Supplemental Material: A kernel-based goodness-of-fit test for censored data.

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1 Proofs

Proof (Proposition 3.1) : Let $T = \min\{X, C\}$ and $\Delta = \mathbb{1}\{T = X\}$ with $X \sim F$ independent of $C \sim G$. Let F_0 be a continuous distribution on $\mathbb{R}$ and define $U = F_0(T)$, then the joint distribution Q of the pair (U, Δ) is given by

$$\begin{aligned}
\mathbb{P}(U \leq u, \Delta = 1) &= \mathbb{P}(\min\{F_0(X), F_0(C)\} \leq u, F_0(X) \leq F_0(C)) \\
&= \mathbb{P}(F_0(X) \leq u, F_0(X) \leq F_0(C)) \\
&= \mathbb{P}(X \leq F_0^{-1}(u), X \leq C) \\
&= \int_0^{F_0^{-1}(u)} \mathbb{P}(x \leq C) dF(x) \\
&= \int_0^{F_0^{-1}(u)} [1 - G(x)] dF(x) \\
&= \int_0^u [1 - G(F_0^{-1}(x))] dF(F_0^{-1}(x)), \qquad (1)
\end{aligned}$$

and

$$\begin{aligned}
\mathbb{P}(U \leq u, \Delta = 0) &= \mathbb{P}(\min\{F_0(X), F_0(C)\} \leq u, F_0(X) > F_0(C)) \\
&= \mathbb{P}(F_0(C) \leq u, F_0(X) > F_0(C)) \\
&= \mathbb{P}(C \leq F_0^{-1}(u), X > C) \\
&= \int_0^{F_0^{-1}(u)} \mathbb{P}(X > c) dG(c) \\
&= \int_0^{F_0^{-1}(u)} [1 - F(c)] dG(c) \\
&= \int_0^u [1 - F(F_0^{-1}(c))] dG(F_0^{-1}(c)), \qquad (2)
\end{aligned}$$

Let $u \in [0, 1]$ be fixed, we define the random variable

$$Z^u(U, \Delta) = \mathbb{1}\{U \leq u\}\Delta + \mathbb{1}\{U \leq u\}(1 - \Delta)\frac{u - U}{1 - U}.$$

By the strong law of large numbers, it holds

$$\hat{F}(u) = \frac{1}{n}\sum_{i=1}^{n} Z_i^u \overset{a.s.}{\to} \mathbb{E}(Z^u),$$

where (computed by using the joint distribution Q of the pair (U, Δ))

$$\begin{aligned}\mathbb{E}(Z^u) &= \int_0^u (1 - GF_0^{-1}(s))dFF_0^{-1}(s) + \int_0^u \frac{u-s}{1-s}(1 - FF_0^{-1}(s))dGF_0^{-1}(s) \\ &= FF_0^{-1}(u) - \left[FF_0^{-1}(u) - \int_0^u FF_0^{-1}(s)dGF_0^{-1}(s)\right] + \int_0^u \frac{u-s}{1-s}(1 - FF_0^{-1}(s))dGF_0^{-1}(s) \\ &= FF_0^{-1}(u) + \int_0^u \frac{1}{1-s}[(1-s)(1 - FF_0^{-1}(u)) - (1-u)(1 - FF_0^{-1}(s))]dGF_0^{-1}(s),\end{aligned}$$

(the second equality follows from integration by parts) which is an unbiased estimator of FF_0^{-1} when $F = F_0$, in which case $E(Z^u) = u$. Observe that in the case of extreme censoring, for example G is delta measure on zero, $\mathbb{E}(Z^u) = u$ which reflects our lack of information. ■

Proof (Lemma 4.4) : By the main theorem of section 5.2.2. of [29], it suffices to prove that $\mathbb{E}(J(U, \Delta), (U', \Delta')^2) < \infty$, where $(U, \Delta), (U', \Delta') \overset{i.i.d.}{\sim} Q$ and that the kernel J is degenerated.

Degeneracy: For the degeneracy, note that

$$\begin{aligned}\mathbb{E}[J((u, \delta), (U, \Delta))] &= \mathbb{E}\left(\int_0^1 \int_0^1 K(x, y)(dx - dh_{u,\delta}(x))(dy - dh_{U,\Delta}(y))\right) \\ &= \mathbb{E}\left(\int_0^1 \psi_{u,\delta}(y)(dy - dh_{U,\Delta}(y))\right),\end{aligned}$$

where $\psi_{u,\delta}(y) = \int_0^1 K(x, y)(dx - dh_{u,\delta}(x))$. By using equation (3), it holds

$$\begin{aligned}\mathbb{E}[J((u, \delta), (U, \Delta))] &= \int_0^1 \psi_{u,\delta}(y)dy - \mathbb{E}\left(\int_U^1 \psi_{u,\delta}(y)\frac{1-\Delta}{1-U}dy + \Delta\psi_{u,\delta}(U)\right) \\ &= \int_0^1 \psi_{u,\delta}(y)dy - \int_0^1 \int_x^1 \psi_{u,\delta}(y)dydG(x) - \int_0^1 (1 - G(x))\psi_{u,\delta}(x)dx \\ &= \int_0^1 \psi_{u,\delta}(y)dy - \int_0^1 G(y)\psi_{u,\delta}(y)dy - \int_0^1 (1 - G(x))\psi_{u,\delta}(x)dx = 0.\end{aligned}$$

$\mathbb{E}(\boldsymbol{J^2}) < \infty$ **:** We continue by checking the finite variance condition, that is $\mathbb{E}(J((U, \Delta), (U', \Delta'))^2) < \infty$. Under assumption 4.1, observe that

$$\begin{aligned}\mathbb{E}(J((U, \Delta), (U', \Delta'))^2) &= \mathbb{E}\left(\left[\int_0^1 \int_0^1 K(x, y)(dx - dh_{U,\Delta}(x))(dy - dh_{U',\Delta'}(y))\right]^2\right) \\ &\leq M^2\left(1 + \mathbb{E}\left(\left(\int_0^1 dh_{U,\Delta}(x)\right)^4\right)\right) \\ &\leq 2M^2, \end{aligned} \tag{3}$$

since $h_{U,\Delta}(x)$ is a cumulative distribution function.

Diagonal: We finalize by analysing the asymptotic behaviour of the diagonal term. Under assumption 4.1, it holds

$$\begin{aligned}\mathbb{E}(J((U, \Delta), (U, \Delta))) &= \mathbb{E}\left(\int_0^1 \int_0^1 K(x, y)(dx - dh_{U,\Delta}(x))(dy - dh_{U,\Delta}(y))\right) \\ &\leq M\left(1 + \mathbb{E}\left(\left(\int_0^1 dh_{U,\Delta}(x)\right)^2\right)\right), \\ &\leq 2M \end{aligned} \tag{4}$$

since $h_{U,\Delta}(x)$ is a distribution function. Then by the strong law of large numbers, the diagonal of the V-statistic converges to

$$n\text{Diag} = \frac{1}{n}\sum_{i=1}^n J((U_i, \Delta_i), (U_i, \Delta_i)) \overset{a.s.}{\to} \mathbb{E}(J((U, \Delta), (U, \Delta))). \tag{5}$$

■

Proof (of Proposition 4.3) : Equation (5) follows easily from equation (3). The unbiasedness of the U-statistic in equation (6) follows from the degeneracy property (proved in Lemma 4.4). ■

1.1 Proof of Proposition 4.2

In this section, instead of proving Proposition 4.2, we prove a even stronger result. Suppose that each data point i generates a (random) probability measure α_i in $[0,1]$ and suppose all these points are independent and thus the measures they represent are also independent. We define the measure α as the expected measure of α_i, i.e. for $A \subseteq [0,1]$ measurable, we define $\alpha(A) = \mathbb{E}(\alpha_i(A))$. In our setting, under the null it holds $\alpha_i([0,x)) = \mathbb{1}\{U < x\}\left(\Delta_i + (1-\Delta)\frac{x-U}{1-U}\right)$, and $\alpha([0,x)) = \mathbb{E}(\alpha_i([0,x))) = x$ for $x \in [0,1]$. Our estimator $\tilde{F}(x)$ corresponds to $n^{-1}\sum_{i=1}\alpha_i([0,x))$. We prove the following theorem.

Theorem 1.1. *Let $K : [0,1]^2 \to \mathbb{R}$ be a kernel such that it exist $M \geq 1$ with $|K(x,y)| \leq M$ for all x, y. Then*

$$\mathbb{E}\left(MMD\left(\frac{1}{n}\sum_i \alpha_i(\cdot), \alpha(\cdot)\right)^2\right) \leq \frac{4M}{n} \tag{6}$$

and, moreover, for all $\varepsilon > 0$ and $n \geq 2$ it holds

$$\mathbb{P}\left(\left|MMD\left(\frac{1}{n}\sum_i \alpha_i, \alpha\right)^2 - \mathbb{E}\left(MMD\left(\frac{1}{n}\sum_i \alpha_i, \alpha\right)\right)^2\right| > \varepsilon\right) \leq \exp\left(-\varepsilon^2 n/(32M^2)\right) \tag{7}$$

Proof:

Denote the signed measure $\beta_i = (\alpha_i - \alpha)$, and for shortness, denote $Z = MMD\left(\frac{1}{n}\sum_i \alpha_i, \alpha\right)$, then

$$Z^2 = \frac{1}{n^2}\int_{[0,1]^2} K(x,y)\sum_{i,j}\beta_i(dx)\beta_j(dx) = \frac{1}{n^2}\sum_{i,j}\int_{[0,1]^2} K(x,y)\beta_i(dx)\beta_j(dx) \tag{8}$$

Using that $\mathbb{E}(Z) \leq \mathbb{E}(Z^2)^{1/2}$, we get

$$\mathbb{E}Z \leq \frac{n-1}{n}\mathbb{E}\int_{[0,1]^2} K(x,y)\beta_1(dx)\beta_2(dy) + \frac{1}{n}\mathbb{E}\int_{[0,1]^2} K(x,y)\beta_1(dx)\beta_1(dy) \tag{9}$$

Note that $|\beta_1(dx)| \leq \alpha_1(dx) + \alpha(dx)$, then

$$\mathbb{E}\int_{[0,1]^2} K(x,y)\beta_1(dx)\beta_1(dy) \leq M\mathbb{E}\int_{[0,1]^2}(\alpha_1(dx) + \alpha(x))(\alpha_1(dx) + \alpha(x)) = 4M \tag{10}$$

Now, we claim that

$$\mathbb{E}\int_{[0,1]^2} K(x,y)\beta_1(dx)\beta_2(dy) = 0 \tag{11}$$

as β_1 and β_2 are i.i.d measures, and for all measurable sets $A \subseteq [0,1]$ it holds $\mathbb{E}(\beta_1(A)) = 0$. To check equation (11) we suppose that $K(x,y) = \sum_{k=1}^N c_k \int_{S_k}(x,y)$ where S_k are rectangles in $[0,1]^2$, i.e. K is a simple function in $[0,1]^2$. For a rectangle $S = [x,x'] \times [y,y'] \in [0,1]^2$ we denote $S^1 = [x,x']$ and $S^2 = [y,y']$. Then

$$\mathbb{E}\int_{[0,1]^2} K(x,y)\beta_1(dx)\beta_2(dy) = \sum_{k=1}^N c_k\mathbb{E}\beta_1(S_k^1)\mathbb{E}\beta_2(S_k^2) = 0$$

then as any arbitrary K can be approximated by simple functions, as our kernel K is bounded by M, and β is the difference of two probability measures, by the dominated convergence theorem equation (11) holds for an arbitrary bounded kernel. This proves the first part of the theorem.

For the second part, i.e. concentration. Let Z' be the random variable Z but replacing the data point j by another j' (which is independent of everything). From equation (8) it holds that $Z^2 - Z'^2$ equals

$$\frac{1}{n^2}\int_{[0,1]^2} K(x,y)[\beta_j(dx)\beta_j(dy) - \beta_{j'}(dx)\beta_{j'}(dy)] + \frac{1}{n^2}\sum_{i\neq j}\int_{[0,1]^2} K(x,y)\beta_i(dx)(\beta_j(dy) - \beta'_j(dy))$$

Using the same argument as equation (10) the the absolute value of the above sum is less or equal than $8M/n$. By the McDiarmid inequality we obtain the result. ■

2 Kernel-based test

In this section, we show results for an extra competitor based on a kernel approach, [4] (see reference in the main file). In particular, this approach considers a kernel density estimate for the survival function, i.e., for $S = 1 - F$ which is obtained by using a slightly modified Kaplan-Meier procedure. Then, the test-statistic is defined as the squared difference between this density estimate and the model density. The implementation of this test was directly derived from the code made available by the authors in [4] (see reference in the main file).

Since this procedure relies on density estimation as an intermediate step, it has been found to be relatively data-inefficient, compared with more direct tests, see e.g. the recent discussion in [13] (in the main file).

For instance, for the periodic hazard experiment and under censoring parameter $\gamma = 1/2$, we obtain a fairly correct estimation of the Type-I error (the null is recovered by considering $\theta_2 = 0$ in the model).

	Type-I error		
α	n=30	n=50	n=100
10 %	11.25	8.65	7
5 %	5.35	4.65	3.6
1 %	1.3	1.1	0.9

Nevertheless for alternatives that are distinguishable by other tests as for example for the periodic hazard setting with censoring parameter $\gamma = 1/2$ and $\theta = (\theta_1, 1)$ and $\theta_1 \in \{0.5, 0.9, 1.5\}$, we obtain the following results

	Power		
$\alpha = 5\%$	n=30	n=50	n=100
$\theta = (0.5, 1)$	22.4	43.4	73.45
$\theta = (0.9, 1)$	3.65	4.95	14.1
$\theta = (1.5, 1)$	3.35	5.4	10.4

From the table above, we observe that the power increases as the sample size increases, nevertheless the power attained by this particular test is clearly inferior compared to all the other competitors. This behaviour is also observed for the parallel and Weibull experiments. Therefore, we omit this test from our comparisons.

3 Proportional hazards experiment

3.1 Type-I error

Estimated Type-I error. In red we observe tests that have an clear incorrect level. In orange, we observe tests that have a questionable incorrect level

3.1.1 Parallel hazards experiment: Censoring 30%

Fixed length-scale 1

	Type-I error						
	MW1	MW2	MW3	Pearson	LR1	LR2	WLR
Sample size n=30							
10 %	10.10	11.70	10.20	14.50	11.15	11.10	19.65
5 %	5.10	6.55	4.85	8.70	5.90	6.60	13.65
1 %	1.15	2.05	1.10	3.05	1.60	1.30	6.30
Sample size n=50							
10 %	10.65	11.70	10.75	13.35	11.30	10.45	17.80
5 %	5.50	6.20	5.75	8.15	6.10	5.80	11.85
1 %	1.20	1.60	1.10	2.65	1.45	1.35	4.60
Sample size n=100							
10 %	11.10	11.60	10.90	11.00	10.95	11.10	13.00
5 %	5.90	6.10	5.75	6.40	5.85	6.00	8.40
1 %	1.25	1.40	1.30	2.20	1.35	1.35	2.85

Adaptive length-scale

Type-I error							
	MW1	MW2	MW3	Pearson	LR1	LR2	WLR
Sample size n=30							
10 %	9.35	10.70	9.70	14.50	11.15	11.10	19.65
5 %	4.40	5.45	4.50	8.70	5.90	6.60	13.65
1 %	0.95	1.45	0.95	3.05	1.60	1.30	6.30
Sample size n=50							
10 %	10.50	11.30	10.55	13.35	11.30	10.45	17.80
5 %	5.35	5.85	5.15	8.15	6.10	5.80	11.85
1 %	0.70	1.20	0.85	2.65	1.45	1.35	4.60
Sample size n=100							
10 %	10.65	10.90	10.70	11.00	10.95	11.10	13.00
5 %	5.50	5.45	5.50	6.40	5.85	6.00	8.40
1 %	1.20	1.40	1.15	2.20	1.35	1.35	2.85

3.1.2 Parallel hazards experiment: Censoring 50%

Fixed length-scale 1

Type-I error							
	MW1	MW2	MW3	Pearson	LR1	LR2	WLR
Sample size n=30							
10 %	10.50	11.80	10.50	15.30	10.50	10.25	19.25
5 %	4.45	5.90	4.50	10.15	5.75	5.70	14.05
1 %	1.05	1.65	1.05	5.30	1.30	1.65	6.35
Sample size n=50							
10 %	10.70	11.45	10.75	14.75	10.65	10.50	16.25
5 %	5.55	6.50	5.70	9.30	5.90	5.10	10.90
1 %	1.35	1.45	1.20	5.10	1.60	1.35	4.40
Sample size n=100							
10 %	12.20	12.70	12.20	13.25	11.45	11.65	14.10
5 %	6.15	6.35	6.00	8.70	5.85	5.80	8.25
1 %	1.45	1.65	1.45	2.20	1.35	1.30	3.15

Adaptive length-scale

Type-I error							
	MW1	MW2	MW3	Pearson	LR1	LR2	WLR
Sample size n=30							
10 %	8.65	10.05	8.40	15.30	10.50	10.25	19.25
5 %	4.55	5.25	4.50	10.15	5.75	5.70	14.05
1 %	1.05	1.30	0.95	5.30	1.30	1.65	6.35
Sample size n=50							
10 %	9.45	10.25	9.40	14.75	10.65	10.50	16.25
5 %	4.85	5.50	4.80	9.30	5.90	5.10	10.90
1 %	1.20	1.40	1.20	5.10	1.60	1.35	4.40
Sample size n=100							
10 %	11.00	11.25	11.05	13.25	11.45	11.65	14.10
5 %	5.40	5.60	5.25	8.70	5.85	5.80	8.25
1 %	1.35	1.50	1.40	2.20	1.35	1.30	3.15

3.2 Parallel hazard functions - Sample size $n = 30$ - Significance $\alpha = 0.05$ - Fixed length-scale 1

rate	MW1		MW2		MW3		Pearson		LR1		LR2		WLR	
cens	30%	50%	30%	50%	30%	50%	30%	50%	30%	50%	30%	50%	30%	50%
0.5	74.30	67.45	78.15	72.05	74.40	67.60	64.90	45.20	88.20	77.45	78.40	65.40	62.70	42.80
0.55	62.55	55.40	66.55	59.75	62.60	55.75	47.70	30.80	77.10	63.25	65.40	51.95	46.15	29.35
0.6	50.30	44.70	55.05	49.15	50.85	44.70	30.80	19.30	62.95	49.85	51.65	37.70	31.30	20.30
0.65	39.00	33.35	43.50	37.80	39.15	33.45	19.40	12.10	48.35	35.65	38.25	26.30	22.30	13.80
0.7	28.20	25.20	32.55	28.55	27.90	25.35	11.85	8.90	34.40	24.65	25.20	18.25	15.85	10.50
0.75	19.35	18.15	22.55	20.80	19.45	17.65	8.30	6.55	22.45	15.90	15.85	12.35	12.60	9.45
0.8	12.90	12.55	15.40	14.95	13.00	12.90	6.30	6.25	14.10	10.15	9.80	8.25	10.20	9.25
0.85	8.50	9.45	10.80	11.65	8.15	9.65	5.60	6.85	9.35	7.20	6.80	6.35	10.15	9.65
0.9	5.90	6.90	7.45	8.85	5.95	6.90	5.85	8.05	6.80	5.10	5.30	4.65	10.20	11.45
0.95	5.25	5.55	6.35	6.90	5.10	5.35	7.10	8.75	5.45	5.10	5.70	4.70	12.00	12.55
1	5.10	4.45	6.55	5.90	4.85	4.50	8.70	10.15	5.90	5.75	6.60	5.70	13.65	14.05
1.05	5.90	4.60	7.45	6.55	5.80	4.90	10.55	12.05	7.65	6.90	7.90	6.75	16.20	15.55
1.1	7.65	5.95	9.25	7.80	7.60	6.05	13.05	15.10	10.25	9.00	9.85	8.75	18.80	17.95
1.15	9.30	7.35	11.30	8.75	9.10	7.30	16.40	17.50	12.90	11.80	13.05	10.95	22.25	20.55
1.2	11.85	9.15	14.60	11.75	12.25	9.00	19.80	19.70	16.55	15.10	15.85	14.45	25.60	23.80
1.25	14.95	11.65	17.45	14.25	15.05	11.40	23.50	21.75	21.15	18.30	19.65	16.90	29.20	27.20
1.3	18.00	14.25	21.40	16.45	18.15	14.15	26.55	24.55	27.55	22.85	24.05	19.65	33.45	29.80
1.35	22.25	16.90	26.50	19.95	22.15	16.75	32.20	26.80	32.80	26.20	28.65	23.15	38.30	32.55
1.4	27.15	19.80	31.65	23.85	27.05	19.75	35.20	30.00	38.25	31.00	33.20	26.60	42.05	36.00
1.45	32.10	23.45	37.10	27.65	32.00	23.15	39.35	32.20	44.15	34.95	38.35	30.70	46.55	38.95
1.5	38.05	26.90	42.05	31.40	37.40	27.15	44.65	34.55	50.25	39.60	42.05	34.35	51.80	42.15
1.55	43.20	30.35	47.25	35.25	43.20	30.70	48.55	38.40	56.10	44.65	46.85	37.55	55.70	45.55
1.6	47.90	34.25	53.50	38.15	48.15	34.05	53.15	40.50	60.90	48.50	51.90	41.55	60.60	49.15
1.65	53.00	37.25	58.00	41.60	53.45	37.05	56.90	44.30	66.15	52.50	56.00	44.95	64.95	52.30
1.7	57.55	40.30	63.15	46.25	58.25	41.30	62.05	47.70	70.00	55.70	60.45	48.85	69.00	55.25
1.75	63.30	44.40	67.55	49.80	63.00	44.70	65.60	50.55	73.95	60.60	63.75	52.10	72.00	59.10
1.8	67.60	47.90	72.00	53.60	67.10	47.80	69.55	53.85	78.50	64.10	67.50	54.45	75.95	61.65
1.85	71.50	51.10	75.25	56.55	71.50	51.30	72.90	56.95	81.85	67.20	71.45	57.60	79.05	64.50
1.9	75.00	54.40	79.00	59.95	74.65	54.65	75.65	59.20	84.60	70.05	74.50	60.35	81.70	67.15
1.95	77.65	57.55	81.65	63.10	77.95	57.25	78.20	61.30	86.95	73.10	78.20	62.95	84.20	69.50
2	80.90	61.20	84.55	66.20	81.10	60.55	81.35	64.20	89.05	75.55	81.70	65.50	86.50	72.00

3.3 Parallel hazard functions - Sample size $n = 30$ - Significance $\alpha = 0.05$ - Adaptive length-scale

rate	MW1		MW2		MW3		Pearson		LR1		LR2		WLR	
cens	30%	50%	30%	50%	30%	50%	30%	50%	30%	50%	30%	50%	30%	50%
0.5	70.65	63.75	73.85	66.95	70.60	63.45	64.90	45.20	88.20	77.45	78.40	65.40	62.70	42.80
0.55	57.80	51.80	61.80	55.60	57.20	51.90	47.70	30.80	77.10	63.25	65.40	51.95	46.15	29.35
0.6	45.05	39.40	49.15	43.45	45.10	39.15	30.80	19.30	62.95	49.85	51.65	37.70	31.30	20.30
0.65	33.45	29.40	37.35	32.45	33.20	29.40	19.40	12.10	48.35	35.65	38.25	26.30	22.30	13.80
0.7	23.15	21.30	25.95	24.30	23.25	21.25	11.85	8.90	34.40	24.65	25.20	18.25	15.85	10.50
0.75	16.05	15.90	18.65	17.60	15.90	15.25	8.30	6.55	22.45	15.90	15.85	12.35	12.60	9.45
0.8	10.80	11.00	12.95	13.20	10.65	11.35	6.30	6.25	14.10	10.15	9.80	8.25	10.20	9.25
0.85	7.00	7.60	8.90	9.10	7.40	7.70	5.60	6.85	9.35	7.20	6.80	6.35	10.15	9.65
0.9	5.55	5.70	6.55	6.95	5.55	5.40	5.85	8.05	6.80	5.10	5.30	4.65	10.20	11.45
0.95	4.75	4.75	5.65	5.50	4.70	4.80	7.10	8.75	5.45	5.10	5.70	4.70	12.00	12.55
1	4.40	4.55	5.45	5.25	4.50	4.50	8.70	10.15	5.90	5.75	6.60	5.70	13.65	14.05
1.05	4.85	4.25	5.80	5.05	4.75	4.35	10.55	12.05	7.65	6.90	7.90	6.75	16.20	15.55
1.1	6.00	4.70	7.50	5.30	5.90	4.75	13.05	15.10	10.25	9.00	9.85	8.75	18.80	17.95
1.15	7.35	5.20	9.10	6.20	7.30	5.00	16.40	17.50	12.90	11.80	13.05	10.95	22.25	20.55
1.2	9.15	6.30	11.00	7.80	9.10	6.20	19.80	19.70	16.55	15.10	15.85	14.45	25.60	23.80
1.25	11.25	8.00	12.95	9.45	10.95	7.90	23.50	21.75	21.15	18.30	19.65	16.90	29.20	27.20
1.3	13.40	9.85	15.75	11.40	13.30	9.45	26.55	24.55	27.55	22.85	24.05	19.65	33.45	29.80
1.35	16.60	11.25	18.85	13.50	16.35	11.75	32.20	26.80	32.80	26.20	28.65	23.15	38.30	32.55
1.4	19.15	13.55	21.35	15.65	19.20	13.55	35.20	30.00	38.25	31.00	33.20	26.60	42.05	36.00
1.45	22.20	15.65	25.85	18.55	21.95	15.65	39.35	32.20	44.15	34.95	38.35	30.70	46.55	38.95
1.5	26.35	17.80	29.30	21.05	26.20	17.90	44.65	34.55	50.25	39.60	42.05	34.35	51.80	42.15
1.55	30.00	20.85	34.25	23.35	30.00	20.70	48.55	38.40	56.10	44.65	46.85	37.55	55.70	45.55
1.6	34.50	23.25	38.45	25.90	34.35	23.40	53.15	40.50	60.90	48.50	51.90	41.55	60.60	49.15
1.65	38.70	26.25	42.55	28.95	39.00	25.45	56.90	44.30	66.15	52.50	56.00	44.95	64.95	52.30
1.7	43.15	28.75	47.15	31.85	43.25	28.85	62.05	47.70	70.00	55.70	60.45	48.85	69.00	55.25
1.75	47.80	31.50	51.80	35.15	47.40	31.45	65.60	50.55	73.95	60.60	63.75	52.10	72.00	59.10
1.8	51.80	34.40	55.95	37.90	51.80	34.30	69.55	53.85	78.50	64.10	67.50	54.45	75.95	61.65
1.85	56.25	37.10	59.65	40.45	55.70	37.15	72.90	56.95	81.85	67.20	71.45	57.60	79.05	64.50
1.9	60.10	39.50	63.55	42.50	59.80	39.30	75.65	59.20	84.60	70.05	74.50	60.35	81.70	67.15
1.95	63.75	41.95	67.15	45.40	63.90	41.65	78.20	61.30	86.95	73.10	78.20	62.95	84.20	69.50
2	67.00	44.40	70.35	48.65	67.50	44.80	81.35	64.20	89.05	75.55	81.70	65.50	86.50	72.00

3.3.1 Parallel hazard functions - Sample size $n = 50$ - Significance $\alpha = 0.05$ - Fixed length-scale 1

rate	MW1		MW2		MW3		Pearson		LR1		LR2		WLR	
cens	30%	50%	30%	50%	30%	50%	30%	50%	30%	50%	30%	50%	30%	50%
0.5	93.65	89.70	94.15	90.80	93.80	89.70	89.70	75.80	97.70	94.45	94.35	88.20	88.90	74.95
0.55	85.55	79.30	87.55	81.25	86.15	79.25	77.05	58.85	94.15	85.95	87.35	76.15	76.00	57.00
0.6	73.10	65.45	76.15	67.85	73.85	65.40	60.25	41.55	84.30	71.95	74.60	59.70	59.50	39.85
0.65	59.40	51.45	62.30	54.00	59.90	51.30	42.75	26.70	70.05	57.70	59.60	44.45	40.75	26.40
0.7	44.75	38.80	47.70	41.05	44.05	38.65	27.35	16.65	54.50	40.90	42.25	31.90	27.20	17.20
0.75	32.40	28.25	34.70	30.40	32.30	28.05	16.55	10.75	37.30	27.30	28.80	21.05	17.85	12.40
0.8	21.20	18.70	23.95	20.45	21.25	18.85	11.00	7.90	24.15	17.50	18.05	13.35	12.65	9.00
0.85	14.25	12.60	15.80	14.30	14.20	12.55	7.20	6.85	14.95	11.05	12.05	8.25	9.80	7.70
0.9	9.00	8.25	10.30	9.25	8.80	8.60	5.50	6.60	9.55	7.25	7.10	5.75	9.50	8.20
0.95	6.40	6.20	7.20	6.95	6.25	6.20	6.30	7.80	6.35	5.85	5.75	5.05	10.10	9.20
1	5.50	5.55	6.20	6.50	5.75	5.70	8.15	9.30	6.10	5.90	5.80	5.10	11.85	10.90
1.05	6.05	6.15	7.15	6.85	6.10	6.05	10.10	10.95	8.05	7.30	6.80	6.75	14.30	12.85
1.1	8.25	6.80	9.45	8.05	8.15	6.75	13.10	13.90	11.90	9.70	9.65	8.90	17.60	16.20
1.15	11.95	9.45	13.30	10.75	11.70	9.50	16.25	16.50	16.60	13.75	13.95	11.85	21.85	19.60
1.2	16.20	12.45	18.30	13.65	16.35	12.60	20.95	20.05	23.75	18.55	18.90	15.35	26.80	22.35
1.25	22.50	16.40	24.35	17.95	22.20	16.40	27.55	23.75	30.35	23.90	25.30	19.75	32.30	26.05
1.3	28.80	20.75	31.25	22.95	28.95	20.80	32.45	28.30	37.20	29.75	31.60	24.85	37.85	30.15
1.35	35.40	26.20	37.85	28.80	35.30	26.15	38.65	32.05	44.90	36.35	38.00	29.50	44.20	35.85
1.4	42.00	31.50	45.85	34.60	42.30	31.60	46.05	36.40	53.95	41.85	43.70	35.95	49.70	40.75
1.45	50.55	36.65	53.25	39.35	50.75	36.50	51.50	39.55	61.35	48.15	50.20	40.50	56.05	44.55
1.5	58.05	42.55	60.80	45.40	58.00	42.65	57.40	44.70	68.60	54.20	57.55	45.00	61.30	50.05
1.55	64.85	48.15	67.45	50.85	64.60	47.90	62.70	49.65	74.30	59.75	63.05	50.85	67.45	55.65
1.6	70.80	53.75	73.25	56.70	70.90	53.75	68.30	54.65	78.90	65.85	68.60	55.20	72.65	60.20
1.65	75.95	58.75	77.70	61.60	76.10	58.75	72.85	58.65	83.40	70.90	73.60	59.25	77.60	64.60
1.7	80.55	63.55	82.05	66.40	80.30	63.35	77.60	62.95	87.10	75.00	77.90	63.70	82.05	68.95
1.75	83.55	67.90	84.95	71.30	83.55	68.45	82.05	66.30	90.10	78.70	81.95	67.00	85.90	72.40
1.8	86.90	73.05	88.90	74.95	86.90	72.55	85.60	70.00	92.55	81.95	85.00	70.55	88.60	76.00
1.85	90.05	76.10	91.30	77.80	90.15	75.70	88.40	73.05	94.55	84.65	87.75	74.55	90.10	79.10
1.9	92.30	78.80	93.10	80.70	92.10	78.75	91.10	76.65	95.80	87.35	90.20	78.10	92.55	81.65
1.95	93.80	81.90	94.65	84.20	93.70	82.00	92.70	79.35	97.10	89.05	92.15	80.75	94.20	84.65
2	95.10	84.20	95.95	86.10	95.20	84.30	94.35	81.90	98.05	90.45	93.70	83.40	95.50	86.40

3.4 Parallel hazard functions - Sample size $n = 50$ - Significance $\alpha = 0.05$ - Adaptive length-scale

rate	MW1		MW2		MW3		Pearson		LR1		LR2		WLR	
cens	30%	50%	30%	50%	30%	50%	30%	50%	30%	50%	30%	50%	30%	50%
0.5	91.45	86.75	92.50	87.80	91.55	86.55	89.70	75.80	97.70	94.45	94.35	88.20	88.90	74.95
0.55	82.50	75.60	84.30	77.85	82.90	75.60	77.05	58.85	94.15	85.95	87.35	76.15	76.00	57.00
0.6	69.60	61.30	71.25	63.80	69.60	61.05	60.25	41.55	84.30	71.95	74.60	59.70	59.50	39.85
0.65	54.35	47.15	56.65	48.85	53.95	47.25	42.75	26.70	70.05	57.70	59.60	44.45	40.75	26.40
0.7	39.95	34.60	42.10	36.80	40.30	34.35	27.35	16.65	54.50	40.90	42.25	31.90	27.20	17.20
0.75	28.30	23.80	30.30	25.80	28.45	24.15	16.55	10.75	37.30	27.30	28.80	21.05	17.85	12.40
0.8	18.75	16.45	20.45	17.60	18.70	16.55	11.00	7.90	24.15	17.50	18.05	13.35	12.65	9.00
0.85	12.25	10.90	14.00	11.75	12.35	10.90	7.20	6.85	14.95	11.05	12.05	8.25	9.80	7.70
0.9	7.45	7.35	8.75	8.00	7.70	7.20	5.50	6.60	9.55	7.25	7.10	5.75	9.50	8.20
0.95	6.00	5.55	6.85	6.00	5.95	5.65	6.30	7.80	6.35	5.85	5.75	5.05	10.10	9.20
1	5.35	4.85	5.85	5.50	5.15	4.80	8.15	9.30	6.10	5.90	5.80	5.10	11.85	10.90
1.05	5.40	5.00	6.15	5.50	5.35	5.10	10.10	10.95	8.05	7.30	6.80	6.75	14.30	12.85
1.1	7.20	5.80	7.90	6.35	6.85	5.60	13.10	13.90	11.90	9.70	9.65	8.90	17.60	16.20
1.15	9.85	7.45	10.70	8.05	9.45	7.35	16.25	16.50	16.60	13.75	13.95	11.85	21.85	19.60
1.2	12.30	9.40	13.10	10.45	12.20	9.35	20.95	20.05	23.75	18.55	18.90	15.35	26.80	22.35
1.25	15.80	12.20	17.25	13.50	15.80	12.40	27.55	23.75	30.35	23.90	25.30	19.75	32.30	26.05
1.3	20.95	15.30	23.05	16.25	20.35	15.40	32.45	28.30	37.20	29.75	31.60	24.85	37.85	30.15
1.35	26.75	18.55	28.80	19.40	27.05	18.05	38.65	32.05	44.90	36.35	38.00	29.50	44.20	35.85
1.4	33.15	22.40	35.55	24.05	32.80	22.20	46.05	36.40	53.95	41.85	43.70	35.95	49.70	40.75
1.45	39.10	27.25	41.30	28.30	39.10	27.30	51.50	39.55	61.35	48.15	50.20	40.50	56.05	44.55
1.5	45.65	31.40	47.95	33.20	45.25	31.20	57.40	44.70	68.60	54.20	57.55	45.00	61.30	50.05
1.55	51.50	36.20	53.80	38.40	51.45	36.05	62.70	49.65	74.30	59.75	63.05	50.85	67.45	55.65
1.6	58.40	41.15	60.25	43.25	57.85	40.80	68.30	54.65	78.90	65.85	68.60	55.20	72.65	60.20
1.65	63.75	45.25	65.50	47.35	63.25	45.80	72.85	58.65	83.40	70.90	73.60	59.25	77.60	64.60
1.7	69.10	49.80	71.55	51.65	68.95	49.45	77.60	62.95	87.10	75.00	77.90	63.70	82.05	68.95
1.75	74.55	54.45	76.05	56.65	73.95	54.65	82.05	66.30	90.10	78.70	81.95	67.00	85.90	72.40
1.8	78.55	59.50	80.05	60.95	78.15	59.15	85.60	70.00	92.55	81.95	85.00	70.55	88.60	76.00
1.85	81.55	62.95	82.60	64.70	81.85	62.95	88.40	73.05	94.55	84.65	87.75	74.55	90.10	79.10
1.9	84.40	66.80	85.30	68.30	84.45	66.90	91.10	76.65	95.80	87.35	90.20	78.10	92.55	81.65
1.95	86.65	69.95	87.60	71.45	86.45	69.90	92.70	79.35	97.10	89.05	92.15	80.75	94.20	84.65
2	89.40	73.05	90.65	74.30	89.30	72.60	94.35	81.90	98.05	90.45	93.70	83.40	95.50	86.40

3.5 Parallel hazard functions - Sample size $n = 100$ - Significance $\alpha = 0.05$ - Fixed length-scale 1

rate	MW1		MW2		MW3		Pearson		LR1		LR2		WLR	
cens	30%	50%	30%	50%	30%	50%	30%	50%	30%	50%	30%	50%	30%	50%
0.5	99.90	99.40	99.90	99.55	99.80	99.45	99.85	98.45	100.00	99.85	99.90	99.35	99.80	98.60
0.55	99.00	97.65	98.95	97.85	98.85	97.75	98.35	93.70	99.80	99.30	99.15	96.85	98.65	94.40
0.6	95.70	91.90	96.00	92.40	95.65	91.90	93.85	82.35	98.70	95.70	96.15	89.15	92.80	82.00
0.65	87.35	81.15	88.35	81.75	87.50	81.35	81.50	64.65	94.35	87.30	87.05	76.30	79.95	62.30
0.7	74.60	66.10	75.55	67.80	74.35	66.10	60.65	43.10	83.90	71.65	73.70	60.40	59.10	42.70
0.75	57.20	48.75	58.45	50.75	56.95	48.95	39.10	25.85	66.60	52.40	54.35	40.90	37.10	25.20
0.8	38.80	32.25	40.15	33.65	38.50	32.10	22.35	14.80	44.85	33.40	35.00	25.45	21.60	15.50
0.85	22.90	19.30	24.35	20.15	23.00	19.45	11.80	8.60	25.70	18.70	19.90	14.75	11.80	9.90
0.9	12.95	11.40	13.75	12.15	13.05	11.40	7.40	6.20	13.95	10.95	11.40	9.25	8.15	7.50
0.95	7.45	7.90	7.95	8.10	7.90	7.90	5.80	6.25	6.70	7.15	6.60	6.40	6.95	6.90
1	5.90	6.15	6.10	6.35	5.75	6.00	6.40	8.70	5.85	5.85	6.00	5.80	8.40	8.25
1.05	7.05	6.90	7.75	7.45	7.00	6.85	9.20	9.80	8.65	7.95	8.05	7.85	10.40	11.30
1.1	11.45	9.80	12.35	10.25	11.65	9.70	13.05	12.95	14.95	12.45	13.45	11.90	15.30	15.30
1.15	19.05	14.65	19.60	15.55	18.70	14.80	18.60	18.05	23.90	19.30	20.55	16.95	22.05	20.10
1.2	28.15	21.00	29.15	22.00	28.00	21.10	26.60	23.95	35.10	27.65	28.60	23.05	30.80	26.10
1.25	39.95	29.10	41.15	30.40	39.50	29.00	36.15	32.25	48.50	36.60	38.40	30.15	39.35	33.00
1.3	50.45	37.70	52.70	39.10	50.95	37.70	46.80	40.05	60.65	47.15	49.05	38.70	50.00	41.15
1.35	63.30	47.45	64.50	49.45	63.05	47.05	56.90	47.80	71.90	56.35	59.05	46.40	59.65	48.20
1.4	72.60	57.05	73.95	58.90	72.40	57.35	66.45	55.05	79.70	66.45	69.30	54.70	68.20	55.90
1.45	80.35	66.30	81.40	66.90	80.40	66.20	75.00	62.30	86.35	73.90	76.45	62.10	77.20	63.95
1.5	86.00	73.70	87.15	74.65	86.05	73.45	82.45	69.55	91.45	81.65	83.25	68.45	83.35	70.30
1.55	90.20	80.10	90.95	81.45	90.50	80.05	87.90	74.45	95.00	86.30	87.60	75.20	89.10	76.50
1.6	94.30	84.55	94.70	85.40	94.15	84.60	91.05	79.90	96.75	88.75	91.35	80.65	92.05	82.20
1.65	96.70	88.35	97.10	89.05	96.65	88.30	94.75	83.75	98.20	91.65	93.90	84.65	94.95	85.85
1.7	98.00	90.60	98.20	91.50	98.10	90.80	96.35	87.30	99.25	94.20	95.80	88.25	96.75	88.95
1.75	98.75	93.20	98.95	93.60	98.75	93.35	98.15	89.80	99.45	96.05	97.50	90.40	97.90	91.95
1.8	99.25	94.85	99.25	95.30	99.30	94.90	98.75	92.55	99.65	96.80	98.25	92.40	98.90	93.35
1.85	99.50	96.40	99.50	96.70	99.50	96.55	99.20	94.45	99.75	97.85	98.75	93.95	99.30	95.15
1.9	99.70	97.25	99.75	97.70	99.75	97.40	99.45	95.80	99.90	98.65	99.40	95.35	99.60	96.15
1.95	99.85	98.15	99.95	98.25	99.85	98.15	99.60	97.20	99.95	99.10	99.50	96.50	99.65	97.35
2	99.95	98.65	99.95	98.85	99.95	98.75	99.70	97.95	99.95	99.25	99.80	97.25	99.85	98.20

3.6 Parallel hazard functions - Sample size $n = 100$ - Significance $\alpha = 0.05$ - Adaptive length-scale

rate	MW1		MW2		MW3		Pearson		LR1		LR2		WLR	
cens	30%	50%	30%	50%	30%	50%	30%	50%	30%	50%	30%	50%	30%	50%
0.5	99.60	99.40	99.60	99.50	99.60	99.40	99.85	98.45	100.00	99.85	99.90	99.35	99.80	98.60
0.55	98.60	96.45	98.90	96.70	98.65	96.25	98.35	93.70	99.80	99.30	99.15	96.85	98.65	94.40
0.6	94.65	89.35	94.90	89.80	94.70	89.50	93.85	82.35	98.70	95.70	96.15	89.15	92.80	82.00
0.65	85.30	77.30	85.80	77.95	85.05	77.00	81.50	64.65	94.35	87.30	87.05	76.30	79.95	62.30
0.7	69.75	61.00	70.70	61.90	69.75	61.25	60.65	43.10	83.90	71.65	73.70	60.40	59.10	42.70
0.75	51.15	43.05	52.20	44.50	51.05	43.00	39.10	25.85	66.60	52.40	54.35	40.90	37.10	25.20
0.8	32.70	27.75	33.50	28.25	32.30	27.50	22.35	14.80	44.85	33.40	35.00	25.45	21.60	15.50
0.85	19.20	16.20	20.05	17.05	19.40	16.25	11.80	8.60	25.70	18.70	19.90	14.75	11.80	9.90
0.9	11.20	10.40	11.95	10.60	11.20	10.25	7.40	6.20	13.95	10.95	11.40	9.25	8.15	7.50
0.95	7.05	7.00	7.35	7.45	7.20	7.45	5.80	6.25	6.70	7.15	6.60	6.40	6.95	6.90
1	5.50	5.40	5.45	5.60	5.50	5.25	6.40	8.70	5.85	5.85	6.00	5.80	8.40	8.25
1.05	5.80	6.20	6.45	6.35	6.40	5.75	9.20	9.80	8.65	7.95	8.05	7.85	10.40	11.30
1.1	9.70	8.80	10.00	9.15	9.45	8.35	13.05	12.95	14.95	12.45	13.45	11.90	15.30	15.30
1.15	14.65	12.10	15.55	12.50	14.80	11.85	18.60	18.05	23.90	19.30	20.55	16.95	22.05	20.10
1.2	22.35	17.25	23.15	17.40	21.90	16.65	26.60	23.95	35.10	27.65	28.60	23.05	30.80	26.10
1.25	31.15	23.20	32.55	24.55	30.75	23.30	36.15	32.25	48.50	36.60	38.40	30.15	39.35	33.00
1.3	41.95	30.90	43.30	31.70	42.05	30.65	46.80	40.05	60.65	47.15	49.05	38.70	50.00	41.15
1.35	52.10	38.80	53.50	39.90	51.70	38.85	56.90	47.80	71.90	56.35	59.05	46.40	59.65	48.20
1.4	61.35	46.85	62.60	48.10	61.70	47.20	66.45	55.05	79.70	66.45	69.30	54.70	68.20	55.90
1.45	71.15	54.80	72.20	55.60	71.10	54.30	75.00	62.30	86.35	73.90	76.45	62.10	77.20	63.95
1.5	78.70	63.45	79.85	63.95	78.85	63.25	82.45	69.55	91.45	81.65	83.25	68.45	83.35	70.30
1.55	84.85	70.05	85.15	69.95	84.50	70.05	87.90	74.45	95.00	86.30	87.60	75.20	89.10	76.50
1.6	89.70	75.20	90.15	76.05	89.80	75.45	91.05	79.90	96.75	88.75	91.35	80.65	92.05	82.20
1.65	92.85	80.20	93.35	80.70	93.00	80.45	94.75	83.75	98.20	91.65	93.90	84.65	94.95	85.85
1.7	95.50	84.50	95.80	85.30	95.70	84.75	96.35	87.30	99.25	94.20	95.80	88.25	96.75	88.95
1.75	97.20	88.20	97.15	88.50	97.05	88.10	98.15	89.80	99.45	96.05	97.50	90.40	97.90	91.95
1.8	98.15	91.00	98.20	91.35	98.30	90.90	98.75	92.55	99.65	96.80	98.25	92.40	98.90	93.35
1.85	98.90	93.20	98.85	93.30	98.85	93.05	99.20	94.45	99.75	97.85	98.75	93.95	99.30	95.15
1.9	99.15	94.60	99.30	94.75	99.20	94.70	99.45	95.80	99.90	98.65	99.40	95.35	99.60	96.15
1.95	99.45	95.70	99.50	95.85	99.50	95.85	99.60	97.20	99.95	99.10	99.50	96.50	99.65	97.35
2	99.65	96.75	99.65	96.75	99.60	96.55	99.70	97.95	99.95	99.25	99.80	97.25	99.85	98.20

3.7 Parallel hazard functions - Sample size $n = 200$ - Significance $\alpha = 0.05$ - Fixed length-scale 1

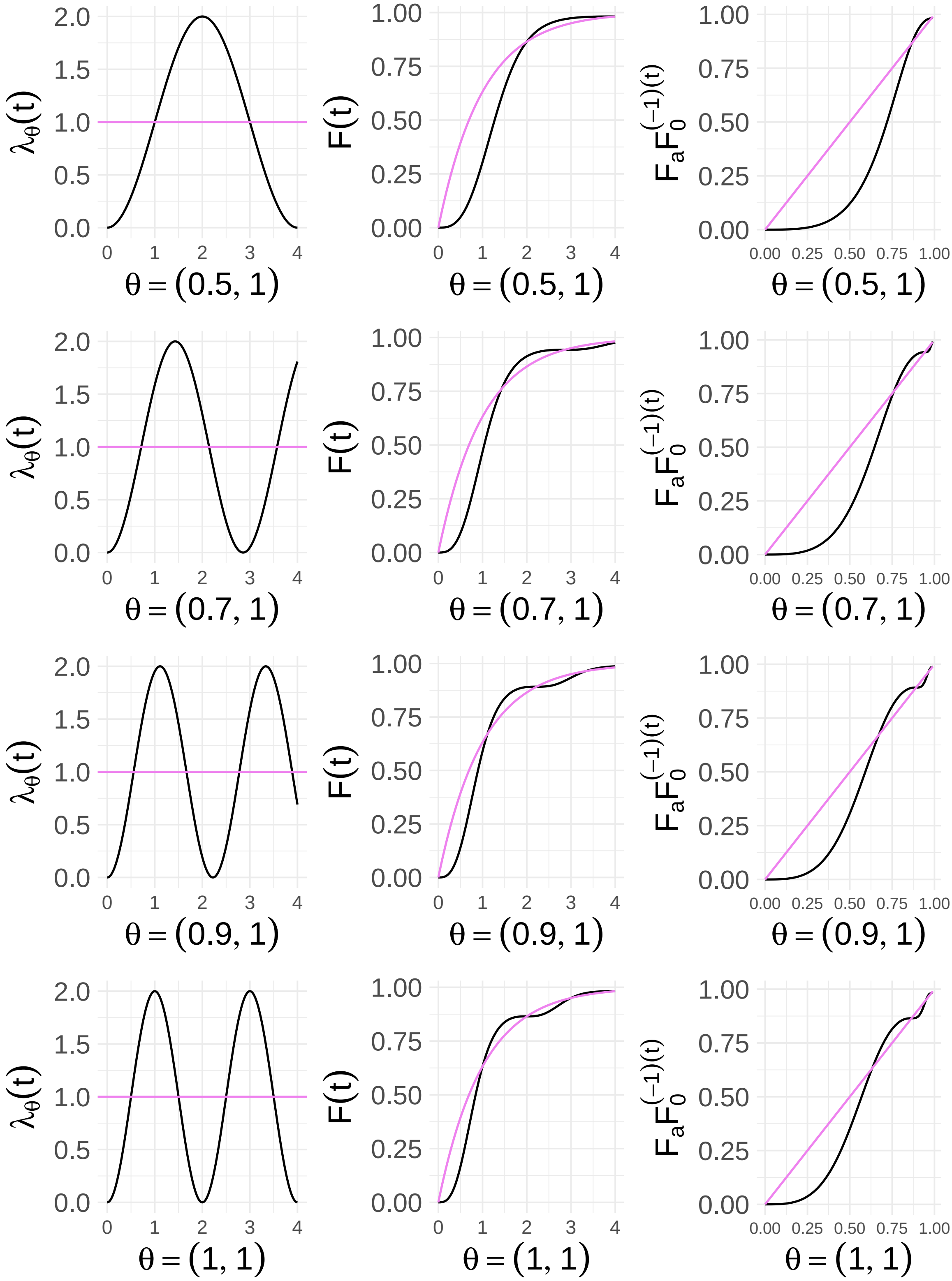
rate	MW1		MW2		MW3		Pearson		LR1		LR2		WLR	
cens	30%	50%	30%	50%	30%	50%	30%	50%	30%	50%	30%	50%	30%	50%
0.5	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
0.55	100.00	99.95	100.00	99.95	100.00	99.95	100.00	99.90	100.00	99.95	100.00	99.95	100.00	99.90
0.6	99.90	99.65	99.90	99.65	99.90	99.60	99.90	99.45	99.90	99.85	99.95	99.55	99.90	99.35
0.65	99.30	98.10	99.35	98.30	99.35	98.25	99.05	95.20	99.85	99.40	99.30	96.80	99.00	94.35
0.7	96.25	91.80	96.10	91.95	96.20	91.30	93.35	80.80	98.85	95.40	95.80	88.10	93.55	80.90
0.75	85.90	77.30	86.50	78.10	86.30	77.40	74.85	54.85	93.00	82.90	84.45	70.75	73.55	55.90
0.8	64.45	54.30	65.25	55.25	64.35	54.35	47.50	31.95	74.35	58.50	60.90	47.10	45.50	32.30
0.85	39.80	32.45	40.80	32.85	39.85	32.50	22.95	16.50	46.15	33.10	36.75	27.65	23.55	17.00
0.9	19.60	17.60	20.10	17.85	19.65	17.60	10.20	7.20	21.50	17.35	17.60	14.05	10.15	9.25
0.95	8.90	8.75	9.05	9.05	8.80	8.95	5.20	4.55	8.30	7.80	7.50	7.15	6.10	6.70
1	4.90	5.05	4.85	5.10	4.75	5.00	5.65	5.50	5.10	4.75	5.10	5.50	6.85	7.30
1.05	7.15	7.45	7.55	7.35	7.60	7.40	7.65	7.80	9.20	9.20	7.95	8.30	10.55	10.65
1.1	17.20	14.20	17.55	14.15	17.00	13.60	14.70	14.00	21.75	17.20	17.95	14.45	17.70	17.00
1.15	33.35	25.05	33.65	25.90	33.10	25.20	26.80	23.10	39.90	31.40	32.55	24.55	29.35	25.30
1.2	50.80	40.05	51.30	40.45	50.55	40.00	42.65	34.85	60.10	47.85	48.15	37.80	45.35	37.65
1.25	68.65	55.35	68.90	55.90	68.20	54.90	58.80	47.90	76.30	63.60	63.70	51.15	62.55	49.90
1.3	81.65	68.60	81.85	69.35	82.15	69.25	73.20	60.75	87.75	75.60	77.25	63.25	75.55	62.55
1.35	90.80	79.50	90.95	79.85	90.90	79.50	85.55	72.05	94.85	83.60	86.80	73.05	85.90	72.20
1.4	95.65	86.40	95.65	86.55	95.65	86.25	91.80	80.90	97.80	90.05	92.75	81.55	92.60	81.10
1.45	98.15	91.40	98.20	91.40	98.20	91.40	96.05	87.95	99.15	94.70	96.95	87.65	96.35	87.30
1.5	99.50	95.85	99.35	95.55	99.30	95.80	98.50	92.10	99.80	97.85	98.35	91.70	98.45	92.25
1.55	99.70	97.95	99.80	98.05	99.75	97.75	99.55	95.50	99.90	98.85	99.25	95.20	99.30	96.10
1.6	99.95	98.90	99.95	98.90	99.95	98.95	99.70	97.70	99.95	99.40	99.75	97.50	99.90	97.80
1.65	99.95	99.55	99.95	99.55	99.95	99.50	99.85	98.80	100.00	99.85	99.90	98.65	99.90	99.05
1.7	100.00	99.75	100.00	99.85	100.00	99.80	99.90	99.45	100.00	99.90	99.95	99.20	100.00	99.65
1.75	100.00	99.90	100.00	99.90	100.00	99.90	100.00	99.75	100.00	99.95	99.95	99.60	100.00	99.70
1.8	100.00	99.95	100.00	99.95	100.00	99.95	100.00	99.90	100.00	100.00	100.00	99.70	100.00	99.85
1.85	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.90	100.00	100.00	100.00	99.90	100.00	99.90
1.9	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.95	100.00	100.00	100.00	99.95	100.00	99.90
1.95	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.95	100.00	100.00	100.00	100.00	100.00	99.95
2	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00

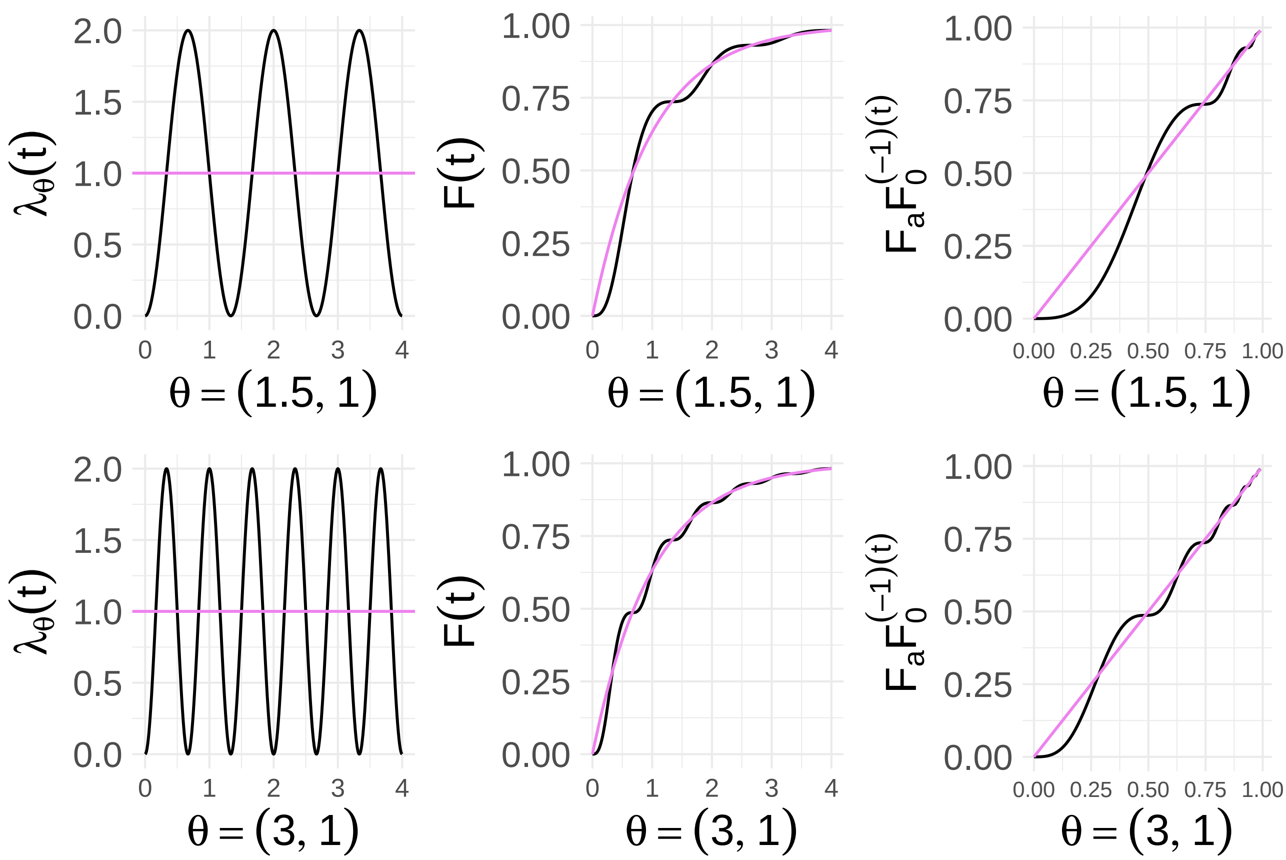
3.8 Parallel hazard functions - Sample size $n = 200$ - Significance $\alpha = 0.05$ - Adaptive length-scale

rate	MW1		MW2		MW3		Pearson		LR1		LR2		WLR	
cens	30%	50%	30%	50%	30%	50%	30%	50%	30%	50%	30%	50%	30%	50%
0.5	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
0.55	100.00	99.95	100.00	99.95	99.95	99.95	100.00	99.90	100.00	99.95	100.00	99.95	100.00	99.90
0.6	99.90	99.65	99.90	99.65	99.90	99.55	99.90	99.45	99.90	99.85	99.95	99.55	99.90	99.35
0.65	99.05	97.05	99.15	97.00	99.10	97.05	99.05	95.20	99.85	99.40	99.30	96.80	99.00	94.35
0.7	94.60	89.05	94.65	89.40	94.70	89.05	93.35	80.80	98.85	95.40	95.80	88.10	93.55	80.90
0.75	81.60	71.85	81.65	72.70	81.70	71.75	74.85	54.85	93.00	82.90	84.45	70.75	73.55	55.90
0.8	58.60	48.25	59.25	48.70	58.15	47.80	47.50	31.95	74.35	58.50	60.90	47.10	45.50	32.30
0.85	33.35	29.10	34.15	29.60	33.45	29.15	22.95	16.50	46.15	33.10	36.75	27.65	23.55	17.00
0.9	15.95	15.45	16.55	15.80	15.85	15.05	10.20	7.20	21.50	17.35	17.60	14.05	10.15	9.25
0.95	7.50	7.60	7.70	7.60	7.70	7.40	5.20	4.55	8.30	7.80	7.50	7.15	6.10	6.70
1	4.95	5.05	4.60	5.10	4.55	5.00	5.65	5.50	5.10	4.75	5.10	5.50	6.85	7.30
1.05	6.10	6.20	6.35	6.35	6.40	6.30	7.65	7.80	9.20	9.20	7.95	8.30	10.55	10.65
1.1	12.80	11.35	13.10	11.60	13.10	11.30	14.70	14.00	21.75	17.20	17.95	14.45	17.70	17.00
1.15	25.80	18.80	26.05	19.50	25.65	19.05	26.80	23.10	39.90	31.40	32.55	24.55	29.35	25.30
1.2	41.50	31.20	41.65	32.15	41.20	31.00	42.65	34.85	60.10	47.85	48.15	37.80	45.35	37.65
1.25	58.10	45.00	58.35	45.60	57.65	44.95	58.80	47.90	76.30	63.60	63.70	51.15	62.55	49.90
1.3	73.15	59.35	73.80	59.05	73.85	59.35	73.20	60.75	87.75	75.60	77.25	63.25	75.55	62.55
1.35	84.45	71.15	85.00	71.70	84.70	71.35	85.55	72.05	94.85	83.60	86.80	73.05	85.90	72.20
1.4	91.90	79.75	92.10	79.95	91.85	79.90	91.80	80.90	97.80	90.05	92.75	81.55	92.60	81.10
1.45	96.00	85.95	96.25	86.05	96.00	86.30	96.05	87.95	99.15	94.70	96.95	87.65	96.35	87.30
1.5	97.95	90.70	98.00	90.75	98.00	90.60	98.50	92.10	99.80	97.85	98.35	91.70	98.45	92.25
1.55	99.25	94.80	99.30	94.80	99.25	94.85	99.55	95.50	99.90	98.85	99.25	95.20	99.30	96.10
1.6	99.65	97.20	99.70	97.20	99.65	97.25	99.70	97.70	99.95	99.40	99.75	97.50	99.90	97.80
1.65	99.90	98.50	99.90	98.55	99.90	98.50	99.85	98.80	100.00	99.85	99.90	98.65	99.90	99.05
1.7	99.90	99.35	99.95	99.25	99.95	99.40	99.90	99.45	100.00	99.90	99.95	99.20	100.00	99.65
1.75	100.00	99.65	100.00	99.70	100.00	99.70	100.00	99.75	100.00	99.95	99.95	99.60	100.00	99.70
1.8	100.00	99.90	100.00	99.95	100.00	99.95	100.00	99.90	100.00	100.00	100.00	99.70	100.00	99.85
1.85	100.00	99.95	100.00	99.95	100.00	99.95	100.00	99.90	100.00	100.00	100.00	99.90	100.00	99.90
1.9	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.95	100.00	100.00	100.00	99.95	100.00	99.90
1.95	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.95	100.00	100.00	100.00	100.00	100.00	99.95
2	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00

4 Periodic hazard functions

Let $\theta = (\theta_1, \theta_2)$ such that $\theta_2 < 1$ and $\theta_1 \in \mathbb{R}$. We consider the family of hazard functions $\lambda_\theta(t) = 1 - \theta_2 \cos(\theta_1 \pi t)$ for fixed $\theta_2 = 1$ and $\theta_1 \in \{0.5, 0.7, 0.9, 1, 1.5, 3\}$. We include plots of the hazard function, distribution function and distribution of the transformed data $U = F_0(X)$ for each combination of alternative and null hypothesis. The null hypothesis, shown in the plots in color pink, is denoted by $F_0(t) = 1 - e^{-t}$ and $\lambda_0(t) = 1$.





4.1 Type I error

For this experiment, the null hypothesis is recovered when $\theta_2 = 0$. In red we observe tests that have an clear incorrect level. In orange, we observe tests that have a questionable incorrect level

4.1.1 Periodic hazards experiment: Censoring parameter $\gamma = 1/2$

Fixed length-scale 1

Type-I error							
	MW1	MW2	MW3	Pearson	LR1	LR2	WLR
Sample size n=30							
10 %	9.30	10.65	9.20	14.75	10.60	10.05	20.95
5 %	4.70	5.60	4.75	9.50	5.45	5.60	14.85
1 %	1.25	2.05	0.95	4.75	1.25	1.65	6.35
Sample size n=50							
10 %	10.55	11.70	10.90	13.95	10.35	10.80	15.50
5 %	5.70	6.70	5.90	8.35	5.55	5.75	9.70
1 %	1.25	1.55	1.35	3.60	1.90	1.80	3.20
Sample size n=100							
10 %	9.35	9.75	9.55	12.50	10.15	9.85	14.50
5 %	4.90	5.35	4.85	7.25	5.50	5.50	8.50
1 %	0.95	1.35	1.15	1.70	1.45	1.10	3.15
Sample size n=200							
10 %	10.80	11.05	10.85	10.10	10.75	10.40	10.90
5 %	5.65	5.95	5.50	4.90	5.60	5.95	6.20
1 %	1.00	1.25	1.10	0.90	1.35	0.85	1.95

Adaptive length-scale

	MW1	MW2	MW3	Pearson	LR1	LR2	WLR
Type-I error							
Sample size n=30							
10 %	10.00	11.05	9.90	14.75	10.60	10.05	20.95
5 %	5.25	6.15	4.85	9.50	5.45	5.60	14.85
1 %	1.05	1.50	0.95	4.75	1.25	1.65	6.35
Sample size n=50							
10 %	9.85	10.70	9.85	13.95	10.35	10.80	15.50
5 %	5.20	5.90	4.95	8.35	5.55	5.75	9.70
1 %	1.25	1.50	1.15	3.60	1.90	1.80	3.20
Sample size n=100							
10 %	9.80	10.30	9.40	12.50	10.15	9.85	14.50
5 %	4.35	4.55	4.10	7.25	5.50	5.50	8.50
1 %	1.10	1.20	1.05	1.70	1.45	1.10	3.15
Sample size n=200							
10 %	10.55	10.90	10.55	10.10	10.75	10.40	10.90
5 %	5.65	5.70	5.70	4.90	5.60	5.95	6.20
1 %	0.65	0.75	0.95	0.90	1.35	0.85	1.95

4.1.2 Periodic hazards experiment: Censoring parameter $\gamma = 2$

Fixed length-scale 1

	MW1	MW2	MW3	Pearson	LR1	LR2	WLR
Type-I error							
Sample size n=30							
10 %	10.05	11.15	9.80	13.60	10.00	10.55	13.85
5 %	5.65	6.55	5.45	9.65	5.05	5.55	9.50
1 %	1.25	2.30	1.05	5.45	1.15	1.55	4.95
Sample size n=50							
10 %	10.80	11.20	10.50	13.95	11.20	10.15	13.75
5 %	5.80	6.50	5.80	9.25	6.00	5.85	9.65
1 %	1.35	1.75	1.40	4.55	1.60	1.60	5.70
Sample size n=100							
10 %	10.45	10.90	10.35	13.55	9.55	10.05	12.05
5 %	5.55	5.85	5.35	9.65	5.45	5.05	7.95
1 %	1.55	1.55	1.35	5.10	1.45	1.45	3.55
Sample size n=200							
10 %	9.85	10.20	10.25	14.15	9.65	10.00	11.30
5 %	4.80	5.25	5.10	9.50	4.75	4.90	6.65
1 %	0.85	1.10	0.85	4.30	1.00	0.90	2.60

Adaptive length-scale

				Type-I error			
	MW1	MW2	MW3	Pearson	LR1	LR2	WLR
Sample size n=30							
10 %	10.25	11.30	10.20	13.60	10.00	10.55	13.85
5 %	5.40	6.35	5.20	9.65	5.05	5.55	9.50
1 %	1.20	1.50	1.40	5.45	1.15	1.55	4.95
Sample size n=50							
10 %	10.55	10.90	10.65	13.95	11.20	10.15	13.75
5 %	5.65	5.90	5.40	9.25	6.00	5.85	9.65
1 %	1.35	1.55	1.35	4.55	1.60	1.60	5.70
Sample size n=100							
10 %	10.30	10.45	10.30	13.55	9.55	10.05	12.05
5 %	5.45	5.45	5.30	9.65	5.45	5.05	7.95
1 %	1.65	1.75	1.50	5.10	1.45	1.45	3.55
Sample size n=200							
10 %	10.25	10.40	10.25	14.15	9.65	10.00	11.30
5 %	4.85	4.95	5.00	9.50	4.75	4.90	6.65
1 %	0.90	0.80	0.85	4.30	1.00	0.90	2.60

4.1.3 Sample size 30; Censoring distribution $G_1(t) = 1 - e^{-1/2t}$

	Fixed length-scale 1						
	Sample size $n = 30$; $\gamma = 1/2$						
	MW1	MW2	MW3	Pearson	LR1	LR2	WLR
$\theta = (0.5, 1)$							
10 %	100.00	100.00	100.00	99.40	89.65	99.95	62.15
5 %	99.95	99.95	99.95	96.95	74.50	99.80	44.60
1 %	99.90	99.95	99.90	76.80	31.25	94.20	17.30
$\theta = (0.7, 1)$							
10 %	99.80	99.90	99.85	95.85	43.95	97.95	40.85
5 %	99.45	99.55	99.45	86.05	27.45	90.25	27.90
1 %	96.45	97.95	96.50	56.55	6.40	49.15	10.70
$\theta = (0.9, 1)$							
10 %	97.25	97.75	97.20	89.35	20.65	80.00	31.40
5 %	93.40	94.45	93.25	76.35	10.80	56.55	20.55
1 %	77.05	82.10	76.70	46.60	1.65	13.00	8.75
$\theta = (1, 1)$							
10 %	93.10	94.25	93.10	87.35	16.20	62.80	31.35
5 %	84.70	86.60	84.55	73.60	8.05	39.55	21.55
1 %	60.00	65.75	59.80	42.05	0.80	6.55	10.30
$\theta = (1.5, 1)$							
10 %	40.05	43.00	40.15	62.95	8.40	18.20	25.20
5 %	26.45	29.95	26.20	46.20	3.85	7.35	17.85
1 %	8.60	11.90	8.40	21.75	0.55	1.00	8.70
$\theta = (3, 1)$							
10 %	11.60	13.20	11.70	20.00	8.95	5.35	19.35
5 %	5.20	7.15	5.25	12.70	4.55	2.20	13.60
1 %	1.50	2.25	1.55	4.90	0.95	0.25	6.70

	Adaptive length-scale						
	Sample size $n = 30$; $\gamma = 1/2$						
	MW1	MW2	MW3	Pearson	LR1	LR2	WLR
$\theta = (0.5, 1)$							
10 %	99.95	99.95	99.95	99.40	89.65	99.95	70.15
5 %	99.95	99.95	99.95	96.95	74.50	99.80	52.40
1 %	99.75	99.80	99.75	76.80	31.25	94.20	21.35
$\theta = (0.7, 1)$							
10 %	99.90	99.90	99.90	95.85	43.95	97.95	46.50
5 %	99.75	99.85	99.75	86.05	27.45	90.25	29.20
1 %	96.30	97.50	96.85	56.55	6.40	49.15	8.60
$\theta = (0.9, 1)$							
10 %	99.25	99.35	99.30	89.35	20.65	80.00	33.75
5 %	97.90	98.40	98.00	76.35	10.80	56.55	20.20
1 %	87.80	89.15	87.70	46.60	1.65	13.00	6.00
$\theta = (1, 1)$							
10 %	98.60	99.05	98.60	87.35	16.20	62.80	30.55
5 %	95.50	96.25	95.30	73.60	8.05	39.55	18.30
1 %	82.00	83.75	82.40	42.05	0.80	6.55	5.35
$\theta = (1.5, 1)$							
10 %	84.85	86.65	84.50	62.95	8.40	18.20	25.20
5 %	71.70	73.65	71.50	46.20	3.85	7.35	15.50
1 %	39.90	43.40	40.85	21.75	0.55	1.00	5.50
$\theta = (3, 1)$							
10 %	23.40	25.80	23.40	20.00	8.95	5.35	18.85
5 %	13.40	15.65	13.60	12.70	4.55	2.20	11.05
1 %	4.00	5.60	4.10	4.90	0.95	0.25	3.40

4.1.4 Sample size 30; Censoring distribution $G_2(t) = 1 - e^{-2t}$

Fixed length-scale 1							
Sample size $n = 30$; $\gamma = 2$							
	MW1	MW2	MW3	Pearson	LR1	LR2	WLR
$\theta = (0.5, 1)$							
10 %	99.95	99.95	99.95	86.00	98.35	99.95	47.55
5 %	99.95	99.95	99.95	62.10	93.30	99.65	24.50
1 %	99.80	99.90	99.85	18.85	62.05	84.00	3.25
$\theta = (0.7, 1)$							
10 %	99.70	99.75	99.70	69.70	82.55	99.40	23.60
5 %	99.45	99.50	99.45	46.10	66.35	96.15	10.80
1 %	96.65	97.65	96.50	18.80	24.05	54.95	3.10
$\theta = (0.9, 1)$							
10 %	98.15	98.35	98.20	60.30	58.95	95.90	13.35
5 %	96.10	96.85	96.15	39.05	36.75	85.85	7.40
1 %	87.60	90.50	87.80	19.20	8.85	28.00	3.55
$\theta = (1, 1)$							
10 %	96.20	96.80	96.15	54.85	45.60	92.20	12.30
5 %	92.60	93.75	92.65	35.25	27.45	75.55	7.45
1 %	77.45	81.65	77.80	17.60	4.65	17.80	3.55
$\theta = (1.5, 1)$							
10 %	65.35	68.30	65.25	44.65	17.25	54.85	9.45
5 %	51.70	56.20	52.10	29.35	7.90	30.55	6.45
1 %	30.75	35.90	30.50	14.70	0.90	1.50	3.60
$\theta = (3, 1)$							
10 %	14.70	16.25	14.55	15.75	7.75	8.65	10.35
5 %	8.90	10.95	8.90	11.45	3.15	2.65	7.25
1 %	2.65	3.95	2.80	5.55	0.40	0.00	3.55

Adaptive length-scale							
Sample size $n = 30$; $\gamma = 2$							
	MW1	MW2	MW3	Pearson	LR1	LR2	WLR
$\theta = (0.5, 1)$							
10 %	99.95	99.95	99.95	86.00	98.35	99.95	52.40
5 %	99.90	99.90	99.90	62.10	93.30	99.65	29.75
1 %	99.55	99.55	99.45	18.85	62.05	84.00	4.85
$\theta = (0.7, 1)$							
10 %	99.80	99.80	99.75	69.70	82.55	99.40	28.25
5 %	99.45	99.60	99.45	46.10	66.35	96.15	12.75
1 %	95.70	95.50	95.90	18.80	24.05	54.95	2.80
$\theta = (0.9, 1)$							
10 %	99.25	99.25	99.20	60.30	58.95	95.90	17.55
5 %	97.80	97.65	97.80	39.05	36.75	85.85	7.55
1 %	89.50	88.45	89.55	19.20	8.85	28.00	2.65
$\theta = (1, 1)$							
10 %	98.45	98.40	98.40	54.85	45.60	92.20	15.15
5 %	96.05	96.05	95.80	35.25	27.45	75.55	7.40
1 %	83.85	81.10	83.50	17.60	4.65	17.80	2.55
$\theta = (1.5, 1)$							
10 %	88.30	88.70	88.20	44.65	17.25	54.85	7.90
5 %	77.55	77.85	77.55	29.35	7.90	30.55	3.60
1 %	51.10	47.00	51.40	14.70	0.90	1.50	1.40
$\theta = (3, 1)$							
10 %	38.85	40.30	38.75	15.75	7.75	8.65	6.30
5 %	22.35	23.80	22.35	11.45	3.15	2.65	3.85
1 %	5.40	6.30	5.10	5.55	0.40	0.00	2.25

4.1.5 Sample size 50; Censoring distribution $G_1(t) = 1 - e^{-1/2t}$

Fixed length-scale 1

Sample size $n = 50$; $\gamma = 1/2$	MW1	MW2	MW3	Pearson	LR1	LR2	WLR
$\theta = (0.5, 1)$							
10 %	100.00	100.00	100.00	100.00	99.25	100.00	94.10
5 %	100.00	100.00	100.00	99.95	97.50	100.00	88.35
1 %	100.00	100.00	100.00	99.85	79.35	99.95	61.25
$\theta = (0.7, 1)$							
10 %	100.00	100.00	100.00	99.95	71.50	99.95	76.10
5 %	100.00	100.00	100.00	99.85	53.75	99.85	60.75
1 %	99.95	99.95	99.90	96.25	17.95	95.15	30.90
$\theta = (0.9, 1)$							
10 %	99.95	99.90	99.95	99.55	36.85	98.15	57.10
5 %	99.80	99.80	99.80	98.70	22.25	91.90	42.00
1 %	96.60	97.40	96.85	87.45	4.50	59.05	18.20
$\theta = (1, 1)$							
10 %	99.45	99.45	99.45	99.45	25.75	92.75	53.15
5 %	98.10	98.45	98.25	97.50	14.45	79.45	38.15
1 %	88.70	90.50	89.00	83.25	2.25	35.60	16.20
$\theta = (1.5, 1)$							
10 %	62.50	64.30	63.10	87.80	11.25	37.00	39.95
5 %	43.60	46.50	43.55	76.70	5.35	19.70	28.10
1 %	16.95	19.85	16.60	49.20	1.05	3.00	12.90
$\theta = (3, 1)$							
10 %	12.65	13.75	12.45	22.20	8.65	7.50	18.30
5 %	6.85	7.80	6.90	14.20	4.75	3.25	12.80
1 %	1.65	2.90	2.05	5.75	1.00	0.30	5.60

Adaptive length-scale

Sample size $n = 50$; $\gamma = 1/2$	MW1	MW2	MW3	Pearson	LR1	LR2	WLR
$\theta = (0.5, 1)$							
10 %	100.00	100.00	100.00	100.00	99.25	100.00	94.10
5 %	100.00	100.00	100.00	99.95	97.50	100.00	88.35
1 %	100.00	100.00	100.00	99.85	79.35	99.95	61.25
$\theta = (0.7, 1)$							
10 %	100.00	100.00	100.00	99.95	71.50	99.95	76.10
5 %	100.00	100.00	100.00	99.85	53.75	99.85	60.75
1 %	99.95	100.00	99.90	96.25	17.95	95.15	30.90
$\theta = (0.9, 1)$							
10 %	100.00	100.00	100.00	99.55	36.85	98.15	57.10
5 %	99.95	99.95	99.95	98.70	22.25	91.90	42.00
1 %	99.55	99.75	99.50	87.45	4.50	59.05	18.20
$\theta = (1, 1)$							
10 %	100.00	100.00	100.00	99.45	25.75	92.75	53.15
5 %	99.85	99.90	99.80	97.50	14.45	79.45	38.15
1 %	98.50	98.75	98.80	83.25	2.25	35.60	16.20
$\theta = (1.5, 1)$							
10 %	98.35	98.45	98.45	87.80	11.25	37.00	39.95
5 %	94.20	94.65	94.20	76.70	5.35	19.70	28.10
1 %	74.65	75.95	74.30	49.20	1.05	3.00	12.90
$\theta = (3, 1)$							
10 %	33.90	35.70	34.45	22.20	8.65	7.50	18.30
5 %	20.30	21.50	19.75	14.20	4.75	3.25	12.80
1 %	6.55	7.20	6.20	5.75	1.00	0.30	5.60

4.1.6 Sample size 50; Censoring distribution $G_2(t) = 1 - e^{-2t}$

Fixed length-scale 1							
Sample size $n = 50$; $\gamma = 2$							
	MW1	MW2	MW3	Pearson	LR1	LR2	WLR
$\theta = (0.5, 1)$							
10 %	100.00	100.00	100.00	99.95	100.00	100.00	94.25
5 %	100.00	100.00	100.00	99.45	99.80	100.00	83.80
1 %	100.00	100.00	100.00	85.00	96.00	99.95	40.55
$\theta = (0.7, 1)$							
10 %	100.00	100.00	100.00	99.00	97.95	100.00	65.80
5 %	100.00	100.00	100.00	95.40	94.20	99.95	44.75
1 %	99.90	99.85	99.85	62.40	68.75	98.65	12.75
$\theta = (0.9, 1)$							
10 %	99.95	99.95	99.95	95.50	84.40	99.95	41.50
5 %	99.75	99.80	99.75	85.45	70.20	99.50	24.50
1 %	98.20	98.75	98.10	47.50	30.70	89.05	5.80
$\theta = (1, 1)$							
10 %	99.80	99.80	99.80	92.80	72.80	99.80	30.20
5 %	99.30	99.30	99.25	81.90	54.30	98.55	17.55
1 %	95.70	96.40	95.35	41.95	17.60	79.20	5.55
$\theta = (1.5, 1)$							
10 %	85.00	86.45	85.10	75.35	29.40	87.35	14.25
5 %	75.35	77.30	75.00	58.75	15.95	69.15	8.60
1 %	51.30	55.40	51.35	29.30	3.70	22.80	3.20
$\theta = (3, 1)$							
10 %	20.05	21.20	20.30	21.55	10.70	17.65	9.25
5 %	13.15	14.60	12.95	14.90	4.35	8.05	5.60
1 %	4.35	5.50	4.55	6.75	0.65	0.70	3.15

Adaptive length-scale							
Sample size $n = 50$; $\gamma = 2$							
	MW1	MW2	MW3	Pearson	LR1	LR2	WLR
$\theta = (0.5, 1)$							
10 %	100.00	100.00	100.00	99.95	100.00	100.00	94.25
5 %	100.00	100.00	100.00	99.45	99.80	100.00	83.80
1 %	99.95	99.95	99.95	85.00	96.00	99.95	40.55
$\theta = (0.7, 1)$							
10 %	100.00	100.00	100.00	99.00	97.95	100.00	65.80
5 %	100.00	100.00	99.95	95.40	94.20	99.95	44.75
1 %	99.95	99.90	99.95	62.40	68.75	98.65	12.75
$\theta = (0.9, 1)$							
10 %	100.00	100.00	100.00	95.50	84.40	99.95	41.50
5 %	99.95	99.95	99.95	85.45	70.20	99.50	24.50
1 %	99.05	99.05	99.15	47.50	30.70	89.05	5.80
$\theta = (1, 1)$							
10 %	99.95	99.95	99.95	92.80	72.80	99.80	30.20
5 %	99.90	99.90	99.95	81.90	54.30	98.55	17.55
1 %	98.50	98.35	98.65	41.95	17.60	79.20	5.55
$\theta = (1.5, 1)$							
10 %	98.85	98.80	98.80	75.35	29.40	87.35	14.25
5 %	96.35	96.50	96.20	58.75	15.95	69.15	8.60
1 %	83.25	81.05	83.10	29.30	3.70	22.80	3.20
$\theta = (3, 1)$							
10 %	63.15	64.40	63.55	21.55	10.70	17.65	9.25
5 %	40.80	42.10	41.05	14.90	4.35	8.05	5.60
1 %	11.80	12.60	12.05	6.75	0.65	0.70	3.15

4.1.7 Sample size 100; Censoring distribution $G_1(t) = 1 - e^{-1/2t}$

Fixed length-scale 1							
Sample size $n = 100$; $\gamma = 1/2$							
	MW1	MW2	MW3	Pearson	LR1	LR2	WLR
$\theta = (0.5, 1)$							
10 %	100.00	100.00	100.00	100.00	100.00	100.00	100.00
5 %	100.00	100.00	100.00	100.00	100.00	100.00	100.00
1 %	100.00	100.00	100.00	100.00	99.85	100.00	99.70
$\theta = (0.7, 1)$							
10 %	100.00	100.00	100.00	100.00	97.00	100.00	99.50
5 %	100.00	100.00	100.00	100.00	92.20	100.00	98.10
1 %	100.00	100.00	100.00	100.00	66.75	100.00	88.95
$\theta = (0.9, 1)$							
10 %	100.00	100.00	100.00	100.00	66.40	100.00	96.40
5 %	100.00	100.00	100.00	100.00	49.25	100.00	91.00
1 %	100.00	100.00	100.00	100.00	19.30	99.65	69.65
$\theta = (1, 1)$							
10 %	100.00	100.00	100.00	100.00	49.65	100.00	93.65
5 %	100.00	100.00	100.00	100.00	33.25	99.85	85.25
1 %	100.00	100.00	100.00	100.00	11.55	94.60	62.30
$\theta = (1.5, 1)$							
10 %	94.90	95.45	95.00	99.85	16.70	77.75	79.60
5 %	84.20	84.65	84.50	99.10	8.95	59.10	65.75
1 %	50.35	53.25	50.65	94.70	1.95	18.70	38.65
$\theta = (3, 1)$							
10 %	13.85	14.90	13.90	28.55	9.80	11.50	27.50
5 %	8.00	8.20	7.80	19.15	5.00	5.80	18.50
1 %	2.15	2.45	2.30	7.25	0.90	0.95	7.90

Adaptive length-scale							
Sample size $n = 100$; $\gamma = 1/2$							
	MW1	MW2	MW3	Pearson	LR1	LR2	WLR
$\theta = (0.5, 1)$							
10 %	100.00	100.00	100.00	100.00	100.00	100.00	100.00
5 %	100.00	100.00	100.00	100.00	100.00	100.00	100.00
1 %	100.00	100.00	100.00	100.00	99.85	100.00	99.70
$\theta = (0.7, 1)$							
10 %	100.00	100.00	100.00	100.00	97.00	100.00	99.50
5 %	100.00	100.00	100.00	100.00	92.20	100.00	98.10
1 %	100.00	100.00	100.00	100.00	66.75	100.00	88.95
$\theta = (0.9, 1)$							
10 %	100.00	100.00	100.00	100.00	66.40	100.00	96.40
5 %	100.00	100.00	100.00	100.00	49.25	100.00	91.00
1 %	100.00	100.00	100.00	100.00	19.30	99.65	69.65
$\theta = (1, 1)$							
10 %	100.00	100.00	100.00	100.00	49.65	100.00	93.65
5 %	100.00	100.00	100.00	100.00	33.25	99.85	85.25
1 %	100.00	100.00	100.00	100.00	11.55	94.60	62.30
$\theta = (1.5, 1)$							
10 %	100.00	100.00	100.00	99.85	16.70	77.75	79.60
5 %	100.00	100.00	100.00	99.10	8.95	59.10	65.75
1 %	99.55	99.70	99.65	94.70	1.95	18.70	38.65
$\theta = (3, 1)$							
10 %	67.00	67.65	67.05	28.55	9.80	11.50	27.50
5 %	43.55	44.65	43.45	19.15	5.00	5.80	18.50
1 %	14.20	15.20	14.55	7.25	0.90	0.95	7.90

4.1.8 Sample size 100; Censoring distribution $G_2(t) = 1 - e^{-2t}$

Fixed length-scale 1							
Sample size $n = 100$; $\gamma = 2$							
	MW1	MW2	MW3	Pearson	LR1	LR2	WLR
$\theta = (0.5, 1)$							
10 %	100.00	100.00	100.00	100.00	100.00	100.00	100.00
5 %	100.00	100.00	100.00	100.00	100.00	100.00	100.00
1 %	100.00	100.00	100.00	100.00	100.00	100.00	99.55
$\theta = (0.7, 1)$							
10 %	100.00	100.00	100.00	100.00	100.00	100.00	99.20
5 %	100.00	100.00	100.00	100.00	99.95	100.00	97.50
1 %	100.00	100.00	100.00	100.00	99.35	100.00	83.95
$\theta = (0.9, 1)$							
10 %	100.00	100.00	100.00	100.00	99.40	100.00	90.65
5 %	100.00	100.00	100.00	100.00	97.80	100.00	79.20
1 %	100.00	100.00	100.00	99.55	85.35	100.00	43.05
$\theta = (1, 1)$							
10 %	100.00	100.00	100.00	100.00	97.25	100.00	79.95
5 %	100.00	100.00	100.00	99.95	92.70	100.00	63.15
1 %	100.00	100.00	100.00	98.55	67.90	100.00	29.10
$\theta = (1.5, 1)$							
10 %	99.35	99.30	99.30	99.00	54.70	99.90	31.10
5 %	97.35	97.40	97.25	96.65	37.60	99.20	17.50
1 %	88.00	88.50	87.75	82.20	12.10	87.85	4.15
$\theta = (3, 1)$							
10 %	27.45	27.95	27.60	34.45	12.20	38.90	14.20
5 %	17.55	18.45	17.50	23.25	5.75	22.45	9.95
1 %	5.80	6.55	6.00	9.80	1.25	3.90	4.55

Adaptive length-scale							
Sample size $n = 100$; $\gamma = 2$							
	MW1	MW2	MW3	Pearson	LR1	LR2	WLR
$\theta = (0.5, 1)$							
10 %	100.00	100.00	100.00	100.00	100.00	100.00	100.00
5 %	100.00	100.00	100.00	100.00	100.00	100.00	100.00
1 %	100.00	100.00	100.00	100.00	100.00	100.00	99.55
$\theta = (0.7, 1)$							
10 %	100.00	100.00	100.00	100.00	100.00	100.00	99.20
5 %	100.00	100.00	100.00	100.00	99.95	100.00	97.50
1 %	100.00	100.00	100.00	100.00	99.35	100.00	83.95
$\theta = (0.9, 1)$							
10 %	100.00	100.00	100.00	100.00	99.40	100.00	90.65
5 %	100.00	100.00	100.00	100.00	97.80	100.00	79.20
1 %	100.00	100.00	100.00	99.55	85.35	100.00	43.05
$\theta = (1, 1)$							
10 %	100.00	100.00	100.00	100.00	97.25	100.00	80.90
5 %	100.00	100.00	100.00	99.95	92.70	100.00	65.30
1 %	100.00	100.00	100.00	98.55	67.90	100.00	27.95
$\theta = (1.5, 1)$							
10 %	100.00	100.00	100.00	99.00	54.70	99.90	35.35
5 %	100.00	100.00	100.00	96.65	37.60	99.20	21.40
1 %	99.85	99.80	99.80	82.20	12.10	87.85	5.30
$\theta = (3, 1)$							
10 %	95.20	95.10	95.15	34.45	12.20	38.90	11.80
5 %	86.70	86.80	86.90	23.25	5.75	22.45	7.20
1 %	49.30	48.45	49.65	9.80	1.25	3.90	2.85

4.1.9 Sample size 200; Censoring distribution $G_1(t) = 1 - e^{-1/2t}$

Fixed length-scale 1							
Sample size $n = 200$; $\gamma = 1/2$							
	MW1	MW2	MW3	Pearson	LR1	LR2	WLR
$\theta = (0.5, 1)$							
10 %	100.00	100.00	100.00	100.00	100.00	100.00	100.00
5 %	100.00	100.00	100.00	100.00	100.00	100.00	100.00
1 %	100.00	100.00	100.00	100.00	100.00	100.00	100.00
$\theta = (0.7, 1)$							
10 %	100.00	100.00	100.00	100.00	100.00	100.00	100.00
5 %	100.00	100.00	100.00	100.00	99.90	100.00	100.00
1 %	100.00	100.00	100.00	100.00	98.80	100.00	100.00
$\theta = (0.9, 1)$							
10 %	100.00	100.00	100.00	100.00	95.05	100.00	100.00
5 %	100.00	100.00	100.00	100.00	88.45	100.00	99.95
1 %	100.00	100.00	100.00	100.00	64.30	100.00	99.75
$\theta = (1, 1)$							
10 %	100.00	100.00	100.00	100.00	83.00	100.00	100.00
5 %	100.00	100.00	100.00	100.00	71.00	100.00	99.75
1 %	100.00	100.00	100.00	100.00	39.95	100.00	99.15
$\theta = (1.5, 1)$							
10 %	99.95	100.00	100.00	100.00	28.50	98.10	99.30
5 %	99.70	99.80	99.80	100.00	17.70	94.40	98.65
1 %	93.80	93.65	93.45	100.00	5.20	73.50	93.40
$\theta = (3, 1)$							
10 %	21.20	21.05	20.95	44.70	10.70	21.85	55.45
5 %	12.00	11.95	11.50	31.70	4.95	11.40	44.05
1 %	3.35	3.60	3.40	14.20	1.00	2.35	24.80

Adaptive length-scale							
Sample size $n = 200$; $\gamma = 1/2$							
	MW1	MW2	MW3	Pearson	LR1	LR2	WLR
$\theta = (0.5, 1)$							
10 %	100.00	100.00	100.00	100.00	100.00	100.00	100.00
5 %	100.00	100.00	100.00	100.00	100.00	100.00	100.00
1 %	100.00	100.00	100.00	100.00	100.00	100.00	100.00
$\theta = (0.7, 1)$							
10 %	100.00	100.00	100.00	100.00	100.00	100.00	100.00
5 %	100.00	100.00	100.00	100.00	99.90	100.00	100.00
1 %	100.00	100.00	100.00	100.00	98.80	100.00	100.00
$\theta = (0.9, 1)$							
10 %	100.00	100.00	100.00	100.00	95.05	100.00	100.00
5 %	100.00	100.00	100.00	100.00	88.45	100.00	99.95
1 %	100.00	100.00	100.00	100.00	64.30	100.00	99.75
$\theta = (1, 1)$							
10 %	100.00	100.00	100.00	100.00	83.00	100.00	100.00
5 %	100.00	100.00	100.00	100.00	71.00	100.00	99.95
1 %	100.00	100.00	100.00	100.00	39.95	100.00	99.30
$\theta = (1.5, 1)$							
10 %	100.00	100.00	100.00	100.00	28.50	98.10	99.25
5 %	100.00	100.00	100.00	100.00	17.70	94.40	98.05
1 %	100.00	100.00	100.00	100.00	5.20	73.50	90.55
$\theta = (3, 1)$							
10 %	98.20	98.35	98.30	44.70	10.70	21.85	44.35
5 %	89.75	89.95	89.85	31.70	4.95	11.40	31.45
1 %	48.90	48.00	47.85	14.20	1.00	2.35	12.85

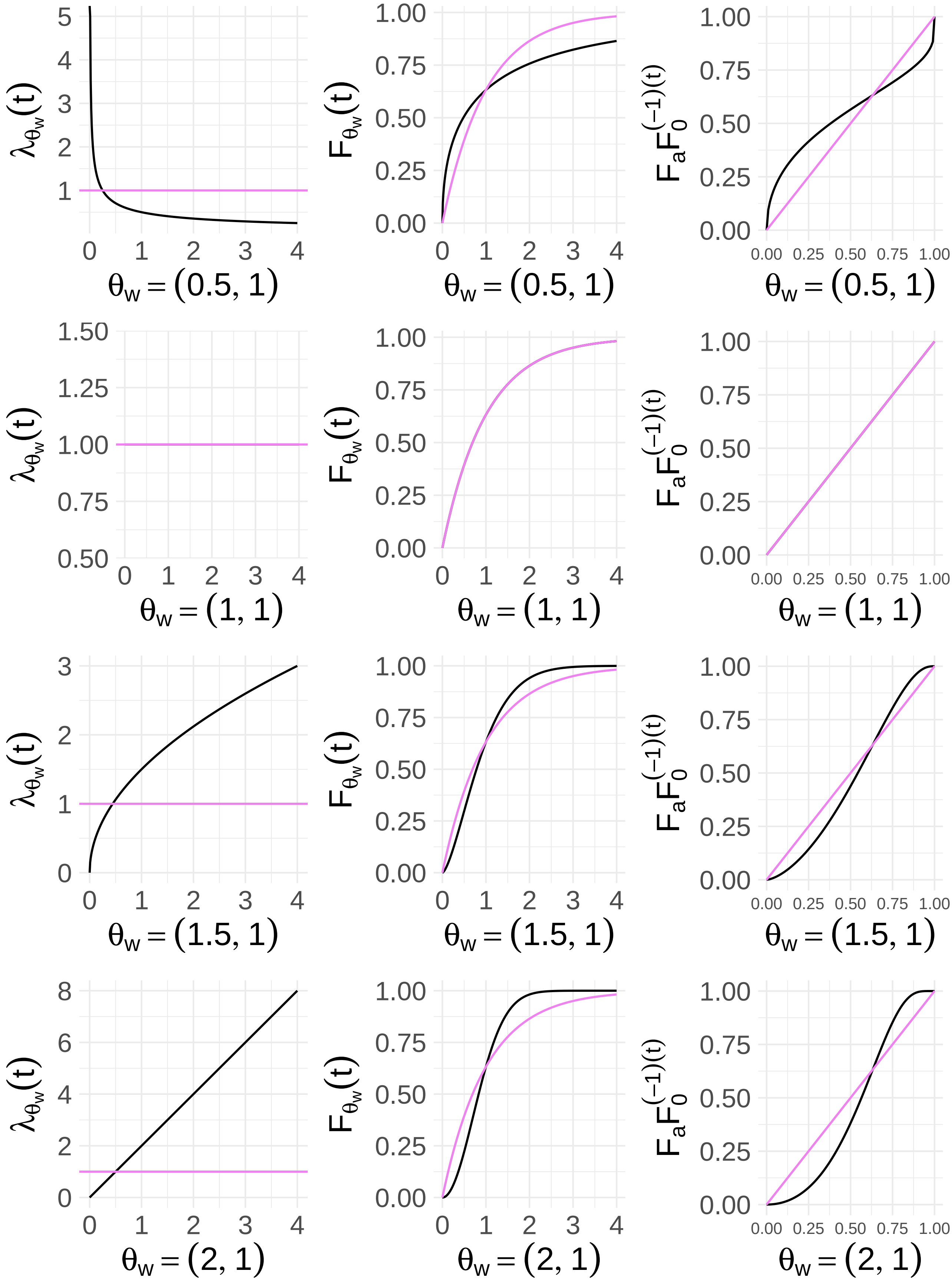
4.1.10 Sample size 200; Censoring distribution $G_2(t) = 1 - e^{-2t}$

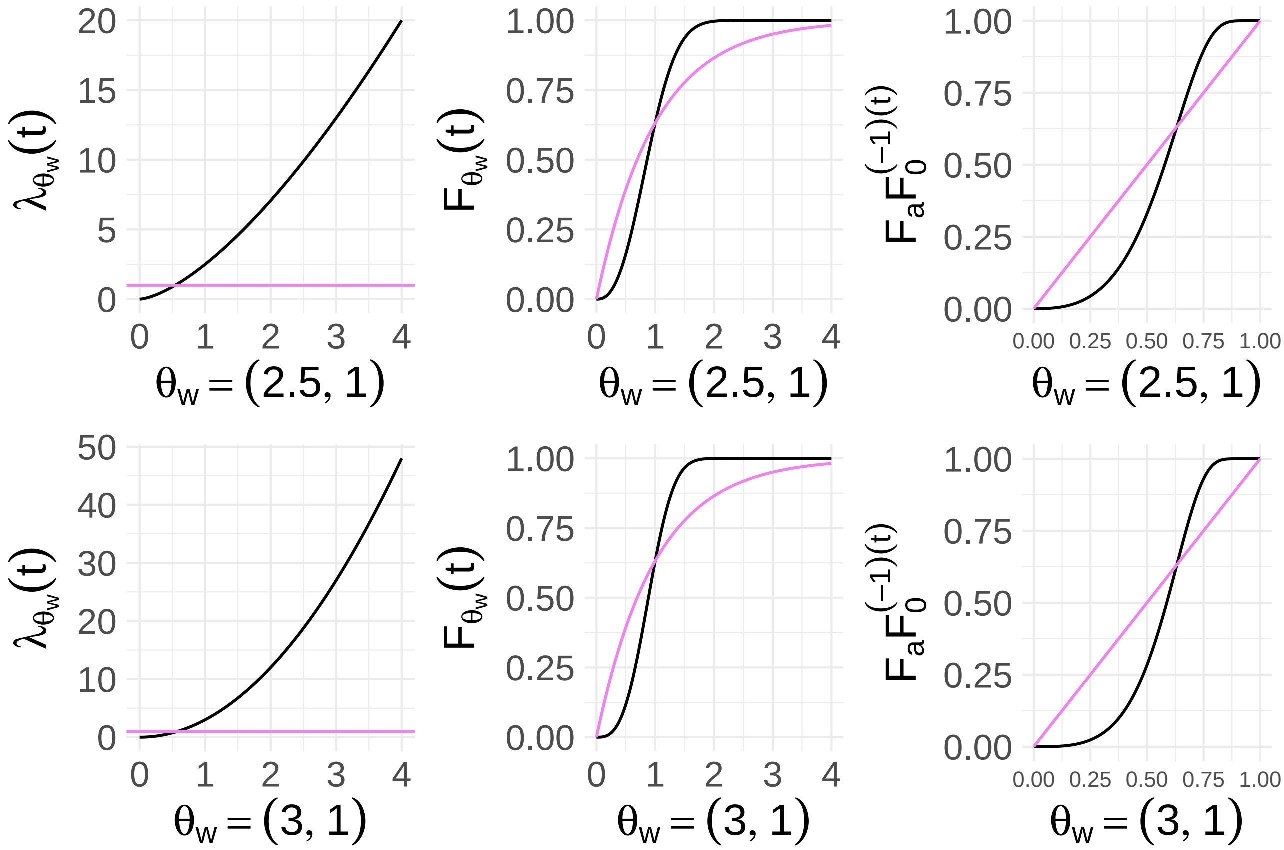
Fixed length-scale 1							
Sample size $n = 200$; $\gamma = 2$							
	MW1	MW2	MW3	Pearson	LR1	LR2	WLR
$\theta = (0.5, 1)$							
10 %	100.00	100.00	100.00	100.00	100.00	100.00	100.00
5 %	100.00	100.00	100.00	100.00	100.00	100.00	100.00
1 %	100.00	100.00	100.00	100.00	100.00	100.00	100.00
$\theta = (0.7, 1)$							
10 %	100.00	100.00	100.00	100.00	100.00	100.00	100.00
5 %	100.00	100.00	100.00	100.00	100.00	100.00	100.00
1 %	100.00	100.00	100.00	100.00	100.00	100.00	100.00
$\theta = (0.9, 1)$							
10 %	100.00	100.00	100.00	100.00	100.00	100.00	99.95
5 %	100.00	100.00	100.00	100.00	100.00	100.00	99.75
1 %	100.00	100.00	100.00	100.00	99.90	100.00	98.25
$\theta = (1, 1)$							
10 %	100.00	100.00	100.00	100.00	99.95	100.00	99.60
5 %	100.00	100.00	100.00	100.00	99.90	100.00	98.50
1 %	100.00	100.00	100.00	100.00	98.95	100.00	91.00
$\theta = (1.5, 1)$							
10 %	100.00	100.00	100.00	100.00	84.75	100.00	75.60
5 %	99.95	100.00	100.00	100.00	72.50	100.00	59.05
1 %	99.75	99.70	99.70	99.95	43.70	99.90	26.90
$\theta = (3, 1)$							
10 %	44.65	44.90	44.65	56.40	18.65	74.90	27.45
5 %	31.45	31.70	30.95	44.05	10.35	56.80	16.95
1 %	13.30	13.50	13.15	22.45	2.20	22.15	6.20

Adaptive length-scale							
Sample size $n = 200$; $\gamma = 2$							
	MW1	MW2	MW3	Pearson	LR1	LR2	WLR
$\theta = (0.5, 1)$							
10 %	100.00	100.00	100.00	100.00	100.00	100.00	100.00
5 %	100.00	100.00	100.00	100.00	100.00	100.00	100.00
1 %	100.00	100.00	100.00	100.00	100.00	100.00	100.00
$\theta = (0.7, 1)$							
10 %	100.00	100.00	100.00	100.00	100.00	100.00	100.00
5 %	100.00	100.00	100.00	100.00	100.00	100.00	100.00
1 %	100.00	100.00	100.00	100.00	100.00	100.00	100.00
$\theta = (0.9, 1)$							
10 %	100.00	100.00	100.00	100.00	100.00	100.00	99.95
5 %	100.00	100.00	100.00	100.00	100.00	100.00	99.75
1 %	100.00	100.00	100.00	100.00	99.90	100.00	98.25
$\theta = (1, 1)$							
10 %	100.00	100.00	100.00	100.00	99.95	100.00	99.80
5 %	100.00	100.00	100.00	100.00	99.90	100.00	98.70
1 %	100.00	100.00	100.00	100.00	98.95	100.00	92.35
$\theta = (1.5, 1)$							
10 %	100.00	100.00	100.00	100.00	84.75	100.00	79.45
5 %	100.00	100.00	100.00	100.00	72.50	100.00	65.70
1 %	100.00	100.00	100.00	99.95	43.70	99.90	30.70
$\theta = (3, 1)$							
10 %	100.00	100.00	100.00	56.40	18.65	74.90	24.00
5 %	99.95	99.95	99.95	44.05	10.35	56.80	15.10
1 %	96.45	96.45	97.05	22.45	2.20	22.15	5.55

5 Weibull hazard functions

Let $\theta_w = (\theta_{w1}, \theta_{w2}) \in \mathbb{R}_+^2$. We consider the hazard functions $\lambda_{\theta_w} = \theta_{w1}/\theta_{w2}(t/\theta_{w2})^{\theta_{w1}-1}$ for fixed $\theta_{w2} = 1$ and $\theta_{w1} \in \{0.5, 1, 1.5, 2, 3\}$. For this model, the null hypothesis is recovered when $\theta_w = (1, 1)$ (shown in pink).





In red we observe tests that have an clear incorrect level. In orange, we observe tests that have a questionable incorrect level.

5.0.1 Sample size 30; Censoring percentage 30%

Weibull hazard functions							
Sample size $n = 30$; Censoring 30%; Fixed length-scale 1							
	MW1	MW2	MW3	Pearson	LR1	LR2	WLR
$\theta_w = (0.5, 1)$							
10 %	40.40	43.85	40.55	70.25	28.55	71.30	70.55
5 %	26.65	31.35	26.45	60.15	20.45	65.15	63.70
1%	7.75	12.75	8.05	42.05	9.60	51.80	52.55
$\theta_w = (1, 1)$							
10 %	8.95	10.50	9.10	13.20	9.45	9.80	18.65
5 %	4.65	6.05	4.80	8.85	5.10	5.20	13.30
1%	1.20	2.05	1.45	3.60	1.60	1.45	6.40
$\theta_w = (1.5, 1)$							
10 %	25.95	28.90	26.10	52.20	3.65	8.45	33.40
5 %	16.75	19.30	16.65	40.80	1.55	3.45	24.10
1%	5.30	7.80	5.35	21.10	0.05	0.25	11.55
$\theta_w = (2, 1)$							
10 %	69.75	72.55	70.00	95.20	1.30	17.60	73.15
5 %	53.45	57.85	53.55	89.65	0.45	5.95	63.10
1%	25.65	30.15	25.45	72.95	0.00	0.20	42.00
$\theta_w = (2.5, 1)$							
10 %	95.05	95.70	95.25	99.90	0.55	28.45	95.75
5 %	87.60	89.90	87.75	99.80	0.00	9.40	92.80
1%	63.20	69.10	63.75	98.20	0.00	0.30	80.35
$\theta_w = (3, 1)$							
10 %	99.90	99.90	99.85	100.00	0.05	43.40	99.55
5 %	98.90	99.25	98.75	100.00	0.00	16.30	99.00
1%	89.60	91.40	89.50	99.90	0.00	0.45	96.20

Weibull hazard functions							
Sample size $n = 30$; Censoring 30%; Adaptive length-scale							
	MW1	MW2	MW3	Pearson	LR1	LR2	WLR
$\theta_w = (0.5, 1)$							
10 %	67.20	70.70	67.30	70.25	28.55	71.30	70.55
5 %	52.30	56.10	52.15	60.15	20.45	65.15	63.70
1%	21.95	29.40	21.95	42.05	9.60	51.80	52.55
$\theta_w = (1, 1)$							
10 %	9.25	10.25	8.95	13.20	9.45	9.80	18.65
5 %	4.40	5.25	4.40	8.85	5.10	5.20	13.30
1%	0.85	1.50	0.95	3.60	1.60	1.45	6.40
$\theta_w = (1.5, 1)$							
10 %	45.35	48.20	45.50	52.20	3.65	8.45	33.40
5 %	32.50	35.80	32.05	40.80	1.55	3.45	24.10
1%	12.70	15.05	12.35	21.10	0.05	0.25	11.55
$\theta_w = (2, 1)$							
10 %	92.75	93.45	92.50	95.20	1.30	17.60	73.15
5 %	85.60	87.50	85.60	89.65	0.45	5.95	63.10
1%	61.25	65.55	60.15	72.95	0.00	0.20	42.00
$\theta_w = (2.5, 1)$							
10 %	99.90	99.95	99.90	99.90	0.55	28.45	95.75
5 %	99.65	99.75	99.60	99.80	0.00	9.40	92.80
1%	95.00	96.15	95.30	98.20	0.00	0.30	80.35
$\theta_w = (3, 1)$							
10 %	100.00	100.00	100.00	100.00	0.05	43.40	99.55
5 %	100.00	100.00	100.00	100.00	0.00	16.30	99.00
1%	99.80	99.95	99.85	99.90	0.00	0.45	96.20

5.0.2 Sample size 30; Censoring percentage 50%

Weibull hazard functions							
Sample size $n = 30$; Censoring 50%; Fixed length-scale 1							
	MW1	MW2	MW3	Pearson	LR1	LR2	WLR
$\theta_w = (0.5, 1)$							
10 %	51.00	54.40	51.00	61.95	51.30	85.05	63.80
5 %	36.05	41.35	36.00	53.60	42.35	80.60	57.50
1%	13.15	19.95	13.10	37.25	27.10	68.40	47.15
$\theta_w = (1, 1)$							
10 %	9.25	10.80	9.20	14.35	9.50	9.75	16.70
5 %	4.45	5.95	4.25	10.05	4.85	5.20	11.05
1%	1.00	1.85	1.10	5.35	1.10	1.10	6.20
$\theta_w = (1.5, 1)$							
10 %	29.35	31.75	29.30	44.30	3.60	15.10	20.65
5 %	19.05	22.15	18.90	32.25	1.30	6.50	14.20
1%	6.95	9.95	7.05	16.70	0.20	0.70	8.05
$\theta_w = (2, 1)$							
10 %	69.95	72.60	69.80	86.25	0.90	32.25	46.25
5 %	54.20	58.50	54.60	77.10	0.20	15.25	36.05
1%	28.35	34.10	27.75	54.40	0.00	1.50	20.70
$\theta_w = (2.5, 1)$							
10 %	93.80	94.95	93.85	99.15	0.40	51.50	76.65
5 %	86.80	88.90	86.50	98.30	0.00	25.15	67.50
1%	62.95	69.00	62.50	91.20	0.00	2.20	47.60
$\theta_w = (3, 1)$							
10 %	99.75	99.85	99.80	100.00	0.10	68.00	92.45
5 %	98.15	98.75	98.25	99.85	0.00	37.20	87.40
1%	88.65	91.35	88.55	98.90	0.00	3.25	74.65

Weibull hazard functions							
Sample size $n = 30$; Censoring 50%; Adaptive length-scale							
	MW1	MW2	MW3	Pearson	LR1	LR2	WLR
$\theta_w = (0.5, 1)$							
10 %	68.20	70.85	67.95	61.95	51.30	85.05	63.80
5 %	52.20	56.40	52.10	53.60	42.35	80.60	57.50
1%	21.00	29.20	21.40	37.25	27.10	68.40	47.15
$\theta_w = (1, 1)$							
10 %	8.85	10.20	8.85	14.35	9.50	9.75	16.70
5 %	4.30	5.35	4.25	10.05	4.85	5.20	11.05
1%	0.65	1.15	0.60	5.35	1.10	1.10	6.20
$\theta_w = (1.5, 1)$							
10 %	41.60	44.25	41.65	44.30	3.60	15.10	20.65
5 %	29.70	32.35	29.80	32.25	1.30	6.50	14.20
1%	11.60	13.15	11.50	16.70	0.20	0.70	8.05
$\theta_w = (2, 1)$							
10 %	87.65	88.75	87.50	86.25	0.90	32.25	46.25
5 %	78.90	80.55	78.75	77.10	0.20	15.25	36.05
1%	53.05	56.95	53.65	54.40	0.00	1.50	20.70
$\theta_w = (2.5, 1)$							
10 %	99.75	99.85	99.80	99.15	0.40	51.50	76.65
5 %	98.65	99.20	98.75	98.30	0.00	25.15	67.50
1%	89.35	91.05	89.30	91.20	0.00	2.20	47.60
$\theta_w = (3, 1)$							
10 %	100.00	100.00	100.00	100.00	0.10	68.00	92.45
5 %	100.00	100.00	100.00	99.85	0.00	37.20	87.40
1%	99.60	99.60	99.60	98.90	0.00	3.25	74.65

5.0.3 Sample size 50; Censoring percentage 30%

Weibull hazard functions							
Sample size $n = 50$; Censoring 30%; Fixed length-scale 1							
	MW1	MW2	MW3	Pearson	LR1	LR2	WLR
$\theta_w = (0.5, 1)$							
10 %	61.40	63.10	61.10	90.35	30.15	85.95	83.95
5 %	46.40	49.25	46.00	84.60	22.75	81.10	78.05
1%	20.95	25.00	20.85	68.05	12.65	68.45	67.00
$\theta_w = (1, 1)$							
10 %	8.85	9.40	8.70	11.55	9.60	9.05	15.80
5 %	4.45	5.15	4.30	6.40	4.40	4.65	9.25
1%	1.05	1.55	1.05	2.05	1.00	1.05	3.65
$\theta_w = (1.5, 1)$							
10 %	38.10	39.80	38.15	74.90	3.55	14.85	45.95
5 %	24.80	26.85	24.95	62.90	1.20	6.20	34.80
1%	8.30	10.00	8.70	38.30	0.05	1.00	18.10
$\theta_w = (2, 1)$							
10 %	91.50	92.40	91.35	99.85	1.10	40.65	94.40
5 %	80.80	82.20	80.55	99.40	0.20	19.60	88.95
1%	49.75	54.45	50.50	96.80	0.00	2.20	73.20
$\theta_w = (2.5, 1)$							
10 %	100.00	99.95	99.95	100.00	0.65	66.90	99.70
5 %	99.15	99.45	99.30	100.00	0.05	40.80	99.65
1%	92.35	93.25	92.25	100.00	0.00	4.80	98.15
$\theta_w = (3, 1)$							
10 %	100.00	100.00	100.00	100.00	0.10	87.35	100.00
5 %	100.00	100.00	100.00	100.00	0.00	62.30	99.95
1%	99.85	99.85	99.85	100.00	0.00	9.35	99.95

Weibull hazard functions							
Sample size $n = 50$; Censoring 30%; Adaptive length-scale							
	MW1	MW2	MW3	Pearson	LR1	LR2	WLR
$\theta_w = (0.5, 1)$							
10 %	90.35	91.30	90.25	90.35	30.15	85.95	83.95
5 %	81.95	84.20	82.50	84.60	22.75	81.10	78.05
1%	53.80	58.05	53.20	68.05	12.65	68.45	67.00
$\theta_w = (1, 1)$							
10 %	8.50	9.05	8.45	11.55	9.60	9.05	15.80
5 %	4.20	4.75	4.40	6.40	4.40	4.65	9.25
1%	1.05	1.25	1.05	2.05	1.00	1.05	3.65
$\theta_w = (1.5, 1)$							
10 %	70.05	71.50	70.10	74.90	3.55	14.85	45.95
5 %	54.70	57.60	55.70	62.90	1.20	6.20	34.80
1%	27.05	29.40	27.70	38.30	0.05	1.00	18.10
$\theta_w = (2, 1)$							
10 %	99.60	99.70	99.60	99.85	1.10	40.65	94.40
5 %	98.85	99.00	98.95	99.40	0.20	19.60	88.95
1%	92.75	93.35	92.75	96.80	0.00	2.20	73.20
$\theta_w = (2.5, 1)$							
10 %	100.00	100.00	100.00	100.00	0.65	66.90	99.70
5 %	100.00	100.00	100.00	100.00	0.05	40.80	99.65
1%	99.95	99.95	99.95	100.00	0.00	4.80	98.15
$\theta_w = (3, 1)$							
10 %	100.00	100.00	100.00	100.00	0.10	87.35	100.00
5 %	100.00	100.00	100.00	100.00	0.00	62.30	99.95
1%	100.00	100.00	100.00	100.00	0.00	9.35	99.95

5.0.4 Sample size 50; Censoring percentage 50%

Weibull hazard functions							
Sample size $n = 50$; Censoring 50%; Fixed length-scale 1							
	MW1	MW2	MW3	Pearson	LR1	LR2	WLR
$\theta_w = (0.5, 1)$							
10 %	74.05	75.60	74.10	82.40	64.50	95.30	76.65
5 %	61.05	63.30	60.85	75.75	55.75	93.00	70.55
1%	31.15	36.15	30.65	58.65	37.00	85.75	57.85
$\theta_w = (1, 1)$							
10 %	8.40	9.30	8.45	14.35	9.15	9.60	15.70
5 %	4.45	5.20	4.50	9.60	4.90	5.05	10.15
1%	1.10	1.65	1.10	4.85	1.20	1.20	4.90
$\theta_w = (1.5, 1)$							
10 %	41.95	43.00	41.40	60.60	3.45	26.85	24.15
5 %	28.20	30.60	28.35	49.40	1.20	14.35	17.15
1%	11.20	13.95	11.25	27.05	0.10	2.15	8.40
$\theta_w = (2, 1)$							
10 %	90.20	91.25	90.40	98.60	1.10	63.95	67.90
5 %	81.45	82.40	80.80	96.65	0.25	41.70	57.05
1%	54.10	58.05	54.30	88.30	0.05	8.60	36.65
$\theta_w = (2.5, 1)$							
10 %	99.90	99.90	99.90	100.00	0.20	87.25	94.40
5 %	98.80	99.20	98.85	99.95	0.05	69.05	90.05
1%	91.60	92.85	91.10	99.75	0.00	18.60	77.40
$\theta_w = (3, 1)$							
10 %	100.00	100.00	100.00	100.00	0.00	96.90	99.35
5 %	100.00	100.00	100.00	100.00	0.00	86.95	98.75
1%	99.70	99.95	99.75	100.00	0.00	34.00	95.30

Weibull hazard functions							
Sample size $n = 50$; Censoring 50%; Adaptive length-scale							
	MW1	MW2	MW3	Pearson	LR1	LR2	WLR
$\theta_w = (0.5, 1)$							
10 %	89.80	90.60	89.75	82.40	64.50	95.30	76.65
5 %	81.15	82.55	81.50	75.75	55.75	93.00	70.55
1%	53.80	59.45	52.95	58.65	37.00	85.75	57.85
$\theta_w = (1, 1)$							
10 %	8.35	8.90	8.35	14.35	9.15	9.60	15.70
5 %	4.15	4.65	4.05	9.60	4.90	5.05	10.15
1%	0.95	1.20	1.00	4.85	1.20	1.20	4.90
$\theta_w = (1.5, 1)$							
10 %	62.90	64.20	62.75	60.60	3.45	26.85	24.15
5 %	49.25	50.85	49.20	49.40	1.20	14.35	17.15
1%	24.20	25.95	24.35	27.05	0.10	2.15	8.40
$\theta_w = (2, 1)$							
10 %	98.65	98.80	98.65	98.60	1.10	63.95	67.90
5 %	97.50	97.60	97.30	96.65	0.25	41.70	57.05
1%	86.85	87.90	86.95	88.30	0.05	8.60	36.65
$\theta_w = (2.5, 1)$							
10 %	100.00	100.00	100.00	100.00	0.20	87.25	94.40
5 %	100.00	100.00	100.00	99.95	0.05	69.05	90.05
1%	99.75	99.85	99.85	99.75	0.00	18.60	77.40
$\theta_w = (3, 1)$							
10 %	100.00	100.00	100.00	100.00	0.00	96.90	99.35
5 %	100.00	100.00	100.00	100.00	0.00	86.95	98.75
1%	100.00	100.00	100.00	100.00	0.00	34.00	95.30

5.0.5 Sample size 100; Censoring percentage 30%

Weibull hazard functions							
Sample size $n = 100$; Censoring 30%; Fixed length-scale 1							
	MW1	MW2	MW3	Pearson	LR1	LR2	WLR
$\theta_w = (0.5, 1)$							
10 %	89.10	89.70	89.20	99.60	36.30	97.20	97.15
5 %	77.30	78.25	76.75	99.00	28.45	95.55	95.20
1%	52.00	55.40	53.10	96.45	16.30	90.00	89.25
$\theta_w = (1, 1)$							
10 %	9.90	10.40	9.95	10.00	10.45	10.10	13.00
5 %	5.30	5.45	5.05	5.45	5.95	5.80	7.30
1%	1.15	1.30	1.00	1.80	1.30	1.30	3.05
$\theta_w = (1.5, 1)$							
10 %	69.95	70.70	69.55	97.10	5.00	39.10	79.05
5 %	52.30	54.05	51.80	94.20	2.35	23.25	66.85
1%	25.45	27.15	25.25	81.70	0.25	4.80	44.25
$\theta_w = (2, 1)$							
10 %	99.85	99.85	99.85	100.00	3.35	85.30	99.95
5 %	99.35	99.35	99.30	100.00	0.80	68.50	99.85
1%	93.20	94.10	93.00	99.95	0.00	25.70	98.60
$\theta_w = (2.5, 1)$							
10 %	100.00	100.00	100.00	100.00	1.35	98.85	100.00
5 %	100.00	100.00	100.00	100.00	0.35	94.05	100.00
1%	100.00	99.95	99.95	100.00	0.00	59.35	100.00
$\theta_w = (3, 1)$							
10 %	100.00	100.00	100.00	100.00	0.35	99.95	100.00
5 %	100.00	100.00	100.00	100.00	0.05	99.90	100.00
1%	100.00	100.00	100.00	100.00	0.00	87.25	100.00

Weibull hazard functions							
Sample size $n = 100$; Censoring 30%; Adaptive length-scale							
	MW1	MW2	MW3	Pearson	LR1	LR2	WLR
$\theta_w = (0.5, 1)$							
10 %	99.85	99.90	99.90	99.60	36.30	97.20	97.15
5 %	99.50	99.65	99.45	99.00	28.45	95.55	95.20
1%	95.20	95.25	94.95	96.45	16.30	90.00	89.25
$\theta_w = (1, 1)$							
10 %	9.50	10.05	9.70	10.00	10.45	10.10	13.00
5 %	5.20	5.65	5.30	5.45	5.95	5.80	7.30
1%	0.95	1.00	0.85	1.80	1.30	1.30	3.05
$\theta_w = (1.5, 1)$							
10 %	95.30	95.75	95.05	97.10	5.00	39.10	79.05
5 %	90.50	91.45	90.55	94.20	2.35	23.25	66.85
1%	71.45	73.00	71.60	81.70	0.25	4.80	44.25
$\theta_w = (2, 1)$							
10 %	100.00	100.00	100.00	100.00	3.35	85.30	99.95
5 %	100.00	100.00	100.00	100.00	0.80	68.50	99.85
1%	99.95	100.00	99.95	99.95	0.00	25.70	98.60
$\theta_w = (2.5, 1)$							
10 %	100.00	100.00	100.00	100.00	1.35	98.85	100.00
5 %	100.00	100.00	100.00	100.00	0.35	94.05	100.00
1%	100.00	100.00	100.00	100.00	0.00	59.35	100.00
$\theta_w = (3, 1)$							
10 %	100.00	100.00	100.00	100.00	0.35	99.95	100.00
5 %	100.00	100.00	100.00	100.00	0.05	99.90	100.00
1%	100.00	100.00	100.00	100.00	0.00	87.25	100.00

5.0.6 Sample size 100; Censoring percentage 50%

Weibull hazard functions							
Sample size $n = 100$; Censoring 50%; Fixed length-scale 1							
	MW1	MW2	MW3	Pearson	LR1	LR2	WLR
$\theta_w = (0.5, 1)$							
10 %	95.35	95.40	95.10	97.95	83.85	99.65	93.10
5 %	89.95	90.50	90.00	96.50	77.25	99.50	90.50
1%	70.55	72.90	70.75	90.80	61.90	98.60	81.45
$\theta_w = (1, 1)$							
10 %	9.60	9.80	9.40	12.20	10.10	9.20	14.00
5 %	5.00	5.30	4.85	6.85	4.75	5.05	8.35
1%	0.85	1.15	0.95	2.40	1.10	1.30	2.55
$\theta_w = (1.5, 1)$							
10 %	71.45	72.05	71.50	90.30	4.80	60.80	43.40
5 %	57.05	58.15	56.90	82.85	1.80	42.75	32.20
1%	30.00	33.20	30.25	60.30	0.10	13.15	14.45
$\theta_w = (2, 1)$							
10 %	99.85	99.85	99.85	100.00	1.70	96.15	94.20
5 %	98.90	98.90	98.90	100.00	0.20	89.90	90.25
1%	93.15	93.20	92.75	99.90	0.00	58.55	76.75
$\theta_w = (2.5, 1)$							
10 %	100.00	100.00	100.00	100.00	0.40	99.90	99.85
5 %	100.00	100.00	100.00	100.00	0.05	99.40	99.80
1%	99.95	99.95	99.95	100.00	0.00	88.40	98.95
$\theta_w = (3, 1)$							
10 %	100.00	100.00	100.00	100.00	0.00	100.00	100.00
5 %	100.00	100.00	100.00	100.00	0.00	99.90	100.00
1%	100.00	100.00	100.00	100.00	0.00	98.25	99.90

Weibull hazard functions							
Sample size $n = 100$; Censoring 50%; Adaptive length-scale							
	MW1	MW2	MW3	Pearson	LR1	LR2	WLR
$\theta_w = (0.5, 1)$							
10 %	99.70	99.70	99.55	97.95	83.85	99.65	93.10
5 %	99.00	99.15	99.00	96.50	77.25	99.50	90.50
1%	93.75	94.70	93.65	90.80	61.90	98.60	81.45
$\theta_w = (1, 1)$							
10 %	9.85	10.30	9.80	12.20	10.10	9.20	14.00
5 %	5.20	5.40	4.80	6.85	4.75	5.05	8.35
1%	0.90	1.05	0.80	2.40	1.10	1.30	2.55
$\theta_w = (1.5, 1)$							
10 %	91.45	91.75	91.60	90.30	4.80	60.80	43.40
5 %	84.40	84.70	84.35	82.85	1.80	42.75	32.20
1%	61.90	63.10	61.80	60.30	0.10	13.15	14.45
$\theta_w = (2, 1)$							
10 %	100.00	100.00	100.00	100.00	1.70	96.15	94.20
5 %	100.00	100.00	100.00	100.00	0.20	89.90	90.25
1%	99.85	99.85	99.85	99.90	0.00	58.55	76.75
$\theta_w = (2.5, 1)$							
10 %	100.00	100.00	100.00	100.00	0.40	99.90	99.85
5 %	100.00	100.00	100.00	100.00	0.05	99.40	99.80
1%	100.00	100.00	100.00	100.00	0.00	88.40	98.95
$\theta_w = (3, 1)$							
10 %	100.00	100.00	100.00	100.00	0.00	100.00	100.00
5 %	100.00	100.00	100.00	100.00	0.00	99.90	100.00
1%	100.00	100.00	100.00	100.00	0.00	98.25	99.90

5.0.7 Sample size 200; Censoring percentage 30%

Weibull hazard functions							
Sample size $n = 200$; Censoring 30%; Fixed length-scale 1							
	MW1	MW2	MW3	Pearson	LR1	LR2	WLR
$\theta_w = (0.5, 1)$							
10 %	99.90	99.95	99.95	100.00	46.05	99.95	99.95
5 %	99.15	99.20	99.05	100.00	36.80	99.90	99.95
1%	93.10	93.70	92.85	100.00	20.90	99.30	99.60
$\theta_w = (1, 1)$							
10 %	9.90	9.80	9.70	9.15	9.85	9.20	11.70
5 %	5.10	5.10	5.05	5.00	5.80	4.95	6.15
1%	0.85	0.90	0.65	1.20	1.30	0.70	1.40
$\theta_w = (1.5, 1)$							
10 %	96.40	96.35	96.25	100.00	6.10	72.65	98.85
5 %	88.25	88.80	88.25	100.00	2.75	56.50	96.85
1%	60.05	62.15	60.85	99.80	0.45	22.40	88.35
$\theta_w = (2, 1)$							
10 %	100.00	100.00	100.00	100.00	5.25	99.75	100.00
5 %	100.00	100.00	100.00	100.00	1.65	98.55	100.00
1%	100.00	100.00	100.00	100.00	0.05	85.50	100.00
$\theta_w = (2.5, 1)$							
10 %	100.00	100.00	100.00	100.00	4.00	100.00	100.00
5 %	100.00	100.00	100.00	100.00	1.00	100.00	100.00
1%	100.00	100.00	100.00	100.00	0.00	99.80	100.00
$\theta_w = (3, 1)$							
10 %	100.00	100.00	100.00	100.00	2.15	100.00	100.00
5 %	100.00	100.00	100.00	100.00	0.20	100.00	100.00
1%	100.00	100.00	100.00	100.00	0.00	100.00	100.00

Weibull hazard functions							
Sample size $n = 200$; Censoring 30%; Adaptive length-scale							
	MW1	MW2	MW3	Pearson	LR1	LR2	WLR
$\theta_w = (0.5, 1)$							
10 %	100.00	100.00	100.00	100.00	46.05	99.95	99.95
5 %	100.00	100.00	100.00	100.00	36.80	99.90	99.95
1%	100.00	100.00	100.00	100.00	20.90	99.30	99.60
$\theta_w = (1, 1)$							
10 %	8.95	9.00	8.95	9.15	9.85	9.20	11.70
5 %	4.75	4.75	4.70	5.00	5.80	4.95	6.15
1%	0.80	0.95	0.90	1.20	1.30	0.70	1.40
$\theta_w = (1.5, 1)$							
10 %	99.95	100.00	99.95	100.00	6.10	72.65	98.85
5 %	99.90	99.85	99.90	100.00	2.75	56.50	96.85
1%	98.70	98.95	98.65	99.80	0.45	22.40	88.35
$\theta_w = (2, 1)$							
10 %	100.00	100.00	100.00	100.00	5.25	99.75	100.00
5 %	100.00	100.00	100.00	100.00	1.65	98.55	100.00
1%	100.00	100.00	100.00	100.00	0.05	85.50	100.00
$\theta_w = (2.5, 1)$							
10 %	100.00	100.00	100.00	100.00	4.00	100.00	100.00
5 %	100.00	100.00	100.00	100.00	1.00	100.00	100.00
1%	100.00	100.00	100.00	100.00	0.00	99.80	100.00
$\theta_w = (3, 1)$							
10 %	100.00	100.00	100.00	100.00	2.15	100.00	100.00
5 %	100.00	100.00	100.00	100.00	0.20	100.00	100.00
1%	100.00	100.00	100.00	100.00	0.00	100.00	100.00

5.0.8 Sample size 200; Censoring percentage 50%

Weibull hazard functions							
Sample size $n = 200$; Censoring 50%; Fixed length-scale 1							
	MW1	MW2	MW3	Pearson	LR1	LR2	WLR
$\theta_w = (0.5, 1)$							
10 %	100.00	100.00	100.00	100.00	97.20	100.00	99.40
5 %	99.85	99.90	99.90	100.00	95.35	100.00	98.95
1%	98.45	98.40	98.40	99.80	88.65	100.00	97.70
$\theta_w = (1, 1)$							
10 %	10.10	10.15	10.00	10.10	10.25	9.35	11.95
5 %	4.70	4.70	4.60	5.40	5.25	4.60	7.05
1%	0.70	0.80	0.65	1.55	1.20	0.90	2.20
$\theta_w = (1.5, 1)$							
10 %	95.50	95.80	95.30	99.65	6.35	90.75	74.65
5 %	88.35	88.60	88.20	99.15	2.15	81.60	62.35
1%	66.45	67.80	66.60	95.80	0.15	50.05	38.80
$\theta_w = (2, 1)$							
10 %	100.00	100.00	100.00	100.00	2.45	100.00	99.95
5 %	100.00	100.00	100.00	100.00	0.50	100.00	99.90
1%	100.00	100.00	100.00	100.00	0.00	98.20	98.85
$\theta_w = (2.5, 1)$							
10 %	100.00	100.00	100.00	100.00	0.30	100.00	100.00
5 %	100.00	100.00	100.00	100.00	0.10	100.00	100.00
1%	100.00	100.00	100.00	100.00	0.00	100.00	100.00
$\theta_w = (3, 1)$							
10 %	100.00	100.00	100.00	100.00	0.20	100.00	100.00
5 %	100.00	100.00	100.00	100.00	0.00	100.00	100.00
1%	100.00	100.00	100.00	100.00	0.00	100.00	100.00

Weibull hazard functions							
Sample size $n = 200$; Censoring 50%; Adaptive length-scale							
	MW1	MW2	MW3	Pearson	LR1	LR2	WLR
$\theta_w = (0.5, 1)$							
10 %	100.00	100.00	100.00	100.00	97.20	100.00	99.40
5 %	100.00	100.00	100.00	100.00	95.35	100.00	98.95
1%	100.00	100.00	100.00	99.80	88.65	100.00	97.70
$\theta_w = (1, 1)$							
10 %	9.40	9.60	9.10	10.10	10.25	9.35	11.95
5 %	4.25	4.40	4.45	5.40	5.25	4.60	7.05
1%	0.75	0.85	0.85	1.55	1.20	0.90	2.20
$\theta_w = (1.5, 1)$							
10 %	99.95	99.95	99.95	99.65	6.35	90.75	74.65
5 %	99.60	99.60	99.70	99.15	2.15	81.60	62.35
1%	95.60	95.65	95.85	95.80	0.15	50.05	38.80
$\theta_w = (2, 1)$							
10 %	100.00	100.00	100.00	100.00	2.45	100.00	99.95
5 %	100.00	100.00	100.00	100.00	0.50	100.00	99.90
1%	100.00	100.00	100.00	100.00	0.00	98.20	98.85
$\theta_w = (2.5, 1)$							
10 %	100.00	100.00	100.00	100.00	0.30	100.00	100.00
5 %	100.00	100.00	100.00	100.00	0.10	100.00	100.00
1%	100.00	100.00	100.00	100.00	0.00	100.00	100.00
$\theta_w = (3, 1)$							
10 %	100.00	100.00	100.00	100.00	0.20	100.00	100.00
5 %	100.00	100.00	100.00	100.00	0.00	100.00	100.00
1%	100.00	100.00	100.00	100.00	0.00	100.00	100.00