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Probabilistic Relational Learner Models Based on Competence Maps

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Abstract. We present a proposal for a computational representation of competence maps that emphasises relationships of inclusion/part-of and specialization/generalization, and a generic approach to the construction of probabilistic relational learner models based on those competence maps, in which conditional probability tables are built on the basis of the kind of relationships between competences and, for the case of inclusion/part-of relationships, on the number of those relationships. We justify the use of *noisy-or* as a substitute for composite conditional tables produced by a competence being part of many other competences. Preliminary testing of both frameworks, for computational representation of competence maps and the construction of probabilistic graphical models from them, suggest coherence with reality.

Keywords: Competence map, relational probabilistic models, learner modelling, conditional probability tables.

1 Introduction

Competence-based education is a world-wide but relatively recent trend [1] which tries to go away from fact memorisation by establishing what students should be capable of doing with their knowledge (i.e. their *competences*), there is still much to do in order to fully implement competence-based education. Most importantly, teachers need to think in terms of competences while teaching and evaluating their students.

Today, a large number of educational programmes are defined in terms of the competences their graduates are going to have, and the competences students are going to develop on each course [3,8,9]. Yet, most of the information regarding competences is held in the design phase, in the documents describing the programmes and course designs. You may find some information regarding competences in some textbooks and online courses, appreciate less competence orientation in learning activities, a little bit of competence-based evaluation by teachers and, finally, almost no information regarding what competences the students have develop (and to which level) in their kardex, and even less information on that in their certificate.

Furthermore, evidence regarding the development of competences do not accumulate, and that is particularly the case for transversal competences, such as problem solving and team collaboration. Many such competences are develop along many courses, specialised in several context (e.g. problem solving in mathematics, in communication, in biology), yet there is no accumulation of evidence regarding the general case, nor we use it to tailor teaching (e.g. a student that has proven to be a good problem solver in other fields of study may need a different teaching that a student who has proven otherwise).

On the particular case of online learning, much of the work of gathering information regarding the development of competences by students can be automatized, so that it is carried out by the learning management system, but it is generally the case it has no information regarding competences, nor information regarding commonalities of competences among several activities dispersed along a few courses.

So our proposal goes on the way of attending these problems by providing the system with detailed information regarding competences, their interrelationships, and their relations to course activities, so that evidence of their development by students can be accumulated, and visualised by anyone interested and proper permissions. Furthermore, we propose mechanisms for evidence propagation, so that knowledge about the student having developed certain level of some competences can be transformed into knowledge concerning the indirect, preliminary development of related competences.

2 Related Work

Most of the work in computational encoding of competences has do to with their description and cataloguing. Consequently, the standard developed on this subject attends precisely these topics [1]. The establishment of relationships between competences is let as optional in the standard, as well as in other tools that support the construction of competence maps, such as COMET [7].

More related work has been focused on the construction of hierarchies of competences (Competence Structures) using relationships of type inclusion/part-of for learner modelling [6], a relation type and goal shared with the work reported here. However, the authors could not find further information regarding the implementation of a belief propagation mechanism that makes use of the hierarchical structure.

3 Competence Maps

Our competence maps are built on a notion of *competence* as the ability to perform a given action in a given context, which demands the mobilization of various cognitive resources (knowledge, skills, attitudes and values) [5], and two kinds of relations among competences: inclusion/part-of, and specialization/generalization. An example of such a competence map is given in Figure 1.

3.1 Inclusion/Part-of

This relation takes into account the observation that competences come in different sizes in educational programmes. There are small competences that clearly decompose into a few attributes, and there are large competences that look like complex agglomeration of attributes, while its process of execution seem to decompose into subprocesses—for example, in the domain of Physics, mathematical competences could be regarded as basic competences, as only their products may be of interest, while physics competences would be described in detail, decomposing them into sub-competences. So in our competence maps we distinguish between *simple competences* that decompose directly into its attributes (knowledge, skills, attitudes and values), and *complex competences* that decompose into *sub-competences*. In the second case, the *super-competence*'s attributes are composed by those associated directly to it, plus the ones associated (directly or indirectly) to its sub-competences.

There is no restriction in the number of (super)competences that include a given (sub)competence. For example, as shown in Figure 1, operating mathematical language to achieve a result to be interpreted in context can be seen as a competence that is a necessary component of problem solving in natural sciences such as Physics and Chemistry.

3.2 Specialization/Generalization

This relation takes into account the observation that there are competences that seem more specialised than others; competences that, on one side, seem to include additional attributes (e.g. 'writing formal letters with Microsoft 365') while, on the other side, seem to lack of generality in comparison to other competences (e.g. 'writing formal letters with a word processor'). So in our competence maps a competence can be specialised or generalised by another competence through the addition[4] of attributes such as knowledge, skills, attitudes or values.

In contrast with the inclusion/part-of relationship, while a competence can be specialized by many other competences, a competence can only specialise a single other competence—in the same way that, in many object oriented programming languages, a class can only specialise another class, not multiple ones.

[4] It could mean *replacement*, but we are not considering it, yet.

3.3 Interplay of Relationships

Both kinds of relationships among competences are not isolated one from the other. If a competence specialises a complex competence, it inherits not only its attributes but also its structure (see example in Figure 1); that is to say, it becomes a complex competence with the same number of sub-competences, each one a specialization of a corresponding sub-competence in the original competence.

3.4 Example

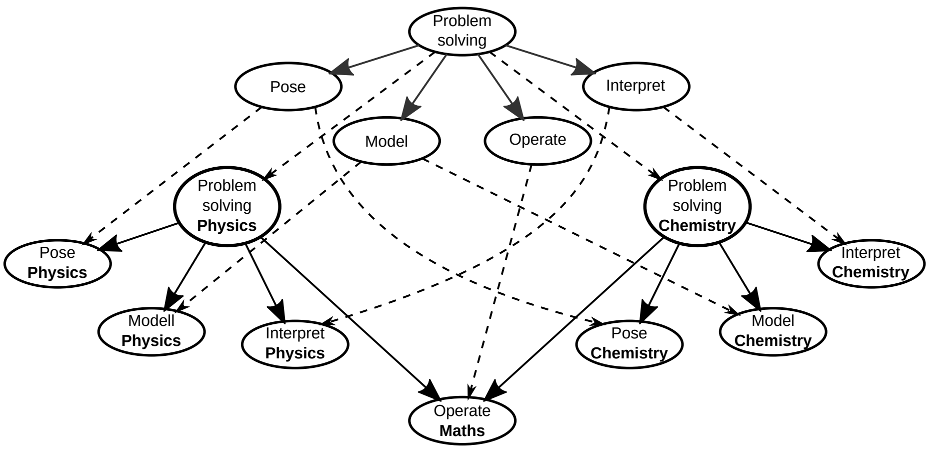

Fig. 1. Example of a competence map for a simplified version of the problem solving competences in Physics and Chemistry.

Figure 1 demonstrates what has been explained so far, using the competence for problem solving to illustrate both kind of relationships and its interplay. The figure includes a complex generic competence for problem solving that gets specialised (dashed lines) into problem-solving competences in the fields of Physics and Chemistry, so both specializations inherit the structure of the generic competence, and their sub-competences (solid lines) specialise the corresponding generic sub-competences of the generic competence for problem solving.

In this example, operating the mathematical language is considered a necessary sub-competence for problem solving in both fields, so this appears connected both to problem-solving in Physics and to problem-solving in Chemistry, and specialises the generic competence for operating in a formal language.

4 Probabilistic Relational Models

A competence map as the one shown above can be used as the base for a probabilistic relational model [4] of the competences being developed by a student, using the relationships between competences for propagating evidences regarding the level of competence developed by a student. For example, if we know a student has demonstrated a high level of competence in problem solving in a few fields (e.g. in physics, mathematics, communication, social conflicts) then we can infer it would not start from zero in problem solving in another field (e.g. in chemistry) but rather exhibit a certain level of development of the problem-solving competence in that field from start. Furthermore, a competence maps as the one shown above allows modelling of the development of a generic problem-solving competence that corresponds to the intuitive sense of the student being generally good at problem solving.

A first approach to developing probabilistic relational model from a competence map would be to base the construction of the conditional tables only on the kind of the relationship and, for the case of the inclusion/part-of relationship, on the number of sub-competences. In that way, the conditional tables can be constructed automatically from a set of principles and rules, as shown below.

4.1 Principles

Levels of competence. According to the educational theory of social constructivism [10] we consider three levels of competence:

- *Low.* The student cannot perform the action on its own nor with the support of others.
- *Medium.* The student can perform the action with the support of others, but not on its own.
- *High.* The student can perform the action on its own, without the support of others.

Conditional probability tables. The definitions of conditional probability tables per relationship type, as well as their composition in cases of multiple connections upwards, is based in the following assumptions:

1. If we believe that someone has achieved a high level of development of a competence, we would expect a better performance in a specialization of such a competence than if we know nothing about the development on the first competence. That is so because the specialization inherits the attributes of the original competence.
2. On the other hand, if we believe that someone has problems with a competence, we would expect they to have similar problems in most of their specializations. That is so because attributes not fully developed in the original competence are also inherited by its specializations.
3. The strength of the relationship between a given competence and another that specializes it does not depend on the existence of other specializations of the same competence.

4. If we believe that someone has achieved a high level of development of a super-competence, we would expect a relatively good performance in each and every one of their sub-competencies (with outstanding performance in some of them, and not so good performance in some others).
5. On the other hand, if we believe that someone has problems performing a super-competence, the difficulty could reside specifically in any of its sub-competences, or accumulate from difficulties in several of them.
6. The strength of the relationship between a super-competence and any of its sub-competences depends on the total number of sub-competences. That is so because the more sub-competencies, the more the performance impact of one of them on the performance of the large competence is diluted (considering an even share of attributes among sub-competences).

4.2 Conditional Tables for Specialization/Generalization

On the basis of principles 1 to 3 we can sketch the conditional probability table for a specialization/generalization relationship:

Table 1. First approach to the conditional probability table for the specialization/generalization relationship.

	Low	**Medium**	**High**
Low	Large	Medium	Small
Medium	Small	Large	Large
High	Very small	Small	Medium

If someone has the capability to perform an action on their own (high level of competence) in certain type of situations, it is likely to be able to do it in any situation that demands some additional resources (knowledge, skills, attitudes and values), although it is more likely that they will need some help, because they may lack some of the additional resources. Yet there is still the possibility that the specialization is so strong that they can perform the action in the new situation, even with support from others.

If someone has the ability to perform an action with support from others in certain kinds of situations, it is unlikely that they can perform it on their own in any kind of situations that demand even more resources. They are likely to need more support than before, or even they may be unable to perform the task at all. Yet, it may still be the case that having most of the additional resources could help them to perform the action, even without help.

If someone is unable to perform an action in a certain type of situations, even with support from others, it is very unlikely that they can perform it in any kind of situations that demand additional resources, unless their having of those additional resources provides them with some lever for compensating their lacks in the original ones. Yet, it seems more unlikely that they can perform the action on their own in the more demanding situations.

There are many ways to assign relative numerical weights to the fuzzy notions of probability used in the table above. We have considered three of them (see Figure 2): a linear distribution (1, 2, 3, 4), that provides a weak differentiation of probabilities; the standard normal distribution (cumulative probability at -3, -2, -1 and 0), that provides a strong differentiation of probabilities; and the binomial distribution with six trials (1, 6, 15, 20) and probability equal to 0.5, as a compromise between the previous ones.

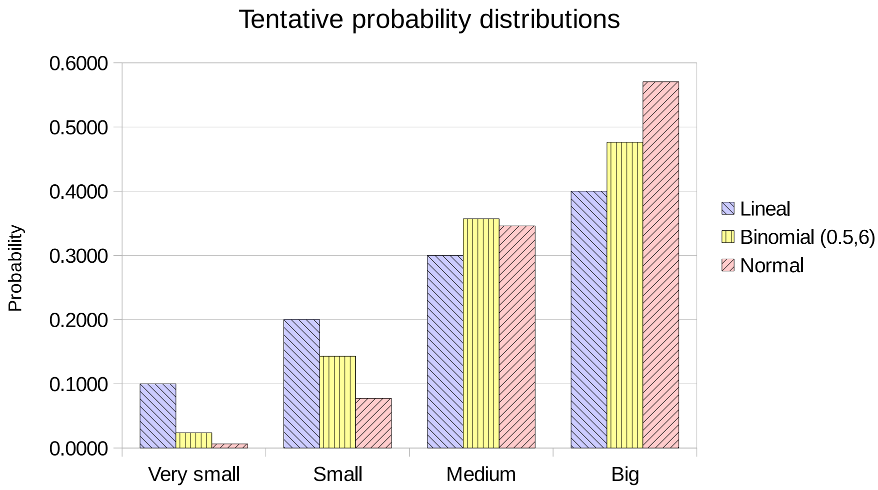

Fig. 2. Linear distribution, binomial distribution with six trials and probability equal to 0.5, and normal distribution.

4.3 Conditional Tables for Inclusion/Part-of

Consider the case of a competence C that includes n sub-competencies which we call $C_1, C_2, ..., C_n$. If someone is able to perform on their own the action associated to the competence in certain kind of situations, this means necessarily that they can perform each of its sub-competences without help. That is,

- $P(C_i(\text{High})|C(\text{High})) = \text{Large}$.
- $P(C_i(\text{Medium})|C(\text{High})) = \text{Small}$.
- $P(C_i(\text{Low})|C(\text{High})) = \text{Very small}$.

On the other hand, if someone is able to perform the action associated with the competence with support from others, but only thus, then at least one of his sub-competences must have an average level and none can have a low level. Yet, in order to do not discard the possibility of a sub-competence with a low level, we can calculate the conditional probabilities as follows:

- $P(C_i(\text{High})|C(\text{Medium})) = (2^{n-1} - 1)/2^n$.
- $P(C_i(\text{Medium})|C(\text{Medium})) = 2^{n-1}/2^n = \frac{1}{2}$.
- $P(C_i(\text{Low})|C(\text{Medium})) = 1/2^n$.

Finally, if a person as a low level in competence C (they cannot perform the associated action, even with support from others) this may be due to problems in the execution of any of C sub-competences. That is, of the 3^n possible permutations of levels of its sub-competencies, only those in which at least one of the sub-competencies has a low level is possible: $3^n - 2^n$. From here we can calculate the conditional probabilities:

- $P(C_i(\text{High})|C(\text{Low})) = (3^{n-1} - 2^{n-1})/(3^n - 2^n)$, because if C_i has a level other than low, at least one of the others must have it.
- $P(C_i(\text{Medium})|C(\text{Low})) = (3^{n-1} - 2^{n-1})/(3^n - 2^n)$, for the same reason.
- $P(C_i(\text{Low})|C(\text{Low}) = 3^{n-1}/(3^n - 2^n)$, because if C_i has a low level, the other sub-competencies can have any level.

The limit of the conditional probabilities as n approaches infinity is $\frac{1}{3}$, so the conditional probability distribution for the case $C(\text{Low})$ is flattened when the number of sub-competencies grows, and it does so rather quickly (with five sub-competences, the difference between conditional probabilities is less than $\frac{7}{100}$).

The full table of conditional probabilities is then as shown in Table 2 below.

Table 2. Conditional probability table for the inclusion/part-of relationship. The top row contains the possible competence levels for the super-competence, while the column on the far left contains the possible competence levels for a sub-competence.

	Low	**Medium**	**High**
Low	$\frac{3^{n-1}}{3^n - 2^n}$	$\frac{1}{2^n}$	Very small
Medium	$\frac{3^{n-1} - 2^{n-1}}{3^n - 2^n}$	$\frac{1}{2}$	Small
High	$\frac{3^{n-1} - 2^{n-1}}{3^n - 2^n}$	$\frac{2^{n-1}-1}{2^n}$	Large

5 Joint Conditional Probability Distributions

The restriction that a competence can only specialize a single other competence reduces the case for a competence to have several relationships upward to the form $< \text{Specialization}, \text{Subpart}_1, \ldots, \text{Subpart}_n >$. Besides, the characteristics of the conditional probability distribution for the inclusion/part-of relation—in the sense that a medium or high level at the super-competence strongly conditions the level at any sub-competence, while a low level at the super-competence weakly conditions the level at any of its sub-competences—suggests we could combine all the conditional probability tables for inclusion/part-of relationships using a *noisy-or* [2].

6 Usage

Given a competence map, corresponding to an educational programme or course, it is possible to create a probabilistic relational model for every student. Its initial state will be given by the probability distributions for the possible levels at the top competences (e.g. uniform distribution) which will be propagated through the whole map using the corresponding conditional probability tables.

Evidence of levels of competence will come mostly at the lower layers of the competence map, as those competences will be associated to more concrete situations, where student performance will be easier to observe: you cannot observe a student using the idea of a generic word processor, but you can see them using LibreOffice 5 Write. The evidence will be propagated to the rest of the map using the same conditional probability tables, so knowledge about more general competences and peer competences—specializations of the same competence; e.g. using Microsoft Word 365, will be accumulated.

In such a way, competence development along all activities in courses could be accumulated in the learner model, which would provide then a more detailed and accurate description of the state of competences development by the learner.

7 Conclusions

In this paper we have briefly described a formalism for the construction of competence maps with a focus in the relationships (specialization/generalization, and inclusion/part-of) between competences, which in turn can be used as the basis for a probabilistic relational learner model. The overall shape of the conditional probability tables has been deduced, with some details, from the levels of competence and the kind of relationship between competences.

In doing so, we have provided a general framework for the development of competence maps and their corresponding probabilistic relational learner models. The specialization/generalization type of relationship between competences leads necessarily to the notion of "generic/abstract competences", with so few attributes (or none) that it could be said that such "competences" cannot be observed in real life—that is so, because any concrete situation would demand more competence attributes (knowledge, skills, attitudes) in order to perform the task—yet they are powerful organizers in competence maps and provide a natural way for evidence propagation in learner models.

Future work includes the development of competence maps for educational programmes based on competences—such as the National High School System of Mexico, particularly in its online implementations at the Virtual University

System at the University of Guadalajara—, the generation of relational probabilistic models from them. We plan to use data from previous courses, evaluated by experts, so to construct learner models from previous students, and then validate their final states against the criteria from the same experts. Finally, our plan includes the development of mechanisms for the recovery of evidence from the activities in online courses, and visualisation of learner models, so to provide real time feedback to teachers and students about the develpment of competences.

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References

1. Competency Data Working Group: 1484.20.1-2007 IEEE Standard for Learning Technology—Data Model for Reusable Competency Definitions. IEEE Standard for Learning Technology, The Institute of Electrical and Electronics Engineers, Inc, New York, NY (Jan 2008)
2. Déz, F.J., Druzdzel, M.J.: Canonical Probabilistic Models for Knowledge Engineering. Technical Report CISIAD-06-01, Universidad Nacional de Educación a Distancia, Madrid (Apr 2007)
3. Fernández-Salinero Miguel, C.: Las competencias en el marco de la convergencia europea: Un nuevo concepto para el diseño de programas educativos. Encounters on Education 7, 131–153 (2006)
4. Koller, D.: Probabilistic Relational Models. In: Džeroski, S., Flach, P. (eds.) Inductive Logic Programming: 9th International Workshop, ILP-99 Bled, Slovenia, June 24–27, 1999 Proceedings, pp. 3–13. Springer Berlin Heidelberg, Berlin, Heidelberg (1999)
5. Perrenoud, P.: Construir competencias ¿es darle la espalda a los saberes? Red U - Revista de Docencia Universitaria 2 (2008)
6. Ramírez, C., Sánchez, E.: A Model for the Representation of Competences Applied to Student's Knowledge Modelling. Research in Computing Science Intelligent Learning Environments(56), 77–85 (2012)
7. Stoof, A., Martens, R.L., van Merriënboer, J.J.G.: Web-based support for constructing competence maps: design and formative evaluation. Educational Technology Research and Development 55(4), 347–368 (2007)
8. Subsecretaría de Educación Media Superior: Reforma Integral de la Educación Media Superior en México: La Creación de un Sistema Nacional de Bachillerato en un marco de diversidad. Tech. rep., Secretaría de Educación Pública, Ciudad de México (2008)
9. U.S. Department of Education: Competency-Based Learning or Personalized Learning, https://www.ed.gov/oii-news/competency-based-learning-or-personalized-learning
10. Vygotskiĭ, L.S., Cole, M.: Mind in Society: The Development of Higher Psychological Processes. Harvard University Press (1978)