# Iris Recognition Using Ordinal Encoding of Log-Euclidean Covariance Matrices

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### Abstract

Iris recognition in less constrained environments is challenging due to the degraded iris images. This paper proposes a novel method fusing multiple cues for iris recognition in the non-ideal imagery. The covariance matrices are used to represent local iris texture property, which capture the correlation of spatial coordinates, intensities, 1st and 2nd-order partial derivatives. The covariance matrices are symmetric positive definite (SPD) which form a Riemannian space rather than a Euclidean one. In the Log-Euclidean framework, the space of SPD matrices is equipped with a linear space structure so that in the logarithmic domain the Euclidean operations are applicable. This enables us to compute the logarithms of covariance matrices, leading to the Log-Euclidean covariance matrices (LECM), which can be handled in common Euclidean operations. The ordinal measure is further used to represent the order relation of iris texture by comparing LECMs at different positions. We finally perform iris matching based on the Hamming distance in which the noise effects are considered. Experiments on challenging databases show the effectiveness of the proposed method.

## 1 Introduction

The iris texture of human beings has unique and stable biological structure which is very suitable for identity recognition [4]. In recent years iris recognition has attracted increasing research interests [3]. Iris recognition in ideal images has achieved considerable advances. However, in order to capture high-quality iris images, the subjects are required to be motionless and be highly cooperative. This greatly limits deployment of iris recognitions in widespread scenarios. Iris recognition in less constrained environments [7, 9] has recently received great research interests, where the subjects are less cooperative or are ideally unaware of the presence of such recognition systems.

The iris images captured in less constrained environments are far from ideal. They are degraded due to large severe occlusion, strong specular reflection, illumination changes and image blurring [9]. The state-ofthe-art work focuses on fusion of multiple image cues and/or multiple modality for improving the identification performance. In [14], the periocular texture and iris texture are independently used to produce matching scores and are then combined for score level fusion. Tan et al. [13] proposed a multi-modal fusion method which combined the features extracted on the iris data and eye data. Santos and Hoyle [11] also fused recognition results of several methods that extract iris texture via wavelet and periocular texture via SIFT descriptors. Proença et al. [10] proposed a recognition method that fuse MPEG-7 color and shape descriptors. They also investigated combination of various classifiers for boosting the recognition performance.

This paper introduces a novel method for representing iris texture by combining various image cues in the non-ideal imagery. We propose the Log-Euclidean covariance matrices (LECM) for modeling the local correlation of multiple cues, e.g., the spatial coordinates, intensities, 1st and 2nd-order image derivatives. Furthermore, we use the ordinal measures for extracting the order relationship of LECM features at different positions. The iris matching algorithm is based on the Hamming distance which considers the noise factors. Section 2 presents in detail the proposed iris recognition method. Section 3 describes the experimental results, followed by the conclusion in section 4.

### 2 Proposed method for iris recognition

An iris recognition system usually consists of several stages of iris imaging, preprocessing as well as iris coding and matching. In this paper we use our method proposed in [5] for preprocessing.

#### 2.1 Overview of the proposed method

Fig. 1 shows an overview of the proposed method. Given an original iris image, we localize the inner and outer iris boundaries, eyelids and specular highlights and then obtain the normalized iris image and the binary mask image [5]. For each pixel we compute a covariance matrix of raw image features in the neighboring region. The covariance matrices are transformed via logarithmic operations from the Riemannian space to an Euclidean space of symmetric matrices. After vectorization of the resulting symmetric matrices, we obtain the Log-Euclidean covariance matrices (LECM) for characterizing the local texture properties. To further model the order relationship of iris texture, we encode the LECM's with ordinal measures. The iris matching method excludes the occluded or corrupted pixels via the mask image obtained previously.



Figure 1: Overview of the proposed method

### 2.2 Log-Euclidean covariance matrices

The covariance matrix captures the correlation of various image cues [8]. The covariance matrices can be computed efficiently via integral images. In addition, it is not sensitive to noise, scale and rotation variation and illumination changes. However, the distance between two covariance matrices involved in [8] are concerned with affine-invariant Riemannian metric which are computationally intensive [2]. In this paper, we propose Log-Euclidean Covariance Matrices for representing the local iris properties.

Let  $I(\mathbf{z}), \mathbf{z} \in \Omega$ , be pixel intensity at the spatial position  $\mathbf{z} = (x, y)$ , where  $\Omega$  denotes the image region and x, y denote the horizontal and vertical coordinates. We first extract the raw features for every pixel in the image

$$\mathbf{f}(\mathbf{z}) = \begin{bmatrix} x \ y \ I(\mathbf{z}) \ I_x(\mathbf{z}) \ I_y(\mathbf{z}) \ I_{xx}(\mathbf{z}) \ I_{yy}(\mathbf{z}) \end{bmatrix}^T \quad (1)$$

where  $I_x$ (resp.  $I_y$ ) denotes the 1st-order partial derivative with respect to x(resp. y) and  $I_{xx}$ (resp.  $I_{yy}$ ) denotes the 2nd-order partial derivative. Other image cues, e.g., gradient orientation or texture, can also be included in the raw features. Define  $\Omega_r(\mathbf{z})$  the image region centered at z

$$\Omega_r(\mathbf{z}) = \left\{ \mathbf{z}' = (x', y') : ||\mathbf{z} - \mathbf{z}'||_{\infty} \le r \right\}$$
(2)

where  $|| \cdot ||_{\infty}$  denotes the Euclidean infinity norm and r is a constant. The covariance matrix  $\mathbf{P}(\mathbf{z})$  at the pixel  $\mathbf{z}$  is computed as

$$\mathbf{P}(\mathbf{z}) = \frac{1}{|\Omega_r(\mathbf{z})|} \sum_{\mathbf{z}' \in \Omega_r(\mathbf{z})} (\mathbf{f}(\mathbf{z}') - \overline{\mathbf{f}}(\mathbf{z})) (\mathbf{f}(\mathbf{z}') - \overline{\mathbf{f}}(\mathbf{z}))^T$$
(3)

where  $|\Omega_r(\mathbf{z})|$  and  $\mathbf{f}(\mathbf{z})$  denote the number and mean vector of the raw features in  $\Omega_r(\mathbf{z})$ , respectively.

From the normalized iris image, we obtain a tensorvalued image  $\mathbf{P}(\mathbf{z}), \mathbf{z} \in \Omega$ , where each pixel is associated with a SPD matrix. Because the space of SPD matrices is a Riemannian space, the common Euclidean operations are not applicable. Thanks to the Log-Euclidean framework, we can avoid computational intensive operations in the Riemannian space and handle the SPD matrices in the common Euclidean way. In the following, we briefly introduce the theory of Log-Euclidean metrics and one may refer to [2] for details. Let S(n) and SPD(n) be the spaces of  $n \times n$  symmetric matrices and  $n \times n$  symmetric positive definite matrices, respectively. For any  $\mathbf{S} \in S(n)$ , the exponential of  $\mathbf{S}$  is defined as

$$\exp(\mathbf{S}) = \mathbf{I} + \mathbf{S} + \dots + \mathbf{S}^k / k! + \dots$$
(4)

where I denotes the identity matrix,  $\mathbf{S}^k$  denotes the matrix power and  $\exp(\mathbf{S}) \in S\mathcal{PD}(n)$ . Conversely, for any matrix  $\mathbf{P} \in S\mathcal{PD}(n)$ , there is a unique matrix  $\mathbf{S} \in S(n)$  for which  $\mathbf{P} = \exp(\mathbf{S})$ . The matrix  $\mathbf{S}$  is called the logarithm of  $\mathbf{P}$  and is represented by  $\mathbf{S} =$ log  $\mathbf{P}$ . The exponential map  $\exp: S(n) \mapsto S\mathcal{PD}(n)$ is diffeomorphism. In the Log-Euclidean framework,  $S\mathcal{PD}(n)$  is Lie group that has a linear space structure; rather than direct , computationally inefficient manipulations in the Riemannian space, the SPD matrices can be equivalently handled in the logarithmic domain with Euclidean operations.

Hence, we compute the logarithm of the SPD matrix  $\mathbf{P}(\mathbf{z})$  (without ambiguity, we may appropriately omit the argument  $\mathbf{z}$ ). The SPD matrix  $\mathbf{P}$  has a unique eigendecomposition  $\mathbf{P} = \mathbf{U} \operatorname{diag} \{\lambda_1, \ldots, \lambda_n\} \mathbf{U}^T$ , where  $\mathbf{U}$ is an orthonormal matrix composed of eigen-vectors of  $\mathbf{P}$  and  $\operatorname{diag} \{\lambda_1, \ldots, \lambda_n\}$  denotes the diagonal matrix with the eigen-values  $\lambda_i$  of  $\mathbf{P}$  as its diagonal entries. Through the eigen-decomposition form of a SPD matrix  $\mathbf{P}$ , its logarithm can be efficiently computed as

$$\log \mathbf{P} = \mathbf{U} \operatorname{diag} \{ \log(\lambda_1), \dots, \log(\lambda_n) \} \mathbf{U}^T$$
 (5)

Due to symmetry,  $\log \mathbf{P}(\mathbf{z})$  has n(n+1)/2 free parameters. We pack the upper triangular part of  $\log \mathbf{P}(\mathbf{z})$ ,

m

obtaining the LECM  $\mathbf{v}(\mathbf{z})$  in vector form

$$\mathbf{v}(\mathbf{z}) = \begin{bmatrix} \log \mathbf{P}_{11} \ \log \mathbf{P}_{21} \ \log \mathbf{P}_{22} \ \dots \ \log \mathbf{P}_{nn} \end{bmatrix}$$
(6)

where  $\log \mathbf{P}_{ij}$  denotes the  $(i, j)^{\text{th}}$  entry of  $\log \mathbf{P}$ . Note that the LECMs can now be handled with common Euclidean operations.

# 2.3 Ordinal encoding of LECM and iris matching

Ordinal measures qualitatively represent the relative order or rank of quantities at varying positions. Sun and Tan [12] first studied the ordinal measures based iris recognition and obtained very promising results. For simplicity, we only consider the order relation of two LECM features that are d pixels apart. Let  $\mathbf{v}(\mathbf{z})$ and  $\mathbf{v}(\mathbf{z}')$  be the LECM features at spatial position  $\mathbf{z} = (x, y)$  and  $\mathbf{z}' = (x + d, y)$ , respectively. We define the following binary code  $b(\mathbf{z})$  for characterizing the local iris texture

$$b(\mathbf{z}) = \sum_{i=1}^{n(n+1)/2} 2^{i-1} H(v_i(\mathbf{z}), v_i(\mathbf{z}'))$$
(7)

where n = 7 denotes the raw feature dimension,  $v_i(\mathbf{z})$ denotes entry *i* in the vector  $\mathbf{v}(\mathbf{z})$  and  $H(v_i(\mathbf{z}), v_i(\mathbf{z}'))$ is the Heaviside step function which equals unitary if  $v_i(\mathbf{z}) > v_i(\mathbf{z}')$  and equals zero otherwise.

Note that  $b(\mathbf{z})$  is computed from the raw features within the square region  $\Omega_r(\mathbf{z})$ . According to the mask image, we can compute the effective iris region rate (EIRR) of  $\eta(\mathbf{z})$  of  $\Omega_r(\mathbf{z})$ . It is computed as the black pixel number divided by the square region area, reflecting the percentage of iris pixles that are not occluded or corrupted. The degree of importance of  $b(\mathbf{z})$  can thus be evaluated by the quantity  $\tilde{\eta}(\mathbf{z}) = \min(\eta(\mathbf{z}), \eta(\mathbf{z}'))$ .

To represent the texture properties of the overall iris image I, we utilize the ordinally encoded LECM features  $b^{I}(\mathbf{z}_{i}), i = 1, \ldots, m$  sampled regularly with horizontal and vertical strides s pixels. Let  $\tilde{\eta}^{I}(\mathbf{z}_{i})$  be the corresponding degree of importance of  $b^{I}(\mathbf{z}_{i})$ . The model of iris image  $I(\mathbf{z})$  can thus be represented by  $\{b^{I}(\mathbf{z}_{i}), \tilde{\eta}^{I}(\mathbf{z}_{i})\}_{i=1,\ldots,m}$ . Let  $\{b^{J}(\mathbf{z}_{i}), \tilde{\eta}^{J}(\mathbf{z}_{i})\}_{i=1,\ldots,m}$  be the model of iris image J. The similarity  $\rho(I, J)$  between the two iris images is defined as

$$\rho(I,J) = \frac{1}{C_{I,J}} \sum_{i=1}^{m} w(\mathbf{z}_i) b^I(\mathbf{z}_i) \oplus b^J(\mathbf{z}_i)$$
(8)

where  $w(\mathbf{z}_i) = \min(\tilde{\eta}^I(\mathbf{z}_i), \tilde{\eta}^J(\mathbf{z}_i)), C_{I,J} = \sum_{i=1}^m w(\mathbf{z}_i)$  and  $\oplus$  denotes the bit-wise exclusive or operation.

### **3** Experiments

We first use the UBIRIS.v2 database [9], one of the most challenging iris databases, for performance evaluation. The iris images were captured under the condition of visible lighting, at-the-distance and on-themove. They are far from ideal due to various noise effects, e.g., illumination changes, obstruction, image blurring, and specular reflections etc. We select a training set (RS) of 500 iris images and a test set (ES) of 1000 images which are not present in the RS.

Iris matchings of one-against-all are performed and two measures are used to evaluate the recognition performance: the Decidability Index (DI) and Equal Error Rates (EER). Higher DI indicates better discriminability of a biometric system while lower EER indicates higher accuracy of a recognition algorithm. Three parameters are involved in our method: r in LECM, d in ordinal encoding and the stride s in dense feature sampling. Fig. 2(a) shows DI versus r and d when s=2(solid red), 4 (dashed blue) and 8 (dot-dashed green), respectively. Overall DI grows when s lowers; for fixed s, DI tends to increase with larger r and d, reaching maximum at some point and then decreases. Fig. 2(b) shows the histogram of the EIRRs of the iris images in the RS; on average 29.3 percent of pixels are occluded or corrupted. This may partly explain why smaller strides are beneficial because with small s we can utilize as many effective regions as possible. Among the top five triples that have highest DI values, we select the triple (s=2, r=8, d=16) with smallest EER for evaluation in the test set. The result as listed in Table 1 is obtained via 3593 intra-class comparisons and 495907 inter-class comparisons. It is better than that obtained by our previous method which ranked among the best in Noisy Iris Challenge Evaluation–Part II (NICE.II) [6].

The CASIA-IrisV3 database [1], obtained under near infrared illumination, is also used for our experiments. In the CASIA-IrisV3, we are interested in the CASIA-Iris-Lamp subset because it contains varying illuminations which causes high nonlinear deformation of iris texture. We also select a training set of 500 images and a test set of 1000 images. Fig. 3 shows the performance of the proposed method in the training set. We see from Fig. 3(a) that though overall DI grows with decrease of s, the degree of growth is small compared to that in the UBIRIS.v2 database. The reason may be that the EIRRs are large as shown in Fig. 3(b), indicating that the portion of effective iris region is large (on average 15.2 percent of pixels are occluded or corrupted) and therefore insignificant improvements are achieved with smaller s. We finally select the triple of s=4, r=12, d=16 for testing and the result is listed in Table 1. The result is



Figure 2: Performance in the RS from UBIRIS.v2



Figure 3: Performance in the RS from CASIA-IrisV3

Table 1: Performance in the test sets

Iris databases (Comparisons Num)	DI	EER
UBIRIS.v2 (Intra:3593,Inter:495907)	1.5572	0.1809
CASIA-IrisV3 (Intra:9366, Inter:490134)	5.3221	0.0008

obtained via 9366 intra-class comparisons and 490134 inter-class comparisons. It is significantly better than that in UBIRIS.v2. We owe this superior performance to the better quality of the iris images in CASIA-IrisV3.

### 4 Conclusion

We present a novel method for iris-based human identification in non-ideal imagery. The main contribution of the paper is combination of multiple image cues, including spatial coordinates, intensities, 1st and 2ndorder derivatives as well as order relationship of iris texture. This combination is achieved via ordinal encoding of the Log-Euclidean covariance matrices. The experiments show the effectiveness of the proposed method. The future work concerns extensive evaluation of our method with a very large number of iris images. Fusion of our method with other recognition modality is also of our interest for boosting the recognition performance.

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