Search Engine Marketing: The Case of Google Auction

by

Rui Eduardo Silva Oliveira

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Faculdade de Economia, Universidade do Porto

Supervised by:

Pedro José Ramos Moreira de Campos

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Biographic note

Rui Oliveira is a master’s student at Faculdade de Economia Porto, Mestrado em Econonomia e Administração de Empresas, who got his bachelor degree in Economics in the same institution.

He started working in a Adclick, S.A., a digital marketing company, at the end of 2013. Since then he has been connected to the same company and changed the role from managerial control to PPC manager. During this journey he has been able to work in the branch company in Brazil for around five months, in São Paulo. Currently he is managing the media acquisition and optimization of the health, fitness and lifestyle department.

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Resumo

O foco principal desta tese são os mecanismos de leilões dos motores de pesquisa da Internet (como Google e Yahoo). . Este foco no Engine Marketing (SEM) deve-se em grande parte ao facto de esta área mover biliões de dólares anualmente. Nesse sentido quisemos criamos um modelo para crar simulações com o objectivo de poder criar estratégias que gerem a maximização dos lucros do ponto de vista dos anunciantes. Com base nesse trabalho conseguimos apurar que a solução que gerou maior bem estar geral foi quando se assistiu a que os anunciantes licitassem exatamente o valor pelo qual avaliam o click, sendo que nos encontramos num leilão selado de segundo preço. No entanto a solução que gera maior bem estar não será de longo prazo porque o interesse individual de cada anunciante para maximizar o seu lucro não coincide com essa licitação generalizada. Através do nosso modelo criamos uma forma simples para a realização de simulações que permitem definir e compreender exatamente como o mecanismo está construído e obter resultados sobre Click-through-rate, Quality Score, cost-per-click e lucro para os anunciantes e o motor de pesquisa. Durante o nosso estudo criamos duas estratégias focadas nas ações que os anunciantes podem tomar com base na informação que possuem e concluímos que uma delas cria uma distribuição no sentido de redistribuir o bem estar geral para os anunciantes em oposição ao motor de pesquisa.

Códigos-JEL:

Palavras-chave:

Abstract

This thesis focuses on the fundamentals of Search Engine Marketing (SEM) and has the purpose of clarifying the mechanism behind the online search auctions, particularly of Google. We constructed a model which simulates the mechanisms used by the search engines to auction the slots available for advertising, when there is a query in the search engine. Focusing on the advertisers point of view we achieved results by which we can affirm that the highest total welfare is achieved by truthful value bidding from all the advertisers. However advertisers will have incentive to change their bids because they will not achieve their individual profit maximization when doing so. Using our model people can create their own simulations and change parameters as they will so they can understand how the mechanism behind search engine auctions actually works. During our study we suggest two different strategies and measure their results and concluding that one of them will actually drag the distribution of welfare in favour of advertisers instead of search engines.

JEL-codes:

Key-words:

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Chapter 1: Introduction

Nowadays whenever anyone is thinking in buying or selling anything they go to a given search engine and proceed by comparing and searching for information about the item they want to purchase. This simple gesture has been responsible for billions and billions of dollars. Companies like Google, Yahoo!, and Bing have been making huge amounts of money because of these searches, Edelman, B. *et al.* (2007). They are today one of the biggest distributors of information. This means that they are able to create an opportunity for companies to communicate with their clients and drive sales as well as influence the minds of millions of people about almost everything. This combination is enough reason to pay attention to search engines and online advertising.

Search engines are platforms that permit advertisers to engage the users with their brands or products, whether they are physical objects, information or even services, as mentioned by Wordstream Blog[1]. The combination of search engines and advertisement is called Search Engine Marketing (SEM). This means that advertisers may connect to consumers using the keywords that they searched for, generally called queries. For every query that a user searches for there are two types of results: paid advertisements, and free content that is ranked by the relevance to the given query, also called organic results.

However Google and others search engines only bill the advertiser when the users click the advertisements, meaning they might be interested in what they are advertising. This is actually one of the Search Engine Marketing's greatest strengths because it gives advertisers a way to impact their costumers in the purchase decision. This billing model is known as pay-per-click (PPC) and the amount they paid is referred as cost-per-click (CPC).

This is actually a complex mechanism that puts together game theory, auctions and economics. Three different types of agents enter in this process that occurs in seconds: the search engine in which the query is made; the user who does the query; and the advertisers who try to make the user click in his advertisement. Every time this process happens there will be an auction. In this auction it will be decided the order by

1 http://www.wordstream.com/ppc accessed 29/09/2016

which the advertisers will appear to the user, which is called the ad position, and how much each advertiser will pay in case there is a click in their ad.

Consequently in this thesis we purpose to examine the economics of search engines. Our main goals will be to understand and shine light on the mechanisms that underlie an ad auction. In addition focusing on the advertisers point of view create strategies that intend to maximize the advertisers' profit.

In order to accomplish our goal, we outline the theoretical foundations of auction and game theory and explore auctions as a pricing mechanism of ads. Furthermore we also investigate Google case of ad auctions and create a model which will help us simulate this auctions. In more detail we intend to simulate results based on numerical methods, in a environment on which various agents interact between each other and have influence in one another's results.

This way we will be able to clarify the mechanisms and strategies that maximize the advertisers' profit and also create a tool that can be used to simulate and help understand the complexity of the Search Engine Marketing industry.

This Thesis consists of six chapters. After this Introduction, in the second chapter we define the main characteristics of Search Engine Marketing, particularly the concepts that are needed to understand the mechanisms and the incentives for each player. Chapter 3 is an overview and of the main auctions, their characteristics, types and mechanisms as well as their final result for both the seller and the buyer. We revise the auction design and which auction should be used in each situation. Chapter 4 is intended to be the link between auctions and its application to search engines. In fact, the Search Engine Marketing industry is evolving and is not a static mechanism. Chapter 5 explores ad auctions as a game and create simulated results from which we try to define strategies for both advertisers and search engines. Chapter 6 contains the main conclusions of the thesis and some future research lines.

Chapter 2: Search Engine Marketing

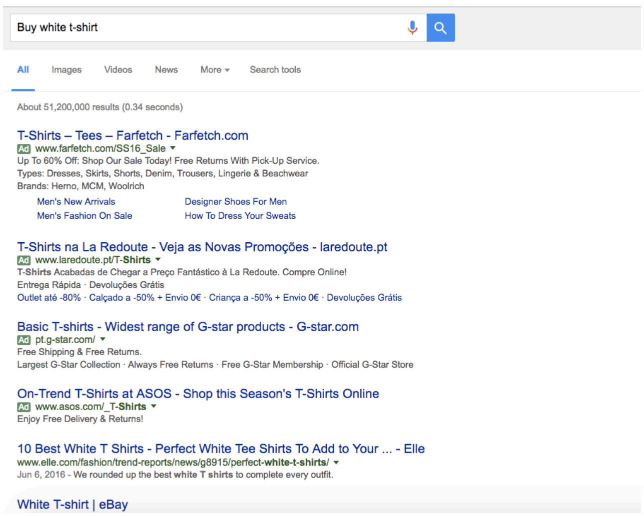
2.1. What is Search Engine Marketing?

According to Boughton (2005) Search Engine Marketing (SEM) is a tool which allows firms to target consumers by placing ads on search engines. This mechanism has proven to be an effective way in acquiring audiences. In contrast with other online advertising systems (e.g. Display advertising cost-per-mile, CPM, which consist of paying for every thousand views), in SEM advertisers only pay when users click on their ad. Search Engine Marketing is then the use of search engines with the objective of advertisement. In fact, by using users' queries we can target them in order to show the most relevant ads based on their searches.

Taking into consideration the Wordstream Blog[2], one of the most well know blogs related to SEM and pay-per-click (PPC), Search Engine Marketing is actually the practice of marketing using advertisements that are shown on search engine results pages. Bidders (the advertisers, usually firms) compete with each other using bids along with performance and relevance metrics that Google, Bing and Yahoo! take into consideration. This all happens at the same time people are looking for products or services, so advertisers appear alongside results for those search queries.

The example in Figure 1 shows that the first four results of the query "Buy white t-shirt" are Paid as confirmed by "ad" label.

Figure 1 - Query "Buy white t-shirts"



[2] http://www.wordstream.com/learn/ppc101 (accessed on 29/09/2016)

2.2. Search Engine Marketing as a platform between three players

We can say search engines serve as a distribution channel of advertisement from advertisers to users. This channel is one of the few that can engage users in the exact moment they decide to make an action. Advertisers fight each other for the users to make them focus on his ad. Additionally they compete with the non-paid results. Those are the ones with no label in figure 1 at the bottom of the page. This is actually one of the Search Engine Marketing's greatest strengths because it gives the advertisers the right moment to impact their consumers in the purchase decision. This billing model is known as pay-per-click (PPC) and the amount each user pays is referred as cost-per-click (CPC).

In order to better explain the way SEM works, first we have identify the agents involved in SEM: the search engine, the users and the advertisers, Hansen, J. (2009). Search engines profits from selling advertisement spaces, from Google institutional site[3]. Advertisers have the chance to impact users who are interested in their products in the exact moment they search for those things or products. On the other hand, users search for answers, services or products they might be interested. This way, they profit the most if they find relevant information.

In addition, these agents define three main relationships within SEM that we should focus on: search engine and users; search engine and advertisers; and advertisers and users.

Search engines have to balance the user's experience about free content that is most relevant to the user's query and create revenue through advertising. There is another important topic related to SEM called Search Engine Optimization (SEO) that focus on getting the most clicks from users and the best position possible for free. The main idea is that Web Pages that produce contents or sell their products or services online will be shown to users by the search engine as organic (also known as non paid results), based on their relevance and ranking to the given query, even without paying the search engine. Therefore search engines pick the best possible solutions from sites all over the web to answer the query that the user made.

In order for the search engines to maximize their profit they have to balance the number of clicks every advertise will get and how much every advertiser is willing to

[3] https://www.google.com/about/company/products/

pay. On the other hand, the advertisers intend to spend the less possible and get the maximum number of clicks.

The relationship between advertiser and user is mediated by the search engine. The search engine is the gateway for the advertiser to get in touch with the user. The user gets the information in searching for it and might interact with the advertisers if he wishes to do it.

Rangaswamy *et al.* (2009) investigated search engines, including in what way business can work with search engines and how high hierarchal executives should foresee the strategic impact of search engines.

2.3. Main Metrics in Search Engine Marketing (SEM)

The above mentioned relationships create interactions. Those can be measured by the means of metrics. The main metrics used in SEM are:

- **number of searches** for a given keyword or query, defined as the number of times the ad is shown (usually called **impressions)**;
- the **number of clicks** on the ad;
- the **click-through-rate** (CTR), defined as the number of clicks divided by the number of impressions.

Since search engines only bills from the clicks, the way to maximize their profit is to balance the price advertisers pay and the CTR. So, the whole process involving these interactions can be viewed as a game. In addition, this game consists in both parts trying to maximize their profits. Focusing on the advertisers, maximizing the profit means to spend the optimal amount given the amount of clicks they can get.

Advertisers' revenue can be achieved thought various ways. It may be fixed or variable. This is crucial because it sets the maximum amount the advertisers will be willing to pay. Also this game is repeatable and changeable each time it happens. As there is a limited space each search engine uses to show ads, it is needed to sort the advertiser by the income they can potentially generate for the search engine. In order to sort advertisers, the search engine needs to know how much they bid for each position, and for this reason the game described is an auction.

The auction is about the position every advertiser will get and also how much each one will pay in case they are clicked. We assume that first positions will get most clicks and for that reason they are the most desired ones. However since there are a limited number of positions available in every auction, they tend to create competitiveness.

In the next chapter we will focus on auctions and in chapter 5 we will continue with this topic by studying the Economics of Online Search.

Chapter 3: Auctions: main features

In this chapter will give an overview and main basics of several types of auctions, and then we will focus on sealed bid auctions.

3.1. Types of auctions

According to Dixit *et al.* (2015) an auction is defined as any transaction where the final price of the object for sale is arrived at by the way of competitive bidding. In this definition three main characteristics of auctions are considered: bids, price and object. By adding another property to this list - value of the object - we will be able to divide all the four properties in two groups: auction rules and auction environment.

Auction rules may be divided according to the way the bid is submitted and the way the final price is defined (Duta, 1999; Varian, 2010; Dixit *et al.*, 2015). The seller has to decide which rules to apply in order to get the highest price.

Open outcry auctions consist of an auction where bidders make their bids in real time. In other words all bidders are able to observe bids as they are being made. This type of auction can use an ascending price (English Auction) or a descending price (Dutch Auction).

The English auction is one of the most famous auctions. In this type of auction, the auctioneer sets a low starting price (reservation price) and calls out successively higher prices. Agents participating on the auction raise the bid for the object as the price is being called by the auctioneer to the point the last bid is placed and no other agent calls the price.

On the other hand, the Dutch Auction starts at extremely high value set by the auctioneer and starts to decrease. When someone bids and signals, he/she is prepared to pay the price and the auction ends. In these two types of auctions, the buyer who gets the good pays exactly the same amount that he bided, and the other bidders do not pay anything.

The second group of auctions is the **Sealed bid Auctions**. In this type of auctions, no one knows the others agents' bids. Bidding is done privately and bidders cannot observe any other bids. There is only one opportunity to bid on the object and the bidder with the highest bid wins. Then the final price is defined by either first price or second price. The first price mechanism says that the winning bid is equal to the price you will pay for the object. On the other hand the second price auction (or Vickrey auction) sets that the price the winner will pay is equal to the amount of the second highest bid, these auctions are extremely useful to determine truthful bids.

In addition to Auction Rules we have to look to **Auction Environment too**. This property concerns the value that the each bidder sets for the good being auctioned.

If the good being auctioned has a market value of, say, 100 €, his Common Value or Objective Value is then 100€ and no bidder will pay more than the market value, unless he/she evaluates the object's value outside its market value. Nevertheless, people may evaluate the object in other ways that are not so direct. That is called Private Value or Subjective Value and is usually associated to emotional value or personal value. For instance if we are auctioning a guitar that has been played by a major rock star, this object will most probably be sold by a price higher than it would if you would go to an shop and buy a new one because each person evaluates that object in his own way. The way agents evaluate the object in auction will influence the final price.

In summary, in the Object-Value auction, the price has a limit and every bidder has the same ceiling price. On the other hand in the Subjective Value each bidder may evaluate the object in his own price.

3.2. Auction design

In order to better describe an auction let us consider a single object auction in which there are n buyers (bidders) with private values $v_1, v_2, \dots v_n$. Every buyer makes at least one bid, $b_1, b_2, \dots b_n$ (only the last bid will count since there are auctions in which the buyer can bid more than once). The challenge is to design the auction to sell the item in order to get the highest payoff. This is a complex theme (Klemperer, 2002) and brings up a difficult question "So what makes a successful auction?" Klemperer (2002) points out that the key concerns are discouraging collusive, entry-deterring and predatory behaviour. He then concludes that the rules to define a good auction design are the same as elementary economics.

According to Varian (2010) the **Auction Design** pursuits two main goals: Pareto efficiency and Profit Maximization. Pareto efficiency means the best overall situation and consists in the agent with the higher value to win the good. Profit Maximization yields the higher expected profit to seller. In other words, in a situation where there is no Pareto efficiency, the maximum social welfare is not achieved. However, Pareto efficiency can be achieved by transferring the good between the winner of the auction and the person with the highest value.

If the seller knows the values for each bidder $v_1,v_2,\ldots,v_n$ the profit maximization is simple, since the seller would go directly to the bidder with the highest value and sell the good by a price ranging from the bidder's reservation price and zero.

A much more complex case is when the seller does not know the buyers value.

By using an English Auction the Pareto Efficiency will be achieved since the bidder with the highest value will get the good but this not guarantee the Profit Maximization because the price that the winner would pay is equal to the second highest value added by the minimum increment. By adding a reservation price equal to the highest value the seller will achieve Profit maximization but it is not a Pareto Efficiency solution due to the fact that sometimes the object would not be sold.

Testing Dutch Auction we conclude that this auction might not be a Pareto efficiency solution because the value depends on the expectations that the highest value bidder makes of the other bidders. In a situation where they under-evaluate the second highest bid they might lose the good.

The same thing happens when we analyse the sealed bid auctions with first price since the bid depends on their expectations to the others bidders' evaluation.

Finally, in the Vickrey Auction, which is equivalent to sealed bid auction with second price, the outcome will be the same as the English Auction since every player is in his best interest to write down his true valuation. For this reason, the Vickrey Auction is extremely employed, particularly on online auctions, and so, we will detail the Vickrey mechanism in the following section.

3.3. Sealed bid first price and second price auctions

As we exposed before the sealed bid auctions may assume two types: first price and second price. The mechanism and the results are different, although they are both static games with incomplete information. In this section, we will detail the analysis of both types of sealed bid auctions with an analytical presentation.

Let us consider a two bidders' auction for one object. The auction description is the following: There is one object and two bidders (i = 1, 2) who value the object as v_1, v_2 and bid it as b_1, b_2.

The winner's payoff is $v_n - b_n$ and the other bidder gets 0 (in case of a tie the winner is decided by a coin flip).

So, the payoff function is:

$$u_i(b_1, b_2; v_1, v_2) = \begin{cases} v_i - b_i & if \quad b_i > b_j \\ \dfrac{v_i - b_i}{2} & if \quad b_i = b_j \\ 0 & if \quad b_i < b_j \end{cases} \tag{3.1}$$

The Bayes-Nash Equilibrium is defined by a pair of strategies, one for each bidder, which solves:

$$\max_{b_i} (v_i - b_i) prob\{b_i > b_j(v_j)\} + \frac{1}{2}(v_i - b_i) prob\{b_i = b_j(v_j)\} \tag{3.2}$$

and holds the following solution:

$$b_i(v_i) = \frac{v_i}{2} \tag{3.3}$$

In the case of a second price auction, the payoff function is different: if the bidder wins the good, then his payoff is the value the bidder gives for the item minus the second highest bid: $v_i - b_j$.

$$u_i(b_1, b_2; v_1, v_2) = \begin{cases} v_i - b_j & if \quad b_i > b_j \\ \dfrac{v_i - b_j}{2} & if \quad b_i = b_j \\ 0 & if \quad b_i < b_j \end{cases} \tag{3.4}$$

Note that each bidder's payoff is not directly related with his own bid because if he wins he will not pay his own bid.

According to Varian (2010) and by the mean of Vickrey (1961), we can prove that the Second Price Sealed bid Auction, or Vickrey Auction, creates the conditions so that the buyers bid their truthful value.

The Bayes-Nash Equilibrium is then defined by a pair of strategies, one for each bidder, which solves:

$$\max_{b_i} \left(v_i - b_j\right) prob\left\{b_i > b_j\left(v_j\right)\right\} + \frac{1}{2}\left(v_i - b_j\right) prob\left\{b_i = b_j\left(v_j\right)\right\} \tag{3.5}$$

Considering two bidders, each one with value v_1 and v_2 also having bids b_1 and b_2 respectively, if $b_2 > b_1$ then the value to bidder 1 is 0. So if $v_1>b_2$, bidder 1 wants to have the highest probability possible to win the auction. On the other hand, if $v_1<b_2$ the bidder wants to have the least probability of winning by a value superior to the one he evaluates the good. So in either case the best option is to bid the truthful value.

To sum up, the Bayes-Nash equilibrium in the Vickrey Auction is:

$$b_i\left(v_i\right) = v_i \tag{3.6}$$

Furthermore we may consider the Vickrey – Clarke - Grooves (Vickrey 1961; Clarke, 1971; Groves, 1973) (VCG) mechanism. This variation of the Vickrey Auction is designed to multiple item auction, having as a goal to achieve the maximum social welfare and consequently Pareto Efficiency.

Let us suppose we are selling a homogeneous set of items. Every buyer can bid for more than one time since the bid consists in quantity and unit price. All bids combinations are considered and then the combination maximizing the sum of bids is the winner. The only condition when considering the best combination is that only one bid per buyer may be used and the total number of items cannot exceed the quantity that the auctioneer was selling. The price they pay is actually not the bid they made but only the net cost they impose to other buyers. At most they will pay as much as they bidded. In other words, in the VCG mechanism the main idea is that the bidders should only pay the net cost they impose to others.

However, VCG mechanism has a difficulty related to budget balance, since the total payment received is lower than the total cost of the goods being auctioned. In fact, the price the player that won the auction has to pay is equal to the difference between the sum of the bids if the player did not participate in the auction and the sum of the bids that won the auction minus his own bid. The first describes the situation if the player is absent or did not participate and the second considers the situation if the player is present.

Chapter 4: The economics of online search

This chapter intends to clarify the main questions about online auctions in search engines, particularly how they work, how they profit from it and how the advertisers are sorted. At the end we give the fundamentals to understand the main mechanisms of search engines auctions.

4.1. Position auctions

Position Auctions relate with Search Engine Marketing (SEM) in the sense that SEM has found in auctions an optimal way of creating revenue for their product: “Internet Searches”. The product is the advertising space in the search engines results page, each time someone searches a given query. Every page has advertising spaces that can be sold, however the profitability does not simply comes from the fact that people see the advertisement. In fact, buyers/advertisers pay each time someone clicks the ad. Position Auctions focus in which way we can sort the advertisers, who wins the auction, and how much they will pay, having under consideration that the seller aims to maximize its profit as is described by Varian (2010).

Firstly every player ranks the positions in the same way. This means everyone prefers to be in the first position rather than in the second. However they might value the positions in a different way. In other words, they all prefer first positions but are not willing to pay the same amount for that position. Since every advertiser sells different things, their expected value for a click is different from the others.

Let us take a simple example concerning an Online Position Auction, similar to Google and Yahoo. Suppose there are S slots available, $s = 1, 2, \dots S$, and x_s is the number of clicks the Advertiser will receive in slot s. As we said before, first slots are preferred to the others so we assume that $x_1 > x_2 > \dots > x_S$.

Taking into consideration the profit from advertising, the advertiser calculates his value per click v_s. When entering in the auction the advertiser places a bid b_s that is the value he is willing to pay for the slot s.

In this type of auction all advertisers that are assigned to a slot are the winners. They will be ranked by their bids. This means the highest bid will get the first slot, the second highest bid the second slot, and so on. On the other hand the advertiser in position one will pay only the price equals to the second highest bid, the advertiser in position two will pay the amount equals to the third highest bid, and so on. This variation of Vickrey Auction is called Generalized Second Price Auction, or GSP.

Suppose we have 2 slots and 2 bidders. Each bidder's value is v_i and his bid is b_i. The first slot gets x_1 clicks and the second gets x_2. The advertiser with the first slot will pay the second highest bid and the second advertiser will pay the reservation price, r.

Imagine we are bidder 1 with valuation v_1. The payoff function for bidder will be:

$$\begin{cases} (v_1 - b_2)\ if\ b_1 > b_2 \\ (v_1 - r)\ if\ b_1 < b_2 \\ 0\ if\ b_1 < r \end{cases} \tag{3.7}$$

which means that the expected payoff will be:

$$Prob(b_1 > b_2)(v_1 - b_2)x_1 + [Prob(b_1 < b_2) - 1](v_1 - r)x_2 \tag{3.8}$$

Rearranging the equation we get that:

$$b_1 x_1 = v_1(x_1 - x_2) + r x_2 \tag{3.9}$$

This should be the bidding method that guarantees that profit is normal at minimum. This is a dominant strategy since both players will want to bid according to this formula, regardless what the other player bids.

The contest in this case is about those extra clicks from staying in first position. In this case you will want to bid the true value from getting more clicks.

According to Varian (2010) there are other ways to rank positions apart from only the bid value. Next we present the Quality Score. This measure of quality is a metric of performance and relevance. In that sense this metric is a way to compare advertisements from different advertisers and how they perform, based on users interaction. Quality Score mainly uses the number of clicks divided by the number of times people see the advertisement (impressions), considering historical click-trough-rate (CTR) values.

Taking this into consideration, a new concept emerges, the Ad Rank. This metric is calculated by multiplying the Quality Score and the bid, in order to create a rank based not only on the advertisers' bid but also taking in consideration the Quality Score component. So the highest rank would get the first position, the second highest rank the second position, and so on. On the other hand the advertiser would only pay the amount needed to maintain his position:

$$p_1 q_1 = b_2 q_2 \ \ or \ \ p_1 = b_2 q_2 / q_1 \qquad (3.10)$$

In this case, if $p_1 > b_2$, this means that player 1 has a lower Quality Score, and the other way around in the case $p_1 < b_2$. This means that advertisers with higher Quality Scores pay less than other advertisers.

The fact that the main component of Ad Rank is historical CTR means that actually this auction is trying to maximize the cost per impression. The advertisers who actually pay more are the ones with higher positions.

According to Edelman *et al.* (2007) the GSP is a variation of the sealed bid second price auction and is compared to VCG mechanism since it is designed to multiple item auctions. The GSP variation is actually what SEM uses in its auctions. In contrast to VCG, the GSP mechanism does not imply truthful bidding and also does not have a dominant strategy.

Since advertisers can change their bid at any point, the online search auctions can be modelled like a continuous and infinitely repeated game. Following Edelman *et al.* (2007), bids can be changed at any time and as often as any advertiser wants and only one bid per keyword can be placed per each advertiser. In the Search Engine Marketing the slots available at any time can only be used once and if they are not filled they are lost. Every time a query is typed in search engines there is a new auction, consequently, advertising slots are not stackable.

In addition, since every advertiser has his own goals and targets that might and will most probably differ from each other, there is no universal measure to fit all purposes. This way the best match is the SEM way of billing, cost-per-click (CPC).

We will consider that there are S slots available s=1,2,…S and N bidders or advertisers, n = 1, 2,…, N. The x_i are the expected clicks for an advertiser placed in position *i*, for $i < j$, $x_i > x_j$.[4] The value associated with users click or in other words, the revenue generated by each click, is the same independently if it comes from ad in slot 1 or S. Clicking in position 1 or S has the same value per click, v_i, to the advertiser.

4.2.The Google Ad Rank

According to Google Support[5], the Ad Rank is a value that is used to determine your ad's position. The key components in this rank are the advertisers' bid, the Quality Score and the expected impact of extensions and other ad formats. Google Ad Rank's formula is not shared or transparent to the users so one cannot calculate their Ad Rank. However their metric is calculated each time your ad is eligible to enter in an auction. So your ad position can change each time depending on your metrics and your competition.

Even though the formula is not known and freely shared by Google, using the aid of companies which manage millions and millions of dollars a month in advertising and spend them on Google's Platform we are able to get an approximation about the weight each component has in the Ad Rank calculation.

So according to the Search Marketing Expo (Seattle, 2016) presentation of Brad Geddes (Geddes, 2016), he points to a distribution of 50% Bid weight, 40% in Quality Score weight and finally 10% of Extension's impact and ad formats. He also discovered that Quality Score is constituted by three main factors: Expected CTR, Ad relevance and Landing page. He also conducted a search in which he found that: the expected CTR has a strong correlation between Average CPCs and Financial impact in the same position; the Ad relevance has a partial correlation to CPCs; and that Landing Pages have the weaker correlation to CPCs but there is a correlation with impression share. Summing up, as Quality Score gets higher so does the ad position which will make the advertiser compete with different advertisers than before and consequently not having a clear direct influence in CPCs. In addition, note the fact that the Quality Score increases and also does impression share.

Google's Ad Rank was not always calculated this way. It is not easy to track all the ways Ad Rank has been calculated in the past. However, in 2013 the Ad Rank was easily calculated and was available to the advertisers. Nowadays, Google has improved

[4] http://blogs.wsj.com/economics/2007/07/19/economics-according-to-google/ (accessed on 29/09/2016).
[5] https://support.google.com/adwords/answer/1752122 (accessed on 29/09/2016).

his calculus and preferred not to dismiss the formula has advertisers need to trust Google. Returning to old Google Ad Rank formula, Larry Kim (CTO of Wordstream and world famous PPC Manager) wrote on 30 of October 2013 that in order to calculate Ad Rank you must multiply your bid by your Quality Score. Given that Google lets you know what is your Quality Score and you control your bid, you were able to calculate and compare your Ad Rank and also using your CPC you could know your competitor Ad Rank. Using the CPC formula which is:

$$CPC = \frac{Adrank_{pos+1}}{Adrank_{your\,pos}} * bid + 0.01 \tag{3.11}$$

Even nowadays you can do this type of logic and get the ratio between you and your competitor Ad Rank. We will follow this logic in the following chapters.

4.3. Other studies about Online Auctions and Search Engine Marketing

In this section we gathered the studies most relevant to our theme in order to have a clear overlook to all the things that have been made regarding this theme in the past.

Concerning the Generalized Second Price Auctions, as the Google Ad Rank Auctions, they were studied by some authors in which we highlight Liu and Chen (2005), Aggarwal *et al.* (2006), Edelman and Ostrovsky (2007), Varian (2007), Bu *et al.* (2008) and Skiera *et al.* (2010).

According to Aggarwal *et al.* (2006) the GSP auction has been working well for Search Engine Marketing. In their work they defend that not all advertisers seek the topmost positions. Base on this fact, there are a variety of factors that contribute to the position preferences of each advertiser. On this study the authors studied the ranking ads system and the pricing model associated with an additional preference of the position in the rank. In more detail they considered the prefix positions auctions where advertiser can specify that is only interested in the positions between the first and one of their choice. On this topic, they concluded that inserting the prefix positions auctions preserves the equilibrium proprieties of a normal VCG.

Liu and Chen (2005) investigate the value of past performance information in the online search engine Auctions where advertisers may have different value per click and the ability to generate clicks is also a differential factor. Focusing on weighted Unit Price Contract auctions, which consists on the bidders making a bid per unit and ending up paying what they bided in case they win. Based on revenue maximizing and efficient they apply this framework to Google's and Yahoo!'s auction design. As the researchers conclude, efficient UPC auctions in which unit-price bids are weighted by expected CTRs, can achieve the first-best efficient allocations. While UPC auctions are not theoretically the optimal form they have practical applicability and reduce bidders' risks. They also proved that auctioneers can get higher results by using appropriated weighted factors based on past performance. In resume they come to the point in which firms that are concerned about assigning advertisement slots to those who value them the most should weight advertisers' unit-price bids by estimates of their future click-through rates. On the other hand, firms that are concerned about total revenue should prejudice more toward low-CTR advertisers than suggested by efficient UPC auctions.

As for Edelman and Ostrovsky (2007)'s work, they examined the Sponsored Search online Auctions of Yahoo! and Google and proved that there was evidence of strategic bidder behaviour in this auctions. They also estimated what could have been the revenue of these search engines if they were able to prevent this behaviour. In this study they propose a mechanism that could decrease the amount of strategizing from the players and in addition grow the revenues of the search engines and also the efficiency of the market. According to this study, the authors believe that the Auctions should be reviewed and rethought in order to change to a VCG mechanism instead of a first price auction as it was at the time in Yahoo!.

Bu *et al.* (2007) claim they introduced forward-looking Nash equilibrium in the GSP auctions. They found one unique solution for this type of auctions and also found the solution which is equivalent to the solution under a VCG mechanism. As said by the authors, the position auction is not an incentive compatible protocol. However the fact that this improvement results in setting the same payoff for everyone equivalent to a VCG protocol solution justifies the practical protocol.

Chapter 5: Application

This chapter focus in creating a simulation of a real SEM auction and also review and examine the results. Based on the concepts and taking off the complexity of some factors in the Google online Auction (like the web page analysis and users' behaviour in site), we created a program in R[6] to help us simulate the auction. The results of the simulation are also discussed.

5.1. Simulation

In this study we will assume randomly generated values in order to achieve and examine patterns that will be generated in this auction. Following the literature, we assume that some factors will be constant and others will be variable. This way we will focus mainly in bidding strategies. Also, we will compare the total gains and the most efficient allocation of resources.

In order to proceed we must explain how advertisers see the auction and what they can actually do. So, advertisers can pick the keywords they are going to bid by using different levels of narrowness and broadness. Those levels are called Match types (Google, 2016). There are four match types in Search Engine Marketing:

- Broad;
- Broad Match modifier;
- Phrase Match;
- Exact Match.

They are different between each in terms of how broad or narrow they are, being the "Broad" the broadest one and "Exact Match" the narrower. When using Broad word match type your query range is any word somewhat related to the query you choose and whenever any word from your query is searched. However when using Broad Match Modifier you can say to the search engine that you want one word to mandatory be the same as you wrote and the others can change. Phrase Match you query must be written in the query in that order but might be together with a longer query. At last, Exact Match means that the query must be exactly the same as the user query. Using Match types you can narrow the range of auctions you actually participate and also improve the relevancy of your ads in relation to the queries you choose.

[6] The R Project, version 3.3.1., available at https://www.r-project.org/ (accessed on 29/06/2016).

In our simulations we will assume that every advertiser is **only using exact match for one determined query**. Every player will have the opportunity to change their bids from one round to another. Every round will have 10000 impressions meaning the same query will be entered 10000 times in the search engine.

So in this case we assume that Quality Score, Extension Score and Bid Score will be calculated based in the proportion of the relative maximum value of each category in a scale from 0 to 10. In the case of Quality Score and Extension Score the calculus are just made one time and based on one variable that is randomly generated between two values. For Quality Score the range is from 1 to 20, and for Extension Score is 1 to 5. Those variables represent Click-through-rate (CTR) and extension-click-rate.

We made two main assumptions: Each Advertiser has a maximum potential CTR. Either he actually achieve that maximum result or not, his Quality Score is based in this potential instead of being indexed to the actual CTR at every round; also the Quality Score metric is reduced to only account for CTR and ignoring all others possible variables assuming they don't create advantages nor handicap to any player.

Even though Extension click rate is not used to calculate nothing directly apart from the score and consequently the Ad Rank, it causes a direct effect on how the advertisers are sorted and how the occupation of the slots will be defined.

In Order to calculate Ad Rank we used three components that are specified by Google: Bid, Quality Score and Ad Extensions (including Ad Extension and Ad format).

To conclude the auction design we define the weight for each Score that is used to calculate the Ad Rank, meaning that we have a way to take in consideration both Quality Score and Extension Score and also Bid Score. This is made using a simple weighted sum:

$$\text{Adrank}_t = \text{Bid Score} * \text{Pond Bid} + \text{Quality Score} * \text{Pond QS} + \text{Extension Score} * \text{Pond ES} \tag{5.1}$$

This value can be changed at any time in our simulation but we will actually assume based on a presentation from Geddes (2016) who assumes that the weights are 50 %, 40 % and 10%, respectively.

To make sure that the weight is correctly applied we use a scale to each component. This scale is from 0 to 10 and his linear.

In our simulation only the bid score will change, because as we assume before that Quality Score and Ad extension impact are constant thought-out this game

We created a simulation with 5 Rounds, each round is equivalent to a set of 10000 impressions, and every round there is a new auction. On this auction 10 players participate in the bidding. They can bid any value higher than zero.

There are 8 available slots. However we can define any number higher than 0. The number of slots does not depend on how much players are bidding.

The Ad Rank is the variable that sorts the players in the auction, and is calculated as described in (5.1). The highest Ad Rank will be awarded the slot 1, in position 1, while the second highest the slot 2 and so on.

Each player values the click they get independently from other players. This value is constant trough the game. The variable is called value per click. This value per click is a random number between 0.05 and 2

The total revenue per player results from multiplying the number of clicks they got by the value per click. In turn, the click for each player is calculated by multiplying the maximum CTR by the weight of position and also by the number of total impressions.

$$Clicks_{player} = Max\ CTR_{Player} * Position\ weight_{playersposition} * impressions \tag{5.2}$$

The variable Max CTR sets the maximum Click-through-rate each player can get and is a number between 1 and 20. Actual CTR is calculated based on the players position. The player in position 1 will get 100% of his Ad Max CTR, while the second position will only get 90% of his own Max CTR. This is the way we found to set an advantage to players with higher Ad Rank.

Players with higher Max CTR will have more incentive to bid higher since a rise in position will get them a higher number of clicks in comparison with lower CTRs.

They have a marginal competitive comparison since they marginally win more clicks than competitors with lower click-trough-rate.

Position weight is not an accurate scale. It is not referred in other studies we saw. There is not an actual scale or study about this fact in particular. As a result, we use it based on experience that the same player in higher positions will get more clicks than in lower positions. We used a distribution that penalize players in much lower positions. This is due to the fact of creating a higher competition for the higher positions

With the goal of creating a comparison, we have developed two different ways to calculate the bid adjustment based on the information that the players have to simulate a true decision making.

The need to automate the process made us thinking of all the possible decisions advertisers could face and create a solution for each hypothesis. This was the premise because we wanted to make as many simulations as possible that we could analyse. Given this problem we deduced from the main calculus of CPC how to achieve a rational decision. This made possible to see changes in the auction in different situations and how advertisers would actually react by trying to bid higher or lower based on their capacity to analyse their profits. We also assumed a constant value during the simulation for the value per click, which we also use to calculate the revenue of each advertiser at any given time. We assume it is constant because even though it is not linear, in this kind of auctions and queries the user use to have a stable and homogenous reaction in the long term.

In this first solution which we will call strategy one, let us start by analysing formula 5.3.

$$\mathrm{CPC}_{t,i} = \frac{\mathrm{Ad\ Rank}_{t,i+1}}{\mathrm{Ad\ Rank}_{t,i}} * \mathrm{Bid}_{t,i} \tag{5.3}$$

Having this in consideration, we can conclude that:

$$\frac{\mathrm{CPC}_{t,i}}{\mathrm{Bid}_{t,i}} = \frac{\mathrm{Ad\ Rank}_{t,i+1}}{\mathrm{Ad\ Rank}_{t,i}} \tag{5.4}$$

And, ceteris paribus,

$$\text{Ad Rank}_{t,i+1} = \text{Ad Rank}_{t+1,i+1} \tag{5.5}$$

$$\text{CPC}_{t+1,i} = \frac{\text{Ad Rank}_{t,i+1}}{\text{Ad Rank}_{t+1,i}} * \text{Bid}_{t+1,i} \tag{5.6}$$

$$\text{Ad Rank}_{t,i} = \text{Bid Score}_{t,i} * \text{pond bid} + \text{Quality Score}_{i} * \text{pond Quality Score} + \text{Extension Score}_{i} * \text{pond Extension Score} \tag{5.7}$$

$$\text{Ad Rank}_{t+1,i} = \text{Bid Score}_{t+1,i} * \text{pond bid} + \text{Quality Score}_{i} * \text{pond Quality Score} + \text{Extension Score}_{i} * \text{pond Extension Score} \tag{5.8}$$

$$\text{Ad Rank}_{t+1,i} = (\text{Bid Score}_{t+1,i} - \text{Bid Score}_{t,i}) * \text{pond bid} + \text{Ad Rank}_{t,i} \tag{5.9}$$

Given this information:

$$\frac{\text{Bid}_{t+1,i}}{\text{CPC}_{t+1,i}} = \frac{\text{Ad Rank}_{t+1,i}}{\text{Ad Rank}_{t,i+1}} \tag{5.10}$$

$$\frac{\text{Bid}_{t+1,i}}{\text{CPC}_{t+1,i}} = \frac{(\text{Bid Score}_{t+1,i} - \text{Bid Score}_{t,i}) * \text{pond bid} + \text{Ad Rank}_{t,i}}{\text{Ad Rank}_{t,i+1}} \tag{5.11}$$

$$\frac{\text{Bid}_{t+1,i}}{\text{CPC}_{t+1,i}} = \frac{(\text{Bid Score}_{t+1,i} - \text{Bid Score}_{t,i}) * \text{pond bid}}{\text{Ad Rank}_{t,i+1}} + \frac{\text{Ad Rank}_{t,i}}{\text{Ad Rank}_{t,i+1}} \tag{5.12}$$

$$\frac{\text{Bid}_{t+1,i}}{\text{CPC}_{t+1,i}} = \frac{(\text{Bid Score}_{t+1,i} - \text{Bid Score}_{t,i}) * \text{pond bid}}{\text{Ad Rank}_{t,i+1}} + \frac{\text{Bid}_{t,i}}{\text{CPC}_{t,i}} \tag{5.13}$$

Based on this the bid adjustment the simulation will be made considering three main hypotheses.

H1: value per click > CPC > 0.95*value per click: There is profit.

H2: CPC < 0.95*value per click: There is profit and may have enough space to get more clicks

H3: value per click < CPC: There is loss.

Given that advertisers have knowledge of their own Quality Score, CPC from the period before and the bid at any moment, and that they have an expectation about the bid ponderation in the Ad Rank calculations, we define, based on this three hypothesis, three different reactions that advertisers will use and their intuition, having in mind they always want to maximize their profit.

Based on this, in hypothesis one the advertiser will try to lower the CPC as much as possible in order to increase his margin per click. The resolution gives us:

$$\text{H1: } \mathrm{bid}_{t+1} = \mathrm{CPC}_t + 0.01 \tag{5.14}$$

In order to achieve this result and trying not to lower Ad Rank less than the advertiser right in the next position we adapt it to:

$$\text{H1: } \mathrm{bid}_{t+1} = \mathrm{CPC}_t * 1.05 \tag{5.15}$$

We also assume that $\frac{(\mathrm{bid}_{t+1,i} - \mathrm{bid}_{t,i}) * \text{pond bid}}{\text{Ad Rank}_{t,i+1}}$ is equal to zero, even though we know it is not. This is due to the lack of knowledge by the advertiser. Instead of assuming a value we preferred to use a margin, 5% in this case, so that the advertiser does not loose his rank and consequently lower his number of clicks.

Given the second hypothesis we assume there might be a chance that the advertisers gets more clicks by outranking the previous round auction winner of the immediately above slot. So the idea is to push the CPC as much as we can in order to discover whether is profitable or not to get more volume. We base our strategy in the information of the ratio between the prices we paid and the bid we made, which is not suited to predict the price we will pay if we outrank the other advertiser:

$$\text{H2: } \mathrm{CPC}_{t+1} = \text{value per click} - 0.01 \tag{5.16}$$

Since we do not know if we are going to achieve an up rise in Ad Rank enough to outrank the competitor, we assume a margin to minimize possible losses.

$$\text{H2: } \mathrm{CPC}_{t+1} = 0.95 * \text{value per click} \tag{5.17}$$

As for the third hypothesis we assume that we are losing money and the main concern is to get at least no losses.

$$\text{H3: } \mathrm{CPC}_{t+1} = \text{value per click} - 0.01 \tag{5.18}$$

Given this we assume the same strategy as in hypothesis two and go for:

$$\text{H3: } \mathrm{CPC}_{t+1} = 0.95 * \text{value per click} \tag{5.19}$$

Using expression 5.13 we can get the bid for the player in the time T, by substituting by the hypothesis in each situation.

$$\mathrm{Bid}_{\mathrm{T,i}} = \frac{\mathrm{Bid}_{\mathrm{T-1,i}}}{\mathrm{CPC}_{\mathrm{T-1,i}}} * \mathrm{CPC}_{\mathrm{T}} \tag{5.20}$$

The second solution, which we will call strategy two, consists on a basic iteration that converges to 0.95 of players' value per click. Basing the bid adjustment to difference between what they paid in the last round, CPC, and what they can pay at maximum, value per click.

$$Bid\ Adjustmnet = (95\% * Value\ per\ click - CPC)/Bid \tag{5.21}$$

We also set a maximum bid adjustment to 25% so that the variations was not so big that created extremely bad results for advertisers that are trying to get in a slot and create big losses.

As for the examining the results we will compare these two strategies each one defined by a the use of bid adjustment, as written before, strategy one based on 5.15, 5.17 and 5.19 and strategy two based on the formula 5.21. In the future we will refer to them as Bid Adjustment 1 and Bid Adjustment 2, respectively.

Finally we present the code in appendix 1.

5.2. Discussion

First, let us present the data and the basic parameters that are in every simulation.

Table 1 – Base parameters of all simulations

All simulations	
# Impressions	10 000
# Rounds	5
# Players	10
# Slots	8
Weighted Bid	50%
Weighted QS	40%
Weighted Extension	10%

In Table 1 are the parameters that are similar in every simulation and remain untouched through all simulations. However there are two other variables that we have defined and have a major role in the auction. Those are Quality Score and value per click. As told before these variables were generated randomly between values of our choice. In the next two tables we present the values that were randomly generated organized by companies and simulations we did. In the Table 2 it is represented the value per click for each company in each simulation. This attribute is constant over the rounds and defines the profit and the revenue each company can make with each click.

Table 2 - Value per click for each company in each simulation

Company	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	1.18	1.89	0.22	1.52	2	1	1.21	0.06	0.06	1.08	1.37	1.19	0.51	0.23	0.86
2	1.36	1.22	0.1	1.49	1.34	1.79	0.08	1.75	1.75	0.52	0.97	0.47	0.54	1.04	0.48
3	0.79	1.83	1.92	0.88	0.76	1.56	0.5	1.25	1.25	0.78	0.92	0.31	0.56	1	1.6
4	1.69	1.87	1.32	1.63	0.56	0.69	1.06	0.1	0.1	0.75	0.31	1.84	0.83	1.41	1.2
5	0.24	1.88	1.12	0.66	0.28	0.13	0.55	2	2	1.42	0.66	0.19	0.55	0.7	1.09
6	1.1	0.07	1.26	0.11	1.68	1.98	1.48	0.4	0.4	1.24	0.95	1.49	0.81	1.2	1.75
7	0.89	1.69	1.29	1.87	0.41	1.83	0.56	0.8	0.8	0.93	1.13	1.85	1.15	0.93	1.57
8	0.84	0.45	1.88	1.74	1.33	0.38	1.17	0.17	0.17	0.24	0.12	1.78	0.59	1.13	0.73
9	0.17	1.33	1.7	1.14	1.79	1.86	1.9	1.6	1.6	0.8	0.84	1.37	0.62	1.45	1.35
10	0.1	0.94	0.69	0.92	0.63	1.01	1.35	1.25	1.25	0.86	1.37	1.86	1.27	1.92	0.05

As one can see from this table there the values range from 0.05 and 2. We do not prefer to have a large range between the values per click because that would create a higher risk of less competition with completely unbalanced and incomparable values that advertisers could offer. That would lead to a more stable position in all the rounds since players could not reach others' Ad Ranks since the bids would have huge differences, since they are directly related to value per click.

In addition we did the same analysis in relation to the Quality Score, one of the most important attributes that remains unchangeable during the auction. Combining this two attributes we have almost 90% of the weight on which Ad Rank will be calculated. That is in fact the reason we picked this two specific metrics: they have a high weight on the Ad Rank and are also unchangeable through the whole process.

Table 3 - Quality Score (rounded) for each company in each simulation

Company	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	5	4	9	9	10	3	10	8	8	2	5	2	3	1	10
2	5	4	10	9	10	1	8	6	6	10	8	4	2	5	10
3	2	1	4	4	1	6	1	10	10	8	4	10	6	10	5
4	1	7	9	9	8	10	2	9	9	8	1	10	10	8	5
5	9	3	8	6	5	2	2	9	9	2	3	2	4	8	1
6	8	5	6	2	5	4	5	1	1	7	3	8	8	10	3
7	5	1	6	4	4	7	1	6	6	9	5	4	3	10	7
8	7	3	8	7	9	8	6	1	1	6	10	1	2	5	9
9	10	8	6	10	1	9	1	4	4	4	1	3	10	10	6
10	5	10	1	5	2	5	2	10	10	7	7	6	1	4	4

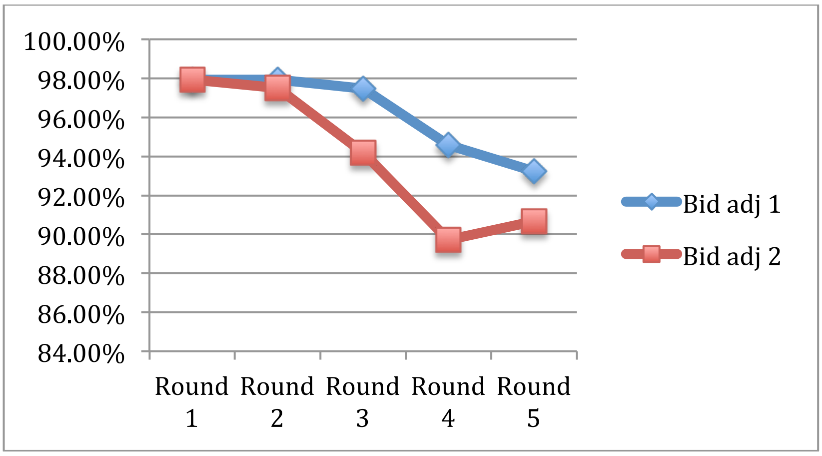
Staring with this data we made fifteen simulations with random values for the variables value per click, CTR and Extensions CTR. Then we calculate the Quality Score, Extension Score, bid, and Bid Score, used to calculate the Ad Rank. After each round there is an adjustment in bids which creates a new Ad Rank. For each round the CPC is calculated as well as CTR and Clicks. Based on this calculus we get advertisers' and search engine's profit.

We tried to find a relationship between the main variables, while comparing both bid adjustment strategies developed in the chapter before. We also calculate the combination that generates the highest total welfare:

$$\textbf{Total Welfare} = \text{Profit for search engine} + \text{Profit to advertisers} \qquad (5.22)$$

As you can see in Figure 2 we compare the overall accomplishment of the total welfare. In fact this results show that in minimum the total welfare is nearly 90% of the best solution. From observing the same figure we can state that bid adjustment 1 creates a better welfare in all the rounds comparing with bid adjustment 2. It is in the first round that the best total welfare is achieved as players bid their true value. This solution is not constant for the rest of the rounds because that does not fit the maximization profit objective of the bidders.

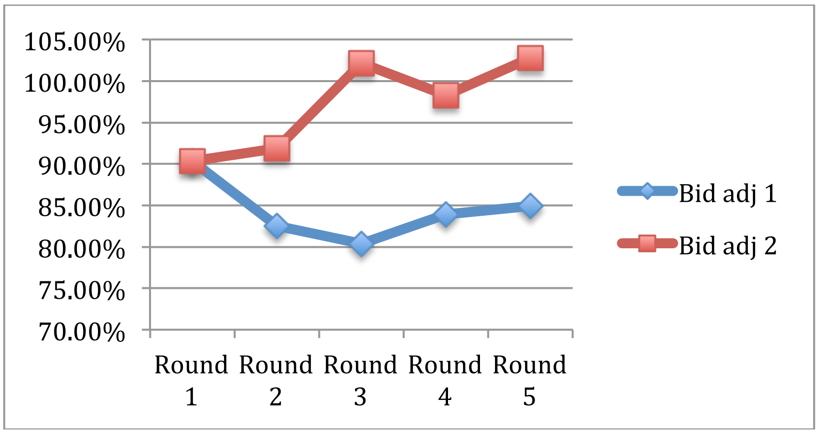
Figure 2 - % of the best allocation of resources by round



Nevertheless, we see that as rounds continue the percentage of the best allocation is decreasing in both types bid adjustments. The biggest difference between these two cases studies is in the distribution between players and search engine. In bid adjustment one, search engine starts to loose part of their profit, Figure 3, in relationship with the advertisers gaining a higher percentage, Figure 4. In other words, as the rounds go on there is a trend that bidders using strategy one gets a higher percentage of total welfare and the search engine looses percentage of total welfare while the total percentage of the best total welfare decreases.

The opposite happens with strategy two which provides a increase in percentage of total welfare to search engine and worsening percentage to the advertisers, also seen in Figure 3 and 4 respectively. In addition, bid adjustment two also has another effect, in overall terms advertisers are losing money. This means that on the long term it is not sustainable strategy, since advertisers with consistent losses would be not incentivized to invest in the search engine.

Figure 3 - % of search engine's profit in relation to the best allocation of resources by round



Examining Figure 3 we can affirm that when the ratio is over 100% that advertisers have losses since at no point in Figure 2 the % of total welfare is 100%. On Figure 4 we see the distribution of profit for the total of advertisers. In Bid adjustment 1 the trend is not clear because in the first 2 rounds there is a gain in percentage of total welfare and from round 3 forward there is a negative trend, which ends with an value in terms of percentage near to the one in round 1. However since the total welfare was decreasing in absolute maters overall advertisers loose money but does not mean that all advertisers have looses.

Figure 4 - % of advertisers' profit in relation to the best allocation of Resources by Round

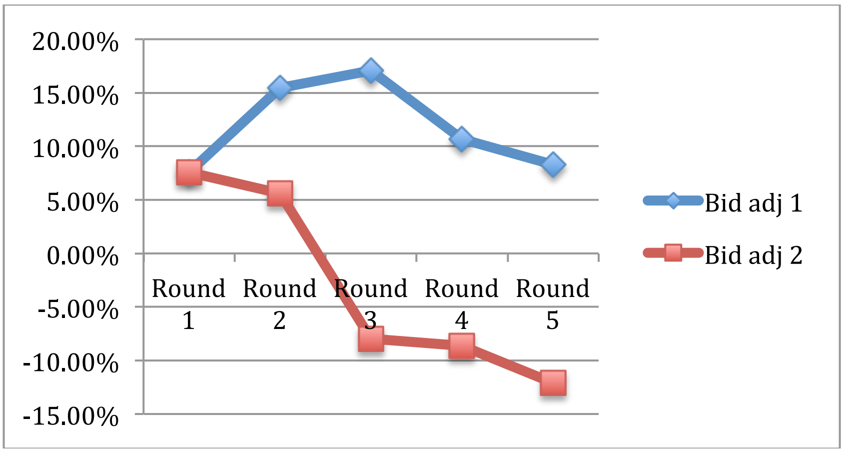
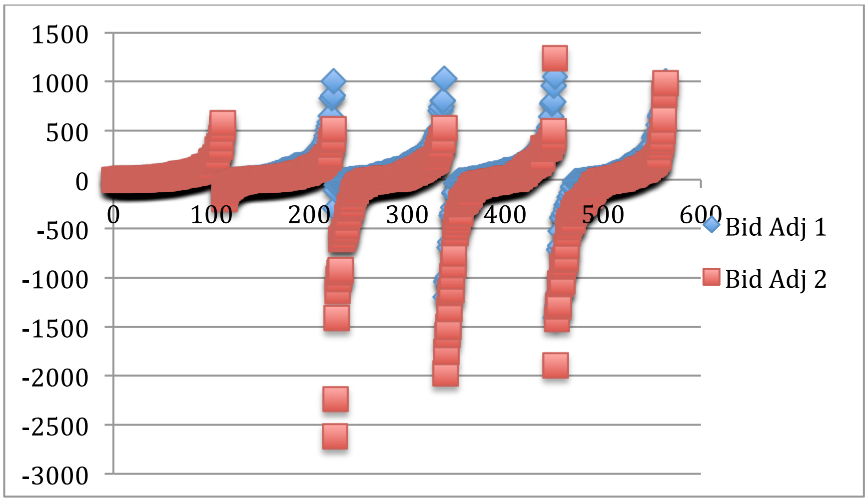


Figure 4 proves however that our second strategy is not the most adequate to create profit for advertisers. Also, we should redesign the second strategy in order to create better results.

In Figure 5 we observe the distribution of profits by round using the two methods. In the first strategy the results are less disperse compared to strategy two. Also, we can see that as rounds go on, we start to observe players with higher profits and players with higher losses too, this justify the facts that in overall changes from round to round generate losses but means that there are a lot of players who generate profits. This changes are mainly due to changes in Ad Rank sort. Players who where having no clicks go up one or more positions due to a raise in bid and end up paying more than their value per click.

Figure 5 - Profit by round Bid Adjustment 1 vs Bid Adjustment 2



We also tried to make a connection between Ad Rank and profit. The question is: would a higher Ad Rank create a higher profit? By logic it would seem to be so because if you bid more, and you are relatively better than other advertisers, you have a higher chance of getting clicks, and so, get more profit even though you might end up paying more for each click in case you are too close to your competitors.

As demonstrated in Figure 7 there is a clear correlation between the Ad Rank and the profit. Advertisers with higher Ad Rank can expect higher results. However Google does not give that metric. In order to look to a metric that we actually can look we studied the ratio between the CPC and Bid which is the same as the ratio between the competitor's Ad Rank in the next position and the player's Ad Rank, as seen in formula 5.4. In addition is the ratio that defines which proportion of your bid you will pay. So, we tried to establish a correlation to profit. From this analysis we could conclude nothing relevant, only observing that the higher the ratio, the closer the advertiser is to the other competitor. Meaning the higher the ratio is the cost-per-click is the cheaper he can, maintaining his position - in order for advertisers to maintain the same number of clicks it is advisable for advertisers to raise the bid a little bit if he has no profit problems.

Figure 6 - Distribution of profit by the ratio between CPC/Bid

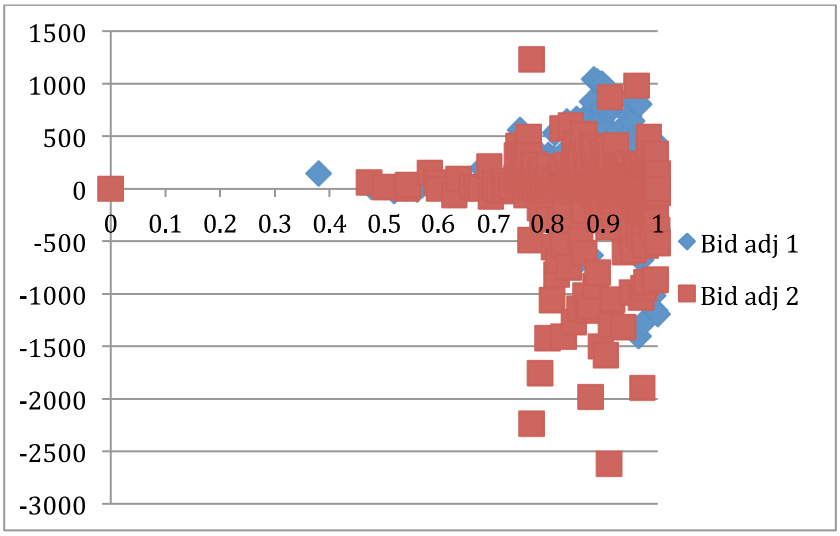
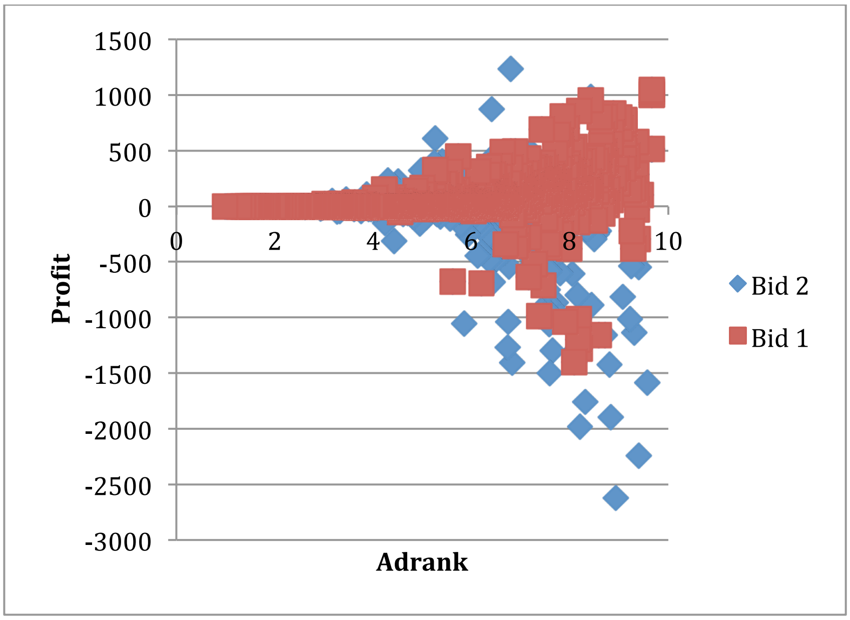
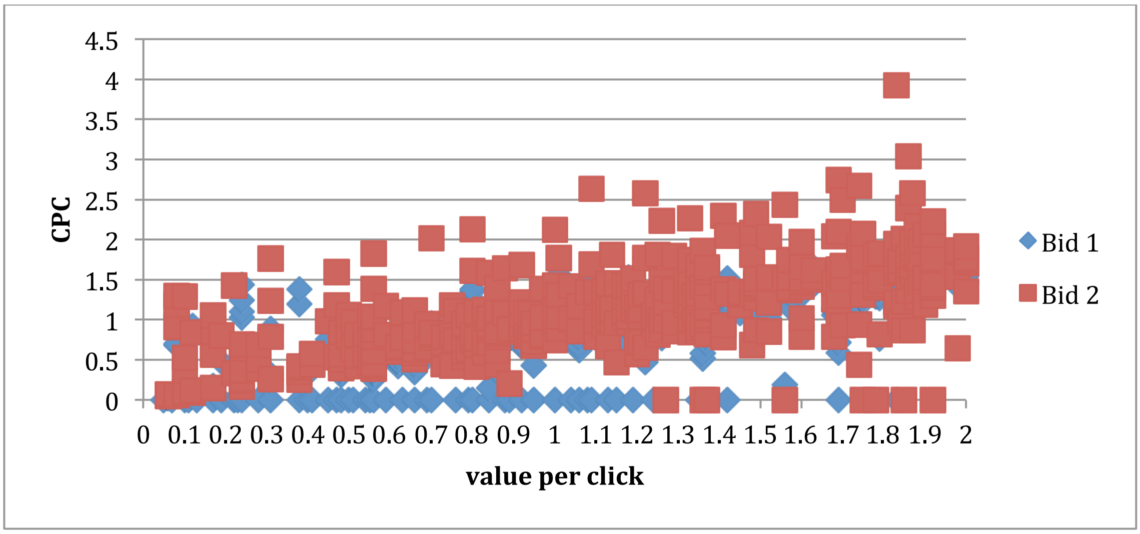


Figure 7 - Profit by Ad Rank



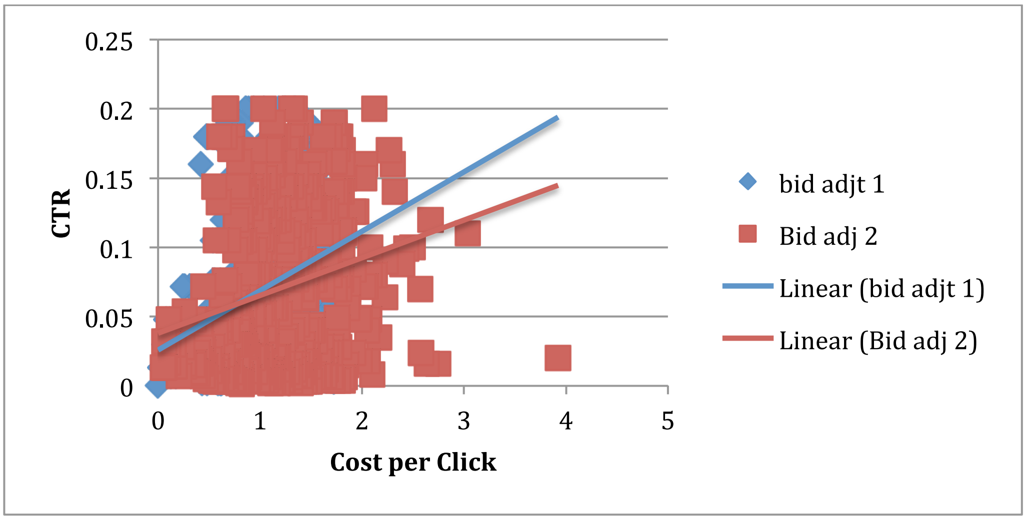
Furthermore we studied, in Figure 8, the relationship between value per click and CPC. In this figure is possible to observe if the CPC is coherent with the value each advertiser is assuming to his value per click. We can actually see every time advertisers loose money and for which values per click that happens the most. We expect that advertisers with higher value per click will have profit more times than advertisers with lower values per click. Because advertisers with higher value per click can bid more than the other players and thus get a clear advantage in the Ad Rank calculus, this way creating more profit than the others. Advertisers, which have a lower value per click, will have also a problem due to existing more players than slots. This means that to get in a slot, Advertisers will have to bid more than their value per click and probably have losses when they go up in the rank, that is the reason there might have higher losses per click in advertisers with lower value per click

Figure 8 - Distribution of CPC by players value per click



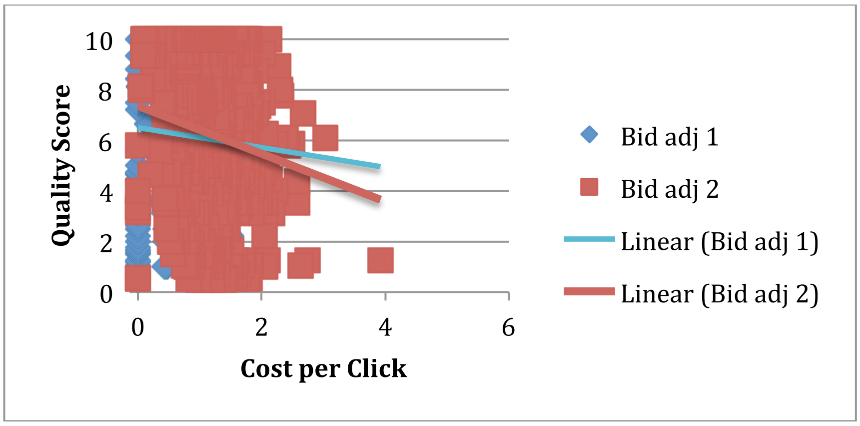
We also compare what happens when the CTR is higher. Figure 9 clearly shows that the higher the CTR the higher the CPC will be. By our knowledge and by the state of art we made before we see that this is contrary to what we have seen in other studies. This is happening most likely because when calculating the Quality Score the CTR part considered only takes in consideration the quality of the ad and the relevancy to the user and does not take in consideration other factors related to the user experience in the site. Although when the bid his the same higher CTR will still generate clicks with a lower CPC.

Figure 9 - Distribution of CPC by CTR



In addition we also studied the relationship between CPC and Quality Score and end up discovering that in our study there is a negative relationship, as expected, between Quality Score and CPC (Figure 10). The higher the Quality Score the lower the CPC. This can be as clear as expected due to the linear correlation between the Quality Score and ad CTR, Extension Score and extension CTR and Bid Score and bid.

Figure 10 - Relationship between Quality Score and CPC



Chapter 6: Conclusions

Taking in consideration what we proposed to do in this thesis, a resume of what we did achieve is made here under the form of concluding remarks.

First, our main goals were to make it more clear how Search Engine Market (SEM) works and in which way we could relate it to economics, auctions and game theory. We were able to do so basing our investigation in past studies which helped us understand the most part and base of this matter of study.

In second place we whished to highlight the mechanism of SEM and we did so by creating a program in R code that simulates a working search engine auction and from which we can understand how it works and reacts. This simulation is not the most accurate if we compare it to real life data. However we hope it helps people that are interested in the matter to understand how changing parameters in this auction affects the results and how the mechanism is constructed, in a simplification of the real model.

In addition, we proposed to create different advertisers behaviours so we could understand how the model would react and what would the results be. Consequently we pick two strategies to try and beat the game, since we only made changes to the advertisers behaviour. In other words, we approached the mechanism trying to use the information available to each advertiser and only information that was available and measure the results. We came to the conclusion that one of our strategies was far better than the other. Since one of them, strategy one which used the ratio based on past information and assumed zero changes from the other advertisers, created higher total welfare and an improvement in the distribution of the welfare in favour of the advertisers. On the other hand, the other strategy that focused on the gap between the value that player won per click and the price they paid had bad results in the sense that it created a decrease in total welfare and an overall shift in the distribution of welfare in favour of the search engine. In our simulations we proved that if every player bided their value per click the total welfare would be almost the best total welfare combination. However we noticed that this doesn't fit the maximization of profit for individual advertisers and so is not a long-term solution and does not generate a stable solution.

In the future we expect to broaden our study in some directions in order to create a more realistic simulator and also approach the search engine point of view in trying to balance its profits with their advertisers purposes. Therefore, in terms of what respects the simulation we would like to introduce in future work a wider range of rounds, players and slots. Also we would like to introduce new features that give more realism to the simulation such as reservation price, each slot could have a reserve price and also a capping by advertiser based on their bid or Ad Rank. As Quality Score is now constant we would like to improve that so it could be calculated every round and for it to take in consideration the past performance of advertisers as well as expect performance. In the same line of thinking we would like to test multiple ways of calculating the scores that are at this moment a proportion of each round highest value in a 0 to 10 score. Finally we would like to approach the strategy of advertisers the other way around using all the info we have to try and discover if the profit maximization by the advertiser could lead us to pattern that we could use to create a strategy.

References

Aggarwal, G., Feldman, J. and Muthukrishnan, S. (2006), “Bidding to the Top: VCG and Equilibria of Position-Based Auctions”

Ayanso, A. and Karimi, A. (2015), “The moderating effects of keyword competition on the determinants of ad position in sponsored search advertising”, Decision Support Systems, 70 42–59.,

Boughton, S. B. (2005), “Perspectives in business”, “Search Engine Marketing” 2.1: 29-33.

Bu, T., X. Deng, and Q. Qi (2008), “Forward looking Nash equilibrium for keyword auction”, Information Processing Letters, 105: 41–46.

Clarke, E. (1971), "Multipart Pricing of Public Goods", Public Choice, 11 (1): 17–33.

Dixit, A., S. Skeath and D. Reiley (2015) “Games of strategy”, 4th ed., New York, W. W. Norton.

Dutta, P. K. (1999), “Strategies and games: Theory and practice”, Cambridge, Mass, MIT Press.

Edelman, B. and Ostrovsky, M. (2007), “Strategic bidder behavior in sponsored search auctions”

Edelman, B., M. Ostrovsky and M. Schwarz (2007). "Internet Advertising and the Generalized Second-Price Auction: Selling Billions of Dollars Worth of Keywords." American Economic Review, 97(1): 242-259.

Groves, T. (1973), "Incentives in Teams", Econometrica, 41 (4): 617–631.

Hansen, J. (2009), “The Economics of search engines – Search, Ad Auctions & Game Theory”, Copenhagen Business School, Applied Economics and Finance Master Thesis.

Klemperer, P. (2002), “What Really Matters in Auction Design”. The Journal Economic Perspective, 16, 1: 169–189

Li, H. and J. G. Riley (2007), “Auction Choice”, International Journal of Industrial Organization, 25: 1269–1298.

Liu, D. and J. Chen, (2005), “Designing online auctions with past performance information”, Decision Support Systems, 42 (3): 1307–1320

Rangaswamy, A., Giles, C. and Seres, S. (2009) “A Strategic Perspective on search engines: Thought Candies for Practitioners and Researchers”, Journal of Interactive Marketing 23 (2009) 49-60

Skiera, B., Eckert, J. and Hinz, O. (2010), “An analysis of the importance of the long tail in search engine marketing”

Varian, H. R. (2007), “Position Auctions”, International Journal of Industrial Organization, 25: 1163–1178.

Varian, H. R. (2010), “Intermediate microeconomics: a modern approach”, 8th ed., New York, W. W. Norton

Vickrey, W. (1961), "Counterspeculation, Auctions, and Competitive Sealed Tenders", The Journal of Finance, 16 (1): 8–37.

Web Sites:

Geddes, B. (2016), The Mad Sentists: The Quality Score under Microscope, http://www.slideshare.net/SearchMarketingExpo/the-mad-scientists-quality-score-under-the-microscope-by-brad-geddes, accessedhttps://support.google.com/adwords/answer/1752122 on 21/09/2016.

Google Support, Ad Extensions, https://support.google.com/adwords/answer/2375499, accessed on 25/09/2016

Google Support, Ad Position, https://support.google.com/adwords/answer/6300

Google Support, Ad Rank, https://support.google.com/adwords/answer/1752122, accessed on 21/09/2016.

Google Support, Ad Relevance, https://support.google.com/adwords/answer/1659752, accessed on 25/09/2016

Google Support, Click-trough-rate, https://support.google.com/adwords/answer/2615875 accessed on 25/09/2106

Google Support, Click, https://support.google.com/adwords/answer/31799, accessed on 25/09/2016

Google Support, https://support.google.com/adwords/answer/6324 accessed on 25/09/2016.

Google Support, Impressions, https://support.google.com/adwords/answer/6320, accessed on 25/09/2016

Google Support, Keywords, https://support.google.com/adwords/answer/6323, accessed on 25/09/2016

Google Support, Landing Page, https://support.google.com/adwords/answer/14086, accessed on 25/09/2016

Google Support, Maximum CPC bid, https://support.google.com/adwords/answer/6326, accessed on 25/09/2016

Google Support, Quality Score, https://support.google.com/adwords/answer/140351, accessed on 25/09/2016

Search engine Land Blog, http://searchengineland.com/new-adwords-Ad Ranking-formula-what-does-it-mean-174946, accessed on 21/09/2016

Wordstream Blog, http://www.wordstream.com/blog, accessed on 25/09/2016

Wordstream Blog, http://www.wordstream.com/ppc accessed 29/09/2016

Appendices

Appendix 1: the R code

```
# Ad Rank and Strategies
types<-2
files<-10
for (y in 1:files) {
print(y)
#start by defining the number of rounds and players
# of rounds
rounds<-5
# of players
players<-10
# of impressions by round
impressions<-10000
# of slots
slots<-8
players_out<-players-slots

#defining the variables
{
CTR<-matrix(0, nrow=rounds, ncol=players)
CPC<-matrix(0, nrow=rounds, ncol=players)
clicks<-matrix(0, nrow=rounds, ncol=players)
cost<-matrix(0, nrow=rounds, ncol=players)
revenue<-matrix(0, nrow=rounds, ncol=players)
profit<-matrix(0, nrow=rounds, ncol=players)
newbid<-matrix(0, nrow=rounds, ncol=players)
bidadjustment<-matrix(0, nrow=rounds, ncol=players)
adrank<-matrix(0, nrow=rounds, ncol=players)
w<-matrix(0, nrow=rounds, ncol=players)
h3<-matrix(0, nrow=rounds, ncol=players)
h2<-matrix(0, nrow=rounds, ncol=players)
h1<-matrix(0, nrow=rounds, ncol=players)
u<-matrix(0, nrow=rounds, ncol=players)
new_bid_score<-matrix(0, nrow=rounds, ncol=players)
}
#defining vectors
{
 qs<-vector()
 extensions_ctr<-vector()
 ad_max_ctr<-vector()
 value_per_click<-vector()
 position_pond<-vector()
}
#given data
{
#initial parameters to change
```

```
#BID: automaticly random bid: bid<-round(runif(players,min,max)) or
 given bid<-c(x,x,x,x,x,x,x,x,x,x)
#this aplies to other parameters to
 #bid<-c(2,2,2,2,2,2,2,2,2,2)
 extensions_ctr<-round(runif(players,1,5))
 ad_max_ctr<-round(runif(players,1,20))
 value_per_click<-round(runif(players,5,200))/100
 bid<-value_per_click
 position_pond<-c(100,90,80,70,40,30,20,10,0,0) #10 players
 #position_pond<-c(100,90,80,70,40,30,20,10,0,0,0,0,0,0,0) #15 players
}
#Calculated fields
{#which ponderation maximizes the advertiser profit? and wich one
 maximizes SE profit
qs=10*(ad_max_ctr/max(ad_max_ctr))
extension_score=10*(extensions_ctr/max(extensions_ctr))
bid_score<-(bid/max(bid))*10
}
#ponderation to Ad Rank
for (l in 1:types) {
{
pond_bid<-0.5
pond_qs<-0.4
pond_extension<-0.1
}
#Ad Rank
{
#adrank<-bid*pond_bid+qs*pond_qs+extension_score*pond_extension #same
 wieght and same scale
adrank<-bid_score*pond_bid+qs*pond_qs+extension_score*pond_extension
 #same wieght and same scale
print(adrank)
}
company<-1:players
# THE MATRIX
{
 order<-order(adrank,decreasing=TRUE)
matrix<-
 rbind(adrank,company,bid,ad_max_ctr,extensions_ctr,value_per_click,bid
 _score)
matrix<-matrix[,order]
matrix<-rbind(position_pond,matrix)
ematriz<-array(0,c(14,players,rounds))
ematriz2<-array(0,c(14,players,rounds))

}
#CYCLES - 1st Round use Matrix Then use other
for (t in 1:1) {

CTR[t,]<-matrix[1,]/100*matrix[5,]/100
```

```
for (i in 1:slots) {
#CPC

u[t,i]<-(matrix[2,i+1]/matrix[2,i])
CPC[t,i]<-(matrix[2,i+1]/matrix[2,i])*matrix[4,i]

}

clicks[t,]<-impressions*CTR[t,]
cost[t,]<-clicks[t,]*CPC[t,]
revenue[t,]<-clicks[t,]*matrix[7,]
profit[t,]<-revenue[t,]-cost[t,]



order<-order(adrank, decreasing=TRUE)
#matrix<-
 rbind(adrank,matrix[3,],matrix[4,],matrix[5,],matrix[6,],matrix[7,])[,
 order]
matrix<-
 rbind(matrix,CTR[t,],CPC[t,],clicks[t,],cost[t,],revenue[t,],profit[t,
 ])
#matrix<-rbind(position_pond,matrix)
print(t)
print(matrix)
ematriz[,,t]<-matrix

w[t,]<-matrix[10,]/matrix[4,] #CPC/Bid
h3[t,]<-w[t,]*0.95*matrix[7,] #achieve 1.05 ROi and try to mantain
 Volume
h2[t,]<-w[t,]*0.95*matrix[7,] #try to achieve maximum clicks even
 thought it migh loose in margin per click
h1[t,]<-w[t,]*1.05*matrix[10,] #reduce the costs to achieve maximum
 Margin per click and mantaining clicks

#bid adjustment
if (l==1) {
 bidadjustment[t,]<-0
for (i in 1:slots) {
 if (matrix[10,i]> matrix[7,i]) bidadjustment[t,i]<-(h3[t,i]-
 matrix[4,i])/matrix[4,i]
 if (matrix[10,i]< 0.95*matrix[7,i]) bidadjustment[t,i]<-(h2[t,i]-
 matrix[4,i])/matrix[4,i]
 if (matrix[10,i]> 0.95*matrix[7,i]) bidadjustment[t,i]<-(h1[t,i]-
 matrix[4,i])/matrix[4,i]
 }
for (i in (slots+1):players) {
 bidadjustment[t,i]<-0.1
 }
if (bidadjustment[t,1]>0) bidadjustment[t,1]<-0
```

```
if (bidadjustment[t,i]>0.25) bidadjustment[t,1]<-0.25
}
if (l==2) {
    bidadjustment[t,]<-(0.95*matrix[7,]-matrix[10,])/matrix[4,]
for (i in 1:players) {
 if (bidadjustment[t,i]> 0.25) bidadjustment[t,i]<-0.25
 if (bidadjustment[t,i]< -0.25)bidadjustment[t,i]<--0.25
 if (bidadjustment[t,1]> 0) bidadjustment[t,1]<-0
 }
}
newbid[t,]<-(1+bidadjustment[t,])*matrix[4,]

new_bid_score[t,]<-(newbid[t,]/max(newbid[t,]))*10

adrank<-
 pond_bid*new_bid_score[t,]+pond_qs*matrix[5,]/max(matrix[5,])*10+pond_
 extension*matrix[6,]/max(matrix[6,])*10

}
for (t in 2:rounds) {
order<-order(adrank, decreasing=TRUE)
matrix<-rbind(adrank,matrix[3,],newbid[t-
 1,],matrix[5,],matrix[6,],matrix[7,],new_bid_score[t-1,])[,order]
matrix<-rbind(position_pond,matrix)

CTR[t,]<-matrix[5,]/100*matrix[1,]/100
#didn't understand this step - Max_ctr i * position ponderation /100
 because it is not in percentage
for (i in 1:slots) {
#CPC

CPC[t,i]<-(matrix[2,i+1]/matrix[2,i])*matrix[4,i]
}

clicks[t,]<-impressions*CTR[t,]
cost[t,]<-clicks[t,]*CPC[t,]
revenue[t,]<-clicks[t,]*matrix[7,]
profit[t,]<-revenue[t,]-cost[t,]

order<-order(adrank, decreasing=TRUE)
matrix<-
 rbind(matrix,CTR[t,],CPC[t,],clicks[t,],cost[t,],revenue[t,],profit[t,
 ])
print(t)
print(matrix)
ematriz[,,t]<-matrix

w[t,]<-matrix[10,]/matrix[4,]
h3[t,]<-w[t,]*0.95*matrix[7,] #achieve 1.05 ROi and try to mantain
 Volume
```

```
h2[t,]<-w[t,]*0.95*matrix[7,] #try to achieve maximum clicks even
 thought it migh loose in margin per click
h1[t,]<-w[t,]*1.05*matrix[10,] #reduce the costs to achieve maximum
 Margin per click and mantaining clicks

#bid adjustment
 bidadjustment[t,]<-0
if (l==1) {
 bidadjustment[t,]<-0
for (i in 1:slots) {
 if (matrix[10,i]> matrix[7,i]) bidadjustment[t,i]<-(h3[t,i]-
 matrix[4,i])/matrix[4,i]
 if (matrix[10,i]< 0.95*matrix[7,i]) bidadjustment[t,i]<-(h2[t,i]-
 matrix[4,i])/matrix[4,i]
 if (matrix[10,i]> 0.95*matrix[7,i]) bidadjustment[t,i]<-(h1[t,i]-
 matrix[4,i])/matrix[4,i]
 }
for (i in (slots+1):players) {
 bidadjustment[t,i]<-0.1
 }
if (bidadjustment[t,1]>0) bidadjustment[t,1]<-0
if (bidadjustment[t,i]>0.25) bidadjustment[t,1]<-0.25
}
if (l==2) {
     bidadjustment[t,]<-(0.95*matrix[7,]-matrix[10,])/matrix[4,]
for (i in 1:players) {
 if (bidadjustment[t,i]> 0.25) bidadjustment[t,i]<-0.25
 if (bidadjustment[t,i]< -0.25)bidadjustment[t,i]<--0.25
 if (bidadjustment[t,1]> 0) bidadjustment[t,1]<-0
 }
}

newbid[t,]<-(1+bidadjustment[t,])*newbid[t-1,]

new_bid_score[t,]<-(newbid[t,]/max(newbid[t,]))*10

adrank<-
 pond_bid*new_bid_score[t,]+pond_qs*matrix[5,]/max(matrix[5,])*10+pond_
 extension*matrix[6,]/max(matrix[6,])*10


write.table(ematriz,file=paste('ematrix1 ',y,'
 ',l,'.csv'),append=FALSE,sep=',',dec=".")#PROBLEMA CTR NAO ESTA
 CORRETA - 0 a mais

}
}
```