A Multi-Objective Model for Inventory and Planned Production Reassignment to Committed Orders with Homogeneity Requirements

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Certain industries are characterized by obtaining non-homogeneous units of the same product. However, customers require homogeneity in some attributes between units of the same and different products requesting in their orders. To commit such orders, an estimation of the homogeneous product to be obtained can be used. Unfortunately, estimations of homogenous product quantities can differ considerably from real distributions. This fact could entail the impossibility of accomplishing the delivery of customer orders in the terms previously committed. To solve this, we propose a multi-objective mathematical programming model to reallocate already available homogeneous products in stock and planned production to committed orders. The main contributions of this model are the consideration of the homogeneity requirement between units of different lines of the same order, the allowance of partial deliveries of order lines, and the specification of some relevant attributes of products to accomplish with the customer homogeneity requirement. Different hypotheses are proved through experiments and statistical analyses applied to a ceramic tile company. The ε-constraint method is used to obtain an implementable solution for the company. The weighted sum method is used when proving other hypotheses that offer some managerial insights to companies.

Keywords: reallocation process; mathematical programming; lack of homogeneity in the product; homogeneity among order lines; deterministic

1. Introduction

Customers usually express requirements in their orders in terms of quantity and delivery date. However, several situations emerge where customers require homogeneity among units of the same product or different products for certain attributes that are relevant for them. These attributes refer to functional or aesthetical reasons because units of the same or different products need to be assembled, packed or presented together. For instance, customer orders in the agricultural sector should be served with units of the same fruit belonging to the same quality, size and weight. This is also valid for the furniture sector, where colour uniformity among units of the same product (e.g. chairs) or among products (e.g. chairs and table) impacts the final value of the products perceived by customers. Thus, colour and grain sorting are necessary.

Another example is the ceramic sector, where the nature of the raw material (clay) and components (frits and enamels) employed during ceramic tile production, and the variability of the environmental conditions during this process, means obtaining units with different tone, gage and quality attributes from a unique production batch (Alemany, Alarcón, Ortiz, & Lario, 2008; Grillo, Alemany, & Ortiz, 2016). In this sector, customers require product homogeneity for quality, tone and gage for all the units that compose an order line. Customers also require gage homogeneity for units of different order lines that are to be jointly installed for functional and aesthetic reasons. To ensure serving customer orders with the required homogeneity, classification stages are included during production processes.

The causes that generate product heterogeneity are mainly uncontrollable because the non-homogeneity of the raw material and components usually coming from the nature or the productive process itself. The above aspects make the homogeneous quantities of each product in planned production batches to be uncertain. In such a way, that only the homogeneous quantities of stocked products are really known. However, the Order Promising Process (OPP) should decide based on both, the uncommitted availability of products in stock and in planned batches, which customer order proposals to be committed and an accurate due date for them (Kiralp & Venkatadri, 2010). For this reason, the distribution of production batches into homogeneous sublots should be estimated during the OPP. However, due to the inherent aforementioned uncertainty, discrepancies between the estimated homogeneous quantities in batches and real ones are quite likely to occur. This circumstance can lead to some orders committed during the OPP not being served as there is not enough quantity of homogeneous product, although enough total quantity exists. This shortage situation can occur even with high stock levels and causes a poor customer service level since it is caused by homogeneity requirements (HR). One solution would be to simply refuse any orders that cannot be served (Fung, Cheung, Lee, & Kwok, 2005). However, this decision could very negatively impact both the customer and the company, so better solutions for the shortage problem are necessary.

One solution for minimising this problem is Shortage Planning (SP), which refers to the activities to be performed if stock becomes unavailable (Framinan & Leisten, 2010). Some examples of SP activities are negotiation with customers (late supply, partial shipments, etc.) and decisions about supply alternatives (outsourcing, substitutive products, etc.). Another possible solution to this problem is reallocating inventories to previously committed orders to improve the customer service level and to increase profits (Alarcón, Alemany, Lario, & Oltra, 2011; Lee, Jung, Eum, Parl, & Nam, 2006). Other strategies to improve customer satisfaction, such as postponement, are not possible in this case. The reason is that postponement attempts to delay product differentiation as much as possible until orders are received (Kisperska-Moron &

Swierczek, 2011) in order to face uncertainty in customised orders. Delayed product differentiation has proven capable of reducing inventory requirements and ensuring high product availability at the same time (Lee, Billington, & Carter, 1993). However, in the problem under study, uncertainty is not on the customer orders' side because we deal with already committed orders and, therefore, known with certainty. On the contrary, uncertainty is on the supply side, because of the final availability of homogeneous quantities cannot be known until they have been produced and classified.

In this paper, a multi-objective mathematical programming (MOILP) model to reallocate available homogeneous stocked and planned quantities that are already committed orders in ceramic companies is proposed. Although some publications have addressed the SP problem in the ceramic sector (Alemany, Alarcón, Oltra, & Lario, 2013b; Alemany, Grillo, Ortiz, & Fuertes-Miquel, 2015; Boza, Alemany, Alarcón, & Cuenca, 2014), none has considered HR among units that comprise different order lines, nor the allowance of partial deliveries of order lines, which are some of the novelties of this proposal. This requires not only the differentiation among the homogeneous sublots from the same batch (as previously done), but also the attributes specification for each sublot. This model pursues maximisation of profits and minimisation of order lines served with delays, plus minimisation of the partial deliveries of order lines. The consideration of the last two objectives, as well as the combination of all the objectives, is another contribution of this paper. Some hypotheses are proposed that provide some managerial insights. The model is executed for a different set of scenarios, whose results are statistically analysed to prove the proposed hypotheses.

The rest of the paper is structured as follows: Section 2 describes the problem under study, while Section 3 presents a literature review on the SP problem. Section 4 introduces the MOILP model, which is validated through an experimental design applied to a ceramic tile company in Section 5. Finally, Section 6 offers the main conclusions and the identified future research lines.

2. Problem description

The starting situation contemplates the existence of orders previously committed to customers by means of the OPP. In an ideal situation where the homogeneous planned and real quantities coincide, customer orders are delivered during execution activities as promised. However, discrepancies between the planned and real homogeneous quantities usually occur due to the uncertainty in the homogeneous quantities of the same product in planned production batches. When this happens, it is necessary to verify that the obtained homogeneous quantities are sufficient to serve already committed orders. If not, it will not be possible to serve all the committed orders as previously planned.

To solve this situation, the reallocation of updated available homogeneous quantities both in stock and planned to already committed orders is proposed to minimise the negative impact for both the company and the customer. This reallocation process should meet not only the committed quantity and due date as usual, but also the HR among the units that comprise an order line in all its attributes, and among the units of different order lines that belong to the same series in the gage attribute.

The characteristics of the company and products, customers, orders and delivery flexibility involved in the problem, as well as the reallocation objectives, are described below.

Company and product characteristics:

- The existence of a ceramic production plant composed of several parallel production lines that work according to a Make-To-Stock strategy is assumed.

- The products, once produced, are classified into homogeneous sublots based on their attributes: quality, tone, and gage.
- The products that can be assembled together belong to the same series (e.g. units of two ceramic tiles products which are combined to form a mosaic floor, or units of ceramic skirting boards and ceramic tiles for paving which are assembled together).

Availability of products:

- The existing stock and planned quantities to be produced in the Master Production Schedule (MPS) are used during the reallocation process, but only for first quality products.
- The stocked quantities at the beginning of the planning horizon are already classified into homogeneous sublots. So, their attributes (tone and gage) are known.
- The production batches defined in the MPS (planned batches) are divided into different homogeneous sublots by an estimated distribution. The sum of all homogeneous sublots of a batch must equal the batch size.

Customers:

- The orders previously committed during the OPP (firm orders) are considered for reallocation.
- Two types of customers are distinguished when reallocating available homogeneous quantities to already committed orders: priority and non-priority customer orders.
- An order can be composed of one order line or more. For each order line, the required product and the demanded quantity are detailed. The same finished

product can be claimed in more than one order line (e.g. two lines of an order can demand the same product if these quantities are to be assembled separately), but only one product can be requested in each order line.

- The committed due date for each order is known and previously agreed on with customers through the OPP. It is the same for all their order lines.
- An order line must be reserved with a homogeneous product so that all units of the product must have the first quality, and the same tone, and gage, but customers do not specify the tone and gage requested in their orders.
- The order lines with the products that belong to the same series must be booked with the products that present the same gage.
- An order can be served only if all the lines that comprise it are served.

Flexibility in delivery:

- A maximum delivery delay is defined for each order. The real delivery date of an order after the reallocation process is comprised during the period defined by the committed due date and the maximum allowed delay.
- Partial deliveries of order lines are allowed. This means that each line of the same order can be delivered on different dates if the maximum number of partial deliveries and the maximum delay defined by the customer for this order are not exceeded.
- No partial deliveries of quantities of an order line are allowed. The entire quantity demanded by a customer in an order line must be served simultaneously.
- The reallocation objectives are: maximisation of obtained profits, minimisation of the order lines served with delays, and minimisation of partial deliveries of order lines.

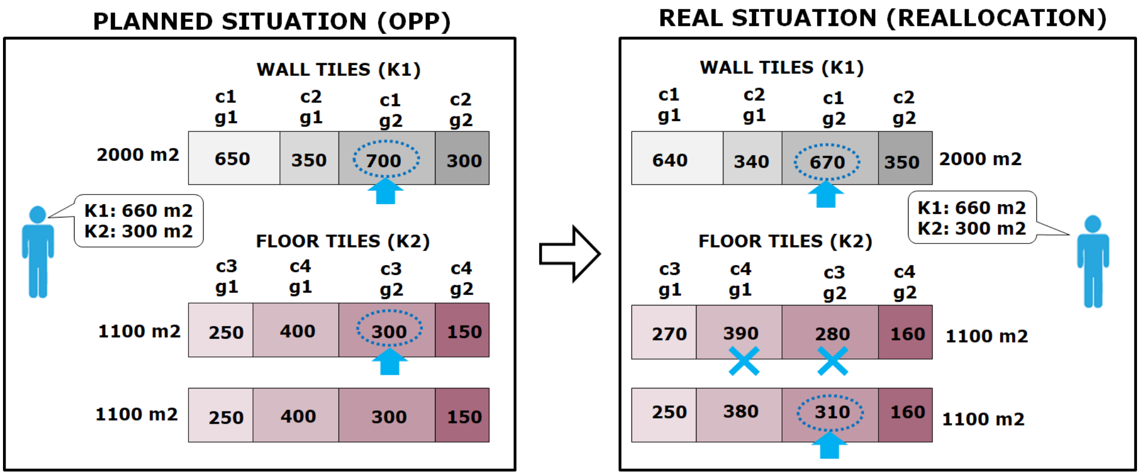
To better understand the problem under study, let's assume two products that belong to the same series: wall tiles (k_1) and floor tiles (k_2). For simplicity, let's assume that each product can be classified into two tones (c_1 and c_2 for k_1; c_3 and c_4 for k_2) and two gages (g_1 and g_2 for both products). This implies that each batch of each product can be classified into four homogeneous sublots. Let's also assume the existence of a planned production batch for k_1 of 2,000 m^2 that the company estimates is divided into four homogeneous sublots of 650, 350, 700 and 300 m^2 with the tone (c_i) and gage (g_i) represented in Figure 1. Finally, let's also assume the existence of two planned production batches, each of 1,100 m^2 for k_2, which the company estimates will be also divided into four homogeneous sublots of 250, 400, 300 and 150 m^2 with the tone (c_i) and gage (g_i) represented in Figure 1.

Based on these planned homogeneous sublots, the ceramic company can commit during the OPP the request of 660 m^2 of k_1 and 300 m^2 of k_2 from a customer order proposal composed of two order lines. Given the HR among the units of the same product, the only possibility of committing this order is for the company to reserve 660 m^2 from the homogeneous sublot of k_1 with tone c_1 and gage g_2 because it is the only homogeneous sublot whose size (700 m^2) is bigger than the required quantity (660 m^2). Since k_1 and k_2 belong to the same series, the homogeneous sublot used to reserve the 300 m^2 of k_2 in the customer order should also be of gage g_2. The only sublot of k_2 with a size that equals or is bigger than 300 m^2 and with gage g_2 are both the sublots of 300 m^2 with tone c_3 and gage g_2. Therefore, the customer order proposal will be committed according to this estimation of the size of the homogeneous sublots.

However, given the inherent uncertainty in such companies, when planned production lots are produced and classified, real homogeneous quantities are likely to differ from the initial estimated ones. This can lead to a situation where if anything is made, the customer order cannot really be served.

Following the previous example, let's assume that once the three planned production batches have finally been manufactured, they are classified to provide the size of the homogeneous sublots depicted as "real situation" in Figure 1. In this new situation, the real homogeneous sublot of k_1 with tone c_1 and gage g_2 is 670 m^2 instead of the previously estimated 700 m^2. Even so, this discrepancy does not affect being able to serve the requested quantity of product k_1 because it is still enough to serve the 660 m^2 requested by the customer. However, for product k_2, the real size of 280 m^2 for the homogeneous sublot with tone c_3 and gage g_2 makes it impossible to serve the initial committed quantity of 300 m^2 with the customer if nothing is done: a shortage situation occurs. Therefore, once the real homogeneous sublots are known, the initial assignment of customer orders becomes infeasible, which renders serving the customer impossible.

Figure 1. Example of the problem under study



If the possibility of orders reallocation to homogeneous sublots exists, we might think about reserving 300 m^2 from the 390 m^2 homogeneous sublot of the first batch of k_2 with tone c_4 and gage g_1. However, this reallocation is not possible because product k_2 delivered to the customer should be of the same gage g_2 as product k_1. If only the first batch of k_2 had been manufactured, the customer order would not have been served. However, if all the quantities of the second batch of k_2 are uncommitted, the 300 m^2 of k2 requested by the customer would be served by reserving them from the homogeneous sublot of k_2 with a size of 310 m^2 and tone c_3 and gage g_2. Without the availability of this second batch of k_2 only in case customer allows some delay, partial deliveries should be contemplated to solve the problem.

If we consider that ceramic companies manage hundreds of customer orders from several order lines and each product presents more than two tones and gages, the task of finding only a feasible solution to this reallocation problem is no trivial one. This reallocation procedure becomes even more complicated when there are one or more objectives to be optimised. In these situations, mathematical programming models have proved their validity.

3. Literature review

A search of publications about mathematical models for SP was performed. As very few publications on this topic were found, the search was extended to mathematical programming models for the OPP that include some characteristic of the problem to be solved. The reason was that, according to Framinan & Leisten (2010), from a modelling point of view, SP deals with relaxing some constraints that have been previously considered in the OPP.

Note that this literature analysis does not intend to provide in-depth details of the features of the reviewed models, but of those closely linked to the problem at hand. Therefore, the employed analysis framework was divided into nine dimensions related to the previously described problem: 1) problem type; 2) availability; 3) manufacturing strategy; 4) customer segmentation; 5) customer orders; 6) homogeneity requirements;

7) flexibility in requirements; 8) objectives; 9) modelling approach. This literature review aims to identify which features have been addressed by existing models, and which represent a gap in the existing literature. The results of this analysis are shown in Tables 1 and 2, where the differences between existing models and the model proposed in this paper are also demonstrated.

The analysis of publications per problem type shows that only three of the 35 analysed articles address the SP problem, while the rest address the OPP. For SP problems, availability refers to the quantities used during the reallocation process. Alemany, Alarcón, Oltra, & Lario (2013b) and Boza, Alemany, Alarcón, & Cuenca (2014) consider the reallocation of available quantities in stock, while Alemany, Grillo, Ortiz, & Fuertes-Miquel (2015) consider the simultaneous reallocation of stocked and planned ones. For OPP problems, availability refers to the availability level checked when promising orders. 26 of the 32 OPP publications use the Available-To-Promise level, while the rest resort to other levels of availability, such as Capable-To-Promise, Deliver-To-Promise or Profitable-To-Promise (Abid, D'amours, & Montreuil, 2004; Baker, 2014; Behdani, Adhitya, Lukszo, & Srinivasan, 2011; Kirche, Kadipasaoglu, & Khumawala, 2005; Manavizadeh, Goodarzi, Rabbani, & Jolai, 2013; Zhang & Tseng, 2009).

When we examined the manufacturing strategy, we found that all the SP publications deal with the Make-To-Stock strategy, while OPP publications use different manufacturing strategies: Make-To-Stock in 41% of publications, Make-To-Order in 56% of them, and Assemble-To-Order in 25%. Percentages sum more than 100% as some references consider more than one manufacturing strategy (Fleischmann & Meyr, 2004; Khataie, Bulgak, & Segovia, 2011; Robinson & Carlson, 2007).

In customer segmentation terms, only six papers consider it when treating some customers as priority (Alemany, Grillo, Ortiz, & Fuertes-Miquel, 2015; Manavizadeh, Goodarzi, Rabbani, & Jolai, 2013; Meyr, 2009; Pibernik & Yadav, 2009), when prioritising those customer orders with an early delivery date (Alemany, Lario, Ortiz, & Gómez, 2013a), or when assigning priority to customers depending on the order size (Jung, 2010). Furthermore, 31.4% of the analysed articles consider multiline orders, while the rest consider single line orders.

On the other hand, 17.1% of the analysed publications have HR among units of the same final product. So, customer orders must be served with homogeneous products. In the ceramic field, homogeneity is measured by the quality, tone and gage attributes of the final product (Alemany, Lario, Ortiz, & Gómez, 2013a; Alemany, Alarcón, Oltra, & Lario, 2013b; Alemany, Grillo, Ortiz, & Fuertes-Miquel, 2015; Boza, Alemany, Alarcón, & Cuenca, 2014). In TFT-LCD production, homogeneity is given by the quality and materials used (Lin, Hong, Wu, & Wang, 2010). In the computers assembly field, different components have specifications that can make assembly compatible or incompatible (Zhao, Ball, & Kotake, 2005). No analysed reference deals with HR among the units that comprise different order lines. However, there are various sectors in which this requirement should be considered, such as the furniture industry or the ceramic sector.

Regarding the flexibility in deliveries requirements, 45.7% of the publications contemplate the possibility of making delayed deliveries. In contrast, none of the analysed articles consider the possibility of making partial deliveries of order lines, which will be another novelty of this proposal.

The analysis of the objectives proposed by previous literature works shows that 51.4% of the publications seek to maximise profits after SP or OPP processes.

However, the minimisation of the order lines served with delays, and the minimisation of the partial deliveries of order lines, are not addressed in the analysed literature.

Table 1. Literature review (Part I)

References	Problem		Availability			Manufacturing strategy			Customer segmentation	Customer orders	Homogeneity requirements		Flexibility in requirements	
	OP	SP	STOCK	MPS	ATP	MTS	MTO	ATO	Yes	ML	HP	HL	DA	POL
Abid, D'amours, & Montreuil (2004)	X						X							
Alemany, Alarcón, Oltra, & Lario (2013b)		X	X			X				X	X		X	
Alemany, Grillo, Ortiz, & Fuertes-Miquel (2015)		X	X	X		X			X	X	X		X	
Alemany, Lario, Ortiz, & Gómez (2013a)	X				X	X			X	X	X			
Baker (2014)	X						X							
Behdani, Adhitya, Lukszo, & Srinivasan (2011)	X						X						X	
Boza, Alemany, Alarcón, & Cuenca (2014)		X	X			X				X	X			
Bui & Sebastian (2010)	X				X		X							
Chen, Zhao, & Ball (2001)	X				X		X							
Chen, Zhao, & Ball (2002)	X				X		X			X				
Cheng & Cheng (2011)	X				X		X							
Chiang & Hsu (2014)	X				X		X			X				
Fleischmann & Meyr (2004)	X				X	X	X	X		X			X	
Gharehgozli, Rabbani, Zaerpour, & Razmi (2008)	X				X		X						X	
Halim & Muthusamy (2012)	X				X	X								
Jung (2010)	X				X	X			X				X	
Khataie, Bulgak, & Segovia (2011)	X				X	X	X						X	
Kirche, Kadipasaoglu, & Khumawala (2005)	X						X							
Lin, Hong, Wu, & Wang (2010)	X				X	X				X	X			
Manavizadeh, Goodarzi, Rabbani, & Jolai (2013)	X						X		X				X	
Meyr (2009)	X				X	X			X				X	
Okongwu, Lauras, Dupont, & Humez (2012)	X				X	X				X			X	
Pibernik (2005)	X				X	X								
Pibernik (2006)	X				X	X							X	
Pibernik & Yadav (2009)	X				X	X			X					
Rabbani, Monshi, & Rafiei (2014)	X				X		X			X			X	
Robinson & Carlson (2007)	X				X	X	X			X				
Tsai & Wang (2009)	X				X			X					X	
Venkatadri, Srinivasan, Montreuil, & Saraswat (2006)	X				X	X							X	
Volling & Spengler (2011)	X				X		X							
Wang, Zhu, & Zhang (2011)	X				X		X							
Xu, Allgor, & Graves (2009)	X				X	X								
Yang & Fung (2012)	X				X	X							X	
Zhang & Tseng (2009)	X						X						X	
Zhao, Ball, & Kotake (2005)	X				X			X			X			
This paper		X	X	X		X			X	X	X	X	X	X

SP: Shortage planning; OP: Order promising; STOCK: Real quantities in stock; MPS: Planned quantities in MPS; ATP: Available-to-promise; MTS: Make-to-stock; MTO: Make-to-order; ATO: Assemble-to-order; ML: Multiline order: HP: Homogeneity between units of the same order line; HL: Homogeneity between units of different order lines; DA: Delivery delay allowed; POL: Partial deliveries of order lines.

Table 2. Literature review (Part II)

References	Objectives			Modelling approach										
	MP	MD	MSOL	LP	MILP	MOILP	NLP	INLP	FMP	HEU	HYB	SIM	SPP	DP
Abid, D’amours, & Montreuil (2004)									X	X				
Alemany, Alarcón, Oltra, & Lario (2013b)	X					X								
Alemany, Grillo, Ortiz, & Fuertes-Miquel (2015)	X								X					
Alemany, Lario, Ortiz, & Gómez (2013a)	X					X								
Baker (2014)								X			X		X	
Behdani, Adhitya, Lukszo, & Srinivasan (2011)												X		
Boza, Alemany, Alarcón, & Cuenca (2014)	X					X								
Bui & Sebastian (2010)	X				X									
Chen, Zhao, & Ball (2001)	X				X									
Chen, Zhao, & Ball (2002)	X				X									
Cheng & Cheng (2011)	X								X	X	X			
Chiang & Hsu (2014)	X			X										
Fleischmann & Meyr (2004)					X									
Gharehgozli, Rabbani, Zaerpour, & Razmi (2008)	X										X			
Halim & Muthusamy (2012)									X					
Jung (2010)				X										
Khataie, Bulgak, & Segovia (2011)	X					X								
Kirche, Kadipasaoglu, & Khumawala (2005)	X				X									
Lin, Hong, Wu, & Wang (2010)	X				X									
Manavizadeh, Goodarzi, Rabbani, & Jolai (2013)					X					X	X			
Meyr (2009)	X				X									
Okongwu, Lauras, Dupont, & Humez (2012)					X									
Pibernik (2005)	X				X									
Pibernik (2006)					X									
Pibernik & Yadav (2009)												X		
Rabbani, Monshi, & Rafiei (2014)							X			X		X		
Robinson & Carlson (2007)					X							X		
Tsai & Wang (2009)	X				X									
Venkatadri, Srinivasan, Montreuil, & Saraswat (2006)				X										
Volling & Spengler (2011)					X							X		
Wang, Zhu, & Zhang (2011)	X				X									
Xu, Allgor, & Graves (2009)					X									
Yang & Fung (2012)	X							X		X	X	X	X	X
Zhang & Tseng (2009)	X				X									
Zhao, Ball, & Kotake (2005)					X									
This paper	X	X	X			X								

MP: Maximise profit; MD: Minimise delayed deliveries; MSOL: Minimise partial deliveries; LP: Linear programming; MILP: Mixed integer linear programming; MOILP: Multi-objective integer linear programming; NLP: Non-linear programming; INLP: mixed integer non-linear programming, FMP: fuzzy mathematical programming; HEU: heuristics/metaheuristics; HYB: hybrid models; SIM: simulation; SPP: stochastic/probabilistic programming; DP: Dynamic programming.

According to the analysed publications, the most widely used modelling approach for this problem is MILP, but other modelling approaches are used by some authors, such as linear programming, non-linear programming, integer non-linear programming, simulation, heuristics and metaheuristics, fuzzy mathematical programming models, multi-objective integer linear programming, stochastic programming and dynamic programming.

To summarise, we conclude that, although there are publications that consider some of the characteristics of the problem, none addresses them all simultaneously. In addition, the joint consideration of the proposed objectives is a novelty as most are not addressed in the literature. Indeed, no publication addresses HR among order lines, nor the allowance of partial deliveries of order lines, which are the main novelties of this proposal. This requires not only the differentiation between homogeneous sublots from the same batch (as previously done), but also the attributes specification for each sublot. These features are the major contributions of the proposed model.

4. Model

This model is referred to hereinafter as the "Homogeneity Multi-Line Shortage-Planning Model" (HML-SP Model).

4.1. Nomenclature

The indices, sets of indices, parameters and decision variables that are subsequently used in the HML-SP Model are described in Table 3. As seen from the definition part of the model, to ensure achieving the HR among the units of the products that belong to the same and different order lines, the novel specification of the tones, gages, and series which characterise each product is necessary. This aspect obliges a new more complex formulation of the whole proposed model compared to others that consider HR and are reported in the literature review section. Furthermore, the modelling of the multiple objectives and the allowance of the partial deliveries of the order lines that belong to the same customer order constitute the other differentiation characteristic.

Table 3. Nomenclature

Indices			
f	Reallocation objective	g	Existing gage of the considered products
o	Customer order already committed	c	Existing tone of the considered products
l, l'	Order line that composes customer orders	m	Production line
k, k'	Finished product	t	Time periods in the reallocation planning horizon ($t = 1, \dots, T$)
s	Series to which a product can belong		
Set of indices			
O_k	Set of orders *o* requesting product *k*	C_k	Set of possible tones *c* for product *k*
L_o	Set of order lines *l* included in order *o*	G_k	Set of possible gages *g* for product *k*
KLO_{ol}	Set defining the product *k* required on order line *l* of order o	S_k	Set defining the serie *s* that product *k* belongs to.
LOK_{ok}	Set of order lines *l* of order o requesting product *k*	KS_s	Set of products *k* that belong to the same serie *s*
Parameters			
w_f	Weight assigned to objective *f* of the HML-SP Model	β_{kcg}	Fraction of the production lot of product *k* expected to have tone *c* and gage *g* after production
p_{ol}	Profit obtained when serving order line *l* of order *o*	$LDmax_o$	Maximum number of time periods that order *o* can be delayed
hc_k	Per unit inventory holding cost of product *k* per period *t*	$DOmax_o$	Maximum number of partial deliveries allowed for order *o*
q_{olk}	Requested quantity of product *k* in order line *l* of order *o*	$stock_{kcg}$	Initial stock of product *k* characterised by tone *c* and gage *g*
rc_o	Cost of rejecting order proposal *o*	mps_{kmt}	Planned quantity of product *k* to be produced on production line *m* during period *t*
no	Total number of orders *o*		
dd_o	Committed due date for order *o*		
nl_o	Number of order lines included in order *o*		
Decision variables			
Y_o	Binary variable takes a value of 1 when the entire order *o* is served, and 0 otherwise		
YL_{ol}	Binary variable takes a value of 1 when order line *l* from order *o* is served, and 0 otherwise		
D_{ot}	Binary variable takes a value of 1 when order *o* is partially or completely delivered during period *t*, and 0 otherwise		
DL_{olt}	Binary variable takes a value of 1 when order line *l* from order *o* is delivered during period *t*, and 0 otherwise		
AD_{ol}	Number of time periods during which the required product quantity in order line *l* of order *o* is reserved until it is delivered		
LDL_{ol}	Number of time periods of delay in the delivery of order line *l* of order *o* in relation to committed due date dd_o		
UDL_{ol}	Binary variable takes a value of 1 when order line *l* from order *o* is served with delay, and 0 otherwise		
$ATP0_{kcg}$	Stock available to promise quantity (ATP) of product *k* with tone *c* and gage *g* after the reallocation of the real and planned available quantities to the committed orders		
ATP_{kcgmt}	Planned available to promise quantity (ATP) of product *k* with tone *c* and gage *g* to be produced on production line *m* during period *t* after the reallocation of the real and planned available quantities to the committed orders		
$U0_{olkcgs}$	Binary variable takes a value of 1 when the quantity of required product *k* on order line *l* of order *o* that belongs to series *s* is reserved from $stock_{kcg}$, and 0 otherwise		
$U_{olkcgmts}$	Binary variable takes a value of 1 when the quantity of required product *k* on order line *l* of order *o* that belongs to series *s* is reserved from planned lot mps_{kmt} with tone *c* and gage *g*, and 0 otherwise		

4.2. HML-SP Model

The HML-SP Model is presented in this subsection. Firstly, the different objective functions are detailed. Secondly, the restrictions given by the characteristics of the problem are formulated.

4.2.1. Objective function

The first objective (1), called Z_P, consists in maximising profits during the reallocation process. Profits are made as the difference between the margin earned by serving order lines and the costs incurred when rejecting orders and holding quantities of product for an order until it meets its committed due date.

$$Max[Z_P] = \sum_{o} \left(\sum_{l \in L_o} \left(p_{ol} \cdot YL_{ol} - \sum_{k \in KLO_{ol}} hc_k \cdot AD_{ol} \cdot q_{olk} \right) - rc_o \cdot (1 - Y_o) \right) \quad (1)$$

The second objective (2), called Z_D, consists in minimising the number of order lines served with delays.

$$Min[Z_D] = \sum_{o} \sum_{l \in L_o} UDL_{ol} \quad (2)$$

The third objective (3), called Z_{PD}, consists in minimising the number of partial deliveries of order lines. For an order, a partial delivery exists if the number of deliveries ($\sum_t D_{ot}$) is higher than one when the order is delivered ($Y_o = 1$). The total number of partial deliveries is calculated as the difference between the total number of deliveries and the number of served orders.

$$Min[Z_{PD}] = \sum_{o} \left(\sum_{t} D_{ot} - Y_o \right) \quad (3)$$

4.2.2. Constraints

Set of constraints (4) establishes that the updated stocked quantity of product *k* with tone *c* and gage *g* equals the initial stock of this product with tone *c* and gage *g*, minus the quantities reserved to serve orders.

$$ATP0_{kcg} = stock_{kcg} - \sum_{o \in O_k} \sum_{l \in LOK_{ok}} \sum_{s \in S_k} q_{olk} \cdot U0_{olkcgs} \qquad \forall k, c \in C_k, g \in G_k \tag{4}$$

Set of constraints (5) indicates that the available planned quantity of product *k* with tone *c* and gage *g* produced on production line *m* during period *t* equals the master production schedule quantity to be produced for this product, production line and period, multiplied by the probability of obtaining tone *c* and gage *g*, minus the quantities reserved to serve orders.

$$ATP_{kcgmt} = \beta_{kcg} \cdot mps_{kmt} - \sum_{o \in K_o} \sum_{l \in LOK_{ok}} \sum_{s \in S_k} q_{olk} \cdot U_{olkcgmts} \qquad \forall k, c \in C_k, g \in G_k, m, t \tag{5}$$

Set of constraints (6) ensures that an order line can be reserved only once to thus avoid the possibility of serving an order line with heterogeneous quantities.

$$\sum_{k \in KLO_{ol}} \sum_{c \in C_k} \sum_{g \in G_k} \sum_{s \in S_k} \left(U0_{olkcgs} + \sum_{m} \sum_{t} U_{olkcgmts}\right) = YL_{ol} \qquad \forall o, l \in L_o \tag{6}$$

Set of constraints (7) indicates that an order can be served only if all its order lines are served. These constraints also act contrariwise.

$$\sum_{l \in L_o} YL_{ol} = nl_o \cdot Y_o \qquad \forall o \tag{7}$$

Sets of constraints (8) – (10) force the real delivery date of an order line to be comprised during the period defined by the committed due date and the maximum delay allowed for that order.

$$\sum_{t} DL_{olt} \cdot t \geq dd_o \cdot YL_{ol} \qquad \forall o, l \in L_o \tag{8}$$

$$\sum_{t} DL_{olt} \cdot t = dd_o \cdot YL_{ol} + LDL_{ol} \qquad \forall o, l \in L_o, \qquad (9)$$

$$LDL_{ol} \leq LDmax_o \cdot UDL_{ol} \qquad \forall o, l \in L_o \qquad (10)$$

Set of constraints (11) ensures that, if an order line is served without delays, then the binary variable that indicates if an order line is delayed equals zero.

$$LDL_{ol} \geq UDL_{ol} \qquad \forall o, l \in L_o \qquad (11)$$

Set of constraints (12) indicates that an order cannot be delivered with delays if it is not served.

$$UDL_{ol} \leq YL_{ol} \qquad \forall o, l \in L_o \qquad (12)$$

Set of constraints (13) ensures that an order line can be served only once if it is served.

$$\sum_{t} DL_{olt} \leq 1 \qquad \forall o, l \in L_o \qquad (13)$$

Set of constraints (14) calculates the number of time periods during which a requested quantity of product is reserved until its real delivery date.

$$AD_{ol} = \sum_{t} DL_{olt} \cdot t - \sum_{k \in KLO_{ol}} \sum_{c \in C_k} \sum_{g \in G_k} \sum_{s \in S_k} \left(U0_{olkcgs} + \sum_{m} \sum_{t} U_{olkcgmts} \cdot t \right) \; \forall o, l \in L_o \qquad (14)$$

Set of constraints (15) ensures that when an order line is served during period *t*, then a partial or complete delivery of that order is made during this period.

$$\sum_{l \in L_o} DL_{olt} \leq D_{ot} \cdot nl_o \qquad \forall o, t \qquad (15)$$

Set of constraints (16) indicates that when an order is completely or partially delivered during period *t*, then at least one line of this order is delivered during that period:

$$\sum_{l \in L_o} DL_{olt} \geq D_{ot} \qquad \forall o, t \tag{16}$$

Set of constraints (17) ensures that the quantity of partial deliveries made for an order is less than or equals the maximum of partial deliveries allowed for that order.

$$\sum_{t} D_{ot} \leq DOmax_o \qquad \forall o \tag{17}$$

Set of constraints (18) ensures that the novelty requirement of two lines or more of the same customer order that belong to the same series *s* must be served with the quantities available with the same gage *g*:

$$\sum_{c} \left(U0_{olkcgs} + \sum_{m} \sum_{t} U_{olkcgmts} \right) = \sum_{c} \left(U0_{ol'k'cgs} + \sum_{m} \sum_{t} U_{ol'k'cgmts} \right)$$

$$\forall o, s, k \in KS_s, k' \in KS_s, l \in LOK_{ok}, l' \in LOK_{ok'}, g \tag{18}$$

Finally, set of constraints (19) shows the definition of the decision variables:

$$\begin{aligned} &\mathrm{AD}_{\mathrm{ol}}, \mathrm{LDL}_{\mathrm{ol}} \quad INTEGER, \\ &\mathrm{ATP0}_{\mathrm{kcg}}, \mathrm{ATP}_{\mathrm{kcgmt}} \quad CONTINUOUS, \\ &Y_o, YL_{ol}, \mathrm{D}_{\mathrm{ot}}, \mathrm{DL}_{\mathrm{olt}}, \mathrm{UDL}_{ol}, \mathrm{U}_{\mathrm{olkcgmts}}, \mathrm{U0}_{\mathrm{olkcgs}} \quad BINARY \end{aligned} \tag{19}$$

4.3. Resolution methodology for the HML-SP Model

MOILP models can be solved by different methods regarding the phase in which decision makers express their preferences about the objectives (Hwang & Masud, 1979). In *a priori* methods, decision makers express their preferences before solving the model, while decision makers select the most satisfying solution from among a set of non-dominated solutions obtained by the model in *a posteriori* methods (Mavrotas, 2009). Thus, in *a posteriori* methods, decision makers express their preferences after solving the model. In this subsection, *a priori* and *a posteriori* methods to solve the HML-SP model are presented. These methods are later applied in Section 5.3.

4.3.1. A priori method: the weighted sum method

The weighted sum method consists in constructing a single global objective function by assigning weights to each objective and summing their results. The sum of the weights assigned to each objective should equal the unit ($w_P + w_D + w_{PD} = 1$). The closer the weight assigned to an objective is to one, the stronger incidence that this objective has on the global objective function. It is necessary to scale each objective value by dividing them between the highest value that they can reach so they acquire values between 0 and 1. The benefit of serving all the committed orders with no cost, the total number of existing order lines, and the total number of allowed deliveries will be the maximum values for objectives Z_P, Z_D, and Z_{PD}, respectively. After applying the weighted sum resolution method, the resulting HML-SP model is formulated as follows:

$$Max[Z] = w_P \cdot \frac{Z_P}{\sum_o \sum_{l \in L_o} p_{ol}} + w_D \cdot \frac{Z_D}{\sum_o nl_o} + w_{PD} \cdot \frac{Z_{PD}}{\sum_o DOmax_o} \qquad (20)$$

Subject to: Equations (4) – (19).

Note that Z_P, Z_D, and Z_{PD} are calculated through Equations (1) – (3).

The disadvantage of this method is that decision makers hardly know what their preferences are and/or how to quantify them (Mavrotas, 2009). So it is difficult to establish weights to objectives. To solve this, a method like the Analytic Hierarchy Process (AHP) can be employed to determine the objectives' weights (Saaty, 1990).

4.3.2. A posteriori method: the ε-constraint method

To transform the multi-objective model into a single-objective model, the ε-constraint method is used (Chankong and Haimes, 1983; Ehrgott, 2005; Mavrotas, 2009) in which one of the objectives is selected as the model's objective function, while the other objectives are considered the model's constraints. In this case, maximisation of profits is maintained as the model's objective function, minimisation of the number of order lines served with delays, and minimisation of partial deliveries of order lines are transformed into the model's constraints. The new model is formulated as follows:

$$Max\ Z = \sum_{o} \left(\sum_{l \in L_o} \left(p_{ol} \cdot YL_{ol} - \sum_{k \in KLO_{ol}} hc_k \cdot AD_{ol} \cdot q_{olk} \right) - rc_o \cdot (1 - Y_o) \right) \tag{21}$$

subject to:

$$\sum_{o} \sum_{l \in L_o} UDL_{ol} \leq \varepsilon_D \tag{22}$$

$$\sum_{o} \left(\sum_{t} D_{ot} - Y_o \right) \leq \varepsilon_{PD} \tag{23}$$

and Equations (4) - (19).

To apply this method, a payoff table that determines the ranges of values that each objective modelled as a constraint can assume needs to be calculated. In this paper, the lexicographic optimisation for the payoff table proposed by Mavrotas (2009) is used that provides with non-dominated solutions. It consists in solving the model each time for only one objective. Then, the model is solved for another objective, forcing the first objective to be equal to its optimal value by means of a constraint. This process is repeated for all the combination of objectives. For example, in a model with two objectives (f_1 and f_2), the optimum value for f_1 is obtained. Then objective f_2 is optimised by considering that f_1 must equal the optimal value obtained in the previous execution. To obtain another non-dominated solution, the process is repeated by firstly solving the model for objective f_2.

Then the grid points (ε_i) obtained when dividing the objective's range of values into equal intervals are used to obtain the non-dominated solutions to the problem. Finally, decision makers select the non-dominated solution that most satisfies them. Note that the payoff table, and therefore the grid points, differ for each data instance.

This approach is more appropriate for obtaining the solution to be implemented into a real company because it obtains non-dominated solutions, among which decision makers can choose. However, if the model needs to be executed for different sets of instances (scenarios), this approach becomes tedious, long and dependent on the decision maker's preference. So the experimental design could not be automated for this last reason. To avoid these disadvantages for the experimental design, an *a priori* method seems more adequate.

5. Experimental design: application to a ceramic tile company

The aims of the numerical tests defined in this section are threefold: 1) to validate the HML-SP Model; 2) to analyse the model's behaviour in different situations for the company under study to provide some managerial insights for the studied case and 3) to check computational efficiency by solving different scenarios. Before analysing these aspects, the data used in the experimentation are described.

5.1. Input data

The experimental design was conducted with data from a major company in the Spanish ceramic sector, and were slightly modified for confidentiality reasons, while maintaining the magnitude order.

A planning horizon of 12-time periods (weeks) was contemplated, which is approximately a 3-month planning. Ten final products were considered and classified into two different tones and three different gages, with six homogeneous subtypes. In addition, each product belonged to a series so, if products from the same series were required in the same order, it was necessary to ensure that all their units were homogeneous for the gage attribute.

There were 150 committed orders (firm orders) for the considered planning horizon. Fifty of these orders were considered priorities. Each order was made up of between one and ten order lines, with an average of 2.31 lines per order, and there were 347 total order lines. For each line that belonged to an order, the final requested product and the demanded quantity were known and ranged from a minimum of 20 m^2 to a maximum of 4,000 m^2, with an average of 150 m^2 per order line. The same final product could be requested on more than one order line of the same order. This is often done if a customer requires a very large amount of a given product and does not require all this quantity to be homogeneous, but only parts of it (for example, large builders).

Each order was associated a committed due date, which was the same for all its order lines. For each order, the maximum delivery delay (one-time periods) and the maximum partial deliveries allowed (two for multiline orders and one for single line orders) were also known.

It was assumed that current stocks were classified according to their attributes and the planned batches of the MPS were known. Current stocks varied by subtype, ranging from 0 m^2 and 3,500 m^2. In addition, the distribution of production batches into homogeneous sublots was estimated.

Table 4 shows the unitary margin, unitary holding cost and unitary rejection cost per product. Note that unitary rejection costs were estimated as 75% of the unitary margin for each product. An increase of 20% in the rejection costs for priority orders was assumed to reflect the company's preference for them to be firstly served.

Table 4. Economic data per product

Final product *k*	Unitary profit (€/m^2)	Unitary rejection cost (€/m^2)	Unitary holding cost (€/m^2·week)
1	7.00	5.25	0.064
2	18.00	13.50	0.052
3	12.00	9.00	0.040
4	10.00	7.50	0.036
5	5.00	3.75	0.036
6	11.00	8.25	0.052

7	13.00	9.75	0.040
8	12.00	9.00	0.036
9	6.00	4.50	0.052
10	15.00	11.25	0.045

Two new data instances were created to assess the complexity of the HML-SP model in light of the different problem sizes and their respective resolution times. A smaller instance was built by considering the data for the first six time periods of the original instance. Similarly, a larger instance was generated by duplicating the data provided by the company and comprised a 24-time period planning horizon. To avoid equality between the data from the first 12 time periods and the other periods, the due dates between the 13^{th} and 24^{th} time periods were randomly attributed.

5.2. Defining the hypotheses

The purpose of the experimental design was to validate the HML-SP Model and to provide some managerial insights as the following hypotheses:

- H1. There may be some conflict with the HML-SP model objectives when obtaining optimum values.
- H2. Given a master plan, the greater the division of a batch into homogeneous sublots (more subtypes) and the more uniform its size, the more difficult it will be to serve the committed orders from the homogeneous product.
- H3. The results should improve if the number of allowed partial deliveries and/or the maximum allowed delay for each order increases as these measures increase the feasible area and, therefore, the possibility of finding better solutions.
- H4. The difficulty of serving orders should grow significantly when considering HR among order lines.

The hypotheses were demonstrated by executing different sets of scenarios and a statistical analysis of the obtained results. For clarity reasons, these demonstrations are explained fairly in Section 5.3.

In addition, an analysis of the model's computational complexity was done in Section 5.4, where the problem size, the resolution time and the GAP for each execution are displayed. For the scenarios in which the optimal solution was not found during the time limit defined as 18,000 seconds, a GAP was obtained and represents the difference between the best-found solution and the best-bound explored one. The average GAP for the original data instance was 0.24%. The GAP varied from 0.00% to 0.63% in the proposed scenarios that came very close to zero. This denotes that the obtained solutions presented in next section are optimum solutions or come very close to them.

5.3. Experimental results to prove the hypotheses

In this subsection, different sets of scenarios were solved with the proposed model to prove the defined hypotheses. The original data instance provided by the company (a 12 time-period planning horizon) was used for all the executions.

5.3.1. Objectives' conflict

A partial correlation analysis of the non-dominated solutions for the HML-SP model was made to prove the existing conflict between the model's objectives (H1). When the model was solved with the ε-constraint method, a payoff table comprised by the non-dominated solutions was needed. To find out these non-dominated (Pareto optimal) solutions (Table 5), lexicographic optimisation, as explained in Section 4.3.2, was employed.

Table 5. Payoff table

#	Z_P	Z_D	Z_{PD}
1	267162.717	78	30
2	267162.717	86	25
3	222613.882	0	0
4	266856.842	149	0

A partial correlation analysis of these solutions can be made to study the relations between the results of the objectives, and to therefore discover if there is any conflict between the different objectives considered in the HML-SP model (Table 6).

Table 6. Partial correlation coefficient

	Z_P	Z_D	Z_{PD}
Z_P	1	0.9996	0.9990
Z_D	0.9996	1	-0.9985
Z_{PD}	0.9990	-0.9985	1

The values of the profits and order lines served with delays positively and perfectly correlated (0.9996 ≈ 1) in such a way that when profits increased, the number of required delayed order lines also increased. Similarly, profits and partial deliveries also perfectly and positively correlated (0.9990 ≈ 1), in such a way that the partial deliveries increased as profits improved. As the purpose of the model was to maximise profits while minimising the number of delayed order lines and partial deliveries, this analysis proved the conflict between maximisation of profits and the other objectives.

The number of delayed order lines and the number of partial deliveries correlated perfectly and negatively (-0.9985 ≈ -1). This means that one of them increased, while the other decreased. As the model intended to minimise both objectives, this result ensured a conflict between them. This proved the existence of conflict among all the proposed objectives and proved Hypothesis H1.

5.3.2. Distribution of batches into homogeneous sublots

Five scenarios were proposed to prove Hypothesis H2, according to which it was more difficult to serve committed orders with homogeneous product when a production lot was divided into more sublots and their size was more uniform. These scenarios (Table 7) differed in the considered distribution of a production batch into homogeneous sublots (β_{kcg}). It was assumed that a maximum of three homogeneous sublots could be obtained by each production batch ($\beta_{k11}; \beta_{k12}; \beta_{k23}$). The homogeneous sublot β_{k11} was defined by tone 1 and gage 1, sublot β_{k12} was defined by tone 1 and gage 2, and finally, the sublot β_{k23} was defined by tone 2 and gage 3.

Table 7. Distribution of batches into homogeneous sublot scenarios

Scenario	β_{k11}(%)	β_{k12}(%)	β_{k23}(%)
1 homogeneous sublot	100	--	--
2 unbalanced homogeneous sublots	70	30	--
3 unbalanced homogeneous sublots	70	20	10
2 balanced homogeneous sublots	50	50	--
3 balanced homogeneous sublots	40	30	30

As explained in the last paragraph of Section 4.3.2, the weighted sum method was employed given its suitability for solving sets of scenarios. To determine the weight distribution between the objectives that comprised the global objective function, AHP was used. This technique is based on the paired comparisons of the elements among which weights were to be distributed.

The scale used to make judgements was the proposed by Saaty (1990), where *1* means that both elements are of the same importance, and *3, 5, 7,* and *9* mean that one element is moderately, strongly, very strongly, or extremely important over another element, respectively. If one of the above numbers is assigned to element *x* when compared with the element *y*, then *y* has the reciprocal value when compared with *x* (Saaty, 1990). With this scale, the pairwise comparison matrix and weight distribution were obtained (Table 8).

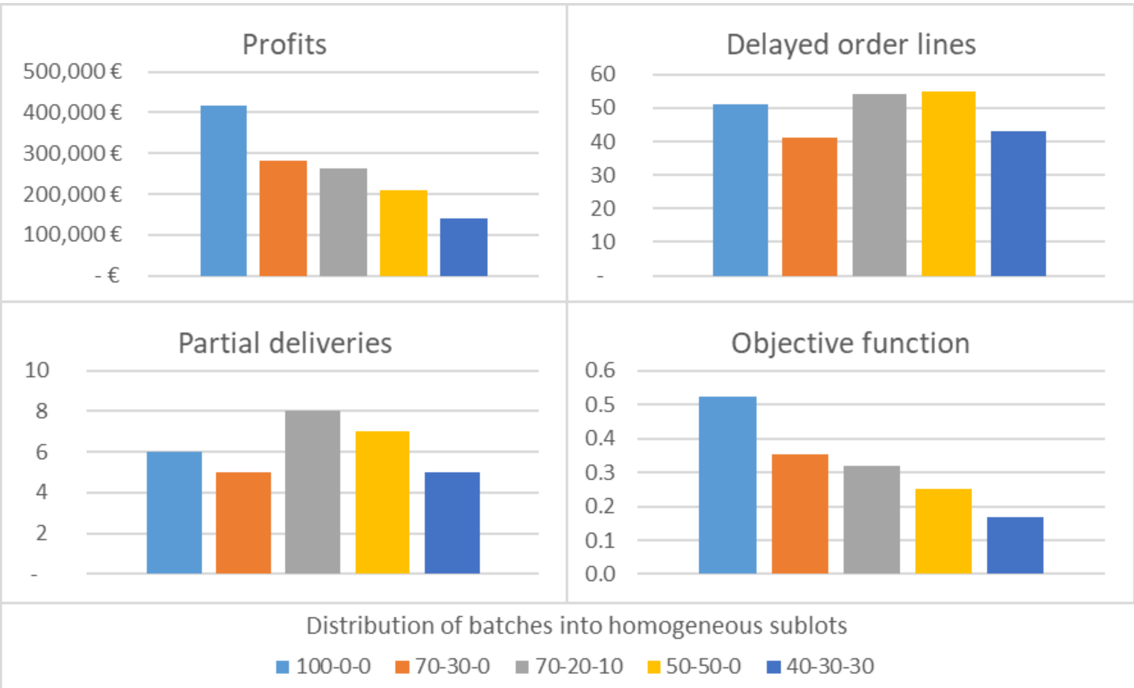
Table 8. Pairwise comparison matrix

	Z_P	Z_D	Z_{PD}	w_f
Z_P	1	5	5	0.66
Z_D	1/5	1	1/3	0.09
Z_{PD}	1/5	3	1	0.25

A maximum delivery delay of one period ($LDmax_o = 1$) and a maximum of two partial deliveries per order ($DOmax_o = 2$) were allowed. Both HR were considered: homogeneity among units of the same order line and among units of different order lines.

The results (Figure 2) show that the values of the profits and the global objective function became worse as the division of a batch into homogeneous sublots increased. This was because it is more difficult to serve orders with homogeneous product when lots were more heterogeneous. Therefore, the profits made in the "One homogeneous sublot" scenario practically duplicated those made in the scenarios where the lack of homogeneity in the product was considered.

Figure 2. Results of the distribution of batches into homogeneous sublot scenarios

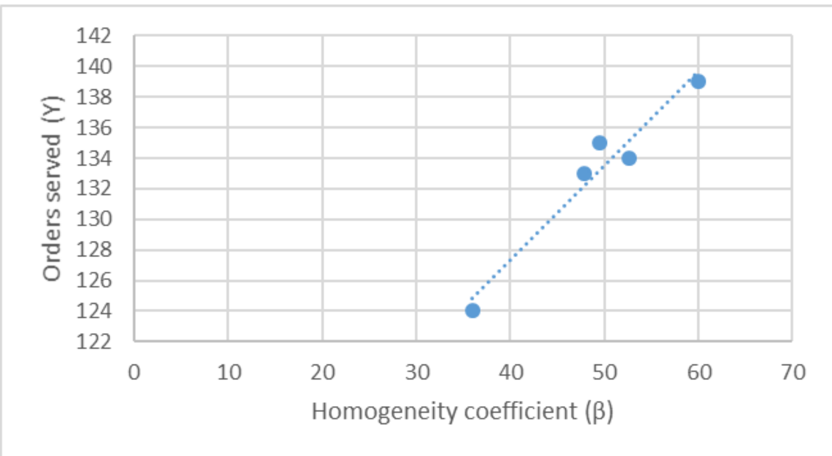


This same logic was not seen in the other objectives partly since the weights assigned to them in the global objective function were relatively small compared to the profits weight. A scatter plot of the distribution of batches into homogeneous sublots and the number of orders served in each scenario (Figure 3) shows how the quantity of served orders decreased as the number of homogeneous sublots increased and consequently their size decreased. To obtain this scatter plot, it was necessary to first transform each homogeneity distribution, composed of three terms (β_{k11}, β_{k12}, and β_{k23}), into a numerical value. The homogeneity coefficient value is supposed to be high when just one homogeneous sublot is obtained from the same production batch and to decrease its value as more sublots are obtained. Similarly, this coefficient should decrease its value as the different obtained sublots are more uniform in size. Thus, AHP was employed again, and conferred much preference to obtain only one homogeneous sublot rather than obtaining two, and even more preference rather than obtaining three sublots in the same lot. The weights obtained with this process ($w_{\beta_{k11}} = 0.60$; $w_{\beta_{k12}} = 0.36$; $w_{\beta_{k23}} = 0.04$) were multiplied to the different terms of each homogeneity distribution to obtain a homogeneity coefficient β (Table 9), which was used in the statistical analysis of the results.

Table 9. Homogeneity coefficient β obtainment

Homogeneity distribution β_{k11}-β_{k12}-β_{k23}	Homogeneity coefficient β
100-00-00	100 * 0.60 + 0 * 0.36 + 0 * 0.04 = 60
70-30-00	70 * 0.60 + 30 * 0.36 + 0 * 0.04 = 53
70-20-10	70 * 0.60 + 20 * 0.36 + 10 * 0.04 = 50
50-50-00	50 * 0.60 + 50 * 0.36 + 0 * 0.04 = 48
40-30-30	40 * 0.60 + 30 * 0.36 + 30 * 0.04 = 36

Figure 3. Scatter plot: Orders served *vs.* β



A correlation coefficient of 0.97 demonstrated the clear relation between the number of orders served and the homogeneity coefficient. In addition, a scatter plot showing the relation between these variables is displayed in Figure 3. Thus, when the homogeneity coefficient rose, the number of served orders also increased. This proved hypothesis H2 and showed the importance of allocating product quantities to customer orders considering HR in those industries characterised by the lack of homogeneity in the product.

5.3.3. Flexibility in order deliveries

This subsection aimed to demonstrate that flexibility in order deliveries impacted the reallocating process. In the HML-SP model, flexibility in deliveries can be modified by allowing more/less partial deliveries per orders ($DOmax_o$) and/or shorter/larger delays ($LDmax_o$) of deliveries. For this reason, two sets of scenarios were proposed to prove the independent effect that partial and delayed deliveries had on the results (Table 10). In all, 22 scenarios were executed.

Table 10. Conditions of "Flexibility in Order Deliveries" scenarios

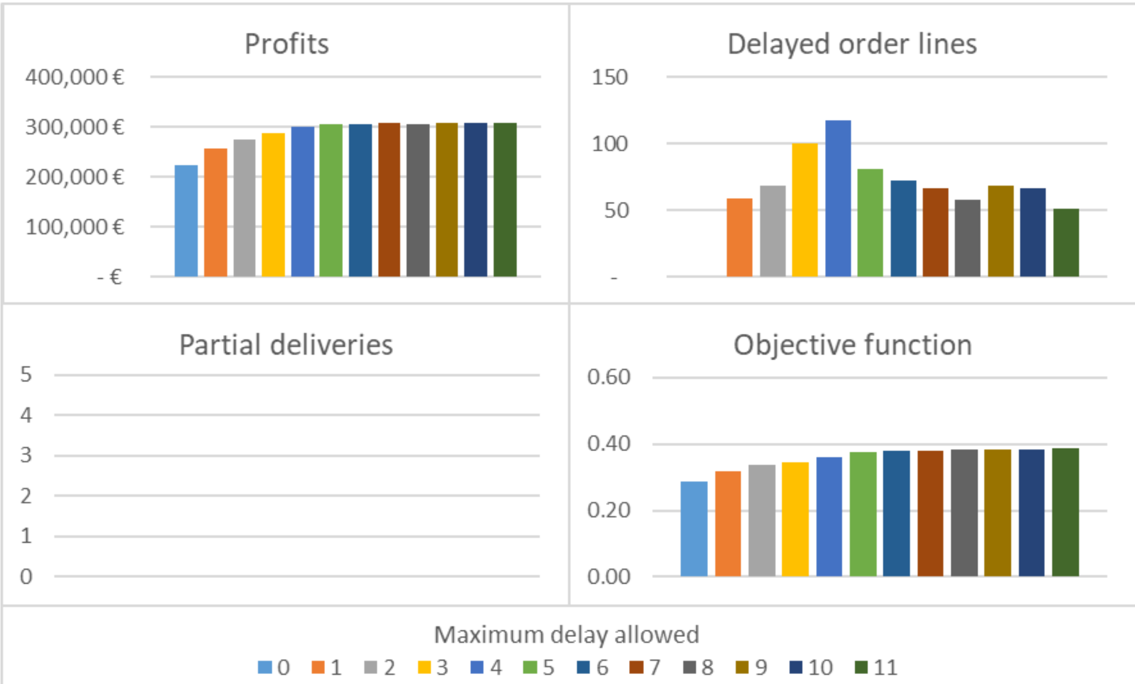
Set of scenarios	Scenario	Number of scenarios	$DOmax_o$	$LDmax_o$
Flexibility in the maximum allowed delay	i periods delay allowed $i \in (0, T-1)$	12	1	$min(i, T - dd_o)$
Flexibility in partial deliveries	j deliveries per order $j \in (1, 10)$	10	$min(j, nl_o)$	$Domax_o - 1$

For these scenarios, the same weight distribution among the objectives was assumed ($w_P = 0.66; w_D = 0.09; w_{PD} = 0.25$), as was the division of production batches into the most usual three unbalanced homogeneous sublots ($\beta_{k11} = 0.7; \beta_{k12} = 0.2; \beta_{k23} = 0.1$).

In the "Flexibility in the Maximum Allowed Delay" scenarios, the maximum delay allowed per order had to equal the minimum between the general maximum delay allowed and the difference between the planning horizon and the due date for this order. This assumption ensured that any order could be served after the planning horizon. Only one delivery was allowed per order to study the independent effect that delays had on the model.

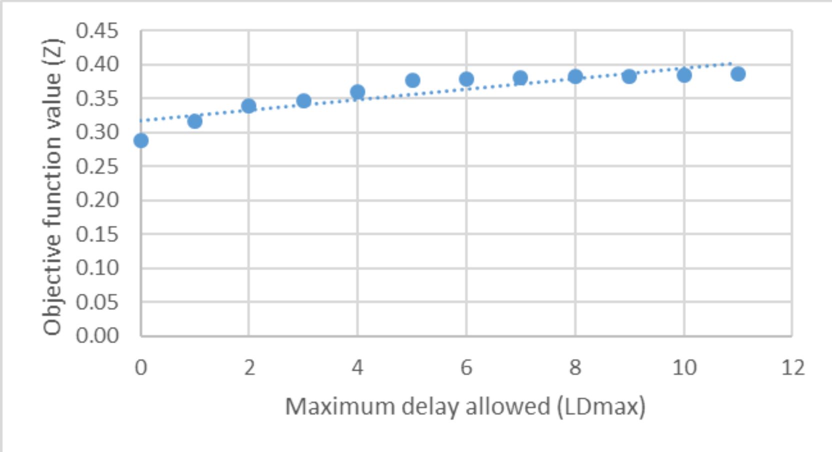
The results showed how the profits and the objective function value improved as the general maximum allowed delay increased (Figure 4). The same relation was not found in the number of order lines served with delay because this objective had a lower weight in the objective function. The part of the partial deliveries in Figure 4 is empty because no partial deliveries were allowed.

Figure 4. Results of the flexibility in the maximum allowed delay scenarios



To statistically prove that the objective function value improved as the maximum delay allowed increased, a correlation analysis of these variables was run. This was proved with a correlation coefficient of 0.90, which determined that both variables would simultaneously improve or worsen. Figure 5 shows a scatter plot of these variables, where their relation can be seen.

Figure 5. Scatter plot: Maximum delay allowed *vs.* Objective function value



In the “Flexibility in Partial Deliveries” set of scenarios, the number of deliveries allowed per order was modified to analyse how this factor impacted on the results of the model. For each scenario it was assumed that the maximum delay allowed per order was equal to the partial deliveries allowed in this scenario, minus one. The objective of this assumption was to ensure that enough delivery periods were available to make us of all the allowed deliveries. In addition, it was assumed that the number of deliveries allowed per order could be at most equal to the number of lines that comprise the order (Table 10).

The results (Figure 6) showed how both profits and the objective function value improved as the number of allowed deliveries per order increased. Besides, the number of order lines served with delays and the number of partial deliveries made did not seem to follow a pattern related to the flexibility in the allowed partial deliveries. As in the other scenarios, it was produced because the weight that the last two objectives had on the global objective function was low compared to maximisation of profits.

Figure 6. Results of the flexibility in partial deliveries scenarios

A correlation analysis between the global objective function value and the number of partial deliveries allowed and a scatter plot between these variables (Figure 7) were done. The relation between these variables was proved by a correlation coefficient of 0.94, which demonstrates that when these variables improve, the value of the other one also increases.

Figure 7. Scatter plot: Partial deliveries allowed *vs.* Objective function value

We hence concluded that delivery flexibilities led to better results for the global objective function of the HML-SP Model, and Hypothesis H3 was demonstrated. So these results can be employed by manufacturers to decide which policy to apply to their customers as to delays and partial deliveries if negotiation is possible.

5.3.4. Flexibility in the homogeneity requirement

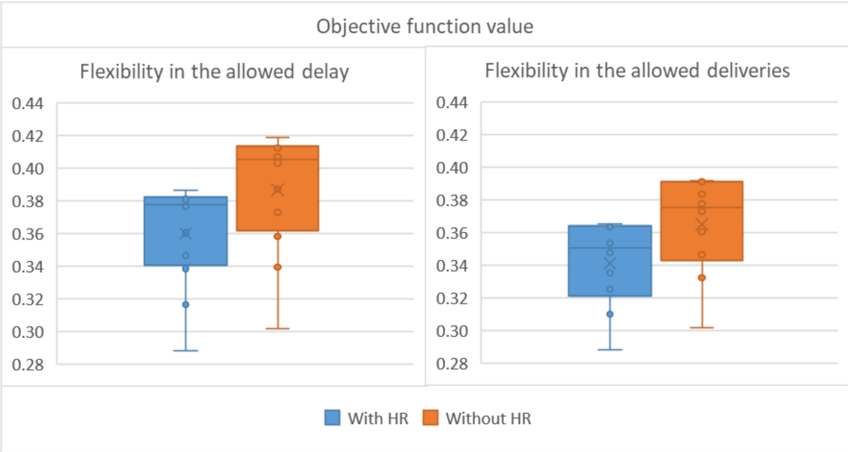
To prove Hypothesis H4, the scenarios solved in Section 5.3.3 when considering HR among units of different order lines were compared to the homologues without considering this requirement. For these scenarios, the real weight distribution among the objectives $(w_P = 0.66; w_D = 0.09; w_{PD} = 0.25)$ and the division of production batches into the most usual three unbalanced homogeneous sublots $(\beta_{k11} = 0.7; \beta_{k12} = 0.2; \beta_{k23} = 0.1)$ were assumed.

Figure 8 shows the comparison of these results for the scenarios proving the flexibility in the maximum allowed delay and the flexibility in partial deliveries. The scenarios that considered HR among order lines obtained worse global objective function values than the homologues scenarios that did not consider this requirement. This was because considering HR implies a reduced feasible area, which hinders the reallocation process.

Figure 8. Results of Flexibility in the homogeneity requirements scenarios

Box and Whiskers plots were used to show the main differences between the results distribution in those cases in which HR among the units of different order lines were or were not considered. For both cases, in the scenarios that proved flexibility in the number of deliveries allowed or in the allowed delay, the obtained values for the objective function were higher when HR was not considered. Also, in the scenarios where the flexibility in the allowed delay is analysed, the range of the objective function values was wider when HR were not considered.

Figure 9. Box and Whiskers plots



We conclude that absence of HR among order lines gave better results for the global objective function of the HML-SP Model and proved Hypothesis H4.

5.4. Computational efficiency

The proposed model was implemented in modelling language MPL® 5.0 and was resolved with solver Gurobi^{TM} 7.0.2. Input data and the values that the decision variables and objectives acquired after resolving the model were stored in a Microsoft Access database. The computer used to solve different scenarios had an Intel® Xeon® CPU E5-2640 v2 with two 2.00 GHz processor, with an installed capacity of 32.0 GB and a 64-bits operating system.

The maximum time resolution was limited to 18,000 seconds (5 h) for each proposed scenario. The model was solved for the different instances comprised by a 6-, 12- or 24-time period planning horizon to determine the model's complexity regarding the size of the problem and its impact on both, the resolution time and the quality of the obtained solutions.

Table 11 shows the problem size for each scenario set, which was evaluated by the number of constraints and the continuous, integer and binary variables. After analysing it, we found that all the scenarios corresponding to the same instance had the same number of continuous, integer and binary variables and these quantities augmented when the instance became bigger (more customer orders and larger planning horizon). The considerable presence of binary variables, which represented between 95-98% of the variables in all instances, should be emphasised. We observed that the number of constraints lowered for all the instances when HR among the units of different order lines were not considered by the model. This confirmed that the model’s size was bigger in those scenarios that included this requirement and comprised more customer orders and larger planning horizon.

Table 11. Problem size

	Planning horizon	Set of scenarios			
		Distribution of batches into homogeneous sublots	Flexibility in order deliveries	Flexibility in HR: with HR	Flexibility in HR: without HR
Constraints	6	5,505	5,505	5,505	4,284
	12	10,795	10,795	10,795	8,896
	24	28,730	28,730	28,730	24,932
Continuous variables	6	1,140	1,140	1,140	1,140
	12	2,220	2,220	2,220	2,220
	24	4,380	4,380	4,380	4,380
Integer variables	6	450	450	450	450
	12	694	694	694	694
	24	1,388	1,388	1,388	1,388
Binary variables	6	28,122	28,122	28,122	28,122
	12	83,842	83,842	83,842	83,842
	24	329,516	329,516	329,516	329,516
Total variables	6	29,712	29,712	29,712	29,712
	12	86,756	86,756	86,756	86,756
	24	335,284	335,284	335,284	335,284

If an optimal solution was not found for a particular scenario during the fixed resolution time (RT) of 18,000 seconds, a GAP was displayed. It represented the difference between the best-found solution and the best-bound explored one. Thus, a

GAP of 0.5% meant that the global objective function value for this solution had to improve by 0.5% to reach the best bound. The resolution time and GAP obtained for each scenario and instance are shown in Table 12.

When using the small instance (the 6-time period planning horizon), an optimal solution was found in 20 of the 27 scenarios, with an average resolution time of 7,895 seconds (132 minutes). With the original instance (the 12-time period planning horizon), the optimal solution was reached only in 11 of the 39 scenarios. Finally, no optimal solution was found for any scenario when solving the large instance (the 24-time period planning horizon), although the GAP was quite small and reached near optimal solutions. These results proved that the size of the instance influenced the time in which to optimally solve the model (Table 12).

Table 12. Resolution time and GAP per scenario and instance

Set of scenarios / Scenario	6 periods PH		12 periods PH		24 periods PH	
	RT (s)	GAP	RT (s)	GAP	RT (s)	GAP
Distribution of batches into homogeneous sublots (HS):						
• 1 HS	663	-	18,000	0.11%	18,000	0.61%
• 2 unbalanced HS	540	-	188	-	18,000	0.64%
• 3 unbalanced HS	429	-	18,000	0.20%	18,000	0.95%
• 2 balanced HS	2,597	-	2,396	-	18,000	0.53%
• 3 balanced HS	895	-	680	-	18,000	0.33%
Flexibility in the maximum delay allowed (with HR):						
• 0 periods of delay	84	-	148	-	18,000	0.25%
• 1 period of delay	137	-	4,054	-	18,000	0.85%
• 2 periods of delay	173	-	4,004	-	18,000	1.47%
• 3 periods of delay	18,000	0.13%	18,000	0.17%	18,000	1.35%
• 4 periods of delay	18,000	0.15%	18,000	0.34%	18,000	1.62%
• 5 periods of delay	18,000	0.21%	18,000	0.49%	18,000	0.76%
• 6 periods of delay	-	-	18,000	0.46%	18,000	0.87%
• 7 periods of delay	-	-	18,000	0.39%	18,000	0.96%
• 8 periods of delay	-	-	18,000	0.51%	18,000	1.15%
• 9 periods of delay	-	-	18,000	0.52%	18,000	0.97%
• 10 periods of delay	-	-	18,000	0.63%	18,000	1.00%
• 11 periods of delay	-	-	18,000	0.43%	18,000	0.97%
Flexibility in the maximum delay allowed (without HR):						
• 0 periods of delay	74	-	1,127	-	18,000	0.37%
• 1 period of delay	137	-	18,000	0.11%	18,000	0.52%
• 2 periods of delay	648	-	18,000	0.15%	18,000	0.43%
• 3 periods of delay	2,389	-	18,000	0.28%	18,000	0.33%
• 4 periods of delay	18,000	0.14%	18,000	0.21%	18,000	0.97%
• 5 periods of delay	18,000	0.16%	18,000	0.17%	18,000	0.33%
• 6 periods of delay	-	-	18,000	0.18%	18,000	0.38%
• 7 periods of delay	-	-	18,000	0.17%	18,000	0.33%

• 8 periods of delay	-	-	18,000	0.19%	18,000	0.38%
• 9 periods of delay	-	-	18,000	0.28%	18,000	0.38%
• 10 periods of delay	-	-	18,000	0.11%	18,000	0.49%
• 11 periods of delay	-	-	18,000	0.12%	18,000	0.27%
Flexibility in partial deliveries (with HR):						
• 1 delivery per order	84	-	148	-	18,000	0.25%
• 2 deliveries per order	492	-	15,470	-	18,000	0.76%
• 3 deliveries per order	2,434	-	18,000	0.50%	18,000	0.95%
• 4 deliveries per order	18,000	0.15%	18,000	0.25%	18,000	1.21%
• 5 deliveries per order	18,000	0.22%	18,000	0.42%	18,000	1.53%
• 6 deliveries per order	18,000	0.24%	18,000	0.37%	18,000	1.59%
• 7 deliveries per order	-	-	18,000	0.52%	18,000	1.65%
• 8 deliveries per order	-	-	18,000	0.28%	18,000	1.20%
• 9 deliveries per order	-	-	18,000	0.23%	18,000	1.16%
• 10 deliveries per order	-	-	18,000	0.26%	18,000	1.25%
Flexibility in partial deliveries (without HR):						
• 1 delivery per order	81	-	1,127	-	18,000	0.37%
• 2 deliveries per order	221	-	4,845	-	18,000	0.27%
• 3 deliveries per order	150	-	18,000	0.52%	18,000	0.23%
• 4 deliveries per order	4,009	-	18,000	0.43%	18,000	0.40%
• 5 deliveries per order	18,000	0.22%	18,000	0.37%	18,000	
• 6 deliveries per order	16,362	-	18,000	0.38%	18,000	0.50%
• 7 deliveries per order	-	-	18,000	0.26%	18,000	
• 8 deliveries per order	-	-	18,000	0.22%	18,000	
• 9 deliveries per order	-	-	18,000	0.17%	18,000	0.40%
• 10 deliveries per order	-	-	18,000	0.22%	18,000	0.30%

The increasing complexity of solving the model with the size of the instance was also seen when comparing the GAP average for all the scenarios. For the small instance, an average GAP of 0.03% was obtained, whereas an average GAP of 0.24% and 0.73% were obtained for the original and large instance respectively. In addition, the GAP of almost all the scenarios with no optimal solution came close to zero, which denotes that the obtained solutions came close to the optimum solution.

When comparing the resolution time and the GAP of each specific scenario, they increased as the instance became bigger. The difference between the average GAP of the scenarios that considered (0.05% for the small instance, 0.31% for the original instance and 1.08% for the large instance) or did not consider (0.02% for the small instance, 0.21% for the original instance and 0.41% for the large instance) HR among units of different order lines demonstrated that the computational efficiency was greater in those scenarios that did not take HR into account.

5.5. Managerial insights

As shown in the previous section, HML-SP model proved to be a suitable tool for decision makers in charge of delivering already committed orders to customers. During this process, it is usual that real quantities of homogeneous sublots do not match the planned ones in LHP contexts. If nothing is made, some orders could not be served with the initial assignation made. Therefore, an efficient resolution method is necessary to reallocate the real homogeneous availabilities to orders to find a satisfactory solution for both, customers and the company.

From the managerial point of view, the HML-SP model allows an optimal or nearly optimal solution to be found within a very acceptable time range for this type of decisions. A maximum 5-hour execution implies that the model can be executed at the end of one period, in which discrepancies in homogeneous sublots are detected, to the next, for which a new solution for delivery is necessary. Proof of the conflicting objectives (H1) indicates that the final reallocation solution of availability to orders should be a trade-off among different objectives. Therefore, no solution exists that simultaneously optimises all the pursued objectives.

From the customers' relationship point of view, the positive impact on objective function when increasing flexibility of partial deliveries and/or of the maximum allowed delay, provides valuable information to negotiate delivery conditions with customers when it is not possible to serve all of them on time. Incrementing profits when allowing flexibility in deliveries can be used to define discounts for customers in case they are not served as promised, but to ensure them still being profitable for company. Management of priority orders/customers can be made by not allowing any delay and/or increasing rejection costs of them.

The negative impact of heterogeneity on lots (H2) shows the importance of investing in technology to obtain more uniform product quantities. Unfortunately, this is not possible for all companies with LHP, especially for those that obtain products directly from nature.

Until a technology solution that eliminates LHP is found, efforts should be made on the planning and product design sides. In line with this, it is very important when defining the master plan and executing the OPP that the heterogeneity in the production lots and customer order sizes and their uncertainty should be taken into account (Mundi et al., 2016). This provides more robust promised conditions with customers, as reflected by the minor reallocations required and the major fulfilment of the initial conditions committed with customers during the OPP.

As Hypothesis H4 proved that the results worsened with HR among units of different order lines, efforts should be made when designing products that are jointly sold to avoid this additional homogeneity requirement.

6. Conclusions and future research lines

The uncertainty inherent to the lack of homogeneity in the product in the ceramic sector constantly leads to discrepancies between planned and available homogeneous quantities. This aspect can result in certain previously committed orders not being served under the conditions previously agreed on as there is not sufficient homogeneous quantity, which entails a shortage situation. To reduce the negative impact on both customers and company profitability, an optimisation model for SP in ceramic sector companies is presented in this article. The reallocation of planned and real available quantities to firm orders is proposed as a solution to possible shortage. Moreover, partial deliveries of order lines and delayed deliveries are allowed if the HR imposed by customers are respected during available quantities reallocation. What all this represents is an attempt to optimise different conflicting objectives. One of the main contributions of this article is to treat the above aspects as we are unaware of any previous study that has jointly addressed all the characteristics of the problem under study. Moreover, partial deliveries of order lines and HR among order lines/products have not been addressed as far as we know.

Two resolution methods are applied to the model, depending on whether it is being used to obtain an implementable solution for the company (ε-constraint method) or to prove the behaviour of the shortage planning process (weighted sum procedure).

In this paper, four hypotheses are proved by applying the model to a ceramic tile company: 1) conflict exists among the model's objectives; 2) worse results are obtained as a batch is divided into many sublots and these are more uniform; 3) the results improve when more flexibility in deliveries is allowed; 4) HR among units of different order lines makes it more difficult to serve orders. The hypotheses were demonstrated by comparing the results obtained with the experiments and by a statistical analysis of these results.

As a future research line, an uncertain modelling of the distribution of batches into homogeneous sublots can be considered. The model proposed in this paper is specifically designed for the ceramic sector as it considers the attributes that characterise it. However, the application of this model can be extended to other sectors by replacing the ceramic attributes with the new sector ones. One example would be to implement the HML-SP Model into the furniture sector where homogeneity among different products that make up a set (chairs, tables, etc.) is also required for raw material (e.g. pine wood, cherry wood, birch wood), colour (e.g. wood, red, white), and quality. In this case the sets or ambiences in furniture sector should be equivalent to the series in the ceramic sector, and each combination of material-colour-quality in the furniture sector should be equivalent to a specific gage.

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