Unknown Input Observer Based Robust Fault Estimation for Systems Corrupted by Partially-Decoupled Disturbances

Zhiwei Gao, *Senior Member, IEEE*, Xiaoxu Liu, *Student Member, IEEE*, and Michael Z. Q. Chen, *Member, IEEE*

Abstract **– Robust fault estimation plays an important role in real-time monitoring, diagnosis and fault-tolerance control. Accordingly, this paper aims to develop an effective fault estimation technique to simultaneously estimate the system states and the concerned faults, while minimizing the influences from process/sensor disturbances. Specifically, an augmented system is constructed by forming an augmented state vector composed of the system states and the concerned faults. Next, an unknown input observer is designed for the augmented system by decoupling the partial disturbances and attenuating the disturbances that cannot be decoupled, leading to a simultaneous estimate of the system states and the concerned faults. In order to be close to the practical engineering situations, the process disturbances in this study are assumed not to be completely decoupled. In the first part of the paper, the existence condition of such an unknown input observer is proposed to facilitate the fault estimation for linear systems subjected to process disturbances. In the second part, robust fault estimation techniques are addressed for Lipschitz nonlinear systems subjected to both process and sensor disturbances. The proposed technique is finally illustrated by the simulation studies of a three-shaft gas turbine engine and a single-link flexible joint robot.**

Keywords**: Fault diagnosis, fault estimation, unknown input observer, augmented system approach, linear matrix inequality.**

I. Introduction

Industrial systems have become more expensive and more complex, which have a higher demand in reliability and safety. Unexpected deviation of characteristic properties or system parameters from standard condition, defined as fault, can possibly induce serious damages and even break down the system. As a result, it is paramount to detect the occurrence of a fault at the early stage, determine the location of the fault, and identify the degree of the severity of the fault. During the last four decades, extensive results were reported on various analytical redundancy based fault diagnosis techniques [1-4] and their applications in aero systems, electro-mechanical systems, energy systems [5-8] and so forth.

The approaches of fault diagnosis can be classified into various categories from different perspectives. In the three-part review paper [9-11], fault diagnosis methods were classified into quantitative model based methods, qualitative model methods, and process history-based methods. In the recent two-part survey paper [12, 13], fault diagnosis approaches were categorized into model-based methods, signal-based methods, knowledge-based methods, and hybrid/active methods. Model-based method has been a popular tool for fault diagnosis, which can provide systematic design solutions, but the diagnosis performance highly relies on the disturbance/noise attenuation ability of the diagnosis algorithms. Observer/filter plays a key role in model-based fault diagnosis methods, which utilizes input and output data to monitor the consistency between the predicted model output and the output of the actual process, leading to a diagnosis decision. In order to attenuate the influences from the disturbances/uncertainties, one solution is to carry out various optimization calculations to make the residual sensitive to faults, but robust against the disturbances/uncertainties [14-17]. The alternative is to utilize decomposition techniques to decouple the process disturbances so that the effects from the disturbances to the residual are removed. One of the known *disturbance decomposition techniques* is differential geometric approach [18-20], which was utilized to perfectly decouple process disturbances for fault detection and isolation. Another popular disturbance decomposition technique is *unknown input observer* (*UIO*), which was proposed in [21] and was then extended for nonlinear systems with Lipschitz constraints [22, 23]. The UIO methods were utilized for either *robust state estimation* [22, 23] or *robust fault diagnosis* [21, 24-30]. Specifically, an UIO based fault detection filter was proposed for linear time-invariant systems in [21]. Meanwhile, UIO techniques were developed in [24-27] for robust fault detection and isolation for a class of nonlinear systems. Moreover, UIO-based *fault/disturbance estimation and reconstruction* were addressed in [28-30]. It is noted that both

Manuscript received April 23, 2015; revised June 28, 2015 and August 22, 2015; accepted October 5, 2015.

Z. Gao (Corresponding Author) and X. Liu are with the Faculty of Engineering and Environment, The University of Northumbria, Newcastle upon Tyne, NE1 8ST, United Kingdom (Tel: +441912437832; Fax: +441912274397; e-mail: zhiwei.gao@northumbria.ac.uk, e-mail: xiaoxu.liu@northumbria.ac.uk).

M. Chen is with Department of Mechanical Engineering, The University of Hong Kong, Hong Kong (e-mail: mzqchen@hku.hk).

the differential geometric decomposition techniques in [18-20] and the UIO decomposition techniques in [21-30] are constrained by some existence conditions, which would become invalid in some scenarios. As a result, there is strong motivation to develop a technique to handle the systems when the disturbances cannot be completely decoupled by using the existing *decomposition techniques* [18-30]. Recently, a few results were reported for systems subjected to partially decoupled process disturbances in [31, 32] by using the UIO techniques, however, which were proposed for state estimates only. Therefore, this motives us to make more effort to investigate fault diagnosis issues for systems subjected to partially decoupled disturbances/uncertainties.

Fault estimation is an advanced fault diagnosis method, which not only can tell when and where the faults occur, but also can provide the sizes and shapes of the faults, which are crucial for on-line fault tolerant control and real-time decision. Fault estimation can be realized by utilizing various observer techniques such as adaptive observers [33], sliding mode observers [34], and augmented system observers (including descriptor observers) [35-37]. It is noticed that the fault estimation techniques in [33-37] attenuate the disturbances by either using optimization methods or high-gain design approaches, rather than using the disturbance decomposition techniques. It is evident that disturbance decomposition can better alleviate the adverse influences from the disturbances. Therefore, it is desirable to decouple the disturbances as much as possible. For systems subjected to the disturbances which cannot be completely decoupled, one can decouple partial disturbance components while attenuating the disturbance components that cannot be decoupled, by using optimization techniques.

In this study, an augmented system is first constructed by forming an augmented state vector composed of the system states and the concerned faults. Second, an UIO is presented for the augmented system to decouple partial process disturbances, and the existence conditions of such an observer are addressed as well. Linear matrix inequality (LMI) optimization technique is utilized to ensure the estimation error dynamics to be stable, and the disturbances that cannot be decoupled to be attenuated, leading to an effective simultaneous estimate of the states and the concerned faults, addressed in Section II. Furthermore, the UIO-based fault estimation methods are extended to Lipschitz nonlinear systems, and further extensions are then made so that the methods can be applied to the systems subjected to both process and sensor disturbances, presented in Section III. Finally, the effectiveness of the proposed methods is demonstrated by two engineering-oriented systems in Section IV: a three-shaft gas turbine engine and a single-link flexible joint robot.

The notations in the presented paper are quite standard. $\mathcal{R}^n$ and $\mathcal{R}^{n\times m}$ stand for n-dimentional Euclidean space and the set of $n\times m$ real matrices, respectively. I_n denotes identity matrix with dimension of $n\times n$. 0 is a scalar zero or a zero matrix with appropriate zero entries. The superscript T represents the transpose of matrices or vectors. The notation $X>Y$ indicates that the symmetric matrix $X-Y$ is positive definite. $Re(s)$ represents the real part of the complex number s. $\|.\|$ denotes standard norm of vectors or matrices, while $\|d\|_{Tf}=(\int_0^{Tf} d^T(\tau)\, d(\tau)d\tau)^{1/2}$. Moreover, $\begin{bmatrix} M_1 & M_2 \\ * & M_3 \end{bmatrix} = \begin{bmatrix} M_1 & M_2 \\ M_2^T & M_3 \end{bmatrix}$.

II. UIO-BASED FAULT ESTIMATION FOR LINEAR SYSTEMS

A. System description and augmented system

Consider a dynamic system in the form of:

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) + B_d d(t) + B_f f(t) \\ y(t) = Cx(t) + Du(t) + D_f f(t) \end{cases} \tag{1}$$

where $x(t)\in\mathcal{R}^n$ represents state vector with initial value of $x(0)\in\mathcal{R}^n$, $u(t)\in\mathcal{R}^m$ and $y(t)\in\mathcal{R}^p$ stand for control input vector and measurement output vector, respectively, $d(t)\in\mathcal{R}^{l_d}$ is a bounded unknown input vector caused by either disturbances or modelling errors, $f\in\mathcal{R}^{l_f}$ is the fault vector involving actuator faults and sensor faults, A, B, C, D, B_d, B_f and D_f are known constant coefficient matrices with appropriate dimensions. For the simplicity of presentation, the symbol t will be omitted in the rest of the paper.

Incipient faults and abrupt faults are the most common faults in industrial processes. Therefore, in this study, the faults concerned are assumed to be either incipient faults or abrupt faults. As a result, the second-order derivative of the fault f should be zero piecewise. In other words, $\ddot{f}=0$, which makes sense from the perspective of engineering practices. On the other hand, one could consider a more general case, that is, the q^{th}-order derivate of the fault is assumed to be zero as shown in [35, 36]. However, in this paper, we concentrate on the case $q=2$ without loss of generality, and the results of the paper can easily be extended to the case when $q\geq 3$.

In addition, $B_d=[B_{d1}\ \ B_{d2}]$, $d=[d_1\ \ d_2]^T$, $d_1\in\mathcal{R}^{l_{d1}}$ and $d_2\in\mathcal{R}^{l_{d2}}$, where d_1 rather than d_2 is assumed to be decoupled, and B_{d1} is of full column rank.

Define an augmented state vector as

$$\bar{x} = [x^T \quad \dot{f}^T \quad f^T]^T \in \mathcal{R}^{\bar{n}} \tag{2}$$

where $\bar{n}=n+2l_f$.

As a result, we can construct an equivalent augmented system as follows:

$$\begin{cases} \dot{\bar{x}} = \bar{A}\bar{x} + \bar{B}u + \bar{B}_d d \\ y = \bar{C}\bar{x} + Du \end{cases} \tag{3}$$

where

$$\bar{A} = \begin{bmatrix} A & 0 & B_f \\ 0 & 0 & 0 \\ 0 & I_{l_f} & 0 \end{bmatrix} \in \mathcal{R}^{\bar{n}\times\bar{n}}, \bar{B} = \begin{bmatrix} B \\ 0 \\ 0 \end{bmatrix} \in \mathcal{R}^{\bar{n}\times m},$$

$$\bar{B}_d = \begin{bmatrix} B_d \\ 0 \\ 0 \end{bmatrix} \in \mathcal{R}^{\bar{n}\times l_d}, \text{ and } \bar{C} = [C \quad 0 \quad D_f] \in \mathcal{R}^{p\times\bar{n}}.$$

Clearly, $\bar{x}$ contains the state vector x, the concerned fault vector f, and its first-order derivative $\dot{f}$. As a result, the three components can be estimated simultaneously if an observer exists for the augmented system (3).

B. Novel unknown input observer (UIO)

Consider the following unknown input observer (UIO):

$$\begin{cases} \dot{\bar{z}} = R\bar{z} + T\bar{B}u + K(y - Du) \\ \hat{\bar{x}} = \bar{z} + H(y - Du) \end{cases} \quad (4)$$

in which $\bar{z} \in \mathcal{R}^{\bar{n}}$ is the state vector of the dynamic system (4) and $\hat{\bar{x}} \in \mathcal{R}^{\bar{n}}$ represents the estimation of $\bar{x} \in \mathcal{R}^{\bar{n}}$, while $R \in \mathcal{R}^{\bar{n}\times\bar{n}}$, $K = K_1 + K_2$, $K_1 \in \mathcal{R}^{\bar{n}\times p}$, $K_2 \in \mathcal{R}^{\bar{n}\times p}$, $T \in \mathcal{R}^{\bar{n}\times m}$ and $H \in \mathcal{R}^{\bar{n}\times p}$ are the gain matrices to be designed.

Letting estimation error $\bar{e} = \bar{x} - \hat{\bar{x}}$, and using the output equation in (4), one has

$$\begin{aligned} \bar{e} &= \bar{x} - \hat{\bar{x}} \\ &= \bar{x} - \bar{z} - H\bar{C}\bar{x} \\ &= (I_{\bar{n}} - H\bar{C})\bar{x} - \bar{z} \end{aligned} \quad (5)$$

Using (3)-(5), the derivative of $\bar{e}$ can thus be calculated as

$$\begin{aligned} \dot{\bar{e}} &= (I_{\bar{n}} - H\bar{C})\dot{\bar{x}} - \dot{\bar{z}} \\ &= (I_{\bar{n}} - H\bar{C})(\bar{A}\bar{x} + \bar{B}u + \bar{B}_d d) - R\bar{z} - T\bar{B}u - K_1\bar{C}\bar{x} \\ &\quad -K_2(y - Du) \\ &= (\bar{A} - H\bar{C}\bar{A} - K_1\bar{C})\bar{e} + (\bar{A} - H\bar{C}\bar{A} - K_1\bar{C} - R)\bar{z} \\ &\quad +[(\bar{A} - H\bar{C}\bar{A} - K_1\bar{C})H - K_2](y - Du) \\ &\quad +[(I_{\bar{n}} - H\bar{C})\bar{B} - T\bar{B}]u + (I_{\bar{n}} - H\bar{C})\bar{B}_{d1}d_1 \\ &\quad +(I_{\bar{n}} - H\bar{C})\bar{B}_{d2}d_2 \end{aligned} \quad (6)$$

where $[\bar{B}_{d1}\ \bar{B}_{d2}] = \bar{B}_d$.

If one can make the following relationships hold,

$$(I_{\bar{n}} - H\bar{C})\bar{B}_{d1} = 0 \quad (7)$$

$$R = \bar{A} - H\bar{C}\bar{A} - K_1\bar{C} \quad (8)$$

$$T = I_{\bar{n}} - H\bar{C} \quad (9)$$

$$K_2 = RH \quad (10)$$

the state estimation error dynamics (6) reduces to

$$\dot{\bar{e}} = R\bar{e} + (I_{\bar{n}} - H\bar{C})\bar{B}_{d2}d_2. \quad (11)$$

From (11), one can see d_1 has been decoupled under the conditions (7)-(10), but d_2 still exists. Therefore, the observer design is transformed into seeking the solution to (7), and designing an algorithm to make the observer system matrix R stable, and minimizing the influence from the unknown input d_2.

It is ready to develop the existence condition of the UIO with original system matrices, and the following lemma is useful for the proof of Theorem 1.

Lemma 1 [2, 21]. The necessary and sufficient conditions for the existence of the UIO (4) for the system (3) are:

(i) $\text{rank}(\bar{C}\bar{B}_{d1}) = \text{rank}(\bar{B}_{d1})$;

(ii) $(\bar{C}, \bar{A}_1)$ is a detectable pair, where $\bar{A}_1 = (I_n - H\bar{C})\bar{A}$.

Remark 1.

(a) Condition (i) in Lemma 1 can ensure equation (7) to be solvable, and a special solution is:

$$H_* = \bar{B}_{d1}[(\bar{C}\bar{B}_{d1})^T(\bar{C}\bar{B}_{d1})]^{-1}(\bar{C}\bar{B}_{d1})^T. \quad (12)$$

(b) Condition (ii) in Lemma 1 is standard for arbitrarily assigning the unstable poles of R. Moreover, the condition (ii) is equivalent to the condition that the transmission zeros from the unknown inputs to the measurements must be stable, *i.e.*,

$$\begin{bmatrix} sI_{\bar{n}} - \bar{A} & \bar{B}_{d1} \\ \bar{C} & 0 \end{bmatrix} \quad (13)$$

is of full column rank for all s with $Re(s) \geq 0$.

Theorem 1. The necessary and sufficient conditions for the existence of the UIO (4) for the system (3) are

(i) $\text{rank}(CB_{d1}) = \text{rank}(B_{d1})$;

(ii) $\begin{bmatrix} A & B_f & B_{d1} \\ C & D_f & 0 \end{bmatrix}$ is of full column rank;

(iii) $\text{rank}\begin{bmatrix} sI_n - A & B_{d1} \\ C & 0 \end{bmatrix} = n + l_{d1}$ for all s with $Re(s) \geq 0$, but $s \neq 0$.

Proof. It is noted that

$$\bar{C}\bar{B}_{d1} = [C \quad 0 \quad D_f]\begin{bmatrix} B_{d1} \\ 0 \\ 0 \end{bmatrix} = CB_{d1}$$

and

$$\text{rank}(\bar{B}_{d1}) = \text{rank}(B_{d1}).$$

Therefore, one can know that condition (i) in Lemma 1, that is $\text{rank}(\bar{C}\bar{B}_{d1}) = \text{rank}(\bar{B}_{d1})$, is equivalent to the condition (i) in Theorem 1, that is, $\text{rank}(CB_{d1}) = \text{rank}(B_{d1})$.

It is noted that

$$\begin{aligned} &\text{rank}\begin{bmatrix} sI_{\bar{n}} - \bar{A} & \bar{B}_{d1} \\ \bar{C} & 0 \end{bmatrix} \\ &= \text{rank}\begin{bmatrix} sI_n - A & 0 & -B_f & B_{d1} \\ 0 & sI_{l_f} & 0 & 0 \\ 0 & -I_{l_f} & sI_{l_f} & 0 \\ C & 0 & D_f & 0 \end{bmatrix} \\ &= \begin{cases} \text{rank}\begin{bmatrix} A & B_f & B_{d1} \\ C & D_f & 0 \end{bmatrix} + l_f, \ s = 0 \\ \text{rank}\begin{bmatrix} sI_n - A & B_{d1} \\ C & 0 \end{bmatrix} + 2l_f, \ s \neq 0 \end{cases} \end{aligned} \quad (14)$$

Therefore, (14) implies that the conditions (ii) and (iii) in Theorem 1 are equivalent to (13) being full of column rank for all s with $Re(s) \geq 0$, which is also equivalent to (ii) in Lemma 1. This completes the proof.

The next step for designing robust observer (4) is to make the matrix R stable and reduce the influence from the disturbance that cannot be decoupled, that is, d_2.

Theorem 2. For system (3), there exists a robust UIO in the form of (4) such that $\|\bar{e}\|_{Tf} \leq r\|d_2\|_{Tf}$, if there exists a positive definite matrix P and matrix Q, such that

$$\begin{bmatrix} I_{\bar{n}} + \bar{A}_1^T P + P\bar{A}_1 - \bar{C}^T Q^T - Q\bar{C} & P(I_{\bar{n}} - H\bar{C})\bar{B}_{d2} \\ * & -r^2 I_{l_{d2}} \end{bmatrix} < 0 \quad (15)$$

where $\bar{A}_1 = (I_n - H\bar{C})\bar{A}$, and $Q = PK_1$.

Proof. Take the following Lyapunov function candidate for error dynamic system (11):

$$V(\bar{e}) = \bar{e}^T P\bar{e}\ . \quad (16)$$

Using (11) and (16), one has

$$\dot{V}(\bar{e}) = \bar{e}^T P\dot{\bar{e}} + \dot{\bar{e}}^T P\bar{e} \\ = \bar{e}^T(\bar{A}_1^T P + P\bar{A}_1 - \bar{C}^T Q^T - Q\bar{C})\bar{e} \\ +2\bar{e}^T P(I_{\bar{n}} - H\bar{C})\bar{B}_{d2}d_{2.} \quad (17)$$

Form (15), one can see

$$I_{\bar{n}} + \bar{A}_1^T P + P\bar{A}_1 - \bar{C}^T Q^T - Q\bar{C} < 0,$$

indicating $\bar{A}_1^T P + P\bar{A}_1 - \bar{C}^T Q^T - Q\bar{C} < 0$.

Apparently, when $d_2 = 0$, one can obtain $\dot{V}(\bar{e}) < 0$, implying that the error dynamics in (11) is asymptotically stable.

Let

$$\Gamma = \int_0^{Tf}(\bar{e}^T\bar{e} - r^2 d_2^T d_2)dt. \quad (18)$$

By using (17) and (18), one has:

$$\Gamma = \int_0^{Tf}\left(\bar{e}^T\bar{e} - r^2 d_2^T d_2 + \dot{V}(\bar{e})\right)dt - \int_0^{Tf}\dot{V}(\bar{e})dt \\ = \int_0^{Tf}[\bar{e}^T(I_{\bar{n}} + \bar{A}_1^T P + P\bar{A}_1 - \bar{C}^T Q^T - Q\bar{C})\bar{e} \\ +2\bar{e}^T P(I_{\bar{n}} - H\bar{C})\bar{B}_{d2}d_2 - \gamma^2 d_2^T d_2]dt - \int_0^{Tf}\dot{V}(\bar{e})dt \\ = \int_0^{Tf}[\bar{e}^T \quad d_2^T]\Pi\begin{bmatrix}\bar{e}\\ d_2\end{bmatrix}dt - \int_0^{Tf}\dot{V}(\bar{e})dt \quad (19)$$

where

$$\Pi = \begin{bmatrix} I_{\bar{n}} + \bar{A}_1^T P + P\bar{A}_1 - \bar{C}^T Q^T - Q\bar{C} & P(I_{\bar{n}} - H\bar{C})\bar{B}_{d2} \\ * & -r^2 I_{l_{d2}} \end{bmatrix}.$$

Under zero initial condition $\bar{e}(0) = 0$, one has

$$\int_0^{Tf}\dot{V}(\bar{e})dt = \bar{e}^T(T_f)P\bar{e}(T_f) - \bar{e}^T(0)P\bar{e}(0) \\ = V(\bar{e}(T_f)) > 0. \quad (20)$$

Since $\Pi < 0$ in terms of (15), and from (19) and (20), one has $\Gamma < 0$, which indicates $\|\bar{e}\|_{Tf} \leq r\|d_2\|_{Tf}$. The proof is completed.

C. Design procedure of the UIO for fault estimation

Based on Theorems 1 and 2, we can summarize the design procedure of the UIO as follows.

Procedure1. The design of robust UIO for fault estimation

i) Construct the augmented system in the form of (3).

ii) Select the matrix H_* in the form of (12).

iii) Solve the LMI (15) to obtain the matrices P and Q, and calculate the gain $K_1 = P^{-1}Q$.

iv) Calculate the other gain matrices R, T and K_2 following the formulae (8)-(10), respectively.

v) Implement the robust UIO (4), and obtain the augmented estimate $\hat{\bar{x}}$, leading to the simultaneous state and fault estimates as follows:

$$\hat{x} = \begin{bmatrix} I_n & 0_{n\times 2l_f}\end{bmatrix}\hat{\bar{x}} \quad (21)$$

$$\hat{f} = \begin{bmatrix} 0_{n\times(n+l_f)} & I_{l_f}\end{bmatrix}\hat{\bar{x}}. \quad (22)$$

III. UIO-BASED FAULT ESTIMATION FOR LIPSCHITZ NONLINEAR SYSTEMS

A. Nonlinear systems subjected to process disturbances

In this subsection, UIO-based fault estimation approaches are to be proposed for Lipschitz nonlinear system subjected to process disturbances. The Lipschitz nonlinear system under consideration is represented as follows:

$$\begin{cases} \dot{x} = Ax + Bu + B_d d + B_f f + \Phi(t,x,u) \\ y = Cx + Du + D_f f \end{cases} \quad (23)$$

where $\Phi(t,x,u) \in \mathcal{R}^n$ is a real nonlinear vector function with Lipschitz constant θ, namely,

$$\|\Phi(t,x,u) - \Phi(t,\hat{x},u)\| \leq \theta\|x - \hat{x}\|, \\ \forall(t,x,u),(t,\hat{x},u) \in \mathcal{R}\times\mathcal{R}^n\times\mathcal{R}^m, \quad (24)$$

and the other symbols are the same as defined as (1). Lipschitz nonlinear systems, locally Lipschitz nonlinear systems at least, can be found in many practical systems. All the results derived for a globally Lipschitz system can be applied to a locally Lipschitz system directly.

Defining an augmented state vector in the form of (2), one can obtain an equivalent augmented system as follows:

$$\begin{cases} \dot{\bar{x}} = \bar{A}\bar{x} + \bar{B}u + \bar{B}_d d + \bar{\Phi}(t,x,u) \\ y = \bar{C}\bar{x} + Du \end{cases} \quad (25)$$

where $\bar{\Phi}(t,x,u) = [\Phi^T(t,x,u) \quad 0 \quad 0]^T \in \mathcal{R}^{\bar{n}}$, and the other symbols are defined the same as those in (3).

A nonlinear UIO is in the form of

$$\begin{cases} \dot{\bar{z}} = R\bar{z} + T\bar{B}u + K(y - Du) + T\bar{\Phi}(t,\hat{x},u) \\ \hat{\bar{x}} = \bar{z} + H(y - Du) \end{cases} \quad (26)$$

where $K = K_1 + K_2$, the gains H, R, T, and K_2 satisfy (7)-(10), and K_1 is to be designed.

The estimation error is defined by (5). In terms of (5), (25) and (26), the estimation error dynamics is represented as

$$\dot{\bar{e}} = (\bar{A} - H\bar{C}\bar{A} - K_1\bar{C})\bar{e} + (\bar{A} - H\bar{C}\bar{A} - K_1\bar{C} - R)\bar{z} \\ +[(\bar{A} - H\bar{C}\bar{A} - K_1\bar{C})H - K_2](y - Du) \\ +[(I_{\bar{n}} - H\bar{C})\bar{B} - T\bar{B}]u + (I_{\bar{n}} - H\bar{C})\bar{B}_{d1}d_1$$

$$+(I_{\bar n}-H\bar C)\bar B_{d2}d_2 + (I_{\bar n}-H\bar C)\widetilde{\Phi} \quad (27)$$

in which $\widetilde{\Phi}=\bar\Phi(t,x,u)-\bar\Phi(t,\hat x,u)$.

Substitution (7)-(10) into (27) yields

$$\dot{\bar e}=R\bar e+(I_{\bar n}-H\bar C)\bar B_{d2}d_2+(I_{\bar n}-H\bar C)\widetilde{\Phi}. \quad (28)$$

It is time to design the observer K_1 to ensure the estimation error dynamics above to be asymptotically stable and satisfy the robust performance index. The following two lemmas are useful in deriving Theorem 3.

Lemma 2 [38]. For any matrices $X\in\mathcal{R}^{s\times t}$ and $Y\in\mathcal{R}^{t\times s}$, a time-varying matrix $F(t)\in\mathcal{R}^{t\times t}$ with $\|F(t)\|\le 1$, and any scalar $\varepsilon>0$, we have:

$$XF(t)Y+Y^TF^T(t)X^T\le\varepsilon^{-1}XX^T+\varepsilon Y^TY.$$

Lemma 3 (Schur complement**)** [39]. Let $S=\begin{bmatrix}S_{11} & S_{12}\\ * & S_{22}\end{bmatrix}$ be a symmetric matrix. $S<0$ is equivalent to $S_{22}<0$ and $S_{11}-S_{12}S_{22}^{-1}S_{12}^T<0$.

Theorem 3. For system (25), there exists a robust observer in the form of (26), such that $\|\bar e\|_{Tf}\le r\|d_2\|_{Tf}$,if there exists a positive definite matrix P and matrix Q, such that

$$\begin{bmatrix}\Lambda & P(I_{\bar n}-H\bar C)\bar B_{d2} & P(I_{\bar n}-H\bar C)\\ * & -r^2I_{l_{d2}} & 0\\ * & * & -\varepsilon I_{\bar n}\end{bmatrix}<0 \quad (29)$$

where $\Lambda=(\varepsilon\theta^2+1)I_{\bar n}+\bar A_1^TP+P\bar A_1-\bar C^T\mathrm{Q}^T-\mathrm{Q}\bar C$,

$\bar A_1=(I_{\bar n}-H\bar C)\bar A$, $Q=PK_1$, ε is a given positive number, r is a performance index, standing for the magnitude of error compared with disturbances.

Proof. Choosing the Lyapunov function in the form of (16), and using (28), and noticing that $R=\bar A_1-K_1\bar C$, one has

$$\begin{aligned}\dot V(\bar e)=&\ \bar e^T(\bar A_1^TP+P\bar A_1-\bar C^TQ^T-Q\bar C)\bar e\\ &+2\bar e^TP(I_{\bar n}-H\bar C)\bar B_{d2}d_2\\ &+\bar e^TP(I_{\bar n}-H\bar C)\widetilde{\Phi} + \widetilde{\Phi}^T(I-H\bar C)^TP\bar e \quad (30)\end{aligned}$$

Applying Lemma 2 to the last two terms in (30) and using (24), one has

$$\begin{aligned}\dot V(\bar e)\le&\ \bar e^T(\bar A_1^TP+P\bar A_1-\bar C^TQ^T-Q\bar C+\varepsilon\theta^2I_{\bar n}\\ &+\varepsilon^{-1}P(I_{\bar n}-H\bar C)(I_{\bar n}-H\bar C)^TP)\bar e\\ &+2\bar e^TP(I_{\bar n}-H\bar C)\bar B_{d2}d_2 \quad (31)\end{aligned}$$

In terms of Lemma 3, one can see that (29) implies

$$\Lambda + \varepsilon^{-1}P(I_{\bar n}-H\bar C)(I_{\bar n}-H\bar C)^TP<0, \quad (32)$$

which is equivalent to

$$\begin{aligned}&\bar A_1^TP+P\bar A_1-\bar C^TQ^T-Q\bar C+\varepsilon\theta^2I_{\bar n}\\ &+\varepsilon^{-1}P(I_{\bar n}-H\bar C)(I_{\bar n}-H\bar C)^TP<0. \quad (33)\end{aligned}$$

When $d_2=0$, from (31) and (33) one has $\dot V(\bar e)<0$, indicating the error dynamics is asymptotically stable.

Letting

$$\Gamma_a=\int_0^{Tf}\left(\bar e^T\bar e-r^2d_2^Td_2+\dot V(\bar e)\right)dt-\int_0^{Tf}\dot V(\bar e)dt \quad (34)$$

and using (31), one has

$$\begin{aligned}\Gamma_a\le&\int_0^{Tf}[\bar e^T(I_{\bar n}+\bar A_1^TP+P\bar A_1-\bar C^TQ^T-Q\bar C\\ &+\varepsilon\theta^2I_{\bar n}+\varepsilon^{-1}P(I_{\bar n}-H\bar C)(I_{\bar n}-H\bar C)^TP)\bar e\\ &+2\bar e^TP(I_{\bar n}-H\bar C)\bar B_{d2}d_2-r^2d_2^Td_2]dt-\int_0^{Tf}\dot V(\bar e)dt\\ =&\int_0^{Tf}[\bar e^T\quad d_2^T]\Omega\begin{bmatrix}\bar e\\ d_2\end{bmatrix}dt-\int_0^{Tf}\dot V(\bar e)dt \quad (35)\end{aligned}$$

where

$$\Omega=\begin{bmatrix}\Lambda+\varepsilon^{-1}P(I_{\bar n}-H\bar C)(I_{\bar n}-H\bar C)^TP & P(I_{\bar n}-H\bar C)\bar B_{d2}\\ * & -r^2I_{l_{d2}}\end{bmatrix}. \quad (36)$$

In terms of Lemma 3, the inequality (29) implies $\Omega<0$. It is also noted that $\int_0^{Tf}\dot V(\bar e)dt=V\left(\bar e(t_f)\right)>0$ under zero initial condition. As a result, from (35) one has $\Gamma_a<0$, implying

$$\|\bar e\|_{Tf}\le r\|d_2\|_{Tf.} \quad (37)$$

This completes the proof.

B. Nonlinear systems with process and sensor disturbances

In this subsection, a more general case is taken into consideration, that is, a Lipschitz nonlinear system corrupted by both process and sensor disturbances, described by

$$\begin{cases}\dot x=Ax+Bu+B_ff+B_dd+\Phi(t,x,u)\\ y=Cx+Du+D_dd_s+D_ff\end{cases} \quad (38)$$

where D_d is constant known matrix, standing for the distribution matrix of the measurement noise $d_s\in\mathcal{R}^s$, and the other symbols are the same as defined before.

Defining an augmented state vector in the form of (2), an equivalent augmented system is given as

$$\begin{cases}\dot{\bar x}=\bar A\bar x+\bar Bu+\bar B_dd+\bar\Phi(t,x,u)\\ y=\bar C\bar x+Du+D_dd_s\end{cases} \quad (39)$$

where the symbols are the same as defined in (25) except for D_d and d_s.

The nonlinear UIO takes the same form as (26). From (26) and (39), one can see the estimation error as

$$\begin{aligned}\bar e&=\bar x-\hat{\bar x}\\ &=\bar x-\bar z-H(y-Du)\\ &=(I_{\bar n}-H\bar C)\bar x-\bar z-HD_dd_{s.} \quad (40)\end{aligned}$$

Furthermore, in terms of (26), (39) and (40), one can obtain the estimation error dynamic equation as follows:

$$\begin{aligned}\dot{\bar e}=&R\bar e+(I_{\bar n}-H\bar C)\bar B_{d2}d_2+(I_{\bar n}-H\bar C)\widetilde{\Phi}\\ &-K_1D_dd_s-HD_d\dot d_s \quad (41)\end{aligned}$$

where $\widetilde{\Phi}=\bar\Phi(t,x,u)-\bar\Phi(t,\hat x,u)$.

To design the parameters of the observer (26), the following theorem is addressed.

Theorem 4. For system (39), there exists a robust observer in the form of (26) such that $\|\bar{e}\|_{Tf} \leq r\|\bar{d}\|_{Tf}$,if there exists a positive definite matrix P and matrix Q, such that

$$\begin{bmatrix} \Lambda & P(I_{\bar{n}} - H\bar{C})\bar{B}_{d2} & P(I_{\bar{n}} - H\bar{C}) & -QD_d & -PHD_d \\ * & -r^2 I_{l_{d2}} & 0 & 0 & 0 \\ * & * & -\varepsilon I_{\bar{n}} & 0 & 0 \\ * & * & * & -r^2 I_s & 0 \\ * & * & * & * & -r^2 I_s \end{bmatrix} < 0 \quad (42)$$

where $\Lambda = (\varepsilon\theta^2 + 1)I_{\bar{n}} + \bar{A}_1^T P + P\bar{A}_1 - \bar{C}^T Q^T - Q\bar{C}$,

$\bar{A}_1 = (I_{\bar{n}} - H\bar{C})\bar{A}$, $Q = PK_1$, $\bar{d} = [d_2^T \quad d_s{}^T \quad \dot{d}_s^T]^T$, ε is a given positive number, and r is a performance index.

Proof. Taking the Lyapunov function in the form of (16), using (41) and the proof methodology of (30) and (31), one has

$$\begin{aligned} \dot{V}(\bar{e}) \leq\ & \bar{e}^T(\bar{A}_1^T P + P\bar{A}_1 - \bar{C}^T Q^T - Q\bar{C} + \varepsilon\theta^2 I_{\bar{n}} \\ & +\varepsilon^{-1}P(I_{\bar{n}} - H\bar{C})(I_{\bar{n}} - H\bar{C})^T P)\,\bar{e} \\ & +2\bar{e}^T P(I - H\bar{C})\bar{B}_{d2}d_2 - 2\bar{e}^T PK_1 D_d d_s \\ & -2\bar{e}^T PHD_d\dot{d}_s. \end{aligned} \quad (43)$$

For $d_2 = 0$, and $d_s = 0$, one can see that the error estimation system (41) is asymptotically stable, similar to the proof in Theorem 3.

Letting

$$\Gamma_b = \int_0^{Tf}\left(\bar{e}^T\bar{e} - r^2\bar{d}^T\bar{d} + \dot{V}(\bar{e})\right)dt - \int_0^{Tf}\dot{V}(\bar{e})dt \quad (44)$$

and using (43), one has

$$\Gamma_b = \int_0^{Tf}[\bar{e}^T \quad d_2^T \quad d_s^T \quad \dot{d}_s^T]\Psi\begin{bmatrix}\bar{e}\\ d_2\\ d_s\\ \dot{d}_s\end{bmatrix}dt - \int_0^{Tf}\dot{V}(\bar{e})dt \quad (45)$$

where

$$\Psi = \begin{bmatrix} \Sigma & P(I_{\bar{n}} - H\bar{C})\bar{B}_{d2} & -QD_d & -PHD_d \\ * & -r^2 I_{l_{d2}} & 0 & 0 \\ * & * & -r^2 I_s & 0 \\ * & * & * & -r^2 I_s \end{bmatrix} \quad (46)$$

$$\Sigma = \Lambda + \varepsilon^{-1}P(I_{\bar{n}} - H\bar{C})(I_{\bar{n}} - H\bar{C})^T P \quad (47)$$

and Λ is defined in (42).

In terms of Lemma 3, the inequality (42) implies $\Psi < 0$. It is also noted that $\int_0^{Tf}\dot{V}(\bar{e})dt = V\left(\bar{e}(t_f)\right) > 0$ under zero initial condition. Therefore, from (45), one has $\Gamma_b < 0$, indicating $\|\bar{e}\|_{Tf} \leq r\|\bar{d}\|_{Tf}$. This completes the proof.

C. Design procedure of the Nonlinear UIO for fault estimation

On the basis of Theorems 3 and 4, we can summarize the design procedure of the robust nonlinear UIO estimator as follows.

Procedure 2. The design of nonlinear UIO for fault estimation

i) Construct the augmented system in the form of (25) or (39), respectively, for systems subjected to either process disturbances or both disturbances in the process and measurement.

ii) Select the matrix H_* in the form of (12).

iii) Solve the LMI (29) or (42) to obtain the matrices P and Q, and calculate the gain $K_1 = P^{-1}Q$.

iv) Calculate the other gain matrices R, T and K_2 following the formulae (8)-(10), respectively.

v) Implement the robust UIO (26), and obtain the augmented estimate $\hat{\bar{x}}$, leading to the simultaneous state and fault estimates in the forms of (21) and (22), respectively.

IV. SIMULATION STUDY

A. Three-shaft gas turbine engine

A three-shaft gas turbine engine can be characterized by a 14-order linearized model:

$$\begin{cases}\dot{x} = Ax + Bu + B_a f_a + B_d d \\ y = Cx + Du + D_a f_a + D_s f_s\end{cases} \quad (48)$$

where the state vector, input vector and output vector are defined respectively as

$$x = [N_L, N_I, N_H, P_{2LM}, P_{2I}, P_2, T_3, P_{4H}, P_{4I}, P_{4M}, W_H, W_C, P_5, T_6]^T,$$

$$u = [W_{FE}, W_{FR}, A_J]^T,$$

$$y = [W_1, W_2, P_6, T_{2LM}, T_{2I}, T_H]^T.$$

The meanings of the symbols above are presented in Table 1. The coefficient matrices $A, B, C,$ and D are provided by [37, 40], which are omitted here for brevity.

In this simulation study, three actuator faults and two sensor faults are to be considered. Therefore, $B_a = B$, $D_s = \begin{bmatrix}1 & 0 & 0 & 0 & 0 & 0\\ 0 & 1 & 0 & 0 & 0 & 0\end{bmatrix}^T$.

Denote $f = [f_a^T \quad f_s^T]^T$, we can have $B_f = [B_a \quad 0]$ and $D_f = [D_a \quad D_s]$, correspondingly.

The unknown input disturbance vector is $d = [d_1^T \quad d_2^T \quad d_3^T]^T$, where $d_1 = 5\sin(10t)$, d_2 is random number between -0.5 to 0.5, and $d_3 = 0.5\sin(50t)$. The control input vector is $u = [2 \quad 2 \quad 2]^T$.

The three actuator faults are:

$$f_{a1} = \begin{cases} 0, & t < 10 \\ t - 10, & 10 \leq t < 15 \\ 20 - t, & 15 \leq t < 20 \\ 0, & t \geq 20 \end{cases} \quad (49)$$

TABLE 1
PARAMETER SYMBOLS OF GAS TURBINE ENGINE

A_J	Nozzle area	T_{2LM}	LP/IP inter-compressor temperature
N_H	HP shaft speed	T_3	Combustor outlet temperature
N_I	IP shaft speed	T_6	Jet pipe outlet temperature
N_L	LP shaft speed	W_1	Fan mass flow
P_{2I}	IP/P inter-compressor pressure	W_2	HP compressor mass flow
P_{2LM}	LP/IP inter-compressor pressure	W_C	Cold stream mass flow
P_{4H}	HP/IP inter-turbine pressure	W_{FE}	Engine fuel
P_{4M}	Post-turbine pressure	W_{FR}	Reheat fuel
P_5	Jet pipe pressure	W_H	Hot stream mass flow
P_6	Nozzle pressure	T_H	Thrust
P_2	Combustor pressure	T_{2I}	LP/HP inter-compressor temperature
P_{4I}	IP/LP inter-turbine pressure		

$$f_{a2}=\begin{cases}0, & t<25\\ 25-t, & 25\le t<30\\ -5, & 30\le t<35\\ t-40, & 35\le t<40\\ 0, & t\ge 40\end{cases} \tag{50}$$

$$f_{a3}=\begin{cases}0, & t<2\\ 0.2\mathrm{t}-0.4, & 2\le t<6\\ 0.1\sin(2t)+0.8, & t\ge 6\end{cases} \tag{51}$$

and the two sensor faults concerned are:

$$f_{s1}=\begin{cases}0, & t<65\\ 1, & t\ge 65\end{cases} \tag{52}$$

$$f_{s2}=\begin{cases}0, & t<40\\ 0.1\sin(0.5\mathrm{t})+0.2\sin(\mathrm{t}+\frac{\pi}{2}), & 40\le t<65\\ 0, & t\ge 65\end{cases} \tag{53}$$

By using the design procedure 1, one obtains the robust UIO in the form of (4) (the obtained observer gains are omitted here for brevity). The unknown input d_1 is decoupled whereas the influences of d_2 and d_3 are attenuated via the designed observer gains. Due to the limit of space, we only present the curves of the three dominant states (*i.e.*, states corresponding to the three dominant poles) and their estimates, shown in Figs. 1−3, showing excellent estimation performance. The estimates of the three actuator faults and two sensor faults are depicted by Figs. 4−8, respectively. It can be seen that the proposed UIO-based fault estimation techniques can successfully estimate abrupt faults, incipient faults and even sinusoidal faults.

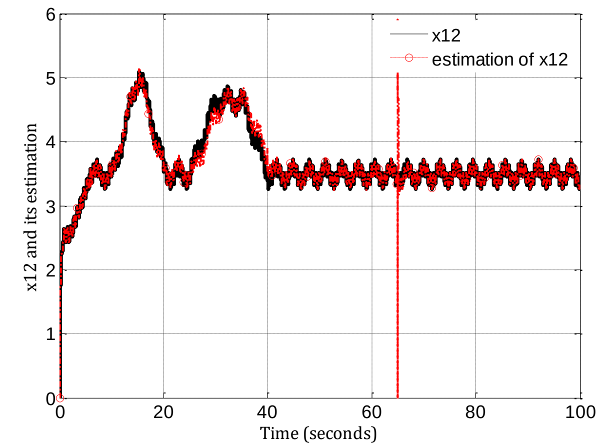

Fig. 1. x_{12} (the 12th state) and its estimation.

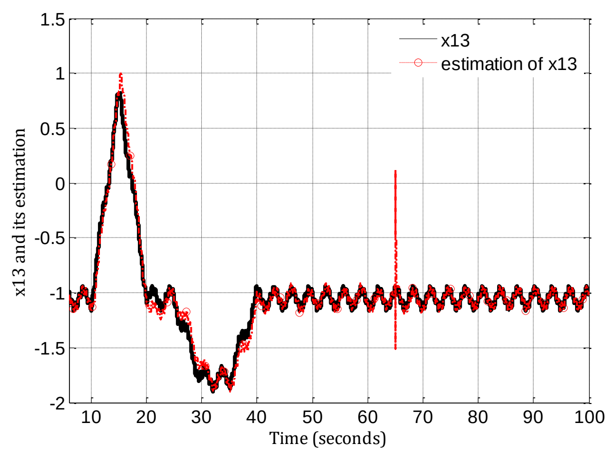

Fig. 2. x_{13} (the 13th state) and its estimation.

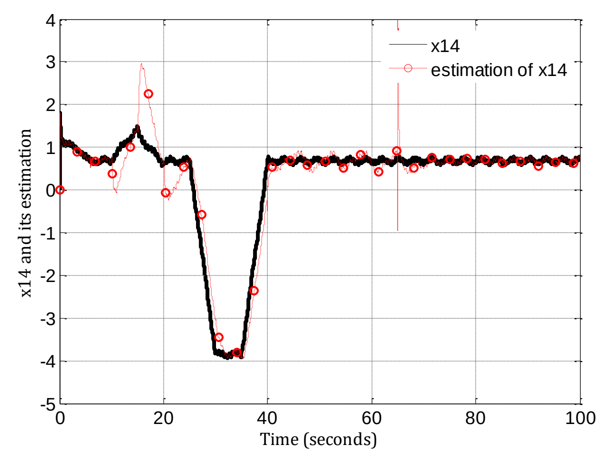

Fig. 3. x_{14} (the 14th state) and its estimation.

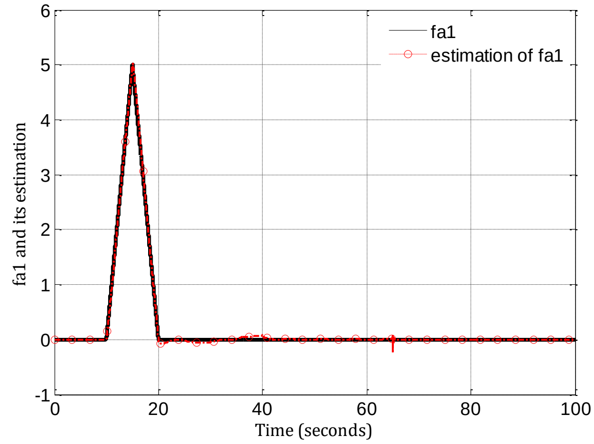

Fig. 4. f_{a1} (engine fuel actuator fault) and its estimation.

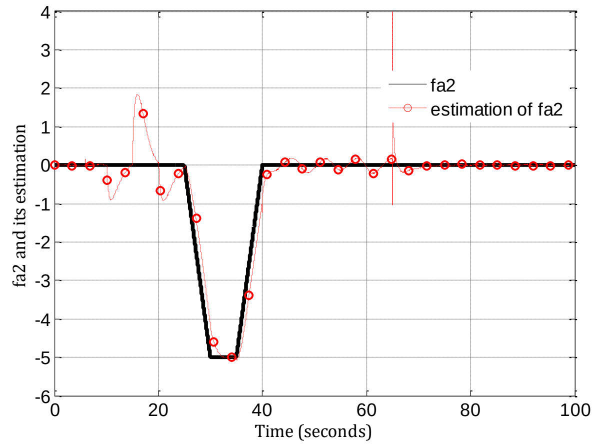

Fig. 5. f_{a2} (reheat fuel actuator fault) and its estimation.

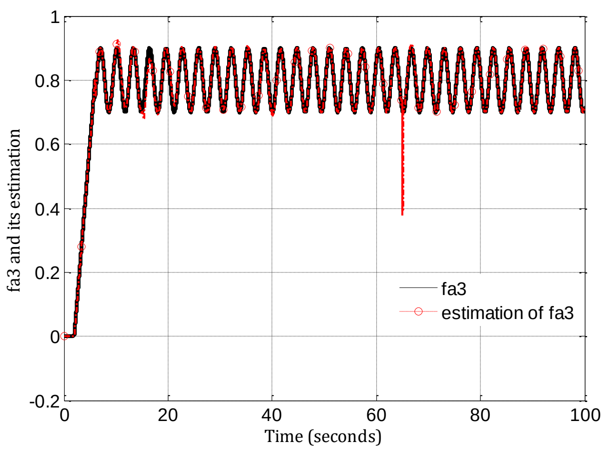

Fig. 6. f_{a3} (nozel area actuator fault) and its estimation.

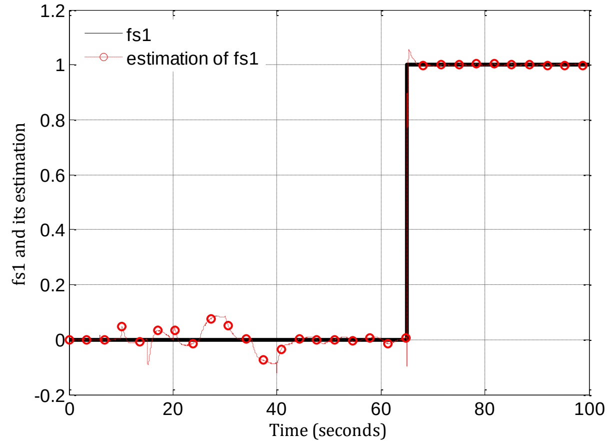

Fig. 7. f_{s1} (fan mass flow sensor fault) and its estimation.

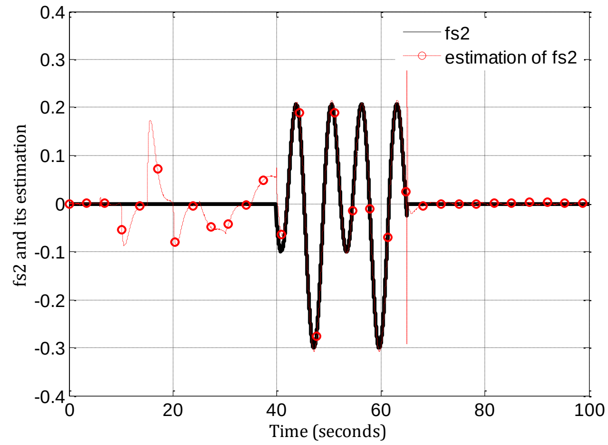

Fig. 8. f_{s2} (HP compressor mass flow sensor fault) and its estimation.

B. Single-link flexible joint robot

The single-link manipulator with revolute joints actuated by a DC motor can be described by a Lipschtiz nonlinear system [41,42]:

$$\begin{cases} \dot{\theta}_m = \omega_m \\ \dot{\omega}_m = \frac{k}{J_m}(\theta_l - \theta_m) - \frac{G}{J_m}\omega_m + \frac{k_\tau}{J_m}u \\ \dot{\theta}_l = \omega_l \\ \dot{\omega}_l = -\frac{k}{J_l}(\theta_l - \theta_m) - \frac{mgh}{J_l}\sin(\theta_l) \end{cases} \tag{54}$$

where J_m represents the inertia of the DC motor, J_l is the inertia of the link, θ_m and θ_l denote the angles of the rotations of the motor and link, respectively, ω_m and ω_l are the angular velocities of the motor and link, respectively, k is torsional spring constant, k_τ is the amplifier gain, G is the viscous friction, m is the pointer mass, g is the gravity constant, and h is the distance from the rotor to the center of the gravity of the link, and u is the control input (DC voltage) to produce the motor torque. Let $x = [\theta_m \quad \omega_m \quad \theta_l \quad 0.1\omega_l]$, the system can be written in the form of (38), where

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -48.6 & -1.25 & 48.6 & 0 \\ 0 & 0 & 0 & 10 \\ 1.95 & 0 & -1.95 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 21.6 \\ 0 \\ 0 \end{bmatrix},$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}, \Phi(x) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -0.333\sin(x_3) \end{bmatrix}.$$

The fault and disturbance distribution matrices are respectively $B_{fa} = B$, and

$$B_d = \begin{bmatrix} -0.2 & 0.01 & -0.02 \\ -0.1 & 0.02 & -0.04 \\ 0.1 & -0.02 & 0.04 \\ 0.2 & 0.02 & -0.04 \end{bmatrix}, D_d = \begin{bmatrix} 0.1 \\ -0.02 \end{bmatrix}.$$

The actuator fault is:

$$f_a = \begin{cases} 1 + 0.1\sin(4t) & t \geq 4 \\ 0.5(t-2) & 2 \leq t < 4, \\ 0 & t < 2 \end{cases}$$

The unknown input disturbances are as follows: $d_1 = 5\sin(10t)$, corrupted by a uniform-random-number signal, $d_2 = 2\sin(10t), d_3 = \sin(20t)$ and $d_s = 0.1\sin(10t)$. The control input is added as $u = 2\sin(2\pi t)$ and the initial state value is given as $x(0) = [0.01 \ \ -5 \ \ 0.01 \ \ 5\ ]^T$.

Choose $r = 0.58$, $\varepsilon = 50$, and using the procedure 2, we can obtain the observer gains as follows:

$$H = \begin{bmatrix} 0.8000 & 0.4000 \\ 0.4000 & 0.2000 \\ -0.4000 & -0.2000 \\ -0.8000 & -0.4000 \\ 0 & 0 \\ 0 & 0 \end{bmatrix},$$

$$T = \begin{bmatrix} 0.2000 & -0.4000 & 0 & 0 & 0 & 0 \\ -0.4000 & 0.8000 & 0 & 0 & 0 & 0 \\ 0.4000 & 0.2000 & 1 & 0 & 0 & 0 \\ 0.8000 & 0.4000 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix},$$

$$K = K_1 + K_2 = \begin{bmatrix} -2851.2 & 5761.3 \\ 396.33 & -910.70 \\ 3172.4 & -6393.8 \\ -1638.4 & 3224.6 \\ -6308.5 & 12617 \\ -14930 & 29859 \end{bmatrix},$$

$$R = \begin{bmatrix} -1595.9 & -7993.9 & -19.440 & 0 & 0 & -8.6400 \\ 210.82 & 1232.3 & 38.880 & 0 & 0 & 17.280 \\ 1781.0 & 8875.5 & 9.7200 & 10 & 0 & 4.3200 \\ -924.22 & -4496.9 & 17.490 & 0 & 0 & 8.6400 \\ -3540.0 & -17541 & 0 & 0 & 0 & 0 \\ -8378.3 & -41513 & 0 & 0 & 1 & 0 \end{bmatrix}.$$

By choosing the above parameters, d_1 is decoupled and the influences of d_2, d_3 and d_s are minimized. The curves displayed in Figs. 9−11 exhibit the estimation performance for angular velocities of the motor and link, and actuator fault respectively.

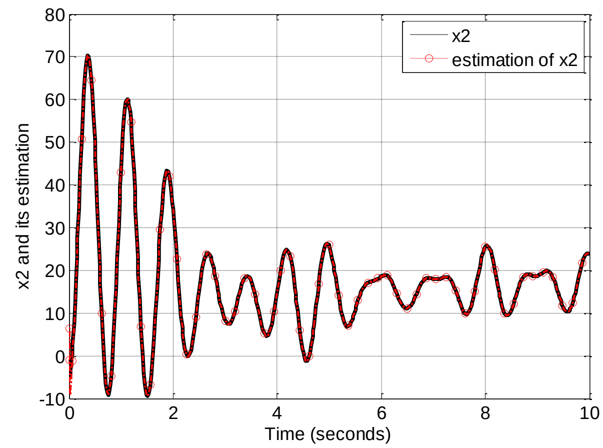

Fig. 9: x_2 (motor angular velocity) and its estimation.

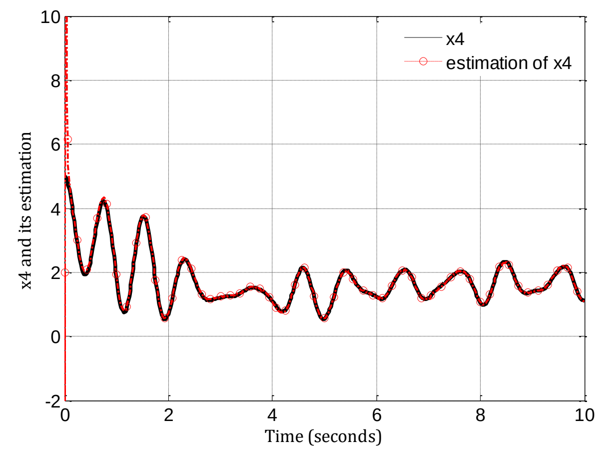

Fig. 10. x_4 (10% link angular velocity) and its estimation.

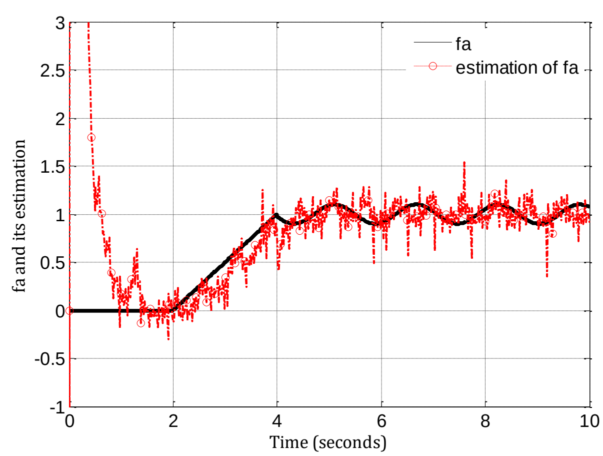

Fig. 11. f_a (DC input voltage actuator fault) and its estimation.

V. CONCLUSION

In this paper, a novel UIO-based simultaneous state and fault estimation technique has been proposed, which can be utilized to handle systems subjected to partially decoupled process disturbances and even sensor disturbances. The design procedures of the estimators for both linear and nonlinear systems are presented. The robustness is ensured by decoupling partial process disturbances with the UIO approach, and attenuating the process disturbances and sensor disturbances that cannot be decoupled, with LMI optimisation technique. The simultaneous estimation is realized with the integration of the system augmentation and the estimator design for the augmented system. The proposed techniques have been illustrated by using two engineering-oriented systems: the three-shaft gas turbine engine and the single link robot. The proposed techniques have great potentials to apply to various engineering systems. It is encouraged to extend the proposed techniques to more complex systems such as nonlinear systems, delay systems, and distributed systems. In addition, from the viewpoint of digital monitoring and real-time implementation, it is of interest to investigate discrete-time systems and delta operator systems [43-45] by taking into account stability, robustness, reliability and optimal operation performance.

ACKNOWLEDGEMENT

The work is partially supported by the PGR scholarship, the sabbatical leave scheme at the E&E faculty in the University of Northumbria, and the Alexander von Humboldt Renewed Fellowship (GRO/1117303 STP).

REFERENCES

[1] A. Willsky, ``A survey of design methods for failure detection in dynamic systems,'' *Automatica*, vol. 12, no. 6, pp.601-611, Nov. 1976.
[2] S. Yin, S. X. Ding, X. Xie and H. Luo, " A review on basic data-driven approaches for industrial process monitoring," *IEEE Trans. Ind. Electron.*, vol. 61, no. 11, pp. 6418-6428, Nov. 2014.
[3] S. Yin, X. Zhu, and O. Kaynak, ``Improved PLS focused on key-performance-indicator-related fault diagnosis,'' *IEEE Trans. Ind. Electron.*, vol. 62, no. 3, pp.1651-1658, Jun. 2015.
[4] S. Yin, and X. Zhu, ``Intelligent particle filter and its application to fault detection of nonlinear systems,'' *IEEE Trans. Ind. Electron.*, vol. 62, no. 6, pp.3852-3861, Jun. 2015.
[5] J. Chen and R. Patton, Robust Model-Based Fault Diagnosis for Dynamic Systems, Boston: Kuwer Academic Publishers, 1999.
[6] K. Liu, Z. Zhu, and D. Stone, ``Parameter estimation for condition monitoring of PMSM stator winding and rotor permanent magnets,'' *IEEE Trans. Ind. Electron.*, vol. 60, no. 12, pp.5902-5913, Dec. 2013.
[7] M. Valtierra-Rodriguez, R. Romero-Troncoso, R. Osornio-Rios, and A. Garcia-Perez, ``Detection and classification of single and combined power quality disturbances using neural networks,'' *IEEE Trans. Ind. Electron.*, vol.61, no.5, pp.2473-2482, May. 2014.
[8] B. Ebrahimi, M. Roshtkhari, J. Faiz, and S. Khatami, ``Advanced eccentricity fault recognition in permanent magnet synchronous motors using stator current signature analysis,'' *IEEE Trans. Ind. Electron.*, vol. 61, no. 4, pp.2041-2052, Apr. 2014.
[9] V. Venkatasubramanian, R. Rengaswamy, K. Yin, and S. Kavuri, ``A review of process fault detection and diagnosis part I: quantitative model-based methods,'' *Comput. Chem. Eng.*, vol. 27, no. 3, pp. 293-311, Mar. 2003.
[10] V. Venkatasubramanian, R. Rengaswamy, and S. Kavuri, ``A review of process fault detection and diagnosis part II: qualitative models and search strategies,'' *Comput. Chem. Eng.*, vol. 27, no. 3, pp. 313-326, Mar. 2003.
[11] V. Venkatasubramanian, R. Rengaswamy, S. Kavuri, and K.Yin ``A review of process fault detection and diagnosis part III: process history based methods,'' *Comput. Chem. Eng.*, vol. 27, no. 3, pp. 313-326, Mar. 2003.
[12] Z. Gao, C. Cecati, and S. X. Ding, ``A survey of fault diagnosis and fault-tolerant techniques part I: fault diagnosis with model- and signal-based approaches,'' *IEEE Trans. Ind. Electron.*, vol. 62, no. 6, pp.3757-3767, Jun. 2015.
[13] Z. Gao, C. Cecati, and S. X. Ding, ``A survey of fault diagnosis and fault-tolerant techniques part II: fault diagnosis with knowledge-based and hybrid/active approaches,'' *IEEE Trans. Ind. Electron.*, vol. 62, no. 6, pp.3768-3774, Jun. 2015.
[14] R. Patton, and J. Chen, ``On eigenstructure assignment for robust fault diagnosis,'' *Int. J. Ada. Contr. Sig. Proc.*, vol. 10, no. 14, pp.1193-1208, Dec. 2000.
[15] M. Zhong, S. X. Ding, J. Lam and H. Wang, "An LMI approach to design robust fault detection filter for uncertain LTI systems," *Automatica*, vol. 39, no. 3, pp. 543-550, Mar. 2003.
[16] B. Chen, and S. Nagarajaiah, ``H_-/H_∞ structural damage detection filter design using an iterative linear matrix inequality approach,'' *Smart Mater. Struct.*, vol. 17, no. 3, pp.1-9, Apr. 2008.
[17] M. Chadli, A. Abdo, and S. X. Ding, ``H_-/H_∞ fault detection filter design for discrete-time Takagi-Sugeno fuzzy system,'' *Automatica*, vol. 49, no. 7, pp.1996-2005, Jul. 2013.
[18] C. De Persis, and A Isidori, ``A geometric approach to non–linear fault detection and isolation,'' *IEEE Trans. Automat. Contr.*, vol. 45, no. 6, pp.853–865, Jun. 2001.
[19] R. Mattone, and A. De Luca, ``Nonlinear fault detection and isolation in a three-tank heating system,'' *IEEE Trans. Contr. Syst. Tech.*, vol. 14, no. 6, Nov. 2006.
[20] P. Castaldi, N. Mimmo, and S. Siman, ``Differential geometry based active fault tolerant control for aircraft,'' *Contr. Eng. Pract.*, vol. 32, no., pp.227-235, Nov.2014.
[21] J. Chen, R. Patton, and H.Y. Zhang, "Design of unknown input observers and robust fault detection filters," *Int. J. Contr.*, vol. 63, no. 1, pp. 85-105, Jan. 1996.
[22] A. Pertew, H. Marquez, and Q. Zhao, ``Design of unknown input observers for lipschitz nonlinear systems,'' *Proc. American Contr. Conf.*, pp.4198-4203, Portland, USA, Jun. 2005.
[23] W. Chen, and M. Saif, ``Unknown input observer design for a class of nonlinear systems: an LMI approach,'' *Proc. American Contr. Conf.*, pp.834-838, Minnesota, USA, Jun. 2006.
[24] Z. Wang, ``Unknown input observer based fault class isolation and estimation,'' *Proc. Chinese Contr. Conf.*, pp.3963-3968, Beijing, China, Jul. 2010
[25] T. Sedighi, A. Koshkouei, and K. Burnham, ``Nonlinear unknown input observer design for nonlinear systems: A new method,'' *Proc. IFAC World Congress*, pp.11067-11072, Milano, Italy, Aug. 2011.
[26] J. Zari, and E. Shokri, ``Robust sensor fault detection based on nonlinear unknown input observer,'' *Measurement*, vo. 48, no. 2, pp.355-367, Feb. 2014.
[27] R. Fonod, D. Henry, C. Charbonnely, and E. Bornschleglz, ``A class of nonlinear unknown input observer for fault diagnosis: application to fault tolerant control of an autonomous spacecraft,'' *Proc. UKACC Int. Conf. Contr.*, pp.19-24, Loughborgh , UK, Jul 2014,
[28] L. Imsland, T. Johansen, H. Grip, and T. Fossen, ``On nonlinear unknown input observers – applied to lateral vehicle velocity estimation on banked roads,'' *Int. J. Contr.*, vol. 80, no. 11, pp.1741-1750, Nov. 2007.
[29] O. Hrizi, B. Boussaid, A. Zouinkhi, and M. Abdelkrim, ``Robust unknown input observer based fast adaptive fault estimation: application to unicycle robot,'' *Proc. Int. Conf. Automation, Contr. Eng. Comput. Sci.*, pp.186-194, Monastir, Tunisia, Mar. 2014.
[30] X. Sun and R. Patton, "Robust actuator multiplicative fault estimation with unknown input decoupling for a wind turbine system," *Proc. Conf. Contr. Fault-Tolerant Syst.*, pp. 263-268, Nice, France, Oct. 2013.
[31] S. Bezzaoucha, B. Marx, D. Maquin, and J. Ragot, ``On the unknown input observer design: A decoupling class approach,'' *Proc. IEEE Int. Conf. Contr. Automat.*, pp.602-607, Santiago, Chile. Dec. 2011.
[32] H. Ting, J. Chang, and Y. Chen, ``Proportional-derivative unknown input observer design using descriptor system approach for non-

minimum phase systems,'' *Int. J. Contr. Automat. Sys*., vol. 9, no. 5, pp.850-856, May 2011.
[33] H. Wang, Z. Huang, and S. Daley, ``On the use of adaptive updating rules for actuator and sensor fault diagnosis,'' *Automatica*, vol. 33, no. 2, pp.217-225, Feb. 1997.
[34] H. Alwi, and C. Edwards, ``Robust fault reconstruction for linear parameter varying systems using sliding mode observers,'' *Int. J. Robust Nonlin. Contr*., vol. 24, no. 14, pp.1947-1968, Sep., 2014.
[35] Z. Gao, S. X. Ding, and Y. Ma, ``Robust fault estimation approach and its application in vehicle lateral dynamic systems,'' *Opt. Contr. App. Meth*., vol. 28, no. 3, pp. 143-156, May 2007.
[36] Z. Gao, and S. X. Ding, ``Actuator fault robust estimation and fault-tolerant control for a class of nonlinear descriptor systems,'' *Automatica*, vol. 43, no. 5, pp. 912-920, May 2007.
[37] Z. Gao, T. Breikin, and H. Wang, "High-gain estimator and fault-tolerant design with application to a gas turbine dynamic system," *IEEE Trans. Contr. Syst. Tech.*, vol. 15, no. 4, pp. 740-753, Apr. 2007.
[38] L. Li, Y. Jia, J. Du and S. Yuan, "Robust $L_2 - L_\infty$ control for uncertain singular systems with time-varying delay," *Prog. Natur. Sci.*, vol. 18, no. 8, pp. 1015-1021, Aug. 2008.
[39] S. Boyd, L. E. Ghaoui, E. Feron and V. Balakrishnan, *Linear matrix inequalities in system and control theory*, Philadelphia: Society for Industrial and Applied Mathematics, 1994.
[40] S. Daley and H. Wang, "Fault detection using reduced order models," *Proceedings of the Institution of Mechanical Engineers, Part I: J. Syst. Contr. Eng.*, vol. 207, no. 3, pp. 153-164, Jul. 1993.
[41] S. Ibrir, W. Xie, and C. Su, "Observer-based control of discrete-time Lipschitzian non-linear systems: application to one-link flexible joint robot," *Int. J. Contr.*, vol. 78, no. 6, pp. 385-395, Apr. 2005.
[42] R. Rajamani and Y. Cho, "Existence and design of observers for nonlinear systems: relation to distance to unobservability," *Int. J. Contr*, vol. 65, no. 5, pp. 717-731, May 1998.
[43] Z. Gao, ``Fault estimation and fault tolerant control for discrete-time dynamic systems,'' *IEEE Trans. Ind. Electron*., vol.62, no.6, pp.3874-3884, Jun. 2015.
[44] Z. Gao, ``Eestimation and compensation for lipschitz nonlinear discrete-time systems subjected to unknown measurement delays,'' *IEEE Trans. Ind. Electron*., vol.62, no.9, pp.5950-5961, Sep. 2015.
[45] X. Xie, S. Yin, H. Gao, and O. Kaynak, ``Asymptotic stablity and stablisation of uncertanin delta operator systems with time-varying delays,'' *IET Contr. Theory Appl*., vol.7, no.8, pp.1071-1078.

Zhiwei Gao (SM'08) received the B.Eng. degree in electric engineering and automation and M.Eng. and Ph.D. degrees in systems engineering from Tianjin University, Tianjin, China, in 1987, 1993, and 1996, respectively. Presently, he works with the Faculty of Engineering and Environment at the University of Northumbria, UK. His research interests include data-driven modelling, estimation and filtering, fault diagnosis, fault-tolerant control, intelligent optimisation, large-scale systems, singular systems, distributed/decentralized estimation and control, renewable energy systems, power electronics and electrical vehicles, bioinformatics and healthcare systems.

Dr. Gao is the associate editor of IEEE TRANSACTIONS ON INDUSTRIAL ELECTRONICS, IEEE TRANSACTIONS ON INDUSTRIAL INFORMATICS, and IEEE TRANSACTIONS ON CONTROL SYSTEMS TECHNOLOGY.

Xiaoxu Liu (S'15) received her B.S. and M.S. degrees in the Department of Mathematics from Northeastern University, Liaoning, China in 2012 and 2014, respectively. She is currently working towards her PhD degree in the Faculty of Engineering and Environment, University of Northumbria at Newcastle upon Tyne, UK. Her research interests include robust fault diagnosis, nonlinear systems, fuzzy modelling, distributed systems, and wind turbine energy systems.

Michael Z. Q. Chen (M'08) received the B.Eng. degree in electrical and electronic engineering from Nanyang Technological University, Singapore in 2003, and the Ph.D. degree in control engineering from Cambridge University, Cambridge, U.K. in 2007. He is currently an Assistant Professor at the Department of Mechanical Engineering, The University of Hong Kong. Dr. Chen is a Fellow of the Cambridge Philosophical Society. He is a Guest Associate Editor for the International Journal of Bifurcation and Chaos.