THE MINERALOGICAL MAGAZINE

AND

JOURNAL

OF

THE MINERALOGICAL SOCIETY.

No. 62. DECEMBER, 1903. Vol. XIII.

On the advantages of the Gnomonic Projection and its use in the Drawing of Crystals.

(*With a table to facilitate its employment.*)

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[Read June 10, 1902.]

OF the various methods employed or suggested for the graphical representation on a plane of points on the surface of a sphere, the most important to the crystallographer are the stereographic and the gnomonic projections. Both are in common use, and both have their particular advantages. Professor Penfield has recently published a series of papers [1] in which he points out the advantages that accrue from the employment of the stereographic projection both in crystallography and cartography. He has devised protractors and scales, which facilitate the construction of the projection of the poles of the faces bounding a crystal or of portions of the earth's surface, and enable the angles between points or contained by zones, i. e. great circles, to be readily determined from measurements on the plane diagram. This projection is especially advan-

[1] 'The stereographic projection and its possibilities from a graphical standpoint.' Amer. Journ. Sci., 1901, ser. 4, vol. xi, pp. 1–24, 115–144; abstract, this vol., p. 203. 'On the use of the stereographic projection for geographical maps and sailing charts.' Amer. Journ. Sci., 1902, ser. 4, vol. xiii, pp. 245–275, 347–376; abstract, this vol., p. 306. 'On the solution of problems in crystallography by means of graphical methods, based upon spherical and plane trigonometry.' Amer. Journ. Sci., 1902, ser. 4, vol. xiv, pp. 249–284; abstract, this vol., p. 306.

tageous for use in the construction of maps, because the meridians and parallels are projected as arcs of circles, and are, therefore, easily drawn. We must not, however, overlook the very considerable merits of the gnomonic projection in crystallographical work, especially in the investigation of complicated zonal relations. The author found it of inestimable service in the study of the crystalline development of calaverite[1], but the ordinary method of plotting, such as is described in the textbooks, is awkward, and a projection is not easy of construction. The purpose of this paper is to show that, by means of the table given at the end, the spherical coordinates of any pole may readily be converted into plane coordinates, and, by means of a particular form of protractor, these poles may be plotted on a plane diagram. Further, it will be seen that, with the aid of the table and protractor, the angles between points or contained by zones may be graphically determined on the plane diagram just as in the case of the stereographic projection.

The gnomonic[2] is a perspective projection in which the point of vision is at the centre of the sphere and the poles are projected on to a plane touching it. The point of contact with the sphere, which is the point where the perpendicular from the centre meets the plane, is called the centre of projection. Since the planes of all great circles pass through the centre of the sphere, zones are represented in the projection by straight lines of infinite length. In this property lies the principal advantage of the method; for the zonal relations subsisting between the faces of a crystal are easily tested by means of a straight-edge. This is obviously a simpler criterion than that in the stereographic projection in which three tautozonal poles must lie on the same circular arc, the common chord of which and the primitive circle must be a diameter of the latter. All poles on the great circle whose plane is parallel to the plane of projection are projected to infinity, and hence only a portion of the sphere can be represented on the same diagram. These poles are distant a right angle from the centre of projection, and therefore the angle between any pair is the same as the angle between the zones connecting them with the centre of projection. We can thus represent them by the corresponding zones. This is effected in practice by ter-

[1] This vol., pp. 122–150. A translation appeared in Zeits. Kryst. Min., 1903, vol. xxxvii, pp. 209–234.

[2] For other discussions of this projection see:—'Crystallography; a Treatise on the Morphology of Crystals,' by N. Story-Maskelyne, Oxford, 1895, pp. 492–499. H. A. Miers, 'The gnomonic projection,' Min. Mag., 1887, vol. vii, pp. 145–149. 'Traité de Cristallographie,' by E. Mallard, Paris, 1879, vol. i, pp. 63–66. 'Projection und graphische Krystallberechnung,' by V. Goldschmidt, Berlin, 1887.

minating the diagram by some arbitrary boundary—perhaps the best, a circle—and marking on a similar one outside it the intersections with the lines representing the zones. A projection will not be of inconvenient size which includes all poles lying within an angular distance of about 65° from the centre of projection. Thus on a single diagram we can represent nearly all the faces forming one termination of the crystal and also the faces in the zone of reference. Since the linear distances from the centre of projection are tangents of the corresponding angle, the resulting diagram suffers considerably from distortion, and cannot

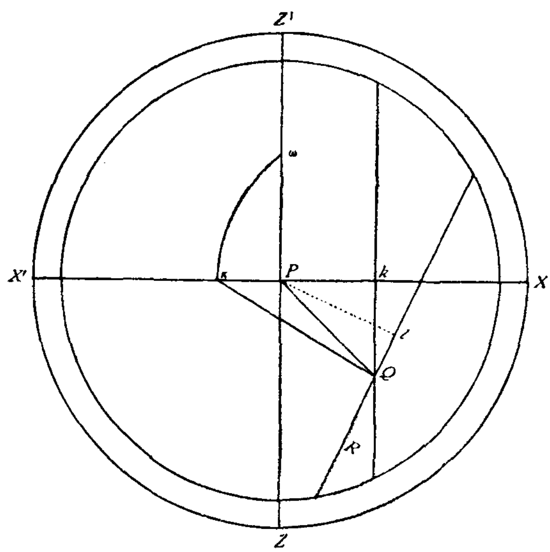

FIG. 1.—Gnomonic projection.

conveniently be used for mapping portions of a sphere. Another disadvantage practically prohibits its employment for this purpose, viz. that parallels of latitude are projected as conic sections, and are only circles in the particular cases when either the north or the south pole is the centre of projection.

Anything like general use of the gnomonic projection has probably been hindered by the difficulty of constructing a diagram from the observations made on a crystal. No difficulty, however, arises if we make use of the methods devised for the measurements of crystals by means of a three-circle goniometer. Measurements are made from poles in a certain zone, known as the zone of reference, as origins, and from the readings of the circles we obtain for any pole its distance from the origin and its azimuth from the zone of reference. In order to assure the observation of all the faces on the crystal, we should make a systematic survey from one particular origin (including with it the one diametrically opposite). The coordinates thus determined are used for locating the poles of the faces on the diagram, whose plane is taken at right angles to the edge of the zone of reference. All poles in this zone are projected to infinity on the diagram, and consequently all zones passing through the same origin appear as parallel lines.

We may now consider what are the corresponding linear coordinates of a point in the diagram. In fig. 1, P, the pole of the zone of reference, is the centre of projection, and Z the origin on the zone of reference; PX is drawn at right angles to PZ; Q is any point, and Qk is drawn parallel to PZ, and consequently at right angles to PX. Qk is therefore the zone passing through Q and the origin, Z.

If the spherical coordinates of the pole, represented by Q, with respect to the origin, Z, be

ρ, the distance from Z,
ϕ, the azimuth from the zone of reference,

the corresponding linear coordinates are easily seen to be

$$kQ = r \cot \rho \operatorname{cosec} \phi,$$
$$Pk = r \cot \phi,$$

where r is the radius of the sphere of projection.

One coordinate, Pk, representing the azimuth, is readily found from a table of tangents; but as yet either the other must be calculated or the position of Q be found graphically. In the latter method[1] we have successively to find a point ω (fig. 1) in PZ such that $P\omega = r$, the radius of the sphere of projection, and a point κ in PX such that $k\kappa = k\omega$; then the plane angle $Q\kappa k$ is the complement of ρ. Such a method, entailing as it does the use of subsidiary points and lines and of several operations, each liable to error, is exceedingly troublesome to carry out in practice, and may result in considerable confusion and inaccuracy, as, indeed, the author is aware from experience.

At the end of this paper is given a table containing the values of $\cot \rho \operatorname{cosec} \phi$ (multiplied by 10 for convenience) for every degree from 90° to 25°. In the columns the azimuths are constant, and in the rows the distances. Since cosec 90° is unity, the first column gives the

[1] Cf. Story-Maskelyne and Miers, loc. cit.

cotangents (multiplied by 10) for every degree, and we may therefore use this column to find the coordinate *Pk*, corresponding to the azimuth.

The use of the table will be made clear from an example. Suppose we wish to locate a pole whose azimuth and distance are respectively 60° 40′ and 55° 10′. Under the column headed 60° we find the value corresponding to 55°; we proceed to subtract the proportionate difference in the column for 10′ and in the row for 40′. Thus we actually obtain 8·09 less 0·05, 8·04, and finally, less 0·05, 7·99. The two plane coordinates are therefore 5·62 and 7·99, multiplied or divided by a constant depending on the length assigned to the radius of the sphere. In

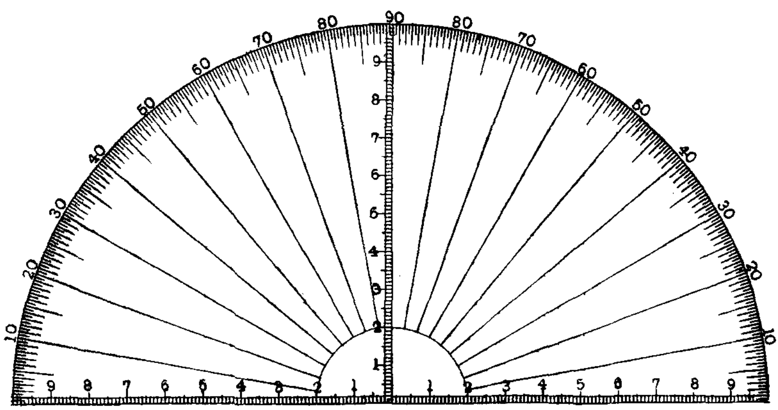

FIG. 2.—Protractor for plotting gnomonic projections.
(Half actual size.)

practice we find the coordinate corresponding to the azimuth first, because there will usually be several poles with the same azimuth. We notice that the plane coordinates increase as the spherical coordinates diminish, and vice versa. All subsidiary work required in the determination of these coordinates may be done on a separate sheet of paper, and the diagram need have no subsidiary lines or points on it at all.

Squared paper, for instance, divided to millimetres, is convenient for plotting the positions of poles, but ordinary blank paper may be used in conjunction with the form of protractor shown in fig. 2. This consists of a strip of celluloid on which is engraved a semicircle. The diameter, which is on the edge of the strip, and the radius at right angles to it are graduated to millimetres. By means of these two scales, mutually at right angles, we can easily plot the poles from their plane coordinates. To avoid parallax, the engraved side of the celluloid strip should be placed next to the paper. If a pencil be used for plotting, allowance must be made for its thickness. It is perhaps best to use a pin or a needle, which will accurately mark a point just under the edge of the strip. A line must be drawn on the sheet of paper corresponding to PX (fig. 1), and a point marked on it to represent P. Then the protractor is moved about so that the shorter scale lies on this line. The purpose of the graduated semicircle is explained below (p. 316).

In applying the above method to measurements on a goniometer with a single circle, it will be necessary to calculate at least one angle. The majority of the poles may, however, then be located from their positions

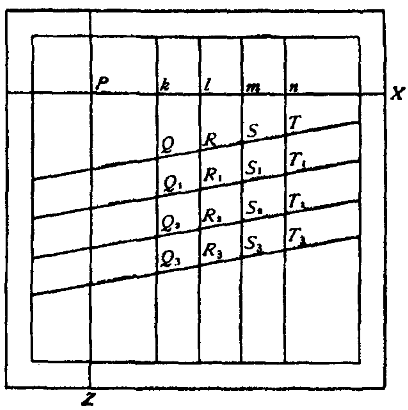

FIG. 3.—Gnomonic projection.

in known zones. In the case of measurements on a goniometer with two circles it may be more convenient to take the centre of projection as origin. In that case the azimuths are the same as the plane angles between the lines on the diagram corresponding to the zones, and the linear distances, e.g. PQ in fig. 1, are proportional to the tangents of the corresponding angle.

We may now proceed to consider a remarkable property of this projection. In fig. 3 we have a series of poles lying in zones passing through the same origin in the zone of reference. Let their azimuths be $\phi, \phi_1, \phi_2, \phi_3$. From what has preceded we see that

$$\frac{km\,.\,ln}{kl\,.\,mn}=\frac{(\cot\phi_2-\cot\phi)(\cot\phi_3-\cot\phi_1)}{(\cot\phi_1-\cot\phi)(\cot\phi_3-\cot\phi_2)}=\frac{\sin(\phi-\phi_2)\sin(\phi_3-\phi_1)}{\sin(\phi-\phi_1)\sin(\phi_3-\phi_2)}=$$

a rational quantity, being the anharmonic ratio of a pencil of four crystallographically possible zones, intersecting in the same pole.

Suppose now that $\{T T_1 T_2 T_3\}$ is the zone of reference. Then n lies at infinity, and we are left with the very simple relation

$$\frac{km}{kl} = \text{a rational quantity.}$$

The same relation also holds with regard to poles in a zone, and accordingly we have

$$\frac{QQ_2}{QQ_1} = \text{a rational quantity.}$$

The principal zones in the diagram are thus equally spaced apart, and the same is true of zones passing through any other origin on the zone of reference. The diagram may therefore be divided into a network of equal parallelograms [1], or in special cases rectangles or squares. A necessary consequence follows that the unit linear distances on parallel lines must be the same. This is a most important property, for we have merely to take measurements on the diagram to determine the simple indices corresponding to any particular face.

We will now show that the angles between poles or contained by zones may be graphically determined from the projection with the aid of the protractor and the table.

To find the angle between two poles, represented by points on the gnomonic projection.—Let Q and R (fig. 1, p. 311) be the two points, and P the centre of projection. Place the protractor on the diagram so that the diameter lies along QR, and at the same time the radius passes over P. The zero point then lies on l. Take the readings for P, Q, and R. If necessary, multiply or divide these by a constant quantity, depending upon the length selected for the radius of the sphere of projection. The reading for P gives the length Pl, and from the table the azimuth of the zone QR can be found. Knowing this, we can further find from the table the angular distances corresponding to the observed linear lengths Ql and Rl. The sum or the difference must be taken according as Q and R lie on opposite sides or on the same side of Pl. In the latter case we have the angle required, but in the former the supplement, because the table gives the distance measured from the origin in the zone of reference.

[1] It may be noted that in the triclinic system P will not be a node of the network.

To find the angle between any two zones, represented by straight lines on the gnomonic projection.—Let the zones intersect in D and meet the zone of reference in p_1 and p_2 (fig. 4 *a*). To make the reasoning clearer the corresponding stereographic (fig. 4 *b*) is placed alongside the gnomonic projection. Draw the zone PD and let it meet the zone of reference in d.

Let the angle Dd be ϕ, the angles p_1d and p_2d be the complements of ρ_1 and ρ_2, and the angles p_1Dd and p_2Dd be D_1 and D_2 respectively. Then we have in the triangles p_1Dd and p_2Dd

$$\tan D_1 = \cot \rho_1 \operatorname{cosec} \phi,$$
$$\tan D_2 = \cot \rho_2 \operatorname{cosec} \phi.$$

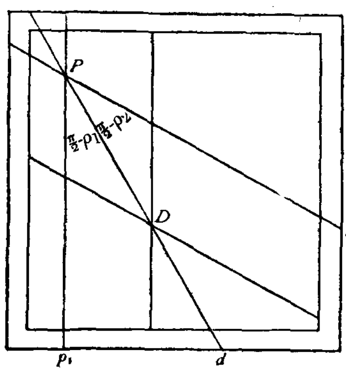

FIG. 4 *a*.—Gnomonic projection.

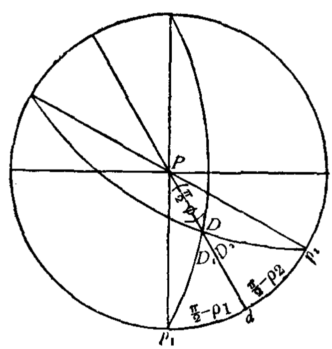

FIG. 4 *b*.—Stereographic projection.

If we now place the protractor on the diagram, so that the zero point is on D and the radius, the reading of which is 90°, passes over the centre of projection, and note the readings of P on the radius scale and those of the two zone-lines on the circular scale, from the first we obtain the length PD, and therefore in the same way as before from the table the angle ϕ, and from the other two we obtain the angles ρ_1 and ρ_2, because the circle is so graduated that the radius, from which we are really measuring, reads 90°. Now the formula given above is the same as that found for the determination of the plane coordinate corresponding to the distance of a pole. We may therefore proceed in the same way. Treating ϕ as an azimuth and ρ as a distance, we get from the table a corresponding quantity, which from the first column we convert into the complement of D_1 or D_2, because this column gives the cotangents. The sum or difference must be taken according as the angles have been measured on opposite sides or on the same side of PD. As before, we have in the latter case the angle required, but in the former its supplement.

The Representation of Crystals.

Another important property of the gnomonic projection is the facility it affords for drawing crystals in any position. It is customary to suppose the eye sufficiently distant from the crystal for the imaginary lines connecting it to different parts of the crystal to be sensibly parallel. Thus all edges which are parallel on the crystal, i. e. which form the intersections of pairs of faces lying in the same zone, are parallel in the drawing, and the zonal relations are apparent from inspection. Now the edges common to pairs of faces in a zone are parallel to its axis, and in the gnomonic projection zone-planes are represented by straight lines. If we project this axis orthogonally on to the same plane of projection, the line passes through the centre of projection and is at right angles to the zone-line. Hence to prepare an orthographic drawing of a crystal on any plane, we have only to project the poles gnomonically on to the same plane, and the edge common to a pair of faces is given by a line at right angles to the zone-line in which their poles lie.

If the projection be made, as suggested earlier in this paper, so that the edge of some zone is perpendicular to the plane of the paper, the resulting drawing gives a correct idea of the distribution of the faces on the portion of the crystal in view, but the appearance is very flat, because the edges common to pairs of faces in the zone mentioned are represented only by points, and there is a consequent want of solidity. To gain this effect, we must suppose the crystal to be slightly rotated about the vertical and horizontal axes successively in the plane of the paper. The faces, which had previously been at right angles to the plane of the paper, now come into view, and the drawing of the crystal presents the solid appearance desired.

We will now consider how we may determine the positions of the poles after rotation. Suppose we have three rectangular directions fixed in space, OX, OY, OZ (fig. 1), of which OY is perpendicular to the plane of the paper and represented by P, the centre of projection. The crystal is rotated about OZ and OX successively. To take a concrete instance, let us suppose these rotations are 20° and 10° respectively, and consider the effect on a pole whose azimuth and distance from Z are 38° 35′ and 49° 26′ respectively, the rotation increasing the azimuth in each case. After the first rotation these become 58° 35′ and 49° 26′, the latter of course being unaltered. We now have to find the new spherical coordinates with respect to X. From the table we find that the plane coordinates with respect to Z are 6·11 and 10·03; with respect to X therefore they are 10·03 and 6·11, or in spherical coordinates 44° 55′ and 66° 38′. After rotation of 10° about OX we have the coordinates, 54° 55′ and 66° 38′, and obtain from the table the corresponding plane coordinates, 7·03 and 5·28. Reversing these, we find that the coordinates which were 12·54 and 13·73, corresponding to 38° 35′ and 49° 26′, are now 5·28 and 7·03. These must be multiplied or divided by a constant depending on the length assigned to the radius of the sphere. Since for the purpose of delineation extreme accuracy in the position of the poles is not essential and the projection should be as compact as possible, 2 cm. will be a convenient length to give to this radius, and the coordinates after rotation are therefore 1·06 and 1·41 cm. parallel to PX and PZ respectively. In practice we should make the transformations on a separate sheet of paper in the following way:—

			Azimuth	Distance	
Before rotation	plane coordinates		12.54	13.73	Measured from Z
	spherical ,,		38° 35′	49° 26′	
After first rotation (20° about OZ)	,, ,,		58° 35′	,, ,,	
	plane ,,		6.11	10.03	
	spherical ,,		44° 55′	66° 38′	Measured from X
After second rotation (10° about OX)	,, ,,		54° 55′	,, ,,	
	plane ,,		7.03	5.28	
	,, ,,	($r = 2$ cm.)	1.41 cm.	1.06 cm.	

After plotting a few poles we may determine the positions of the remainder with sufficient accuracy as the intersections of zones. Four poles, no three of which are tautozonal, are theoretically enough; but it is advisable to plot a few more as a check on the accuracy of the work.

This method applies equally well to all systems. In plotting *ab initio*, the edge of some zone is taken perpendicular to the plane of the paper. Where possible, it would naturally be the zone whose edge in the resulting drawing is most foreshortened, so that the rotations which have first to be applied are not large. If this zone is [100,001], the face (010) is represented by P in all systems save the triclinic. When the axes are at right angles, the faces (100) and (001) would naturally be represented by X and Z. In any system we should plot the new positions of the faces (010), (110), ($\bar{1}$10), (011), (01$\bar{1}$), (111), ($\bar{1}$11), (11$\bar{1}$), ($\bar{1}$1$\bar{1}$). These would suffice to give accurately the positions of the initial zone-lines, and the remainder may be determined by graphical construction. In the general case, in which these particular poles are not for some reason available, we select in the most convenient way eight poles, which lie by pairs on four zone-lines intersecting in a ninth.

Since the rotations are usually not large, it will not be possible to plot on a sheet of convenient size the poles representing faces in the zone, whose edge was originally perpendicular to the plane of the paper, unless the radius of the sphere is taken too small. For instance, if we employ the rotations suggested by Haidinger, namely, $18° 26'$ and $7° 11'$, the distances of (001) and (100) from the centre of projection after rotation, assuming that these poles originally coincided with Z and Y, are $7{\cdot}9\, r$ and $3{\cdot}0\, r$, where r is the radius of the sphere. We can, however, easily obtain the direction of the edge common to the faces lying in this zone in another way. P is its pole in the altered position. If P' be the new position of P, PP' will be the orthogonal projection of the axis of the zone, and therefore gives the direction of the edge in question. Other zone-lines passing through poles on this zone may be determined by means of poles on them, which are located in accessible portions of the diagram.

We can further determine from the gnomonic projection the proper length on the drawing for any edge. It is the actual length on the crystal multiplied by the sine of the angle it makes with the normal to the plane of the paper. This angle is the same as the azimuth of the zone, whose axis is parallel to the particular edge, as given earlier in this paper (p. 312). Hence if we know the actual length of any edge we can readily find its projected length. This is a very useful check on the accuracy of the work.

In this method twin crystals present no greater difficulties than simple crystals. The poles of corresponding faces on the two individuals lie on a line passing through the pole of the twin-axis and make equal angles with it. We have shown above how to find the angle between the faces represented by two points. After plotting a few poles in this way, the remainder may as before be determined from the intersections of zones.

The method here described is precisely the same as that of Mohs and Haidinger[1] as regards the resulting drawing; but these authors obtain

[1] Haidinger, 'Account of the method of drawing crystals in true perspective, followed in the Treatise on Mineralogy of Professor Mohs.' Mem. Wernerian Soc. Edinburgh, 1824-5, vol. v, pp. 485-508.

the directions of the edges in another way. The projections of the three fundamental axes and the unit intercepts made on them have first to be determined. This is a troublesome process *ab initio.* In practice a crystallograph is used, an instrument consisting of three coplanar arms inclined to one another at the proper angles and graduated according to the several projected unit lengths. For the cubic system we have at once the fundamental factors required; for any other system we must introduce modifications, maybe only in the units, but in the case of the monoclinic and triclinic systems in both units and angles. The determination of the fundamental factors for the second individual of a twin presents considerable difficulties. When we have at length drawn the axes and marked off the unit lengths on each, we find the position of any face from the relative intercepts it makes on the axes. If we suppose two faces meet one axis in the same point, the direction of the common edge is given by the line connecting this point to the intersection of the two lines, belonging to each face, which join the points where they meet the other axes. In practice many difficulties and possibilities of error arise. If the crystal is at all complicated, confusion may occur from multiplicity of lines that must be drawn for the determination of the various edges. We have further to remember, that an intercept is derived directly not from the corresponding index, but from its inverse; and we must be careful of the signs. None of these difficulties are encountered if the gnomonic projection be employed to give the directions of the edges. Then, again, in this method we may select the rotations most suitable for displaying the symmetry of the faces and their disposition on a crystal. If a crystallograph be used, we are confined to particular angles, usually those suggested by Haidinger, namely, $\tan^{-1}\frac{1}{3}$ (18° 26′) and $\sin^{-1}\frac{1}{8}$ (7° 11′).

Another method frequently employed is that of Naumann[1], which only differs from the preceding in that the paper is supposed to move with the crystal in the second rotation, or, as Naumann expresses it, the eye moves up[2]. The paper is now inclined to the line of sight and the projection is clinographic. Its principal recommendation is the simple practical rule[3] whereby the projections of the axes and the unit intercepts on each may be determined; otherwise it has no superior merits, and the after-procedure is precisely the same as in the orthographic method.

To illustrate the method described above, a combination of the cube,

[1] 'Lehrbuch der reinen und angewandten Krystallographie,' Leipzig, 1830, vol. ii, pp. 400–483. [2] Loc. cit., p. 401. [3] Loc. cit., p. 403.

TABLE FOR USE IN PLOTTING GNOMONIC PROJECTIONS. By G. F. Herbert Smith, M.A.

(*Mineralogical Magazine*, No. 62, Vol. XIII, 1903.)

Cosecant Azimuths. | Cosecant Azimuths.

Cotangent Distances.

	90°	89°	88°	87°	86°	85°
90°	0·00	0·00	0·00	0·00	0·00	0·00
89°	0·17	0·17	0·17	0·17	0·17	0·18
88°	0·35	0·35	0·35	0·35	0·35	0·35
87°	0·52	0·52	0·52	0·52	0·53	0·53
86°	0·70	0·70	0·70	0·70	0·70	0·70
85°	0·87	0·88	0·88	0·88	0·88	0·88
84°	1·05	1·05	1·05	1·05	1·05	1·06
83°	1·23	1·23	1·23	1·23	1·23	1·23
82°	1·41	1·41	1·41	1·41	1·41	1·41
81°	1·58	1·58	1·58	1·59	1·59	1·59
80°	1·76	1·76	1·76	1·77	1·77	1·77
79°	1·94	1·94	1·94	1·95	1·95	1·95
78°	2·13	2·13	2·13	2·13	2·13	2·13
77°	2·31	2·31	2·31	2·31	2·31	2·32
76°	2·49	2·49	2·49	2·50	2·50	2·50
75°	2·68	2·68	2·68	2·68	2·69	2·69
74°	2·87	2·87	2·87	2·87	2·87	2·88
73°	3·06	3·06	3·06	3·06	3·06	3·07
72°	3·25	3·25	3·25	3·25	3·26	3·26
71°	3·44	3·44	3·45	3·45	3·45	3·46
70°	3·64	3·64	3·64	3·64	3·65	3·65
69°	3·84	3·84	3·84	3·84	3·85	3·85
68°	4·04	4·04	4·04	4·05	4·05	4·06
67°	4·24	4·25	4·25	4·25	4·26	4·26
66°	4·45	4·45	4·45	4·46	4·46	4·47
65°	4·66	4·66	4·67	4·67	4·67	4·68
64°	4·88	4·88	4·88	4·88	4·89	4·90
63°	5·10	5·10	5·10	5·10	5·11	5·11
62°	5·32	5·32	5·32	5·32	5·33	5·34
61°	5·54	5·54	5·55	5·55	5·56	5·56
60°	5·77	5·77	5·77	5·78	5·79	5·80
59°	6·01	6·01	6·01	6·02	6·02	6·03
58°	6·25	6·25	6·25	6·26	6·26	6·27
57°	6·49	6·50	6·50	6·50	6·51	6·52
56°	6·75	6·75	6·75	6·75	6·76	6·77
55°	7·00	7·00	7·01	7·01	7·02	7·03
54°	7·27	7·27	7·27	7·28	7·28	7·29
53°	7·54	7·54	7·54	7·55	7·55	7·56
52°	7·81	7·81	7·81	7·82	7·83	7·84
51°	8·10	8·10	8·10	8·11	8·12	8·13
50°	8·39	8·39	8·39	8·40	8·41	8·42
49°	8·69	8·69	8·70	8·70	8·71	8·73
48°	9·00	9·01	9·01	9·02	9·03	9·04
47°	9·33	9·33	9·33	9·34	9·35	9·36
46°	9·66	9·66	9·67	9·67	9·68	9·69
45°	10·00	10·00	10·01	10·01	10·02	10·04
44°	10·36	10·36	10·37	10·37	10·38	10·39
43°	10·72	10·73	10·73	10·74	10·75	10·76
42°	11·11	11·11	11·12	11·12	11·13	11·15
41°	11·50	11·51	11·51	11·52	11·53	11·55
40°	11·92	11·92	11·93	11·93	11·95	11·96
39°	12·35	12·35	12·36	12·37	12·38	12·40
38°	12·80	12·80	12·81	12·82	12·83	12·85
37°	13·27	13·27	13·28	13·29	13·30	13·32
36°	13·76	13·77	13·77	13·78	13·80	13·82
35°	14·28	14·28	14·29	14·30	14·32	14·34
34°	14·83	14·83	14·84	14·85	14·86	14·88
33°	15·40	15·40	15·41	15·42	15·44	15·46
32°	16·00	16·01	16·01	16·03	16·04	16·06
31°	16·64	16·65	16·65	16·67	16·68	16·71
30°	17·32	17·32	17·33	17·34	17·36	17·39
29°	18·04	18·04	18·05	18·07	18·08	18·11
28°	18·81	18·81	18·82	18·83	18·85	18·88
27°	19·63	19·63	19·64	19·65	19·67	19·70
26°	20·50	20·51	20·51	20·53	20·55	20·58
25°	21·45	21·45	21·46	21·47	21·50	21·53

	84°	83°	82°	81°	80°
90°	0·00	0·00	0·00	0·00	0·00
89°	0·18	0·18	0·18	0·18	0·18
88°	0·35	0·35	0·35	0·35	0·35
87°	0·53	0·53	0·53	0·53	0·53
86°	0·70	0·70	0·71	0·71	0·71
85°	0·88	0·88	0·88	0·89	0·89
84°	1·06	1·06	1·06	1·06	1·07
83°	1·23	1·24	1·24	1·24	1·25
82°	1·41	1·42	1·42	1·42	1·43
81°	1·59	1·60	1·60	1·60	1·61
80°	1·77	1·78	1·78	1·79	1·79
79°	1·95	1·96	1·96	1·97	1·97
78°	2·14	2·14	2·15	2·15	2·16
77°	2·32	2·33	2·33	2·34	2·34
76°	2·51	2·51	2·52	2·52	2·53
75°	2·69	2·70	2·71	2·71	2·72
74°	2·88	2·89	2·90	2·90	2·91
73°	3·07	3·08	3·09	3·10	3·10
72°	3·27	3·27	3·28	3·29	3·30
71°	3·46	3·47	3·48	3·49	3·50
70°	3·66	3·67	3·68	3·69	3·70
69°	3·86	3·87	3·88	3·89	3·90
68°	4·06	4·07	4·08	4·09	4·10
67°	4·27	4·28	4·29	4·30	4·31
66°	4·48	4·49	4·50	4·51	4·52
65°	4·69	4·70	4·71	4·72	4·73
64°	4·90	4·91	4·93	4·94	4·95
63°	5·12	5·13	5·15	5·16	5·17
62°	5·35	5·36	5·37	5·38	5·40
61°	5·57	5·58	5·60	5·61	5·63
60°	5·81	5·82	5·83	5·85	5·86
59°	6·04	6·05	6·07	6·08	6·10
58°	6·28	6·30	6·31	6·33	6·35
57°	6·53	6·54	6·56	6·57	6·59
56°	6·78	6·80	6·81	6·83	6·85
55°	7·04	7·05	7·07	7·09	7·11
54°	7·31	7·32	7·34	7·36	7·38
53°	7·58	7·59	7·61	7·63	7·65
52°	7·86	7·87	7·89	7·91	7·93
51°	8·14	8·16	8·18	8·20	8·22
50°	8·44	8·45	8·47	8·50	8·52
49°	8·74	8·76	8·78	8·80	8·83
48°	9·05	9·07	9·09	9·12	9·14
47°	9·38	9·40	9·42	9·44	9·47
46°	9·71	9·73	9·75	9·78	9·81
45°	10·06	10·08	10·10	10·12	10·15
44°	10·41	10·43	10·46	10·48	10·51
43°	10·78	10·80	10·83	10·86	10·89
42°	11·17	11·19	11·22	11·24	11·27
41°	11·57	11·59	11·62	11·65	11·68
40°	11·98	12·01	12·03	12·07	12·10
39°	12·42	12·44	12·47	12·50	12·54
38°	12·87	12·90	12·93	12·96	13·00
37°	13·34	13·37	13·40	13·44	13·48
36°	12·84	13·87	13·90	13·94	13·98
35°	14·36	14·39	14·42	14·46	14·50
34°	14·91	14·94	14·97	15·01	15·05
33°	15·48	15·51	15·55	15·59	15·64
32°	16·09	16·12	16·16	16·20	16·25
31°	16·73	16·77	16·81	16·85	16·90
30°	17·42	17·45	17·49	17·54	17·59
29°	18·14	18·18	18·22	18·27	18·32
28°	18·91	18·95	18·99	19·04	19·10
27°	19·73	19·77	19·82	19·87	19·93
26°	20·62	20·66	20·70	20·76	20·82
25°	21·56	21·61	21·66	21·71	21·78

	79°	78°	77°	76°	75°
90°	0·00	0·00	0·00	0·00	0·00
89°	0·18	0·18	0·18	0·18	0·18
88°	0·36	0·36	0·36	0·36	0·36
87°	0·53	0·54	0·54	0·54	0·54
86°	0·71	0·71	0·72	0·72	0·72
85°	0·89	0·89	0·90	0·90	0·91
84°	1·07	1·08	1·08	1·08	1·09
83°	1·25	1·26	1·26	1·27	1·27
82°	1·43	1·44	1·44	1·45	1·45
81°	1·61	1·62	1·63	1·63	1·64
80°	1·80	1·80	1·81	1·82	1·83
79°	1·98	1·99	1·99	2·00	2·01
78°	2·17	2·17	2·18	2·19	2·20
77°	2·35	2·36	2·37	2·38	2·39
76°	2·54	2·55	2·56	2·57	2·58
75°	2·73	2·74	2·75	2·76	2·77
74°	2·92	2·93	2·94	2·96	2·97
73°	3·11	3·13	3·14	3·15	3·17
72°	3·31	3·32	3·33	3·35	3·36
71°	3·51	3·52	3·53	3·55	3·56
70°	3·71	3·72	3·74	3·75	3·77
69°	3·91	3·92	3·94	3·96	3·97
68°	4·12	4·13	4·15	4·16	4·18
67°	4·32	4·34	4·36	4·37	4·39
66°	4·54	4·55	4·57	4·59	4·61
65°	4·75	4·77	4·79	4·81	4·83
64°	4·97	4·99	5·01	5·03	5·05
63°	5·19	5·21	5·23	5·25	5·28
62°	5·42	5·44	5·46	5·48	5·50
61°	5·65	5·67	5·69	5·71	5·74
60°	5·88	5·90	5·93	5·95	5·98
59°	6·12	6·14	6·17	6·19	6·22
58°	6·37	6·39	6·41	6·44	6·47
57°	6·62	6·64	6·66	6·69	6·72
56°	6·87	6·90	6·92	6·95	6·98
55°	7·13	7·16	7·19	7·22	7·25
54°	7·40	7·43	7·46	7·49	7·52
53°	7·68	7·70	7·73	7·77	7·80
52°	7·96	7·99	8·02	8·05	8·09
51°	8·25	8·28	8·31	8·35	8·38
50°	8·55	8·58	8·61	8·65	8·69
49°	8·86	8·89	8·92	8·96	9·00
48°	9·17	9·21	9·24	9·28	9·32
47°	9·50	9·53	9·57	9·61	9·65
46°	9·84	9·87	9·91	9·95	10·00
45°	10·19	10·22	10·26	10·31	10·35
44°	10·55	10·58	10·63	10·67	10·72
43°	10·92	10·96	11·01	11·05	11·10
42°	11·31	11·35	11·40	11·45	11·50
41°	11·72	11·76	11·81	11·86	11·91
40°	12·14	12·18	12·23	12·28	12·34
39°	12·58	12·62	12·67	12·73	12·78
38°	13·04	13·09	13·14	13·19	13·25
37°	13·52	13·57	13·62	13·68	13·74
36°	14·02	14·07	14·13	14·19	14·25
35°	14·55	14·60	14·66	14·72	14·79
34°	15·10	15·16	15·22	15·28	15·35
33°	15·69	15·74	15·80	15·87	15·94
32°	16·30	16·36	16·42	16·49	16·57
31°	16·95	17·01	17·08	17·15	17·23
30°	17·64	17·71	17·78	17·85	17·93
29°	18·38	18·44	18·52	18·59	18·68
28°	19·16	19·23	19·30	19·38	19·47
27°	19·99	20·06	20·14	20·23	20·32
26°	20·89	20·96	21·04	21·13	21·23
25°	21·85	21·92	22·01	22·10	22·20

	74°	73°	72°	71°	70°
90°	0·00	0·00	0·00	0·00	0·00
89°	0·18	0·18	0·18	0·18	0·19
88°	0·36	0·37	0·37	0·37	0·37
87°	0·55	0·55	0·55	0·55	0·56
86°	0·73	0·73	0·74	0·74	0·74
85°	0·91	0·91	0·92	0·92	0·93
84°	1·09	1·10	1·11	1·11	1·12
83°	1·28	1·28	1·29	1·30	1·31
82°	1·46	1·47	1·48	1·49	1·50
81°	1·65	1·66	1·67	1·68	1·69
80°	1·83	1·84	1·85	1·86	1·88
79°	2·02	2·03	2·04	2·06	2·07
78°	2·21	2·22	2·24	2·25	2·26
77°	2·40	2·41	2·43	2·44	2·46
76°	2·59	2·61	2·62	2·64	2·65
75°	2·79	2·80	2·82	2·83	2·85
74°	2·98	3·00	3·01	3·03	3·05
73°	3·18	3·20	3·21	3·23	3·25
72°	3·38	3·40	3·42	3·44	3·46
71°	3·58	3·60	3·62	3·64	3·66
70°	3·79	3·81	3·83	3·85	3·87
69°	3·99	4·01	4·04	4·06	4·08
68°	4·20	4·22	4·25	4·27	4·30
67°	4·42	4·44	4·46	4·49	4·52
66°	4·63	4·66	4·68	4·71	4·74
65°	4·85	4·88	4·90	4·93	4·96
64°	5·07	5·10	5·13	5·16	5·19
63°	5·30	5·33	5·36	5·39	5·42
62°	5·53	5·56	5·59	5·62	5·66
61°	5·77	5·80	5·83	5·86	5·90
60°	6·01	6·04	6·07	6·11	6·14
59°	6·25	6·28	6·32	6·35	6·39
58°	6·50	6·53	6·57	6·61	6·65
57°	6·76	6·79	6·83	6·87	6·91
56°	7·02	7·05	7·09	7·13	7·18
55°	7·28	7·32	7·36	7·41	7·45
54°	7·56	7·60	7·64	7·68	7·73
53°	7·84	7·88	7·92	7·97	8·02
52°	8·13	8·17	8·21	8·26	8·31
51°	8·42	8·47	8·51	8·56	8·62
50°	8·73	8·77	8·82	8·87	8·93
49°	9·04	9·09	9·14	9·19	9·25
48°	9·37	9·42	9·47	9·52	9·58
47°	9·70	9·75	9·80	9·86	9·92
46°	10·05	10·10	10·15	10·21	10·28
45°	10·40	10·46	10·51	10·58	10·64
44°	10·77	10·83	10·89	10·95	11·02
43°	11·16	11·21	11·28	11·34	11·41
42°	11·55	11·61	11·68	11·75	11·82
41°	11·97	12·03	12·10	12·17	12·24
40°	12·40	12·46	12·53	12·60	12·68
39°	12·85	12·91	12·98	13·06	13·14
38°	13·32	13·38	13·46	13·54	13·62
37°	13·81	13·88	13·95	14·04	14·12
36°	14·32	14·39	14·47	14·56	14·65
35°	14·86	14·93	15·02	15·10	15·20
34°	15·42	15·50	15·59	15·68	15·78
33°	16·02	16·10	16·19	16·29	16·39
32°	16·65	16·73	16·83	16·93	17·03
31°	17·31	17·40	17·50	17·60	17·71
30°	18·02	18·11	18·21	18·32	18·43
29°	18·77	18·86	18·97	19·08	19·20
28°	19·56	19·67	19·77	19·89	20·01
27°	20·42	20·52	20·64	20·76	20·89
26°	21·33	21·44	21·56	21·68	21·82
25°	22·31	22·42	22·55	22·68	22·82

	69°	68°	67°	66°	65°
90°	0·00	0·00	0·00	0·00	0·00
89°	0·19	0·19	0·19	0·19	0·19
88°	0·37	0·38	0·38	0·38	0·39
87°	0·56	0·57	0·57	0·57	0·58
86°	0·75	0·75	0·76	0·77	0·77
85°	0·94	0·94	0·95	0·96	0·97
84°	1·13	1·13	1·14	1·15	1·16
83°	1·32	1·32	1·33	1·34	1·35
82°	1·51	1·52	1·53	1·54	1·55
81°	1·70	1·71	1·72	1·73	1·75
80°	1·89	1·90	1·92	1·93	1·95
79°	2·08	2·10	2·11	2·13	2·14
78°	2·28	2·29	2·31	2·33	2·35
77°	2·47	2·49	2·51	2·53	2·55
76°	2·67	2·69	2·71	2·73	2·75
75°	2·87	2·89	2·91	2·93	2·96
74°	3·07	3·09	3·12	3·14	3·16
73°	3·27	3·30	3·32	3·35	3·37
72°	3·48	3·50	3·53	3·56	3·59
71°	3·69	3·71	3·74	3·77	3·80
70°	3·90	3·93	3·95	3·98	4·02
69°	4·11	4·14	4·17	4·20	4·24
68°	4·33	4·36	4·39	4·42	4·46
67°	4·55	4·58	4·61	4·65	4·68
66°	4·77	4·80	4·84	4·87	4·91
65°	4·99	5·03	5·07	5·10	5·14
64°	5·22	5·26	5·30	5·34	5·38
63°	5·46	5·50	5·54	5·58	5·62
62°	5·70	5·73	5·78	5·82	5·87
61°	5·94	5·98	6·02	6·07	6·12
60°	6·18	6·23	6·27	6·32	6·37
59°	6·44	6·48	6·53	6·58	6·63
58°	6·69	6·74	6·79	6·84	6·89
57°	6·96	7·00	7·05	7·11	7·17
56°	7·22	7·27	7·33	7·38	7·44
55°	7·50	7·55	7·61	7·66	7·73
54°	7·78	7·84	7·89	7·95	8·02
53°	8·07	8·13	8·19	8·25	8·31
52°	8·37	8·43	8·49	8·55	8·62
51°	8·67	8·73	8·80	8·86	8·93
50°	8·99	9·05	9·12	9·18	9·26
49°	9·31	9·38	9·44	9·52	9·59
48°	9·64	9·71	9·78	9·86	9·93
47°	9·99	10·06	10·13	10·21	10·29
46°	10·34	10·42	10·49	10·57	10·66
45°	10·71	10·79	10·86	10·95	11·03
44°	11·09	11·17	11·25	11·34	11·43
43°	11·49	11·57	11·65	11·74	11·83
42°	11·90	11·98	12·06	12·16	12·26
41°	12·32	12·41	12·50	12·59	12·69
40°	12·77	12·85	12·95	13·05	13·15
39°	13·23	13·32	13·42	13·52	13·63
38°	13·71	13·80	13·90	14·01	14·12
37°	14·21	14·31	14·42	14·53	14·64
36°	14·74	14·84	14·95	15·07	15·19
35°	15·30	15·40	15·51	15·63	15·76
34°	15·88	15·99	16·11	16·23	16·36
33°	16·49	16·61	16·73	16·86	16·99
32°	17·14	17·26	17·39	17·52	17·66
31°	17·83	17·95	18·08	18·22	18·37
30°	18·55	18·68	18·82	18·96	19·11
29°	19·32	19·46	19·60	19·75	19·91
28°	20·15	20·28	20·43	20·59	20·75
27°	21·02	21·17	21·32	21·48	21·65
26°	21·96	22·11	22·27	22·44	22·62
25°	22·97	23·13	23·30	23·47	23·66

	64°	63°	62°	61°	60°
90°	0·00	0·00	0·00	0·00	0·00
89°	0·19	0·20	0·20	0·20	0·20
88°	0·39	0·39	0·40	0·40	0·40
87°	0·58	0·59	0·59	0·60	0·60
86°	0·78	0·78	0·79	0·80	0·81
85°	0·97	0·98	0·99	1·00	1·01
84°	1·17	1·18	1·19	1·20	1·21
83°	1·37	1·38	1·39	1·40	1·42
82°	1·56	1·58	1·59	1·61	1·62
81°	1·76	1·78	1·79	1·81	1·83
80°	1·96	1·98	2·00	2·02	2·04
79°	2·16	2·18	2·20	2·22	2·24
78°	2·36	2·39	2·41	2·43	2·45
77°	2·57	2·59	2·61	2·64	2·67
76°	2·77	2·80	2·82	2·85	2·88
75°	2·98	3·01	3·03	3·06	3·09
74°	3·19	3·22	3·25	3·28	3·31
73°	3·40	3·43	3·46	3·50	3·53
72°	3·62	3·65	3·68	3·71	3·75
71°	3·83	3·86	3·90	3·94	3·98
70°	4·05	4·08	4·12	4·16	4·20
69°	4·27	4·31	4·35	4·39	4·43
68°	4·50	4·53	4·58	4·62	4·67
67°	4·72	4·76	4·81	4·85	4·90
66°	4·95	5·00	5·04	5·09	5·14
65°	5·19	5·23	5·28	5·33	5·38
64°	5·43	5·47	5·52	5·58	5·63
63°	5·67	5·72	5·77	5·83	5·88
62°	5·91	5·97	6·02	6·08	6·14
61°	6·17	6·22	6·28	6·34	6·40
60°	6·42	6·48	6·54	6·60	6·67
59°	6·69	6·74	6·81	6·87	6·94
58°	6·95	7·01	7·08	7·14	7·22
57°	7·23	7·29	7·36	7·42	7·50
56°	7·50	7·57	7·64	7·71	7·79
55°	7·79	7·86	7·93	8·01	8·09
54°	8·08	8·15	8·23	8·31	8·39
53°	8·38	8·46	8·53	8·62	8·70
52°	8·69	8·77	8·85	8·93	9·02
51°	9·01	9·09	9·17	9·26	9·35
50°	9·34	9·42	9·50	9·59	9·69
49°	9·67	9·76	9·85	9·94	10·04
48°	10·02	10·11	10·20	10·29	10·40
47°	10·38	10·47	10·56	10·66	10·77
46°	10·74	10·84	10·94	11·04	11·15
45°	11·13	11·22	11·33	11·43	11·55
44°	11·52	11·62	11·73	11·84	11·96
43°	11·93	12·04	12·14	12·26	12·38
42°	12·36	12·46	12·58	12·70	12·82
41°	12·80	12·91	13·03	13·15	13·28
40°	13·26	13·38	13·50	13·63	13·76
39°	13·74	13·86	13·99	14·12	14·26
38°	14·24	14·37	14·50	14·63	14·78
37°	14·76	14·89	15·03	15·17	15·32
36°	15·31	15·45	15·59	15·74	15·89
35°	15·89	16·03	16·17	16·33	16·49
34°	16·49	16·64	16·79	16·95	17·12
33°	17·13	17·28	17·44	17·60	17·78
32°	17·81	17·96	18·12	18·30	18·48
31°	18·52	18·68	18·85	19·03	19·22
30°	19·27	19·44	19·62	19·80	20·00
29°	20·07	20·25	20·43	20·63	20·83
28°	20·93	21·11	21·30	21·50	21·72
27°	21·84	22·03	22·23	22·44	22·66
26°	22·81	23·01	23·22	23·44	23·67
25°	23·86	24·07	24·29	24·52	24·76

	59°	58°	57°	56°	55°
90°	0·00	0·00	0·00	0·00	0·00
89°	0·20	0·21	0·21	0·21	0·21
88°	0·41	0·41	0·42	0·42	0·43
87°	0·61	0·62	0·62	0·63	0·64
86°	0·82	0·82	0·83	0·84	0·85
85°	1·02	1·03	1·04	1·06	1·07
84°	1·23	1·24	1·25	1·27	1·28
83°	1·43	1·45	1·46	1·48	1·50
82°	1·64	1·66	1·68	1·70	1·72
81°	1·85	1·87	1·89	1·91	1·93
80°	2·06	2·08	2·10	2·13	2·15
79°	2·27	2·29	2·32	2·34	2·37
78°	2·48	2·50	2·53	2·56	2·59
77°	2·69	2·72	2·75	2·78	2·82
76°	2·91	2·94	2·97	3·01	3·04
75°	3·13	3·16	3·19	3·23	3·27
74°	3·35	3·38	3·42	3·46	3·50
73°	3·57	3·61	3·65	3·69	3·73
72°	3·79	3·83	3·87	3·92	3·97
71°	4·02	4·06	4·11	4·15	4·20
70°	4·25	4·29	4·34	4·39	4·44
69°	4·48	4·53	4·58	4·63	4·69
68°	4·71	4·76	4·82	4·87	4·93
67°	4·95	5·01	5·06	5·12	5·18
66°	5·19	5·25	5·31	5·37	5·44
65°	5·44	5·50	5·56	5·62	5·69
64°	5·69	5·75	5·82	5·88	5·95
63°	5·94	6·01	6·08	6·15	6·22
62°	6·20	6·27	6·34	6·41	6·49
61°	6·47	6·54	6·61	6·69	6·77
60°	6·74	6·81	6·88	6·96	7·05
59°	7·01	7·09	7·16	7·25	7·34
58°	7·29	7·37	7·45	7·54	7·63
57°	7·58	7·66	7·74	7·83	7·93
56°	7·87	7·95	8·04	8·14	8·23
55°	8·17	8·26	8·35	8·45	8·55
54°	8·48	8·57	8·66	8·76	8·87
53°	8·79	8·89	8·98	9·09	9·20
52°	9·11	9·21	9·32	9·42	9·54
51°	9·45	9·55	9·66	9·77	9·89
50°	9·79	9·89	10·01	10·12	10·24
49°	10·14	10·25	10·36	10·49	10·61
48°	10·50	10·62	10·74	10·86	10·99
47°	10·88	11·00	11·12	11·25	11·38
46°	11·27	11·39	11·51	11·65	11·79
45°	11·67	11·79	11·92	12·06	12·21
44°	12·08	12·21	12·35	12·49	12·64
43°	12·51	12·64	12·79	12·94	13·09
42°	12·96	13·10	13·24	13·40	13·56
41°	13·42	13·56	13·72	13·88	14·04
40°	13·90	14·05	14·21	14·38	14·55
39°	14·41	14·56	14·72	14·90	15·08
38°	14·93	15·09	15·26	15·44	15·63
37°	15·48	15·65	15·82	16·01	16·20
36°	16·06	16·23	16·41	16·60	16·80
35°	16·66	16·84	17·03	17·23	17·43
34°	17·30	17·48	17·68	17·88	18·10
33°	17·96	18·16	18·36	18·57	18·80
32°	18·67	18·87	19·08	19·30	19·54
31°	19·42	19·62	19·84	20·07	20·32
30°	20·21	20·42	20·65	20·89	21·14
29°	21·05	21·27	21·51	21·76	22·02
28°	21·94	22·18	22·43	22·69	22·96
27°	22·90	23·14	23·40	23·67	23·96
26°	23·92	24·18	24·45	24·73	25·03
25°	25·02	25·29	25·57	25·87	26·18

	54°	53°	52°	51°	50°
90°	0·00	0·00	0·00	0·00	0·00
89°	0·22	0·22	0·22	0·22	0·23
88°	0·43	0·44	0·44	0·45	0·46
87°	0·65	0·66	0·67	0·67	0·68
86°	0·86	0·88	0·89	0·90	0·91
85°	1·08	1·10	1·11	1·13	1·14
84°	1·30	1·32	1·33	1·35	1·37
83°	1·52	1·54	1·56	1·58	1·60
82°	1·74	1·76	1·78	1·81	1·84
81°	1·96	1·98	2·01	2·04	2·07
80°	2·18	2·21	2·24	2·27	2·30
79°	2·40	2·43	2·47	2·50	2·54
78°	2·63	2·66	2·70	2·74	2·77
77°	2·85	2·89	2·93	2·97	3·01
76°	3·08	3·12	3·16	3·21	3·25
75°	3·31	3·36	3·40	3·45	3·50
74°	3·54	3·59	3·64	3·69	3·74
73°	3·78	3·83	3·88	3·93	3·99
72°	4·02	4·07	4·12	4·18	4·24
71°	4·26	4·31	4·37	4·43	4·49
70°	4·50	4·56	4·62	4·68	4·75
69°	4·74	4·81	4·87	4·94	5·01
68°	4·99	5·06	5·13	5·20	5·27
67°	5·25	5·31	5·39	5·46	5·54
66°	5·50	5·57	5·65	5·73	5·81
65°	5·76	5·84	5·92	6·00	6·09
64°	6·03	6·11	6·19	6·28	6·37
63°	6·30	6·38	6·47	6·56	6·65
62°	6·57	6·66	6·75	6·84	6·94
61°	6·85	6·94	7·03	7·13	7·24
60°	7·14	7·23	7·33	7·43	7·54
59°	7·43	7·52	7·62	7·73	7·84
58°	7·72	7·82	7·93	8·04	8·16
57°	8·03	8·13	8·24	8·36	8·48
56°	8·34	8·45	8·56	8·68	8·81
55°	8·65	8·77	8·89	9·01	9·14
54°	8·98	9·10	9·22	9·35	9·48
53°	9·31	9·44	9·56	9·70	9·84
52°	9·66	9·78	9·91	10·05	10·20
51°	10·01	10·14	10·28	10·42	10·57
50°	10·37	10·51	10·65	10·80	10·95
49°	10·75	10·88	11·03	11·19	11·35
48°	11·13	11·27	11·43	11·59	11·75
47°	11·53	11·68	11·83	12·00	12·17
46°	11·94	12·09	12·25	12·43	12·61
45°	12·36	12·52	12·69	12·87	13·05
44°	12·80	12·97	13·14	13·32	13·52
43°	13·25	13·43	13·61	13·80	14·00
42°	13·73	13·91	14·09	14·29	14·50
41°	14·22	14·40	14·60	14·80	15·02
40°	14·73	14·92	15·12	15·34	15·56
39°	15·26	15·46	15·67	15·89	16·12
38°	15·82	16·03	16·24	16·47	16·71
37°	16·40	16·62	16·84	17·08	17·32
36°	17·01	17·23	17·47	17·71	17·97
35°	17·65	17·88	18·12	18·38	18·64
34°	18·32	18·56	18·81	19·08	19·35
33°	19·03	19·28	19·54	19·81	20·10
32°	19·78	20·04	20·31	20·59	20·89
31°	20·57	20·84	21·12	21·42	21·73
30°	21·41	21·69	21·98	22·29	22·61
29°	22·30	22·59	22·89	23·21	23·55
28°	23·25	23·55	23·87	24·20	24·55
27°	24·26	24·57	24·91	25·25	25·62
26°	25·34	25·67	26·02	26·38	26·77
25°	26·51	26·85	27·21	27·59	27·99

	49°	48°	47°	46°	45°
90°	0·00	0·00	0·00	0·00	0·00
89°	0·23	0·23	0·24	0·24	0·25
88°	0·46	0·47	0·48	0·49	0·49
87°	0·69	0·71	0·72	0·73	0·74
86°	0·93	0·94	0·96	0·97	0·99
85°	1·16	1·18	1·20	1·22	1·24
84°	1·39	1·41	1·44	1·46	1·49
83°	1·63	1·65	1·68	1·71	1·74
82°	1·86	1·89	1·92	1·95	1·99
81°	2·10	2·13	2·17	2·20	2·24
80°	2·34	2·37	2·41	2·45	2·49
79°	2·58	2·62	2·66	2·70	2·75
78°	2·82	2·86	2·91	2·95	3·01
77°	3·06	3·11	3·16	3·21	3·26
76°	3·30	3·36	3·41	3·47	3·53
75°	3·55	3·61	3·66	3·72	3·79
74°	3·80	3·86	3·92	3·99	4·06
73°	4·05	4·11	4·18	4·25	4·32
72°	4·31	4·37	4·44	4·52	4·59
71°	4·56	4·63	4·71	4·79	4·87
70°	4·82	4·90	4·98	5·06	5·15
69°	5·09	5·17	5·25	5·34	5·43
68°	5·35	5·44	5·52	5·62	5·71
67°	5·62	5·71	5·80	5·90	6·00
66°	5·90	5·99	6·09	6·19	6·30
65°	6·18	6·27	6·38	6·48	6·59
64°	6·46	6·56	6·67	6·78	6·90
63°	6·75	6·86	6·97	7·08	7·21
62°	7·05	7·15	7·27	7·39	7·52
61°	7·34	7·46	7·58	7·71	7·84
60°	7·65	7·77	7·89	8·03	8·16
59°	7·96	8·09	8·22	8·35	8·50
58°	8·28	8·41	8·54	8·69	8·84
57°	8·60	8·74	8·88	9·03	9·18
56°	8·94	9·08	9·22	9·38	9·54
55°	9·28	9·42	9·57	9·73	9·90
54°	9·63	9·78	9·93	10·10	10·27
53°	9·98	10·14	10·30	10·48	10·66
52°	10·35	10·51	10·68	10·86	11·05
51°	10·73	10·90	11·07	11·26	11·45
50°	11·12	11·29	11·47	11·66	11·87
49°	11·52	11·70	11·89	12·08	12·29
48°	11·93	12·12	12·31	12·52	12·73
47°	12·36	12·55	12·75	12·96	13·19
46°	12·80	12·99	13·20	13·42	13·66
45°	13·25	13·46	13·67	13·90	14·14
44°	13·72	13·93	14·16	14·40	14·64
43°	14·21	14·43	14·66	14·91	15·17
42°	14·72	14·94	15·19	15·44	15·71
41°	15·24	15·48	15·73	15·99	16·27
40°	15·79	16·04	16·30	16·57	16·85
39°	16·36	16·62	16·88	17·17	17·46
38°	16·96	17·22	17·50	17·79	18·10
37°	17·58	17·86	18·14	18·45	18·77
36°	18·24	18·52	18·82	19·13	19·46
35°	18·92	19·22	19·53	19·85	20·20
34°	19·64	19·95	20·27	20·61	20·97
33°	20·40	20·72	21·05	21·41	21·78
32°	21·20	21·53	21·88	22·25	22·63
31°	22·05	22·40	22·76	23·14	23·54
30°	22·95	23·31	23·68	24·08	24·49
29°	23·90	24·28	24·67	25·08	25·51
28°	24·92	25·31	25·72	26·15	26·60
27°	26·00	26·41	26·83	27·28	27·75
26°	27·17	27·59	28·03	28·50	29·00
25°	28·42	28·86	29·32	29·81	30·33

	44°	43°	42°	41°	40°
90°	0·00	0·00	0·00	0·00	0·00
89°	0·25	0·26	0·26	0·27	0·27
88°	0·50	0·51	0·52	0·53	0·54
87°	0·76	0·77	0·78	0·80	0·81
86°	1·01	1·03	1·05	1·07	1·09
85°	1·26	1·28	1·31	1·33	1·36
84°	1·51	1·54	1·57	1·60	1·64
83°	1·77	1·80	1·83	1·87	1·91
82°	2·02	2·06	2·10	2·14	2·19
81°	2·28	2·32	2·37	2·41	2·46
80°	2·54	2·59	2·64	2·69	2·74
79°	2·80	2·85	2·90	2·96	3·02
78°	3·06	3·12	3·18	3·24	3·31
77°	3·32	3·39	3·45	3·52	3·59
76°	3·59	3·66	3·73	3·80	3·88
75°	3·86	3·93	4·00	4·08	4·17
74°	4·13	4·20	4·29	4·37	4·46
73°	4·40	4·48	4·57	4·66	4·76
72°	4·68	4·76	4·86	4·95	5·05
71°	4·96	5·05	5·15	5·25	5·36
70°	5·24	5·34	5·44	5·55	5·66
69°	5·53	5·63	5·74	5·85	5·97
68°	5·82	5·92	6·04	6·16	6·29
67°	6·11	6·22	6·34	6·47	6·60
66°	6·41	6·53	6·65	6·79	6·93
65°	6·71	6·84	6·97	7·11	7·25
64°	7·02	7·15	7·29	7·43	7·59
63°	7·33	7·47	7·61	7·77	7·93
62°	7·65	7·80	7·95	8·10	8·27
61°	7·98	8·13	8·28	8·45	8·62
60°	8·31	8·47	8·63	8·80	8·98
59°	8·65	8·81	8·98	9·16	9·35
58°	9·00	9·16	9·34	9·52	9·72
57°	9·35	9·52	9·71	9·90	10·10
56°	9·71	9·89	10·08	10·28	10·49
55°	10·08	10·27	10·46	10·67	10·89
54°	10·46	10·65	10·86	11·07	11·30
53°	10·85	11·06	11·26	11·49	11·72
52°	11·25	11·46	11·68	11·91	12·15
51°	11·66	11·87	12·10	12·34	12·60
50°	12·08	12·30	12·54	12·79	13·05
49°	12·51	12·75	12·99	13·25	13·52
48°	12·96	13·20	13·46	13·73	14·01
47°	13·42	13·67	13·94	14·21	14·51
46°	13·90	14·16	14·43	14·72	15·02
45°	14·40	14·66	14·94	15·24	15·56
44°	14·91	15·18	15·48	15·78	16·11
43°	15·44	15·72	16·03	16·35	16·68
42°	15·99	16·28	16·60	16·93	17·28
41°	16·56	16·87	17·19	17·53	17·90
40°	17·16	17·47	17·81	18·17	18·54
39°	17·78	18·11	18·46	18·82	19·21
38°	18·43	18·77	19·13	19·51	19·91
37°	19·10	19·46	19·83	20·23	20·64
36°	19·81	20·18	20·57	20·98	21·41
35°	20·56	20·94	21·34	21·77	22·22
34°	21·34	21·74	22·16	22·60	23·06
33°	22·17	22·58	23·01	23·47	23·96
32°	23·04	23·47	23·92	24·39	24·90
31°	23·96	24·40	24·87	25·37	25·89
30°	24·93	25·40	25·89	26·40	26·95
29°	25·97	26·45	26·96	27·50	28·07
28°	27·07	27·58	28·11	28·67	29·26
27°	28·25	28·78	29·33	29·91	30·53
26°	29·52	30·06	30·64	31·25	31·90
25°	30·87	31·44	32·06	32·69	33·36

	39°	38°	37°	36°	35°
90°	0·00	0·00	0·00	0·00	0·00
89°	0·28	0·28	0·29	0·30	0·30
88°	0·55	0·57	0·58	0·60	0·61
87°	0·83	0·85	0·87	0·89	0·91
86°	1·11	1·14	1·16	1·19	1·22
85°	1·39	1·42	1·45	1·49	1·53
84°	1·67	1·71	1·75	1·79	1·83
83°	1·95	1·99	2·04	2·09	2·14
82°	2·23	2·28	2·34	2·39	2·45
81°	2·52	2·57	2·63	2·69	2·76
80°	2·80	2·86	2·93	3·00	3·07
79°	3·09	3·16	3·23	3·31	3·39
78°	3·38	3·45	3·53	3·62	3·71
77°	3·67	3·75	3·84	3·93	4·02
76°	3·96	4·05	4·14	4·24	4·35
75°	4·26	4·35	4·45	4·56	4·67
74°	4·56	4·66	4·76	4·88	5·00
73°	4·86	4·97	5·08	5·20	5·33
72°	5·16	5·28	5·40	5·53	5·66
71°	5·47	5·59	5·72	5·86	6·00
70°	5·78	5·91	6·05	6·19	6·35
69°	6·10	6·23	6·38	6·53	6·69
68°	6·42	6·56	6·71	6·87	7·04
67°	6·74	6·89	7·05	7·22	7·40
66°	7·07	7·23	7·40	7·57	7·76
65°	7·41	7·57	7·75	7·93	8·13
64°	7·75	7·92	8·10	8·30	8·50
63°	8·10	8·28	8·47	8·67	8·88
62°	8·45	8·64	8·83	9·05	9·27
61°	8·81	9·00	9·21	9·43	9·66
60°	9·17	9·38	9·59	9·82	10·07
59°	9·55	9·76	9·98	10·22	10·48
58°	9·93	10·15	10·38	10·63	10·89
57°	10·32	10·55	10·79	11·05	11·32
56°	10·72	10·96	11·21	11·48	11·76
55°	11·13	11·37	11·64	11·91	12·21
54°	11·54	11·80	12·07	12·36	12·67
53°	11·98	12·24	12·52	12·82	13·14
52°	12·41	12·69	12·98	13·29	13·62
51°	12·87	13·15	13·46	13·78	14·12
50°	13·33	13·63	13·94	14·28	14·63
49°	13·81	14·12	14·44	14·79	15·16
48°	14·31	14·63	14·96	15·32	15·70
47°	14·82	15·15	15·50	15·86	16·26
46°	15·34	15·69	16·05	16·43	16·84
45°	15·89	16·24	16·62	17·01	17·43
44°	16·45	16·82	17·21	17·62	18·05
43°	17·04	17·42	17·82	18·24	18·70
42°	17·65	18·04	18·45	18·89	19·36
41°	18·28	18·69	19·12	19·57	20·06
40°	18·94	19·36	19·80	20·28	20·78
39°	19·62	20·06	20·52	21·01	21·53
38°	20·34	20·79	21·27	21·78	22·32
37°	21·09	21·55	22·05	22·58	23·14
36°	21·87	22·36	22·87	23·42	24·00
35°	22·69	23·20	23·73	24·30	24·90
34°	23·56	24·08	24·63	25·22	25·85
33°	24·47	25·01	25·59	26·20	26·85
32°	25·43	25·99	26·59	27·23	27·90
31°	26·45	27·03	27·65	28·31	29·02
30°	27·52	28·13	28·78	29·47	30·20
29°	28·67	29·30	29·98	30·69	31·45
28°	29·89	30·55	31·25	32·00	32·79
27°	31·19	31·88	32·61	33·39	34·22
26°	32·58	33·30	34·07	34·89	35·75
25°	34·08	34·83	35·63	36·48	37·39

	34°	33°	32°	31°	30°
90°	0·00	0·00	0·00	0·00	0·00
89°	0·31	0·32	0·33	0·34	0·35
88°	0·62	0·64	0·66	0·68	0·70
87°	0·94	0·96	0·99	1·02	1·05
86°	1·25	1·28	1·32	1·36	1·40
85°	1·56	1·61	1·65	1·70	1·75
84°	1·88	1·93	1·98	2·04	2·10
83°	2·20	2·25	2·32	2·38	2·46
82°	2·51	2·58	2·65	2·73	2·81
81°	2·83	2·91	2·99	3·08	3·17
80°	3·15	3·24	3·33	3·42	3·53
79°	3·48	3·57	3·67	3·77	3·89
78°	3·80	3·90	4·01	4·13	4·25
77°	4·13	4·24	4·36	4·48	4·62
76°	4·46	4·58	4·70	4·84	4·99
75°	4·79	4·92	5·06	5·20	5·36
74°	5·13	5·26	5·41	5·57	5·73
73°	5·47	5·61	5·77	5·94	6·11
72°	5·81	5·97	6·13	6·31	6·50
71°	6·16	6·32	6·50	6·69	6·89
70°	6·51	6·68	6·87	7·07	7·28
69°	6·86	7·05	7·24	7·45	7·68
68°	7·23	7·42	7·62	7·84	8·08
67°	7·59	7·79	8·01	8·24	8·49
66°	7·96	8·17	8·40	8·64	8·90
65°	8·34	8·56	8·80	9·05	9·33
64°	8·72	8·95	9·20	9·47	9·75
63°	9·11	9·36	9·62	9·89	10·19
62°	9·51	9·76	10·03	10·32	10·63
61°	9·91	10·18	10·46	10·76	11·09
60°	10·32	10·60	10·89	11·21	11·55
59°	10·75	11·03	11·34	11·67	12·02
58°	11·17	11·48	11·79	12·13	12·50
57°	11·61	11·92	12·25	12·61	12·99
56°	12·06	12·38	12·73	13·10	13·49
55°	12·52	12·86	13·21	13·60	14·00
54°	12·99	13·34	13·71	14·11	14·53
53°	13·48	13·84	14·22	14·63	15·07
52°	13·97	14·34	14·74	15·17	15·63
51°	14·48	14·87	15·28	15·72	16·20
50°	15·01	15·41	15·83	16·29	16·78
49°	15·55	15·96	16·40	16·88	17·39
48°	16·10	16·53	16·99	17·48	18·01
47°	16·68	17·12	17·60	18·11	18·65
46°	17·27	17·73	18·22	18·75	19·31
45°	17·88	18·36	18·87	19·42	20·00
44°	18·52	19·01	19·54	20·11	20·71
43°	19·18	19·69	20·24	20·82	21·45
42°	19·86	20·39	20·96	21·56	22·21
41°	20·57	21·12	21·71	22·34	23·01
40°	21·31	21·88	22·49	23·14	23·84
39°	22·08	22·67	23·30	23·98	24·70
38°	22·89	23·50	24·15	24·85	25·60
37°	23·73	24·37	25·04	25·77	26·54
36°	24·61	25·27	25·97	26·72	27·53
35°	25·54	26·22	26·95	27·73	28·56
34°	26·51	27·22	27·98	28·79	29·65
33°	27·54	28·27	29·06	29·90	30·80
32°	28·62	29·38	30·20	31·07	32·01
31°	29·76	30·56	31·41	32·31	33·29
30°	30·97	31·80	32·69	33·63	34·64
29°	32·26	33·12	34·04	35·08	36·08
28°	33·63	34·53	35·49	36·52	37·61
27°	35·10	36·04	37·04	38·11	39·25
26°	36·67	37·65	38·69	39·81	41·01
25°	38·35	39·37	40·47	41·64	42·89

	29°	28°	27°	26°	25°	
90°	0·00	0·00	0·00	0·00	0·00	90°
89°	0·36	0·37	0·38	0·40	0·41	89°
88°	0·72	0·74	0·77	0·80	0·83	88°
87°	1·08	1·12	1·15	1·20	1·24	87°
86°	1·44	1·49	1·54	1·60	1·65	86°
85°	1·80	1·86	1·93	2·00	2·07	85°
84°	2·17	2·24	2·32	2·40	2·49	84°
83°	2·53	2·62	2·70	2·80	2·91	83°
82°	2·90	2·99	3·10	3·21	3·33	82°
81°	3·27	3·37	3·49	3·61	3·75	81°
80°	3·64	3·76	3·88	4·02	4·17	80°
79°	4·01	4·14	4·28	4·43	4·60	79°
78°	4·38	4·53	4·68	4·85	5·03	78°
77°	4·76	4·92	5·09	5·27	5·46	77°
76°	5·14	5·31	5·49	5·69	5·90	76°
75°	5·53	5·71	5·90	6·11	6·34	75°
74°	5·91	6·11	6·32	6·54	6·78	74°
73°	6·31	6·51	6·73	6·97	7·23	73°
72°	6·70	6·92	7·16	7·41	7·69	72°
71°	7·10	7·33	7·58	7·85	8·15	71°
70°	7·51	7·75	8·02	8·30	8·61	70°
69°	7·92	8·18	8·46	8·76	9·08	69°
68°	8·33	8·61	8·90	9·22	9·56	68°
67°	8·76	9·04	9·35	9·68	10·04	67°
66°	9·18	9·48	9·81	10·16	10·53	66°
65°	9·62	9·93	10·27	10·64	11·04	65°
64°	10·06	10·39	10·74	11·13	11·54	64°
63°	10·51	10·85	11·22	11·62	12·06	63°
62°	10·97	11·33	11·71	12·13	12·58	62°
61°	11·43	11·81	12·21	12·64	13·12	61°
60°	11·91	12·30	12·72	13·17	13·66	60°
59°	12·39	12·80	13·23	13·71	14·22	59°
58°	12·89	13·31	13·76	14·25	14·79	58°
57°	13·40	13·83	14·30	14·81	15·37	57°
56°	13·91	14·37	14·86	15·39	15·96	56°
55°	14·44	14·91	15·42	15·97	16·57	55°
54°	14·99	15·48	16·00	16·57	17·19	54°
53°	15·54	16·05	16·60	17·19	17·83	53°
52°	16·12	16·64	17·21	17·82	18·49	52°
51°	16·70	17·25	17·84	18·47	19·16	51°
50°	17·31	17·87	18·48	19·14	19·85	50°
49°	17·93	18·52	19·15	19·83	20·57	49°
48°	18·57	19·18	19·83	20·54	21·31	48°
47°	19·23	19·86	20·54	21·27	22·07	47°
46°	19·92	20·57	21·27	22·03	22·85	46°
45°	20·63	21·30	22·03	22·81	23·66	45°
44°	21·36	22·06	22·81	23·62	24·50	44°
43°	22·12	22·84	23·62	24·46	25·37	43°
42°	22·91	23·66	24·46	25·34	26·28	42°
41°	23·73	24·50	25·34	26·24	27·22	41°
40°	24·58	25·89	26·25	27·19	28·20	40°
39°	25·47	26·30	27·20	28·17	29·22	39°
38°	26·40	27·26	28·19	29·20	30·29	38°
37°	27·37	28·27	29·23	30·27	31·40	37°
36°	28·39	29·32	30·32	31·40	32·57	36°
35°	29·46	30·42	31·46	32·58	33·79	35°
34°	30·58	31·58	32·66	33·82	35·08	34°
33°	31·76	32·80	33·92	35·13	36·44	33°
32°	33·01	34·09	35·25	36·51	37·87	32°
31°	34·33	35·45	36·66	37·97	39·38	31°
30°	35·73	36·89	38·15	39·51	40·98	30°
29°	37·21	38·43	39·74	41·15	42·69	29°
28°	38·79	40·06	41·43	42·90	44·50	28°
27°	40·48	41·80	43·23	44·77	46·44	27°
26°	42·29	43·67	45·16	46·77	48·51	26°
25°	44·23	45·68	47·24	48·92	50·74	25°

Cotangent Distances.

octahedron, and dodecahedron is drawn in fig. 5. We first draw the gnomonic projection with the pole of a cube face coincident with the centre of projection. In the actual drawing the radius of the sphere was given a length of 2 cm. We now give the system rotations of 20° and 10° about the vertical and horizontal axes, respectively, in the plane of the paper, as shown in the figure, and obtain another projection.

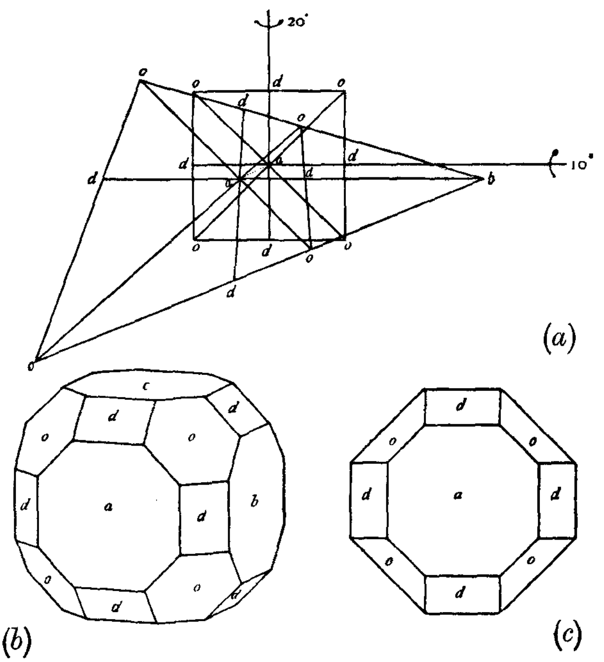

FIG. 5.—Plan (*c*) and orthographic drawing (*b*) derived from the gnomonic projection (*a*) of a crystal.

From these projections we prepare drawings of the crystal in the two positions. The drawings represent crystals of exactly the same size, the lengths of the projections of the various edges having been calculated in the manner described above.

The table appended to this paper gives the values of $10 \cot \rho \operatorname{cosec} \phi$ for every degree from 90° to 25°, where ρ and ϕ represent the distance and azimuth of a pole measured from an origin in the zone, whose edge is at right angles to the plane of the paper, the azimuth being measured from this zone. The purpose of the table is explained above (p. 313).