°C recorded at 15:00.
   (c) The temperature rose between 01:00 and 15:00, and dropped between 15:00 and 24:00.
   (d) Between 13:00 and 15:00

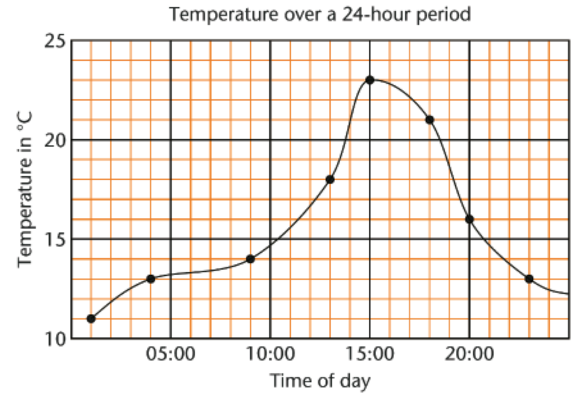
(c) In which year were there the most elephants on the game farm?
(d) Is the data in this graph discrete or continuous?
(e) How many elephants do you think there were on the game reserve in 1995?
(f) The following data shows the number of elephants at a different game reserve. Copy the graph from page 189 on grid paper and plot this information on the grid:

Year	1960	1970	1980	1990	2000	2010
Elephants	30	25	20	15	20	35

(g) Would you say that the second game reserve had more elephants than the first game reserve between 1960 and 2010? Explain your answer.

SHOWING INCREASE AND DECREASE ON GRAPHS

The graph below shows the temperature over a 24-hour period in a town in the Free State. The graph was drawn by connecting the points that show actual temperature readings.



1. (a) Do you think the above temperatures were recorded on a summer day or a winter day?
   (b) At what time of the day was the highest temperature recorded, and what was this temperature?
   (c) During what part of the day did the temperature rise, and during what part did the temperature drop?
   (d) During what part of the period when the temperature was rising, did it rise most rapidly?
   (e) During what part of the day did the temperature drop most rapidly?

(e) Between 18:00 and 20:00

The main purpose of the above and the following questions in this section is to develop an awareness of rate of change (gradient), and how the rate of change is visible in the graph.

2. See answers on LB page 191 alongside.

2. Here are descriptions of the temperature changes on five different days:

Day A: It is already warm early in the morning. The temperature does not change much during the day but late in the afternoon a breeze causes the temperature to drop quite sharply.

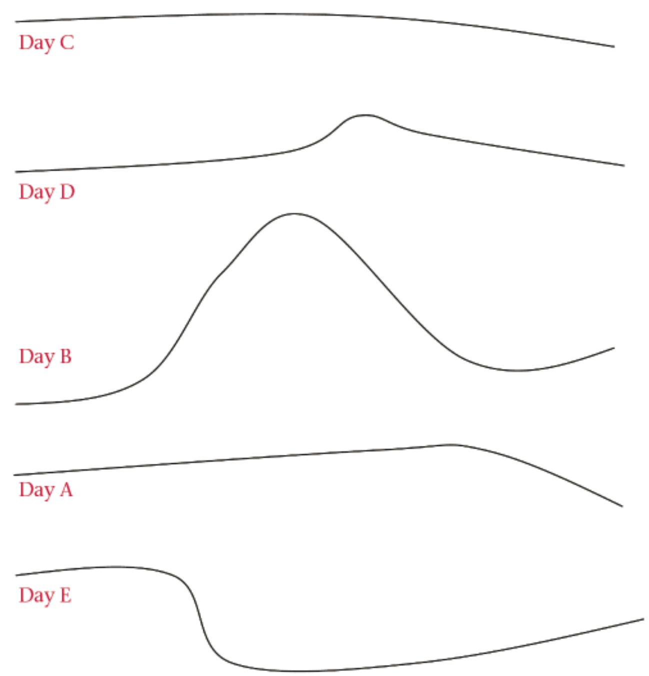
Day B: It is very cold early in the morning but it gets quite hot soon after the sun rises. By midday a cold wind comes up and the temperature drops until late in the afternoon. The wind then stops and it gets warmer again into the evening.

Day C: It is warm in the early morning and the temperature remains about the same until midday. Then the temperature drops slowly during the afternoon.

Day D: It is cold in the early morning and it remains cold for the whole day, except for a short time after lunch when the sun comes out for a while.

Day E: It is warm early in the morning, but the temperature drops sharply soon after sunrise and remains low until mid-afternoon, when it slowly warms up a little.

The shapes of some temperature graphs for 24-hour periods, starting early in the morning, are given below. Write which of the above days is possibly represented by the graph.

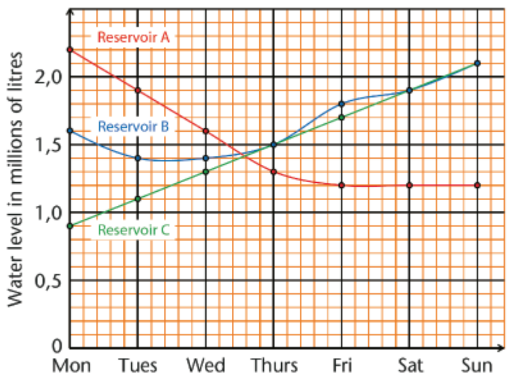


CHAPTER 18: GRAPHS 191

Answers

3. (a) On Wednesday, Thursday, Friday and Saturday
   (b) On Thursday, it increases from 1,5 million to 1,8 million litres.
   (c) The amount of change is different each day.
   (d) The water level increases by 0,2 million litres each day.
   (e) The water level remains constant at 1,2 million litres.

Water is supplied to a township from three reservoirs. The amount of water in each reservoir is measured each day at 08:00. The water level in reservoir A is represented in red on the graph below, and the water levels in reservoirs B and C are represented in blue and green, respectively.



The daily water levels in the three reservoirs, in millions of litres, are also given in the following table:

	Mon.	Tue.	Wed.	Thu.	Fri.	Sat.	Sun.
Reservoir A	2,2	1,9	1,6	1,3	1,2	1,2	1,2
Reservoir B	1,6	1,4	1,4	1,5	1,8	1,9	2,1
Reservoir C	0,9	1,1	1,3	1,5	1,7	1,9	2,1

3. You may use the graph or the table, or both, to find the answers to the questions below.
   (a) On which days does the water level in reservoir B increase from one day to the next?
   (b) On which of these days does the water level in reservoir B increase most, and by how much does it increase from that day to the next?
   (c) By how much does the water level in reservoir B change each day?
   (d) By how much does the water level in reservoir C change each day?
   (e) Describe the water level situation from Friday to Sunday, in reservoir A.

4. During a certain day, these changes occur in the temperature at a certain place:
   Between 00:00 and 03:00, the temperature drops by 2 °C.
   Between 03:00 and 06:00, the temperature drops by 3 °C.
   Between 06:00 and 10:00, the temperature remains constant.
   Between 10:00 and 12:00, the temperature rises by 3 °C.
   Between 12:00 and 16:00, the temperature remains constant.
   Between 16:00 and 18:00, the temperature drops by 4 °C.

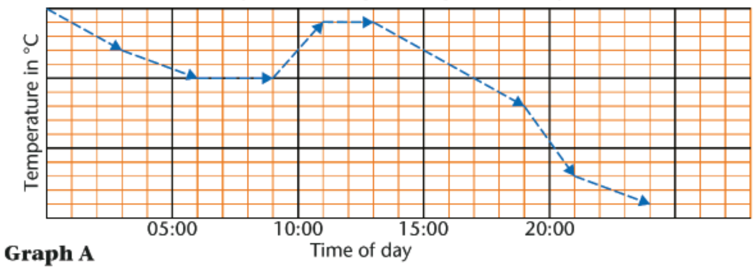
Answers

4. Graph B
5. See the answers on LB pages 193 alongside and LB page 194 on the following page.

Between 18:00 and 22:00, the temperature drops by 5 °C.
Between 22:00 and 24:00, the temperature remains constant.

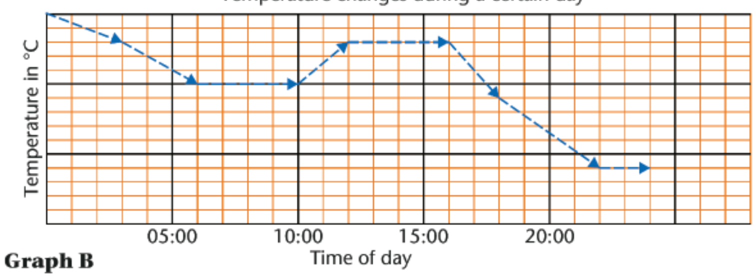
Which of the graphs below show the above temperature changes?

Temperature changes during a certain day



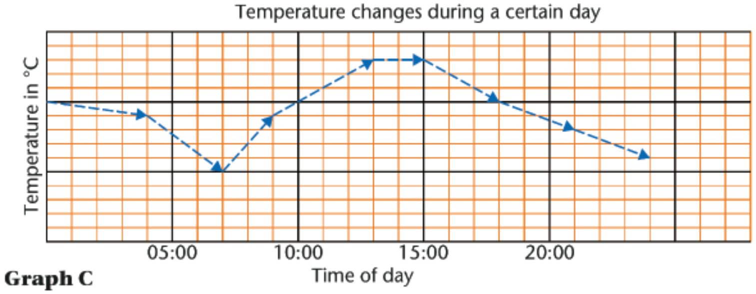
Graph A

Temperature changes during a certain day



Graph B

Temperature changes during a certain day



Graph C

5. Write a verbal description, like in question 4, of the temperature changes shown in graph A in question 4, by copying and completing the following descriptions:
   Between 00:00 and 03:00, the temperature drops by 3°C.
   Between 03:00 and 06:00, the temperature drops by 2°C.

Answers

6. See the answers on LB page 194 alongside.
7. (a) By 6°C
   (b) By 5°C
   (c) The 6°C drop from 13:00 to 19:00 took six hours, while the 5°C drop from 19:00 to 21:00 happened in only two hours. The temperature dropped more rapidly (sharply) in the period 19:00 to 21:00 than in the period 13:00 to 19:00.
8. (a) By 4°C
   (b) By 4°C
   (c) The 4°C increase from 07:00 to 09:00 took two hours, while the 4°C increase from 09:00 to 13:00 took four hours. In the period 07:00 to 09:00, the temperature increased twice as fast as during the interval 09:00 to 13:00.
9. (a) By 4°C
   (b) By 5°C
   (c) The temperature drops more rapidly from 16:00 to 18:00 than from 18:00 to 22:00. 5°C in four hours is slower than 4°C in two hours.

Between 06:00 and 09:00, the temperature remains constant.

Between 09:00 and 11:00, the temperature rises by 4°C.

Between 11:00 and 13:00, the temperature remains constant.

Between 13:00 and 19:00, the temperature drops by 6°C.

Between 19:00 and 21:00, the temperature drops by 5°C.

Between 21:00 and 24:00, the temperature drops by 2°C.

6. Write a verbal description, like in question 4, of the temperature changes shown in graph C in question 4, by copying and completing the following descriptions:

Between 00:00 and 04:00, the temperature drops by 1°C.

Between 04:00 and 07:00, the temperature drops by 4°C.

Between 07:00 and 09:00, the temperature rises by 4°C.

Between 09:00 and 13:00, the temperature rises by 4°C.

Between 13:00 and 15:00, the temperature remains constant.

Between 15:00 and 18:00, the temperature drops by 3°C.

Between 18:00 and 21:00, the temperature drops by 2°C.

Between 21:00 and 24:00, the temperature drops by 2°C.

7. Look at graph A in question 4.
   (a) By how much does the temperature drop from 13:00 to 19:00?
   (b) By how much does the temperature drop from 19:00 to 21:00?
   (c) When does the temperature drop most rapidly, from 13:00 to 19:00 or from 19:00 to 21:00? Explain your answer.
8. Look at graph C in question 4.
   (a) By how much does the temperature increase from 07:00 to 09:00?
   (b) By how much does the temperature increase from 09:00 to 13:00?
   (c) When does the temperature increase more rapidly, from 07:00 to 09:00 or from 09:00 to 13:00? Explain your answer.
9. Look at graph B in question 4.
   (a) By how much does the temperature drop from 16:00 to 18:00?
   (b) By how much does the temperature drop from 18:00 to 22:00?
   (c) When does the temperature drop more rapidly, from 16:00 to 18:00 or from 18:00 to 22:00? Explain your answer.

194 MATHEMATICS GRADE 9: TERM 3

18.2 Change at different rates

Teaching guidelines

Explain that as the unit on the horizontal axis grows, if the unit on the vertical axis increases as well, it is a graph of an increasing situation.

Use the table in question 4 to explain what we mean by a constant rate of change.

Answers

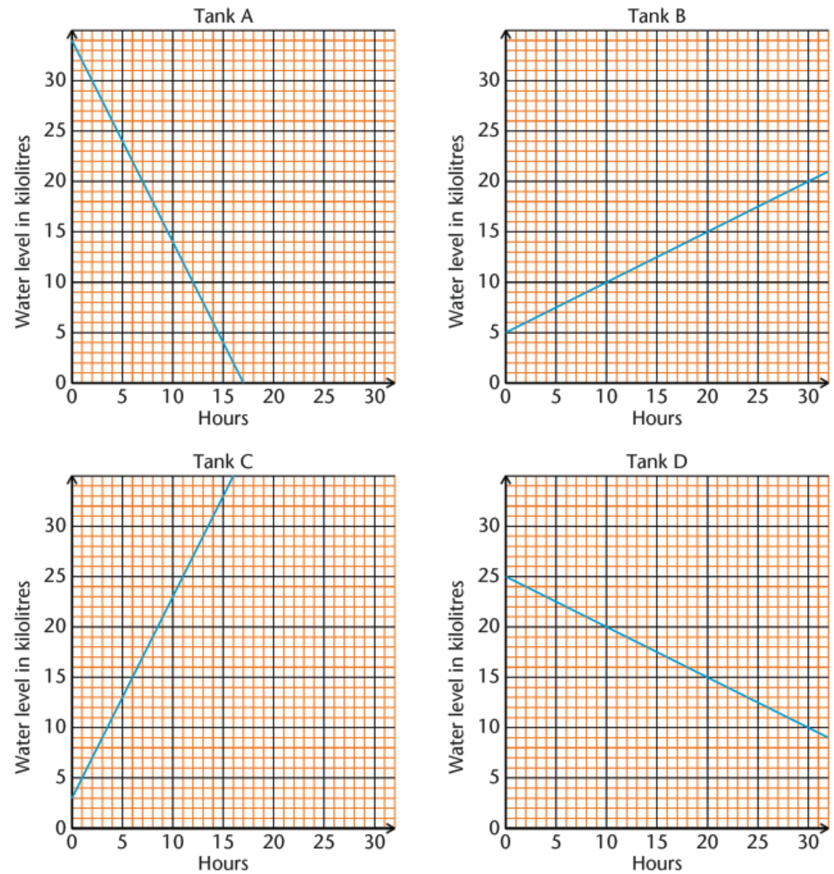
1. (a) B, C, E and F
   (b) A and D
2. Tank A: 34 kl Tank B: 5 kl Tank C: 3 kilolitres
   Tank D: 25 kl Tank E: 0 kl Tank F: 0 kilolitres
3. (a) Tank A. Different explanations are possible. For example: Tank A loses almost 35 kilolitres in less than 20 hours while tank D lost only 10 kilolitres after 20 hours.
   (b) Tank B. Different explanations are possible. For example: Tank B gains 15 litres in 30 hours while the other tanks that gain water gained 15 litres in much less time.

18.2 Change at different rates

The water levels in kilolitres (kl) in different water storage tanks over a period of 30 hours are represented on the graphs below and on the next page.

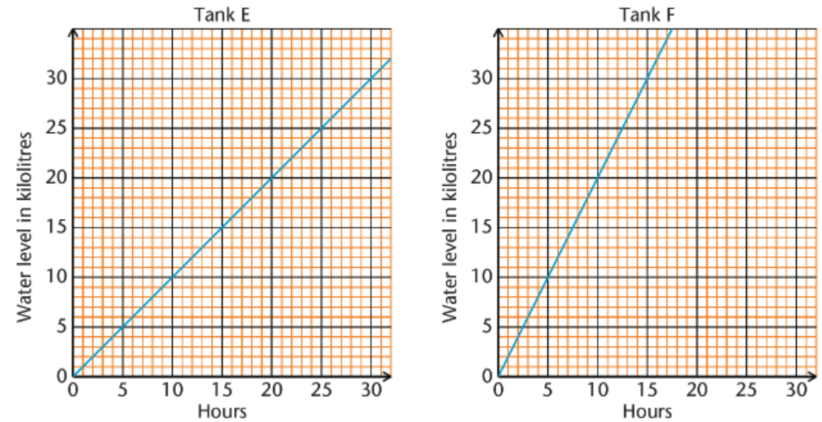
1 kilolitre = 1 000 litres

1. (a) In which tanks does the water level rise during the 30-hour period?
   (b) In which tanks does the water level drop during the 30-hour period?
2. How much water is there at the start of the 30-hour period, in each of the tanks?
3. (a) Which tank is losing water most rapidly? Explain your answer.
   (b) Which tank is gaining water most slowly? Explain your answer.



Mathematical notes

4. See the answers on LB page 196 alongside.



4. Copy and complete the table. Use negative numbers for decreases.

	Change over each hour	Change over any period of five hours
Tank A	–2 kilolitres	–10 kilolitres
Tank B	0,5 kilolitres	2,5 kilolitres
Tank C	2 kilolitres	10 kilolitres
Tank D	–0,5 kilolitres	–2,5 kilolitres
Tank E	1 kilolitres	5 kilolitres
Tank F	2 kilolitres	10 kilolitres

If a constant stream of water is pumped into a tank so that the water level is increased by 3 kilolitres in each hour, we say:

Water is pumped into the tank at a **constant rate** of **3 kilolitres per hour**.

5. (a) Tank G contains 12 kilolitres at the beginning of a 30-hour period. Water is then pumped into it at a constant rate of 3 kilolitres per hour. On graph paper, draw a dotted line graph to show the water level in Tank G.
   (b) Tank H also contains 12 kilolitres at the beginning of a 30-hour period. Water is then pumped into it at a constant rate of 1,5 kilolitres per hour. Return to the graph you drew for question 5(a) and draw a solid line graph to show the water level in Tank H.

Answers

5. (a) See the graph on LB page 197 alongside.
   (b) See the graph on LB page 197 alongside.
6. See the answers on LB page 197 alongside.

18.3 Draw graphs from tables of ordered pairs

Mathematical background

A set of ordered pairs defines a relation or a function. The word "ordered" means there is a first and a second number.

- The first number is always the input number (or x, the independent variable).
- The second number in the ordered pair is always the output number (or y, the dependent variable).

A graph is another way of representing the relationship between the input and output values of a relationship.

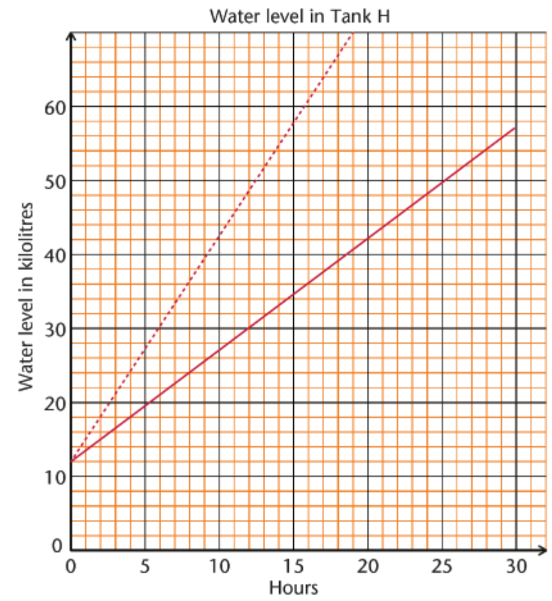
If we let a horizontal and a vertical number line cross each other at right angles at the point 0, we can describe the position of any point in terms of both lines. We call these lines axes (singular: axis).

The position of each point on the coordinate plane created by the axes is described by an ordered pair. The first number is called the x-coordinate and the second is called the y-coordinate.

To describe the position of a point (3; 6), we find the numbers on the axes. On the horizontal axis the point lies in line with 3 and on the vertical axis it lies in line with 6. Therefore, we describe the position of the point by the ordered pair (3; 6).

Misconceptions

Learners switch the order of the pair when plotting the points, reading the first coordinate on the vertical axis and the second on the horizontal instead of the other way around. Or they write down the coordinates in the wrong order and thus make a mistake.



6. Copy and complete the table for Tanks G and H over the 30-hour period:

Hours	0	5	10	15	20	25	30
Kilolitres in Tank G	12	27	42	57	72	87	102
Kilolitres in Tank H	12	19,5	27	34,5	42	49,5	57

18.3 Draw graphs from tables of ordered pairs

A "coordinate" graph shows the relationship between two variables; the dependent and independent variable in a function. The value of the dependent variable depends on the value given to the independent variable, hence its name. Sometimes there is no pattern to the relationship between the two variables and sometimes there is. In Grade 9 we will focus on graphs where there is a pattern to the relationship. Specifically, we will focus on graphs of linear functions. The graph of a linear function is a straight line.

GRAPHS OF FUNCTIONS WITH CONSTANT DIFFERENCES

Teaching guidelines

The constant difference of a function is determined by finding the difference between two consecutive function values. It can also be determined between any two function values by then dividing by the difference between the corresponding values of the independent variable. For example, 8 and $8\frac{1}{2}$ are consecutive function values for function A, as are $8\frac{1}{2}$ and 9. The difference in each case is $\frac{1}{2}$. If we only had the values for $x = 2$ and $x = 8$, namely 9 and 12, the difference between the function values would be $12 - 9 = 3$. The constant difference between consecutive function values could then be calculated by $\frac{12 - 9}{8 - 2} = \frac{1}{2}$.

Answers

1. See the answers on LB page 198 alongside.
2. See the answers on LB page 198 alongside and LB page 199 on the following page.

GRAPHS OF FUNCTIONS WITH CONSTANT DIFFERENCES

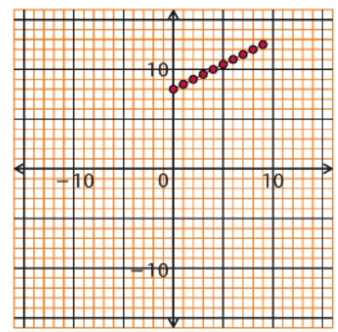
1. Copy and complete the following table:

x	0	1	2	3	4	5	6	7	8	9
Function A	8	$8\frac{1}{2}$	9	$9\frac{1}{2}$	10	$10\frac{1}{2}$	11	$11\frac{1}{2}$	12	$12\frac{1}{2}$
Function B	4	5	6	7	8	9	10	11	12	13
Function C	0	$1\frac{1}{2}$	3	$4\frac{1}{2}$	6	$7\frac{1}{2}$	9	$10\frac{1}{2}$	12	$13\frac{1}{2}$
Function D	–4	–2	0	2	4	6	8	10	12	14

2. Represent each of the functions in question 1 with a graph (like the ones shown below), by plotting the points on grid paper. You may join the points in each case and write down the constant difference between the function values.

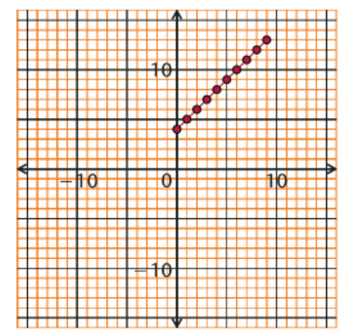
Function A

Constant difference = $\frac{1}{2}$



Function B

Constant difference = 1



Answers

3. (a) The bigger the constant difference, the steeper the graph.
   (b) Learners' own answer (for example, it is like climbing stairs – the bigger the "steps" the steeper the stairs.)
4. (a) See the answers on LB page 199 alongside.
   (b) The differences between the consecutive terms are the same as the coefficient of x in the expression.
   (c) The difference will be 4.

18.4 Gradient

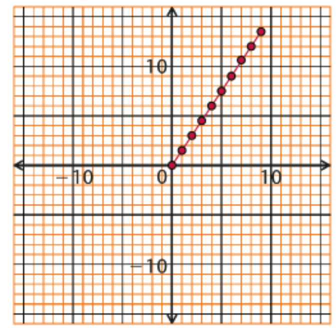
Mathematical background

The gradient is defined as the quotient of the vertical change between any two points on the graph divided by the horizontal change between the same two points.

In the case of linear functions, the gradient is constant, meaning it is the same over any interval or between any two points on the graph.

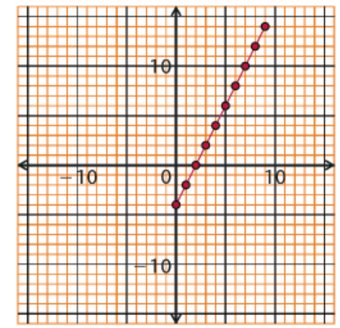
Function C

Constant difference = $1\frac{1}{2}$



Function D

Constant difference = 2



3. Some of the graphs you have drawn "go upwards" (or downwards) quickly, like a steep hill or mountain; others "go up" (or down) slowly.
   (a) Is there a link between the constant difference and the "steepness" of the graph?
   (b) Try to explain why this is the case.
4. (a) Copy and complete the following table:

x	1	2	3	4	5	6	7	8	9	10
$2x + 3$	5	7	9	11	13	15	17	19	21	23
$5x + 4$	9	14	19	24	29	34	39	44	49	54
$3x + 3$	6	9	12	15	18	21	24	27	30	33

   (b) Determine the difference between consecutive terms in each of the above three number sequences. What do you notice about this difference?
   (c) What difference between consecutive terms would you expect in the output numbers for $4x + 5$, if the input numbers are the natural numbers 1; 2; 3; .....?

18.4 Gradient

The "steepness" or **slope** of a line can be indicated by a number, as described below. This number is called the **gradient** of the line.

The gradient is the vertical change divided by the horizontal change as you move from left to right on the line.

$$\text{Gradient} = \frac{\text{vertical change}}{\text{horizontal change}}$$

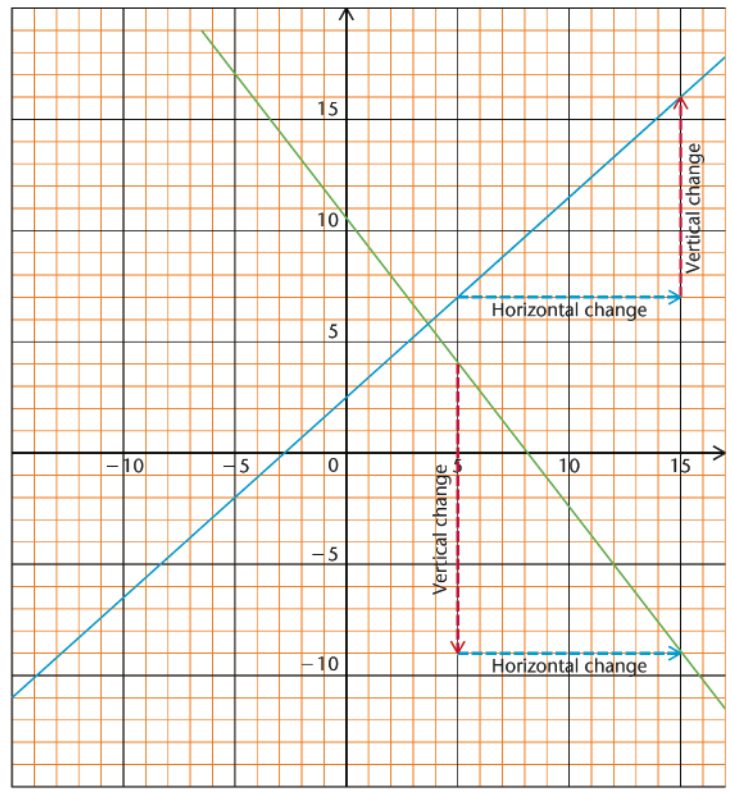
Teaching guidelines

Learners should get used to the concept of the gradient as vertical change over horizontal change. Let them try to determine how to find the vertical and horizontal differences.

Stress that the horizontal direction is always read from left to right, in direction of the increasing *x* values, and taken as positive.

Misconceptions

Learners find the gradient the wrong way around, dividing the horizontal change by the vertical change.



The gradient of the blue line above is $\frac{9}{10} = 0{,}9$.

The gradient of the green line is $\frac{-13}{10} = -1{,}3$.

Note that the horizontal change is always taken to be positive (moving to the right), but the vertical change can be positive (if it is upwards) or negative (if it is downwards).

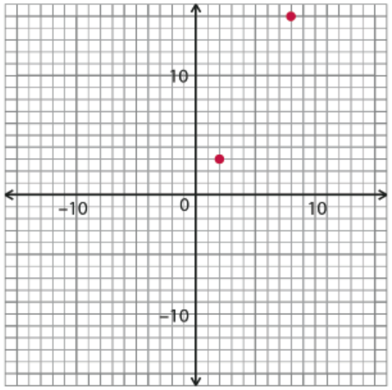
1. A certain line passes through the points (2; 3) and (8; 15). A straight line is drawn through the two points.

200 MATHEMATICS GRADE 9: TERM 3

Answers

1. (a) This is an open question, with the purpose of inducing learners to think. Some learners may actually succeed in finding the gradient, which is 2.
   (b) See the answers on LB page 201 alongside.
   (c) The horizontal change is from 2 to 8, that is six units to the right.
   The vertical change is upwards from 3 to 15, which is 12 units.
   (d) gradient $= \frac{15-3}{8-2} = \frac{12}{6} = 2$
2. See the answers on LB page 201 alongside.
3. The gradients are 2 and –2. Let learners compare the correct gradients with the formulae of the functions to see where they can see the gradient in the formula.

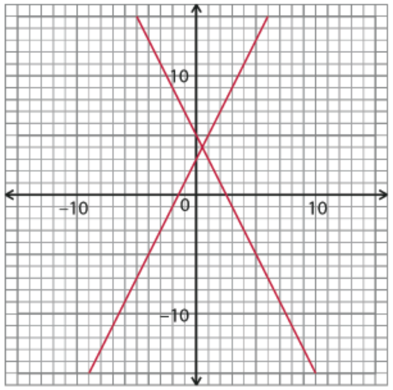
(a) Try to think of a way in which you can work out the gradient of the line that passes through the two points.
(b) On a graph sheet, plot the two points shown on the graph sheet below.



(c) What horizontal change and vertical change is needed to move from the point (2; 3) to the point (8; 15)? You may draw arrows on your graph to help you to think clearly about this.
(d) Work out the gradient of the line that passes through the two points.

2. Copy and complete the table and plot graphs of $y = 2x + 3$ and $y = -2x + 5$ on a graph sheet.

x	–3	1	3	5
$2x + 3$	–3	5	9	13
$-2x + 5$	11	3	–1	–5



3. Work out the gradients of the graphs of $y = 2x + 3$ and $y = -2x + 5$. You may use the coordinates of any of the points you have plotted.

Teaching guidelines

Explain to learners that the gradient is measured between any two points on the graph and that it is indicated by the symbol m.

It is defined as the vertical change between any two points divided by the horizontal change between the same two points. This means that if we have any two points $(x_1; y_1)$ and $(x_2; y_2)$, the gradient between the points will be given by:

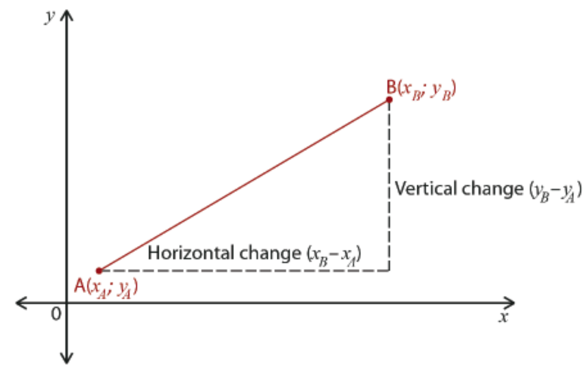
gradient, $m = \frac{y_2 - y_1}{x_2 - x_1}$.

It does not matter which points are used to calculate the gradient, as long as the two points lie on the line of which we are calculating the gradient.

Work through the examples with learners. Stress that they should make sure to divide vertical change by horizontal change and not mix them up.

Talk about graphs that have $m = 0$ and that this happens when the line is parallel to the x-axis. All the y values are the same, so $y_2 - y_1 = 0$ for all y values, therefore $m = 0$.

Suppose the coordinates of point A are $(x_A; y_A)$ and the coordinates of B are $(x_B; y_B)$.



The gradient of line AB is: $m_{AB} = \frac{\textit{Vertical change}}{\textit{Horizontal change}} = \frac{y_A - y_B}{x_A - x_B}$

In summary:
If you have two points A $(x_A; y_A)$ and B $(x_B; y_B)$ then the formula for the gradient is: $m = \frac{y_A - y_B}{x_A - x_B}$

Examples of finding the gradient between two points

Calculate the gradient of the line that goes through the points:

(a) A(2; 5) and B(4; 1)

$$\begin{aligned} m &= \frac{y_A - y_B}{x_A - x_B} \\ &= \frac{5 - 1}{2 - 4} \\ &= \frac{4}{-2} \\ &= -2 \end{aligned}$$

(b) C(2; 2) and D(−6; 0)

$$\begin{aligned} m &= \frac{y_C - y_D}{x_C - x_D} \\ &= \frac{2 - 0}{2 - (-6)} \\ &= \frac{2}{8} \\ &= \frac{1}{4} \end{aligned}$$

(c) A(0; −1) and B(1; 1)

$$\begin{aligned} m &= \frac{y_A - y_B}{x_A - x_B} \\ &= \frac{-1 - 1}{0 - 1} \\ &= \frac{-2}{-1} \\ &= 2 \end{aligned}$$

The gradient of a straight line is the same everywhere, so it does not matter which two points you use to determine the gradient.

Answers

1. (a) $m_{AB} = \frac{12-10}{6-2} = \frac{2}{4} = \frac{1}{2}$

   (b) $m_{CD} = \frac{-3-3}{-2-1} = \frac{-6}{-3} = 2$

   (c) $m_{EF} = \frac{-1-3}{4-0} = \frac{-4}{4} = -1$

   (d) $m_{GH} = \frac{4-2}{4-5} = \frac{2}{-1} = -2$

   $m_{GI} = \frac{8-2}{2-5} = \frac{6}{-3} = -2$

2. See the answers on LB page 203 alongside.

18.5 Finding the formula for a graph

TABLES AND FORMULAE

Teaching guidelines

Discuss with learners that the equations of all the straight lines are similar; in the simplified form they all have one term in x and a constant value: $y = mx + c$.

Learners would know by now that m represents the gradient of the function. Take them back to LB page 201, question 2 and let them try to find what the constants 3 and 5 represent. Show them where the lines intersect the y-axis. The x–value on the vertical axis is 0. So if 0 is substituted into the equations we get 3 and 5, the values where the vertical axis is crossed.

Answers

1. (a) See the answers on LB page 203 alongside and LB page 204 on the following page.

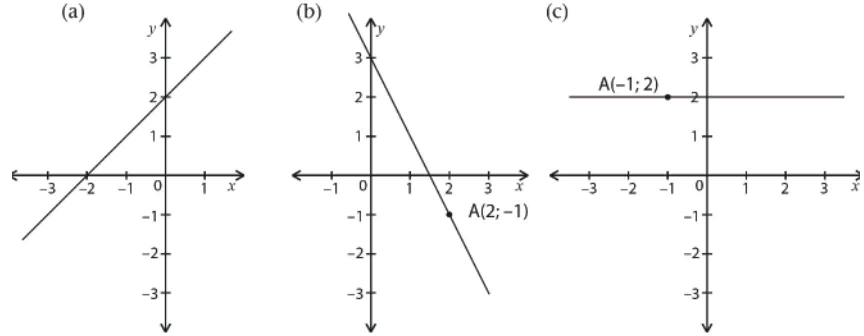
DETERMINE THE GRADIENT

Do the following task:

1. Determine the gradient of the lines that go through the following points:
   (a) A(2; 10) and B(6; 12) (b) C(1; 3) and D(–2; –3)
   (c) E(0; 3) and F(4; –1) (d) G(5; 2), H(4; 4) and I(2; 8)

2. Determine the gradient of the following lines:

(a) (b) (c)



(a) $m = 1$ (b) $m = -2$ (c) $m = 0$

18.5 Finding the formula for a graph

TABLES AND FORMULAE

1. Each table below and on the next page shows values for a relationship represented by one of these rules:

$y = -2x + 3$ $y = 2x - 5$ $y = -3x + 5$ $y = -3(x + 2)$
$y = 3x + 2$ $y = 5(x - 2)$ $y = 2x + 3$ $y = 2x + 5$
$y = -3x + 6$ $y = 5x + 10$ $y = 5x - 10$ $y = -x + 3$

   (a) Copy and complete the following tables by extending the patterns in the output values:

A.

x	0	1	2	3	4	5	6	7
y	2	5	8	11	14	17	20	23

B.

x	0	1	2	3	4	5	6	7
y	3	1	–1	–3	–5	–7	–9	–11

Answers

1. (b) A. Add 3 to previous term or multiply x by 3 and add 2. $y = 3x + 2$
   B. Subtract 2 from previous term or multiply x by –2 and add 3. $y = -2x + 3$
   C. Add 5 to previous term or multiply x by 5 and add –10. $y = 5x - 10$
   D. Add 2 to previous term or multiply x by 2 and add –5. $y = 2x - 5$
   E. Subtract 3 from previous term or multiply x by –3 and add 6. $y = -3x + 6$
   F. Subtract 1 from previous term or multiply x by –1 and add 3. $y = -x + 3$
   G. Add 2 to previous term or multiply x by 2 and add 3. $y = 2x + 3$

Teaching guidelines

Spend time making sure that learners understand the concepts **gradient** and ***y*-intercept** and that they can recognise them in the equation of a straight line.

Talk about the standard form of an equation and the other ways in which the equation could be given:

$ax + by + c = 0$
$by = ax + c$
$ax + by = c$

Show learners how to change the equation to the standard form and that the gradient and y-intercept can be read from the standard form directly.

C.

x	0	1	2	3	4	5	6	7
y	–10	–5	0	5	10	15	20	25

D.

x	0	1	2	3	4	5	6	7
y	–5	–3	–1	1	3	5	7	9

E.

x	0	1	2	3	4	5	6	7
y	6	3	0	–3	–6	–9	–12	–15

F.

x	0	1	2	3	4	5	6	7
y	3	2	1	0	–1	–2	–3	–4

G.

x	0	1	2	3	4	5	6	7
y	3	5	7	9	11	13	15	17

(b) For each table, describe what you did to produce more output values. Also write down the rule (formula) that corresponds to the table.

You may have noticed that the equations of straight lines look similar.

The equation of a straight line is $y = mx + c$, where:

- m tells us the **gradient** of the line
- c tells us where the line crosses the y-axis
- it is called the ***y*-intercept** and it has the coordinates (0; c)
- the line $y = 3x + 4$ has a **gradient of 3** and the y-intercept is (**0; 4**)
- the equation of a line with a **gradient of –2** and y-intercept of (**0;10**) is $y = -2x + 10$
- the line $y = 2x$ has a **gradient of 2** and the y-intercept is (**0; 0**)
- the line $y = 5$ has a **gradient of 0** and the y-intercept is (**0; 5**)

Gradient means the steepness or slope of the line.
The point where a line crosses one of the axes is called the **intercept**.

What is the gradient and y-intercept of the line $2y = 6x + 10$?

If you said $m = 6$ and $c = 10$, you would be wrong. The equation is not in standard form. The equation must be written in standard form before you can read off the values of the gradient and the y-intercept.

$2y = 6x + 10$ → Divide both sides by 2
$y = 3x + 5$

Therefore, the **gradient is 3** and the y-intercept is (**0; 5**).

The **standard form** of a straight line graph is $y = mx + c$. On one side there should only be a "y" (with a coefficient of 1).

Teaching guidelines

Discuss what the gradient will look like if the lines are:

- increasing
- decreasing
- parallel to the horizontal axis
- parallel to the vertical axis.

Explain that the gradient is undefined if the line is vertical because in that case all the x values on the line are the same, which means that when we determine the gradient by the formula for the gradient, we will have to divide by 0, which is undefined.

Answers

2. See the answers on LB page 205 alongside.

3. (a) $y = -2x + 5$; $m = -2$; $(0; 5)$
   (b) $y = -3x - 4$; $m = -3$; $(0; -4)$
   (c) $y = 3x - 4$; $m = 3$; $(0; -4)$
   (d) $y = \frac{1}{3}x - 2$; $m = \frac{1}{3}$; $(0; -2)$
   (e) $y = x + 4$; $m = 1$; $(0; 4)$
   (f) $y = 3x - 2$; $m = 3$; $(0; -2)$
   (g) $y = \frac{1}{4}x + 6$; $m = \frac{1}{4}$; $(0; 6)$
   (h) $y = -12$; $m = 0$; $(0; -12)$
   (i) $x = 15$; $m =$ undefined; no y-intercept

DETERMINE THE EQUATION OF A STRAIGHT LINE

Teaching guidelines

To determine the equation of a straight line we only need to find the values of m, the gradient and c, the y-intercept. How easy this will be will depend on the information we have. We could have:

- the gradient and the y-intercept, for example, the line goes through (0; 5) and has gradient 3. Then the equation is $y = 3x + 5$.
- the gradient and a point on the graph, not the -intercept. For example, the line has gradient 2 and goes through (1; 4). We substitute the point into $y = 2x + c$ so we get $4 = 2(1) + c$ and solve for c. So $y = 2x + 2$. (See also, example 3 on LB page 206.)
- We could have any two points as in examples 1 and 2 on LB pages 205 and 206.

If $m > 0$, the line will be increasing.
If $m < 0$, the line will be decreasing.
If the line is horizontal, $m = 0$.
If the line is vertical, m is undefined.

2. Copy and complete the following table:

Equation	Gradient	y-intercept
$y = 3x + 5$	$m = 3$	(0; 5)
$y = \frac{x}{2} - 7$	$m = \frac{1}{2}$	(0; –7)
$y = 2 - 3x$	$m = -3$	(0; 2)
$-y = 5x - 10$	$m = -5$	(0; 10)
$y = 3$	$m = 0$	(0; 3)
$y = x$	1	(0; 0)
$y = -2x - 7$	–2	(0; –7)

3. Write each of the following equations in standard form and then determine the gradient and y-intercept:
   (a) $2y + 4x = 10$ (b) $-3x = y + 4$ (c) $3x - 4 = y$
   (d) $3y + 6 = x$ (e) $y = -3x + 4y - 12$ (f) $y = 3x - 2$
   (g) $y = \frac{1}{4}x + 6$ (h) $y = -12$ (i) $x = 15$

DETERMINE THE EQUATION OF A STRAIGHT LINE

The equation of a straight line is $y = mx + c$. If you need to determine the equation of a straight line, then all you need to know are the values of m and c.

If you know the values of two points on the graph, then you can determine the gradient using the formula: $m = \frac{y_A - y_B}{x_A - x_B}$

Once you know the gradient you can calculate the value of the y-intercept using substitution.

Example 1: Determine the equation of the straight line that goes through (1; 1) and (5; 13).

Step 1: Calculate the gradient.

$$m = \frac{y_A - y_B}{x_A - x_B} = \frac{1 - 13}{1 - 5} = \frac{-12}{-4} = 3$$

Step 2: Since you now know $m = 3$ you can substitute it into the equation $y = mx + c$.
Therefore, $y = 3x + c$.

Answers

1. (a) $y = 5x - 5$
   (b) $y = 5$
   (c) $y = -2x$
   (d) $y = -4x + 10$
   (e) $y = \frac{1}{2}x + \frac{5}{2}$
   (f) $y = \frac{1}{2}x + 3$
2. (a) $y = 5x - 8$
   (b) $y = -2x$
   (c) $y = -10x + 7$

Step 3: To determine c you need to substitute the coordinates of a point on the line into the equation. (It can be either of the points that were given, so choose the easier one.)

Substitute (5; 13) into $y = 3x + c$

$$(13) = 3(5) + c$$
$$13 = 15 + c$$
$$13 - 15 = c$$
$$-2 = c$$

Step 4: Write down the equation: $y = 3x - 2$.

Example 2: Determine the equation of the line that passes through (4; –1) and (7; 5).

Information	m (Gradient)	c (y-intercept)	$y = mx + c$ (Equation)
(4; –1) **(7; 5)**	$m = \frac{y_A - y_B}{x_A - x_B}$ $= \frac{-1-5}{4-7}$ $= \frac{-6}{-3}$ $= 2$	Substitute $m = 2$ and (7; 5) $y = mx + c$ $y = 2x + c$ $(5) = 2(7) + c$ $5 = 14 + c$ $-9 = c$	$y = 2x - 9$

Example 3: Determine the equation of the line with a gradient of 4 passing through (2; 6).

Information	m (gradient)	c (y-intercept)	$y = mx + c$ (equation)
$\boldsymbol{m = 4}$ **(2; 6)**	$m = 4$	Substitute $m = 4$ and (2; 6) $y = mx + c$ $y = 4x + c$ $6 = 4(2) + c$ $-2 = c$	$y = 4x - 2$

You may want to set your work out as shown in Examples 2 and 3 above.

1. Determine the equation of the each of the straight lines passing through the points given.
   (a) (3; 10) and (2; 5)   (b) (–4; 5) and (2; 5)   (c) (0; 0) and (4; –8)
   (d) $(1\frac{1}{2}; 4)$ and $(-\frac{1}{2}; 12)$ (e) (3; 4) and (–7; –1)   (f) (0; 3) and (–14; –4)
2. Determine the equation of the straight line with:
   (a) a gradient of 5 and passing through the point (1; –3)
   (b) a gradient of –2 passing through the point (0; 0)
   (c) a y-intercept of 7 passing through the point (1; –3)

Answers

3. See the answers on LB page 207 alongside.

18.6 *x*- and *y*-intercepts

Teaching guidelines

Learners should know by now that all points that lie on the vertical axis have an *x*-coordinate equal to 0 as the point has not moved to the left or the right of the origin, in other words, they are all at 0 in the horizontal direction.

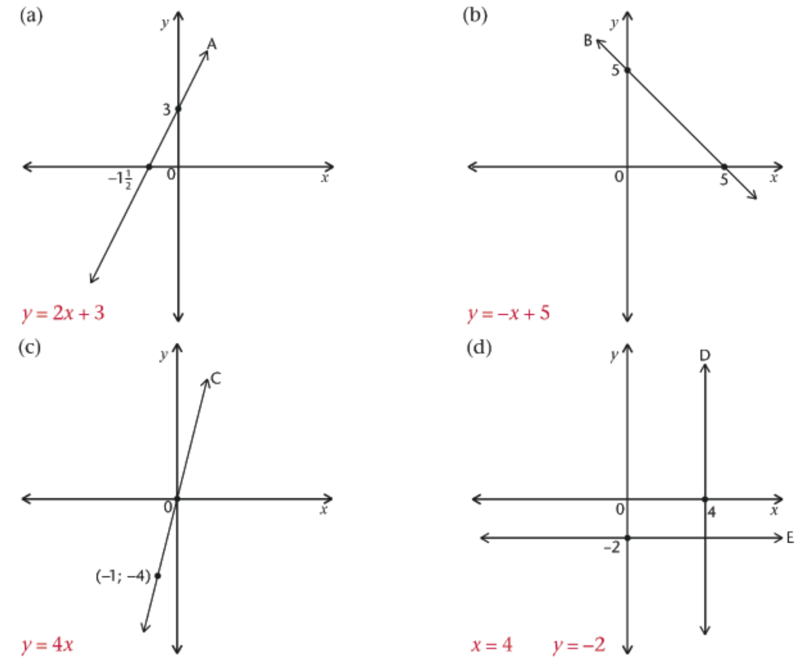
In the same way, all the points on the horizontal axis have their *y*-coordinate equal to 0.

These facts mean that the *y*-intercepts will all be (0; *y*) and the *x*-intercepts (*x*; 0).

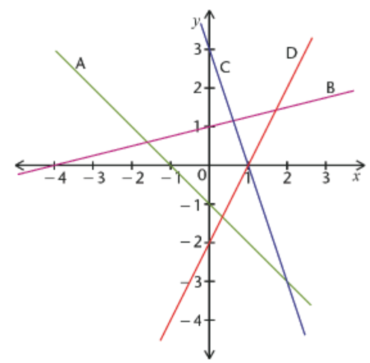
Answers

1. See the answers on LB page 207 alongside.
2. They all have $y = 0$.

3. Determine the equations of the straight lines. Question (d) is a challenge.



18.6 *x*- and *y*-intercepts



1. Copy the table and write down the coordinates of the points where each line cuts the two axes:

	x-intercept	*y*-intercept
A	(–1; 0)	(0; –1)
B	(–4; 0)	(0; 1)
C	(1; 0)	(0; 3)
D	(1; 0)	(0; –2)

2. What do all the *x*-intercepts have in common?

Answers

3. They all have $x = 0$.
4. (a) $(-4; 0)$ and $(0; 12)$ (b) $(3; 0)$ and $(0; -3)$
   (c) $(-2; 0)$ and $(0; -4)$ (d) $(-2; 0)$ and $(0; 6)$
   (e) $(5; 0)$ and $(0; 10)$ (f) $(-\frac{1}{2}; 0)$ and $(0; 13)$

VERTICAL AND HORIZONTAL LINES

Teaching guidelines

Work through the questions with learners and explain again why $m = 0$ for horizontal lines and m is undefined for vertical lines.

Answers

1. They all have $x = 2$.
2. Accept any points with $x = 2$, for example, $(2; 5)$
3. Yes
4. Accept any five points, for example, $(-1; 3)$

18.7 Graphs of non-linear functions

Teaching guidelines

Learners should simply calculate function values for x-values from –10 to 10 in a table and compare them with the given points. For example:

x	–7	–6	–5	–4	–3	–2	–1	0	1	2	3	4	5	6	7	8
x^2	49	36	25	16	9	4	1	0	1	4	9	16	25	36	49	64
$x^2 + 130$	179	166	155	...												

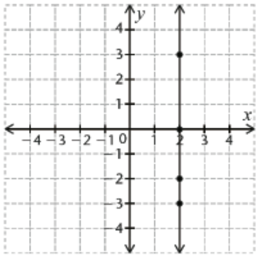
Answers

See the answers on LB page 208 alongside.

3. What do all the y-intercepts have in common?
4. Determine the coordinates of the intercepts of the following straight line graphs:
   (a) $y = 3x + 12$ (b) $y = x - 3$
   (c) $y = -2x - 4$ (d) $2y = 6x + 12$
   (e) $4x + 2y = 20$ (f) $13 - y = -26x$

VERTICAL AND HORIZONTAL LINES

Some special lines are so easy that you do not need any fancy methods to draw them or get their equation; you can just look at them.



1. What do the following coordinate pairs have in common?
   $(2; 3)$, $(2; -2)$, $(2; 0)$ and $(2; -3)$
2. Write down two more points that have an x-coordinate of 2.

If you plot these points on a set of axes you will see that they form a **vertical line**. The equation of the line is $x = 2$.

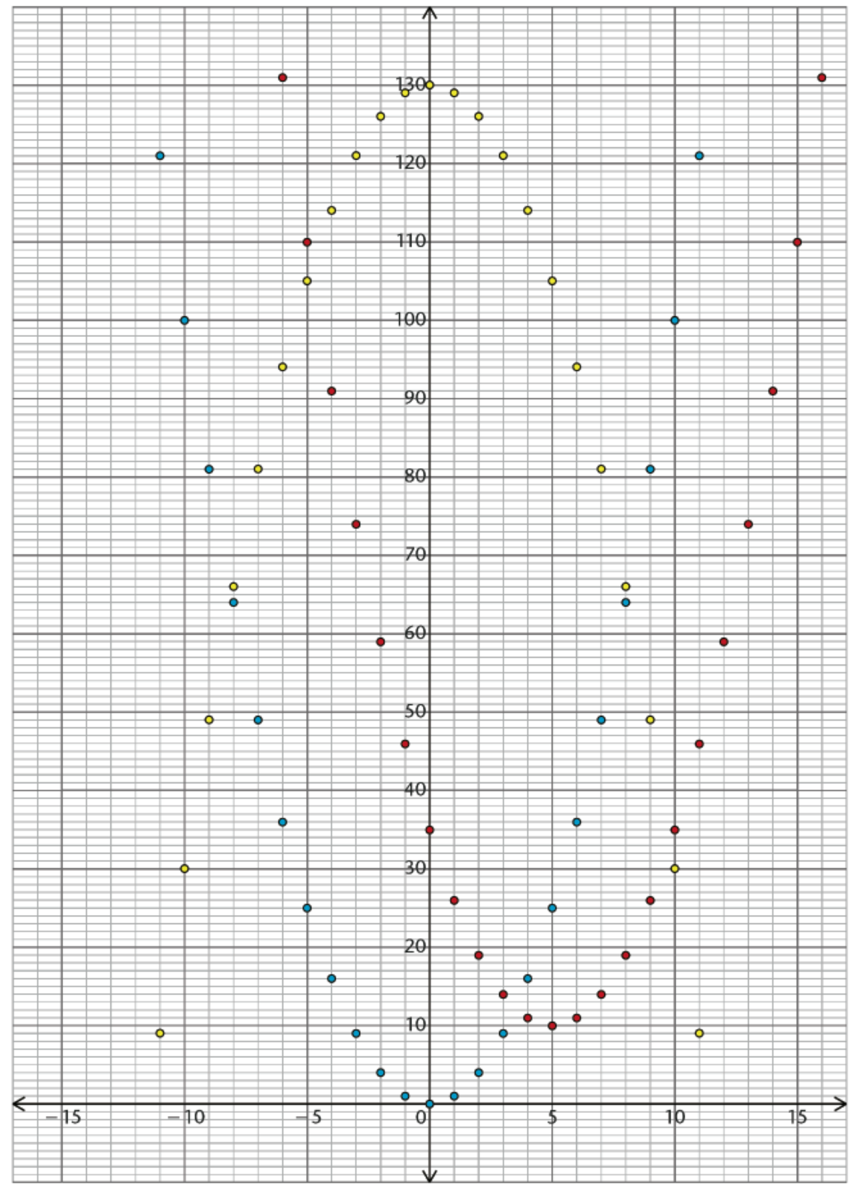
3. Will the two extra points you wrote down (question 2) also be on the line?
4. Write down five coordinate pairs with $x = -1$.

18.7 Graphs of non-linear functions

Some of the following relationships are represented by graphs on the next page. Identify which of the relationships are represented by which set of points on the graph. You may use the table below to help you to answer this question. For example, you may calculate some output numbers by using the formulas and record this in tables.

$y = -x^2$ $y = (-x)^2$ $y = x^2 + 130$ $y = (x - 5)^2 + 10$
$y = x^2$ $y = -x^2 + 130$ $y = 130 - x^2$ $y = x^2 - 10x + 35$

(a) Set of points in yellow $y = -x^2 + 130$ or $y = 130 - x^2$
(b) Set of points in blue $y = x^2$ or $y = (-x)^2$
(c) Set of points in red $y = (x - 5)^2 + 10$ or $y = x^2 - 10x + 35$



130
120
110
100
90
80
70
60
50
40
30
20
10
0
– 15
– 10
– 5
5
10
15

Learner Book Overview		
Sections in this chapter	**Content**	**Pages in Learner Book**
19.1 Surface area	Revising the surface area of prisms; investigating the surface area of cylinders; calculating the area of cylinders	Pages 210 to 212
19.2 Volume	Revising formulae for calculating the volume of prisms; calculating the volume of prisms; formulae for calculating the volume of cylinders	Pages 212 to 215
19.3 Capacity	Revising the concept of capacity; calculating the capacity of a hollow block	Pages 215 to 216
19.4 Doubling dimensions and the effect on volume	Investigating the effect of doubling the dimensions of a prism on the volume; investigating the effect of doubling the dimensions of a cylinder on the volume	Pages 216 to 218

CAPS time allocation	5 hours
CAPS content specification	Page 146

Mathematical background

The **surface area** of an object is the sum of the areas of all the faces and is measured in square units. A formula that can be used to calculate the surface area of a prism or a cylinder is:

Surface area = 2 × area of the base + the perimeter × the height of the prism.

The **volume** of an object is the amount of space it takes up. The formula to calculate the volume of a prism or a cylinder is:

Volume = area of base × height. Three dimensions are involved (namely length, breadth and height), so volume is measured in cubic units, such as cubic centimetres (cm^3) and cubic metres (m^3).

- A cubic centimetre is a cube with sides that are 1 cm each.
- A cubic metre is a cube with sides that are 1 m each.

The difference between volume and capacity is:

- volume is the amount of space an object takes up
- capacity is the amount of content a container can hold if it is filled to capacity.

An object can have both volume and capacity.

If volume is measured in cubic centimetres (cm^3) the measure of capacity is millilitres. A millilitre is the content of a cube with sides 1 cm.
If the dimensions of an object are doubled, the volume increases by a factor of eight.

19.1 Surface area

SURFACE AREA OF PRISMS

Teaching guidelines

Learners can either calculate the sum of the areas of all the faces or use the formula:
Surface area (SA) = 2 × area of the base + the perimeter × the height of the prism.

Learners should keep the following in mind:

- in the case of a cube, it is easy to simply multiply the area of a face by six.
- the formula to calculate the area of a triangle = $\frac{1}{2}bh$.
- the hypotenuse of a right-angled triangle is calculated using the Theorem of Pythagoras.

Answers

1. $SA = 6 \times$ area of face
   $= 6 \times 10^2$ cm^2
   $= 600$ cm^2

2. SA = Sum of area of all faces
   $= 2(12 \times 8 + 8 \times 5 + 5 \times 12)$ cm^2
   $= 2(96 + 40 + 60) = 392$ cm^2

3. $SA = 2 \times 25^2 + (4 \times 25 \times 100)$
   $= 1\,250 + 10\,000$
   $= 11\,250$ mm^2

4. $SA = 2(\frac{1}{2} \times 60 \times 40) + 2(50 \times 70) + (60 \times 70)$
   $= 2\,400 + 7\,000 + 4\,200$
   $= 13\,600$ mm^2

5. $x = \sqrt{(10^2 + 10^2)} = \sqrt{200} = 14{,}14...$
   Area bases $= 2 \times \frac{1}{2} \times 10 \times 10 = 100$ m^2
   Area lateral faces
   $= 3 \times (14{,}14... + 10 + 10)$
   $= 102{,}43$ m^2
   Total $SA = 100 + 102{,}43 = 202{,}43$ m^2

6. $x = \sqrt{(2^2 + (1{,}5)^2)} = \sqrt{(6{,}25)} = 2{,}5$ m
   Area bases $= 2 \times \frac{1}{2} \times 2 \times 1{,}5 = 3$ m^2
   Area lateral faces
   $= 4 \times (1{,}5 + 2 + 2{,}5)$
   $= 24$ m^2
   Total $SA = 3 + 24 = 27$ m^2

Chapter 19
Surface area, volume and capacity of 3D objects

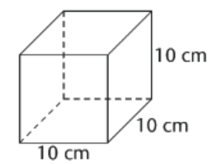
19.1 Surface area

SURFACE AREA OF PRISMS

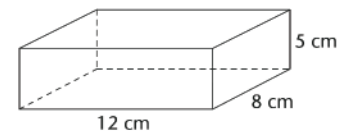
The **surface area** of an object is the total area of all of its faces added together.
You learnt the following formula in previous grades:

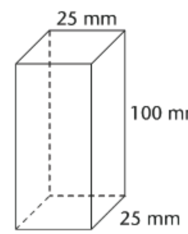
- Surface area of a prism = Sum of the areas of all its faces

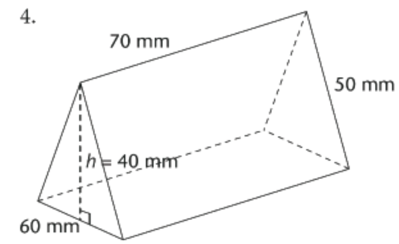
Calculate the surface area of the following objects to revise what you should already know:

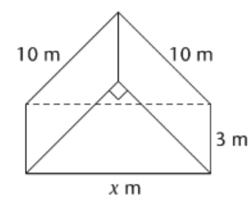
1.


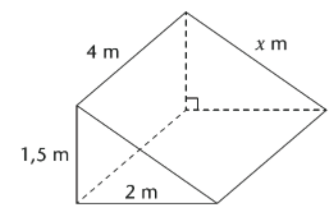
(We use SA for surface area.)

2.


3.


4.


5.


6.


Answers

7. Area front and back faces

$= 2(12 \times 14 + 5 \times 6)\ \text{cm}^2$

$= 396\ \text{cm}^2$

Area side faces $= 2 \times 12 \times 6$

$= 144\ \text{cm}^2$

Area top and base $= 2 \times 6 \times 20$

$= 240\ \text{cm}^2$

Total $SA = 396 + 144 + 240$

$= 780\ \text{cm}^2$

8. Height isosceles triangle

$= \sqrt{(10^2 - 6^2)} = \sqrt{64} = 8\ \text{cm}$

Area front $= 2 \times (144 + \frac{1}{2} \times 12 \times 8)$

$= 384\ \text{cm}^2$

Area side face $= 2[25 \times (12 + 10)]$

$= 1\ 100\ \text{cm}^2$

Area base $= 12 \times 25$

$= 300\ \text{cm}^2$

Total $SA = 384 + 1\ 100 + 300$

$= 1\ 784\ \text{cm}^2$

INVESTIGATING THE SURFACE AREA OF CYLINDERS

Teaching guidelines

Take a rectangular piece of paper and roll it up to form a cylinder. Let learners imagine a base and a top. The rolled-up paper forms the curved surface of the cylinder. Unroll the piece of paper to demonstrate that the curved surface of a cylinder is a rectangle of which the length equals the circumference of the base.

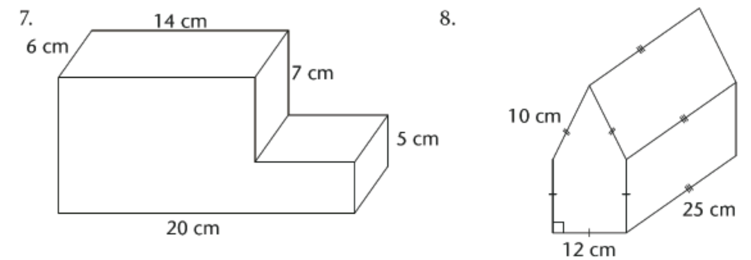
The formula used above to calculate the surface area of a prism can also be used for a cylinder. Simply replace the perimeter of the prism with the circumference of the cylinder:

Surface area = 2 × area of the base + the circumference × the height of the cylinder

$= 2\pi r^2 + 2\pi rh$ which can be factorised to give

$= 2\pi r(r + h)$

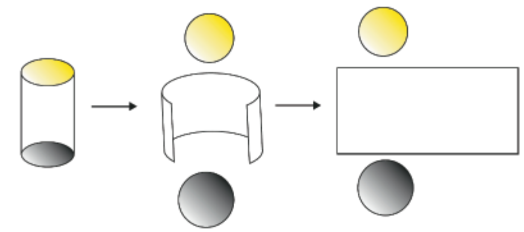
The only information we need is the height of the cylinder and the radius of the base.



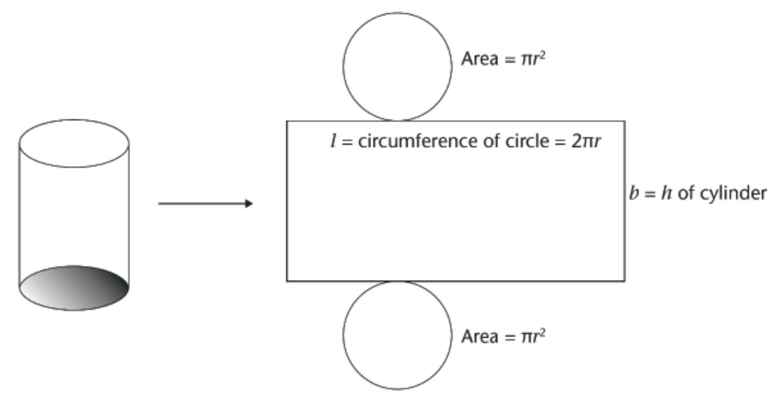
INVESTIGATING THE SURFACE AREA OF CYLINDERS

In order to calculate the surface area of a cylinder, you need to know what shape the surfaces of the cylinder are.

The surfaces of the top and base of a cylinder are made up of circles. The curved surface between the top and base of a cylinder can be unrolled to create a rectangle.



So, the net of a cylinder looks like this:



CALCULATING THE SURFACE AREA OF CYLINDERS

Teaching guidelines

The questions in this section require learners to think about what they need to apply the formula. In questions 2 and 3 they need to calculate the radius first. There may be learners who don't realise that they can find r by solving the equation r = circumference ÷ 2π. Remind learners to use either 3,14 or $\frac{22}{7}$ for π.

They also need to think carefully about using the formula. In question 4, they need not add the area of the base, which will change the formula.

Answers

1. A. $SA = 2\pi r(r + h)$
$= 2 \times 3{,}14 \times 6 \times (6 + 6)$
$= 452{,}16 \text{ cm}^2$

B. $SA = 2\pi r(r + h)$
$= 2(3{,}14) \times 4 \times (4 + 8)$
$= 301{,}44 \text{ m}^2$

2. $r = 25{,}12 \div (2\pi) = 4$ cm
$SA = 2\pi r(r + h) = 25{,}12(4 + 60) = 1\ 607{,}68 \text{ cm}^2$
3. $r = 12{,}56 \div 2\pi = 12{,}56 \div (2 \times 3{,}14) = 2$ m
$SA = 2\pi r(r + h) = 12{,}56(2 + 5) = 87{,}92 \text{ m}^2$
4. $SA = 2\pi r \times h + \pi r^2 = 2 \times 3{,}14 \times 3{,}5 \times 8 + 3{,}14 \times (3{,}5)2 = 214{,}305 \text{ m}^2$
22 litres of paint are required. The calculation results in 21,43 litres; 21 litres will not be enough. 22 litres will allow for enough paint including any wastage or spills.

19.2 Volume

FORMULAE FOR VOLUME OF PRISMS

Teaching guidelines

Remind learners how to calculate the volume of a cube or cuboid (prism), i.e. area of the base times the height.

How the area of the base is calculated depends on the shape of the base, for example:
- a square: $A = s^2$
- a rectangle: $A = lb$
- a triangle: $A = \frac{1}{2}bh$

but make sure that learners differentiate between the height of the base of the triangular base and the height of the prism.

In the same way, the area of any polygonal base has to be calculated first and then multiplied by the height of the prism.

Volume is measured in cubic units, for example centimetres (cm^3) and cubic metres (m^3).

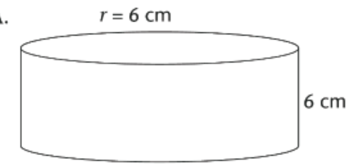
Surface area of a cylinder = Area of all its surfaces
= Area of top + Area of base + Area of curved surface
$= \pi r^2 + \pi r^2 + (l \times b)$
$= 2\pi r^2 + (2\pi r \times h)$
$= 2\pi r(r + h)$

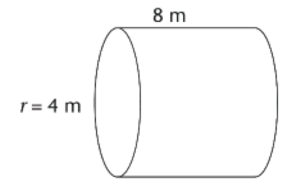
Can you explain why the length of the rectangle is equal to the circumference of the top or base of the cylinder?

CALCULATING THE SURFACE AREA OF CYLINDERS

From the formula above, you can see that we need only know the radius (r) and the height (h) of a cylinder in order to work out its surface area.

1. Calculate the surface areas of the following objects. Use π = 3,14 and round off all your answers to two decimal places.

A.


B.


2. Calculate the surface area of a cylinder if its height is 60 cm and the circumference of its base is 25,12 cm.
3. Calculate the surface area of a cylinder if its height is 5 m and the circumference of its base is 12,56 m.
4. The outside of a cylindrical structure at a factory must be painted. Its radius is 3,5 m and its height is 8 m. How many litres of paint must be bought if 1 litre covers 10 m^2? (The bottom of the structure will not be painted.)

19.2 Volume

The **volume** of an object is the amount of space it occupies. We usually measure volume in cubic units, such as mm^3, cm^3 and m^3.

To convert between cubic units, remember:
$1 \text{ cm}^3 = 1\ 000 \text{ mm}^3$
$1 \text{ m}^3 = 1\ 000\ 000 \text{ cm}^3$

FORMULAE FOR VOLUME OF PRISMS

The general formula for the volume of a prism is:
Volume of a prism = Area of base × height.

In case of a triangular prism, do not confuse the height of the base of the triangle (h_b) with the height of the prism (h_p).

212 MATHEMATICS GRADE 9: TERM 3

CALCULATING THE VOLUME OF PRISMS

Teaching guidelines

In question 1 the calculations are straightforward applications of the formulae.

In question 2(b) learners have to solve the equation $V = s^3$ where V is known, therefore they have to think about how to find s. They should know that the cube root of V has to be calculated.

In question 3 they divide the prism into two parts, calculate the volumes and add them. They should get: $V = 14 \times 6 \times 12 + 6 \times 6 \times 5 = (14 \times 12 + 5 \times 6) \times 6$ when factorised. They could also see the object as a prism with length 20, breadth 6 and height 12 from which a piece 6 by 6 by 7 has been cut. Then the calculation would be: $V = 20 \times 6 \times 12 - 6 \times 6 \times 7$.

Learners should see that on LB page 214 in question 3B the base of the prism is a combined figure consisting of a square and an isosceles triangle. The height of the triangle has to be calculated using the Theorem of Pythagoras. The hypotenuse of the triangle is 10 and the one right-angled side is 6. Learners may know without calculation that the third side is 8. This is a good opportunity to discuss and revise the 3, 4, 5 right-angled triangle.

Answers

1. A. $V = 12 \times 8 \times 5$
   $= 480\ \text{cm}^3$

   B. $V = \frac{1}{2} \times 2 \times 1{,}5 \times 4$
   $= 6\ \text{m}^3$

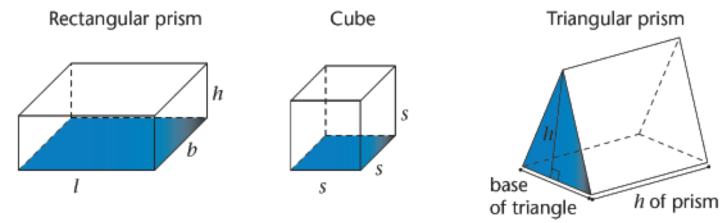
   C. $V = 15 \times 15 \times 15$
   $= 3\,375\ \text{m}^3$

   D. $V = \frac{1}{2} \times 60 \times 40 \times 70$
   $= 84\,000\ \text{mm}^3$

2. (a) $V = 32 \times 12$
   $= 384\ \text{m}^3$

   (b) Length of edge $= \sqrt[3]{216}$
   $= 6\ \text{m}$

Therefore, the formulas to work out the volumes of the following prisms are:



$V = (l \times b) \times h$

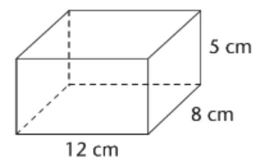
$V = (s \times s) \times s$
$= s^3$

$V = (\frac{1}{2}\text{ base} \times h_b) \times h_p$

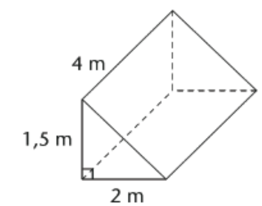
CALCULATING THE VOLUME OF PRISMS

1. Calculate the volumes of the following prisms:

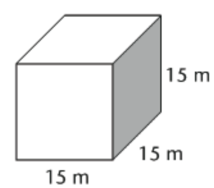
A.



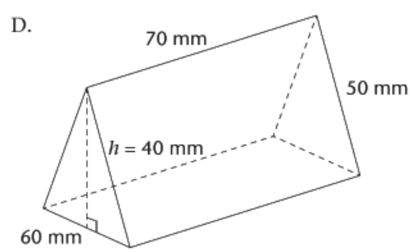
B.



C.



D.



2. (a) The area of the base of a rectangular prism is 32 m² and its height is 12 m. What is its volume?
   (b) The volume of a cube is 216 m³. What is the length of one of its edges?

Answers

3. A. V = Area of base × height
$= (14 \times 12 + 5 \times 6) \times 6$
$= 1\ 188\ \text{cm}^3$

B. V = Area of base × height
$= (144 + \frac{1}{2} \times 12 \times 8) \times 25$
$= 4\ 800\ \text{cm}^3$

VOLUME OF CYLINDERS

Teaching guidelines

Remind learners how to calculate the volume of a cube or cuboid (prism), i.e. area of the base times the height.

Explain that the same formula must be applied to find the volume of a cylinder: V = area of base × height where the area of the base = πr^2.

Learners must be careful that they do not inadvertently use the diameter and not the radius. If the diameter is given, they must divide it by 2.

Answers

1. A. $V = \pi r^2 \times h$
$= 3{,}14 \times 5 \times 5 \times 15$
$= 1\ 177{,}5\ \text{cm}^3$

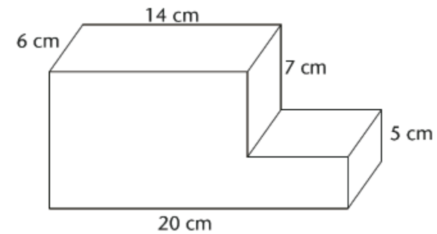
B. $V = \pi r^2 \times h$
$= 3{,}14 \times 9 \times 12$
$= 339{,}12\ \text{cm}^3$

C. $V = \pi r^2 \times h$
$= 3{,}14 \times 7{,}5 \times 7{,}5 \times 10$
$= 1\ 766{,}25\ \text{m}^3$

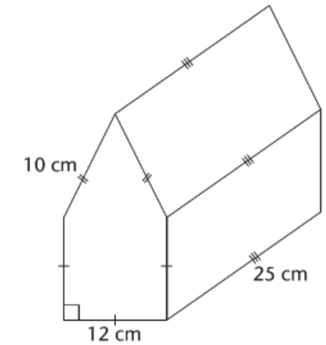
D. $V = \pi r^2 \times h$
$= 3{,}14 \times 7 \times 7 \times 9$
$= 1\ 384{,}74\ \text{m}^3$

3. Calculate the volume of the following objects:

A.



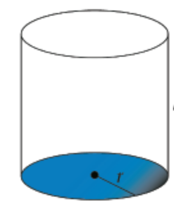
B.



VOLUME OF CYLINDERS

You also calculate the volume of a cylinder by multiplying the area of the base by the height of the cylinder. The base of a cylinder is circular, therefore:

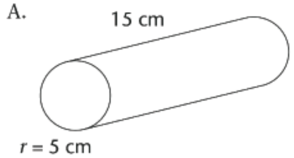
Volume of a cylinder = Area of base × h
$= \pi r^2 \times h$



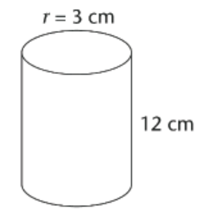
Area of circle = πr^2

1. Calculate the volume of the following cylinders. Use $\pi = 3{,}14$ and round off all answers to two decimal places.

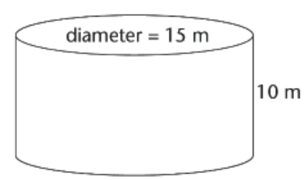
A.



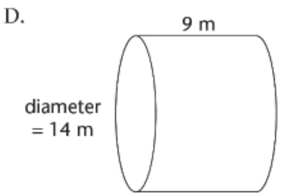
B.



C.



D.



Answers

2. (a) $V = \frac{22}{7} \times 14 \times 14 \times 20$
$= 22 \times 2 \times 14 \times 20$
$= 12\ 320\ \text{cm}^3$

(b) $V = \frac{22}{7} \times 7 \times 7 \times 35$
$= 22 \times 7 \times 35$
$= 5\ 390\ \text{cm}^3$

(c) $V = \frac{22}{7} \times 14 \times 14 \times 50$
$= 22 \times 100 \times 14$
$= 30\ 800\ \text{cm}^3$

(d) $V = \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 35$
$= 11 \times 7 \times 5$
$= 385\ \text{cm}^3$

3. See the answers on LB page 215 alongside.

19.3 Capacity

Teaching guidelines

The volume is the amount of space that an object takes up, while capacity is the amount of content that a container can hold if it is filled to the brim.

An object can have both volume and capacity.

Volume is measured in cubic centimetres (cm^3) and capacity is measured in millilitres. A millilitre is the content of a cube with sides that are 1 cm each. So 1 ml = 1 cm^3 and 1 000 ml = 1 ℓ.

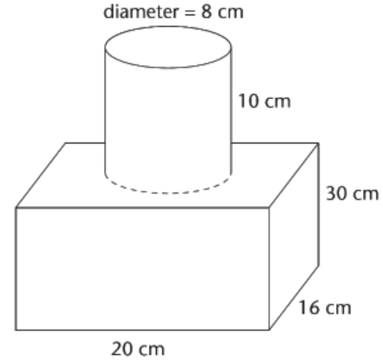
The following conversions apply:

$1\ \text{m}^3 = 100 \times 100 \times 100\ \text{cm}^3$
$= 1\ 000\ 000\ \text{cm}^3$
$= 1\ 000\ 000$ ml
$= 1\ 000$ ℓ
= 1 kilolitre (kl)

1 m = 100 cm
$1\ \text{cm}^3 = 1$ ml
1 000 ml = 1 ℓ
1 000 ℓ = 1 kl

2. Without using a calculator, calculate the volume of cylinders with the measurements given below. Use $\pi = \frac{22}{7}$.
(a) $r = 14$ cm; $h = 20$ cm
(b) $r = 7$ cm; $h = 35$ cm
(c) diameter = 28 cm; $h = 50$ cm
(d) diameter = 7 cm; $h = 10$ cm

3. Calculate the volume of the following object. Use a calculator and round off all answers to two decimal places.



V cylinder $= \pi r^2 h$
$= 3{,}14 \times 4 \times 4 \times 10$
$= 502{,}40\ \text{cm}^3$

V prism $= lbh$
$= 20 \times 16 \times 30$
$= 9\ 600\ \text{cm}^3$

V object $= 502{,}40 + 9\ 600$
$= 10\ 102{,}40\ \text{cm}^3$

19.3 Capacity

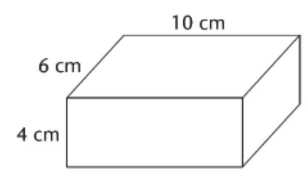
Remember that the **capacity** of an object is the amount of space *inside* the object. You can think of the capacity of an object as the amount of liquid that the object can hold.

The **volume** of an object is the amount of space that the object itself takes up.

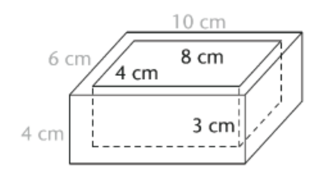
The volume of a solid block of wood is 10 cm × 6 cm × 4 cm = 240 cm^3.

The same block of wood is carved out to make a hollow container with inside measurements of 8 cm × 4 cm × 3 cm. (Its walls are 1 cm thick.) The amount of space inside the container must be calculated using the *inside* measurements. So, the capacity of the container is 8 cm × 4 cm × 3 cm = 96 cm^3.

A. Solid block with outside measurements



B. Hollowed block with inside measurements



Answers

1. $V = 96$ ml
2. Capacity $= 5 \times 9 \times 3{,}5\ \text{cm}^3 = 157{,}5$ ml
3. Capacity $= 9 \times 4 \times 2\ \text{m}^3 = 72$ kl

19.4 Doubling dimensions and the effect on volume

DOUBLING THE DIMENSIONS OF A PRISM

Teaching guidelines

You can show learners how doubling dimensions influences the volume of a prism.

For example, for a cuboid: length l, breadth b and height h; $V = lbh$.

- Double the length to $2l$, then the new volume is $V_1 = 2lbh = 2V$.
- Double the length to $2l$ and the breadth to $2b$, then the new volume is $V_2 = 2l \times 2b \times h = 4lbh = 4V$.
- Double all the dimensions to $2l$, $2b$ and $2h$, then the new volume is $V_3 = 2l \times 2b \times 2h = 8lbh = 8V$.

Misconceptions

Learners think that doubling the dimensions will lead to doubling of the volume.

Answers

1. $V = 3 \times 5 \times 2 = 30\ \text{cm}^3$ $V = 3 \times 10 \times 2 = 60\ \text{cm}^3$
   $V = 6 \times 10 \times 2 = 120\ \text{cm}^3$ $V = 6 \times 10 \times 4 = 240\ \text{cm}^3$
2. See the answers on LB page 216 alongside.

1. Write, in ml, the volume of water that would fill container B.
2. If the walls and bottom of container B were 0,5 cm thick, what would its capacity be? Write the answer in ml.
3. The inside measurements of a swimming pool are 9 m × 4 m × 2 m. What is the capacity of the pool in kl?

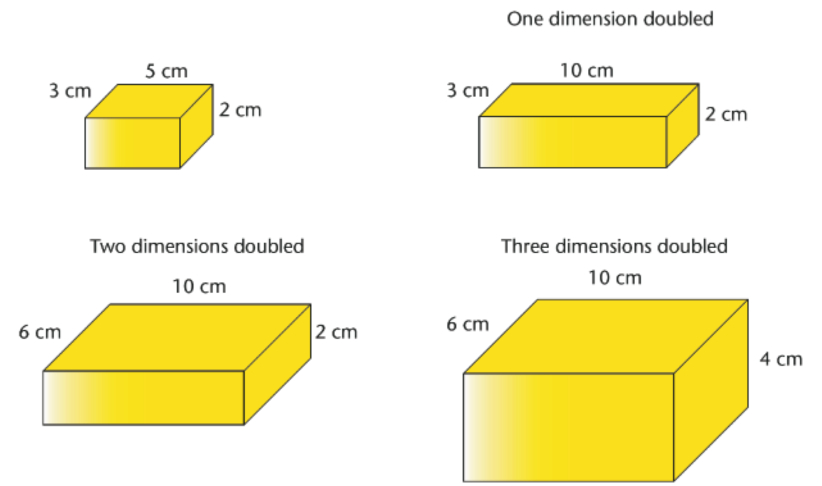
Remember:
$1\ \text{cm}^3 = 1$ ml
$1\ \text{m}^3 = 1$ kl

19.4 Doubling dimensions and the effect on volume

DOUBLING THE DIMENSIONS OF A PRISM

The first prism below measures 5 cm × 3 cm × 2 cm. The other diagrams show the prism with one or more of its dimensions doubled.

1. Work out the volume of each prism.



2. Copy and complete the following:
   (a) When one dimension of a prism is doubled, the volume doubles.
   (b) When two dimensions of a prism are doubled, the volume increases by 4 times.
   (c) When all three dimensions of a prism are doubled, the volume increases by 8 times.

Answers

3. See the answers on LB page 217 alongside.

DOUBLING THE DIMENSIONS OF A CYLINDER

Teaching guidelines

The same principles apply to the doubling of the dimensions of a cylinder. For a cylinder with radius r and height h, the volume is $V = \pi r^2 h$.

- Double the height to $2h$, then the new volume is $V_1 = \pi r^2 \times 2h = 2\pi r^2 h = 2V$.
- Double the radius to $2r$, then the new volume is $V_2 = \pi(2r)^2 \times h = 4\pi r^2 h = 4V$.
- Double the height to $2h$ and the radius to $2r$, then the new volume is $V_3 = \pi(2r)^2 \times 2h = \pi \times 4r^2 \times 2h = 8\ \pi r^2 h = 8V$.

Misconceptions

Learners think that doubling the dimensions will lead to doubling of the volume.

Answers

1. $V = \frac{22}{7} \times 7 \times 7 \times 2 = 308 \text{ cm}^3$ $\quad V = \frac{22}{7} \times 7 \times 7 \times 4 = 616 \text{ cm}^3$

   $V = \frac{22}{7} \times 14 \times 14 \times 2 = 1\,232 \text{ cm}^3$ $\quad V = \frac{22}{7} \times 14 \times 14 \times 4 = 2\,464 \text{ cm}^3$

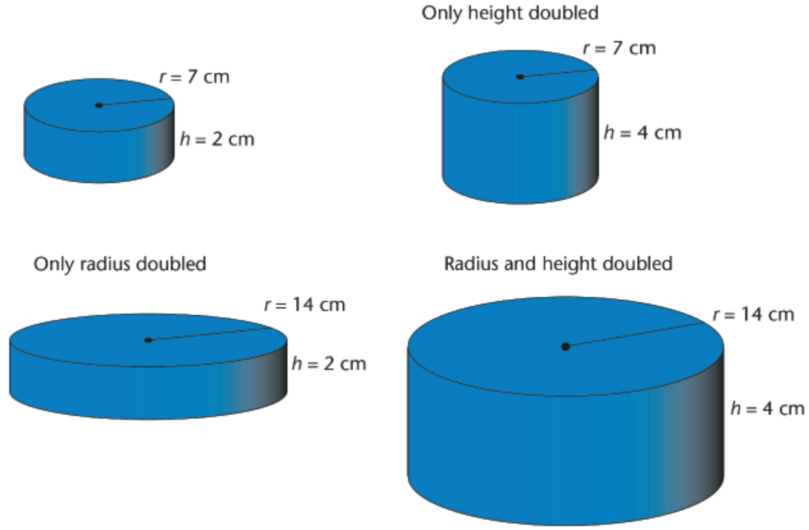
2. See the answers on LB page 217 alongside.
3. See the answers on LB page 217 alongside.

3. The volume of a prism is 80 cm³. What is its volume if:
   (a) its length is doubled? 160 cm³
   (b) its length and breadth are doubled? 320 cm³
   (c) its length, breadth and height are doubled? 640 cm³

DOUBLING THE DIMENSIONS OF A CYLINDER

The first cylinder below has a radius of 7 cm and a height of 2 cm. The other diagrams show the cylinder with one or more of its dimensions doubled.

1. Work out the volume of each cylinder:



2. Copy and complete the following:
   (a) When the height of a cylinder is doubled, the volume doubles.
   (b) When the radius of a cylinder is doubled, the volume increases by 4 times.
   (c) When height and radius of cylinder are doubled, the volume increases by 8 times.

3. The volume of a cylinder is 462 cm³. What is its volume if:
   (a) its height is doubled? 924 cm³
   (b) its radius is doubled? 1 848 cm³
   (c) its height and radius are doubled? 3 696 cm³

Answers

4. (a) See the answers on LB page 218 alongside.

   (b) I looked at how the dimensions changed and worked out the new volume. The volume changes depending on which measurements change:

   - doubling the height gives twice the volume
   - doubling the radius gives four times the volume, and
   - doubling the height and the radius gives eight times the volume.

4. (a) Study the following tables and copy them. Without using the formulas to calculate volume, complete the last column in each table. (Hint: Identify which dimensions are doubled each time, then work out the volume accordingly.)

Rectangular prism			
Length (l) in m	Breadth (b) in m	Height (h) in m	Volume (V) in m^3
4	2	1	8
4	4	1	16
8	2	1	16
8	2	2	32
8	4	2	64

Cylinder		
Radius (r) in m	Height (h) in m	Volume (V) in m^3
3,5	4	154
7	4	616
3,5	8	308
7	8	1 232

(b) Explain how you worked out the answers in the tables.

Grade 9 Term 3 Chapter 20 Transformation geometry

Learner Book Overview		
Sections in this chapter	**Content**	**Pages in Learner Book**
20.1 Points on a coordinate system	The Cartesian coordinate system; origin; coordinates; ordered pairs; quadrants	Pages 219 to 220
20.2 Reflection (flip)	Reflection of points in the *x*-axis, *y*-axis and the line $y = x$; reflection of geometric figures in the *x*-axis, *y*-axis and the line $y = x$	Pages 220 to 223
20.3 Translation (slide)	Translation of points on a coordinate system; translation of geometric figures on a coordinate system	Pages 223 to 227
20.4 Enlargement (expansion) and reduction (shrinking)	Scale factor; influence of scale factor on perimeter and area of geometric figures; the origin as centre of enlargement and reduction	Pages 227 to 234

CAPS time allocation	9 hours
CAPS content specification	Page 147

Mathematical background

- **Transformation geometry** deals with the operations that may be used on a figure to affect its position, size or shape, or any combination thereof.
- The **figure** (also referred to as the **object**) is the original shape before transformation is applied.
- The **image** is the shape which appears after transformation has been applied to the figure.
- Transformations that affect the **position** of a figure are translations, reflections and rotations:
  - During a **translation** every point in the figure is shifted in the **same direction** over the **same distance**.
  - During a **reflection** every point in the figure is flipped **perpendicularly** over a **line of reflection** (mirror line) so that the point and its image are the **same distance** from the line of reflection.
  - During a **rotation** every point in the figure is turned **clockwise** or **anti-clockwise** through the same angle about a fixed point, the **centre of rotation**, so that the point and its image are the **same distance** from the centre of rotation.
- Transformations that affect the **size** of a figure are enlargements and reductions:
  - During an **enlargement** every side of a figure is multiplied by a positive number bigger than 1 to produce an image larger than the figure.
  - During a **reduction** every side of a figure is multiplied by a positive number smaller that 1 to produce an image smaller than the figure.
- Transformations that affect the **shape** of a figure are shears and stretches:
  - During a **shear** all points along a fixed line remain fixed while all other points are shifted parallel to the fixed line, for example, turning a square into a parallelogram.
  - During a **stretch** all points along a fixed line remain fixed while all other points are stretched away from the fixed line, for example, turning a square into a rectangle.

20.1 Points on a coordinate system

Background information

- A **coordinate system** consists of numbered horizontal and vertical lines that are used to describe position.
    - The **point of intersection** of the numbered lines is called the **origin**.
    - The **horizontal numbered line** is called the ***x*-axis**.
    - The **vertical numbered line** is called the ***y*-axis**.
    - The coordinate system is divided into **four quadrants** by the system of axes.

      first quadrant: $x > 0$ and $y > 0$ second quadrant: $x < 0$ and $y > 0$

      third quadrant: $x < 0$ and $y < 0$ fourth quadrant: $x > 0$ and $y < 0$

Teaching guidelines

- The **position of a point** in a coordinate system is described by using an **ordered pair of coordinates (*x*; *y*)**. Refer to LB page 219 alongside.
- The ordered pair of coordinates A(4; 3) indicates that:
    - the value of the *x*-coordinate of point A is 4, which means that point A lies four units to the right of the *y*-axis, and
    - the value of the *y*-coordinate of point A is 3, which means that point A lies three units above the *x*-axis.
- The ordered pair of coordinates B(4; –3) indicates that:
    - the value of the *x*-coordinate of point B is 4, which means that point B lies four units to the right of the *y*-axis, and
    - the value of the *y*-coordinate of point A is –3, which means that point B lies three units below the *x*-axis.

Misconceptions

Learners often plot the coordinates of a point the wrong way around. The problem can be solved if the following strategy is followed:

- **ALWAYS start** at the **origin**.
- **From the origin**, perform the **horizontal translation**: If *x* is positive, move to the right. If *x* is negative, move to the left.
- **From that point on the *x*-axis**, perform the **vertical translation**:

  If *y* is positive, move upwards. If *y* is negative, move downwards.

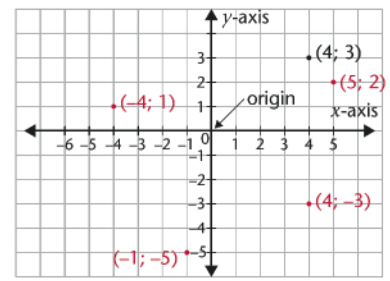
CHAPTER 20
Transformation geometry

20.1 Points on a coordinate system

A rectangular coordinate system is also called a **Cartesian coordinate system**. It consists of a horizontal *x*-axis and a vertical *y*-axis.

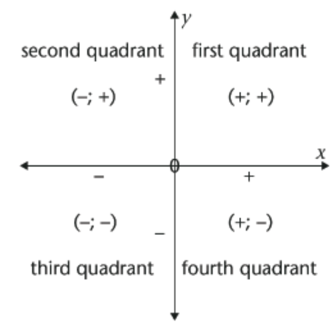
The intersection of the axes is called the **origin**, and represents the point (0; 0).

Any point can be represented on a coordinate system using an *x*-value and a *y*-value. These numbers are called **coordinates**, and describe the position of the point with reference to the two axes.



The coordinates of a point are always written in a certain order:

- The horizontal distance from the origin (*x*-coordinate) is written first.
- The vertical distance from the origin (*y*-coordinate) is written second.
- These numbers, called an **ordered pair**, are separated by a semi-colon (;) and are placed between brackets. Here is an example of an ordered pair: (4; 3) (see on the coordinate system above).
- The *x*-axis and *y*-axis divide the coordinate system into four sections called **quadrants**. The diagram alongside shows how the quadrants are numbered, and also whether the *x*- and *y*-coordinates are negative or positive in each quadrant.



CHAPTER 20: TRANSFORMATION GEOMETRY 219

Answers

1. See the answers on LB page 220 alongside.
2. See the points given on first coordinate system on LB page 219 on the previous page.

20.2 Reflection (flip)

REFLECTING POINTS IN THE x-AXIS, y-AXIS AND THE LINE $y = x$

Background information

- **Transformations** are operations that may be used on a figure to affect its position, size or shape, or any combination thereof. Figures can, **without changing their size or shape**, be moved around by using reflection, translation or rotation.
- A **reflection** is a "flip-over" across a mirror line called the **line of reflection**. These are the **properties of reflection**:
  - An object and its image lie on **opposite sides** of the line of reflection.
  - The **distance** from the original point to the line of reflection is the same as the distance from the image point to the line of reflection.
  - The line that connects the original point to its image point is always **perpendicular** to the line of reflection.
  - When a figure is reflected, the figure and its image are **congruent**.
- Any line on the coordinate system can be a line of reflection, including the ***x*-axis**, the ***y*-axis** and the line $y = x$.

Teaching guidelines

- Learners recall the properties of reflection listed above.
- Remind learners the symbol ⊥ means "is perpendicular to".

Answers

1. See the answers on the coordinate system on LB page 220 alongside.

---

1. In which quadrant will the following points be plotted?
   (a) (–4; 1) second quadrant (b) (–1; –5) third quadrant
   (c) (4; –3) fourth quadrant (d) (5; 2) first quadrant
2. Copy the first coordinate system given on the previous page and plot the points in question 1 on it.

When a point is translated to a different position on a coordinate system, the new position is called the image of the point. We use the prime symbol (') to indicate an image. For example, the image of A is indicated by A' (read as "A prime"). If the coordinates of A are labelled as $(x; y)$, the coordinates of A' can be labelled as $(x'; y')$.

We write A → A' and $(x; y) \rightarrow (x'; y')$ to indicate that A is mapped to A'.

20.2 Reflection (flip)

The **mirror image** or **reflection** of a point is on the opposite side of a **line of reflection**.

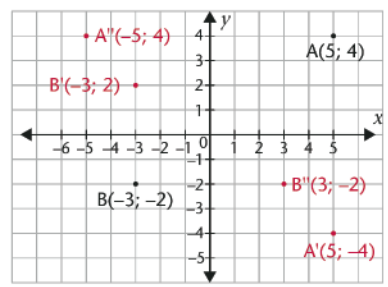
"Reflecting a point in the x-axis" means that the x-axis is the line of reflection.

The original point and its mirror image are the same distance away from the line of reflection, and the line that joins the point and its image is perpendicular to the line of reflection.

Any line on the coordinate system can be a line of reflection, including the x-axis, the y-axis and the line $y = x$.

REFLECTING POINTS IN THE x-AXIS, y-AXIS AND THE LINE $y = x$

1. The points A(5; 4) and B(–3; –2) are plotted on a coordinate system. Copy the coordinate system.
   (a) Reflect points A and B in the x-axis and write down the coordinates of the images.
   (b) Reflect points A and B in the y-axis and write down the coordinates of the images.
   (c) Compare the coordinates of the original points with those of its images. What do you notice?



When reflected in the x-axis, the y-value changes sign and the x-value stays the same. When reflected in the y-axis, the x-value changes sign and the y-value stays the same.

Background information (continued)

- For a **reflection in the x-axis**, the x-coordinate stays the same and the sign of the y-coordinate changes. We write: $(x; y) \rightarrow (x; -y)$ or $x' = x$ and $y' = -y$.

  Example: (4; –3) → (4; 3)
- For a **reflection in the y-axis**, the sign of the x-coordinate changes and the y-coordinate stays the same. We write: $(x; y) \rightarrow (-x; y)$ or $x' = -x$ and $y' = y$.

  Example: (4; –3) → (–4; –3)
- For a **reflection in the line $y = x$**, the values of the x- and y-coordinates are interchanged. We write: $(x; y) \rightarrow (y; x)$ or $x' = y$ and $y' = x$.

  Example: (4; –3) → (–3; 4)

Teaching guidelines (continued)

At the end of question 3, learners should be able to interpret the background information listed above.

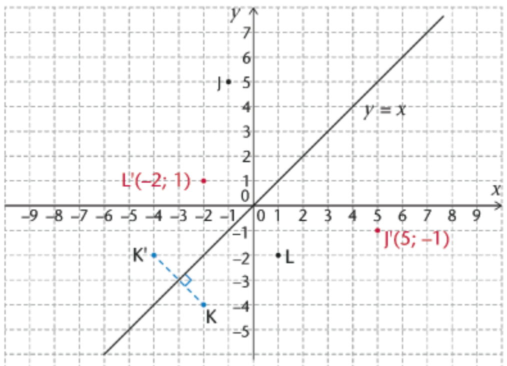
Answers

2. See the first table on LB page 221 alongside.
3. (a) See the coordinate system on LB page 221 alongside.
   (b) J'(5; –1)
   K'(–4; –2)
   L'(–2; 1)
   (c) The x- and y-values are interchanged.
   (d) See the second table on LB page 221 alongside.

2. Copy the table and write down the coordinates of the images of the following reflected points:

Point	Reflection in the x-axis	Reflection in the y-axis
(–131; 24)	(–131; –24)	(131; 24)
(–459; –795)	(–459; 795)	(459; –795)
$(x; y)$	$(x; -y)$	$(-x; y)$

3. The points J(–1; 5), K(–2; –4) and L(1; –2) are plotted on the coordinate system. K' is the reflection of point K in the line $y = x$. This means that the line $y = x$ is the line of reflection.
   (a) Reflect J and L in the line $y = x$.
   (b) Write down the coordinates of the images of the points.
   (c) What do you notice about the coordinates of the images of the points in (b) above?
   (d) Use your observation in (c) above to copy and complete this table.



Point	Coordinates of the image of the point reflected in $y = x$
(–1 001; –402)	(–402; –1 001)
(459; –795)	(–795; 459)
(–342; 31)	(31; –342)
(21; 67)	(67; 21)
$(x; y)$	$(y; x)$

While doing the previous activity, you may have noticed the following:

- For a reflection in the y-axis, the sign of the x-coordinate changes and the y-coordinate stays the same: $(x; y) \rightarrow (-x; y)$ or $x' = -x$ and $y' = y$, for example: (–3; 4) → (3; 4)

Answers

4. For a reflection in the line $y = -x$, the coordinates of the image of the point $(x; y)$ are $y' = -x$ and $x' = -y$.

   For example: $(-3; 4) \rightarrow (-4; 3)$

5. (a) $(-5; -2)$ (b) $(2; -5)$
   (c) $(-2; 5)$ (d) $(5; 2)$

REFLECTING GEOMETRIC FIGURES

Teaching guideline

Refer to the background information above. The same principles apply when geometric figures are reflected.

Answers

1. (a) See the coordinate system on LB page 222 alongside.
   (b) See the table on LB page 222 alongside.
   (c) They are exactly the same size and shape. They are congruent.

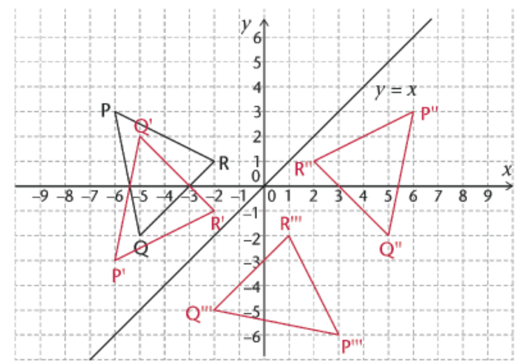
- For a reflection in the x-axis, the sign of the y-coordinate changes and the x-coordinate stays the same: $(x; y) \rightarrow (x; -y)$ or $x' = x$ and $y' = -y$, for example: $(-3; 4) \rightarrow (-3; -4)$
- For a reflection in the line $y = x$, the values of the x- and y-coordinates are interchanged: $(x; y) \rightarrow (y; x)$ or $x' = y$ and $y' = x$, for example: $(-3; 4) \rightarrow (4; -3)$.

4. Investigate the effect of reflection in the line $y = -x$ on the coordinates of a point.
5. A is the point $(5; -2)$. Write the coordinates of the mirror images of A if the point is reflected in:
   (a) the y-axis (b) the line $y = -x$
   (c) the line $y = x$ (d) the x-axis

REFLECTING GEOMETRIC FIGURES

The same principles, as above, apply when reflecting geometric figures.

1. (a) On graph paper, reflect ΔPQR in the x-axis, in the y-axis and in the line $y = x$ in the coordinate system (first reflect the vertices and then join the reflected points).



   (b) Copy the table below. Look at your completed reflections in question 1(a), and write down the coordinates of the image points in the table.

Vertices of triangle	Reflection in the x-axis	Reflection in the y-axis	Reflection in the line $y = x$
P(–6; 3)	P'(–6; –3)	P''(6; 3)	P'''(3; –6)
Q(–5; –2)	Q'(–5; 2)	Q''(5; –2)	Q'''(–2; –5)
R(–2; 1)	R'(–2; –1)	R''(2; 1)	R'''(1; –2)

   (c) What do you notice about ΔPQR, ΔP'Q'R', ΔP''Q''R'' and ΔP'''Q'''R'''?

Answers

2. See the coordinate system on LB page 223 alongside.
3. (a) A'(1; –4), B'(–6; –1), C'(–2; 1), D'(7; –2)
   (b) A''(–1; 4), B''(6; 1), C''(2; –1), D''(–7; 2)
   (c) A'''(4; 1), B'''(1; –6), C'''(–1; –2), D'''(2; 7)
4. See the answers on LB page 223 alongside.

20.3 Translation (slide)

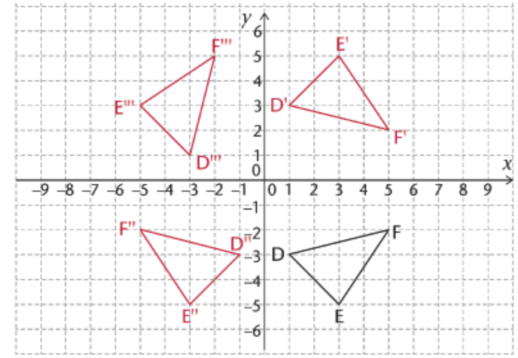
Background information

- A **translation** is a "slide" in a certain direction over a certain distance. These are the **properties of translation**:
  - The line segments that connect any original point in the object to its image are all **equal in length**.
  - The line segments that connect any original point in the object to its image are all **parallel**.
  - When a figure is translated, the figure and its image are **congruent**.

Teaching guidelines

- Learners recall the properties of translation.
- Remind learners that:
  - **parallel lines** are marked with arrowheads on the line segments
  - **equal line segments** are marked with tally marks on the line segments.

2. On graph paper, reflect ΔDEF in the x-axis, in the y-axis and in the line $y = x$.



3. A quadrilateral has the following vertices: A(1; 4), B(–6; 1), C(–2; –1) and D(7; 2). Without performing the actual reflections, write down the coordinates of the vertices of the image when the quadrilateral is:
   (a) reflected in the x-axis
   (b) reflected in the y-axis
   (c) reflected in the line $y = x$

4. In each case, state around which line the point was reflected.
   (a) (–4; 5) → (–4; –5) Reflection in x-axis
   (b) (2; –3) → (–2; –3) Reflection in y-axis
   (c) (–13; –3) → (–3; –13) Reflection in line $y = x$
   (d) (1; 16) → (16; 1) Reflection in line $y = x$
   (e) (12; –8) → (–12; –8) Reflection in y-axis
   (f) (–7; –5) → (–5; –7) Reflection in line $y = x$
   (g) (2; –3) → (–2; –3) Reflection in y-axis

20.3 Translation (slide)

Remember: A translation of a point or geometric figure on a coordinate system means moving or sliding the point in a vertical direction, in a horizontal direction, or in both a vertical and horizontal direction.

TRANSLATING POINTS HORIZONTALLY OR VERTICALLY ON A COORDINATE SYSTEM

Background information

A **translation** can take place in the following directions:

- **horizontally** to the left or to the right
- **vertically** upwards or downwards
- **horizontally** to the left/right **and vertically** upwards/downwards.

Teaching guidelines

Discuss how to translate a point from the origin so that it ends up:

- on the x-axis
- on the y-axis
- inside the first quadrant
- inside the second quadrant
- inside the third quadrant
- inside the fourth quadrant.

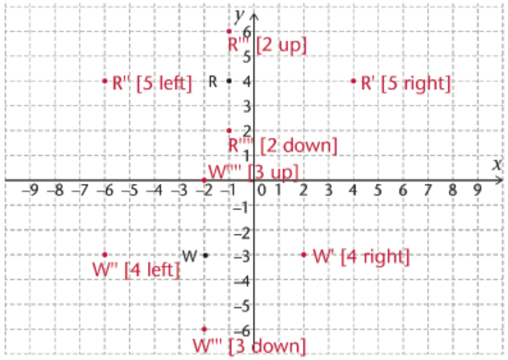
Answers

1. (a) See the coordinate system on LB page 224 alongside.
   (b) See the coordinate system on LB page 224 alongside.
   (c) See the answers on LB page 224 alongside.
   (d) See the answers on LB page 224 alongside and LB page 225 on the following page.

TRANSLATING POINTS HORIZONTALLY OR VERTICALLY ON A COORDINATE SYSTEM

1. Points R and W are plotted on a coordinate system.
   (a) On graph paper, plot the image of point R after a translation of:
   - five units to the right
   - five units to the left
   - two units up
   - two units down

   (b) On the same graph paper, plot the image of point W after a translation of:
   - four units to the right
   - four units to the left
   - three units up
   - three units down



   (c) Look at your completed translations in (a) and (b) above. Copy and complete the following table by writing down the coordinates of the original points and their images after each translation.

Coordinates of original points	**R(–1; 4)**	**W(–2; –3)**
Coordinates of image after a translation to the **right**	(4; 4)	(2; –3)
Coordinates of image after a translation to the **left**	(–6; 4)	(–6; –3)
Coordinates of image after a translation **up**	(–1; 6)	(–2; 0)
Coordinates of image after a translation **down**	(–1; 2)	(–2; –6)

   (d) Look at your completed table in (c) above. Choose the correct answers below to make each statement true:
   - For translations to the **right or left**, the (x-value/y-value) changes and the (x-value/y-value) stays the same.
   - For translations **up or down**, the (x-value/y-value) changes and the (x-value/y-value) stays the same.
   - For translations to the **right**, (add/subtract) the number of translated units (to/from) the x-value.

Background information (continued)

- For a **horizontal translation through the distance p**, the:
  - x-coordinate increases by p units and the y-coordinate stays the same if the slide is to the right ($p > 0$)
  - x-coordinate decreases by p units and the y-coordinate stays the same if the slide is to the left ($p < 0$).

  We write $(x; y) \rightarrow (x + p; y)$ or $x' = x + p$ and $y' = y$
- For a **vertical translation through the distance q**, the:
  - x-coordinate stays the same and the y-coordinate increases by q units if the slide is upwards ($q > 0$)
  - x-coordinate stays the same and the y-coordinate decreases by q units if the slide is downwards ($q < 0$).

  We write $(x; y) \rightarrow (x; y + q)$ or $x' = x$ and $y' = y + q$

Teaching guidelines (continued)

At the end of question 3 on LB page 225, learners should be able to interpret the background information listed above.

Answers

2. See the first table on LB page 225 alongside.
3. See the second table on LB page 225 alongside.

- For translations to the **left**, (add/subtract) the number of translated units (to/from) the x-value.
- For translations **up**, (add/subtract) the number of translated units (to/from) the y-value.
- For translations **down**, (add/subtract) the number of translated units (to/from) the y-value.

2. Copy the table and write down the coordinates of each image after the following translations:

Point	Three units to the right	Four units to the left	Two units up	Five units down
(3; 5)	(6; 5)	(–1; 5)	(3; 7)	(3; 0)
(–13; 42)	(–10; 42)	(–17; 42)	(–13; 44)	(–13; 37)
(–59; –95)	(–56; –95)	(–63; –95)	(–59; –93)	(–59; –100)
$(x; y)$	$(x + 3; y)$	$(x - 4; y)$	$(x; y + 2)$	$(x; y - 5)$

3. Copy the table and write down the coordinates of each image after the following translations:

Point	Four units to the right and three units up	Two units to the left and one unit up	One unit to the right and five units down	Six units to the left and two units down
(4; 2)	(8; 5)	(2; 3)	(5; –3)	(–2; 0)
(–32; 21)	(–28; 24)	(–34; 22)	(–31; 16)	(–38; 19)
(–68; –57)	(–64; –54)	(–70; –56)	(–67; –62)	(–74; –59)
$(x; y)$	$(x + 4; y + 3)$	$(x - 2; y + 1)$	$(x + 1; y - 5)$	$(x - 6; y - 2)$

While doing the previous activity, you may have noticed the following:

- For a horizontal translation through the distance p, the x-coordinate increases by the distance p if the slide is to the right, and decreases by the distance p if the slide is to the left. We may write $x' = x + p$, with $p > 0$ for a translation to the right, and $p < 0$ for a translation to the left. The y-coordinate remains the same, so $(x; y) \rightarrow (x + p; y)$.
- For a vertical translation through the distance q, the y-coordinate increases by the distance q if the slide is upwards, and decreases by the distance q if the slide is downwards. We may write $y' = y + q$, with $q > 0$ for a translation vertically upwards, and $q < 0$ for a translation vertically downwards. The x-coordinate remains the same, so $(x; y) \rightarrow (x; y + q)$.

CHAPTER 20: TRANSFORMATION GEOMETRY 225

TRANSLATION OF GEOMETRIC FIGURES ON A COORDINATE SYSTEM

Teaching guidelines

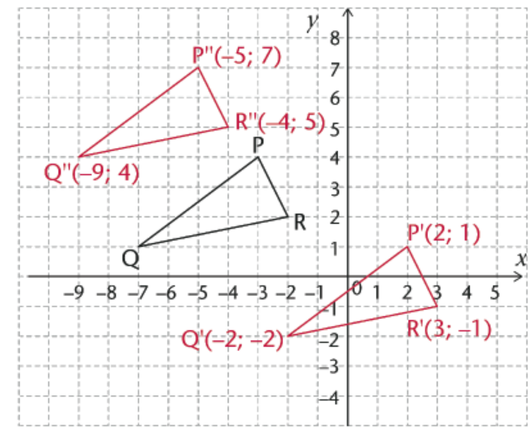
Refer to the background information above. The same principles apply when geometric figures are translated.

Answers

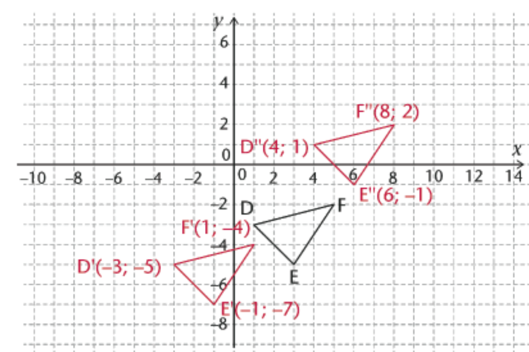
1. (a) See ΔP′Q′R′ on the first coordinate system on LB page 226 alongside.
   (b) See ΔP″Q″R″ on the first coordinate system on LB page 226 alongside.
   (c) Yes
2. (a) See ΔD′E′F′ on the second coordinate system on LB page 226 alongside.
   (b) See ΔD″E″F″ on the second coordinate system on LB page 226 alongside.
   (c) Yes
3. (a) K'(2; 4), L'(3; 0), M'(8; –1), N'(11; 5)
   (b) K'(0; 0), L'(1; –4), M'(6; –5), N'(9; 1)
   (c) K'(–9; –1), L'(–8; –5), M'(–3; –6), N'(0; 0)
   (d) K'(–7; 9), L'(–6; 5), M'(–1; 4), N'(2; 10)

TRANSLATION OF GEOMETRIC FIGURES ON A COORDINATE SYSTEM

1. (a) On graph paper, translate ΔPQR five units to the right and three units down.
   (b) Translate ΔPQR two units to the left and three units up.
   (c) Are all the triangles congruent?



2. (a) On graph paper, translate ΔDEF four units to the left and two units down.
   (b) Translate ΔDEF three units to the right and four units up.
   (c) Are all the triangles congruent?



3. The vertices of a quadrilateral have the following coordinates: K(–5; 2), L(–4; –2), M(1; –3) and N(4; 3). Write down the coordinates of the image of the quadrilateral after the following translations:
   (a) seven units to the right and two units up
   (b) five units to the right and two units down
   (c) four units to the left and three units down
   (d) two units to the left and seven units up

226 MATHEMATICS GRADE 9: TERM 3

Answers

4. (a) Two units down
   (b) Ten units to the left and seven units up
   (c) Thirteen units to the right and nine units up
   (d) Six units to the right and two units down
   (e) Ten units to the left and eight units up

20.4 Enlargement (expansion) and reduction (shrinking)

WHAT ARE ENLARGEMENTS AND REDUCTIONS?

Background information

- Some transformations **change the size but not the shape** of a figure.
  - An **enlargement** is the image of a figure which was made **bigger** by multiplying the lengths of all its sides by a number.
  - A **reduction** is the image of a figure which was made **smaller** by multiplying the lengths of all its sides by a number.
- A figure is only an enlargement or reduction of another figure if all the corresponding sides between the two figures are in **proportion**. This means that all the sides of the original figure are multiplied by the same number.
- The number by which the sides of a figure are multiplied is called the **scale factor**. Scale factor = $\frac{\text{side length of image}}{\text{length of corresponding side of original figure}}$
  - If the scale factor is a **positive number bigger than 1**, the image is an **enlargement** of the original figure.
  - If the scale factor is a **positive number smaller than 1**, the image is a **reduction** of the original figure.
- The original figure and its enlarged image are **similar**.
- The **perimeter of the image** is equal to the product of the perimeter of the original figure and the scale factor.
- The **area of the image** is equal to the product of the area of the original figure and the square of the scale factor.

Teaching guidelines

Use the diagrams on LB page 227 to revise the background information listed above.

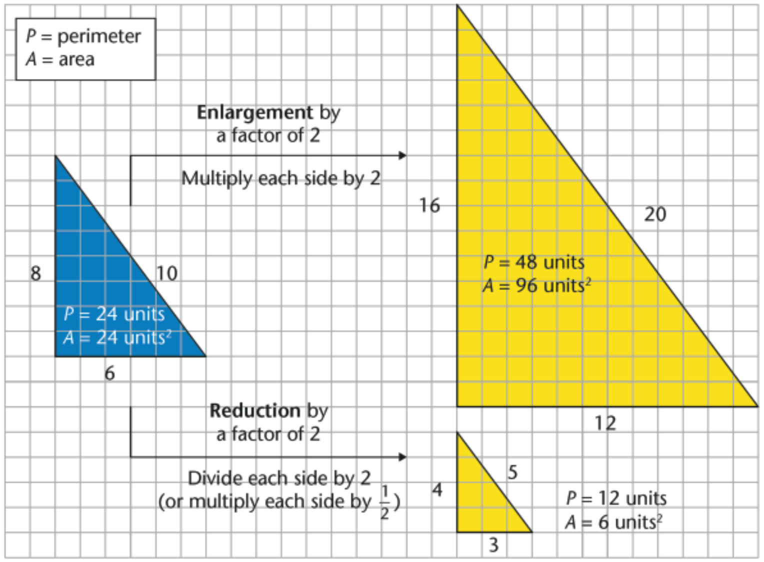
4. Describe the translation if the coordinates of the original point and the image point are:
   (a) (–2; –3) → (–2; –5) (b) (4; –7) → (–6; 0)
   (c) (3; 11) → (16; 20) (d) (–1; –2) → (5; –4)
   (e) (8; –11) → (–2; –3)

20.4 Enlargement (expansion) and reduction (shrinking)

WHAT ARE ENLARGEMENTS AND REDUCTIONS?

You will remember the following from Grade 8:

- An image is an enlargement or reduction of the original figure only if all the corresponding sides between the two figures are in **proportion**. This means that *all* the sides of the original figure are multiplied by the same number (the **scale factor**) to produce the image.
- Scale factor = $\frac{\text{side length of image}}{\text{length of corresponding side of original figure}}$
  - If the scale factor is > 1, the image is an enlargement.
  - If the scale factor is < 1, the image is a reduction.
- The original figure and its enlarged or reduced image are **similar**.
- Perimeter of image = Perimeter of original figure × scale factor
- Area of image = Area of original figure × (scale factor)^2



Background information (continued)

- **"Enlarge a figure by a scale factor of 2"** means the following:
  - Side of image = 2 × side of original figure.
  - Each **side of the image** will be **two times longer** than the corresponding side of the original figure.
  - The **perimeter of the image** will be **two times longer** than the perimeter of the original figure.
  - The **area of the image** will be 2^2 = **four times bigger** than the area of the original figure.
- **"Reduce a figure by a scale factor of 2"** means the following:
  - Side of image = $\frac{1}{2}$ × side of original figure
  - Each **side of the image** will be **two times shorter** than the corresponding side of the original figure.
  - The **perimeter of the image** will be **two times shorter** than the perimeter of the original figure.
  - The **area of the image** will be 2^2 = **four times smaller** than the area of the original figure, i.e. $\frac{1}{4}$ of the area of the original figure.

Teaching guidelines (continued)

Use the diagrams on LB page 227 to explain the background information listed above.

Misconceptions

Some learners misinterpret the meaning of "**reduce a figure by a scale factor of 2**". When calculating, they multiply the side of the original figure by 2 instead of $\frac{1}{2}$.

PRACTISE WORKING WITH ENLARGEMENTS AND REDUCTIONS

Teaching guidelines

Learners find the scale factor of some original figures and their images.

Answers

1. (a) See the answer on LB page 228 alongside.

Sometimes the terminology used for enlargements and reductions can be confusing. Make sure you understand the following examples. Refer to the diagram on the previous page.

"**Enlarge** a figure by a scale factor of 2" means:

- $\frac{\text{side length of image}}{\text{length of corresponding side of original figure}} = 2$.
- Each side of the original figure must be *multiplied* by 2.
- Each side of the image will be two times *longer* than its corresponding side in the original figure.
- The perimeter of the image will be two times *longer* than the perimeter of the original figure.
- The area of the image will be 2^2 times (2 × 2 = 4 times) *bigger* than the area of the original figure.

"**Reduce** a figure by a scale factor of 2" means:

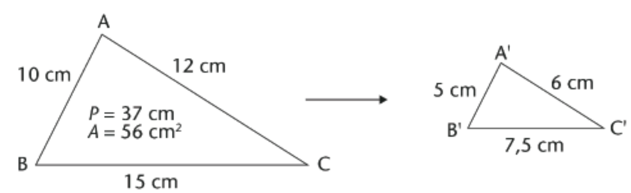
- $\frac{\text{side length of image}}{\text{length of corresponding side of original figure}} = 0{,}5$.
- Each side of the original figure must be *multiplied* by $\frac{1}{2}$ (or *divided* by 2).
- Each side of the image will be two times *shorter* than its corresponding side in the original figure.
- The perimeter of the image will be two times *shorter* than the perimeter of the original figure.
- The area of the image will be 2^2 times (2 × 2 = 4 times) *smaller* than the area of the original figure. (Or, area of image = $(\frac{1}{2})^2 = \frac{1}{4}$ of the area of the original figure.)

Note that the multiplicative inverse of 2 is $\frac{1}{2}$.

PRACTISE WORKING WITH ENLARGEMENTS AND REDUCTIONS

1. Work out the scale factor of each original figure and its image:

(a)



Scale factor = $\frac{1}{2}$

Note on question 2

Learners use the scale factor to:

- express the perimeter of the image as a fraction of the perimeter of the original figure
- calculate the perimeter of the image.

Note on question 3

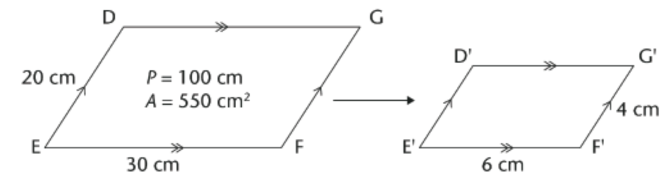
Learners use the scale factor to:

- express the area of the image as a fraction of the area of the original figure
- calculate the area of the image.

Answers

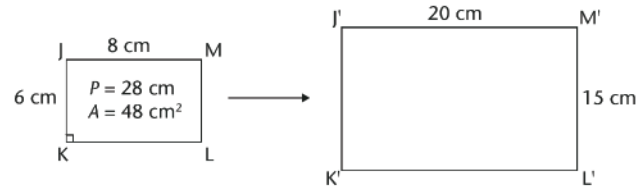
1. (b) See the answer on LB page 229 alongside.
   (c) See the answer on LB page 229 alongside.
2. (a) The perimeter of △A'B'C' is half of △ABC. Perimeter △A'B'C' = 18,5 cm.
   (b) The perimeter of D'E'F'G' is one fifth of DEFG. Perimeter D'E'F'G' = 20 cm.
   (c) The perimeter of J'K'L'M' is $2\frac{1}{2}$ times JKLM. Perimeter J'K'L'M' = 70 cm.
3. (a) The area of △A'B'C' is one quarter or $(\frac{1}{2})^2$ of △ABC. Area △A'B'C' = 14 cm^2.
   (b) The area of D'E'F'G' is one twenty-fifth or $(\frac{1}{5})^2$ of DEFG. Area D'E'F'G' = 22 cm^2.
   (c) The area of J'K'L'M' is 6,25 [or $(\frac{5}{2})^2 = \frac{25}{4}$] times JKLM. Area J'K'L'M' = 300 cm^2.
4. 60 cm
5. (a) 6 cm
   (b) 2 cm^2
6. (a) $\frac{3}{2} = 1\frac{1}{2}$
   (b) 31,5 cm^2
7. (a) $\frac{1}{2}$
   (b) 11 cm

(b)



Scale factor = $\frac{1}{5}$

(c)



Scale factor = $\frac{5}{2} = 2\frac{1}{2}$

2. For each set of figures in question 1, write down by how many times the *perimeter* of each image is longer or shorter than the perimeter of the original image. Also write down the perimeter of each image.
3. For each set of figures in question 1, write down by how many times the *area* of each image is bigger or smaller than the area of the original image. Also write down the area of each image.
4. The perimeter of rectangle DEFG = 20 cm. Write down the perimeter of the rectangle D'E'F'G' if the scale factor is 3.
5. The perimeter of quadrilateral PQRS = 30 cm and its area is 50 cm^2.
   (a) Find the perimeter of P'Q'R'S' if the scale factor is $\frac{1}{5}$.
   (b) Determine the area of quadrilateral P'Q'R'S' if the scale factor is $\frac{1}{5}$.
6. The perimeter of ΔDEF = 17 cm and the perimeter of ΔD'E'F' = 25,5 cm.
   (a) What is the scale factor of enlargement?
   (b) What is the area of ΔD'E'F' if the area of ΔDEF = 14 cm^2?
7. The area of ΔABC = 20 cm^2 and the area of ΔA'B'C' = 5 cm^2.
   (a) What is the scale factor of reduction?
   (b) What is the perimeter of the image if the perimeter of ΔABC = 22 cm?

INVESTIGATING ENLARGEMENT AND REDUCTION

Background information

- If an enlargement or reduction is drawn on a coordinate system and the corresponding vertices of the original figure and the image are joined by straight lines, all those lines cross at a common point called the **centre of enlargement or reduction**.
- The centre of enlargement or reduction can be **any point** on the coordinate system.
- In this chapter the **origin** is always used as the centre of enlargement or reduction.

Teaching guidelines

Use the diagram on LB page 230 to explain the concept of the centre of enlargement or reduction.

Answers

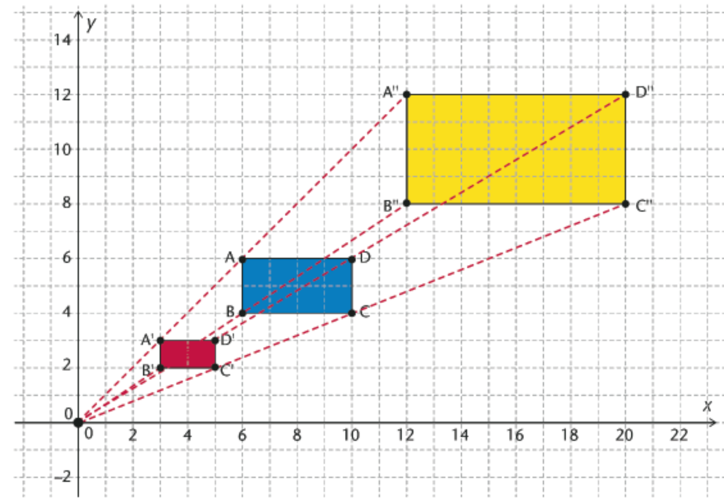
1. (a) Yes. It is enlarged by a factor of 2.
   (b) Yes. The scale factor is $\frac{1}{2}$.
2. (a) See the drawing on LB page 230 alongside.
   (b) They pass through the corresponding points on ABCD, A'B'C'D' and A''B''C''D''.

INVESTIGATING ENLARGEMENT AND REDUCTION

When we do enlargements or reductions on a coordinate system, we use one point from which to perform the enlargement or reduction. This point is known as the **centre of enlargement or reduction.**

The centre of enlargement or reduction can be any point on the coordinate system. In this chapter, we will always use the **origin** as the centre of enlargement or reduction.

Rectangle ABCD, rectangle A'B'C'D' and rectangle A"B"C"D" are plotted on a coordinate system as shown below:



1. (a) Is rectangle A"B"C"D" an enlargement of rectangle ABCD? Explain your answer.
   (b) Is rectangle A'B'C'D' a reduction of rectangle ABCD? Explain your answer.
2. (a) The origin is the centre of enlargement and reduction. Draw four line segments to join the origin with A", B", C" and D".
   (b) What do you notice about these line segments?

Background information (continued)

- When an **enlargement** is drawn **on a coordinate system**:
  - the **line** that joins the centre of enlargement to a vertex of the original figure also passes through the corresponding vertex of the enlarged image
  - the **coordinates** of the vertex of the enlarged image are equal to the scale factor × the coordinates of the corresponding vertex of the original figure. We write:

    $(x; y) \rightarrow (kx; ky)$ or $x' = kx$ and $y' = ky$ where k is the scale factor and $k > 1$
- When a **reduction** is drawn **on a coordinate system**:
  - the **line** that joins the centre of reduction to a vertex of the original figure also passes through the corresponding vertex of the reduced image
  - the **coordinates** of the vertex of the reduced image are equal to the scale factor × the coordinates of the corresponding vertex of the original figure. We write:

    $(x; y) \rightarrow (kx; ky)$ or $x' = kx$ and $y' = ky$ where k is the scale factor and $0 < k < 1$

Teaching guidelines (continued)

At the end of question 3 on LB page 231 learners should be able to interpret the background information listed above.

Answers

3. (a) See the table on LB page 231 alongside.

   (b) The x- and y-values of A'B'C'D' are half of those of ABCD.

   The x- and y-values of A''B''C''D'' are double those of ABCD.

3. (a) Copy the following table and list the coordinates of the images to complete it:

Vertices of ABCD	Vertices of A'B'C'D'	Vertices of A"B"C"D"
A(6; 6)	A'(3; 3)	A"(12; 12)
B(6; 4)	B'(3; 2)	B"(12; 8)
C(10; 4)	C'(5; 2)	C"(20; 8)
D(10; 6)	D'(5; 3)	D"(20; 12)

(b) What do you notice about the coordinates of the vertices of the original rectangle and the coordinates of the vertices of the image?

From the previous activity, you should have found the following:

On a coordinate system, the line that joins the centre of an enlargement or reduction to a vertex of the original figure also passes through the corresponding vertex of the enlarged or reduced image.

The coordinates of a vertex of the enlarged or reduced image are equal to the scale factor × the coordinates of the corresponding vertex of the original figure.

For example:

B(6; 4) → B' (3; 2): The coordinates of B' are $\frac{1}{2}$ the coordinates of B. Note that the scale factor is $\frac{1}{2}$.

B(6; 4) → B" (12; 8): The coordinates of B" are two times the coordinates of B. Note that the scale factor is 2.

In general, we therefore use the following notation to describe the enlargement or reduction with respect to the origin:

$(x; y) \rightarrow (kx; ky)$ or $(x'; y') = (kx; ky)$ where k is the scale factor.

If $0 < k < 1$, the image is a reduction.

If $k > 1$, the image is an enlargement.

PRACTISE

Teaching guidelines

Learners use the origin as the centre of enlargement or reduction to solve problems on enlargement and reduction.

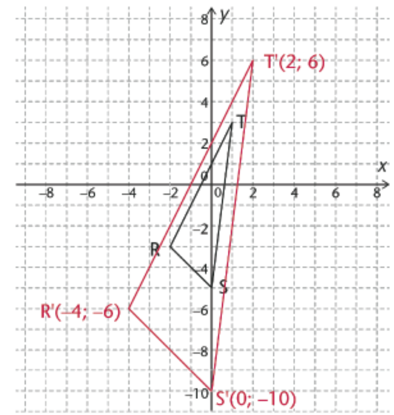
Answers

1. (a) See the first coordinate system on LB page 232 alongside.
   (b) See the second coordinate system on LB page 232 alongside.

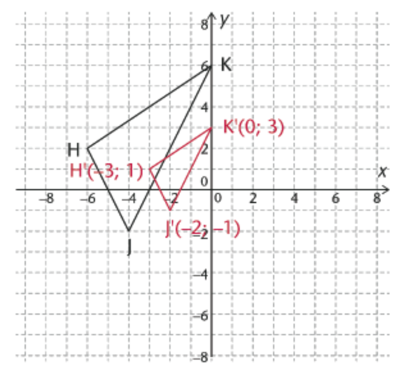
PRACTISE

1. On grid paper, draw the enlarged or reduced images of the following figures according to the scale factor given. In each case, use the **origin as the centre of enlargement or reduction**.

   (a) Scale factor = 2



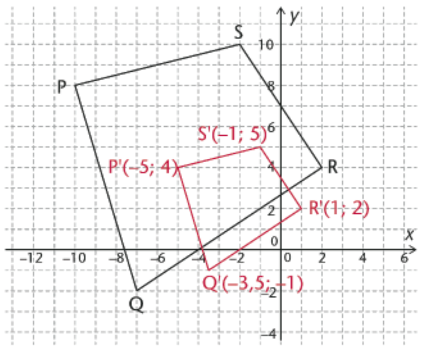
   (b) Scale factor = $\frac{1}{2}$



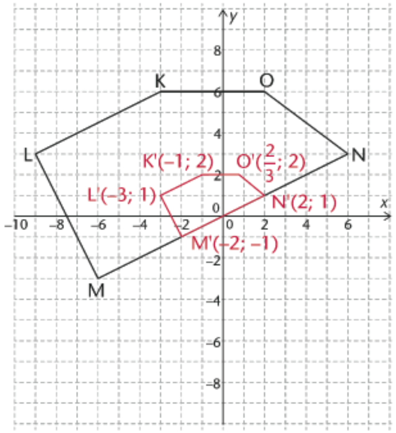
Answers

1. (c) See the first coordinate system on LB page 233 alongside.
   (d) See the second coordinate system on LB page 233 alongside.
2. A'(–4; 8), B'(–8; –4), C'(8; –6), D'(4; 2)
3. P'(–16; 0), Q'(10; 18), R'(24; –9), S'(8; –16)

(c) Scale factor = $\frac{1}{2}$



(d) Scale factor = $\frac{1}{3}$



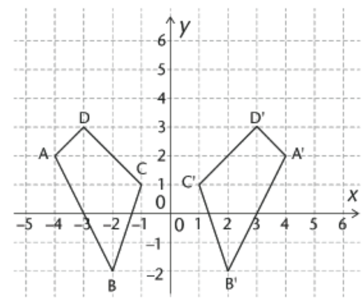
2. A quadrilateral has the following vertices: A(–2; 4), B(–4; –2), C(4; –3) and D(2; 1). Determine the coordinates of the enlarged image if the scale factor = 2.
3. A quadrilateral has the following vertices: P(–4; 0), Q(2,5; 4,5), R(6; –2,25) and S(2; –4). Determine the coordinates of the enlarged image if the scale factor = 4.

Answers

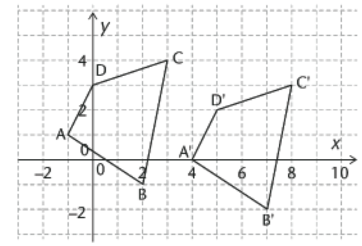
4. D'(3; –2), E'(2; –3), F'(–2; 1), G'(–1; –1)
5. K'(2; $-\frac{1}{2}$), L'(1; $-\frac{3}{2}$), M'(–2; –1) and N'($-\frac{3}{2}$; $\frac{5}{2}$)
6. (a) Move two units to the right and five units up.
   (b) Reflection in the y-axis OR move eight units to the right.
   (c) Reflection in the line $y = x$ OR move one unit to the right and one unit down.
   (d) Reflection in the x-axis OR move two units down.
   (e) Enlargement, scale factor = 2 OR move four units to the right and two units down.
   (f) Reduction, scale factor = $\frac{1}{4}$ OR move nine units to the left and 12 units up.
   (g) Move five units to the left and four units down.
7. (a) Reflection in the y-axis.
   (b) Move five units to the right and one unit down.

4. A quadrilateral has the following vertices: D(6; –4), E(4; –6), F(–4; 2) and G(–2; –2). Determine the coordinates of the reduced image if the scale factor = $\frac{1}{2}$.
5. A quadrilateral has the following vertices: K(8; –2), L(4; –6), M(–8; –4) and N(–6; 10). Determine the coordinates of the reduced image if the scale factor = $\frac{1}{4}$.
6. Describe the following transformations:
   (a) A(7; –5) → A'(9; 0)
   (b) A(–4; 6) → A'(4; 6)
   (c) A(–3; –2) → A'(–2; –3)
   (d) A(8; 1) → A'(8; –1)
   (e) A(4; –2) → A'(8; –4)
   (f) A(12; –16) → A'(3; –4)
   (g) A(2; –1) → A'(–3; –5)
7. Describe each of the following transformations:

(a)



(b)



Grade 9 Term 3 Chapter 21 Geometry of 3D objects

Learner Book Overview		
Sections in this chapter	**Content**	**Pages in Learner Book**
21.1 Classifying 3D objects	Classification of prisms, pyramids, cylinders, spheres and cones	Pages 235 to 237
21.2 Nets and models of prisms and pyramids	Nets and models of prisms and pyramids	Pages 237 to 238
21.3 Platonic solids	Tetrahedron; hexahedron; octahedron; dodecahedron; icosahedron	Pages 238 to 241
21.4 Euler's formula	Number of faces + number of vertices – number of edges = 2	Pages 242 to 243
21.5 Cylinders	Properties of cylinders; nets of cylinders	Pages 243 to 245
21.6 Spheres	Properties of spheres; approximated nets of spheres	Pages 245 to 247

CAPS time allocation	9 hours
CAPS content specification	Page 148

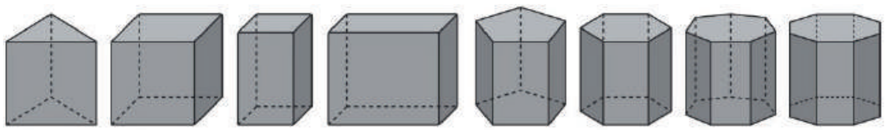
Mathematical background

- A **3D object** takes up space. The family of 3D objects can be classified into different groups, for example 3D objects with:
  - **only flat surfaces**, which include **prisms**, **pyramids** and the **Platonic solids** (such objects are called **polyhedra**)
  - **flat and curved surfaces**, which include **cylinders** and **cones**
  - **only curved surfaces**, which include **spheres** (balls).
- A **face** is any flat surface of a 3D object.
- A **polyhedron** has **faces** which are all polygons (2D figures), an **edge** where two faces meet and a **vertex** where three or more edges meet.
  - **Prisms** have two identical parallel faces, called the bases and between them lateral faces that join the two bases. The base can be any polygon. The lateral faces are parallelograms. A **right prism** is one where the lateral faces are all rectangles perpendicular to the base.
  - **Pyramids** have only one base that can be any polygon. The lateral faces are triangles that meet in a point called the apex. A **right pyramid** is one where all the lateral faces are identical isosceles triangles.
  - **Platonic solids** are polyhedra that have all their faces identical regular polygons. There are only five Platonic solids.
- **Euler's formula** gives the relationship between the number of faces (F), vertices (V) and edges (E) of a polyhedron: $F + V - E = 2$ or $F + V = 2 + E$.
- A **net of an object** is the 2D pattern obtained when an object is cut open and flattened out. There may be more than one net for a given polyhedron.

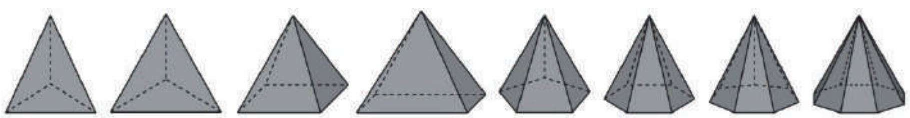
21.1 Classifying 3D objects

Background information

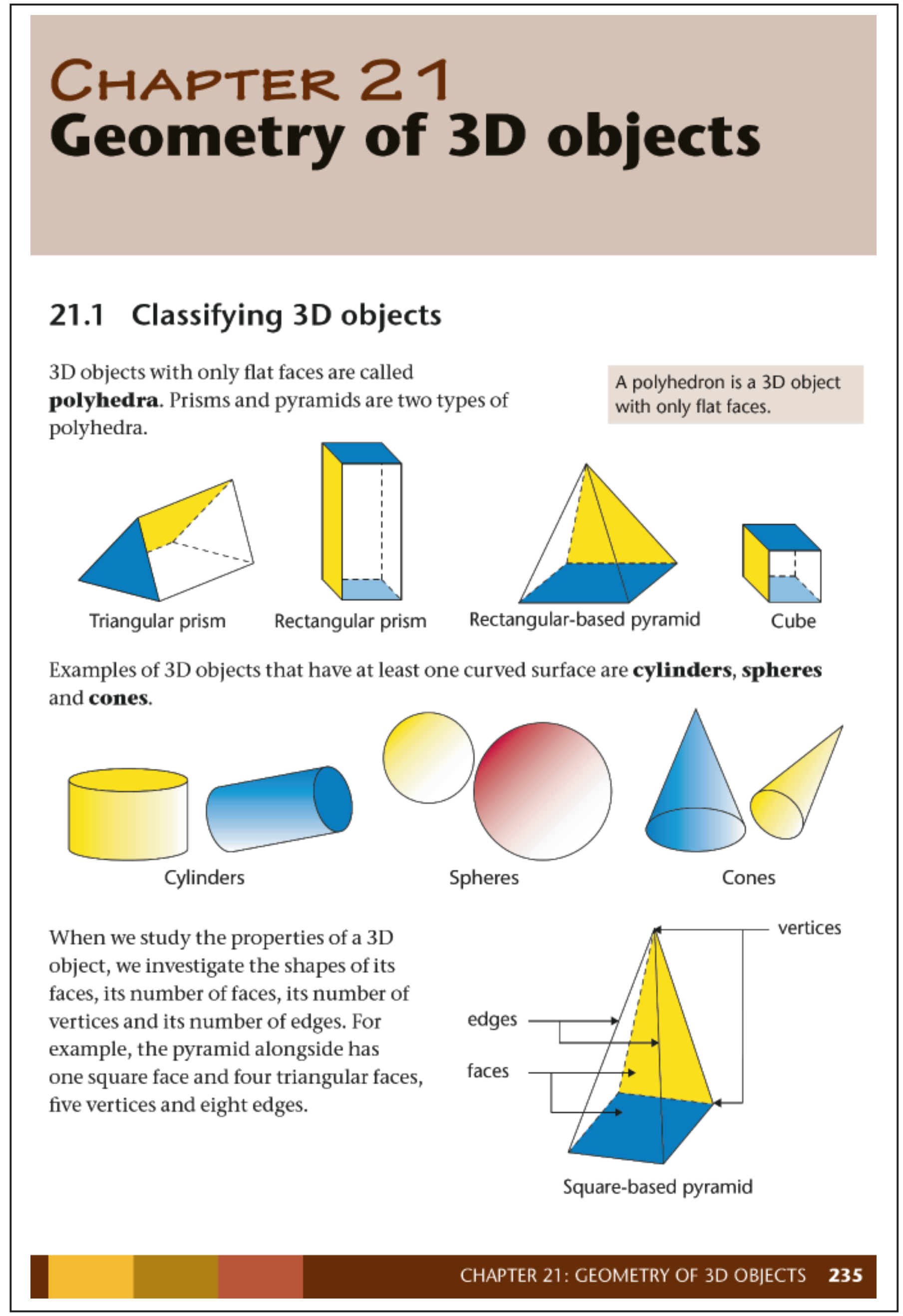
- A **polyhedron** is a 3D object with only flat surfaces.
  - A flat surface of a polyhedron is called a **face**.
  - An **edge** of a polyhedron is formed where two faces meet.
  - A **vertex** of a polyhedron is formed where three or more edges meet.
- **Prisms** have two identical parallel faces (called the bases) and between them lateral faces that join the two bases.
  - **Irregular prisms** have different types of parallelograms as laterals faces.
  - **Right prisms** have only rectangles as lateral faces, for example:



- **Pyramids** have only one base that can be any polygon. The lateral faces are triangles that meet in a point called the apex.
  - **Irregular pyramids** have different types of triangles as lateral faces.
  - **Right pyramids** have only isosceles triangles as lateral faces, for example:



- **Cylinders** have two identical circular faces that are parallel to each other, with a curved surface formed by lines that join corresponding points on the two circles.
- **Cones** have a circle as their base. The curved surface between the apex (vertex at the top) and the circle is formed by lines joining the apex and the circle.
- **Spheres** have one curved surface with every point on its surface the same distance from its centre.



CHAPTER 21
Geometry of 3D objects

21.1 Classifying 3D objects

3D objects with only flat faces are called **polyhedra**. Prisms and pyramids are two types of polyhedra.

A polyhedron is a 3D object with only flat faces.

Examples of 3D objects that have at least one curved surface are **cylinders**, **spheres** and **cones**.

When we study the properties of a 3D object, we investigate the shapes of its faces, its number of faces, its number of vertices and its number of edges. For example, the pyramid alongside has one square face and four triangular faces, five vertices and eight edges.

CHAPTER 21: GEOMETRY OF 3D OBJECTS 235

Teaching guidelines

Revise the features of polyhedra, cylinders, cones and spheres listed under background information on the previous page.

Point out that prisms and pyramids can be named in two different ways, for example:

- an **octagonal prism** is also referred to as an **octagon-based prism**
- a **pentagonal pyramid** is also referred to as a **pentagon-based pyramid**.

Point out that:

- a **hexahedron** (cube) is a rectangular prism with all its faces identical **squares**
- a **tetrahedron** is a triangular pyramid with all its faces identical **triangles**.

CLASSIFYING AND DESCRIBING 3D OBJECTS

Teaching guidelines

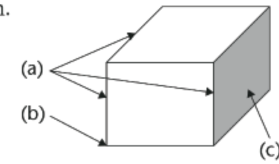
Learners classify different 3D objects according to their types of faces, number of faces, number of edges and number of vertices.

Answers

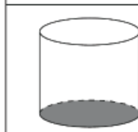
1. See the answers on LB page 236 alongside.
2. See the completed table on LB page 236 alongside.

CLASSIFYING AND DESCRIBING 3D OBJECTS

1. Write down the labels for parts (a) to (c) on the prism.
   (a) Edges
   (b) Vertex
   (c) Face



2. Copy and complete the following table:

	3D object	Name of the object	Number of faces and shape of faces	Number of vertices
(a)		Triangular prism	two triangles and three rectangles	6
(b)		Triangular pyramid/ tetrahedron	4 triangles	4
(c)		Cube/ Hexahedron	six squares	8
(d)		Rectangular-based pyramid	one rectangle and four triangles	5
(e)		Cone	1 circle and 1 curved surface	1
(f)		Cylinder	2 circles and 1 curved surface	0
(g)		Rectangular prism	2 squares and 4 rectangles	8

236 MATHEMATICS GRADE 9: TERM 3

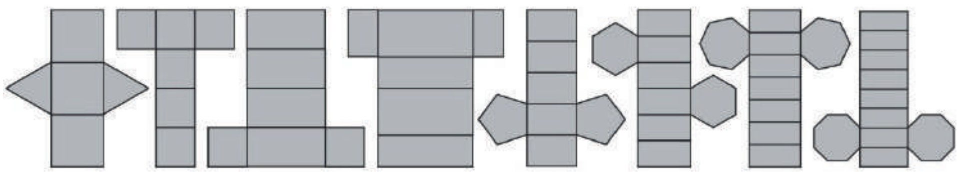
Answers

3. See the answers on LB page 237 alongside.

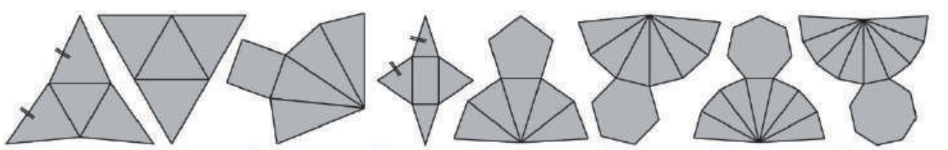
21.2 Nets and models of prisms and pyramids

Background information

- A **net** is an arrangement of polygons (2D figures) connected at their edges to form a flat pattern which can be folded up to make a model of a 3D object.
- A **variety of nets** can be constructed for a particular 3D object.
- **Nets of right prisms** may consist of a column of rectangles (or squares) with two identical polygons attached on either side.



  - Number of rectangles = Number of sides of the polygon
  - Width of a rectangle = Length of a side of the polygon
  - Length of a rectangle = Height of the prism
- **Nets of regular right pyramids** may consist of a "sector" of identical triangles with a polygon attached to the base of one triangle.

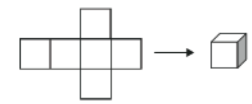


  - Number of triangles = Number of sides of the polygon
  - Base of a triangle = Length of a side of the polygon
  - Height of a triangle = Slant height of the pyramid
- Refer to the nets of the triangular prisms and the rectangular pyramid above. If the net is constructed with the **lateral faces surrounding the polygon**, adjacent sides of adjacent lateral faces must be equal in length.

3. Say whether each statement below is true or false:
   (a) A cylinder is a polyhedron. False
   (b) A triangular-based pyramid has four triangular faces. True
   (c) A cube is also known as a hexahedron. True
   (d) A triangular-based pyramid has six vertices. False
   (e) A pyramid is a 3D object. True

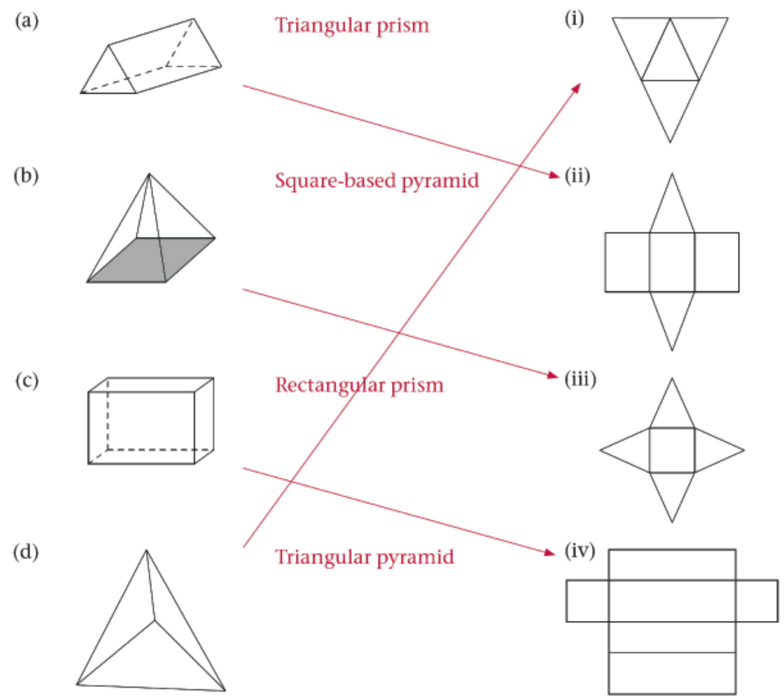
21.2 Nets and models of prisms and pyramids

A **net** is a flat pattern that can be used to represent a 3D object. The net can be folded up to create a model of the 3D object.



Net of a cube

1. Name each object below and match it with its net.



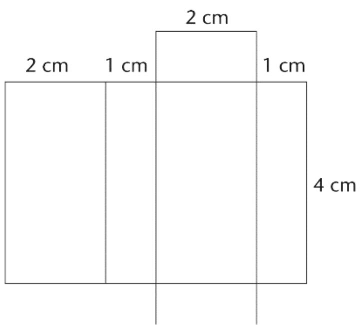
CHAPTER 21: GEOMETRY OF 3D OBJECTS 237

Teaching guidelines

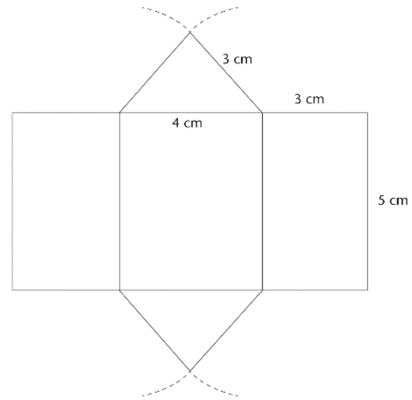
Learners discuss how to construct nets of right prisms and regular right pyramids.

Answers

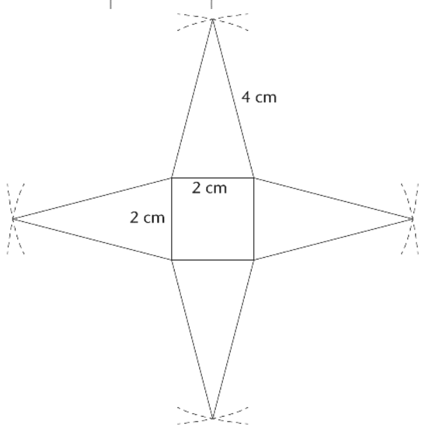
1. See the answers on LB page 237 on the previous page.
2. (a)



(b)



(c)



3. Learners construct models of the objects in question 2, but double all the measurements.

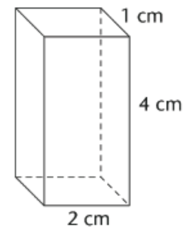
21.3 Platonic solids

Background information

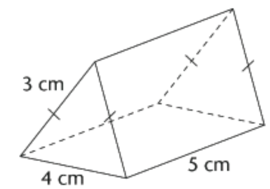
- A **polygon** is a 2D figure completely enclosed by three or more straight sides.

2. Construct an accurate net for each of the following 3D objects:

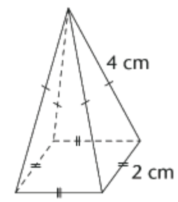
(a)



(b)



(c)

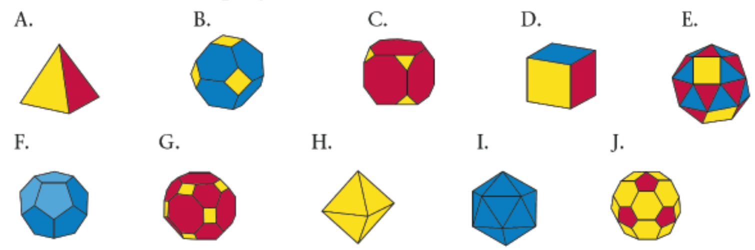


3. Construct models of the objects in question 2, but double all the measurements.

21.3 Platonic solids

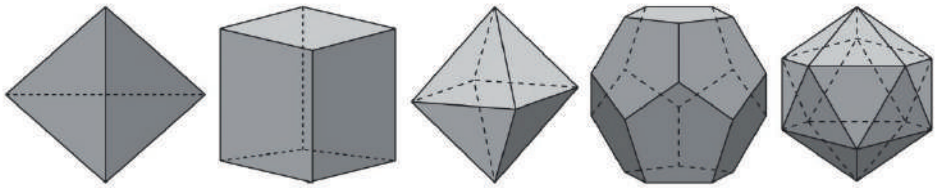
A **Platonic solid** is a 3D object which has identical faces, and all of the faces are identical regular polygons. This means that all its faces are the same shape and size and all the vertices are identical.

1. Which of the following objects are Platonic solids? D, F, H, I



2. How many Platonic solids are there in question 1?

- A **regular polygon** is a 2D figure with all its sides equal in length and all its interior angles equal in size.
- A **Platonic solid** is a 3D object with all of its faces identical regular polygons.



Tetrahedron Hexahedron (cube) Octahedron Dodecahedron Icosahedron

Teaching guidelines

Make sure that learners understand the definition of a Platonic solid.

Answers

1. See the answer on LB page 238 on the previous page.
2. Four

ONLY FIVE PLATONIC SOLIDS?

Background information

- The **five** Platonic solids are named after their number of faces.
- Each polyhedron has at least **three** faces that meet at each vertex.
- To form a shape like a pyramid at each vertex, the **sum of the angles at each vertex** must be less than 360°.

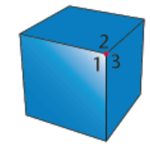
Teaching guidelines

The ideal would be for the learners to draw, cut and fold the different objects.

ONLY FIVE PLATONIC SOLIDS?

You can use your knowledge about angles to prove that the five Platonic solids are the only 3D objects that can be made from identical regular polygons. Keep the following facts in mind:

- A 3D object has *at least* three faces that meet at each vertex.
- The sum of the angles that meet at a vertex must be less than 360°. If it is equal to 360°, it will form a flat surface. If it is greater than 360°, the faces will overlap.
- Each Platonic solid is made up of one type of regular polygon only.



What 3D objects can you make from equilateral triangles?

We use the following reasoning:

Size of each interior angle = 180° ÷ 3 = 60°

$\therefore$ three triangles = 3 × 60° = 180° [< 360°]

four triangles = 4 × 60° = 240° [< 360°]

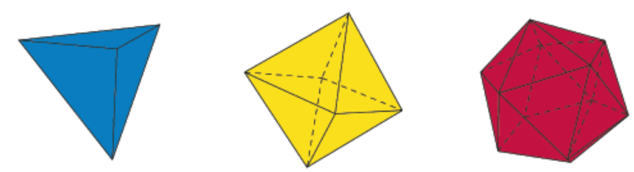
five triangles = 5 × 60° = 300° [< 360°]

six triangles = 6 × 60° = 360°

Any more than five triangles will be equal to or more than 360° and will therefore form a flat surface or overlap.

This means that we can make three 3D objects from equilateral triangles:

- If three triangles are at each vertex, it will form a **tetrahedron**.
- If four triangles are at each vertex, it will form an **octahedron**.
- If five triangles are at each vertex, it will form an **icosahedron**.



Tetrahedron Octahedron Icosahedron

What 3D objects can you make from squares?

- Each **interior angle of a square** is 360° ÷ 4 = 90°.
- **Three squares** around a vertex will form a **hexahedron (cube)**.
- Four squares around a vertex will form a flat surface.
- See the answers to questions on LB page 240 alongside.

What 3D objects can you make from regular pentagons?

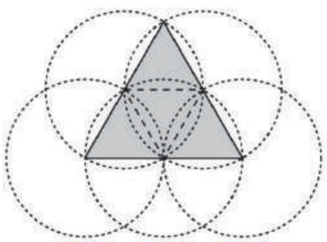
- Each **interior angle of a regular pentagon** is 3 × 180° ÷ 5 = 108°.
- **Three regular pentagons** around a vertex will form a **dodecahedron**.
- Four regular pentagons around a vertex will exceed 360°.
- See the answers to questions on LB page 240 alongside.

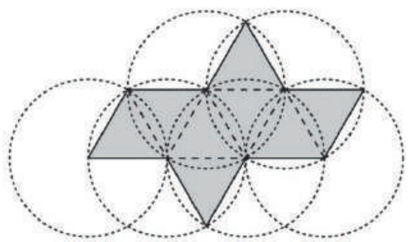
What 3D objects can you make from regular hexagons?

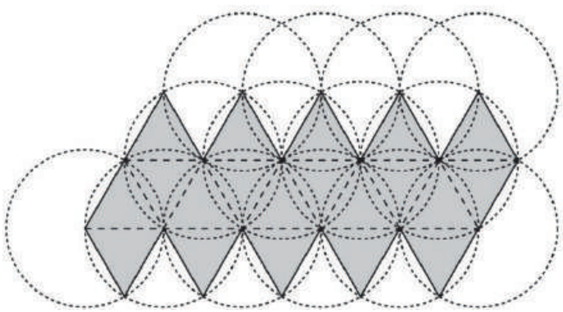
- Each **interior angle of a regular hexagon** is 4 × 180° ÷ 6 = 120°.
- Three regular hexagons around a vertex will form a flat surface.
- See the answers to questions on LB page 240 alongside.

Mathematical notes

Circles can be used to construct nets for tetrahedrons, octahedrons and icosahedrons.







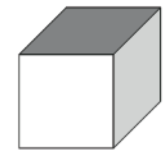
What 3D objects can you make from squares?

Complete the statements:

Size of each interior angle = 2(180°) ÷ 4 = 90°

∴ three squares = 3 × 90° = 270° [<360°]

four squares = 4 × 90° = 360°



Hexahedron (cube)

Therefore, we can make only one 3D object using squares. This 3D object is called a **hexahedron** (**or cube**).

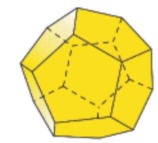
What 3D objects can you make from regular pentagons?

Complete the statements:

Size of each interior angle = 3(180°) ÷ 5 = 108°

∴ three pentagons = 3 × 108° = 324° [<360°]

four pentagons = 4 × 108° = 432°



Dodecahedron

Therefore, we can make only one 3D object using regular pentagons. This 3D object is called a **dodecahedron**.

What 3D objects can you make from regular hexagons?

Complete the statements:

Size of each interior angle = 4(180°) ÷ 6 = 120°

∴ three hexagons = 3 × 120° = 360°

Three hexagons will already form a flat surface. Therefore, it is impossible to make a 3D object from regular hexagons.

Also, the interior angles of polygons with more than six sides are bigger than those of a hexagon, so it is not possible to make 3D objects from any other regular polygons.

Therefore, the five Platonic solids already mentioned (tetrahedron, octahedron, icosahedron, hexahedron and dodecahedron) are the only ones that can be made of identical regular polygons. Each of these solids is named after the number of faces it has.

PROPERTIES OF THE PLATONIC SOLIDS

Background information

The following are properties of a Platonic solid:

- the **shape of its faces**
- its **number of faces** (which will determine the **name of the solid**)
- its **number of edges**
- its **number of vertices**.

Teaching guidelines

- The **shapes of the faces** of the five Platonic solids are **triangles, squares, triangles, pentagons and triangles** respectively. Learners could memorise this sequence.
- The **number of faces** can be deduced from the names of the Platonic solids:
  - "**tetra**" means four
  - "**hexa**" means six
  - "**octa**" means eight
  - "**dodeca**" means twelve
  - "**icosa**" means twenty.
- The **number of edges** can be found if the number of faces and their shapes are known:

  Number of edges = (Number of faces × Number of sides per face) ÷ 2

  Example: An octahedron has eight triangular faces.

  Number of edges = (8 faces × 3 sides per triangle) ÷ 2
  = 24 ÷ 2
  = 12 edges

  Note: If the net of an octahedron is constructed, two sides of two separate triangles are joined together to form one edge of the solid, therefore we divide by two.
- The **number of vertices** is always two more than the difference between the number of edges and the number of faces.

  Example: An octahedron has 12 edges and 8 faces.

  Number of vertices = (12 – 8) + 2
  = 6 vertices

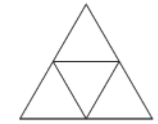
Answers

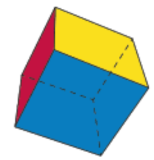
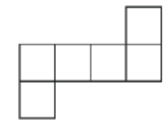
1. to 5. See the answers on LB page 241 alongside.

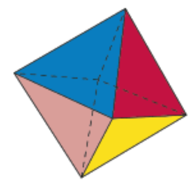
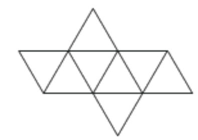
PROPERTIES OF THE PLATONIC SOLIDS

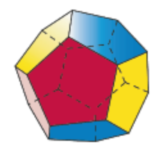
Copy and complete the information about each of the following Platonic solids:

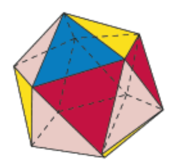
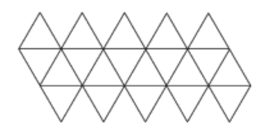
1.

Name: Tetrahedron or triangular pyramid
Shape of the faces: Triangles
Number of faces: 4
Number of edges: 6
Number of vertices: 4


2.

Name: Hexahedron or cube
Shape of the faces: Squares
Number of faces: 6
Number of edges: 12
Number of vertices: 8


3.

Name: Octahedron
Shape of the faces: Triangles
Number of faces: 8
Number of edges: 12
Number of vertices: 6


4.

Name: Dodecahedron
Shape of the faces: Pentagon
Number of faces: 12
Edges: 30
Vertices: 20


5.

Name: Icosahedron
Shape of the faces: Triangles
Number of faces: 20
Edges: 30
Vertices: 12


CHAPTER 21: GEOMETRY OF 3D OBJECTS 241

21.4 Euler's formula

EULER'S FORMULA AND PLATONIC SOLIDS

Background information

Euler's formula states that, for any polyhedron:

- number of **faces** (F) + number of **vertices** (V) – number of **edges** (E) = 2
- $F + V - E = 2$ or $F + V = E + 2$

Teaching guidelines

Learners investigate whether Euler's formula applies to Platonic solids.

Note on question 1

If all variables in Euler's formula are written down on one side of the equals sign, the variable **E**, as in **E**uler, is the one to be subtracted from the sum of others to get an answer of 2.

Answers

1. See the completed table on LB page 242 alongside.
2. See the answer on LB page 242 alongside.
3. (a) $E = F + V - 2 = 25 + 13 - 2 = 36$
   (b) $F = E - V + 2 = 23 - 11 + 2 = 14$
   (c) $V = E - F + 2 = 12 - 8 + 2 = 6$

EULER'S FORMULA AND OTHER POLYHEDRA

Background information

Euler's formula applies to all other polyhedra.

Teaching guidelines

Learners investigate whether Euler's formula applies to all polyhedra.

Answers

1. (a) See the answer on LB page 242 alongside.
   (b) See the answer on LB page 242 alongside.

21.4 Euler's formula

EULER'S FORMULA AND PLATONIC SOLIDS

1. You learnt about Euler's formula in Grade 8. Copy and complete the following table to investigate whether or not Euler's formula holds true for Platonic solids:

	Name	Shape of faces	No. of faces (F)	No. of vertices (V)	No. of edges (E)	F + V – E
	Tetrahedron	Triangles	4	4	6	2
	Cube or hexahedron	Squares	6	8	12	2
	Octahedron	Triangles	8	6	12	2
	Dodecahedron	Pentagons	12	20	30	2
	Icosahedron	Triangles	20	12	30	2

2. Complete Euler's formula for polyhedra: F + V = E + 2 or F + V – E = 2

3. Apply Euler's formula to each of the following:
   (a) A polyhedron has 25 faces and 13 vertices. How many edges will it have?
   (b) A polyhedron has 11 vertices and 23 edges. How many faces does it have?
   (c) A polyhedron has eight faces and 12 edges. How many vertices does it have?

EULER'S FORMULA AND OTHER POLYHEDRA

1. Is each of the following statements true or false?
   (a) A polyhedron with ten vertices and 15 edges must have seven faces. True
   (b) A polyhedron will always have more edges than either faces or vertices. True

Answers

1. (c) See the answer on LB page 243 alongside.
   (d) See the answer on LB page 243 alongside.
2. See the completed table on LB page 243 alongside.
3. (a) 12
   (b) 20
   (c) 90
   (d) 60
   (e) Yes: $F + V - E = (12 + 20) + 60 - 90 = 2$

21.5 Cylinders

PROPERTIES OF CYLINDERS

Background information

- **Cylinders** have two identical circular faces that are parallel to each other, with a curved surface formed by lines that join corresponding points on the two circles.
- A cylinder has the following **properties**:
  - it has **two circular faces** and **one curved surface**
  - it has **no vertices**
  - it has **two edges**.
- **Right cylinders** are cylinders with two parallel circular bases joined perpendicularly by a curved surface.

Teaching guidelines

Learners discuss the properties of right cylinders.

Answers

1. See the answer on LB page 243 alongside.

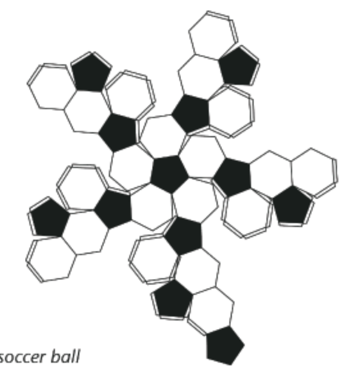
(c) A polyhedron with five faces must have six edges. False

(d) A pyramid will always have the same number of faces and vertices. True

2. Copy and complete the following table:

	No. of faces (F)	No. of vertices (V)	No. of edges (E)	Name of polyhedron	Shapes of faces
(a)	6	8	12	Rectangular prism	Rectangles
(b)	7	7	12	Hexagonal pyramid	Triangles and hexagon
(c)	4	4	6	Tetrahedron	Triangles
(d)	5	6	9	Triangular prism	Triangles and rectangles

3. A soccer ball consists of pentagons and hexagons.
   (a) How many pentagons does it consist of?
   (b) How many hexagons does it consist of?
   (c) How many edges does it have?
   (d) How many vertices does it have?
   (e) Does Euler's formula apply to soccer balls too?

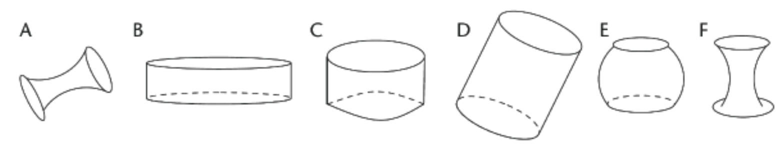


Net of a soccer ball

21.5 Cylinders

PROPERTIES OF CYLINDERS

1. Which of the following 3D objects are cylinders? B and D



Answers

2. See the answer on LB page 244 alongside.
3. See the answers on LB page 244 alongside.

NETS OF CYLINDERS

Background information

- The **curved surface of a right cylinder** is the rectangle which could be bent around to form a pipe which fits the two circular faces at its ends.
- **Nets of right cylinders** consist of a rectangle with two identical circles attached on either side.
  - length of the rectangle = circumference of a circle
  - breadth of the rectangle = height of the cylinder

Teaching guidelines

Learners investigate features of the net of a cylinder.

Answers

1. See the answer on LB page 244 alongside.
2. See the answers on LB pages 244 alongside and LB page 245 on following page.

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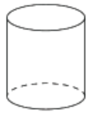
2. Write down the statement or statements below that are true only for cylinders and not for the other objects shown in question 1:

- ☐ It is a 3D object.
- ☐ It has a curved surface.
- ☐ It has two circular bases that are parallel to each other.
- ☐ It has two flat circular bases and a curved surface.
- ☐ The radius of its curved surface is equal from the top to the bottom between the bases.
- ☑ It has two circular bases opposite each other, joined by a curved surface whose radius is equal from the top to the bottom between the bases.

3. Look at the cylinder alongside and write down the:



(a) number and shape of faces: 2 circles and 1 curved surface

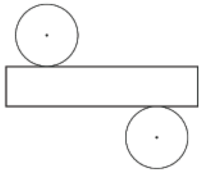
(b) number of vertices: 0

(c) number of edges: 2

NETS OF CYLINDERS

In Chapter 19, you learnt about the net of a cylinder. If you cut the curved surface of a cylinder vertically and flatten it, it will be the shape of a rectangle.

1. Explain why the length of the rectangular face is equal to the circumference of the base.

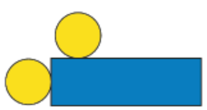


The circumference of the base is the distance around one of the circular faces of the cylinder. One side of the rectangle must fit exactly around the circle without overlap or shortages.

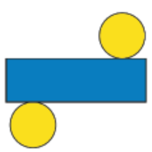
2. Will each of the following nets form a cylinder?

A.

B.



C.



A: No B: No C: Yes

Answers

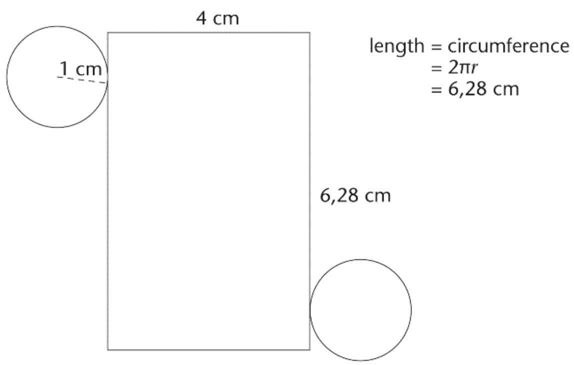
3. (a) Length of rect. surface = Circumference of base = $2\pi r = 2\left(\frac{22}{7}\right)(3) = 18{,}86$ cm

(b) Length of rect. surface = Circumference of base = $2\pi r = 2\left(\frac{22}{7}\right)(5) = 31{,}43$ cm

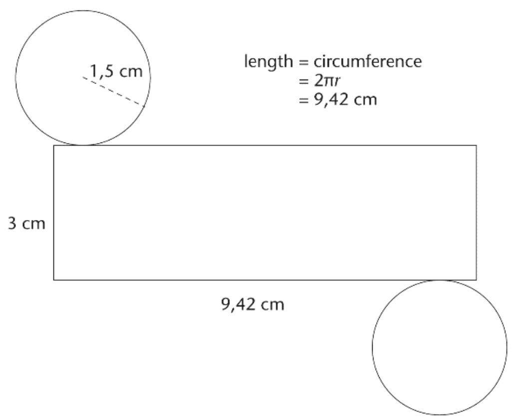
(c) Length of rect. surface = Circumference of base = $2\pi r = 2\left(\frac{22}{7}\right)(4) = 25{,}14$ cm

(d) Length of rect. surface = Circumference of base = $2\pi r = 2\left(\frac{22}{7}\right)(4{,}5) = 28{,}29$ cm

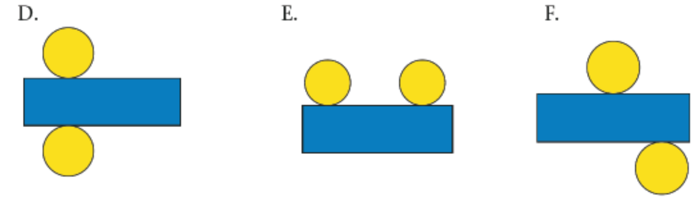
4. (a)



(b)



5. Learners' own work

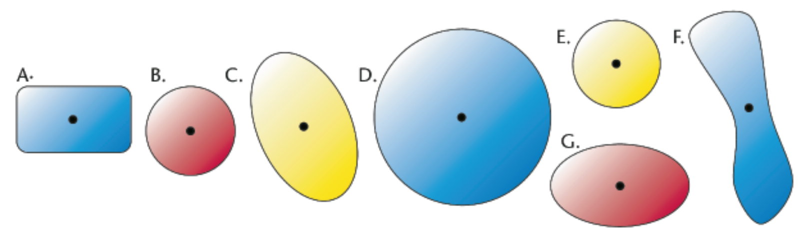


D: Yes E: No F: Yes

3. In each of the following questions, use $\pi = \frac{22}{7}$ and round off your answer to two decimal places to do the calculations.
   (a) If the radius of a cylinder is 3 cm, what is the length of the rectangular surface of the cylinder?
   (b) If the radius of a cylinder is 5 cm, what is the length of the rectangular surface of the cylinder?
   (c) If the diameter of a cylinder is 8 cm, what is the length of the rectangular surface of the cylinder?
   (d) If the diameter of a cylinder is 9 cm, what is the length of the rectangular surface of the cylinder?
4. Use a ruler and a set of compasses to construct the following nets as accurately as possible. Show the measurements on each net.
   (a) Net of a cylinder with a radius of 1 cm and a height of 4 cm
   (b) Net of a cylinder with a radius of 1,5 cm and a height of 3 cm
5. Construct models of the cylinders in question 4 but double the measurements.

21.6 Spheres

1. Which of the following 3D objects are spheres?



B, D, E

21.6 Spheres

Background information

- **Spheres** have one curved surface with every point on its surface the same distance from its centre.
- A sphere has the following **properties**:
  - it has **one curved surface**
  - it has **no vertices**
  - it has **no edges**.

Teaching guidelines

At the end of question 2 on LB page 246, learners should be able to define a sphere and list its properties.

Answers

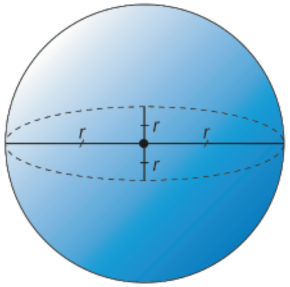
1. See the answer on LB page 245 on the previous page.
2. See the answer on LB page 246 alongside.
3. See the answers on LB page 246 alongside.
4. See the answers on LB page 246 alongside.

2. Write down the property or properties below that are true for spheres only and not for the other objects shown in question 1:
   - ☐ It is a 3D object.
   - ☐ It has one curved surface.
   - ☐ It has no bases.
   - ☐ It has no vertices.
   - ☐ It has no edges.
   - ☑ The distance from its centre to any point on its surface is always equal.

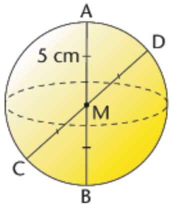
3. Complete the following information for a sphere:
   (a) Number and shape of faces: 1 curved surface
   (b) Number of vertices: 0
   (c) Number of edges: 0

From your study of spheres in the above activity, you should have found the following:

A **sphere** is a round 3D object with only one curved surface and the distance from its centre to any point on its surface is always equal. It has no vertices or edges.



4. Trace the sphere alongside, and write down the length of:
   (a) the radius: 5 cm
   (b) the diameter: 10 cm
   (c) MD: 5 cm
   (d) CD: 10 cm



5. The drawing on the following page shows part of a sphere with a diameter of 100 km. Imagine that you are at point M, at the centre inside the sphere. People A, B and C are all at different places on the surface of the sphere.

Answers

5. (a) None of them. They are all on the surface and are therefore the same distance away from the centre.

   (b) Diameter = 100 km ∴ radius = 50 km; so person C is 50 km away from M.

NET OF A SPHERE

Background information

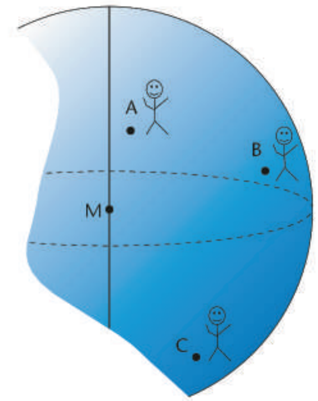
A **sphere** curves in two directions, therefore it is impossible to design a flat net for a sphere.

Teaching guidelines

Learners use copies of the net on LB page 247 to make an approximated model of a sphere.

(a) Which of the people – A, B or C – is closest to you?

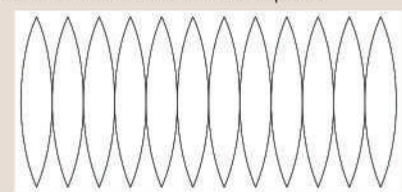
(b) How far away is person C from you?



NET OF A SPHERE

It is impossible to make a perfect sphere (ball or globe) from a flat sheet of paper. Paper can curve in one direction, but cannot curve in two directions at the same time. So, all spheres made from paper or card will be approximations. This is the best net we can make of a sphere.

Can you make your own paper model of a sphere?



WORKSHEET

Answers

1. See the answers on LB page 248 alongside.
2. See the table on LB page 248 alongside.
3. Learners follow the instructions given on LB page 248 alongside.

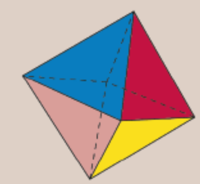
WORKSHEET

1. Grade 9 learners were asked to represent a 3D object and give the class clues as to which polyhedron they represent. Name their objects:
   (a) Amy: I have six faces and they are all the same size. Cube
   (b) John: I have six faces and 12 edges. I am not a cube. Rectangular prism
   (c) Onke: I have three faces. I also have two edges. Cylinder
   (d) Tessa: I have eight edges and I have five vertices. Square-based pyramid
   (e) Mandlakazi: I have six edges and four vertices. Tetrahedron or triangular pyramid
   (f) Chiquita: I have eight faces and I am a Platonic solid. Octahedron
   (g) Seni: I do not have any edges. Sphere
   (h) Mpu: My faces are made only of regular pentagons. Dodecahedron

2. Copy the table and write down the required information about each object below.

A.

B.



	Object A	Object B
Name	Cylinder	Octahedron
Number of faces	3	8
Shape/s of faces	2 circles and 1 curved surface (or 1 rectangle)	8 equilateral triangles
Number of edges	2	12
Number of vertices	0	6
Does Euler's formula work?	No	Yes
Is it a Platonic solid?	No	Yes

3. (a) On a separate sheet of paper, construct a net of a cylinder with a diameter of 7 cm and a height of 10 cm.
   (b) Fold your net to make a model of the cylinder.

Term 4

Chapter 22: Collect, organise and summarise data **293**
22.1 Collecting data 294
22.2 Organising data 297
22.3 Summarising data 299
Chapter 23: Representing data **303**
23.1 Bar graphs and double bar graphs 304
23.2 Histograms 306
23.3 Pie charts 308
23.4 Broken-line graphs 310
23.5 Scatter plots 312
Chapter 24: Interpret, analyse and report on data **319**
24.1 Which graph is best? 320
24.2 The effects of summary statistics on how data is reported 323
24.3 Misleading graphs 324
24.4 Analysing extreme values and outliers 327
Chapter 25: Probability **331**
25.1 Simple events 332
25.2 Compound events 336

Learner Book Overview		
Sections in this chapter	**Content**	**Pages in Learner Book**
22.1 Collecting data	Random sampling; development of a questionnaire	Pages 249 to 251
22.2 Organising data	Tally tables; frequency tables; class intervals; stem-and-leaf displays; grouped data	Pages 251 to 254
22.3 Summarising data	Mode; median; mean; range; comparison of sets of data; outliers (extreme values)	Pages 254 to 256

CAPS time allocation	4 hours
CAPS content specification	Page 149

Mathematical background

Data handling is the part of Mathematics that deals with numbers and facts that we collect about the world around us. Data can be many different things, for example, people's opinions on politics or the success rates of treating people with a certain kind of medicine. We use data to help us make decisions and solve problems about the world around us.

The **data handling cycle** consists of the following phases:

- **Pose a question**: Identify a real-life problem and pose (formulate) a question that requires the collection of data.
- **Collect data**: Identify the data source (the population), which is the whole group you are asking the question about. If the population is too large to handle, select a smaller group (the sample) to represent the population. Find the most suitable method to collect the data, for example, through observation (by watching something closely) or by using a questionnaire (a list of questions) or data bases. Decide whether to use a data collection sheet (during observation) or a questionnaire (during interviews) to collect the data. In Grade 9, learners need to decide which data collection methods are best in certain situations and they need to justify their choices.
- **Classify and organise data**: Identify whether the data is categorical (words) or numerical (numbers) and whether the numerical data is discrete (fixed numbers) or continuous (measurements). Sort the data into categories or into ungrouped or grouped intervals. Organise the data using tallies in frequency tables as well as stem-and-leaf displays.
- **Summarise data**: Find the mode, median and mean, which are measures of central tendency (balance), the range, which is a measure of spread (width) and outliers (extremes) of the data set.
- **Represent data**: Draw a graph of the data, for example a bar graph, double bar graph, histogram, pie chart, broken-line graph or scatter plot.
- **Interpret and analyse data**: Ask questions about the data and identify and describe trends or patterns in the data in order to draw conclusions about the data.
- **Report on the data**: Explain what the data tells about the problem or question and predict how the data can be used to solve problems about the world around us.

22.1 Collecting data

Background information

The **population** of a statistical investigation is the whole group of people or things that you want to find out about.

- The **size of a population** depends on what you need to find out.
- The **larger a population**, the more difficult it becomes to involve every member of that population in the data collection process. In such cases a **sample** (smaller group) can be chosen from the population to represent the whole population.
- The sample should represent all the **features of the whole population** and should be chosen with care.

A **random sample** usually reflects all the features of a whole population. During random sampling every member of the population has an **equal chance** of being chosen. The following are **random sampling methods**:

- **Simple random sampling**: Choose the sample by drawing names from a bag that contains the names of all the learners in the school.
- **Systematic random sampling**: Choose a number from 1 to 20 at random, say 14. Then choose every fourteenth name on an alphabetical list of all the learners in the school.
- **Stratified random sampling**: Determine the ratio of boys to girls in the school. Now choose a random sample from the whole school in that ratio.
- **Cluster random sampling**: Divide the learners in the school into grades and each grade into classes. Choose one class at random from each grade.

Bias is a term which refers to how far the information gained from a sample lies from the information hidden by the population. During the data handling cycle, bias can already surface during the sampling process. The following are **biased sampling methods** and usually lead to the collection of unreliable data:

- **Convenience sampling**: Choose only your friends to be part of the sample.
- **Self-selection sampling**: Ask who in school would like to be part of the sample.
- **Quota sampling**: Choose only boys in Grade 12 to be part of the sample.

Teaching guidelines

- Discuss the concepts of population and sample.
- Discuss the random sampling methods listed above.
- Discuss the concept of bias as well as the biased sampling methods listed above.

CHAPTER 22
Collect, organise and summarise data

22.1 Collecting data

Avoiding bias when selecting a sample

The methods that we use to collect data must help us to make sure that the data is reliable. This means that it is data that we can trust.

Data cannot be trusted unless it has been collected in a way that makes sure that every member of the population had the same chance of being selected in the sample.

It is not practical to taste all the oranges on a tree to know whether the oranges are sweet. Only a small number of oranges can be tested, otherwise the farmer would have too few oranges to sell. The oranges that are tested are called a **sample**, and all the oranges harvested from the tree are called the **population**.

Sample bias occurs when the particular section of the population from which the sample is drawn does not represent that population. The way to avoid sample bias is to take a **random** sample. A sample is random if **every member of the population has the same chance** of being selected. A random sample of the orange trees means that every tree should have a chance of being selected for the sample. Every person in the country should have a chance of being selected for the housing survey in a random sample.

An example of sample bias would be to survey only the people in Limpopo about their views on housing provision when you want to know the views of the whole country. For the sample to provide information on the population as a whole, each person in the country should have the same chance of being part of the survey.

Data can be collected through questionnaires, through observation and through access to databases.

How to develop a good questionnaire

The questionnaire also has an important role in making sure that the information you collect is reliable. You should aim to get a high number of respondents and accurate information. If not enough people fill in the questionnaire, then you won't know whether the information you get reflects the real situation. Sampling techniques and rules developed by statisticians determine the numbers needed.

Background information (continued)

A **questionnaire** is a sheet with questions used to collect data. The person who answers these questions is called the **respondent**. When a questionnaire is constructed, different **types of questions** can be asked:

- close-ended questions with "yes" or "no" responses (answers)
- closed-ended questions with multiple responses (multiple choice questions)
- closed-ended questions that ask for a rating such as "never", "sometimes", "always"
- open-ended questions where the respondents may enter their own view or information.

If you use the wrong method or instrument for collecting data, the data may be **flawed**. This could lead to unreliable conclusions and predictions.

The type of questions that you choose depends on the data you want to collect. These are **hints to construct a questionnaire**:

- Ask clear, short and accurate questions.
- Start with easy questions.
- Avoid leading questions which tell the respondent what the answer should be.
- Avoid biased questions which favour someone or something.
- Avoid offensive questions which are too personal or upsetting.
- Give clear instructions.
- Make the questionnaire as short as possible.

Multiple choice questions should have the following features:

- The options should **not overlap**.
- There should be **no gaps** between the options.
- The options should cover **all possible answers**.

THINK ABOUT DATA COLLECTION AND DEVELOP A QUESTIONNAIRE

Teaching guidelines

Learners discuss the information on questionnaires provided on LB page 250 alongside.

Answers

1. (a) Anonymous questionnaires may be distributed and collected. This information could inform the school policy regarding support for the learners and the community in terms of school lunches.

There are some important points to consider when designing a questionnaire. Two of the most important points are that the questions are **clear and accurate** and that people find the questionnaire relatively **easy to complete**.

1. Keep in mind the length of the questionnaire and the time that it takes to complete. Your participants are more likely to complete a short questionnaire that is quick and easy to complete. Exclude unnecessary information.
2. Write down a selection of questions that you think will provide the information that you want.
3. Check the wording for each question.
4. Order the items so that they are in a logical sequence. It might make sense to have the easiest questions first, but in some cases the more general questions should come first and the more specific questions towards the end of the questionnaire.
5. Then try the questionnaire out on a partner. Ask the following questions:
   - Is this question necessary? What information will be provided by the answer?
   - How easy will it be for the respondent to answer this question? How much time will it take to answer the question?
   - Do the questions ask for sensitive information? Will people want to answer the question? Will the respondent answer the question honestly?
   - Can the question be answered quickly?
6. Decide how the answers should be provided. Questions may require **open-ended** responses or **closed-ended** responses, as described below.

In an **open-ended** question, the person responds in his or her own words. Through his or her own words important information can be gained; the person is therefore free to write what he or she likes. A disadvantage is that you might not get the information you want and that it might take a long time to answer.

In a **closed-ended** question, the respondents are given some options to choose from. They tick the box which most closely represents their response. These options can be constructed in categories. For example, age may be categorised as follows:

Under 10 ☐ From 10 to 14 ☐ From 15 to 19 ☐ 20 and older ☐

Answers (continued)

(b) A questionnaire could be administered that asks the tellers to complete the pertinent questions about their conditions. Following the questionnaire, some interviews could be conducted.

(c) A questionnaire could be administered to clients as they enter the clinic. Every tenth person could be sampled. Another sampling procedure is to interview a client who arrives at the clinic every 20 minutes during one day.

(d) Observations of the pre-school children during their free time would be useful. A schedule and a tally table to record how many children engage with each activity would be appropriate.

(e) This information can be obtained from a database, for example, the statistics department of the Department of Basic Education.

2. A possible answer:

Put a tick next to your age group.	☐ 12–14 ☐ 15–18 ☐ 19–20 ☐ older than 20
Male or female?	☐ male ☐ female
What kind of food do you eat after school?	☐ chicken ☐ hamburger ☐ chips ☐ toasted sandwich ☐ other (please specify) ___________
What type of music would you like to hear in a fast-food shop?	☐ hip hop ☐ house ☐ pop ☐ rock ☐ other (please specify) ___________

THINK ABOUT DATA COLLECTION AND DEVELOP A QUESTIONNAIRE

1. Which method for collecting data would be most appropriate for each of the cases below? Give reasons for your choice.
   (a) The number of learners who bring lunch to school. What are the contents of the school lunch?
   (b) Whether or not the tellers at a supermarket chain are happy with their conditions of work.
   (c) Whether or not the clients of a clinic are satisfied with the professional conduct of the medical staff.
   (d) The types of activities preschool children choose during their free time.
   (e) The number of Grade 9 learners in the Gauteng North district.

2. You are doing some market research for a new fast-food shop near the high school. The owners of the shop want to find out what kind of food and music the target market likes. The target market is learners from the high school. Develop a questionnaire to collect this information.

22.2 Organising data

There is a difference between **data** and **information**. Data is unorganised facts. When data is organised and analysed so that people can make decisions, it may be called information. Data can be organised in many different ways. Some methods are described below.

Data can be organised by making a **tally table**. Here is an example of a tally table showing the numbers of learners in a class that participate in different sports:

Sport	Tally marks
Soccer	𝍸 𝍸 𝍸 𝍸 𝍸
Athletics	𝍸 ///
Netball	𝍸 𝍸 𝍸 𝍸 /
Chess	𝍸 /

The above data can also be organised in a **frequency table**:

Sport	Frequency
Soccer	25
Athletics	8
Netball	21
Chess	6

22.2 Organising data

Background information

The way data is organised depends on the **type of data** and what we want to find out from the data.

- **Categorical data** is data in the form of words, for example, your birth month.
- **Numerical data** is either **discrete data**, which are fixed numbers, for example the number of siblings per family, or **continuous data**, which are measurements, for example your height or weight.

Before data is organised it has to be:

- **classified** as categorical, discrete or continuous data
- **sorted** into categories or class intervals.

Data can be **organised** in ordinary tables, tally tables, frequency tables and stem-and-leaf displays.

- **Ordinary tables** show **raw data**, which is data as it is collected, before any sorting is done.
- **Tally tables** show **categories or intervals of data** in column 1 and **tally marks** (|) in clusters of five (卌) in column 2.
- **Frequency tables** show **categories or intervals of data** in column 1 and frequencies (total number of counts per category) in column 2.
- A **stem-and-leaf display** shows numerical data listed in two columns separated by a vertical line. The **stem column** (on the left of the vertical line) shows the tens and hundreds of all the data values in numerical order, including those which are missing from the sequence. The **leaf column** (on the right of the vertical line) shows the units of all the data values in numerical order and in line with their relevant stems, which means that something like 23 **|** 0 5 represents the data values 230 and 235.

Teaching guidelines

Learners discuss the following ways to organise data:

- tally tables, which can be used to collect data during observations (watching something and recording the data as it becomes available)
- frequency tables
- stem-and-leaf displays.

Numerical data sets with many items are often grouped into equal **class intervals** and represented in a table of frequencies for the different class intervals. This is very useful since it makes it easy to see how the data is spread out.

Here is an example of grouped data showing the heights of all the learners in a school. To make a frequency table for numerical data, the data has to be arranged from smallest to biggest first.

Height in m	Number of learners (frequency)
< 1,20 m	13
1,20 m – 1,30 m	28
1,30 m – 1,40 m	57
1,40 m – 1,50 m	164
1,50 m – 1,60 m	274
1,60 m – 1,70 m	198
1,70 m – 1,80 m	73
> 1,80 m	13

A value equal to the **lower boundary** of a class interval is counted in that interval.
For example, a length of 1,60 m is counted in the interval 1,60 – 1,70, and not in the interval 1,50 – 1,60 m.
However, 1,599 m is less than 1,60 m, so it belongs in the interval 1,50 m – 1,60 m.

A **stem-and-leaf display** is a useful way to organise numerical data. It also shows you what the "shape" of the data is like. Here is an example of a stem-and-leaf display:

Key: 35 | 4 means 354

34	0 4
35	4 8 8
36	0 1 6 8
37	1 3 5 8 8 8 9
38	2 4 9
39	0 3 4 4 5 6 9
40	0 3 7
41	1

The above stem-and-leaf display represents the following data about the masses in grams of the chickens in a sample of six-week-old chickens on a chicken farm:

399	378	382	360	396	389	344	411	378	394
394	354	375	378	400	371	379	358	366	403
358	395	390	340	393	384	361	407	373	368

To make a stem-and-leaf display, it helps to first arrange the data from smallest to largest, as shown on the next page, for the above data set.

252 MATHEMATICS GRADE 9: TERM 4

WORKING WITH GROUPED DATA

Background information

- Sets of **numerical data** are usually grouped when the set contains a large number of data values or the data values are very different in magnitude (size).
- A **class** of data is one of the **groups** in a collection of grouped data.
- The **class limits** are the **two values** which define the two ends of a class.
- The **class interval** is the **width** of a class, i.e. the difference between the two class limits.
- The class interval can be chosen in different ways, for example in the answer to question 1 the upper limit was chosen to be excluded, but this is not always the case.
- Data is grouped into **intervals** to make it easier to handle.
- Class intervals should **neither overlap nor have any gaps** between them.
- Once data is grouped, the **original data values** cannot be found again.

Teaching guidelines

Learners revise the information on grouped data provided at the top of LB page 253 alongside.

Answers

1.

Number of calls	Tally marks	Frequency
0–40	/	1
40– 80	~~////~~	5
80–120	~~////~~ ~~////~~ //	12
120–160	~~////~~ ///	8
160–200	////	4
200–240	/	1
Total		**31**

340	344	354	358	358	360	361	366	368	371
373	375	378	378	378	379	382	384	389	390
393	394	394	395	396	399	400	403	407	411

The same data set is displayed in two slightly different ways below:

			379		399		
			378		396		
			378		395		
		368	378		394		
	358	366	375	389	394	407	
344	358	361	373	384	393	403	
340	354	360	371	382	390	400	411

In this display, the width of each class interval is 10, as in the stem-and-leaf display above.

		384		
		382	399	
		379	396	
	368	378	395	
	366	378	394	
	361	378	394	411
354	360	375	393	407
344	358	373	390	403
340	358	371	389	400

In this display, the width of each class interval is 15.

WORKING WITH GROUPED DATA

1. An organisation called Auto Rescue recorded the following numbers of calls from motorists each day for roadside service during March 2014:

28	122	217	130	120	86	80	90	120	140
70	40	145	187	113	90	68	174	194	170
100	75	104	97	75	123	100	82	109	120
81									

Set up a tally and frequency table for this set of data values, in intervals of width 40.

2. When geologists go out into the field they make sure they have their rulers and measurement instruments in their bags. They also have their "inbuilt rulers", for example their handspans. A handspan is the distance from the tip of the thumb to the tip of the fifth finger on an outstretched hand. Measure your handspan against the ruler! This frequency table shows the handspans of different Grade 9 learners, in cm.

Handspan of Grade 9 learners (cm)	Frequency
15–18	7
18–21	9
21–24	10
24 and greater	4

Answers

2. (a) 7 + 9 + 10 + 4 = 30 handspans were measured altogether.
   (b) 7 + 9 = 16 handspans are less than 21 cm wide.
   (c) 9 + 10 + 4 = 23 handspans are 18 cm or wider.
   (d) In the interval 18–21 cm.

22.3 Summarising data

Background information

We **summarise** data by finding a few numbers that, together, show us more about the whole data set. Some of these numbers involve all data values. Others involve only a few.

Measures of central tendency tell us more about the **balance** in a data set.

- The **mode** shows the **most common data value** in an *ordered data set*. It is equal to the data value with the highest frequency. A data set can have **more than one mode**. If the frequencies of all data values are equal, the data set has **no mode**.
- The **median** separates an *ordered data set* into **an upper half and a lower half**. If the data set consists of an **odd** number of data values, the median is the **data value in the middle** of the *ordered data set*. If the data set consists of an **even** number of data values, the median is the **value between the two data values in the middle** of the *ordered data set*.
- The **mean** is the "**average**" of a data set and is calculated by adding all the data values and dividing the total by the number of data values.

Measures of spread tell us how **far apart** the data values in an *ordered data set* lie.

- The **range** is the difference between the largest and smallest data values in an ordered data set.

An **outlier** is a data value that is much lower or higher than any other data values in the data set. It lies an abnormal distance from other values in a random sample taken from a population.

ORGANISE, SUMMARISE AND COMPARE SOME DATA

Teaching guidelines

Learners revise the concepts of mode, median, mean and range.

(a) How many learner handspans were measured altogether?
(b) How many learner handspans are less than 21cm wide?
(c) How many handspans are 18 cm or wider?
(d) In which interval would you place a handspan of 18 cm?

22.3 Summarising data

The mean, median, mode and range are single numbers that provide some information about a data set, without listing all the data values.

The **mode** is the value that occurs most frequently. To find the mode, look for the number or category that is listed in the data set most often. Some data sets have more than one mode, and some may have none.

The **median** is the number that separates the set of values into an upper half and a lower half. The median can be found by arranging the values from small to big or big to small. If the data set consists of an even number of items, the median is the sum of the two middle values divided by 2.

The **mean** (average) of a set of numerical data is the sum of the values divided by the number of values in the data set.

Mean = the sum of the values ÷ the number of values.

The **range** is a number that tells us how spread out the data values are. It is the difference between the largest and smallest values.

The mean, median and mode do not work equally well for all sets of data. It depends on the kind of data, and also on whether the data is evenly spread out or not.

ORGANISE, SUMMARISE AND COMPARE SOME DATA

1. A researcher analyses data about the people who are suffering from three different types of the flu virus: A, B and C. The ages of the people in the different groups are:

   Type A: 60, 80, 75, 87, 88, 49, 94, 84, 59, 86, 82, 62, 79, 89 and 78.

   Type B: 27, 39, 43, 29, 36, 70, 56, 25, 54, 36, 66, 45, 33, 46, 14 and 41.

   Type C: 33, 48, 64, 15, 31, 20, 70, 21, 18, 49, 21, 19, 57, 23, 29 and 20.

254 MATHEMATICS GRADE 9: TERM 4

Answers

1. Type A: Key: 4 | 9 means 49 years

3	
4	9
5	9
6	0 2
7	5 8 9
8	0 2 4 6 7 8 9
9	4

- range: 94 – 49 = 45 years
- mean = 76,8 years
- median = 80 years

The data is not evenly spread out – the data values are clustered around the ages in the 70s and 80s. The range is 45 years, but there are only a few values less than 70 years of age. This type of virus affects mostly elderly people.

Type B:

1	4
2	5 7 9
3	3 6 6 9
4	1 3 5 6
5	4 6
6	6
7	0

- range: 70 – 14 = 56 years
- median = 40 years
- mean = 41,25 years

The data is evenly spread out, and the data values are clustered in the middle. People in their 30s and 40s are the most affected by this virus, but it also affects older and younger people.

For each group:

- Draw a stem-and-leaf plot.
- Calculate the range, mean and median of the ages.
- Look at the shape of the stem-and-leaf displays as well as the summary measures. Discuss the spread of the data in each case, and compare the three different groups.

Work and report on your work.

2. Copy the table and fill in the statistic (mode, mean or median) that would best summarise each data set, and indicate the central tendency of the data:

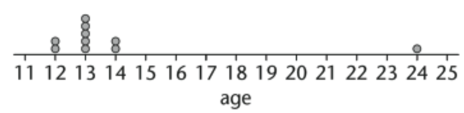
Data set	Best measure of central tendency
The shoe sizes of boys in Grade 9	mode
An evenly spread set of measurement values, such as the heights of learners in a class	mean and median
A set of measurement values with a few very low values and mostly high values	median
The number of siblings each person in your class has	mode
The sizes of properties in a town, where most people live in small apartments or RDP houses, and a few live on large properties	median

EXTREME VALUES OR OUTLIERS

An **extreme value** or **outlier** is a data value that lies an abnormal distance from other values in a random sample from a population. Sometimes there are reasons why this data value is so different to the others. It is important to comment on the possible reasons.

When you are summarising data (and also when you analyse data), you need to decide whether or not an outlier makes sense in the context you are looking at.

It is possible that an outlier does not make sense, as it lies too far away and is an unreasonable measurement. Then you need to think about the fact that this data value may be an error. For example:



Answers (continued)

Type C:

1	5 8 9
2	0 0 1 1 3 9
3	1 3
4	8 9
5	7
6	4
7	0

- range: 70 – 15 = 55 years
- mean = 33,63 years
- median = 26 years

The data is not evenly spread out, as it is closely clustered in the 10s and 20s, and there are a few higher values. Type C of the virus affects mostly young people.

2. See the table on LB page 255 alongside.

EXTREME VALUES OR OUTLIERS

Teaching guidelines

Learners revise the concept of outlier.

Point out to learners that the table indicates the total population to the nearest 1 000 so their answer should also be to the nearest 1 000.

(LB page 255 repeat)

For each group:

- Draw a stem-and-leaf plot.
- Calculate the range, mean and median of the ages.
- Look at the shape of the stem-and-leaf displays as well as the summary measures. Discuss the spread of the data in each case, and compare the three different groups.

Work and report on your work.

2. Copy the table and fill in the statistic (mode, mean or median) that would best summarise each data set, and indicate the central tendency of the data:

Data set	Best measure of central tendency
The shoe sizes of boys in Grade 9	mode
An evenly spread set of measurement values, such as the heights of learners in a class	mean and median
A set of measurement values with a few very low values and mostly high values	median
The number of siblings each person in your class has	mode
The sizes of properties in a town, where most people live in small apartments or RDP houses, and a few live on large properties	median

EXTREME VALUES OR OUTLIERS

An **extreme value** or **outlier** is a data value that lies an abnormal distance from other values in a random sample from a population. Sometimes there are reasons why this data value is so different to the others. It is important to comment on the possible reasons.

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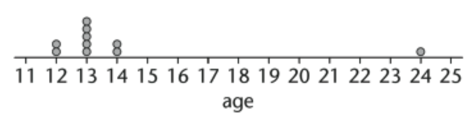


Answers

1. (a) 17 909 000 people
   (b) See the dot plot on LB page 256 alongside.
   (c) 84 000; 1 185 000; 1 288 000; 1 950 000; 2 067 000; 2 171 000; 12 523 000; 12 935 000; 15 263 000; 18 498 000; 22 894 000; 43 739 000; 50 110 000; 66 020 000

   The median is (12 523 000 + 12 935 000) ÷ 2 = 12 729 000
   (d) 66 020 000 – 84 000 = 65 936 000
   (e) The median, as there are many countries clustered below 20 000 000 and only a few above that value.
2. There is a very wide spread of values from 170 to 19 650. Most of the values are below 10 000.

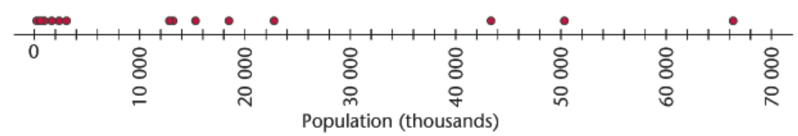
In this case, the value of 24 years old could be an unreasonable value. This depends on the context of the survey.

You will learn more about extreme values and outliers in Chapter 24.

Use this information about 14 countries to answer the questions that follow:

Country	Total population (in 1 000s)	Total annual national income per person (US$)	Percentage of income spent on health
Angola	18 498	4 830	4,6
Botswana	1 950	13 310	10,3
DRC	66 020	280	2,0
Lesotho	2 067	1 970	8,2
Malawi	15 263	810	6,2
Mauritius	1 288	12 580	5,7
Mozambique	22 894	770	5,7
Namibia	2 171	6 250	5,9
Seychelles	84	19 650	4,0
South Africa	50 110	9 790	8,5
Swaziland	1 185	5 000	6,3
Tanzania	43 739	1 260	5,1
Zambia	12 935	1 230	4,8
Zimbabwe	12 523	170	Not available

1. Look at the total population for each country.
   (a) Calculate the mean of the data.
   (b) Copy the number line below and draw a dot plot on the number line to show the data.



   (c) Find the median of the data.
   (d) What is the range of the data?
   (e) Which measure of central tendency do you think represents the data more accurately? Explain.
2. Look at the *Total annual national income per person in US dollars* column. Comment on the spread of the data.

Learner Book Overview		
Sections in this chapter	**Content**	**Pages in Learner Book**
23.1 Bar graphs and double bar graphs	Revision of bar graphs and double bar graphs; drawing bar graphs and double bar graphs	Pages 257 to 259
23.2 Histograms	Revision of histograms; representing data in histograms	Pages 259 to 260
23.3 Pie charts	Drawing pie charts	Pages 260 to 261
23.4 Broken-line graphs	Drawing broken-line graphs	Pages 262 to 263
23.5 Scatter plots	Understanding and constructing scatter plots; positive correlation; negative correlation; no correlation; the relationship between arm span and height	Pages 263 to 268

CAPS time allocation	3 hours
CAPS content specification	Page 150

Mathematical background

- In Chapter 22 we covered the following **phases in the data handling cycle**:
  - **Pose a question** about a real-life problem that requires the collection of data.
  - **Collect and record data** on data recording sheets during observations and on questionnaires during interviews.
  - **Classify, sort and organise data** in categories or intervals on frequency tables and stem-and-leaf displays.
  - **Summarise data** by finding the mode, median, mean and range of the data set and taking note of outliers.
- In this chapter the focus is on **representing data**, which is the next phase in the data handling cycle.
- Data is represented by **drawing a picture** of the tabulated data. This is done for the following reasons:
  - A picture makes information **easier to understand**.
  - It is easier to **identify patterns and upward, downward and cyclic trends** in a picture than in a table.
  - People pay more attention to visual information that is presented in an **aesthetically pleasing manner** in the media.
- A large **variety of statistical displays** are used by statisticians and in the media to convey information, for example:
  - dot plots, pictographs, bar graphs, double bar graphs, pie charts, line and broken-line graphs and scatter plots
  - histograms, frequency polygons, ogives (cumulative frequency polygons), regression functions, normal distributions, and so on.
- Graphs focused on here are **bar graphs**, **double bar graphs**, **histograms**, **pie charts**, **broken-line graphs** and **scatter plots**.

23.1 Bar graphs and double bar graphs

REVISING BAR GRAPHS AND DOUBLE BAR GRAPHS

Background information

A **bar graph** usually shows categories of data along the horizontal axis, and the frequency of each category along the vertical axis. Here are some **features of a bar graph**:

- It represents only one set of data.
- It represents either categorical or ungrouped numerical data.
- It never represents grouped numerical data.
- It uses bars to show the frequencies of the different categories.
- The heights or lengths of the bars represent the frequencies of the different categories.
- The title (heading) tells what the bar graph is about.

A **double bar graph** shows two sets of data with matching categories on the same set of axes. Here are some **features of a double bar graph**:

- It represents two sets of data with matching categories.
- It represents either categorical or ungrouped numerical data.
- It never represents grouped numerical data.
- It uses separate pairs of bars to show the frequencies of matching categories.
- The heights or lengths of the bars represent the frequencies of the matching categories.
- The title (heading) tells what the double bar graph is about.
- The key explains the colours used to distinguish the two sets of data.

Vertical bar graphs are usually used to show change over time at discrete times, for example, absentees per day of the week.

Horizontal bar graphs are usually used to compare or rank items at one point in time, for example, absentees per grade on a specific day.

Teaching guidelines

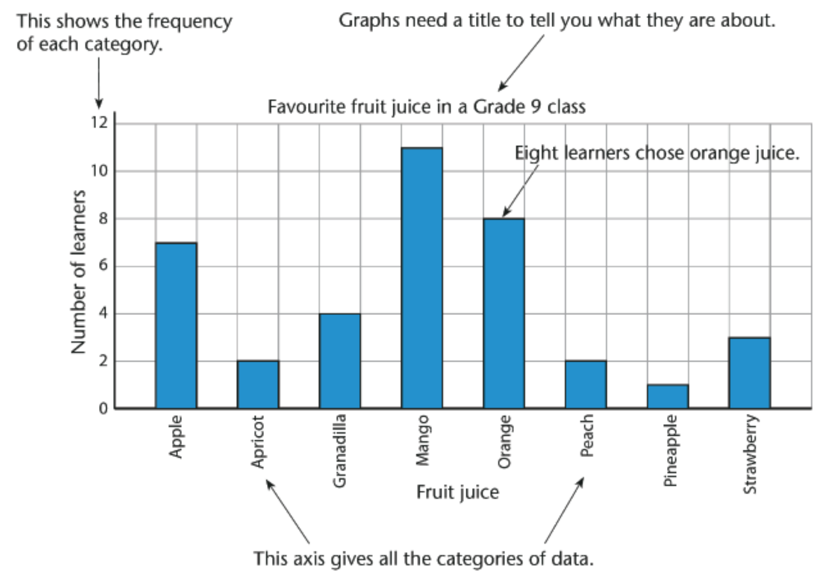
Learners revise the properties of bar graphs and double bar graphs.

CHAPTER 23
Representing data

23.1 Bar graphs and double bar graphs

REVISING BAR GRAPHS AND DOUBLE BAR GRAPHS

A **bar graph** shows categories of data along the horizontal axis, and the frequency of each category along the vertical axis. An example is given below.



A **double bar graph** shows two sets of data in the same categories on the same set of axes. This is useful when we need to show two groups within each category.

DRAWING BAR GRAPHS AND DOUBLE BAR GRAPHS

Background information

Bar graphs are suitable to compare frequencies in a set of data, identify patterns in the data and describe trends shown by the data. When drawing a bar graph, remember the following:

- All bars should be equally wide.
- Gaps between bars should be equally wide, but narrower than the bars, unless a bar is missing from the sequence.
- The first bar should not touch the frequency axis.
- A title should explain what the bar graph is about.

Double bar graphs are suitable to compare frequencies of matching categories in two data sets, identify similar and different patterns in the two data sets and describe similar and different trends shown by the two data sets. When drawing a double bar graph, remember the following:

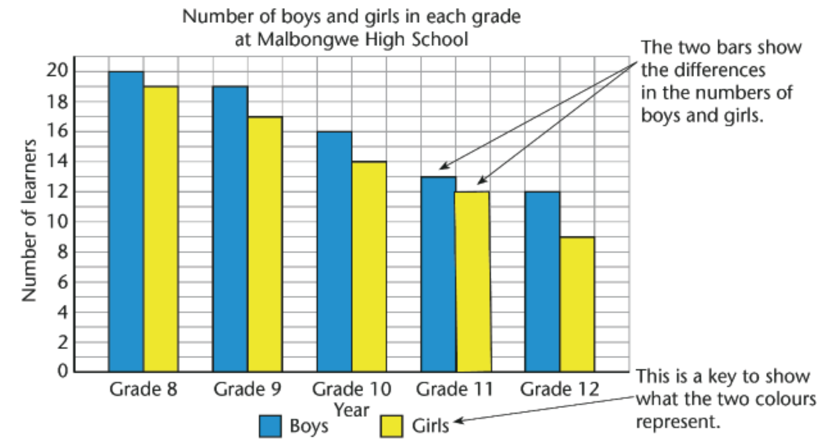
- All bars should be equally wide.
- Gaps between pairs of bars should be equally wide, but narrower than the bars, unless a bar is missing from one or both sequences.
- The first bar should not touch the frequency axis.
- A title should explain what the double bar graph is about.
- A key (legend) should explain the colours used to distinguish the two sets of data.

Teaching guidelines

Learners discuss important facts to remember when they draw bar graphs and double bar graphs.

Answers

1. (a) Data such as the rates for males and females, data for different countries within the areas, the level of obesity, the number of diseases related to obesity that are common in the areas, factors which cause the obesity. Data for some parts of the world are excluded, for example, high-income countries in north Asia.
   (b) Learners' own answers, for example: the highest figure is 70% for North America in 2008. So obesity seems to be related to high income.
   (c) See the double bar graph at the top of the next TG page.



DRAWING BAR GRAPHS AND DOUBLE BAR GRAPHS

1. Obese (very overweight) people have many health problems. It is a concern all around the world. Health researchers analysed the change over 28 years in the numbers of people who are overweight and obese in different areas of the world. The following table summarises some of the data:

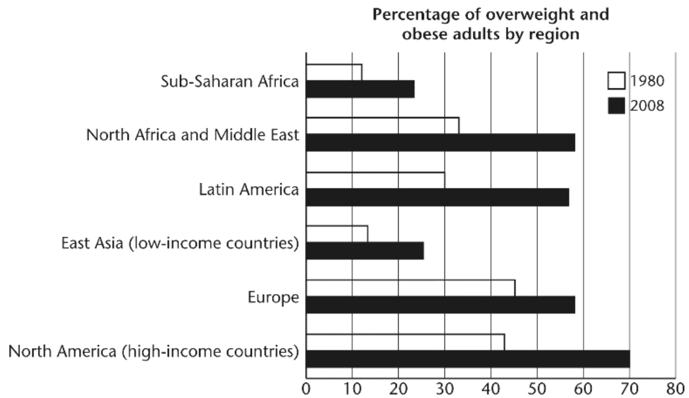
   Percentage of population that is overweight and obese

	1980	2008
Sub-Saharan Africa	12%	23%
North Africa and Middle East	33%	58%
Latin America	30%	57%
East Asia (low-income countries)	13%	25%
Europe	45%	58%
North America (high-income countries)	43%	70%

   (a) The table summarises "some" of the data. What would some other important data be? Think of as many things as you can.
   (b) Which data stands out the most for you in the table above? Give your personal opinion.
   (c) On grid paper, plot a double bar graph to compare the data for the areas, and for the two years. Remember to give your graph a key.
   (d) Look carefully at the comparisons that the graph makes. Has your opinion of the most interesting differences changed, now that you see the double bar graph? Explain.

Answers

1. (c)



(d) Learners' own answers. They should notice that the comparison between the two years is more noticeable between parts of the world such as North Africa and Latin America, which are developing countries. In fact, these areas have almost caught up with the relatively wealthy Europe and North America. Also, the figures for sub-Saharan Africa are low, but have almost doubled since 1980! Use this for a class discussion, focusing on what the double bar graph helps us to analyse in the data.

(e) Sample answer: The highest obesity figures are for wealthy western countries such as those in Europe and North America. However, the rate of increase of obesity in poorer countries has increased dramatically: from 33% to 58% (25%) in North Africa and the Middle East. The data shows a worrying increase (almost doubling) in sub-Saharan Africa, which indicates a problem for the future.

23.2 Histograms

REVISING HISTOGRAMS

Background information

A **histogram** shows grouped numerical data on the horizontal axis and the frequencies of data in different class intervals on the vertical axis. Here are some **features of a histogram**:

- It represents only one set of data.
- It represents numerical data, which is grouped in class intervals.
- Each class interval is used for a range of data values.

(e) In some countries, the obesity problem has been labelled "Obesity in the face of poverty". Write a short report on the data and your double bar graph to support this argument.

23.2 Histograms

REVISING HISTOGRAMS

A histogram is a graph of the frequencies of data in different **class intervals**, as demonstrated in the example below. Each class interval is used for a range of values. The different class intervals are consecutive and cannot have values that overlap. The data may result from counting or from measurement.

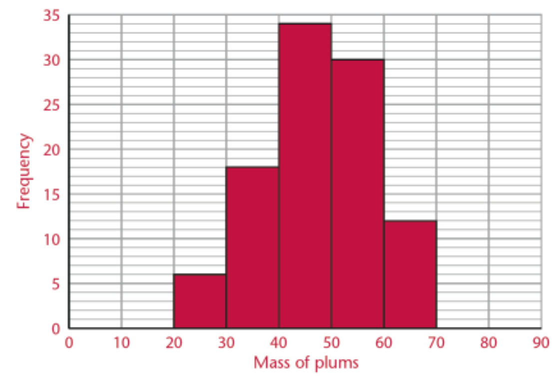
A histogram looks somewhat like a bar graph, but is normally drawn without gaps between the bars.

REPRESENTING DATA IN HISTOGRAMS

1. (a) A fruit farmer wants to know which of his trees are producing good plums and which trees need to be replaced. He collects 100 plums each from two trees and measures their masses. The data below gives the mass of plums from the first tree:

Mass of plums (g)	20–29	30–39	40–49	50–59	60–69
Frequency	6	18	34	30	12

Copy the grid below and use it to represent the data in a histogram.



(b) Now draw another histogram to represent the following data giving the mass of the same type of plums from another tree in the orchard:

Mass of plums (g)	20–29	30–39	40–49	50–59	60–69
Frequency	3	14	26	36	21

- The different class intervals are consecutive and cannot have values that overlap. This means that either the upper boundary or the lower boundary of each class interval is excluded from that interval.
- It uses bars to show the frequencies of the different class intervals. There are no gaps between the bars because the class intervals are consecutive.
- The heights of the bars represent the frequencies of the different class intervals.

Teaching guidelines

Learners revise the properties of histograms.

REPRESENTING DATA IN HISTOGRAMS

Background information

Histograms are suitable to compare frequencies of categories of grouped numerical data, identify patterns in grouped numerical data and describe trends shown by grouped numerical data. When drawing a histogram, remember the following:

- All bars should be equally wide if all class intervals are equally wide.
- There are no gaps between bars unless the frequency of a specific class interval is equal to 0.
- The first bar should not touch the frequency axis.
- The title should explain what the histogram is about.

Teaching guidelines

Learners discuss important facts to remember when they draw histograms.

Misconceptions

"The upper class limit is always excluded from an interval." This is NOT the case. In some cases, the lower class limit is excluded. The strategy used is determined by the question we want to answer, for example:

- If we want to know how many learners scored **50% or more** in a test, 50 should be the **lower class limit** of the class interval **50–60**.
- If we want to know how many learners scored **50% or less**, 50 should be the **upper class limit** of the class interval **40–50**.

Answers

1. (a) See the histogram on LB page 259 alongside.
   (b) See the histogram, labelled as 1(b), at the top of the next TG page.
   (c) The second tree produces fewer small plums (less than 40 g) and more plums that are bigger than 50 g. So the second tree should be kept and the first one should be replaced.

(LB page 259 repeat)

(e) In some countries, the obesity problem has been labelled "Obesity in the face of poverty". Write a short report on the data and your double bar graph to support this argument.

23.2 Histograms

REVISING HISTOGRAMS

A histogram is a graph of the frequencies of data in different **class intervals**, as demonstrated in the example below. Each class interval is used for a range of values. The different class intervals are consecutive and cannot have values that overlap. The data may result from counting or from measurement.

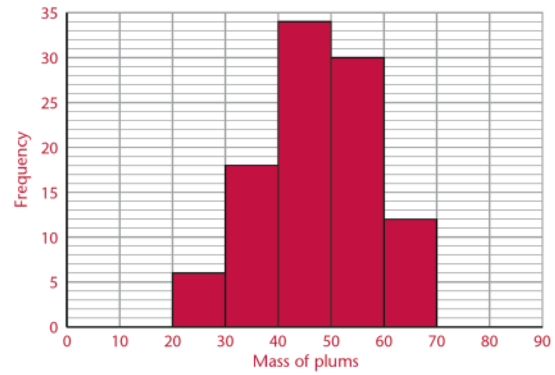
A histogram looks somewhat like a bar graph, but is normally drawn without gaps between the bars.

REPRESENTING DATA IN HISTOGRAMS

1. (a) A fruit farmer wants to know which of his trees are producing good plums and which trees need to be replaced. He collects 100 plums each from two trees and measures their masses. The data below gives the mass of plums from the first tree:

Mass of plums (g)	20–29	30–39	40–49	50–59	60–69
Frequency	6	18	34	30	12

Copy the grid below and use it to represent the data in a histogram.

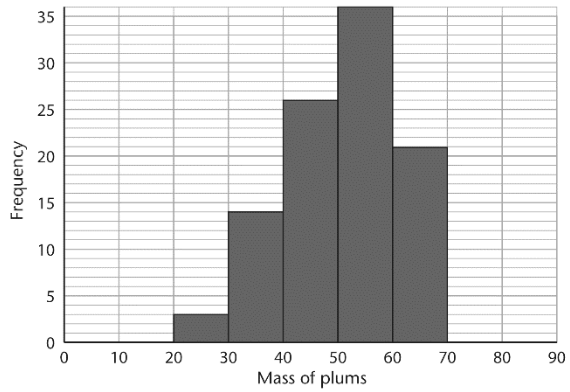


(b) Now draw another histogram to represent the following data giving the mass of the same type of plums from another tree in the orchard:

Mass of plums (g)	20–29	30–39	40–49	50–59	60–69
Frequency	3	14	26	36	21

Answers

1. (b)



2. (a) See the histogram on LB page 260 alongside.

   (b) Mean $= \frac{(57{,}33)}{28} = 2{,}0475 \approx 2{,}05$. Median $= \frac{(1{,}91) + (1{,}99)}{2} = 1{,}95$

   (c) In this data the mean and median birth weights are below the population mean and median.

23.3 Pie charts

Background information

A **pie chart** consists of a circle divided into sectors (slices). Here are some features of a pie chart:

- It represents only one set of data.
- It can be used to display any type of data.
- It uses sectors of the same circle to compare the frequencies of the different categories within a data set. Bigger categories of data have bigger slices of the circle.
- It shows how the data set is divided up into different categories and what fraction of the data set each category represents. The whole chart shows how much each category contributes to the whole.
- The sizes of the sectors are proportional to the frequencies of the different categories, i.e. the fraction or percentage of the whole that the category forms.
- It works best if the frequencies of the different categories in the data set are expressed as percentages.
- The title (heading) shows what the data set is about.
- The key (legend) shows the category that each sector represents.

Teaching guidelines

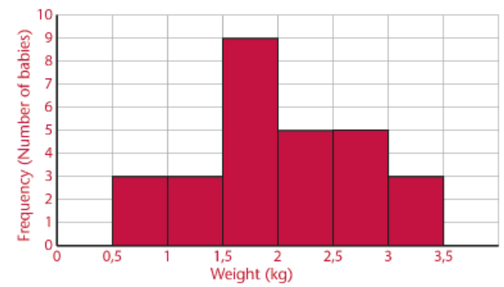
Learners revise the properties of pie charts.

(c) Study the two histograms and then comment on the number of plums produced by the two trees.

2. (a) Use the example below to draw a histogram to represent the data in the table below. Group the data in intervals of 0,5 kg.

Birth weights (kg) of 28 babies at a clinic

3,3	1,34	2,88	2,54	1,87	2,06	2,72
1,89	0,85	1,99	2,43	1,66	2,45	1,62
1,91	1,20	2,45	1,38	0,9	2,65	2,88
1,75	2,11	3,2	1,74	0,6	3,1	1,86



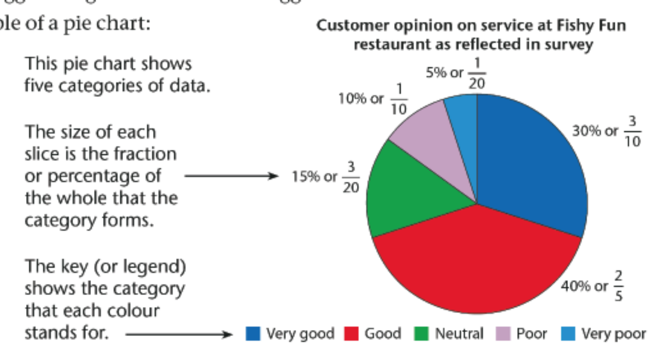
(b) Calculate the mean and median of the data.

(c) Records from the whole country show that the birth weight of babies ranges from 0,5 kg to 4,5 kg, and the mean birth weight is 3,18 kg. Use the graph and the mean and median to write a short report on the data from the clinic.

23.3 Pie charts

A **pie chart** consists of a circle divided into sectors (slices). Each sector shows one category of data. Bigger categories of data have bigger slices of the circle.

Here is an example of a pie chart:

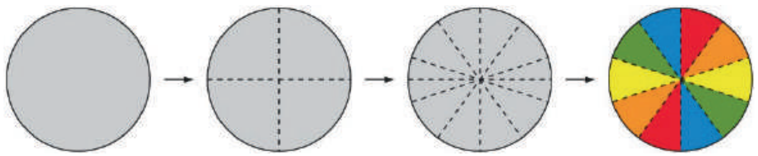


DRAWING PIE CHARTS

Background information

Pie charts are suitable to compare frequencies of categories of any type of data within a data set. They are **not suitable** to identify patterns in the data or describe trends shown by the data. When drawing a pie chart, remember the following:

- The first category on the pie chart usually starts at a vertical line at 12 o'clock and turns in a clockwise direction.
- To divide a circle into ten equal slices using estimation, first divide it into quarters and then divide each quarter into $2\frac{1}{2}$ slices.



- To draw an accurate pie chart, each fraction can be converted to degrees by multiplying it by 360°. A protractor can then be used to construct the different sectors accurately, starting at 12 o'clock and moving in a clockwise direction.
- If a key is not provided, each sector should be accompanied by a **description of the category** and **the percentage it represents**.

Teaching guidelines

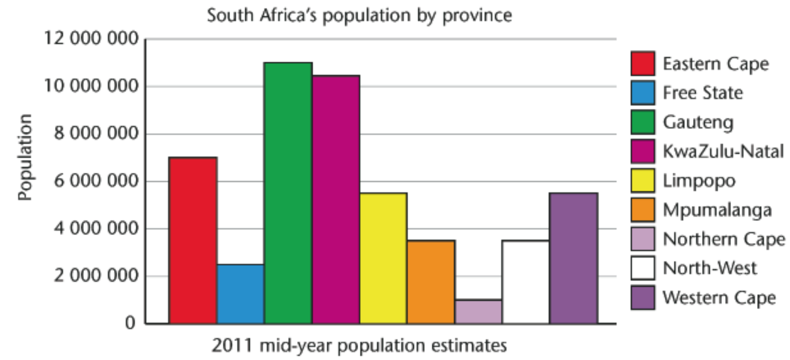
Discuss important facts to remember when drawing pie charts.

Answers

1. (a) See the first table on LB page 261 alongside.
   (b) 50 000 000 or 50 million
   (c) See the second table on LB page 261 alongside.
   (d) The pie chart drawn above is a difficult one to draw, as some sectors are tiny, and shows learners the drawbacks of drawing a pie chart with many sectors.
   (e) The bar graph shows the populations in each province as numbers and not as percentages or fractions. It is easy to compare the numbers by the height of the bars and to see that the highest population is in Gauteng at close to 12 million, and the lowest in Northern Cape at around 1 000 000. The pie chart shows how each population makes up the total of the country's population. It isn't possible to read off the numbers (but the numbers could be given on the bar chart). It is difficult to see the figure for the Northern Cape, as it is only 2% of the population. Which graph is better? Learners must provide a reason for their choice.

DRAWING PIE CHARTS

1. The following bar graph shows the population of South Africa by province.



(a) Copy the table and write down the figures in the graph correct to the nearest 500 000.

Province	E Cape	FS	Gau	KZN	Lim	Mpum	NC	NW	WC
Population (× 1 000)	7 000	2 500	11 000	10 500	5 500	3 500	1 000	3 500	5 500

(b) What is the total of the rounded off numbers?

(c) Work out the percentage of the whole for each province.

Province	E Cape	FS	Gau	KZN	Lim	Mpum	NC	NW	WC
Percentage of total	14%	5%	22%	21%	11%	7%	2%	7%	11%

(d) Draw a pie chart showing the data in the completed table. (Estimate the sizes of the slices.)

(e) Write a short report explaining the difference in the way the data is represented in the pie chart and the bar graph. Which do you think is a better method to show this data?

23.4 Broken-line graphs

BROKEN-LINE GRAPHS

Background information

A **broken-line graph** uses successive points, joined by line segments, to show the frequencies of different categories in a set of data that changes continuously over time. Here are some features of a broken-line graph:

- It represents only one set of data.
- It represents numerical data collected at specific moments in time over a period of time, for example, per month over a period of one year.
- The horizontal axis shows categories of moments in time in consecutive order.
- The vertical axis shows the frequencies of the different categories by using points. The heights of these points above the horizontal axis match the frequencies of the different categories.
- The order of the categories is indicated on the graph by joining successive points using line segments.
- The title shows what the broken-line graph is about.

Teaching guidelines

Learners use the information provided on LB page 262 to revise the properties of broken-line graphs.

Background information (continued)

Broken-line graphs are suitable for identifying patterns in a data set and describing trends shown by the data set. When drawing a broken-line graph, remember the following:

- All categories of time, from start to end, should be shown at equal distances from each other on the horizontal axis.
- The heights of the points above the horizontal axis should match the frequencies of the different categories of time.
- The first category should not lie on the frequency axis.
- A title should explain what the broken-line graph is about.

Teaching guidelines

Learners discuss important facts to remember when they draw broken-line graphs.

23.4 Broken-line graphs

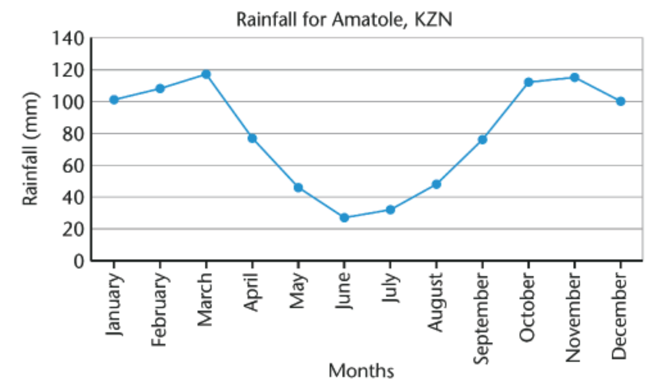
BROKEN-LINE GRAPHS

Broken-line graphs are used to represent data that changes continuously over time. For example, the rainfall for a whole month is captured as one data point, even though the rain is spread out over the month, and it rains on some days and not on others. Broken line graphs are useful to identify and display trends.

Here is some data that can be represented with broken-line graphs:

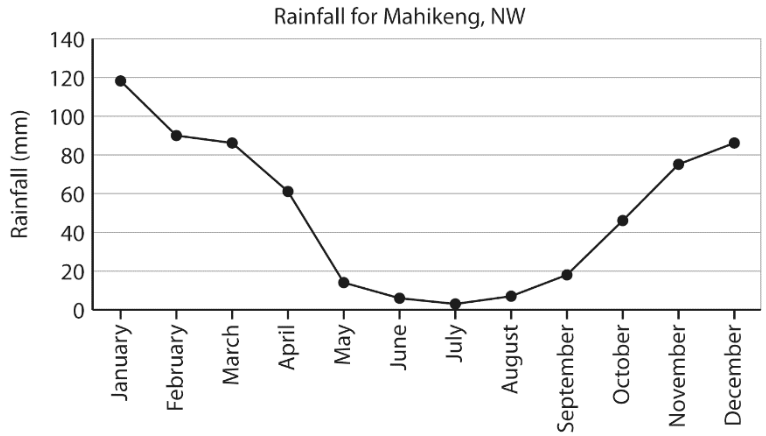
Rainfall at three locations in South Africa in 2012			
	Amatole, KZN	Mahikeng, NW	Ceres, WC
	Rainfall (mm)	Rainfall (mm)	Rainfall (mm)
January	101	118	27
February	108	90	23
March	117	86	41
April	77	61	60
May	46	14	130
June	27	6	168
July	32	3	152
August	48	7	162
September	76	18	88
October	112	46	60
November	115	75	41
December	100	86	36

Here is a broken line graph for the Amatole rainfall data:

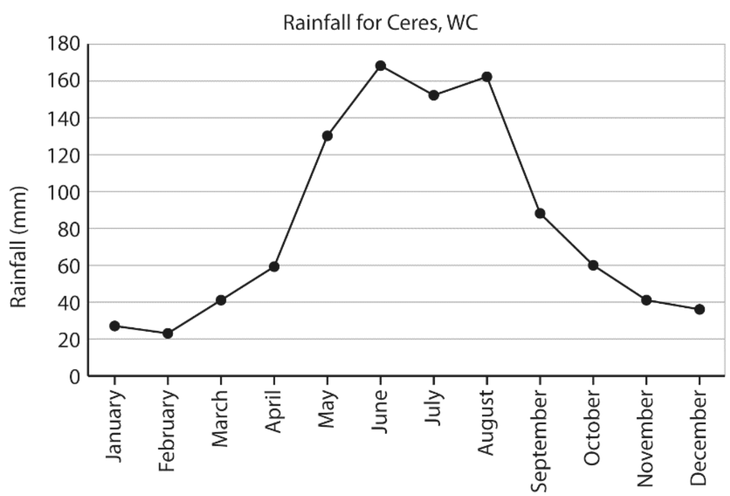


Answers

1. May, June, July, and August
2. January, February, March, October, November, and December
3. June or July
4. Temperature, wind, other precipitation (snow, hail, etc.)
5.



6.



7. In Ceres, the high rainfall period is during the winter months (May – September). In Mahikeng, the high rainfall period is during the summer months (November – March). The high rainfall months in Ceres have more rainfall than the high rainfall months in Mahikeng.

1. During which four months does Amatole have the least rain?
2. During which six months does Amatole have the most rain?
3. During which months would you plan a hike if you were only considering the rainfall patterns?
4. What other factors should you consider when planning a hike in this region?
5. Make a broken-line graph for the Mahikeng rainfall data.
6. Make a broken-line graph for the Ceres rainfall data.
7. Write a few lines on the difference in rainfall patterns between Ceres and Mahikeng.
8. Draw a combined broken-line graph with the information from all three regions on one graph.

23.5 Scatter plots

UNDERSTANDING AND CONSTRUCTING SCATTER PLOTS

Scatter plots show how two sets of numerical data are related. Matching pairs of numbers are treated as coordinates and are plotted as a single point. All the points, made up of two data items each, show a scattering across the graph.

1. This table shows a data set with **two** variables. Study the information in the table.
2. Copy the number lines on the next page and make a dot for each learner's mark for each subject.

Learners	Mathematics marks	Natural Sciences marks
Zinzi	**25**	**26**
John	23	25
Palesa	**22**	**25**
Siza	21	23
Eric	20	23
Chokocha	19	21
Gabriel	17	20
Simon	16	19
Miriam	15	18
Frederik	15	16
Sibusiso	**12**	**15**
Meshack	11	13
Duma	11	12
Samuel	10	12
Lola	10	11
Thandile	9	10
Jabulani	**8**	**10**
Manare	7	9
Marlene	7	7
Mary	5	7

Note on question 8

A **combined broken-line graph** shows two or more sets of data in the same categories on the same set of axes. Some features of a combined broken-line graph are:

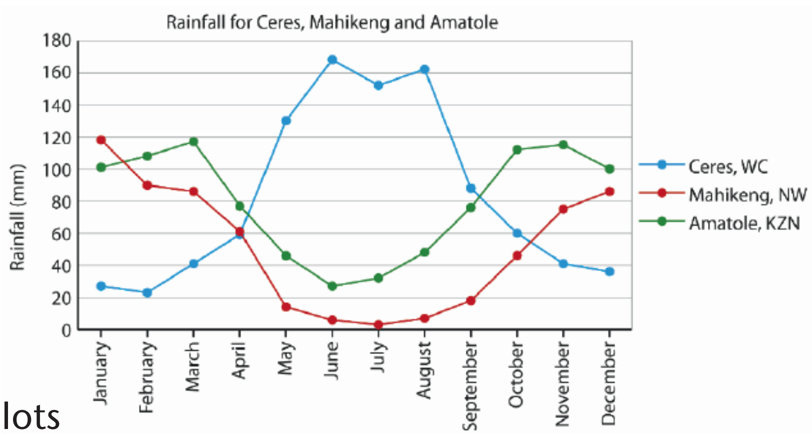
- It represents two or more sets of data with matching categories.
- It represents numerical data collected over a period of time.
- The horizontal axis shows categories of moments in time in consecutive order.
- The vertical axis shows the frequencies of the different categories by using points. The heights of these points above the horizontal axis match the frequencies of the different categories for each data set.
- The order of the categories of the different data sets is indicated on the graphs by joining successive points using line segments.
- The title shows what the combined broken-line graph is about.
- The key explains the colours used to distinguish the sets of data.

Teaching guidelines

Discuss important facts to remember when drawing combined broken-line graphs.

Answers

8.



23.5 Scatter plots

UNDERSTANDING AND CONSTRUCTING SCATTER PLOTS

Background information

A **scatter plot** uses ordered pairs of numerical data, which are plotted as points on a coordinate system, to show how two sets of data are related. Here are some features of a scatter plot:

- It shows two sets of related numerical data as a cloud of points across the

(LB page 263 repeat)

1. During which four months does Amatole have the least rain?
2. During which six months does Amatole have the most rain?
3. During which months would you plan a hike if you were only considering the rainfall patterns?
4. What other factors should you consider when planning a hike in this region?
5. Make a broken-line graph for the Mahikeng rainfall data.
6. Make a broken-line graph for the Ceres rainfall data.
7. Write a few lines on the difference in rainfall patterns between Ceres and Mahikeng.
8. Draw a combined broken-line graph with the information from all three regions on one graph.

23.5 Scatter plots

UNDERSTANDING AND CONSTRUCTING SCATTER PLOTS

Scatter plots show how two sets of numerical data are related. Matching pairs of numbers are treated as coordinates and are plotted as a single point. All the points, made up of two data items each, show a scattering across the graph.

1. This table shows a data set with **two** variables. Study the information in the table.
2. Copy the number lines on the next page and make a dot for each learner's mark for each subject.

Learners	Mathematics marks	Natural Sciences marks
Zinzi	**25**	**26**
John	23	25
Palesa	**22**	**25**
Siza	21	23
Eric	20	23
Chokocha	19	21
Gabriel	17	20
Simon	16	19
Miriam	15	18
Frederik	15	16
Sibusiso	**12**	**15**
Meshack	11	13
Duma	11	12
Samuel	10	12
Lola	10	11
Thandile	9	10
Jabulani	**8**	**10**
Manare	7	9
Marlene	7	7
Mary	5	7

coordinate system.

- The horizontal axis represents the data values of the first data set.
- The vertical axis represents the data values of the second data set.
- Each point on the coordinate system represents two related data values obtained from the two data sets.

Teaching guidelines

Use questions 1 to 5 on LB page 263 on the previous page and LB page 264 alongside to discuss the features of a scatter plot.

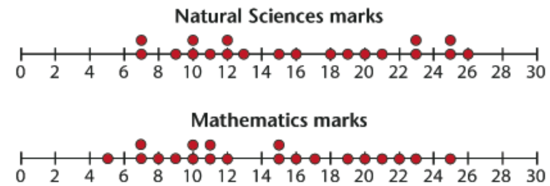
Answers

1. Learners study the information in the table on LB page 263.
2. See the two dot plots on LB page 264 alongside.
3. See the scatter plot on LB page 264 alongside.
4. See the dot labelled "S" on the scatter plot on LB page 264 alongside.
5. See the dots labelled "Z", "P", "J" and "M" on the scatter plot on LB page 264 alongside.

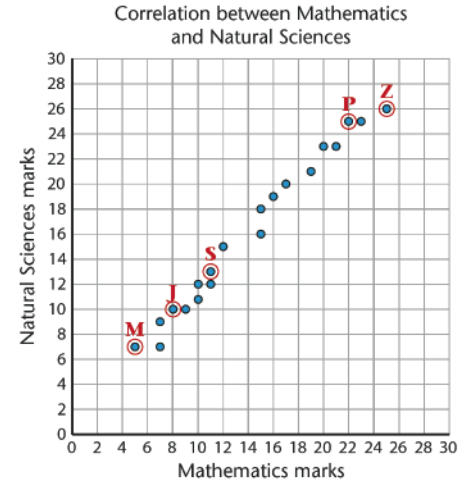
Background information (continued)

Correlation show whether and how strongly pairs of matching data values in two data sets are related. This relationship between the two sets of data is described in terms of the shape, direction and strength shown by the cloud of points on the scatter plot.

- The **shape** of the cloud is described by using words such as:
  - "**linear**" when the points are scattered around an imaginary line called the **trend line**
  - "**curved**" when the points are scattered in the shape of a curve.
- The **direction** shown by the cloud is described by using words such as:
  - "**positive**" when an increase in a data value from one data set matches an increase in the corresponding data value from the other data set
  - "**negative**" when an increase in a data value from one data set matches a decrease in the corresponding data value from the other data set.
- The **strength** of the relationship shown by the cloud is described by using words such as:
  - "**weak**" when the points are scattered wide around the trend line
  - "**strong**" when the points are scattered closely around the trend line
  - "**perfect**" when all the points lie on the trend line.
- When there is **no correlation** between two data sets the points are scattered all over the coordinate system.



3. What if you were to show both sets of marks on the same graph, instead of a separate number line for each set? The graph below shows a scatter plot that represents both sets of data. Each dot represents one learner. Copy the scatter plot.



The scatter plot shows the **relationship** between the Natural Sciences mark and the Mathematics mark.

4. Find the dot for Sibusiso in the data set. He obtained a mark of 12 for the Mathematics test and a mark of 15 for Natural Sciences. Find 12 on the horizontal axis. Follow the vertical line up until you reach a blue dot. Find 15 on the vertical axis. Follow the line horizontally until you reach the same blue dot. This blue dot represents the two marks that belong to Sibusiso. On your scatter plot, circle the blue dot and label it "S".
5. Find the data points for Zinzi, Palesa, Jabulani and Mary. On your scatter plot, circle them and label them Z, P, J and M.

264 MATHEMATICS GRADE 9: TERM 4

Teaching guidelines (continued)

Discuss the concept of correlation as explained in the background information on the previous page. Point out that the scatter plot on LB page 264 on previous page shows:

- a **linear correlation** between Mathematics marks and Natural Sciences marks because all the points are scattered around an imaginary line
- a **positive correlation** between Mathematics marks and Natural Sciences marks because a higher mark in Mathematics corresponds to a higher mark in Natural Sciences
- a **strong correlation** between Mathematics marks and Natural Sciences marks because all the points lie relatively close to the imaginary trend line.

Answers

6. Learners study the table on LB page 265 and the first scatter plot on LB page 266.
7. See the dot labelled "E" on the first scatter plot on LB page 266.
8. See the dot labelled "S" on the first scatter plot on LB page 266.
9. Eric has 20 for Mathematics and 10 for Art, and Samuel's marks are the other way around.
10. See the labelled dots "Z", "E", "M", "F", "S" and "Ma" on the first scatter plot on LB page 266 on the next page.

In the example on page 264, a higher Mathematics mark corresponds to a higher Natural Sciences mark. We say there is a **positive correlation** between the Mathematics marks and the Natural Sciences marks.

6. Study this data set and the scatter plot of the data given on the next page. Copy the scatter plot.

Learner	Mathematics marks	Art marks
Zinzi	**25**	**7**
John	23	7
Jabulani	22	9
Siza	21	10
Eric	**20**	**10**
Chokocha	19	11
Gabriel	17	12
Simon	16	12
Miriam	**15**	**15**
Frederik	**15**	**15**
Sibusiso	12	16
Mishack	11	17
Duma	11	19
Samuel	**10**	**20**
Lola	10	21
Thandile	9	23
Palesa	8	23
Manare	7	25
Marlene	7	25
Mary	**5**	**26**

7. Find Eric in the table. Note his marks for Mathematics and Art. Find the dot that represents his marks on the scatter plot. Encircle it and label it E.
8. Find Samuel in the table. Note his marks for Mathematics and Art. Find the dot that represents his marks. Encircle it and label it S.
9. Compare the two sets of marks for Eric and for Samuel. What do you notice about the marks?
10. On your scatter plot, find the data points on the scatter plot for Zinzi, Eric, Miriam, Frederik, Samuel and Mary. Circle the points and label them Z, E, M, F, S and Ma.

Answers

11. Lower Art marks correspond to higher Mathematics marks.

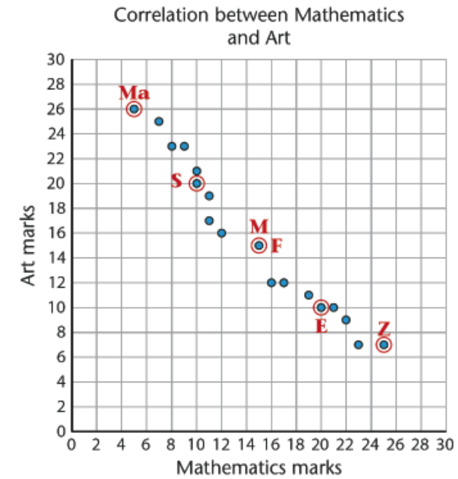
Teaching guidelines (continued)

Point out that the first scatter plot on LB page 266 shows:

- a **linear correlation** between Mathematics marks and Art marks because all the points are scattered around an imaginary line
- a **negative correlation** between Mathematics marks and Art marks because a higher mark in Mathematics corresponds to a lower mark in Art
- a **strong correlation** between Mathematics marks and Art marks because all the points lie relatively close to the imaginary trend line.

Point out that the second scatter plot on LB page 266 alongside shows no correlation because the points are scattered all over the coordinate system.

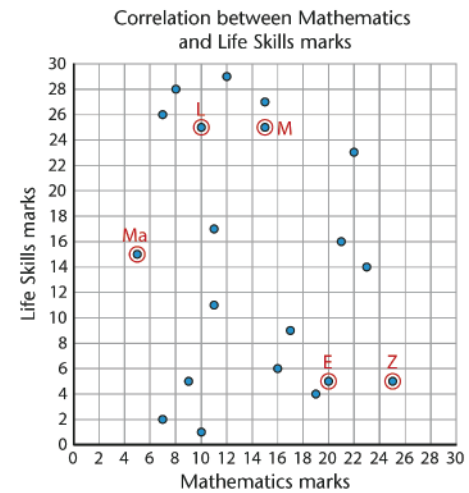
11. What do you notice about the pattern of marks in Mathematics and Art for this data set?



A **negative correlation** is a correlation in which an increase in the value of one piece of data tends to be matched by the decrease in the other set of data. Learners who obtain a high mark for Mathematics appear to obtain a low mark for Art. We say there is a negative correlation between the Mathematics and Art scores for this data set.

A correlation is an assessment of how strongly two sets of data appear to be connected. Two sets of data may be correlated or may show **no correlation**.

Here is the scatter plot for the Mathematics and Life Skills marks of the same group of learners. The table for this data is given on the next page.



266 MATHEMATICS GRADE 9: TERM 4

Answers

12. Learners study the second scatter plot on LB page 266 on the previous page and the data table on LB page 267 alongside.
13. See the labelled dots "Z", "E", "M", "L" and "Ma" on the second scatter plot on LB page 266 on the previous page.
14. There is no pattern that we can see in the relationship, there seems to be no correlation between the Mathematics marks and the Life Skills marks.

THE RELATIONSHIP BETWEEN ARM SPAN AND HEIGHT

Background information

To **draw a scatter plot**, remember the following:

- Scatter plots show two related data sets on a coordinate system.
- The horizontal axis represents the first data set.
- The vertical axis represents the second data set.
- To avoid misleading scatter plots, start each axis at 0 and use similar scales on both axes.
- Plot each point by starting at the origin and moving to the right to find the first data value on the horizontal axis. From that point, move upwards to find the point in line with the second data value on the vertical axis.

Teaching guidelines

Learners discuss important facts to remember when they draw scatter plots.

12. Study the scatter plot on the previous page and the data table below. Copy the scatter plot.

13. On your scatter plot, find the data points on the scatter plot for Zinzi, Eric, Miriam, Lola and Mary. Circle the points and label them Z, E, M, L and Ma.

14. What do you notice about the pattern of marks in Mathematics and Life Skills for this data set?

Learner	Mathematics	Life Skills
Zinzi	25	5
John	23	14
Jabulani	22	23
Siza	21	16
Eric	20	5
Chokocha	19	4
Gabriel	17	9
Simon	16	6
Miriam	15	25
Frederik	15	27
Sibusiso	12	29
Meshack	11	17
Duma	11	11
Samuel	10	1
Lola	10	25
Thandile	9	5
Palesa	8	28
Manare	7	26
Marlene	7	2
Mary	5	15

THE RELATIONSHIP BETWEEN ARM SPAN AND HEIGHT

The idea that a person's arm span (the distance from the tip of the middle finger on one hand to the tip of the middle finger on the other hand when the arms are stretched out sideways), is the same as one's height has been explored many times.

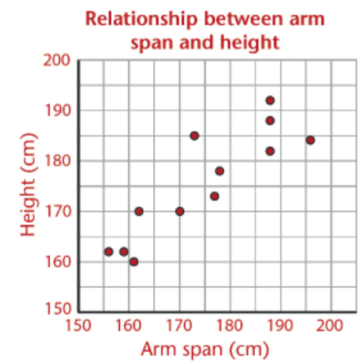
A data set for 13 people is given on the next page.

Answers

1. Learners draw the scatter plot such as the one shown on LB page 268 alongside.
2. There is a positive correlation between height and arm span (but the correlation is not very strong).

1. Make a scatter plot of this data on a grid like the one below.
   For example, take Cilla's arm span. Find 156 on the horizontal axis. Follow a vertical line up. Then on the vertical axis find 162. Follow a horizontal line across. Where the two points meet, draw a dot.

Person	Arm span	Height
Cilla	156	162
Meshack	159	162
Tony	161	160
Ellen	162	170
Karin	170	170
Sibongile	173	185
Gabriel	177	173
Alpheus	178	178
Mfiki	188	188
Nathi	188	182
Manare	188	192
Khanyi	196	184



2. What would you say about the correlation between the arm span and the height?

Learner Book Overview		
Sections in this chapter	**Content**	**Pages in Learner Book**
24.1 Which graph is best?	Advantages of statistical graphs; the best representation for given situations	Pages 269 to 271
24.2 The effects of summary statistics on how data is reported	Advantages and disadvantages of the mean, median and mode as summary statistics	Pages 272 to 273
24.3 Misleading graphs	Strategies used to create misleading graphs; analysis of graphs	Pages 273 to 276
24.4 Analysing extreme values and outliers	The influence of outliers on data sets; finding outliers	Pages 276 to 279

CAPS time allocation	3,5 hours
CAPS content specification	Page 151

Mathematical background

In this chapter the focus is on interpretation and analysis of data and reporting on findings. By looking at reported data and analysing the whole data handling cycle for the data, learners will develop and practise some critical data analysis skills.

- To **interpret data** means to extract information directly from the data.
- To **analyse data** means to investigate the given data in order to find patterns and trends shown by the data.
  - **Patterns** can be cyclic, for example, data may show that learners tend to be absent from school on Mondays.
  - **Trends** are increasing or decreasing patterns, for example, the percentage of girls per grade increases in higher grades.
- To **report on data** means to summarise the most important findings from the data in a short paragraph.
- **Bias** is a term which refers to how far the information gained from a sample lies from the information hidden by the population. During the data handling cycle bias can already surface during the sampling process.
  - A **biased sampling method** is a method that tends to give non-representative samples. Such samples under-represent or over-represent some characteristics of the population. For biased sampling methods, refer to the background information provided for LB page 249.
  - An **unbiased sampling method** is a method than tends to give representative samples. Such samples give a true reflection of the characteristics of the population. For unbiased sampling methods, refer to the background information provided for LB page 249.
- **Misleading data** is data which is manipulated in order to favour a specific opinion. This usually occurs when statistical displays in newspapers and financial reports are presented in such a way that the data reflects a positive picture of one party and/or a negative picture of the opposition. It can be done in a variety of ways, for example, starting the scale on the frequency axis at a point other than 0; using three-dimensional displays to give a better impression of a specific category; or using different scales on the frequency axes of two displays.

24.1 Which graph is best?

Background information

Learners need to be able to choose the most suitable data representation in a given situation. Here are some examples of the **advantages and disadvantages of using tables and different types of graphs**.

	Advantages	**Disadvantages**
Tables	They can show any type of data. They show more information than graphs.	Patterns and particular trends are not as easy to see.
Pie charts	They can show any type of data. They show how categories relate to each other and how categories relate to the whole.	They do not show the quantities involved.
Bar graphs	They can show categorical or discrete data. They show the amount of quantities involved. They allow us to compare the quantities of different categories.	They cannot show grouped numerical data. They do not show the relationship between categories very clearly. They can be easily manipulated to give false impressions.
Double bar graphs	They compare quantities for two or more data sets, for example, related data for males and females.	They can be easily manipulated to give false impressions.
Histograms	They represent numerical data that is grouped into equal class intervals. They show the way the data is spread out.	They can only be used to represent grouped numerical data.
Broken-line graphs	They show patterns or trends in quantities over time.	They can be easily manipulated to give false impressions.

Teaching guidelines

Learners discuss the advantages and disadvantages of different statistical displays.

CHAPTER 24
Interpret, analyse and report on data

24.1 Which graph is best?

You have learnt that certain types of graphs are best for displaying certain kinds of information. The type of graph depends mostly on the type of data that needs to be represented. Here is a summary of the advantages of different types of graphs:

Tables show more information than graphs but the patterns are not as easy to see. They do not give a visual impression of particular trends.

Pie charts show a whole divided into parts. They show how the parts relate to each other and how the parts relate to a whole. They do not show the quantities involved.

Bar graphs show the amounts or quantities involved but do not show the relationship as effectively as pie charts. They are useful for showing **quantitative** data. Bar charts allow us to compare the quantities of different categories, for example, the sales of different items.

A **double-bar graph** is used to compare two or more things for each category. For example, we could use a double-bar graph to compare the differences between males and females.

Histograms are used to represent numerical data that is grouped into equal class intervals. Histograms are useful to show the way the data is spread out.

Broken-line graphs show trends or changes in quantities over time.

CHOOSE THE BEST REPRESENTATION

Teaching guidelines

Learners choose the best way of representing data in a variety of situations and explain their answers.

Answers

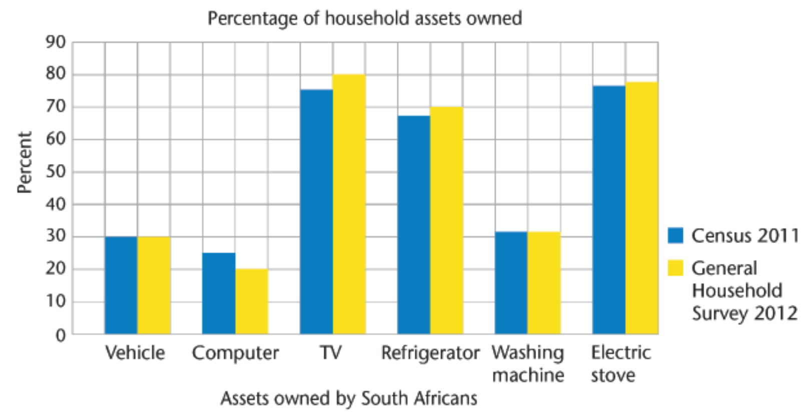
1. (a) A broken-line graph – shows trends over time
   (b) A double bar graph – compares two categories of data
   (c) A pie chart shows proportions. You could also use a bar graph if there are many categories.
   (d) A bar graph would show the quantities effectively. A pie chart would show the percentage (of all the crops produced) that each crop represents.
   (e) A pie chart shows parts of the whole. You could also use a bar graph if there are many categories.
   (f) Broken-line graph – shows trends over time
2. (a) The two surveys show very similar results, and they are only one year apart, but it does show that the data is likely to be reliable.
   (b) Yes, it is useful to see which items most people regard as the most important items to buy; in this case electric stoves and television sets.
   (c) Yes, it is useful, particularly to manufacturers and sellers, to see that only 30% of people own a vehicle, 30% own a washing machine and around 20% own a computer. This shows that there is room for increased sales of these items.
   (d)

	Percentage of people (census 2011)	**Percentage of people (GHS 2012)**
Vehicle	30%	30%
Computer	25%	20%
Television	75%	80%
Refrigerator	68%	70%
Washing machine	31%	31%
Electric stove	77%	78%

   (e) It is not as easy to see the differences between the numbers in the table at a glance.

CHOOSE THE BEST REPRESENTATION

1. Which kind of graph is best to represent each of the following? Explain your answers.
   (a) Showing the value of the rand against the US dollar over several years
   (b) Comparing the monthly sales of six different makes of car in 2014 and 2015
   (c) The proportion of people of different age groups in a town
   (d) The quantities of different crops produced on a farm
   (e) The percentages of different goods sold to make up the total sales for a shop
   (f) The change in HIV infection rates over time
2. This graph was published by Statistics South Africa to show the assets owned by South Africans. The blue bar shows the Census 2011 results and the yellow bar shows the General Household Survey 2012 results.



   Give reasons for your answers to the following questions:
   (a) Is it useful to show the differences in the results of Census 2011 and the General Household Survey 2012?
   (b) Is it useful to collect data on assets that people own?
   (c) Is it useful to show that lower percentages of people own certain assets?
   (d) The different coloured bars represent the two different surveys. Draw up a table to show the data in table form. (Read the percentages as accurately as you can from the graph and round off the data to the nearest whole number for the table.)
   (e) Does the table show the data as effectively as the double bar chart? Give your own opinion.

Answers

3. (a) A pie chart is a good way to compare the percentages.
   (b) A broken-line graph will show the change over time effectively.
   (c) A pie chart is a good way to compare the parts or proportions.
   (d) A double bar graph would show the different divisions of the categories at the three time periods.

3. The table below shows the employment status of people ages 15–64 years in South Africa. Discuss some ways of representing the data (e.g. graphs). Justify your answers.

	Jul–Sept 2012	Apr–June 2013	Jul–Sep 2013
	Number of people (thousands)		
Population 15–64 years old	33 017	33 352	33 464
Labour force	18 313	18 444	18 638
Employed	**13 645**	**13 720**	**14 028**
Formal sector (non-agricultural)	9 663	9 694	10 008
Informal sector (non-agricultural)	2 197	2 221	2 182
Agriculture	661	712	706
Private households	1 124	1 093	1 132
Unemployed	**4 668**	**4 723**	**4 609**
Not economically active	**14 705**	**14 908**	**14 826**
Discouraged work-seekers	2 170	2 365	2 240
Other (not economically active)	12 535	12 543	12 586
Unemployment rate (%)	25,5	25,6	24,7

(a) The percentages of the employed, unemployed, and not economically active people in July–September 2013
(b) The change in the employment rates over three time periods
(c) The proportions of employed people who work in the formal sector, informal sector, agriculture and private households
(d) The numbers of the employed and unemployed over the three time periods

24.2 The effects of summary statistics on how data is reported

Background information

In summarising data, some measures are more appropriate for different types of data. Here are some **advantages and disadvantages of some summary statistics**:

Mean: This is the "average" of a data set.

- Advantages: It is useful for describing any type of numerical data. It involves all values in a data set.
- Disadvantages: It can only be used for numerical data. It is not reliable if the data set is too spread out. It is affected by extreme values (outliers) in the data set.

Median: This is the middle value of an ordered data set.

- Advantage: It is not affected by extreme values (outliers) in the data set.
- Disadvantages: It can only be used for numerical data. It takes a long time to calculate for large sets of numerical data.

Mode: This is the most common value in a data set.

- Advantage: It can be used for categorical and numerical data.
- Disadvantages: Some data sets have more than one mode, while other data sets may have no mode.

Teaching guidelines

Learners use the example on LB page 272 to discuss advantages and disadvantages of measures of central tendency.

24.2 The effects of summary statistics on how data is reported

Information articles often use averages to report information. The articles might not use the exact terms for average that you have learnt about: the mean, median and mode. Instead, they may use terms such as "most". However, it is important to be sure about the kind of average to which a report refers, because an average gives us different information.

- Remember that the **mean** is useful for describing a set of measurement values, but can also be used for other numerical data sets. The word "average" usually refers to the "mean" if it is not explained further. The mean is not reliable if a data set is too spread out.
- The **median** is the value in the middle of a data set when it is arranged in order. Half the values in the data set are lower than the median and half of them are higher than the median. The median is often the average used when data values are not uniformly distributed, because the mean is affected by extreme values in the data set, while the median is not. For example, house prices vary widely, so the median would be a better description of the data than the mean. When the median is given in a report, the writer should state that he or she is using the median or middle value.
- The **mode** is the number that occurs most often in a set of data. For example, if we collect data about people's favourite colours, the data set would be a list of colours, and the mode would be the colour that comes up most often. The mode can also be used for numbers. Not all data sets have a mode, because sometimes none of the numbers occurs more than once.

Example: The standard way of reporting house prices in South Africa and internationally is the median house price, which is used by economists in financial reports. The median is regarded as more useful than the mean house price because the sale of a few expensive houses would increase the mean, but would not affect the median.

If a bank gives bonds for eight houses to the value of R100 000, and for two houses to the value of R1 million, then the mean would be R280 000. This does not seem to be an accurate reflection of the value of the houses, because it is distorted by the higher values. The median house price would be R100 000, which is an accurate reflection of the prices.

Remember that the median is the middle point, and half of the values fall below the median, and half above. If the median is lower than the mean, this shows us that there are high values that are distorting the mean.

USING DIFFERENT SUMMARY STATISTICS

Teaching guidelines

Learners answer some questions involving the mean, median and mode.

Answers

1. (a) mean (b) mode (c) mode
   (d) mean (e) mode
2. (a) The mean tends to be shifted upwards if there are extreme values. In this case, there are a few higher salaries, so the mean is shifted upwards, while the median shows that half of the salaries will be below R5 000.
   (b) The median is generally a better indicator of the real situation when the data is not evenly spread out.

24.3 Misleading graphs

Background information

Information represented in graphs is often manipulated to emphasise a particular result because the writer simply wants to make an argument more obvious to the reader. Here are some examples of **techniques that can be used to create misleading graphs**.

Changing the scale of the frequency axis:

- Expanding the scale on the frequency axis increases the spaces between the numbers. This results in bigger differences between frequencies of categories and more dramatic patterns and trends shown by the graph.
- Compressing the scale on the frequency axis decreases the spaces between the numbers. This results in smaller differences between frequencies of categories and less dramatic patterns and trends shown by the data.

Starting the frequency scale at a point other than 0:

- This technique is often used to magnify patterns and trends shown by the data.

Using 2D figures or 3D objects to represent data:

- Frequencies of categories are represented by height. Using 2D figures emphasises area rather than height.
- Frequencies of categories are represented by height. Using 3D objects emphasises volume rather than height.

USING DIFFERENT SUMMARY STATISTICS

1. What kind of average is used in each of these statements?
   (a) The average family has 2,6 children.
   (b) Most families have three children.
   (c) Most people prefer red cars.
   (d) The average height for women is 1,62 m.
   (e) More people shop after work than at any other time during the day.
2. The mean monthly salary of all the staff at company ABC is R8 000 per month, but the median salary is R5 000.
   (a) Explain why the two summary statistics are so different.
   (b) Which summary statistic gives a better idea of the salaries at the company? Give reasons for your answer.

24.3 Misleading graphs

The media (i.e. newspapers, magazines and television), regularly use graphs to show information. Unfortunately, the information is often manipulated to emphasise a particular result. This may be because the writer simply wants to make his or her argument more obvious to the reader.

Changing the scale of the axis

If you change the scale of the vertical axis on bar graphs and line graphs, you will change the way the graphs look. For a bar graph, the larger the spaces between the numbers on the vertical axis, the bigger the difference between the bars. The smaller the spaces between the numbers on the axis, the smaller the difference in the height of the bars. The same is true for a line graph which will either have sharp points or be much flatter, depending on how you have changed the scale.

Example: The two broken-line graphs on the next page show the same sales data for a business over a period of six months. Which graph gives the more accurate impression?

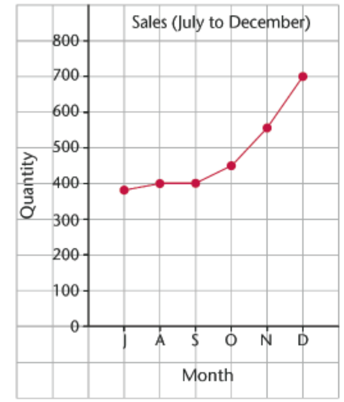
Teaching guidelines

Use the example on LB pages 273 and 274 to illustrate how changing the scale on the frequency axis and/or starting it at a point other than 0 can be used to create misleading graphs.

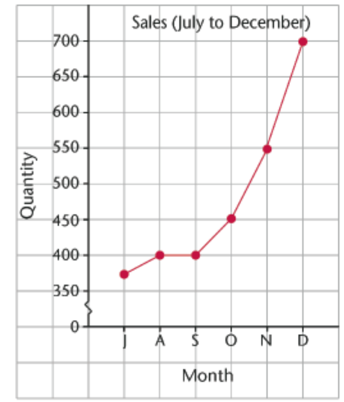
Mathematical notes

The scale on the frequency axis of the first broken-line graph on LB page 274 starts at 0 and increases in increments of 100 per block. The graph shows a moderate increase in sales over the period of six months.

The scale on the frequency axis of the second broken-line graph on LB page 274 does not start at 0 and increases in increments of 100 per two blocks and therefore the graph shows a steeper increase in sales over the period of six months.



Graph A



Graph B

Graph B has a different scale on the vertical axis. The vertical axis does not start at 0 and so **two** blocks on the vertical axis represent 100 items instead of only **one** block, as in Graph A. This makes it look as if the sales increased rapidly over the six months.

Note that it is not necessarily wrong to change the scale on the axes or not to start at 0. For example, graphs showing stock exchange fluctuations rarely show the origin on the graph and stockbrokers are taught to interpret the graphs in that form. Sometimes small changes in data values have important effects and in these cases, it may be valid to change the scale to show these.

274 MATHEMATICS GRADE 9: TERM 4

ANALYSING GRAPHS

Teaching guidelines

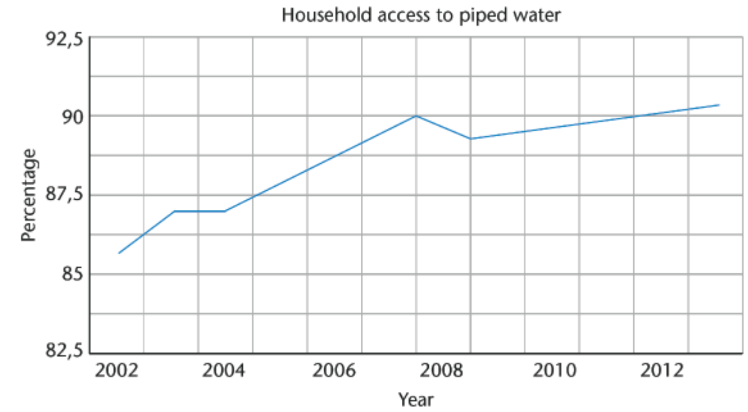
Learners analyse graphs in order to identify techniques that are used to mislead.

Answers

1. (a) The axis does not show zero and emphasises small changes in the percentages over ten years. The large spaces between percentage points make the increases appear large. However, Statistics SA is not misleading the reader, because increases in access of nearly 6% over ten years indicates that progress has been made.
   (b) The scale could be increased so that the increases are even steeper.
   (c) The scale on the vertical axis could be reduced so that the graph is flatter.
2. The graph is misleading because the overall size of the 2014 house gives the impression that the sales have increased more than they actually have. The height of a house represents the number of sales; the three-dimensional pictures of the houses emphasise volume rather than height.

ANALYSING GRAPHS

1. This graph from Statistics South Africa shows the increase in the percentage of households that had access to piped water over a ten-year period.



   (a) Comment on the scale used on the vertical axis. Is this a misleading graph?
   (b) How could you redraw the graph so that the differences on the graph are more noticeable?
   (c) How could you draw the graph so that the differences are less noticeable?

2. In this graph the height of the houses represents the number of sales.

Do you think that this graph is misleading? Give reason(s) for your answer.

Answers

3. Graph B is drawn correctly because the pictures are spaced out so that they line up, showing that there are four of each kind of fruit. Graph A is misleading because there appear to be more apples than bananas and pears.

24.4 Analysing extreme values and outliers

Background information

- An **outlier** is a data value that is very different from all (or most) of the other data values in a data set.
- When data is displayed on **scatter plots**, outliers become clearly noticeable.

Teaching guidelines

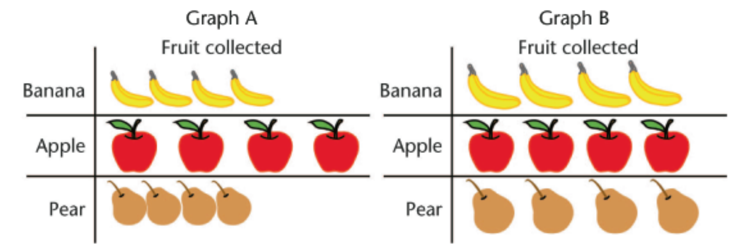
Learners interpret outliers that appear on some scatter plots.

Note on question 1

The scatter plot on LB page 276 shows:

- a **linear correlation** between Mathematics marks and History marks because the majority of points are scattered around an imaginary line running from top left to bottom right
- a **negative correlation** between Mathematics marks and History marks because a higher mark in Mathematics corresponds to a lower mark in History and vice versa
- a **weak correlation** between Mathematics marks and History marks because the points are widely scattered around the imaginary trend line
- **outliers** at the points that represent marks scored by Sara and Raphael because these points lie noticeably further away from the trend line.

3. Look at the two graphs below:

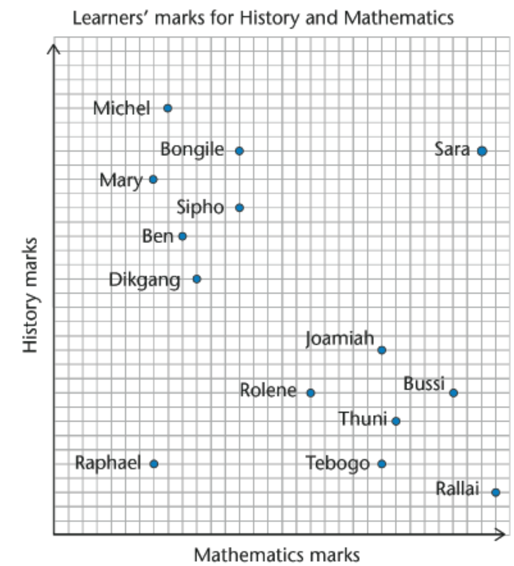


Which graph do you think is drawn correctly? Explain your answer.

24.4 Analysing extreme values and outliers

A data item that is very different from all (or most) of the other items in a data set is called an **outlier**.

It is sometimes difficult to notice outliers in numerical data. However, outliers often become clearly noticeable when data is displayed with graphs.



Answers

1. Sara and Raphael's performances can be regarded as outliers. Most people who did well in History, fared poorly in Mathematics. Most people who did well in Mathematics, fared poorly in History. Sara is the only one who did well in both Mathematics and History. Raphael is the only one who did poorly in both Mathematics and History.

Note on question 2

The scatter plot on LB page 277 shows:

- a **linear correlation** between load weight and fuel consumption because the majority of points are scattered around the trend line
- a **positive correlation** between load weight and fuel consumption because a higher load weight corresponds to a higher fuel consumption and vice versa
- a **strong correlation** between load weight and fuel consumption because the points lie closely around the trend line
- an **outlier** one point below the trend line which lies noticeably further away from the trend line than the other points.

Answers

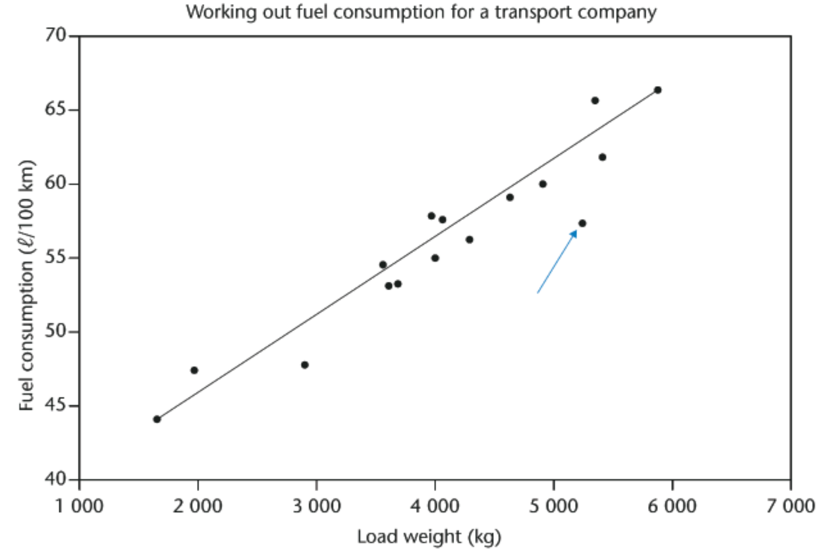
2. (a) The load weight and the fuel consumption
   (b) Job number 7: 5 357 kg and 58,7 litres/100 km

1. The scatter plot on the previous page shows the performance of a group of learners in Mathematics and History. Which of the points on the scatter plot can be regarded as outliers? Give reasons for your answer.

Outliers in data sets can be very important. We need to decide if there is a particular reason for the value being so different to the others. Sometimes it gives us important information. In some cases, the data collected for that point could be wrong.

The scatter plot below is for data collected by a transport company.

Working out fuel consumption for a transport company



The company uses just one type of truck. Before each transport job, the company has to specify the price for the job. In order to specify a price before a job, the company needs to estimate how much their costs will be for doing the job. One of the main costs is the cost of fuel, and the main factor influencing the amount of fuel used is the distance. The load weight also plays a role: the greater the load weight, the higher the fuel consumption (litres/100 km).

The table on the next page gives information that was recorded for previous transport jobs. The jobs are numbered from 1 to 16 and for each job the values of the four variables *distance*, *load weight*, *amount of fuel used* and *fuel consumption rate* are given.

2. (a) Which of the four variables are represented on the scatter plot given above?
   (b) What are the values of these two variables for the point indicated by the blue arrow on the scatter plot?

Answers

3. (a) Greater load weights cause higher fuel consumption.
   (b) The fuel consumption in the case of job 7 is considerably lower than for other jobs with comparable load weights.
4. It is possible that the driver drives differently to the other drivers, and manages to achieve a lower fuel consumption.

FIND OUTLIERS

Teaching guidelines

Learners find outliers in the tabulated data on LB page 278 alongside and LB page 279 on the following page.

Job number	Distance (km)	Load weight (kg)	Fuel used (ℓ)	Fuel consumption (ℓ/100 km)
1	1 304	5 445	879	67,4
2	1 320	2 954	639	48,4
3	1 151	4 705	698	60,6
4	1 371	4 378	787	57,4
5	325	3 673	176	54,2
6	1 630	5 995	1 113	68,3
7	1 023	5 357	600	58,7
8	620	4 988	382	61,6
9	73	1 992	35	47,9
10	1 071	5 529	680	63,5
11	370	4 140	218	58,9
12	1 423	4 062	843	59,2
13	394	4 068	221	56,1
14	1 536	1 678	682	44,4
15	1 633	3 736	887	54,3
16	435	3 644	241	55,4

3. (a) Consider the scatter plot and the data set. What is the effect of load weight on fuel consumption?
   (b) Is job 7 an exception in this respect? Explain your answer.
4. Further investigations revealed that the driver for jobs 2 and 7 was the same person, and that he was not the driver for any other jobs. What may this indicate?

FIND OUTLIERS

Researchers collected data on the population of some African countries (including the Seychelles), which included the income per person and the percentage of the income spent on health.

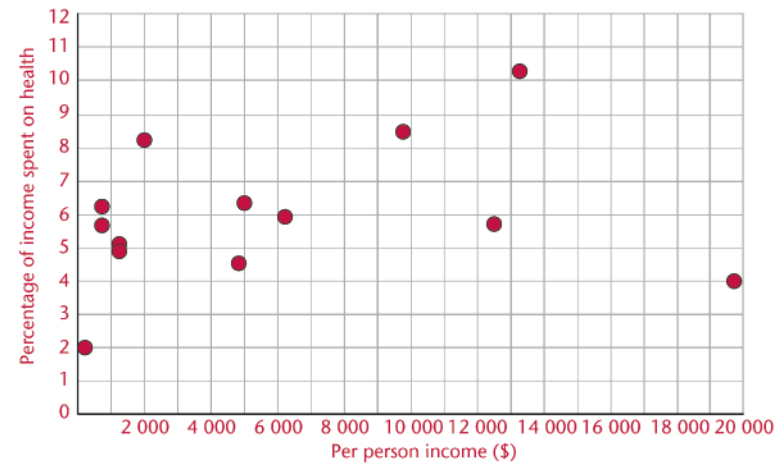
Country	Total population (in 1 000s)	Total annual national income per person (US$)	Percentage of income spent on health
Angola	18 498	4 830	4,6
Botswana	1 950	13 310	10,3
DRC	66 020	280	2,0

Answers

1. The variables are: population of the country, the country's income per person and the percentage of the income spent on health.
2. The populations of the different countries vary considerably. A high income per person makes the demand on the state less.
3. See the scatter plot on LB page 279 alongside.
4. There is a general increase in the percentage spent on health with the increase in per person income, but this is not very strong. Some of the lower-income countries spend fairly high percentages on health. This could be because health is a priority. The Seychelles is an outlier, with a low percentage spent on health even though personal incomes are the highest of all the countries. This may be because there is mostly private health care in a wealthy country.

Country	Total population (in 1 000s)	Total annual national income per person (US$)	Percentage of income spent on health
Lesotho	2 067	1 970	8,2
Malawi	15 263	810	6,2
Mauritius	1 288	12 580	5,7
Mozambique	22 894	770	5,7
Namibia	2 171	6 250	5,9
Seychelles	84	19 650	4,0
South Africa	50 110	9 790	8,5
Swaziland	1 185	5 000	6,3
Tanzania	43 739	1 260	5,1
Zambia	12 935	1 230	4,8

1. What are the three variables in this table?
2. Why do you think it is important to look at income per person in this case, rather than the total income?
3. On graph paper, plot the points for the national income per person and the percentage spent on health care for each country.



4. Write a short report on the data in the table and what the scatter plot shows you about the data. Comment on the general trend and any outliers.

Grade 9 Term 4 Chapter 25 Probability

Learner Book Overview		
Sections in this chapter	**Content**	**Pages in Learner Book**
25.1 Simple events	Events; trials; experiments; frequency of an event; relative frequency of an event; probability of an event; equally likely outcomes	Pages 280 to 284
25.2 Compound events	Compound events; independent events; two-way tables; tree diagrams	Pages 284 to 286

CAPS time allocation	4,5 hours
CAPS content specification	Pages 152 to 153

Mathematical background

- An **event** is something that may or may not happen when an action is performed.
- A **trial** is an action which may lead to a result, for example, rolling a die.
  - A **possible outcome** is any of the possible results of a trial, for example, scoring a 1 or 2 or 3 or 4 or 5 or 6 when a die is rolled.
  - A **favourable outcome** is any of the possible outcomes which favour a specific event, for example, scoring a factor of 6 when a die is rolled has four favourable outcomes: 1 or 2 or 3 or 6.
  - An **actual outcome** is the actual result of a single trial, for example rolling a die and scoring 3.
  - **Equally likely outcomes** of an event are outcomes of which any is as readily to occur as another, for example, scoring less than 7 when rolling a die.
- An **experiment** is a series of trials performed one after the other, for example, rolling a die ten times in a row.
- The **frequency of an outcome** is the number of times it happens, for example, the number of times 3 was scored when a die was rolled ten times.
  - The **expected frequency of an outcome** is the number of times one expects the outcome to occur during an experiment.
  - The **actual frequency of an outcome** is the number of times the outcome actually occurs during an experiment.
- The **probability of an outcome** is a measure of how likely that outcome is.
  - The **(theoretical) probability** of an outcome is given by the ratio $\frac{\text{number of favourable outcomes}}{\text{number of possible outcomes}}$, for example, the theoretical probability of scoring a prime number when rolling a die is $\frac{3}{6} = \frac{1}{2}$, or 0,5 or 50% because there are three favourable outcomes (2, 3, 5) and six possible outcomes (1, 2, 3, 4, 5, 6).
  - The **experimental probability (relative frequency) of an outcome** is given by the ratio $\frac{\text{number of times the outcome happens}}{\text{total number of trials}}$ and can be expressed as a fraction, a decimal number or a percentage.
- It takes many trials before the relative frequency of an outcome approaches the probability of the outcome.

25.1 Simple events

REVISION

Background information

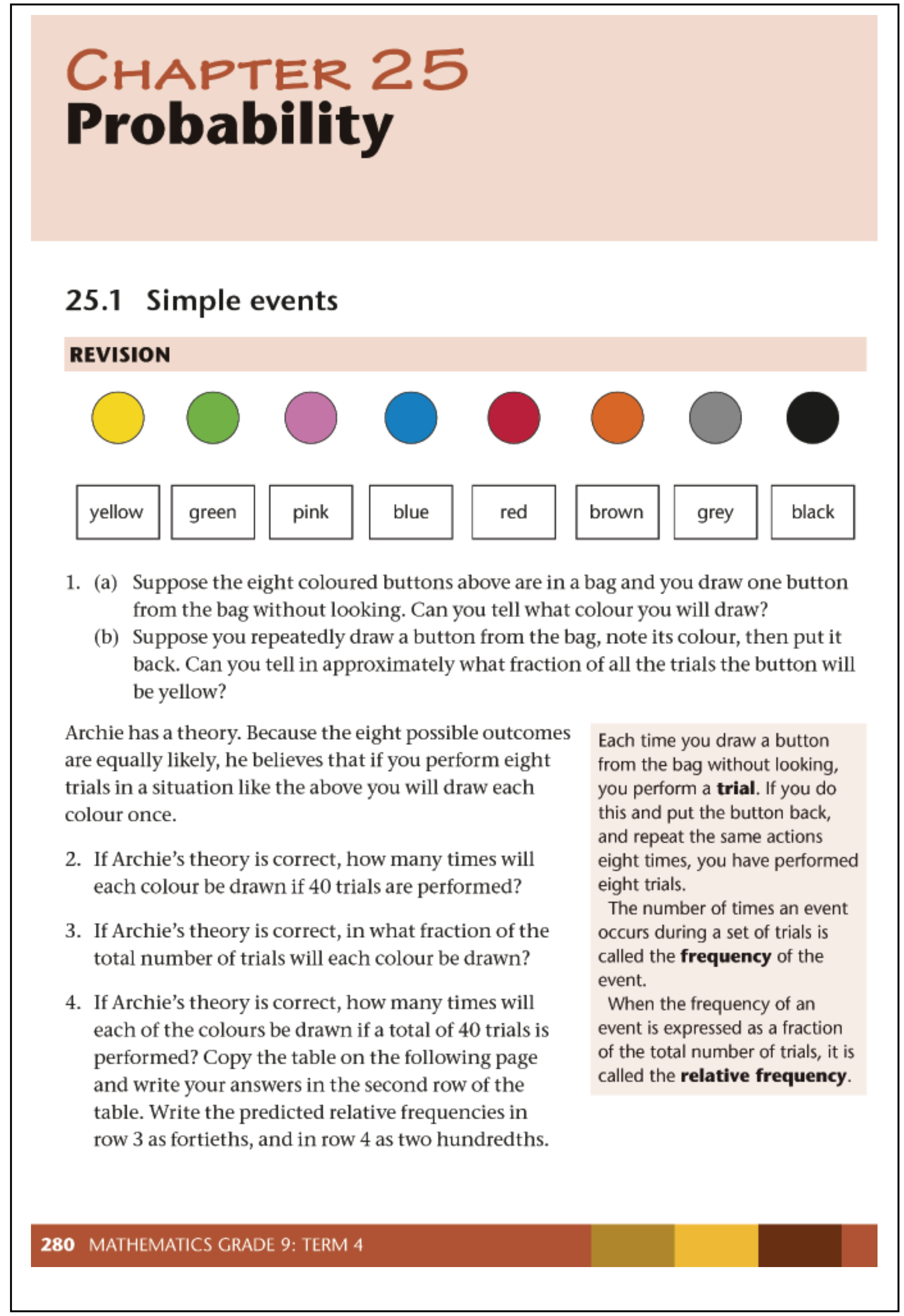
- An **event** is something that may or may not happen when an action is performed.
- A **trial** is an action which may lead to a result, for example, drawing a button from a bag without looking.
- An **experiment** is a series of trials performed one after the other, for example, drawing a button from a bag without looking and recording its colour before putting it back, and repeating the same action eight times.
- The **frequency of an event** is the number of times that event occurs during an experiment (a set of trials).
- The **relative frequency (experimental probability) of an event** is given by the ratio $\frac{\text{number of times the event occurs}}{\text{total number of trials}}$ and can be expressed as a fraction of the total number of trials.

Teaching guidelines

Revise the concepts of probability listed above.

Answers

1. (a) No
   (b) One eighth of the trials. It is in order if some learners do not give this answer. The issue is addressed in the text and questions below.
2. Each colour will be drawn five times if Archie's theory is correct.
3. One eighth of the total number of trials.
4. See the answers on LB page 281 on the following page.



CHAPTER 25
Probability

25.1 Simple events

REVISION

yellow	green	pink	blue	red	brown	grey	black

1. (a) Suppose the eight coloured buttons above are in a bag and you draw one button from the bag without looking. Can you tell what colour you will draw?
   (b) Suppose you repeatedly draw a button from the bag, note its colour, then put it back. Can you tell in approximately what fraction of all the trials the button will be yellow?

Archie has a theory. Because the eight possible outcomes are equally likely, he believes that if you perform eight trials in a situation like the above you will draw each colour once.

2. If Archie's theory is correct, how many times will each colour be drawn if 40 trials are performed?
3. If Archie's theory is correct, in what fraction of the total number of trials will each colour be drawn?
4. If Archie's theory is correct, how many times will each of the colours be drawn if a total of 40 trials is performed? Copy the table on the following page and write your answers in the second row of the table. Write the predicted relative frequencies in row 3 as fortieths, and in row 4 as two hundredths.

Each time you draw a button from the bag without looking, you perform a **trial**. If you do this and put the button back, and repeat the same actions eight times, you have performed eight trials.
The number of times an event occurs during a set of trials is called the **frequency** of the event.
When the frequency of an event is expressed as a fraction of the total number of trials, it is called the **relative frequency**.

280 MATHEMATICS GRADE 9: TERM 4

Background information (continued)

- The **probability of an outcome** is a measure of how likely that outcome is.
- The **(theoretical) probability of an outcome** = $\frac{\text{number of favourable outcomes}}{\text{number of possible outcomes}}$

  Example: The probability of scoring a prime number when rolling a die is $\frac{3}{6} = \frac{1}{2}$, or 0,5 or 50% because there are three favourable outcomes (2, 3, 5) and six possible outcomes (1, 2, 3, 4, 5, 6).
- **Equally likely outcomes** have an equal chance of occurring. Outcomes are said to be equally likely when there is no reason to believe that any outcome will occur more often than any other outcome.
- **If all the outcomes of a trial are equally likely**, the following statement is true:

  $$\text{Probability of an outcome} = \frac{1}{\text{total number of equally likely outcomes}}$$

  Example: If a yellow, a green, a pink, a blue, a red, a brown, a grey and a black button are put in a bag and one button is drawn from the bag at random, the probability that the button will be green is $\frac{1}{8}$.

Teaching guidelines (continued)

At the end of question 6 on LB page 281 learners should realise that Archie's theory is incorrect.

Answers

5. (a) Learners follow the instructions on LB page 281 alongside.
   (b) Learners follow the instructions on LB page 281 alongside.
6. (a) 40
   (b) If the total is not 40, the learner has made a mistake somewhere.

Colour	Yellow	Green	Pink	Blue	Red	Brown	Grey	Black
Frequencies predicted by Archie	5	5	5	5	5	5	5	5
Relative frequencies predicted by Archie expressed in fortieths	$\frac{5}{40}$	$\frac{5}{40}$	$\frac{5}{40}$	$\frac{5}{40}$	$\frac{5}{40}$	$\frac{5}{40}$	$\frac{5}{40}$	$\frac{5}{40}$
Relative frequencies predicted by Archie expressed in two hundredths	$\frac{25}{200}$	$\frac{25}{200}$	$\frac{25}{200}$	$\frac{25}{200}$	$\frac{25}{200}$	$\frac{25}{200}$	$\frac{25}{200}$	$\frac{25}{200}$

The relative frequency for each colour that Archie predicted is called the **probability** of drawing that colour. If all the outcomes are equally likely, then:

$$\text{probability of an outcome} = \frac{1}{\text{the total number of equally - likely outcomes}}$$

You will now investigate whether or not Archie's theory is correct.

5. (a) Make eight small cards and write the name of one of the above colours on each card, so that you have cards with the eight colour names. Perform eight trials to check whether or not Archie's theory is correct. Copy the table below and record your results (your tally marks 1 and your frequencies 1) in the relevant row of the table.
   (b) Find out what any four of your classmates found when they did the experiment. Enter their results in your table too (Friend 1, 2, 3 and 4 frequencies).

Table for the results of the experiments

Colour	Yellow	Green	Pink	Blue	Red	Brown	Grey	Black
Your tally marks (1)								
Your frequencies (1)								
Friend 1 frequencies								
Friend 2 frequencies								
Friend 3 frequencies								
Friend 4 frequencies								
Total frequencies for 5 experiments								

6. (a) What was the total number of trials in the five experiments you recorded in the table?
   (b) What is the total of the frequencies for the different colours, in the last row of your table?

Background information (continued)

It takes **many trials** before the relative frequency of an outcome approaches the probability of the outcome.

Teaching guidelines (continued)

At the end of question 8 on LB page 282 learners should realise that Bettina's theory is correct.

Answers

7. No, the results will show that it is not correct.
8. (a) Learners complete the first table on LB page 282 as instructed.
   (b) Bettina's theory is correct.

INVESTIGATE WHAT HAPPENS WHEN MORE TRIALS ARE DONE

Background information

- The **expected frequency of an outcome** is the number of times one expects the outcome to occur during an experiment.
- The **actual frequency of an outcome** is the number of times the outcome actually occurs during an experiment.
- The **more trials are performed**, the closer the relative frequency (experimental probability) will get to the (theoretical) probability of an event.

Teaching guidelines

Learners complete questions 1 to 5 of the investigation on LB pages 282 to 283 to determine whether Jayden's theory is correct.

Answers

1. Learners complete the second table on LB page 282 as instructed.

7. Is Archie's theory correct?

Bettina has a different theory to Archie's. She believes that if one does many trials with the eight buttons in a bag, each colour will be drawn in **approximately** one-eighth of the cases. In other words, Bettina believes that the relative frequency of each outcome will be close to the probability of that outcome, but may not be equal to it.

8. (a) You and your four classmates performed 40 trials in total. Copy the table below and enter the results in the second row of the table. Also express each frequency as a fraction of 40, in fortieths and in two hundredths.

Colour	Yellow	Green	Pink	Blue	Red	Brown	Grey	Black
Actual frequencies obtained in your experiments (40 trials)								
Relative frequencies as fortieths								
Relative frequencies as two hundredths								
Probability as two hundredths								

   (b) Do your experiments show that Bettina's theory is correct or not?

Jayden believes that when more trials are performed, the relative frequencies will get closer to the probabilities.

You will now do an investigation to find out whether Jayden's theory is true.

INVESTIGATE WHAT HAPPENS WHEN MORE TRIALS ARE DONE

1. Perform 40 trials by drawing one card at a time from eight small cards with the names of the colours written on them, and enter your results in the second and third rows of a table like the one shown below.

Colour	Yellow	Green	Pink	Blue	Red	Brown	Grey	Black
Tally marks								
Frequencies								
Relative frequencies as fortieths								
Relative frequencies as two hundredths								
Probabilities as two hundredths								

Background information (continued)

- The **range of the relative frequencies** of all the different outcomes of an experiment is the difference between the largest and smallest relative frequencies.
- When only a **small number of trials** are done, the actual relative frequencies for different outcomes may differ a lot from the probabilities of the outcomes. This means that, for only a few trials, the range of the relative frequencies of equally likely outcomes could be equal to a large number.
- When **many trials are done**, the actual relative frequencies of the different outcomes are quite close to the probabilities of the outcomes. This means that, for many trials, the range of the relative frequencies of equally likely outcomes should be equal to a small number.

Teaching guidelines (continued)

At the end of question 5 on LB page 283, learners should realise that Jayden's theory is correct.

Note on question 5

If all the different outcomes of a trial are equally likely, the range of their relative frequencies will approach the value of 0 if many trials are performed because all the relative frequencies will approach the same value.

Answers

2. Learners follow the instructions on LB page 283 alongside.
3. Learners follow the instructions on LB page 283 alongside.
4. Learners follow the instructions on LB page 283 alongside.
5. The range for the 200 trials will be smaller than the ranges for most or all of the five sets of 40 trials. This would support Jayden's theory.

2. Make a copy of the table on page 282, but leave out the row for tally marks, the row for the relative frequencies as fortieths and the row for the probabilities, on a loose sheet of paper. Exchange it with a classmate. Copy the following Tables 1 and 2 and enter the results of your classmate on Table 1 and 2. Also enter your own results for question 1 on the tables.
3. Get hold of the data reports of three other classmates, and enter these on the tables as well.
4. Add the frequencies of the various colours in the five sets of data for 40 trials each, and calculate the relative frequencies expressed as two hundredths.
5. Is the range of relative frequencies for 200 trials smaller than the ranges for the five different sets of 40 trials each? What does this indicate with respect to Jayden's theory?

When only a small number of trials are done, the actual relative frequencies for different outcomes may differ a lot from the probabilities of the outcomes.

When many trials are done, the actual relative frequencies of the different outcomes are quite close to the probabilities of the outcomes.

Table 1: Frequencies for five sets of 40 trials each

Colour	Yellow	Green	Pink	Blue	Red	Brown	Grey	Black
Frequencies for your own 40 trials in question 1								
Frequencies for 40 trials by classmate 1								
Frequencies for 40 trials by classmate 2								
Frequencies for 40 trials by classmate 3								
Frequencies for 40 trials by classmate 4								
Total frequencies for 200 trials								
Relative frequencies for 200 trials as two hundredths								

CHAPTER 25: PROBABILITY **283**

Note on question 6

The purpose of this question is to illustrate the usefulness of tree diagrams.

Answers

6. 333; 335; 353; 355; 533; 535; 553; 555 – that is eight different numbers.

25.2 Compound events

TOSSING A COIN AND GIVING BIRTH

Background information

- **Compound events** are two or more events happening at once.
  Example: Two coins are tossed together.
- **Independent events** are events such that the probability of one event occurring in no way affects the probability of the other event occurring.
  Example: A die is rolled and a coin is flipped simultaneously.

Teaching guidelines

Discuss the concepts of compound events and independent events.

Answers

1. (a) heads or tails
   (b) $\frac{1}{2}$
   (c) heads or tails
   (d) $\frac{1}{2}$
2. (a) $\frac{1}{4}$
   (b) No it does not mean that.
   (c) No, it only means that each of the four outcomes will happen approximately 25 times.

Table 2: Relative frequencies for each of the five sets of 40 trials each (expressed as two hundredths)

Colour	Yellow	Green	Pink	Blue	Red	Brown	Grey	Black
Relative frequencies for your own 40 trials								
Relative frequencies for 40 trials by classmate 1								
Relative frequencies for 40 trials by classmate 2								
Relative frequencies for 40 trials by classmate 3								
Relative frequencies for 40 trials by classmate 4								

6. How many different three-digit numbers can be formed with the symbols 3 and 5, if no other symbols are used? You may use one, two or three of the symbols in each number, and you may repeat the same symbol.

25.2 Compound events

TOSSING A COIN AND GIVING BIRTH

1. Simon threw a coin and the outcome was heads. He will now throw the coin again.
   (a) What are the possible outcomes?
   (b) What is the probability of each of the possible outcomes?
   (c) What are the possible outcomes if Simon throws the coin for a third time?
   (d) What is the probability of each of the possible outcomes for the third throw?

What happens when a coin is thrown for a second time has nothing to do with what happened when it was thrown the first time.

The first throw and the second throws are called **independent events**, i.e. what happened on the first throw cannot influence what will happen on the second throw.

2. (a) If an event has four different equally-likely outcomes, what is the probability of each of the four outcomes?
   (b) Does that mean that if the event is repeated four times, each of the four outcomes will happen once?
   (c) Does your answer in (a) mean that if the event is repeated 100 times, each of the four outcomes will happen 25 times?

Background information

- A **two-way table** is used to organise the outcomes of two independent events in a table.
- A **tree diagram** is a statistical display that uses branches to show all the possible outcomes of a trial or an experiment.

Teaching guidelines

Discuss the concepts of two-way tables and tree diagrams.

Note on question 3

- The headings of the **rows** show the possible outcomes of the **first** event.
- The headings of the **columns** show the possible outcomes of the **second** event.

Note on question 4

- The **first column** shows the possible outcomes for the **first coin**.
- The **second column** shows the possible outcomes for the **second coin** for each outcome of the **first coin**.
- The **third column** shows the possible outcomes for the **third** coin for each outcome of the **first two coins**.

Answers

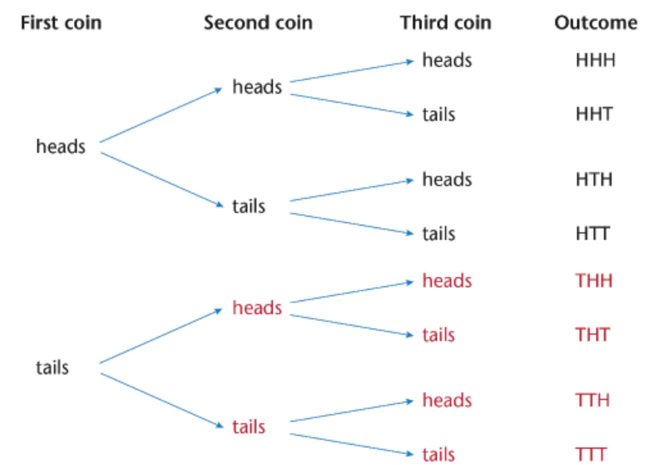
3. (a) See the answers on LB page 285 alongside.
   (b) They are equally likely.
   (c) $\frac{1}{4}$
   (d) $\frac{1}{2}$
4. See the completed tree diagram on LB page 285 alongside.
5. (a) They are equally likely.
   (b) $\frac{1}{8}$
   (c) $\frac{3}{8}$

3. (a) What are the possible outcomes when two coins are thrown? Copy and complete the **two-way table** below to answer this question. One possible outcome is already given.

	Heads	Tails
Heads	H H	H T
Tails	T H	T T

(b) Do you think these four outcomes are equally likely?
(c) What is the probability of each of the four outcomes?
(d) What is the probability of getting a head and a tail?

4. Let us consider the possible outcomes if three coins are thrown.
Below is a tree diagram that can help you figure out what the different possible outcomes are. Complete the diagram by filling in the missing information.



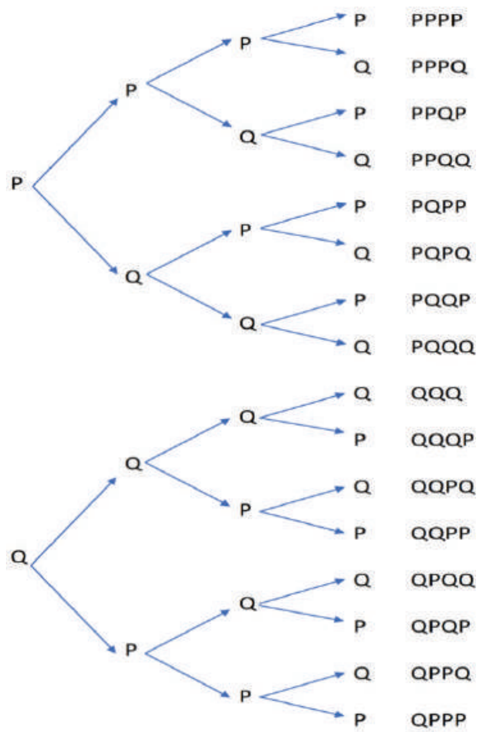
5. (a) Do you think the eight different outcomes in question 4 are equally likely?
(b) What is the probability of each of the eight outcomes?
(c) What is the probability of throwing two heads and one tail?

6. In question 6 on page 284 you were asked to write down the various numbers that can be formed by using symbols 3 and 5. Think of all the four-letter codes that you can form by using only two letters, P and Q. Any letter can be used more than once in one code. First think about how you will go about finding all the possibilities in a systematic way, and then try to set up a tree diagram to help you.

CHAPTER 25: PROBABILITY 285

Answers

6. (a)



(b) $\frac{2}{16}$

(c) $\frac{6}{16}$

7. (a) See the answers on LB page 286 alongside.

(b) BB; BG; GB; GG

(c) $\frac{2}{4}$ or $\frac{1}{2}$

(d) $\frac{1}{4}$ There is only one "boy first, girl second" option.

8. $\frac{1}{2}$ The first event has already occurred. The second event is independent of the first and has only two possible outcomes, either boy or girl.

9. (a) $\frac{1}{4}$ (b) $\frac{1}{8}$

(a) Draw a tree diagram to help you to solve this problem. List all the outcomes.

(b) If the codes are formed by randomly choosing the letters, what is the probability that the code will consist of the same letter being used four times?

(c) What is the probability that the code will consist of two Ps and two Qs?

When a woman is pregnant, the baby can be a boy or a girl. Suppose we make the assumption that the two possibilities are equally likely, so the probability of a boy is $\frac{1}{2}$ and the probability of a girl is $\frac{1}{2}$.

7. (a) Copy and complete this two-way table to show the possible outcomes of the gender of the two children in a family.

	Boy	Girl
Boy	B B	B G
Girl	G B	G G

(b) List the possible outcomes.

(c) What is the probability that the two children in the family will be of the same gender?

(d) What is the probability that the eldest child will be a boy and that they will then have a girl?

8. A certain woman already has a boy. She now expects a second child. What is the probability of it being a boy again, if we make the assumption that a baby being a boy or a girl are equally likely events?

The assumption that a boy or a girl being born are equally likely events may not actually be true. However, probabilities can only be calculated and used to make predictions if it is assumed that outcomes are equally likely.

9. (a) A woman gets married and plans to have a baby in one year and another baby in the next year. What is the probability that both babies will be girls?

(b) A woman gets married and plans to have a baby in each of the first three years of the marriage. What is the probability that she will have a boy in the first year, and girls in the second and third years?