Scientists reinvent equations governing formation of snowflakes, raindrops and Saturn's rings

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Snowflake and planetary rings abstraction. Credit: Modified from media generated by Artistic 2 and DaVinci2 models on Deep Dream Generator, prompted by Nicolas Posunko/Skoltech PR

Skoltech researchers have proposed novel mathematical equations that describe the behavior of aggregating particles in fluids. This bears on natural and engineering processes as diverse as rain and snow formation, the emergence of planetary rings, and the flow of fluids and powders in pipes.

Reported in *Physical Review Letters*, the new equations eliminate the need for juggling two sets of equations that had to be used in conjunction, which led to unacceptable errors for some applications.

Fluid aggregation is involved in many processes. In the atmosphere, water droplets agglomerate into rain, and ice microcrystals into snow. In space, particles orbiting giant planets come together to form rings like those of Saturn.

In human technology, this phenomenon is relevant for painting with aerosols, powder transport, controlled explosions, and more.

Understanding, predicting, and manipulating these processes depends on the scientists and engineers being able to use an accurate mathematical model of aggregation in fluid.

In the early 20th century, Polish physicist Marian Smoluchowski came up with a set of equations that describe aggregation processes in terms of the number of aggregates of different sizes and aggregation rates—the kinetic coefficients showing how fast the aggregates merge to form larger entities.

The classical Smoluchowski equations, however, deal with uniform systems without any space inhomogeneities and fluxes. This is certainly too idealistic a model for real-life aggregation processes.

To describe aggregating particle behavior in the actual atmosphere, in orbit, or at industrial facilities, one has to merge the Smoluchowski equations with the Euler equations or, more generally, with Navier-

Stokes equations. These are two fundamental descriptions of fluid motion dating back to the middle of the 18th and the middle of the 19th centuries, respectively.

The resulting mathematical formulation is a "mechanical" hybrid of two parts that do not always go well together, potentially causing unacceptably large and sometimes even qualitative errors.

A way out has been proposed in the paper in *Physical Review Letters* by Skoltech Senior Research Scientist Alexander Osinsky and Professor Nikolay Brilliantov from the Institute's AI Center.

Rather than continue seeking ways to marry the old equations, the researchers report a mathematically rigorous derivation of new hydrodynamic equations with unfamiliar coefficients obtained from the first principles.

"Surprisingly, these are neither the reaction-rate, nor the transport coefficients familiar from the Navier-Stokes equations, but a combination of both in the form of kinetic coefficients of a new nature," Brilliantov commented.

"They are as fundamental for aggregating fluids as viscosity and thermal conductivity are for ordinary fluids. Using extensive computer simulations, we have shown the accuracy and relevance of our novel Smoluchowski-Euler hydrodynamic equations with the new coefficients for some of the technologically important aggregating fluids."

The new equations will increase the precision of the models used in the analysis of air pollution by solid particles, rapid granular flows, as well as in powder technology and, potentially, in aircraft and car design.

More information: A. I. Osinsky et al, Hydrodynamic Equations for

Space-Inhomogeneous Aggregating Fluids with First-Principle Kinetic Coefficients, *Physical Review Letters* (2024). DOI: 10.1103/PhysRevLett.133.217201

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