On the Possibility of Supertasks in General Relativity

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Abstract Malament-Hogarth spacetimes are the sort of models within general relativity that seem to allow for the possibility of supertasks. There are various ways in which these spacetimes might be considered physically problematic. Here, we examine these criticisms and investigate the prospect of escaping them.

1 Introduction

Within general relativity, some models seem to allow for the completion of an infinite number of tasks (a supertask) in a finite amount of time.[1] Such models—called Malament-Hogarth (M-H) spacetimes—may have significant implications for the foundations of computability theory. For example, some have argued that the supertasks enabled by M-H spacetimes refute the physical Church's Thesis by allowing for the computation of a Turing non-computable function [11]. M-H spacetimes can be defined as follows [10].

Definition We say (M, g_{ab}) is an *M-H spacetime* if there is a future-directed timelike half-curve $\gamma \subset M$ and a point $p \in M$ such that $\int_\gamma d\tau = \infty$ and $\gamma \subset I^-(p)$.

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[1]What we mean by an "infinite number of tasks" is left deliberately vague; it is notoriously difficult to be careful about such things. An excellent introduction to supertasks as well as a survey of the relevant literature is given in [4]. For the background structure of general relativity, see [9] and [16].

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Fig. 1 The spacetime
$(\mathbb{R}^2 - \{q\}, \Omega^2 \eta_{ab})$
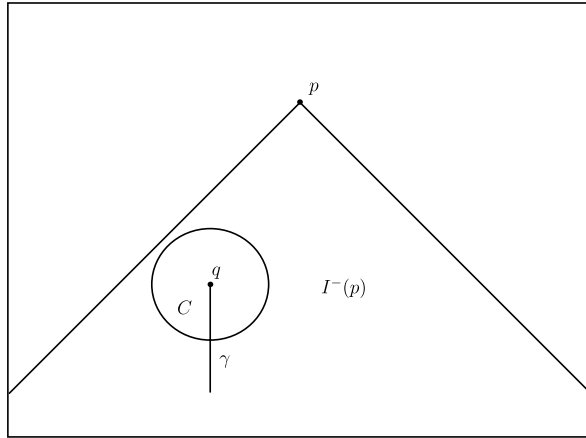
Here, the curve γ represents the worldline of some observer. Because γ has infinite proper time, she may complete an infinite number of tasks. But, at every point in γ, it is possible to send a signal to the point p. Because there always exists a curve γ' with future endpoint p which has finite proper time, we can think of γ as the "sender" and γ' as the "receiver" of a signal. In this way, the receiver may complete an infinite number of tasks in a finite time. To illustrate these basic features of the definition, it helps to examine various instantiations. Here is a toy example [3]:

Example Let $(\mathbb{R}^2, \eta_{ab})$ be two-dimensional Minkwski spacetime. Consider a compact set $C \subset \mathbb{R}^2$, a point $q \in C$, and point p such that $C \subset I^-(p)$. Now let Ω be a strictly positive smooth function on $\mathbb{R}^2 - \{q\}$ such that, $\Omega = 1$ outside of C and, as q is approached, Ω becomes infinite. $(\mathbb{R}^2 - \{q\}, \Omega^2 \eta_{ab})$ is an M-H spacetime with any future-directed timelike curve γ approaching q representing the sender and p as a point on the worldline of the receiver where $\gamma \subset I^-(p)$ (see Fig. 1).

Now we want to consider physical constraints that may be used to evaluate whether there exist M-H spacetimes in which a supertask may be plausibly completed. We want to assess not only the physical reasonableness of the M-H spacetime in question, but also the physical implementability of the supertask within that spacetime. To do this, we will need a way to represent such setups within M-H spacetimes. Consider the following definition.

Definition Let (M, g_{ab}) be an M-H spacetime. Let $\gamma \subset M$ be a future-directed timelike half-curve such that $\int_\gamma d\tau = \infty$. Let $p \in M$ be such that $\gamma \subset I^-(p)$. We say the ordered triple $((M, g_{ab}), p, \gamma)$ is an *M-H setup*.

We note that there is, associated with every M-H spacetime, an infinite number of M-H setups. What follows are characterizations of certain conditions, many of which are found in [3], that an M-H setup $((M, g_{ab}), p, \gamma)$ must satisfy in order for it to be considered physically reasonable.

Fig. 2 "Rolled" Minkowski spacetime
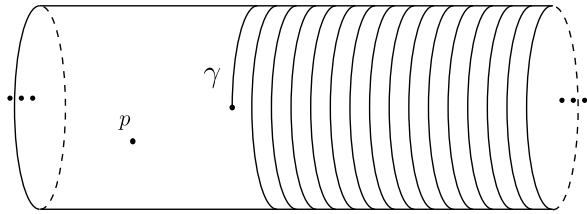
2 Conditions of Physical Reasonableness

The first condition, inextendibility, is one which is regularly used to rule out seemingly artificial models. Roughly, the idea is to require that spacetime extend out "as far as it can." Formally, we have the following.

Definition A spacetime (M, g_{ab}) is *inextendible* if there is no isometric embedding $\theta : M \to M'$ into a spacetime (M', g'_{ab}) such that M is a proper subset of M'.

Though the metaphysics involved can be questioned, we will follow general practice and assume that, for any spacetime to be physically reasonable, it must be inextendible.[2]

Next, consider the following M-H setup.

Example Let $(\mathbb{S} \times \mathbb{R}, \eta_{ab})$ be two-dimensional Minkowski spacetime which has been "rolled up" along the time axis. Let γ be any future-inextendible timelike curve and let p be any point in $\mathbb{S} \times \mathbb{R}$. The triple $((\mathbb{S} \times \mathbb{R}, \eta_{ab}), p, \gamma)$ is an M-H setup because $I^-(p)$ covers the entire manifold (see Fig. 2).

Given this example, it may seem that M-H setups are too easily constructed if causal misbehavior is permitted. This is the first worry noted by Earman and Norton [3] and the intuition can be realized in the following easily proved proposition.

Proposition *If a spacetime contains closed timelike curves, then it is an M-H spacetime.*

Proof Let (M, g_{ab}) be a spacetime. Let $\gamma \subset M$ be a closed timelike curve. Let $\gamma_\infty \subset M$ be the curve that results from staring at a point $p \in \gamma$ and looping around γ an infinite number of times. Clearly γ_∞ is, by construction, a timelike half-curve such that $\int_{\gamma_\infty} d\tau = \infty$. Of course, $\gamma_\infty \subset I^-(p)$. So (M, g_{ab}) is an M-H spacetime. □

Whether acausal spacetimes should be classified as physically unreasonable is the topic of another paper. But for the time being, we can nonetheless require that M-H spacetimes have well-behaved causal structure. There is one caveat, however. We cannot become too causally strict or we risk labeling *all* M-H spacetimes as physically unreasonable. We have the following result.

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[2]See [2, pp. 31–33] for a discussion.

Proposition (Malament) *If a spacetime is globally hyperbolic, then it is not an M-H spacetime.*

Proof Given in [2, p. 122].   □

So, in order to entertain the notion of M-H spacetimes, one must be willing to welcome at least a bit causal mischief. The task is to find some causal condition which is strong enough to exclude trivial M-H spacetimes but not too strong so as to eliminate all of them. Following [3], our preferred condition will be stable causality.

Definition A spacetime (M, g_{ab}) is *stably causal* if there exists a non-vanishing timelike vector field ξ^a on M such that the spacetime $(M, g_{ab} + \xi_a \xi_b)$ contains no closed timelike curves.

A second concern given in [3] has to do with possible infinite blueshifts recorded by the receiver. An informal argument summarizing the distress is recounted here:

> During her lifetime, [the sender] measures an infinite number of vibrations of her source, each vibration taking the same amount of her proper time. [The receiver] must agree that an infinite number of vibrations take place. But within a finite amount of his proper time, [the receiver] receives an infinite number of light signals from [the sender], each announcing the completion of a vibration. For this to happen, [the receiver] must receive the signals in ever decreasing intervals of his proper time. Thus, [the receiver] will perceive the frequency of [the sender's] source to increase without bound [3, p. 30].

There is a technical result offered in support of the informal argument given above [3, p. 31]. The precise formulation of the result need not concern us here. For our purposes, it is enough to note that arbitrary M-H setups do not satisfy the antecedent assumptions of the given result.[3] Thus, although the result nicely labels some M-H setups (those satisfying its assumptions) as blueshift delinquent, the lemma says nothing concerning setups falling outside its scope. Accordingly, we would like to find a condition that, if satisfied, would seem to label an M-H setup without blueshift complications as such. We have the following.

Definition Let $((M, g_{ab}), p, \gamma)$ be an M-H setup. Let $\{q_i\}$ be a countably infinite sequence of points in γ such that, for all i, the distance (as measured in proper time along γ) between q_i and q_{i+1} is the same. We say that $((M, g_{ab}), p, \gamma)$ satisfies the *bounded blueshift condition* if (i) there exists a timelike curve γ' with future endpoint p such that there is a future-directed null geodesic K_i from each q_i to some $q_i' \in \gamma'$ and (ii) there is a $\delta > 0$ such that, for all i, the quantity $(V'^a k_a')_{|q_i'} / (V^a k_a)_{|q_i}$ is less

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[3]For example, it is not always possible to find a family of null geodesics connecting points in the worldline of the sender to the worldline of the receiver. In addition, even assuming that such a family of null geodesics exists, there is no guarantee that they will form an integral submanifold or that the order of emission from the sender will match the order of reception by the receiver.

than δ where $V^a_{|q_i}$ and $V'^a_{|q'_i}$ are, respectively, the unit tangent to γ at q_i and the unit tangent to γ' at q'_i and $k^a_{|q_i}$ and $k'^a_{|q'_i}$ are, respectively, the tangents to K_i at q_i and q'_i.

We want to emphasize that it is not clear from the outset that any M-H setups satisfy the bounded blueshift condition. In fact, citing their proposition and numerous examples, Earman and Norton seem to think that the divergent blueshift problem is essentially inescapable [3, p. 33].

Next, we consider a third worry given in [3]. Some spacetimes, such as the toy example mentioned above, do not satisfy the so-called "energy conditions". For each spacetime (M, g_{ab}), we associate via Einstein's equation, a stress-energy tensor T_{ab} defined on all of M. The energy conditions are constraints on placed on T_{ab} which are thought to be satisfied by all physically reasonable matter sources. We will define two of these energy conditions here.[4]

Definition A spacetime (M, g_{ab}) satisfies the *dominant energy condition* if, for all future directed timelike vectors ξ^a, then $T^a_b \xi^b$ is a future directed causal vector where T_{ab} is the stress-energy tensor associated with (M, g_{ab}).

Definition A spacetime (M, g_{ab}) satisfies the *strong energy condition* if, for all unit timelike vectors ξ^a, then $T_{ab}\xi^a\xi^b \geq -\frac{1}{2}T^a_a$ where T_{ab} is the stress-energy tensor associated with (M, g_{ab}).

Next, we note that in some M-H setups $((M, g_{ab}), p, \gamma)$, the curve γ is a geodesic but that this is not always the case. Consider a setup generated from anti-de Sitter spacetime.

Example Let t and x be standard coordinates on $\mathbb{R}^2$. Consider the spacetime $(\mathbb{R}^2, g_{ab})$ where $g_{ab} = \cosh^2 x \nabla_a t \nabla_b t - \nabla_a x \nabla_b x$. Here, the null cones rapidly widen as x increases in absolute value. Thus, for any point $p \in \mathbb{R}^2$, there are future-directed timelike curves γ in $I^-(p)$ with infinite proper time (see Fig. 3).

In the example above, it can be shown that not only is γ not a geodesic, but its total (integrated) acceleration is infinite [3, p. 36]. Given such examples, Earman and Norton demand a finite bound on the total acceleration of γ. This seems sensible—a rocket ship which carries a finite amount of fuel cannot traverse a curve with infinite total acceleration (assuming it cannot refuel along the way) [1]. So, we have the following.

Definition We say that the M-H setup $((M, g_{ab}), p, \gamma)$ satisfies the *finite acceleration condition* if $\int_\gamma a\, d\tau < \infty$ where a is the scalar value of acceleration defined at each point in γ.

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[4]A third energy condition, not defined here is called the "weak energy condition" and is satisfied whenever the dominant energy condition is. For more on the energy conditions, see [3].

Fig. 3 Anti-de Sitter spacetime
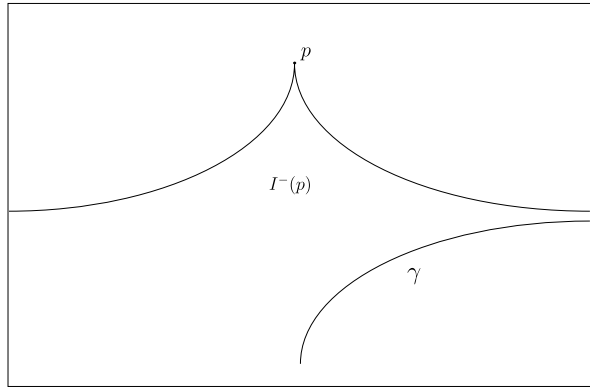
In an M-H setup $((M, g_{ab}), p, \gamma)$, the sender γ is represented by the worldline of a point particle. However, it has been emphasized that such representations are not realistic [3]. Instead of a single worldline γ, one must be concerned with a congruence Γ of such worldlines which more appropriately represents the sender. But, if the sender is to carry out her tasks, we must verify that the tidal forces on such a congruence Γ remain bounded. Associated with each spacetime (M, g_{ab}) is a Riemann curvature tensor R^a_{bcd} defined on all of M. Because the tidal forces are proportional to the Riemann curvature tensor, we feel the following constraint is a reasonable one to place on spacetime:

Definition A spacetime (M, g_{ab}) satisfies the *tidal force condition* if there exists a $k > 0$ such that the Kretschmann curvature scalar $R_{abcd} R^{abcd}$ is less than k and greater than $-k$ at all points in M.

Earman and Norton have a sixth worry. They contend that the receiver in an M-H setup can *always* question the reliability of the signal sent by the sender. The concern has its roots in the following precise claim [3, p. 37].

Claim Let $((M, g_{ab}), p, \gamma)$ be an M-H setup. If $\Sigma \subset M$ is a spacelike hypersurface such that $\gamma \subset I^+(\Sigma)$, then $p \notin \text{int}[D^+(\Sigma)]$.[5]

The idea is that Σ is a spacelike surface from which the physical situation on all of $D^+(\Sigma)$ can be determined. If γ were in $D^+(\Sigma)$, then initial conditions on Σ would guarantee the completion of the sender's tasks and signals. Clearly, $\gamma \subset D^+(\Sigma)$ implies $\gamma \subset I^+(\Sigma)$. However, as Earman and Norton point out, if $p \notin \text{int}[D^+(\Sigma)]$ then "events at p or at points arbitrarily close to p are subject to nondeterministic influences" [3, p. 38]. So, if the above claim is true, there would always be room for the

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[5]Here, as in [3, p. 35], we define the future domain of dependence $D^+(\Sigma)$ of Σ as the set of all points $p \in M$ such that every causal curve which passes through p without past endpoint meets Σ. This agrees with the formulation given in [9].

receiver to doubt the reliability of the sender's signal. Earman and Norton have of-
fered a proof of the claim so it would seem that the matter has been settled. However,
we note here that the claim is false as presented.[6]

The formulation of the claim can be altered so as to make it true by assuming that
Σ is achronal; it is likely that this was the intended theorem. But we note that this
additional assumption significantly weakens the result and we question its imposition.

In some texts, the future domain of dependence $D^+(\Sigma)$ of a set Σ is only defined
when Σ is achronal [16]. But, we want to make it very clear that this is done for
mathematical convenience only [5, p. 438]. The domain of dependence can be and,
in places such as [3] and [9], has been defined for arbitrary sets. So, we emphasize
that the achronality of Σ is *not* implicit in the formulation of the claim above.

One attempt to justify the achronality assumption is to argue that non-achronal
spacelike hypersurfaces somehow signal causal misbehavior. But such an argument
is inadequate. While it is true that some causally delinquent spacetimes do not admit
any achronal spacelike hypersurfaces (e.g. Gödel spacetime), *every spacetime* has
innumerable spacelike hypersurfaces which are *not* achronal.[7] So the existence of
such surfaces in no way flags causal mischief.

Besides, we argue that any worries concerning unruly causal structure should be
formulated as a condition satisfied by the whole of spacetime. We have, for example,
already stipulated that (for our purposes) an M-H spacetime must be stably causal to
be considered physically reasonable. In our opinion, this seems to be enough to rule
out causal pathologies.

Consequently, we feel that the claim above can be used to formulate a condition
on M-H setups ensuring signal reliability. We have the following.

Definition The M-H setup $((M, g_{ab}), p, \gamma)$ satisfies the *signal reliability condi-
tion* if there is a spacelike hypersurface $\Sigma \subset I^-(p)$ such that $\gamma \subset D^+(\Sigma)$ and
$p \in \text{int}[D^+(\Sigma)]$.

Here, we have required that not only γ be in $I^+(\Sigma)$ but that it be in $D^+(\Sigma)$ as
well. We have also insisted that Σ be in $I^-(p)$. Together, these constraints ensure
that the receiver at p has the resources to determine the behavior of γ. In addition,
we demand that $p \in \text{int}[D^+(\Sigma)]$. This secures the determination, from p, of events
sufficiently close to p. Hence, the reliability of the sender's signal is guaranteed.

Clearly, the rolled Minkowski spacetime example given above satisfies the signal
reliability condition. However, it is not clear from the outset that the condition is
satisfied by any causally well-behaved M-H setup. All examples considered in [3] do
not. This includes Reissner-Nordström spacetime.

Example (Reissner-Nordström spacetime)[8] This spacetime represents a spherically
symmetric electrically charged body (see the conformal diagram given in Fig. 4). In

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[6]For a counterexample, the interested reader may consult the proposition below.

[7]For an example in Minkowski spacetime, see [8, p. 246].

[8]Details can be found in [9, pp. 156–161].

Fig. 4 Conformal diagram of Reissner-Nordström spacetime
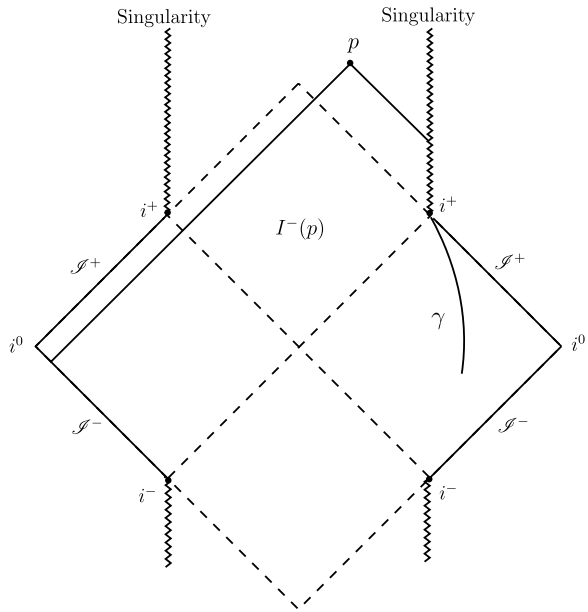
the diagram, the future-directed timelike curve γ approaches the singularity at i^+ and is such that $\int_\gamma d\tau = \infty$. However, the point p is such that $\gamma \subset I^-(p)$.

3 An Example M-H Setup

We have outlined seven conditions regarding the physicality of M-H setups. These conditions seem to be substantial in that no M-H setup presented in the literature escapes them all. Accordingly, one might be tempted to conclude that there do not exist physically reasonable M-H setups. However, we suggest here that such a conclusion may be unwarranted. Indeed, we now present a spacetime which avoids *all* of the problems discussed above. In lieu of a formal existence proof of such a model, we offer an argument sketch accompanied with diagrams which, we hope, will better convey the basic structure of the example in mind.

Proposition *There exists an M-H setup $((M, g_{ab}), p, \gamma)$ such that all of the following hold*:

  (i) *(M, g_{ab}) is inextendible.*
 (ii) *(M, g_{ab}) is stably causal.*
(iii) *$((M, g_{ab}), p, \gamma)$ satisfies the bounded blueshift condition.*
 (iv) *(M, g_{ab}) satisfies the dominant and strong energy conditions.*
  (v) *$((M, g_{ab}), p, \gamma)$ satisfies the finite acceleration condition.*
 (vi) *(M, g_{ab}) satisfies the tidal force condition.*
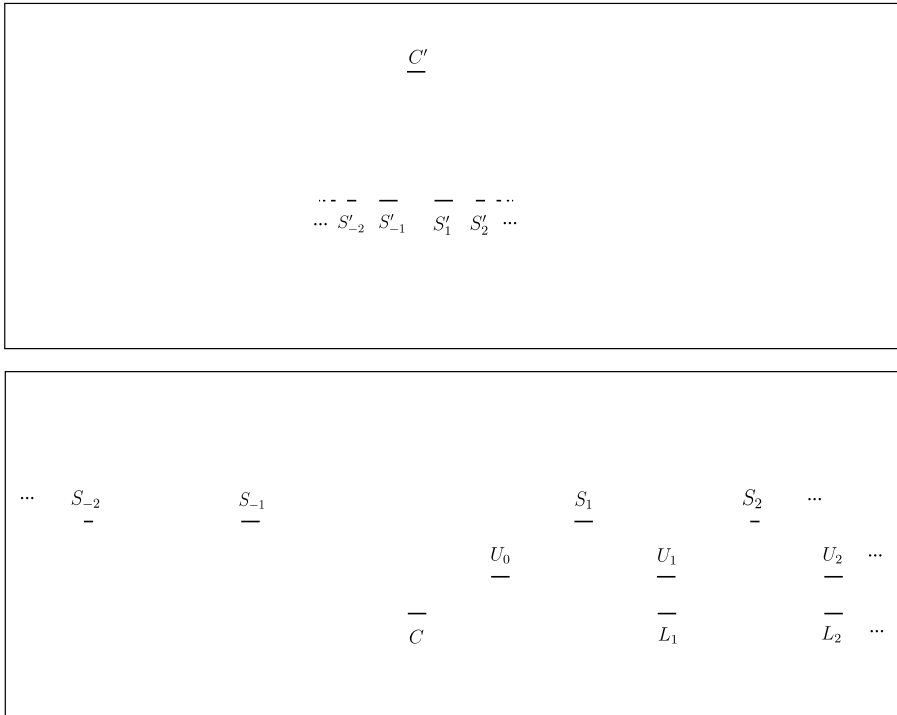(vii) *$((M, g_{ab}), p, \gamma)$ satisfies the signal reliability condition.*

Fig. 5 The spacetime above is $(M', \eta'_{ab|M'})$. The spacetime below is $(M, \eta_{ab|M})$ where, for each $i \in \mathbb{N}$, the upper edge of U_i is identified with the lower edge of L_{i+1} and vice versa. The two spacetimes are connected to make (M'', g_{ab}) by making the following identifications: For all $j \in \mathbb{Z}$, identify the upper edge of S_j with the lower edge of S'_j and vice versa. Make similar identifications with the sets C and C'. Drawn to scale

Proof Sketch Here, we provide a two-dimensional example. It is not difficult to see that the example generalizes to spacetimes of dimension four.

Let $(\mathbb{R}^2, \eta_{ab})$ be Minkowski spacetime in standard t, x coordinates (here $\eta_{ab} = \nabla_a t \nabla_b t - \nabla_a x \nabla_b x$). Now, cut slits S_i, U_i, L_i, and C as shown in the bottom portion of Fig. 5. For each $i \in \mathbb{N}$ identify the upper edge of U_i with the lower edge of L_{i+1} (excluding boundary points). Similarly, identify the lower edge of U_i with the upper edge of L_{i+1}. Let $(M, \eta_{ab|M})$ be the resulting spacetime.

Now consider another copy of Minkowski spacetime $(\mathbb{R}^2, \eta_{ab})$. Now, cut slits S'_i and C' as shown in the top portion of Fig. 5. Let $(M', \eta_{ab|M'})$ be the resulting spacetime. Now we attach the spacetimes $(M, \eta_{ab|M})$ and $(M', \eta_{ab|M'})$ as follows: For all $j \in \mathbb{Z}$, identify the upper edge of S_j with the lower edge of S'_j (excluding boundary points). Similarly, identify the lower edge of S_j with the upper edge of S'_j. Make similar identifications with the sets C and C'. Let the resulting spacetime be (M'', g_{ab}).

Let p and γ be as shown in Fig. 6. Note that γ continues until the lower edge of U_0 which, by construction, is identified with the upper edge of L_1. Similarly, the curve γ continues until it reaches the lower edge of U_1 which is identified with the upper edge of L_2. The process continues ad infinitum. Thus, it is clear that $\int_\gamma d\tau = \infty$. It
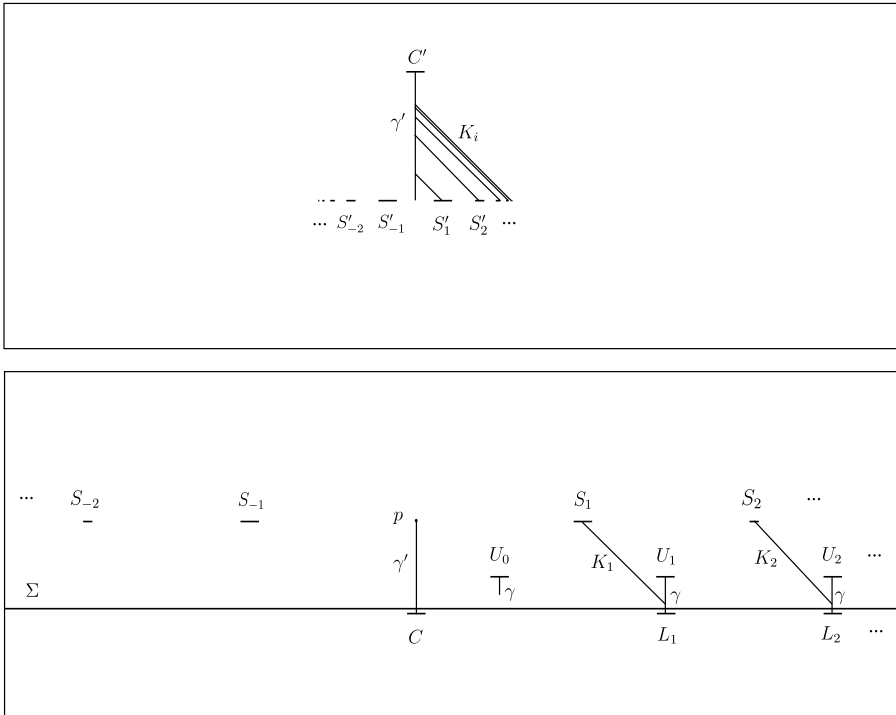
Fig. 6 The spacetime (M'', g_{ab}) with $p, \gamma, \gamma', \Sigma$, and K_i indicated. Drawn to scale

can be verified that there exists a future directed timelike curve from every point in γ to the lower edge of S_j for some j. So there is a future directed timelike curve from any point in γ to the upper edge of S'_j for some j. But clearly, the lower edge of C, and hence the upper edge of C' is in $I^-(p)$. So, the upper edge of every S_j is in $I^-(p)$. So, $\gamma \subset I^-(p)$ and (M'', g_{ab}) is an M-H spacetime.

Now we show that $((M'', g_{ab}), p, \gamma)$ satisfies conditions (i)–(vii). By construction, (M'', g_{ab}) is inextendible and locally carries the Minkowski metric. So, (i), (iv), and (vi) are easily satisfied. Because the curve γ is a geodesic, condition (v) is satisfied as well. Let Σ be the spacelike hypersurface as shown in Fig. 6. Every past-inextendible causal curve from γ intersects Σ. So, $\gamma \subset D^+(\Sigma)$. One can also note that, for any point in Σ, there is a future-directed timelike curve from that point to some S_j or to p. So, this implies that $\Sigma \subset I^-(p)$. Also, it an be easily verified that $p \in \text{int}[D^+(\Sigma)]$. So, (M'', g_{ab}) satisfies condition (vii).

Let K_i and γ' be as shown in Fig. 6. Without too much trouble, it is possible to verify that $(V'^a k_a)/(V^a k_a) = 1$ where V^a and V'^a are, respectively, the unit tangent to γ and γ' and k^a is the tangent to the K_i. So $((M'', g_{ab}), p, \gamma)$ satisfies condition (iii). Note also that the order of signal transmission matches the order of signal reception.

Finally, one can verify that (M'', g_{ab}) is stably causal therefore satisfying condition (ii). Here, we simply show that it contains no closed timelike curves. We first

claim that any future-directed timelike curve γ_s with past endpoint $s \in M'$ cannot return to s. It is easy to see that if γ_s never leaves M', it cannot return to s. And if γ_s does leave M', it must do so in one of two ways: either through one of the S_i' or through C'. Clearly, if γ leaves through C' or one of the S_i', and hence enters M, it can never leave M. So, there is no closed timelike curve through any point in M'. Now consider any point $s \in M$. We also claim that any future-directed timelike curve γ_s with past endpoint $s \in M$ cannot return to s. Because of the previous argument, such a curve cannot enter C or any S_i and be closed. If γ_s enters some L_i, then it must exit some U_i and can never return to any point to the past of any L_i (including s). If γ_s enters some U_i, it may exit through any L_j where $i < j$. But, for all $i < j$ the timelike future of L_j does not contain U_i. So, γ_s cannot return to s. It is clear that any γ_s which remains in M but does not enter any U_i or L_i cannot be closed.   □

4 Conclusion

We want to make it very clear that we do not feel that the example just presented is physically reasonable. We simply emphasize that the potential problems outlined above fail to rule it out. In addition, it must be remembered that the proposition can have physical relevance even if the particular cut and paste model constructed in the proof sketch does not. Geroch explains:

> The space-times obtained by cutting and patching are not normally considered as serious models of our universe. However, the mere existence of a space-time having certain global features suggests that there are many models—some perhaps quite reaonable physically—with similar properties [6, p. 78].

Of course, one can consider other requirements in addition to those presented here. Here, we briefly note some of these.

One way of ruling out our example above is to insist that spacetime be "hole-free" [7]. Intuitively, this condition guarantees that, for any spacelike surface Σ, the set $D^+(\Sigma)$ is "as large as it can be". We note here, however, that it is not clear that all physically reasonable spacetimes are hole-free [12, 14]. But, even granting this assumption, it is not as if a no-go theorem is then straightforwardly obtained.

A different sort of objection is hinted at by Earman and Norton. Following [15], they believe that although the sender, by construction, has "time enough" to complete its tasks, a physically reasonable M-H setup must guarantee that there is "world enough" as well. The problem is that a computational device with a limited amount of computational space does not seem to have the resources to complete a supertask [3]. We acknowledge that this concern deserves attention, but for now, it is unclear how exactly to formalize such a worry.

Next, Norton[9] has pointed out the possibility of energy related problems even for M-H setups satisfying the bounded blueshift condition. The concern is that if the receiver, in a finite time, receives an infinite number of signals from the sender and if each signal has the same energy on arrival, then the total energy received may

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[9]Private communication.

diverge. This may well be a serious problem. But, we also note that if the M-H setup satisfies the signal reliability condition, the sender need not send an infinite number of signals to the receiver in order to complete certain supertasks—at most one is needed. Imagine the sender checking a conjecture of number theory for '1', for '2', etc. If a counterexample to the conjecture is found, the sender sends one signal to the receiver. Otherwise, no signal is ever sent.[10] Since the signal (or non-signal as the case may be) is trustworthy in the sense of the signal reliability condition, the receiver would seem to be able to settle the conjecture.

Finally, one might insist that the receiver must "bring about" the supertask instead simply finding it already completed. Consider an M-H setup $((M, g_{ab}), p, \gamma)$. In order for a supertask is to be completed at p, then at some point q (where $\gamma \cup \{p\} \subset I^+(q)$), the receiver must manufacture and initialize a computational device. But, it seems there would be no reason whatsoever for the receiver to manufacture and initialize the device at q if there was no assurance at q that a supertask at may be completed at p. In other words, it seems that part of what it means to say that the receiver brings about a supertask is that she is be able to predict, from q, the relevant spacetime structure to the causal future of q.[11] And, given that (M, g_{ab}) necessarily fails to be globally hyperbolic, the receiver seems to be incapable of such predictions. (We note, however, that there is a sense in which every observer in every spacetime is unable to make sure predictions concerning her casual future [13].)

This objection is certainly significant as a practical matter but we argue that it does not undermine the physical implementability of supertasks. It may be that the receiver at q, not knowing with certainty the spacetime structure to her casual future, decides to manufacture and initialize a computational device anyway. She simply bets that at some later point p, the intended supertask may be completed. If her hopes are realized, wouldn't we want to say that she, in some way, brought about the outcome?

In sum, there are indeed other worries concerning the physical reasonableness of M-H setups. But, given the resilience of examples like the one presented above, it is not yet clear that such potential problems will turn out to prohibit the physical implementability of supertasks.

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[10]This setup is suggested in [3, p. 29].

[11]I am grateful to Robert Geroch for this point.

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