ADAM HARPER TO RECEIVE 2019 SASTRA RAMANUJAN PRIZE

The 2019 SASTRA Ramanujan Prize will be awarded to Dr. ADAM HARPER of the University of Warwick, England. This annual prize is for outstanding contributions by individuals not exceeding the age of 32 in areas of mathematics influenced by Ramanujan in a broad sense. The age limit has been set at 32 because Ramanujan achieved so much in his brief life of 32 years. The prize will be awarded at the International Conference in Number Theory during December 21-22, 2019, at SASTRA University in Kumbakonam (Ramanujan's hometown) in South India.

Adam Harper has made path-breaking contributions to analytic and probabilistic number theory by establishing a number of deep and surprising results. His fundamental researches, both individually and in collaboration, cover the theory of the Riemann zeta function, random multiplicative functions, S-unit equations, smooth numbers, the large sieve, and the recent highly innovative "pretentious" approach to number theory. In establishing these results, he has shown mastery over probabilistic methods which he has used with remarkable effect in analytic number theory.

Even as a second year PhD student at Cambridge University, Harper disproved a famous conjecture on sums of random multiplicative functions. It was widely believed that sums of random multiplicative functions ought to have a normal distribution (Gaussian law). In his PhD thesis, Harper demonstrated that sums of random multiplicative functions taken over integers in a large interval $[1, x]$, *do not* behave like sums of random functions. Bob Hough, as a doctoral student under Kannan Soundararajan, had shown that if the sum up to x is taken over integers with a prescribed number of prime factors k (k, arbitrary but fixed), then the Gaussian law holds. Harper first extended Hough's result using the theory of martingales, and showed that the Gaussian law holds even when if k varies with x, as long as k is small in comparision with $\log\log x$. However, "almost all" integers n in $[1, x]$ have about $\log\log x$ prime factors, so this improvement over Hough's result does not apply to the "almost all" situation. Harper stunned everyone by showing that when k is like a constant times $\log\log x$, then the distribution is not Gaussian, thereby disproving the conjecture. This work of Harper published in 2013 *Crelle's Journal*, led to an explosion of activity in this area.

In connection with the distribution of random multiplicative functions with values on the unit circle, there is a surprising conjecture of Helson (2010) that the cancellation would yield a sum of size much smaller that $\sqrt{x}$. Harper has recently shown that the precise order of this error is $\sqrt{x}/(\log\log x)^{1/4}$, by using ideas from multiplicative chaos and the theory of probability. Harper's method also applies to the sizes of character sums, and therefore is of great importance.

Another great advance due to Harper concerns estimates for the higher moments of the Riemann zeta function on the critical line. This problem going back to Hardy and Littlewood, is of fundamental importance in analytic number theory. There is a conjectured asymptotic formula by Keating and Snaith for

$$\int_0^T |\zeta(\frac{1}{2} + it)|^{2k} dt.$$

This conjecture is known to be true for $k = 1$ and $k = 2$, but has remained open for larger values of k. In 2009, assuming the Riemann Hypothesis, Soundararajan obtained almost the correct order of magnitude. But Harper, using the Riemann Hypothesis was able to get the correct order of magnitude upper bound $O(T \log^{k^2} T)$ for all values of k as per the conjectured formula. Here also, Harper's deep understanding of probability techniques was crucial.

Harper also made a significant impact on a problem concerning values of $\zeta(\frac{1}{2} + it)$ in short intervals posed by Fyodorov, Hiary and Keating, who conjectured an asymptotic for the maximum value on almost all intervals of length one. Harper settled the upper bound part of this question in the affirmative by estimating second moments on short intervals analogous to his work on random multiplicative functions. Harper's magnificent survey article on this topic based on his Bourbaki lecture is to appear in *Asterisque*.

Harper has also established very deep results on S-unit equations, namely equations of the form $a + 1 = b$, where a and b are integers all of whose prime factors come from a finite set S. Of special interest, and importance, is the case where a and b are smooth numbers, namely integers all of whose prime factors are small; typically one considers integers up to x all of whose prime factors are $< y$, and allow y to vary with x. Smooth numbers are of fundamental importance in various number theoretic problems. Another remarkable paper published in *Compositio Math.* (2016) deserves mention; there he investigates exponential sums over smooth numbers and establishes minor arc estimates which were peviously known by assuming the Generalized Riemann Hypothesis. This had major implications on the solubility of equations like $a + 1 = b$ in smooth numbers a, b, which has ramifications in transcendental number theory and the $abc - conjecture$.

For smooth numbers, a natural conjecture is that they are uniformly distributed in arithmetic progressions. The Bombieri-Vinogradov and the Barban-Davenport-Halberstam theorems concern the uniform distribution of primes in arithmetic progressions; Harper has the strongest analogues of these theorems for smooth numbers.

In the last few years, a far reaching technique in analytic number theory is the "pretentious approach" due to Granville and Soundararajan. It is based on a 1974 theorem of Halasz concerning the classification of multiplicative functions. In 2013 Harper gave a different proof of Halasz' theorem which threw an entirely new light on the "pretentious approach". Consequently, Harper forged a major collaboration with Granville and Soundararajan on Halasz' theorem and its consequences, resulting in two papers in *Compositio Math (2019)* and *Proceedings of the AMS (2018)*.

Harper's breadth is evidenced by his publications spanning several areas in number theory. Of note are Harper's joint paper with Ben Green on inverse problems pertaining to the large sieve that appeared in *Geometric and Functional Analysis* in 2014, and two papers on prime number races (jointly with Youness Lamzouri in *Probability and Related Fields (2018)* and another jointly with Kevin Ford and Lamzouri to appear in *Mathematische Annalen*).

Adam Harper was born in Lowestoft in the United Kingdom. He did a four year MMath Course at Exeter College, Oxford University and won the Oxford Junior Mathematics Prize. He completed his PhD in 2012 at Cambridge University under the guidance of Professor Ben Green, and as a PhD student won the Smith Essay Prize. He was a Post-Doctoral Fellow with Professor Andrew Granville at CRM Montréal during 2012-13, following which he was a Research Fellow at Jesus College, Cambridge University during 2013-16. He returned to Montréal in 2018 as Simons CRM Visiting Professor. He is currently an Assistant Professor at the University of Warwick.

The SASTRA Ramanujan Prize Committee for 2019 comprised Professors Krishnaswami Alladi (University of Florida - Chair), David Bressoud (Macalester College), Kevin Ford (University of Illinois, Urbana), Gerhard Frey (University of Essen), Robert Tijdeman (University of Leiden), Shouwu Zhang (Princeton University), and Maryna Viazovska (Ecole Polytechnique, Lausanne). Once again, the SASTRA Ramanujan Prize has recognized highly influential and groundbreaking work by a very young mathematician.

Krishnaswami Alladi
Chair - SASTRA Ramanujan Prize Committee