VERTICAL PROJECTILE MOTION (LIVE) 08 APRIL 2015

Section A: Summary Notes and Examples

Equations of Motion

When an object is thrown, projected or shot upwards or downwards, it is said to be a projectile. Any object that is being acted upon **only** by the force of gravity is said to be in a state of **free fall**. Projectiles experience a constant gravitational acceleration, 'g'of **9,8 m.s^{-2} downwards**, irrespective of whether it is moving upwards, downwards or it is at its maximum height.

Note: At maximum height, ***velocity is equal to zero*** *but* ***acceleration is 9,8 m.s^{-2} downwards.***

Due to objects always having the same acceleration (9,8 m.s^{-2} down) then whatever time it takes to reach the top of its motion is the time it takes to return to its original position.

The motion of a projectile can be described by the following equations:

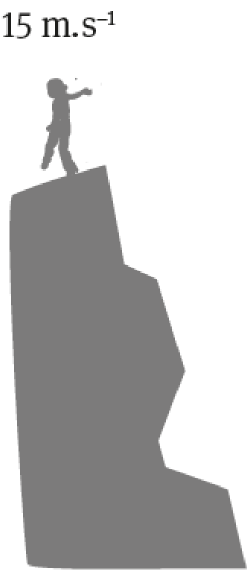
$v_f = v_i + a\Delta t$	$v_i = initial\ velocity$
$v_f{}^2 = v_i{}^2 + 2a\Delta x$	$v_f = final\ velocity$
$\Delta x = v_i\Delta t + \frac{1}{2}a\Delta t^2$	$a = acceleration$
$\Delta x = \left(\frac{v_i + v_f}{2}\right)\Delta t$	$\Delta x = displacement\ (change\ in\ position)$
	$\Delta t = time$

Remember that velocity, displacement (change in position) and acceleration are all vectors; therefore they have both **magnitude** and **direction**.

Always choose a direction as positive at the beginning of a problem and keep the same sign convention throughout the question.

Example 1

A boy is standing on the top of a cliff. He throws a small rock vertically upwards with a velocity of 15 m.s^{-1}.
It takes 4 s for the rock to hit the ground below.



1. Calculate the velocity of the rock as it hits the ground.

2. Calculate how long it takes for the rock to reach its maximum height.

3. Calculate the height of the cliff.

Solution

1. Take up as positive

v_f = ?

v_i = 15 m.s^{-1}

a = -9,8 m.s^{-1}

Δt = 4 s

$v_f = v_i + a\Delta t$

= 15 + (-9,8)(4)

= - 24,2

= 24,2 m.s^{-1} down

2. $v_f = 0$ (max height)

$v_i = 15\ m.s^{-1}$

$a = -9,8\ m.s^{-1}$

$\Delta t = ?$

$v_f = v_i + a\Delta t$

$0 = 15 + (-9,8)(\Delta t)$

$-15 = (-9,8)\Delta t$

$\therefore \Delta t = 1,53\ s$

3. $v_f = -24,2\ m.s^{-1}$

$v_i = 15\ m.s^{-1}$

$a = -9,8\ m.s^{-1}$

$\Delta t = 4\ s$

$\Delta y = ?$

$$\Delta y = v_i \Delta t + \frac{1}{2} a \Delta t^2$$

$$\Delta y = (15)(4) + \frac{1}{2}(-9,8)(4)^2$$

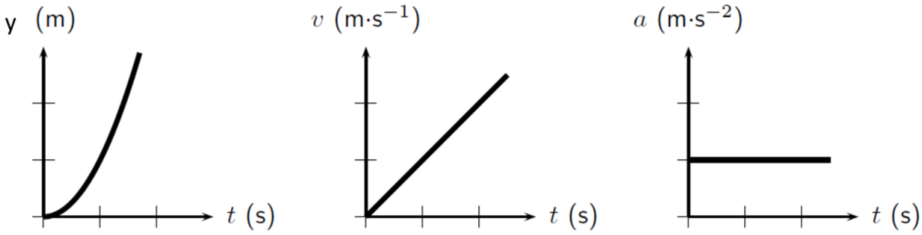
$$= -18,4$$

The height of the cliff is 18,4 m.

Graphs

The motion of a projectile can also be describe using graphs of motion. The three types of graphs are: *position versus time, velocity versus time and acceleration versus time* graphs.

In general, ***position versus time*** graphs will always be curved due to the object always accelerating due to gravity. ***Velocity versus time*** graphs will be straight lines, not parallel to the time axis and ***acceleration versus time*** graphs will always be straight lines parallel to the time axis.



Graphs of motion are useful in describing the motion of an object and can be used to calculate information about the motion.

The gradient of a ***position versus time*** graph gives the **velocity** of the object. However the gradient needs to be calculated using a tangent to a specific point on the graph as the graph is curved.

The gradient of a ***velocity versus time*** graph gives the **acceleration** of the object. This should be equal to 9,8 $m.s^{-2}$ as the object is experiencing an acceleration due to gravity.

The area between the graph and time axis of an ***acceleration versus time*** graph gives the **average velocity** of the object.

The area between the graph and the time axis of a ***velocity versus time*** graph gives the **displacement** of the object.

Example 2

Consider the rock being thrown by the boy again in Example 1. Use the information calculated in the example above to draw the following sketch graphs:

1. Position vs time
2. Velocity vs time
3. Acceleration vs time.

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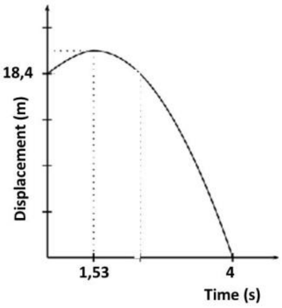
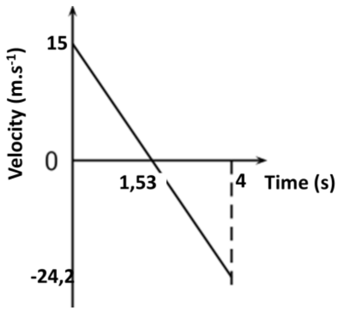
Solution

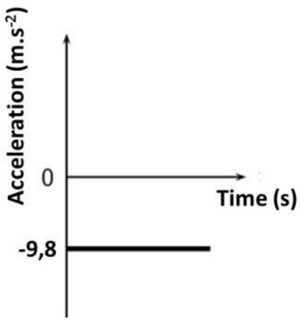
Position – time graph



Velocity – time graph



Acceleration – time graph

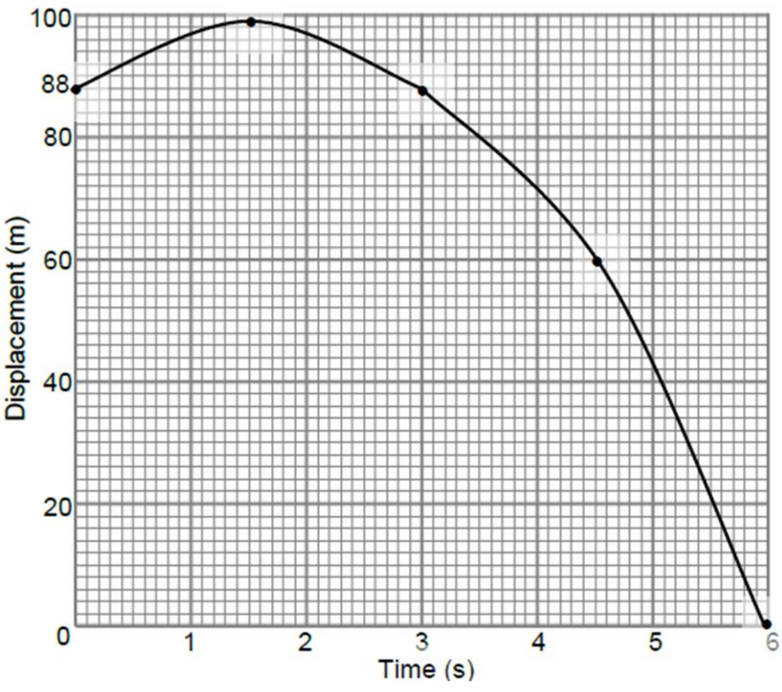


Section B: Practice Questions

Question 1

(Taken from DOE Exemplar 2008)

A hot-air balloon is rising vertically at constant velocity. When the balloon is at a height of 88 m above the ground, a stone is released from it. The **position-time** graph below represents the motion of the stone from the moment it is released from the balloon until it strikes the ground. Ignore the effect of air resistance.



Use the information from the graph to answer the following questions:

1.1. Calculate the velocity of the hot-air balloon at the instance the stone is released. (6)

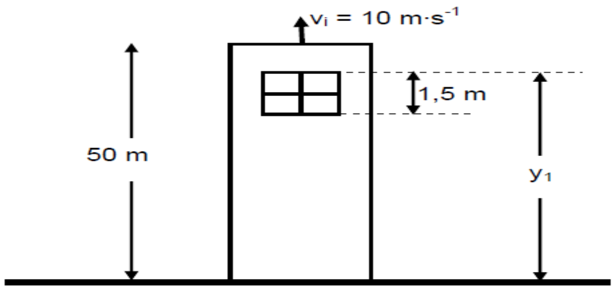
1.2. Draw a sketch graph of velocity versus time for the motion of the stone from the moment it is released from the balloon until it strikes the ground. Indicate the respective values of the intercepts on your velocity-time graph (3)

[9]

Question 2

(Adapted from Feb 2012, P1, Question 3)

A stone is thrown vertically upward at a velocity of 10 $m \cdot s^{-1}$ from the top of a tower of height 50 m. After some time the stone passes the edge of the tower and strikes the ground below the tower. Ignore the effects of friction.



2.1 Draw a labelled free-body diagram showing the force(s) acting on the stone during its motion. (1)

2.2 Calculate the:

2.2.1 Time taken by the stone to reach its maximum height above the ground (4)

2.2.2 Maximum height that the stone reaches above the ground (4)

2.3 USING THE GROUND AS REFERENCE (zero position), sketch a position time graph for the entire motion of the stone. (3)

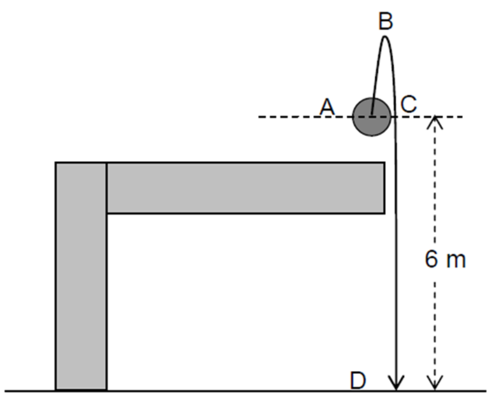
2.4 On its way down, the stone takes 0,1 s to pass a window of length 1,5 m, as shown in the diagram above. Calculate the distance (y1) from the top of the window to the ground. (5)

[19]

Question 3

(Taken from DOE Preparatory Examination 2008)

Marshall stands on a platform and kicks a soccer ball from 6 m above the ground (position A) vertically upwards into the air with an initial velocity of 4 $m.s^{-1}$. The ball hits the ground (position D) after 1,6 seconds. The motion of the ball is represented in the diagram below. Ignore the effects of air resistance.



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3.1. Calculate the maximum height (position B) the ball reaches above the ground. (5)

3.2. Calculate the time take for the ball to reach maximum height (position B). (3)

3.3. Draw a sketch graph of position versus time for the motion of the ball from the moment it was kicked until it hits the ground. Use point A as the reference point (zero position). Indicate ALL relevant position and time values at positions A, B, C and D. (5)

[13]

Question 4

(Taken from DOE November 2009)

The following extract comes from an article in school newspaper

The Laws of Physics are Accurate

Two construction workers, Alex and Pete, were arguing about whether a smaller brick would hit the ground quicker than a larger brick when both are released from the same height.

Alex said that the larger brick should hit the ground first. Pete argued that the smaller brick would hit the ground first.

4.1. Are their statements correct? Give a reason for your answer. (3)

4.2. A group of Physical Sciences learners decide to test Alex's and Pete's hypothesis. They drop two bricks, one small and the other much larger from one of the floors of the school building.

4.2.1. Write down TWO precautions they should take to ensure that the result is reliable. (2)

4.2.2. Give a reason why, despite all the necessary precautions, they might not get the correct result. (1)

4.3. In another experiment, the learners **drop** a brick A from a height of 8 m. After 0,6 s, they **throw** a second brick B downwards from the same height. Both bricks, A and B, hit the ground at the same time.

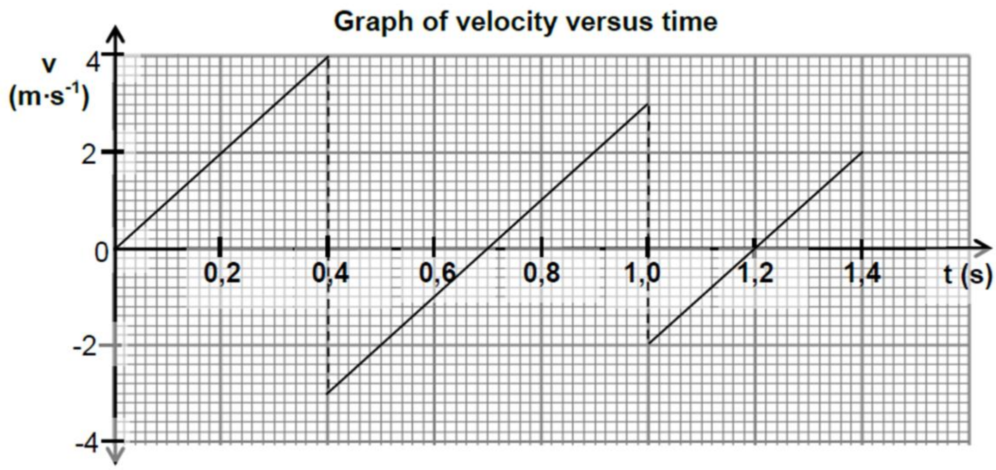
Ignore the effects of friction and calculate the speed at which brick B was thrown. (7)

[13]

Question 5

(Taken from DOE November 2009)

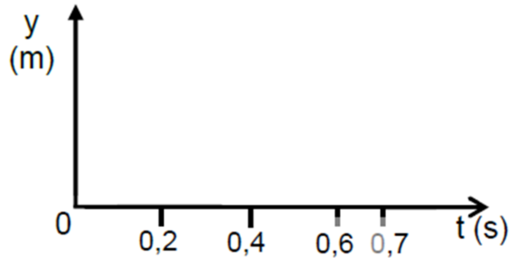
A ball is released from a certain height. The velocity-time graph below represents the motion of the ball as it bounces vertically on a concrete floor. The interaction time of the ball with the floor is negligibly small and is thus ignored.



5.1. Describe the changes, if any, in velocity and acceleration of the ball from t = 0s to t = 0,4 s. (4)

5.2. Without using the equations of motion, calculate the height from which the ball has been dropped initially. (4)

5.3. Copy the set of axes below.



Use the given velocity versus time graph for the motion of the ball to sketch the corresponding position-time graph for the time interval 0s to 0,7s. (3)

5.4. Is the first collision of the ball with the floor elastic or inelastic? Give a reason for your answer. (2)

[13]

Question 6

(Taken from DOE Feb – March 2010)

A supervisor, 1,8 m tall, visits a construction site. A brick resting at the edge of a roof 50 m above the ground suddenly falls. At the instance when the brick has fallen 30 m the supervisor sees the brick coming down directly towards him from above.

Ignore the effects of friction and take the downwards motion as positive.

6.1. Calculate the speed of the brick after it has fallen 30 m. (3)

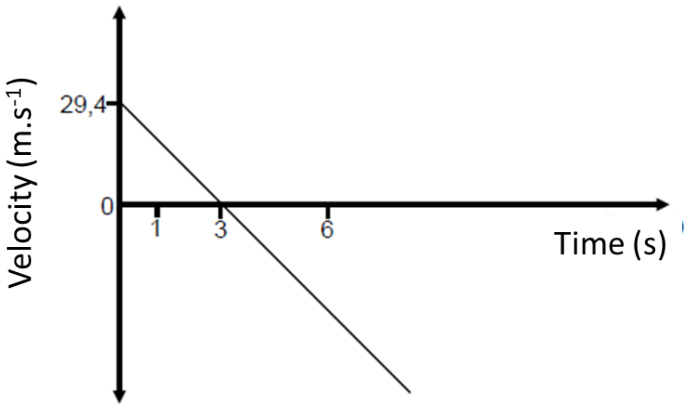
6.2. The average reaction time of a human being is 0,4 s. With the aid of a suitable calculation, determine whether the supervisor will be able to avoid being hit by the brick. (6)

[9]

Question 7

(Taken from DOE November 2010)

A man fires a projectile X vertically upwards at a velocity of 29,4 $m.s^{-1}$ from the EDGE of a cliff of height 100 m. After some time the projectile lands on the ground below the cliff. The velocity-time graph below (not drawn to scale) represents the motion of projectile X. (Ignore the effects of friction)



7.1. Use the graph to determine the time that the projectile takes to reach its maximum height. (A calculation is not required) (1)

7.2. Calculate the maximum height that projectile X reaches above the ground. (4)

7.3. Sketch the position-time graph for projectile X for the period t = 0 s to t = 6 s. USE THE EDGE OF THE CLIFF AS THE ZERO POSITION.

Indicate the following on the graph:

- The time when projectile X reaches its maximum height
- The time when projectile X reaches the edge of the cliff (4)

7.4. One second (1 s) after projectile X is fired, the man's friend fires a second projectile Y upwards at a velocity of 49 $m.s^{-1}$ FROM THE GROUND BELOW THE CLIFF. The first projectile X, passes projectile Y 5,23 s after projectile X is fired. (Ignore the effects of friction)

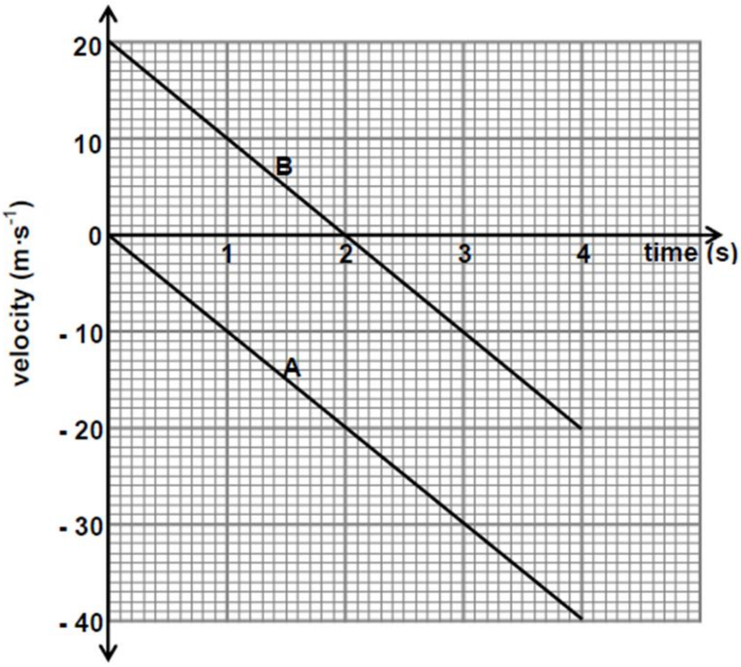
Calculate the velocity of projectile X at the instant it passes projectile Y. (5)

[14]

Question 8

(Taken from DOE Feb-March 2011)

The velocity-time graph shown below represents the motion of two objects, A and B, released from the same height. Object A is released from REST and at the same instant object B is PROJECTED vertically upwards. (Ignore the effects of friction).



8.1. Object A undergoes a constant acceleration. Give a reason for this statement by referring to the graph. (No calculations are required) (2)

8.2. At what time/times is the **speed** of object B equal to 10 $m.s^{-1}$? (2)

8.3. Object A strikes the ground after 4 s. USE EQUATIONS OF MOTION to calculate the height from which the objects were released. (3)

8.4. What physical quantity is represented by the area between the graph and the time axis for each of the graphs A and B? (2)

8.5. Calculate, WITHOUT USING EQUATIONS OF MOTION, the distance between objects A and B at t = 1s. (5)

[14]

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Section C: Solutions

Question 1

1.1. Use entire motion of the stone - Take up as positive

$$\Delta y = v_i \Delta t + \frac{1}{2} a \Delta t^2 \checkmark$$

$$-88\checkmark = v_i(6)\checkmark + \frac{1}{2}(-9{,}8)(6)^2\checkmark$$

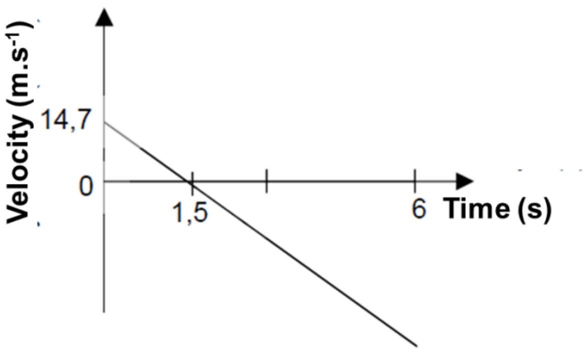
$$-88 = 6v_i - 176{,}4$$

$$\therefore 6v_i = 88{,}4$$

$$\therefore v_i = 14{,}73\ m.s^{-1}\ upwards\ \checkmark$$

$$v_{balloon} = v_{i\ stone} = 14{,}73\ m.s^{-1}\checkmark$$ (6)

1.2. Take up as positive

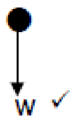


Checklist – Criteria for graph

- Graph is a straight line that intercepts x-axis at 1,5 s ✓
- Maximum velocity after 6 s ✓
- Initial velocity indicated as 14.7 $m.s^{-1}$ ✓ (3)

Question 2

2.1.



(1)

2.2.1 <u>Upward positive</u>

$v_f = v_i + a\Delta t$ ✓

0 = 10 + (-9.8) Δt✓

Δt = 1.02 s✓ (3)

2.2.2. $v_f^2 = v_i^2 + 2a\Delta y$✓

$0^2 = 10^2 + 2(-9.8)\Delta y$✓

Δy = 5,1 m

Height = 50 + 5,1✓ = 55,1 m. ✓ (4)

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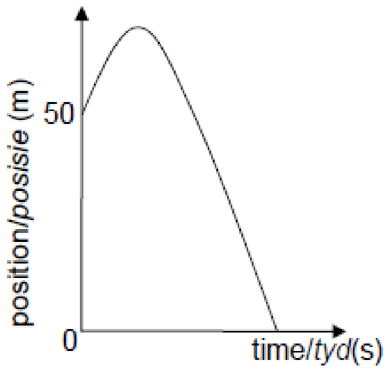
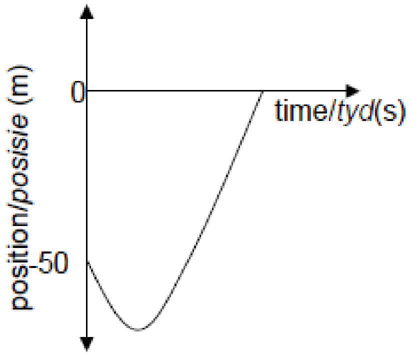
2.3





Criteria for graph/Kriteria vir grafiek	Marks/ *Punte*
Correct shape/*Korrekte vorm*	✓
Final position lower than initial position.	✓
Graph ends on x axis./*Grafiek eindig op x-as.*	✓

2.4.

$\Delta y = v_i \Delta t + \frac{1}{2} a \Delta t^2$ ✓

$1{,}5$ ✓ $= v_i (0{,}1) + \frac{1}{2}(9{,}8)(0{,}1)^2$ ✓

$\therefore v_i = 14{,}51 \text{ m·s}^{-1}$

From maximum height/*Van maksimum hoogte:*

$v_f^2 = v_i^2 + 2a\Delta y$ ✓

$14{,}51^2$ ✓ $= (0)^2 + 2(9{,}8)\Delta y$ ✓

$\therefore \Delta y = 10{,}74 \text{ m}$

Height/*Hoogte* $= 55{,}1 - 10{,}74$
$= 44{,}36 \text{ m}$ ✓

(5)

Question 3

3.1. Take up as positive

$v_f^2 = v_i^2 + 2a\Delta y$ ✓

$(0)^2$ ✓ $= (4)^2 + 2(-9{,}8)\Delta y$ ✓

$\therefore 19{,}6\ \Delta y = 16$

$\therefore \Delta y = 0{,}82\ m\ upwards$ ✓

$\therefore \Delta y_{above\ the\ ground} = 6 + 0{,}82$
$= 6{,}82\ m$ ✓

(5)

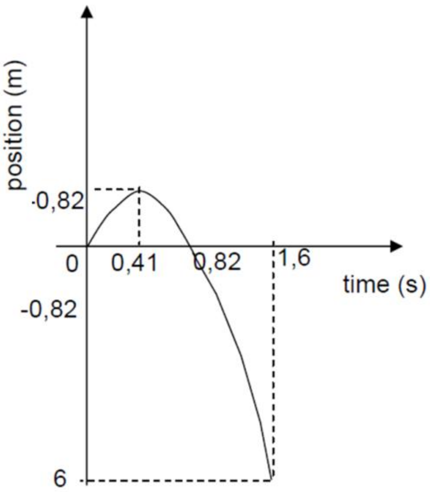
3.2. Take up as positive

$v_f = v_i + a\Delta t$ ✓

$0 = 4 + (-9{,}8)\Delta t$ ✓

$\therefore \Delta t = 0{,}41\ s$ ✓ (3)

3.3.



Checklist – Criteria for graph

- Correct shape for graph 0 – 0,41 s✓
- Correct shape of graph 0,41 – 1,6 s✓
- Coordinates 0,41s : 0,82 m for highest position ✓
- Coordinates 0s : 0,82 m indicated ✓
- Coordinates 1,6s : 6m indicated✓

(5)

Question 4

4.1. *Due to the way the question is asked there are 4 possible ways to answer the question.*

Option 1:
Both are incorrect ✓
The bricks will experience the same gravitational acceleration ✓and thus will reach the ground at the same time✓

Option 2:
Pete is correct and Alex is wrong✓
The smaller brick experiences less air resistance, thus has a larger acceleration ✓and reaches the ground first✓

Option 3:
Alex is correct and Pete is wrong✓
In the presence of air resistance, the larger brick, with larger mass, experiences a larger net force downwards, thus larger acceleration ✓and reaches the ground first. ✓

Option 4:
Both are correct✓
Peter is correct: The smaller brick experiences less air resistance, thus has a larger acceleration and reaches the ground first✓ (3)

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Alex is correct: I n the presence of air resistance, the larger brick, with larger mass, experiences a larger net force downwards, thus larger acceleration and reaches the ground first.✓

4.2.1. Any two of the following are accepted

- Ensure that both bricks are dropped from the same height
- Ensure that both bricks are dropped at the same time
- Repeat the experiment several times and use the average of the results
- Make sure that $v_i = 0$ for both bricks
- Make sure that there is no strong wind
- Use bricks made of the same material

(2)

4.2.2. External forces may be present e.g. friction, air resistance, strong wind blowing. ✓

(1)

4.3. Take down as positive

For brick A:

$\Delta y = v_i \Delta t + \frac{1}{2} a \Delta t^2$✓

8✓ $= (0)\Delta t + \frac{1}{2}(9{,}8)\Delta t^2$✓

$8 = 4{,}9 \Delta t^2$

$\therefore \Delta t = 1{,}28\ s$

For brick B:

$\Delta t = 1{,}28 - 0{,}6$

$= 0{,}68\ s$ ✓

$\Delta y = v_i \Delta t + \frac{1}{2} a \Delta t^2$

8✓ $= v_i(0{,}68) + \frac{1}{2}(9{,}8)(0{,}68)^2$✓

$\therefore v_i = 8{,}43\ m.s^{-1}$✓ (7)

Question 5

5.1. t = 0 s: ball starts from rest ($0\ m.s^{-1}$) ✓

t = 0 s – 0,4 s: falls at constant acceleration ✓

t = 0,4 s: reaches the floor at $4 m.s^{-1}$ downwards ✓and then bounces back at $3\ m.s^{-1}$ upwards ✓

(4)

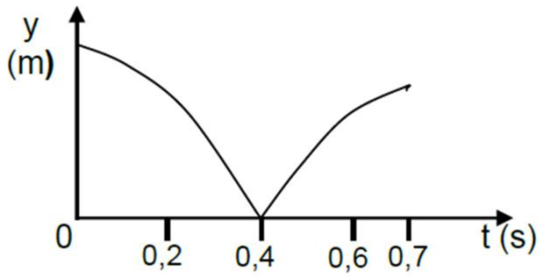
5.2. $\Delta y = area\ of\ triangle$✓

$= \frac{1}{2} bh$

$= \frac{1}{2}(0{,}4)$✓(4)✓

$= 0{,}8\ m$✓ (4)

5.3



Checklist – Criteria for graph

- Correct shape (as shown on graph) ✓
- Zero position at 0,4 s✓
- Maximum position of 2nd bounce smaller than that of 1st bounce. ✓

(3)

5.4. Inelastic ✓

Change in speed / kinetic energy during collision.✓ (2)

Question 6

6.1. Velocity after 30 m

$v_f^2 = v_i^2 + 2a\Delta y$✓

$= 0 + 2(9,8)(30)$✓

$= 588$

$\therefore\ v_f = 24,25\ m.s^{-1}$✓

(3)

6.2. Velocity after a further 18,2 m

$v_f^2 = v_i^2 + 2a\Delta y$✓

$= (24,25)^2 + 2(9,8)(20 - 1,8)$✓

$= 944,7825$

$\therefore\ v_f = 30,74\ m.s^{-1}$

$v_f = v_i + a\Delta t$✓

$30,74 = 24,25 + (9,8)\Delta t$✓

$\therefore (9,8)\Delta t = 6,49$

$\therefore\ \Delta t = 0,66\ s$✓

He will not be struck - his reaction time is shorter than the time for the brick to reach his head. ✓ (6)

Question 7

7.1. 3 seconds (1)

7.2. $Area\ between\ graph\ and\ time\ axis = \Delta y$

$$\Delta y = \frac{1}{2}bh\ ✓$$

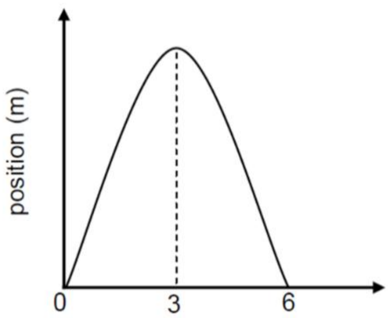
$$= \frac{1}{2}(3)(29,4)✓$$

$$= 44,1\ m$$

$\therefore maximum\ height = 100$ ✓ $+ 44,1$

$= 144,4\ m$ ✓ (4)

7.3.



Checklist – Criteria for graph

- Correct shape (as shown on graph) ✓
- t = 3 s at maximum height ✓
- t = 6 s at y = 0 m✓
- Graphs starts at y = 0 m and t = 0 s. ✓

(4)

7.4. Up is positive

$v_f = v_i + a\Delta t$ ✓

$= 29{,}4$✓ $+ (-9{,}8)(5{,}23)$✓

$= -21\ 85$

$= 21{,}85\ m.s^{-1}$✓$downwards$✓ (5)

Question 8

8.1. Gradient of graph is constant ✓✓ (2)

8.2. At t = 1 s ✓and t = 3 s ✓ (2)

8.3. Up is the positive direction (taken from graph)

$\Delta y = v_i\Delta t + \frac{1}{2}a\Delta t^2$✓

$= (0)(4) + \frac{1}{2}(10)(4)^2$✓ $(a = 10\ m.s^{-2}\ from\ gradient\ of\ graph)$

$= 80\ m$ ✓ (3)

8.4. Displacement ✓✓ (2)

8.5. Distance covered by object B

$\Delta y = \frac{1}{2}bh + lb$ ✓

$= \frac{1}{2}(1)(10) + (10)(1\ )$✓

$= 15\ m$

Distance covered by object A

$\Delta y = \frac{1}{2}bh$✓

$= \frac{1}{2}(1)(-10)$✓

$= -5m$

$Distance\ between\ A\ and\ B = 15 - (-5)$

$= 20\ m$ ✓ (5)