SURFACE TENSION

Definition

In the fall a fisherman's boat is often surrounded by fallen leaves that are lying on the water. The boat floats, because it is partially immersed in the water and the resulting buoyant force balances its weight, as Section 11.6 discusses. The leaves, however, float for a different reason. They are not immersed in the water, so the weight of a leaf is not balanced by a buoyant force. Instead, the force balancing a leaf's weight arises because of the surface tension of the water. Surface tension is a property that allows the surface of a liquid to behave somewhat as a trampoline does. When a person stands on a trampoline, the trampoline stretches downward a bit and, in so doing, exerts an upward elastic force on the person. This upward force balances the person's weight. The surface of the water behaves in a similar way. In Figure 1, for instance, you can see the indentations in the water surface made by the feet of an insect known as a water strider, because it can stride or walk on the surface just as a person can walk on a trampoline.

Figure 1 The indentations in the water surface made by a water strider are readily seen in this photograph. The surface tension of the water allows the insect to walk on the water without sinking. (© Hermann Eisenbess/Photo Researchers)

Figure 2 illustrates the molecular basis for surface tension by considering the attractive forces that molecules in a liquid exert on one another. Part *a* shows a molecule within the bulk liquid, so that it is surrounded on all sides by other molecules. The surrounding molecules attract the central molecule equally in all directions, leading to a zero net force. In contrast, part *b* shows a molecule in the surface. Since there are no molecules of the liquid above the surface, this molecule experiences a net attractive force pointing toward the liquid interior. This net attractive force causes the liquid surface to contract toward the interior until repulsive collisional forces from the other molecules halt the contraction at the point when the surface area is a minimum. If the liquid is not acted upon by external forces, a liquid sample forms a sphere, which has the minimum surface area for a given volume. Nearly spherical drops of water are a familiar sight, for example, when the external forces are negligible.

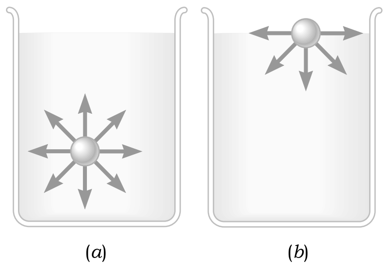

Figure 2 (*a*) A molecule within the bulk liquid is surrounded on all sides by other molecules, which attract it equally in all directions, leading to a zero net force. (*b*) A molecule in the surface experiences a net attractive force pointing toward the liquid interior, because there are no molecules of the liquid above the surface.

To help us define the surface tension we use the apparatus shown in Figure 3. It consists of a C-shaped wire frame, on which is mounted a wire that can slide with negligible friction. The frame and sliding wire contain a thin film of liquid. Because surface tension causes the liquid surface to contract, a force **F** is needed to move the slider to the right and extend the surface. The ***surface tension*** is denoted by the Greek letter gamma (γ) and, as indicated by Equation 1, is the magnitude F of the force per unit length over which it acts. Table 1 gives the value of the surface tension for some typical materials.

■ **DEFINITION OF SURFACE TENSION**

The surface tension γ is the magnitude F of the force exerted parallel to the surface of a liquid divided by the length L of the line over which the force acts:

$$\gamma = \frac{F}{L} \tag{1}$$

SI Unit of Surface Tension: N/m

For the specific case illustrated in Figure 3, there is an upper surface and a lower surface, as the blow-up drawing indicates. Thus, the force **F** acts along a total length of $L = 2\ell$, where ℓ is the length of the slider. Example 1 deals with a demonstration of the effects of surface tension that you can try yourself.

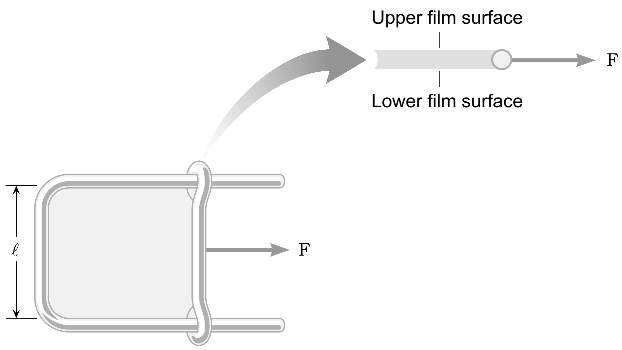

Figure 3 This apparatus, consisting of a C-shaped wire frame and a wire slider, can be used to measure the surface tension of a liquid.

Table 1 Surface Tensions of Common Liquids

Liquid	Surface Tension γ (N/m)
Benzene (20 °C)	0.029
Blood (37 °C)	0.058
Glycerin (20 °C)	0.063
Mercury (20 °C)	0.47
Water (20 °C)	0.073
Water (100 °C)	0.059

EXAMPLE 1 • **Floating a Needle on the Surface of Water**

A needle has a length of 3.2 cm. When placed gently on the surface of the water (γ = 0.073 N/m) in a glass, this needle will float if it is not too heavy. What is the weight of the heaviest needle that can be used in this demonstration?

Reasoning As the end view in Figure 4 shows, three forces act on the needle, its weight **W** and the two forces $\mathbf{F}_1$ and $\mathbf{F}_2$ due to the surface tension of the water. The forces $\mathbf{F}_1$ and $\mathbf{F}_2$ result from the surface tension acting along the length of the needle on either side. According to Equation 1, they have the same magnitude $F_1 = F_2 = \gamma L$, where $\gamma = 0.073$ N/m is the surface tension of water and L is the length of the needle. $\mathbf{F}_1$ and $\mathbf{F}_2$ are each tangent to the indented water surface that is formed when the needle presses on the surface, with the result that each acts at an angle θ with respect to the vertical. The needle floats in equilibrium. Therefore, the net force $\Sigma\mathbf{F}$ acting on the needle is zero. In the vertical direction this means that the sum of the vertical components of $\mathbf{F}_1$, $\mathbf{F}_2$, and **W** equals zero.

Solution Applying the fact that the net force acting on the needle is zero we have

$$\Sigma\mathbf{F} = 0 \qquad (4.9)$$

$$-W + \underbrace{(\gamma L)\cos\theta}_{\substack{\text{Vertical component}\\ \text{of } \mathbf{F}_1}} + \underbrace{(\gamma L)\cos\theta}_{\substack{\text{Vertical component}\\ \text{of } \mathbf{F}_2}} = 0$$

$$W = 2(\gamma L)\cos\theta$$

In other words, the sum of the vertical components of $\mathbf{F}_1$ and $\mathbf{F}_2$ balances the weight of the needle. The forces due to the surface tension will balance the largest weight when they point completely vertically and $\theta = 0°$. Therefore, the weight of the heaviest needle that can be used in this demonstration is

$$W = 2(\gamma L)\cos\theta = 2(0.073\text{ N/m})(0.032\text{ m})\cos 0° = \boxed{4.7\times10^{-3}\text{ N}}$$

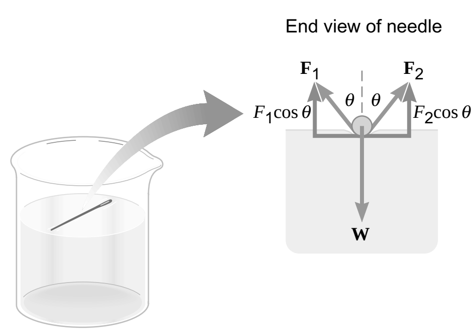

Figure 4 A needle can float on a water surface, because the surface tension of the water can lead to forces strong enough to support the needle's weight.

Capillary Action

We have seen that surface tension arises because of the intermolecular forces of attraction that molecules in a liquid exert on one another. These forces, which are between like molecules, are called ***cohesive forces***. A liquid, however, is often in contact with a solid surface, such as glass. Then additional forces of attraction come into play. They occur between molecules of the liquid and molecules of the solid surface and, being between unlike molecules, are called ***adhesive forces***.

Consider a tube with a very small diameter, which is called a capillary. When a capillary, open at both ends, is inserted into a liquid, the result of the competition between cohesive and adhesive forces can be observed. For instance, Figure 5 shows a glass capillary inserted into water. In this case, the adhesive forces are stronger than the cohesive forces, so that the water molecules are attracted to the glass more strongly than to each other. The result is that the water surface curves upward against the glass. It is said that the water "wets" the glass. The surface tension leads to a force **F** acting on the circular boundary between the water and the glass. This force is oriented at an angle ϕ, which is determined by the competition between the cohesive and adhesive forces. The vertical component of **F** pulls the water up into the tube to a height h. At this height the vertical component of **F** balances the weight of the column of water of length h.

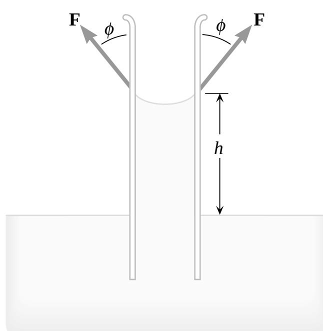

Figure 5 Water rises in a glass capillary due to the surface tension of the water and the fact that the water wets the glass surface.

Figure 6 shows a glass capillary inserted into mercury, a situation in which the adhesive forces are weaker than the cohesive forces. The mercury atoms are attracted to each other more strongly than they are to the glass. As a result, the mercury surface curves downward against the glass and the mercury does not "wet" the glass. Now, in contrast to the situation illustrated in Figure 5, the surface tension leads to a force **F**, the vertical component of which pulls the mercury down a distance h in the tube. The behavior of the liquids in both Figures 5 and 6 is called ***capillary action***.

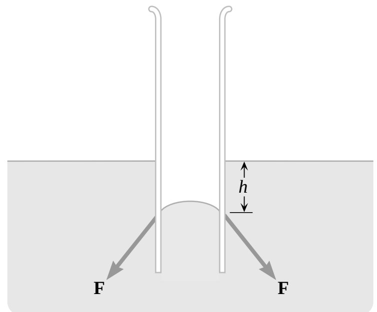

Figure 6 Mercury falls in a glass capillary due to the surface tension of the mercury and the fact that the mercury does not wet the glass surface.

Pressure Inside a Soap Bubble and a Liquid Drop

Anyone who's blown up a balloon has probably noticed that the air pressure inside the balloon is greater than on the outside. For instance, if the balloon is suddenly released, the greater inner pressure forces the air out, propelling the balloon much like a rocket. The reason for the greater pressure is that the tension in the stretched rubber tends to contract the balloon. To counteract this tendency, the balloon has a greater interior air pressure acting to expand the balloon.

A soap bubble (see Figure 7*a*) has two spherical surfaces (inside and outside) with a thin layer of liquid in-between. Like a balloon, the pressure inside a soap bubble is greater than that on the outside. As we will see shortly, this difference in pressure depends on the surface tension γ of the liquid and the radius *R* of the bubble. For the sake of simplicity, let's assume that there is no pressure on the outside of the bubble ($P_o = 0$). Now, imagine that the stationary soap bubble is cut into two halves. Being at rest, each half has no acceleration and so is in equilibrium. According to Newton's second law of motion (see Section 4.11), a zero acceleration implies that the net force acting on each half must be zero ($\Sigma \mathbf{F} = 0$). We will now use this equilibrium relation to obtain an expression relating the interior pressure to the surface tension and the radius of the bubble.

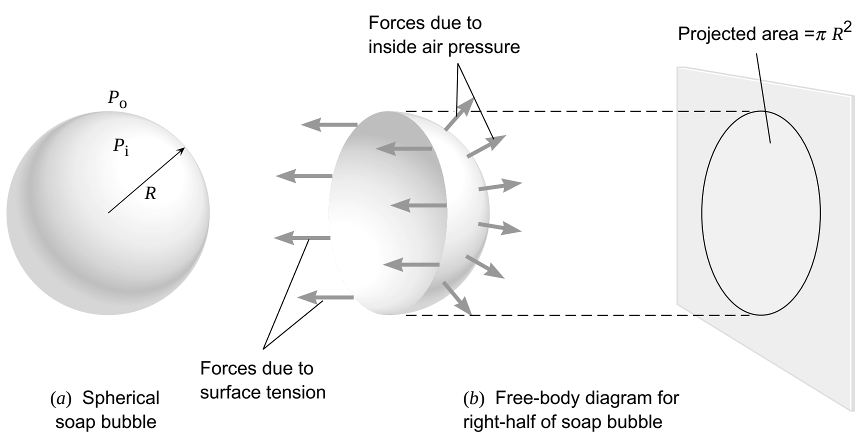

Figure 7 (*a*) The inner and outer pressures on the spherical soap bubble are P_i and P_o, respectively. (*b*) The forces pointing to the left are due to the surface tension. The forces pointing perpendicular to the hemispherical surface are due to the air pressure inside the bubble.

Figure 7*b* shows a free-body diagram for the right half of the bubble, on which two forces act. First, there is the force due to the surface tension in the film. This force is exerted on the right half of the bubble by the left half. The surface tension force points to the left and acts all along the circular edge of the hemispherical film. The magnitude of the force due to each surface of the film is the product of the surface tension γ and the circumference ($2\pi R$) of the circular edge, or $\gamma(2\pi R)$. The total force due to the inner and outer surfaces is twice this amount or $-2\gamma(2\pi R)$. We have included the minus sign to denote that this force points to the left in the drawing. We have also assumed the film to be sufficiently thin enough that its inner and outer radii are nearly the same. Second, there is a force caused by the air pressure inside the bubble. At each point on the surface of the bubble, the force due to the air pressure is perpendicular to the surface and is directed outward. Figure 7*b* shows this force at six points on the surface. When these forces are added to obtain the total force due to the air pressure, all the components cancel, except those pointing to the right. The total force due to all the components pointing to the right is equal to the product of the pressure P_i inside the bubble times the circular cross-sectional area of the hemisphere, or $P_i(\pi R^2)$. Using these expressions for the forces due to the surface tension and air pressure, we can write Newton's second law of motion as:

$$\Sigma \mathbf{F} = 0 \tag{4.9}$$

$$\underbrace{-2\gamma(2\pi R)}_{\substack{\text{Force due to} \\ \text{surface tension}}} + \underbrace{P_i(\pi R^2)}_{\substack{\text{Force due to} \\ \text{pressure inside} \\ \text{bubble}}} = 0$$

Solving this equation for the pressure inside the bubble gives $P_i = 4\gamma/R$. In general, the pressure P_o outside the bubble is not zero. However, this result still gives the difference between the inside and outside pressures, so that we have

$$P_i - P_o = \frac{4\gamma}{R} \qquad \text{(Spherical soap bubble)} \tag{2}$$

This result tells us that the difference in pressure depends on the surface tension and the radius of the sphere. What is surprising is that a greater pressure exists inside a smaller soap bubble (smaller value of *R*) than inside a larger one.

A spherical drop of liquid, like a drop of water, has only *one* surface, rather than two surfaces, for there is no air within it. Thus, the force due to the surface tension is only one-half as large as that in a bubble. Consequently, the difference in pressure between the inside and outside of a liquid drop is one-half of that for a soap bubble:

$$P_i - P_o = \frac{2\gamma}{R} \qquad \text{(Spherical liquid drop)} \tag{3}$$

Equation 3 is known as Laplace's law for a spherical liquid drop, after the French physicist and mathematician Marquis Pierre Simon deLaplace (1749–1827). This result also holds for a spherical bubble in a liquid, such as a gas bubble inside a glass of beer. However, the surface tension γ is that of the surrounding liquid in which the trapped bubble resides. Example 2 illustrates the pressure difference for a soap bubble and a liquid drop.

EXAMPLE 2 • A Soap Bubble and a Liquid Drop

(a) A student, using a circular loop of wire and a pan of soapy water, produces a soap bubble whose radius is 1.0 mm. The surface tension of the soapy water is $\gamma = 2.5 \times 10^{-2}$ N/m. Determine the pressure difference between the inside and outside of the bubble. (b) The same soapy water is used to produce a spherical droplet whose radius is one-half that of the bubble, or 0.50 mm. Find the pressure difference between the inside and outside of the droplet.

Reasoning If the bubble and drop had the same radius, we would expect that the pressure difference between the inside and outside of the bubble to be twice as large as that for the drop. The reason is that the bubble has two surfaces, whereas the drop has only one. Thus, the bubble would have twice the force due to surface tension, and so the pressure inside the bubble would have to be twice as large to counteract this larger force. In fact, however, the bubble has twice the radius compared to the drop. The doubled radius means that the bubble has one-half the pressure difference. Consequently, we expect the larger bubble and smaller drop to have the same pressure difference.

Solution

(a) The pressure difference, $P_i - P_o$, between the inside and outside of the soap bubble is given by Equation 2 as

$$P_i - P_o = \frac{4\gamma}{R} = \frac{4\left(2.5\times10^{-2}\,\text{N/m}\right)}{1.0\times10^{-3}\,\text{m}} = \boxed{1.0\times10^{2}\,\text{N/m}^2}$$

(b) The pressure difference, $P_i - P_o$, between the inside and outside of the drop is given by Equation 3 as

$$P_i - P_o = \frac{2\gamma}{R} = \frac{2\left(2.5\times10^{-2}\,\text{N/m}\right)}{0.50\times10^{-3}\,\text{m}} = \boxed{1.0\times10^{2}\,\text{N/m}^2}$$

PROBLEMS

1. A sliding wire of length 3.5 cm is pulling a liquid film, as Figure 3 shows. The pulling force exerted by the wire is 4.4×10^{-3} N. From the table of surface tensions, determine the film material.

2. A circular ring (radius = 5.0 cm) is used to determine the surface tension of a liquid. The plane of the ring is positioned so that it is parallel to the surface of the liquid. The ring is immersed in the liquid and then pulled upward, so a film is formed between the ring and the liquid. In addition to the ring's weight, an upward force of 3.6×10^{-2} N is required to lift the ring to the point where it just breaks free of the surface. What is the surface tension of the liquid?

3. Suppose that the C-shaped wire frame in Figure 3 is rotated clockwise by 90°, so that the sliding wire would fall down if it were free to do so. The sliding wire has a length of 3.5 cm and a mass of 0.20 g. There is no friction between the sliding wire and the vertical sides of the C-shaped wire. If the sliding wire is in equilibrium, what is the surface tension of the film?

4. Two soap bubbles (γ = 0.025 N/m) have radii of 2.0 and 4.5 mm. For each bubble, determine the difference between the inside and outside pressures.

5. A small bubble of air in water (γ = 0.073 N/m) has a radius of 0.10 mm. Find the difference in pressures between the inside and outside of the bubble.

*6. A drop of oil (γ = 0.0320 N/m) has a radius of 0.0100 mm. The drop is located a distance of 2.55 m below the surface of fresh water. The atmospheric pressure above the water is 1.01×10^{5} Pa. (a) What is the absolute pressure in the water at this depth? (b) Determine the absolute pressure inside the oil drop.

**7. Suppose that a bubble has the shape of a long cylinder, rather than that of a sphere. Determine an expression for the difference between the inside and outside pressures; express your answer in terms of the surface tension γ and the radius R of the cylinder. (*Hint: Review the reasoning that was used to obtain the difference in pressures for a spherical soap bubble. For the cylindrical bubble, "cut" the cylinder into two halves by slicing along a line that is parallel to the axis of the cylinder.*)