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Learning Finite-Dimensional Representations For Koopman Operators

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Abstract

Recent efforts in data-driven modeling of nonlinear dynamical systems have been focused on methods for learning linear embeddings of the dynamics as liftings in high-dimensional spaces. To this end, Koopman operators which are infinite-dimensional linear maps on the space of functions are introduced, with the potentials for full representations of the dynamical systems (Mauroy et al., 2020). The main idea is to use the linearity of the new formalism for facilitating the study of various features of the system and employing this alternative formulation in different applications. While this approach provides numerous potentials and benefits, unless one can obtain a suitable finite-dimensional approximation for the Koopman operator, the infinite-dimensional nature of this operator hinders its practical use in the current settings. Efforts have been done to seek this finite-dimensional approximation using methods such DMD, Hankel-DMD, extended DMD, or closed-form solutions that approximate Koopman operators using state measurements. These linearization methods are only locally accurate and depend strongly on the choice of the observable functions to provide suitable accuracy. Moreover, many of these approaches are not utilizing the data efficiently by enforcing naive finite-dimensional approximation on the Koopman operator.

Motivated by these issues, we propose a general formulation for learning an infinite-dimensional representation of the Koopman operators for a discrete-time autonomous system. The learning problem can be formulated as a constrained optimization problem to enforce available side information on the latent Koopman operator. We introduce representer theorems which hold under certain but general conditions. This allows to reformulate the problem in a finite-dimensional setting without loss of any precision and hence utilize the data efficiently and optimally. Following this, we consider different cases of regularization and constraints, e.g., the Frobenius norm, the operator norm, and rank. Then, we derive the aforementioned finite-dimensional version of the problem in a computationally tractable way.

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References

Alexandre Mauroy, Y Susuki, and I Mezić. *The Koopman Operator in Systems and Control*. Springer, 2020.