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# $\beta^3$ -IRT: A New Item Response Model and its Applications - Supplementary Material

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## 1 Experiment results of abilities on CLUSTERS dataset

Table 1 shows the comparison between abilities and several popular classifier evaluation metrics on the experiments of CLUSTERS dataset. Table 2 gives the Spearman's rank correlation between these metrics in Table 1.

Table 1: Comparison between Ability and other Classifier Performance Metrics (CLUSTERS)

|      | Avg.   | Resp.  | Ability | Accuracy | F1     | Brier  | Log loss | AUC |
|------|--------|--------|---------|----------|--------|--------|----------|-----|
| DT   | 0.7113 | 0.7226 | 0.7175  | 0.7154   | 0.2456 | 1.0114 | 0.7596   |     |
| NB   | 0.7217 | 0.7388 | 0.75    | 0.7487   | 0.2221 | 1.0718 | 0.7682   |     |
| MLP  | 0.7195 | 0.7263 | 0.7375  | 0.7342   | 0.2233 | 1.0092 | 0.7652   |     |
| Ada. | 0.5478 | 0.4623 | 0.725   | 0.7277   | 0.2148 | 0.6204 | 0.7571   |     |
| RF   | 0.7206 | 0.7741 | 0.7275  | 0.7241   | 0.2304 | 5.7305 | 0.7648   |     |
| LDA  | 0.7251 | 0.7488 | 0.745   | 0.745    | 0.2244 | 1.1623 | 0.7683   |     |
| QDA  | 0.7255 | 0.7549 | 0.7475  | 0.7469   | 0.2242 | 1.1802 | 0.768    |     |
| LR   | 0.7017 | 0.6848 | 0.7375  | 0.7328   | 0.2141 | 0.8071 | 0.7677   |     |
| KNN  | 0.7212 | 0.7899 | 0.7325  | 0.7332   | 0.2389 | 6.8574 | 0.7582   |     |

Table 2: Spearman's Rank Correlation between Ability and other Classifier Performance Metrics (CLUSTERS)

|          | Avg.    | Resp.   | Ability | Accuracy | F1      | Brier   | Log loss | AUC |
|----------|---------|---------|---------|----------|---------|---------|----------|-----|
| Avg.     | 1.0     | 0.75    | 0.7197  | 0.7333   | -0.3167 | -0.7333 | 0.6667   |     |
| Ability  | 0.75    | 1.0     | 0.2678  | 0.2833   | -0.6    | -0.9833 | 0.1667   |     |
| Accuracy | 0.7197  | 0.2678  | 1.0     | 0.954    | 0.3766  | -0.1925 | 0.8619   |     |
| F1       | 0.7333  | 0.2833  | 0.954   | 1.0      | 0.3167  | -0.2    | 0.7333   |     |
| Brier    | -0.3167 | -0.6    | 0.3766  | 0.3167   | 1.0     | 0.6833  | 0.2167   |     |
| Log loss | -0.7333 | -0.9833 | -0.1925 | -0.2     | 0.6833  | 1.0     | -0.1333  |     |
| AUC      | 0.6667  | 0.1667  | 0.8619  | 0.7333   | 0.2167  | -0.1333 | 1.0      |     |

## 2 Complete experiment results of student answers

The Table 3 shows complete experiment results of student answers and details of datasets which has been described and partially shown in Section 4 of the main body of the paper.

Table 3: Student answer datasets (Log-loss) for continuous (student's average) and first attempt performance.

| course | students | questions | answers | continuous           |               | first attempts       |               |
|--------|----------|-----------|---------|----------------------|---------------|----------------------|---------------|
|        |          |           |         | $\beta^3$ -IRT       | 2PL-ND        | $\beta^3$ -IRT       | 2PL-ND        |
| 1      | 4460     | 190       | 8619    | <b>0.631 ± 0.003</b> | 0.713 ± 0.004 | <b>0.623 ± 0.004</b> | 0.699 ± 0.005 |
| 2      | 55001    | 532       | 423665  | <b>0.630 ± 0.022</b> | 0.972 ± 0.081 | <b>0.623 ± 0.023</b> | 0.953 ± 0.060 |
| 3      | 64261    | 663       | 313522  | <b>0.617 ± 0.004</b> | 0.695 ± 0.004 | <b>0.628 ± 0.024</b> | 0.760 ± 0.086 |
| 4      | 53152    | 959       | 466135  | <b>0.671 ± 0.004</b> | 0.742 ± 0.009 | <b>0.669 ± 0.004</b> | 0.731 ± 0.007 |
| 5      | 27269    | 438       | 484601  | <b>0.594 ± 0.004</b> | 0.692 ± 0.008 | <b>0.597 ± 0.004</b> | 0.696 ± 0.013 |
| 6      | 17431    | 354       | 140546  | <b>0.661 ± 0.009</b> | 0.899 ± 0.039 | <b>0.651 ± 0.009</b> | 0.892 ± 0.030 |
| 7      | 11765    | 366       | 45924   | <b>0.630 ± 0.007</b> | 0.795 ± 0.020 | <b>0.632 ± 0.007</b> | 0.791 ± 0.015 |
| 8      | 4121     | 134       | 8165    | <b>0.648 ± 0.014</b> | 0.941 ± 0.044 | <b>0.641 ± 0.023</b> | 0.967 ± 0.059 |
| 9      | 4389     | 259       | 16110   | <b>0.657 ± 0.011</b> | 0.941 ± 0.030 | <b>0.660 ± 0.011</b> | 0.931 ± 0.032 |
| 10     | 11331    | 265       | 117131  | <b>0.649 ± 0.007</b> | 0.847 ± 0.030 | <b>0.655 ± 0.009</b> | 0.841 ± 0.032 |
| 11     | 10071    | 240       | 31274   | <b>0.633 ± 0.016</b> | 0.889 ± 0.051 | <b>0.630 ± 0.012</b> | 0.891 ± 0.067 |
| 12     | 9089     | 65        | 37816   | <b>0.650 ± 0.013</b> | 0.938 ± 0.063 | <b>0.662 ± 0.016</b> | 0.883 ± 0.051 |
| 13     | 1130     | 3         | 1926    | <b>0.697 ± 0.066</b> | 1.002 ± 0.218 | <b>0.659 ± 0.086</b> | 1.023 ± 0.423 |
| 14     | 78700    | 1011      | 1090037 | <b>0.642 ± 0.028</b> | 0.936 ± 0.074 | <b>0.623 ± 0.028</b> | 0.909 ± 0.057 |
| 15     | 68250    | 931       | 866785  | <b>0.588 ± 0.002</b> | 0.650 ± 0.003 | <b>0.584 ± 0.003</b> | 0.642 ± 0.003 |
| 16     | 2621     | 175       | 4962    | <b>0.605 ± 0.002</b> | 0.674 ± 0.003 | <b>0.603 ± 0.002</b> | 0.663 ± 0.002 |
| 17     | 50386    | 902       | 764195  | <b>0.603 ± 0.002</b> | 0.665 ± 0.003 | <b>0.596 ± 0.003</b> | 0.657 ± 0.003 |
| 18     | 28952    | 463       | 266205  | <b>0.598 ± 0.006</b> | 0.725 ± 0.008 | <b>0.608 ± 0.005</b> | 0.729 ± 0.011 |
| 19     | 8190     | 24        | 28780   | <b>0.651 ± 0.015</b> | 0.923 ± 0.064 | <b>0.644 ± 0.020</b> | 0.934 ± 0.074 |
| 20     | 3342     | 118       | 7857    | <b>0.640 ± 0.021</b> | 0.959 ± 0.060 | <b>0.636 ± 0.018</b> | 0.933 ± 0.040 |
| 21     | 1819     | 50        | 14896   | <b>0.639 ± 0.016</b> | 0.949 ± 0.072 | <b>0.650 ± 0.014</b> | 0.968 ± 0.094 |
| 22     | 7432     | 208       | 15351   | <b>0.629 ± 0.020</b> | 0.935 ± 0.050 | <b>0.622 ± 0.016</b> | 0.931 ± 0.060 |
| 23     | 57034    | 442       | 405037  | <b>0.602 ± 0.004</b> | 0.692 ± 0.011 | <b>0.609 ± 0.004</b> | 0.682 ± 0.005 |
| 24     | 4466     | 160       | 12839   | <b>0.657 ± 0.014</b> | 0.950 ± 0.046 | <b>0.652 ± 0.011</b> | 0.950 ± 0.044 |
| 25     | 4317     | 199       | 12842   | <b>0.642 ± 0.015</b> | 0.917 ± 0.034 | <b>0.627 ± 0.010</b> | 0.871 ± 0.038 |
| 26     | 2288     | 59        | 24226   | <b>0.572 ± 0.011</b> | 0.836 ± 0.045 | <b>0.593 ± 0.014</b> | 0.874 ± 0.038 |
| 27     | 2169     | 56        | 4501    | <b>0.662 ± 0.022</b> | 0.998 ± 0.093 | <b>0.647 ± 0.028</b> | 0.971 ± 0.073 |
| 28     | 156676   | 4475      | 2737867 | <b>0.603 ± 0.001</b> | 0.647 ± 0.002 | <b>0.603 ± 0.001</b> | 0.645 ± 0.002 |
| 29     | 1274     | 31        | 16865   | <b>0.553 ± 0.021</b> | 0.916 ± 0.056 | <b>0.558 ± 0.017</b> | 0.856 ± 0.051 |
| 30     | 2918     | 214       | 6480    | <b>0.646 ± 0.019</b> | 1.001 ± 0.075 | <b>0.647 ± 0.019</b> | 0.979 ± 0.048 |
| 31     | 9894     | 42        | 34277   | <b>0.647 ± 0.014</b> | 0.911 ± 0.060 | <b>0.634 ± 0.014</b> | 0.918 ± 0.053 |
| 32     | 158871   | 2529      | 2446221 | <b>0.578 ± 0.001</b> | 0.627 ± 0.001 | <b>0.578 ± 0.002</b> | 0.626 ± 0.002 |
| 33     | 2527     | 93        | 4989    | <b>0.663 ± 0.021</b> | 0.929 ± 0.060 | <b>0.674 ± 0.025</b> | 0.993 ± 0.074 |