→ ~~1030~~

→ 1148

Self-generating Sequences

In this section are collected a number of sequences which are generated by simple and yet unusual recurrence rules. We have (rather arbitrarily) called them "self-generating". For yet another kind of recurrence rule, see the (next) section, sequences generated by sieves.

In the first two examples let $A = \{a_0 = 1, a_1, a_2, \cdots\}$ be a sequence of 1's and 2's.

(1) We obtain a new sequence A' from A by replacing each 1 in A by the finite sequence $U = 1, 2$ and each 2 by $V = 2, 1$. ~~For example~~ E.g. if $A = \{1, 1, 1, \cancel{1,} \cdots\}$ then $A' = \{1, 2, 1, 2, 1, 2, \ldots\}$. Now impose the restriction that $A = A'$. ~~This determines A uniquely, and~~ ~~we find~~ then A must be $\{1, 2, 2, 1, 2, 1, 1, 2, 2, 1, \cdots\}$, which is sequence 1285.

Similarly with the same U, and $V = 1, 1, 2$ or $1, 2, 2$ we obtain sequences 1030 and 1468 respectively.

(2) ~~Alternatively~~ Let $A'' = \{a_0'', a_1'', a_2'', \ldots\}$, where a_n'' is the length of the n^{th} run in A. (A <u>run</u> is a maximal string of identical symbols.) ~~Example if A =~~ E.g. if $A = \{1, 2, 1, 2, 1, 2, \cdots\}$ then $A'' = \{1, 1, 1, 1, 1, 1, \ldots\}$. The restriction that $A = A''$ ~~again determines~~ forces A to be ~~uniquely, and A =~~ $\{1, 2, 2, 1, 1, 2, 1, 2, 2, 1, \ldots\}$, which is sequence 2.

In the remaining examples $A = \{a_0 = 1, a_1, a_2, \cdots\}$ is a ~~monotonically~~ nondecreasing sequence of integers.

(3) Let b_n be the number of times ~~the~~ n occurs in the sequence A, for $n = 1, 2, \cdots$. E.g. if $b_n = n$, then $A = \{1, 2, 2, 3, 3, 3, 4, 4, 4, 4, \ldots\}$, sequence 2024. Or if $b_n = a_{n-1}$, then $A = \{1, 2, 2, 3, 3, 4, 4, 4, 5, 5, 5, 6, 6, 6, 6, \cdots\}$, sequence 1462. In the latter sequence let c_n be the greatest integer m such that $a_{m-1} = n$. Then $\{c_1, c_2, \cdots\} = \{1, 3, 5, 8, 11, 15, \ldots\}$ is sequence 1463.

19, 23, 28, 33,

(4) Choose a_n to be the ~~smallest~~ least integer such that A contains 1 odd, then 2 even, then 3 odd, then 4 even, $\cdots$, numbers. Then $A = \{1, 2, 4, 5, 7, 9, 10, 12, 14, 16, \cdots\}$, sequence 1614.

(5) Choose a_{n+1} so that $a_{n+1} - a_n =$ ~~smallest~~ least positive integer m not of the form

$$a_i - a_j \quad \text{for} \quad 0 \le j < i \le n.$$

Then $A = \{1, 2, 4, 8, 13, 21, \underbrace{\cdots}_{31, 45, 60, 76,}\}$, sequence 1148.

Or choose a_{n+1} so that $a_{n+1} - a_{n-1} = m$, and we obtain sequence 1149.

Choose $a_0 = 1$, $a_1 = 2$, $a_{2n} = 2a_{2n-1}$, and $a_{2n+1} - a_{2n} =$ ~~smallest~~ least ~~integer~~ positive integer not of the form

$$a_i - a_j \quad \text{for} \quad 0 \le j < i \le 2n.$$

Then $A = \{1, 2, 4, 8, 16, 21, \underbrace{\cdots}_{42, 51, 102, 112,}\}$, sequence 1856.

Choose $a_0 = 1$, $a_1 = 2$ and $a_n =$ least integer which can be written uniquely as $a_i + a_j$, with $0 \le i < j \le n$. Then $A = \{1, 2, 3, 4, 6, 8, 11, 13, 16, 18, \ldots\}$, sequence 2858. Similarly starting from

[redacted]

1, 3 and 2, 3 we obtain sequences 2859 and 1857 respectively.