On investment decisions in liberalized electricity markets : the impact of price caps at the spot market

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On investment decisions in liberalized electricity markets: the impact of price caps at the spot market

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Abstract

We analyze the impact of a uniform price cap at electricity spot markets on firms investment decisions and on welfare. Since investment decisions for those markets are taken in the long run, fluctuating demand at the spot market eventually gives rise to high price spikes in case of binding capacities. Those price spikes are considered to send accurate signals for investment in generation capacities, limiting those spikes by price caps is thought to reduce firms' investment incentives.

We are able to show that this is not true for the case of strategic investment behavior. More specifically we analyze a market game where firms choose capacities prior to a spot market which is subject to fluctuating or uncertain demand. We derive, that appropriately chosen price caps do always increase firms investment incentives under imperfect competition. We furthermore characterize the optimal price cap. Based on the theoretical framework, we empirically analyze the impact of uniform price caps on the German electricity market.

Keywords: Investment incentives, price caps, fluctuating demand, electricity markets.

JEL Classification: D43, L13, D41, D42, D81

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1 Introduction

In recent years price caps have been proposed (and also been used) in order to combat the exercise of market power in liberalized electricity markets.[1] Practitioners and policy makers seem to perceive them as a powerful tool to discipline market power. In the words of a Federal Energy Regulatory Commissioner: "If you cap those prices, you eliminate any incentive to withhold ... you may well sell into the market at a capped price as long as you're covering your running cost and making a reasonable profit".[2] Traditional theoretical results confirm this view. In a static framework, price caps have been shown to hinder the exercise of market power[3], since they reduce firms incentives to manipulate prices by withholding production. When evaluating the impact of price caps in electricity markets, however, not only competitiveness at the spot market, for given investment decisions matter. Prior to producing electricity, firms have to build generation facilities (power plants). When evaluating the desirability of price caps at the spot market firms' incentives to invest in such facilities have to be taken into account, as an increasing number of experts in the field points out.

At electricity spot markets, firms face highly fluctuating demand, which has to be served by installed capacities. This gives rise to highly volatile prices at those markets, typically they range from 0€/MWh to more than 1000€/MWh[4]. As many experts argue, especially in hours of high demand and tight supply, electricity markets might very likely be subject to the exercise of market power.[5] Price caps have thus been proposed and are currently used in order to limit market power and reduce extreme price spikes at electricity wholesale markets. In the short run, for fixed investment choice, those price caps indeed are welfare enhancing. However, when taking into account firms investment decisions in liberalized electricity markets, desirability of those price caps is much less clear cut and the questions involved are not yet well explored in the literature.

[1]See e.g. Stoft (2002), for the case of electricity markets. They have also been proposed in other liberalized utilities, first by Littlechild (1983) during the British Telecom privatization.

[2]Cf. Walsh (2001).

[3]This is true both for the case of symmetric information between firms and regulator regarding cost of production as analyzed recently by Pint (1992), or Biglaiser and Riordan (2000), but also under asymmetric information as analyzed in the literature on incentive regulation, for a survey of the seminal contributions see Laffont and Tirole (1993), or Laffont and Tirole (2001).

[4]E.g. wholesale prices at the German market ranged from 0€/MWh to 2489€/MWh in 2006.

[5]Exercise of market power at electricity spot markets has received much attention in the literature, compare e.g. Borenstein, Bushnell and Wolak (2002), or Joskow and Kahn (2002) for California, Wolfram (1999) for the United Kingdom, or Schwarz and Lang (2006) for Germany.

Recently economists have seriously questioned the desirability of price caps in electricity markets, arguing that they considerably distort firms investment decisions. Their argument is clear cut, originating in the findings of the peak load pricing literature[6]: In a market without price caps, high price spikes during hours of high demand and scarce supply do send accurate price signals for investment decisions. Very high price spikes signal scarcity of supply and will thus lead to increased investment activities. In equilibrium market prices will be such as to sustain an optimal investment level. Price caps at the spot market distort those price signals and lower firms investment incentives, putting seriously into question the desirability of such price caps. In the literature this is often referred to as a problem of "missing money". As Paul Joskow (2007) summarizes in a recent contribution: "As a result, many economists assume that the primary source of the missing money problem must be the price caps and related market power mitigation procedures imposed by regulators. That is, that the efforts to mitigate market power have had the effect of suppressing energy prices too much, especially during scarcity conditions when prices should be very high." Indisputably, taking into account the feed back of a price cap for spot market prices on firms investment choice is indispensable when evaluating its desirability. The above argument is not based on the analysis of strategically interacting firms making their investment decisions, but relies on results obtained under the assumption of perfectly competitive investment behavior[7].

If firms are assumed to behave strategically when producing at the spot market (only then the use of price caps is justifiable, compare Joskow, and Tirole (2007)), it seems most natural to consider firms to also behave strategically when making their investment decisions. That is, we analyze strategic firms' investment incentives prior to a spot market with fluctuating or uncertain demand (giving rise to price spikes and scarcity pricing) and the impact of a uniform price cap at that spot market. As we will show, appropriately chosen price caps (contrary to what has is currently known for the case of perfect competition) do always increase strategic firms' investment decisions and overall welfare.

To the best of our knowledge, this has not yet been analyzed in the literature. Most of the contributions on price caps analyzes static frameworks, among those most focus on the analysis of cost uncertainty. The case of symmetric information between firms and regulator regarding cost of production have been analyzed for example by Pint (1992), or Biglaiser and Riordan (2000). The literature on incentive regulation analyzes the case of

[6] For a survey see Crew and Kleindorfer(1986)

[7] The above argument basically rephrases the well known marginal revenue equals marginal cost for the case of investment decisions.

asymmetric information in a static framework, for a survey of the seminal contributions see Laffont and Tirole (1993), or Laffont and Tirole (2001). All those approaches do not take into account firms investment incentives prior to competing on the regulated spot markets. As Cowan (2002) and more recently Joskow and Tirole (2007), or Hogan (2006) point out, those incentives are crucial, however, when evaluating the desirability of such price caps, since they cap price spikes in those markets and thus distort investment decisions.

Another strand of literature focuses on optimal timing of investment decisions in stochastic continuous time models. The seminal contributions of Dixit (1991) and Dixit and Pindyk (1994), for the case of perfect competition, has been extended by Dobbs (2004), and in a recent contribution Roques and Savva (2006) analyze the case of strategically interacting firms. In this literature, demand typically follows a Brownian process, capacities are assumed to be always fully utilized[8]. Roques and Savva (2006) show that strategic firms delay their investment decisions, relative to the case of perfect competition (that is, the price trigger for capacity addition is higher under strategic interaction, than for a perfectly competitive industry). They find that introduction of a price cap can reduce this problem and show that the optimal price cap in their framework is given by the investment trigger–price of a competitive industry. Development of industry demand in those models of optimal timing is best interpreted as changes in the level of average market demand where changed price levels trigger instantaneous capacity adjustment. Those contributions thus provide very rich insights on the dynamical timing of strategic firms' investment decisions. They abstract, however, from scarcity pricing and price spikes due to fluctuating demand at the spot markets as analyzed in our contribution.

We consider a general two stage framework where firms first decide on investment in capacities, and then compete at the spot market. Demand at the spot market is either uncertain or the good is economically non–storable and demand fluctuates. We examine the effect of a price cap at the spot market on firms investment incentives in an oligopolistic industry. Our analysis shows, that under imperfect competition[9] accurately chosen price caps are always desirable. In particular, we identify a range of high price caps that always lead to an increase of investment and welfare as compared to the situation without price cap. We then derive the optimal price cap and show that it can be uniquely characterized under rather weak regularity conditions. We show that the optimal cap is increasing in the

[8] This allows to disregard interaction at the spot market.

[9] As we approach the case of perfect competition our results match the already known finding that any binding price cap reduces both Investment and overall welfare.

number of firms on the market.

After having derived the theoretical results, we fit the model to the German electricity market in order to also empirically contribute to the debate on price caps in electricity markets. We find that price caps below 260 €/MWh increase industry capacity for any market structure based on up to 20 firms. Especially for concentrated markets (up to 4 firms) surprisingly low price caps ranging at around 130 €/MWh lead to a dramatic increase of industry capacities (for the case of duopoly capacity increases by 30%). Price caps maximizing overall welfare are even lower, for concentrated markets (up to 4 firms) they range at around 110 €/MWh. Both our theoretical and our empirical results thus suggest that for appropriately chosen price caps, welfare can be increased both from a short run and a long run perspective.

The article is organized as follows: In section 2 we introduce the model. Section 3 contains our results. In section 3.1 we show existence and uniqueness of the equilibrium, section 3.2 characterizes desirable and section 3.3 optimal price caps. In section 4 we empirically analyze capacity choice in the German electricity market. Section 5 concludes.

2 The Model

We analyze a two stage market game where firms have to choose capacities under demand uncertainty, and make output choices after market conditions unraveled. Market prices are subject to a price cap $\bar{p}$. We denote by $q = (q_1, \ldots, q_n)$ the vector of outputs of the n firms, and by $Q = \sum_{i=1}^{n} q_i$ total quantity produced in the market.

Inverse Demand is given by the function $P(Q, \theta)$, which depends on $Q \in \mathbb{R}^+$, and the random variable $\theta \in \mathbb{R}$ which represents uncertainty. The random variable $\theta \in \mathbb{R}$ is distributed according to a distribution $F(\theta)$ with bounded support $\theta \in [\underline{\theta}, \overline{\theta}]$.[10] All firms have the same constant[11] marginal production cost in any demand scenario, denoted by c.

We denote by $\overline{Q}(\theta)$ the quantity where Prices start to be zero.[12] The following two assumptions on demand and cost for each realization of uncertainty $\theta \in \mathbb{R}$ have to be

[10]While F has bounded support, it will be convenient to assume that $P(Q, \theta)$ is defined for all $\theta \in \mathbb{R}$ and $Q \in \mathbb{R}_+$.

[11]The existence and uniqueness results of this article obtain also for non–decreasing marginal cost functions, it would considerably increase the notational burden however, since marginal cost could hit price-caps for some output levels but not for others.

[12]In case prices remain positive we can set $\overline{Q}(\theta) = \infty$. In order to ensure a bounded solution we then have to assume $\lim_{Q \to \infty} P(Q, \theta) < c$ for each $\theta \in (-\infty, \infty]$.

satisfied only for quantities $0 \leq q_i \leq Q < \overline{Q}(\theta)$.

Assumption 1 *(i) Inverse demand $P(Q,\theta)$ is twice continuously differentiable*[13] *in Q with $P_q(Q,\theta) < 0$ and $P_q(Q,\theta) + P_{qq}(Q,\theta)q_i < 0$.*

(ii) $P(Q,\theta)$ is differentiable in θ, and $P_\theta(Q,\theta) > 0$.

(iii) $P(Q,\theta)q_i$ is (differentiable) strict supermodular in q_i and θ, i. e. $P_\theta(Q,\theta) + P_{q\theta}(Q,\theta)q_i > 0$.

The situation we want to analyze is captured by the following two stage game. At stage one firms simultaneously build up capacities $x = (x_1, \ldots, x_n)$. Capacity choices are observed by all firms. Cost of investment $K(x_i)$ is the same for all firms and satisfies

Assumption 2 (Investment Cost) *Investment cost $K(x_i)$ is twice continuously differentiable, with $K_x(x_i) \geq 0$ and $K_{xx}(x_i) \geq 0$.*

At stage two, facing the capacity constraints inherited from stage one, firms simultaneously choose outputs at the spot market. Since demand uncertainty unravels prior to the output decision, produced quantities depend on the realized demand scenario. We assume that in case the price cap is binding the firms' sales are as equal as possible.[14] We denote individual quantities produced in demand scenario θ by $q(\theta) = (q_1(\theta), \ldots, q_n(\theta))$, and the aggregate quantity by $Q(\theta) = \sum_{i=1}^n q_i(\theta)$.

Finally, we state firm i's stage one expected profit from operating in a market with price cap $\bar{p}$ if capacities are given by x and firms plan to choose feasible[15] production schedules $q(\theta)$ for all θ.

$$\pi_i(x, q, \bar{p}) = \int_{\underline{\theta}}^{\overline{\theta}} [\min\{\bar{p}, P(Q(\theta), \theta)\} - c]\, q_i(\theta)\, dF(\theta) - K(x_i). \tag{1}$$

We consider only those cases where, in absence of a price cap, investment is *gainful*, i. e. $K_x(0) < \mathbf{E}_\theta[P(0,\theta) - c]$. Note whenever this condition is not satisfied, firms do not invest in capacity, independently of price caps at the spot markets.

Furthermore we will consider only those price caps which give rise to positive capacity choice. For price caps very close to marginal cost of production, firms will not find it

[13] We denote the derivative of a function $g(x,y)$ with respect to the argument x, by $g_x(x,y)$, the second derivative with respect to that argument by $g_{xx}(x,y)$, and the cross derivative by $g_{xy}(x,y)$.

[14] Note that this does not imply that no asymmetric equilibria of the game exist. It does imply, however, that *if* firms *produce* equal quantities, they always *sell* equal quantities if the price cap is binding.

[15] That is, $0 \leq q_i(\theta) \leq x_i$ for all $\theta \in [-\infty, \infty]$, $i = 1, \ldots, n$.

profitable to invests, since they cannot recover cost of investment. We thus restrict attention to price caps $\bar{p} \geq \bar{\rho}^0 \geq c$. The lowest price cap which yields positive[16] investment $\bar{\rho}^0$ is defined by:

$$\bar{\rho}^0 = \left\{\bar{p} : \mathbf{E}_\theta[\min\{\bar{\rho}^0, P(0,\theta)\} - c] = K_x(0)\right\}$$

We are interested in pure strategy Nash equilibria of the game. For any fixed price cap $\bar{p}$ we denote an equilibrium by $x^*(\bar{p})$ and the corresponding industry capacity by $X^*(\bar{p})$.

3 Results

This section contains all our results. When we analyze welfare effects we will refer to total welfare (W) and consumer welfare (CW), which is calculated under the assumption of efficient rationing of consumers.

3.1 Equilibrium Analysis

In this section we characterize equilibrium capacities in the market game under demand uncertainty. Moreover, we characterize the solution under perfect competition.

Theorem 1 (Equilibrium Investment) *The market game has a unique equilibrium $x^*(\bar{p})$ which is symmetric. It is uniquely characterized by*

$$X^*(\bar{p}) = \left\{ X : \int\limits_{\tilde{\theta}^X(x,\bar{p})}^{\tilde{\theta}^{\bar{p}}(x,\bar{p})} \left[P\left(X,\theta\right) + P_q\left(X,\theta\right)\frac{X}{n} - c \right] dF\left(\theta\right) + \int\limits_{\tilde{\theta}^{\bar{p}}(x,\bar{p})}^{\overline{\theta}} \left(\bar{p} - c\right) dF(\theta) = K_x\left(\frac{X}{n}\right) \right\}, \tag{2}$$

where $\tilde{\theta}^X(x,\bar{p}) \in [\underline{\theta}, \overline{\theta}]$ is the demand scenario from which on firms are capacity constrained and $\tilde{\theta}^{\bar{p}}(x,\bar{p}) \in [\underline{\theta}, \overline{\theta}]$ is the demand scenario when capacity and the price cap start to be binding.

Proof See appendix 5. □

In order to provide some intuition for the equilibrium–characterization in theorem 1, we first determine the range of relevant price caps, that is, those price caps which are binding at least for some realizations of demand. We denote by $\bar{\rho}^\infty$ that price cap which in

[16] To be more precise $\bar{\rho}^0$ yields zero investment, only slightly higher price caps yield positive investment.

equilibrium is reached [17] only for the highest demand realization $\overline{\theta}$, it is defined by:

$$\bar{\rho}^{\infty} = \left\{ \bar{p} : \bar{p} = P(X^*(\bar{p}), \overline{\theta}) \right\}.$$

All higher price caps $\bar{p} \geq \bar{\rho}^{\infty}$ are never binding, in other words, the demand realization where the price cap starts to be binding $\tilde{\theta}^{\bar{p}}(x, \bar{\rho}^{\infty})$ is at or above the range of relevant demand realizations.[18] Formally this corresponds to $\tilde{\theta}^{\bar{p}}(x, \bar{\rho}^{\infty}) = \overline{\theta}$, which leads to the elimination of the second term on the LHS of (2). For the absence of price caps, i.e never–binding price caps, the resulting market game has already been analyzed in Grimm Zoettl (2008), the first order condition have a straightforward intuition: The LHS is expected marginal revenue of capacity choice X, whereas the RHS is just marginal cost of capacity choice. Note that on the LHS expectation is taken only over those demand scenarios where the additional unit installed would actually be used (i. e. the lower limit of integration is $\tilde{\theta}^X(x)$, from where on capacity is binding, and not $\underline{\theta}$). The reason is that additional capacity only contributes to marginal revenue in scenarios where it is actually used.

We thus observe that only price caps in the range $\bar{p} \in [\bar{\rho}^0, \bar{\rho}^{\infty}]$ yield truly interior solutions where firms in equilibrium choose positive capacities and the price cap is binding for some realizations of demand. Characterization of the corresponding equilibrium in expression (2) again equates expected marginal revenue of investment with its marginal cost. As in the case without binding price cap, when calculating expected marginal revenue, the firm considers only those demand scenarios where the additional capacity is actually being used (i.e. integration starts at $\tilde{\theta}^X$). Marginal revenue in scenario θ is given by the well known expression $(P - P_q x - c)$ until the price cap is binding at $\theta^{\bar{p}}$, from there on, marginal revenue equals to $(\bar{p} - c)$. Whenever the price cap starts to be binding before capacity is binding, the left integral of the first order condition (2) cancels, since $\tilde{\theta}^x(x) = \tilde{\theta}^{\bar{p}}(x, \bar{\rho}^{\infty})$. Then firms choose their capacities such that the expected value of $\bar{p} - c$ over all demand realizations where capacity is binding equals to marginal cost of investment K_x.

[17] To be more precise, $\bar{\rho}^{\infty}$ is never binding, but slightly lower price caps are, for high demand realizations.

[18] Remember θ has been defined on $\mathbb{R}$, the distribution of uncertainty has the support $[\underline{\theta}, \overline{\theta}]$, however.

After having derived the market solution for strategic firms we now state the benchmark case of perfect competition.

THEOREM 2 (EQUILIBRIUM UNDER PERFECT COMPETITION) *In a perfectly competitive market the unique industry capacity choice $\widehat{X}(\bar{p})$ is characterized by:*

$$\widehat{X}(\bar{p}) = \left\{ X : \int_{\hat{\theta}^{X}(x,\bar{p})}^{\hat{\theta}^{\bar{p}}(x,\bar{p})} \left(P\left(X,\theta\right)-c\right) dF\left(\theta\right) + \int_{\hat{\theta}^{\bar{p}}(x,\bar{p})}^{\bar{\theta}} \left(\bar{p}-c\right) dF(\theta) = K_x\left(\frac{X}{n}\right) \right\},$$

where $\hat{\theta}^{X}\left(x\right)$ is the demand scenario where capacity and $\hat{\theta}^{\bar{p}}\left(x,\bar{p}\right)$ the scenario where both price cap and capacity are binding.
In a perfectly competitive industry, the welfare maximum is implemented without price cap at the spot market. All binding price caps strictly reduce capacity and total welfare.

PROOF See appendix 5 □

It is worthwhile to notice that under perfect competition the price cap can only be met if production is already binding (that is, if prices do not remain at marginal cost level). In the oligopoly case, on the contrary, equilibrium prices can hit the price cap before the capacity starts to be binding due to strategic withholding at the spot market. The lowest non binding price cap under perfect competition, $\widehat{\bar{p}^{\infty}} = P(\widehat{X}(\widehat{\bar{p}^{\infty}}), \bar{\theta})$, will play a crucial role in our welfare analysis in the following section, it is simply given by the market price obtained under perfect competition for the highest possible demand realization. Under perfect competition overall welfare is maximized if no price cap is set. Distortion of this market outcome by binding price caps will always reduce equilibrium capacity choices and welfare. We can thus conclude that for a perfectly competitive industry price caps are never desirable in our framework[19]. For strategically behaving firms this is not true, however, as we show in the subsequent section. If firms choose capacities strategically, appropriately chosen price caps always do increase both equilibrium capacities and overall welfare.

3.2 Desirable price caps under imperfect competition

As derived in theorem 2, under perfect competition, introduction of a binding price cap always reduce industry capacity and overall welfare. In this section we will characterize binding price caps which always increase both industry capacity choice and overall welfare for the case of strategically behaving firms. The following theorem characterizes an interval

[19]This is in line with current contributions regarding price caps at electricity wholesale markets, compare for example Joskow (2007)

of binding price caps which always enhances investment and welfare as compared to the situation without price cap.

Theorem 3 (Investment and Welfare Enhancing Price Caps) .

(i) Denote by $\overline{MR}$ *the highest marginal equilibrium revenue without price cap*[20]*. A price cap* $\bar{p} \in [\overline{MR}, \bar{\rho}^{\infty}]$ *increases equilibrium investment, consumer surplus and total welfare, and lowers average prices as compared to the market game without price cap.*

(ii) Industry capacity and welfare under imperfect competition, for any price cap $\bar{p} \geq 0$*, remain strictly below the first best solution .*

Proof See appendix 5. □

Given the equilibrium characterization in theorem 1, the intuition is as follows: Any price cap at or above the highest possible marginal revenue in an equilibrium without price cap increases marginal spot market profits of investment in *all* scenarios where it is binding (compared to the situation without price cap). In the new equilibrium with binding price cap, expected marginal spot market profits must again be equal to marginal cost of investment (K_x). This can only be achieved by decreasing marginal spot market profits in scenarios with non–binding price cap, i.e. by increasing capacities. The above findings thus generalize the already known findings of the impact of a price cap on capacity in a deterministic world with a single spot market. Part of its intuition can be extended to the case of uncertain demand. [21]

Notice, however, that unlike in the deterministic case, we cannot make statements about monotonicity, in particular we cannot characterize the best price cap in $[\overline{MR}, \bar{\rho}^{\infty}]$ without further assumptions on the distribution of uncertainty.[22] That is, any price cap in the interval $[\overline{MR}, \bar{\rho}^{\infty})$ is strictly better than no price cap. We are furthermore able to show that even though the price caps characterized in theorem 3 increase welfare and industry

[20] I.e. $\overline{MR} = \max_\theta \{P(X^*(\bar{\rho}^{\infty}), \theta) + P_q(X^*(\bar{\rho}^{\infty}), \theta)\frac{X^*(\bar{\rho}^{\infty})}{n}\} - c$

[21] Remember, with a single deterministic spot market, in the absence of a price cap, marginal profits of the single spot market have to equal marginal cost of investment (i.e. $P(X^*) - P_q(X^*)/n - c = K_x$). All price caps at or above marginal revenue will increase capacity choice. The optimal price cap in the deterministic framework is suited to implement the first best outcome and is given by marginal revenue, i.e. $\bar{p} = P(X^*) - P_q(X^*)/n = c + K_x$), i.e. the optimal price cap equals total marginal cost. With uncertain demand at the spot market this logic can be applied only to the highest demand realizations, typically the first best outcome cannot be reached, however.

[22] We address this issue in theorem 5.

capacity, it is impossible under uncertainty to implement the welfare maximizing solution by setting any price-cap.

In the subsequent theorem we provide a characterization of the desirable price caps established in theorem 3 in terms of the highest market price reached under perfect competition. Under relatively mild assumptions on the demand function[23] the highest possible market price reached under perfect competition $\widehat{\bar{p}^{\infty}}$ could lend itself as a welfare enhancing price cap:

Theorem 4 (Price Cap at the Highest Competitive Price) *Suppose demand can be decomposed such that $P(X,\theta) = a(\theta) + b(\theta)\widetilde{P}(X)$. A price cap $\bar{p} \in \left[\widehat{\bar{p}^{\infty}}, \bar{\rho}^{\infty}\right]$ strictly increases equilibrium investment, consumer surplus and total welfare, and lowers average prices as compared to the market game without price cap.*

Proof See appendix 5. □

Note that from a policy maker's perspective the result is actually very interesting. For example in electricity markets, regulators typically have quite detailed information on demand and cost. Thus, calculating $\widehat{\bar{p}^{\infty}}$ by modeling the *competitive benchmark* seems relatively easy, while basically nothing is known for the case of market power. The theorem nicely connects the two scenarios of perfect competition and oligopoly, giving a clear cut policy result.

Let us finally notice that theorems 3 and 4 do not contradict the result of theorem 2, where we established that under perfect competition price caps are never desirable. As we approach the case of perfect competition ($n \to \infty$), the range of desirable binding price caps as characterized in both theorems vanishes.

3.3 Optimal Price Caps

As we have shown in section 3.1, we can always identify a range of binding price caps that strictly increase equilibrium capacity choice of strategic firms and thus are desirable from a welfare point of view. We have not yet determined however which price cap performs best in this regard. In this section we thus characterize the price caps which maximize industry capacity choice for a given market structure.

[23]These do not seem necessary, but they allow for a very intuitive proof.

In the following theorem we characterize the price cap that maximizes equilibrium investment. In order to do so we need to make regularity assumptions on the distribution of θ.[24]

Theorem 5 (Optimal Price Cap) *Suppose that demand can be decomposed such that $P(X,\theta) = \theta + \widetilde{P}(X)$ and that the hazard rate $h(\theta) := \frac{f(\theta)}{1-F(\theta)}$ is increasing.*[25] *Then there exists a unique price cap $\bar{p}^*$ which maximizes investment, it is uniquely characterized by:*

$$-P_q(X^*(\bar{p}^*), \tilde{\theta}^{\bar{p}}) \frac{X^*(\bar{p}^*)}{n} = \frac{1 - F(\tilde{\theta}^{\bar{p}})}{f(\tilde{\theta}^{\bar{p}})}. \tag{3}$$

$\bar{p}^*$ is increasing in the number of firms n, it is furthermore an upper bound for the welfare maximizing price cap.

Proof See appendix 5. $\square$

Let us point out the trade off that has to be solved at the investment maximizing price cap. Recall from theorem 1 that firms choose the quantity that just equates expected marginal revenue of investment with marginal cost. Lowering a given price cap affects expected marginal revenue and thus, the firms' investment decisions. We observe two effects: On the one hand, lowering the price cap by an increment decreases marginal revenue in all scenarios where the price cap has been binding. The expected marginal loss equals $[1-F(\tilde{\theta}^{\bar{p}})]$. On the other hand, the price cap becomes binding also in lower demand scenarios and thereby marginally increases expected revenue by $-P_q(X^*, \tilde{\theta}^{\bar{p}}) x_i f(\tilde{\theta}^{\bar{p}})$.[26] Condition (3) as stated in theorem 5 balances those two effects. Note that the first effect lowers investment incentives, while the second effect encourages investment. At high price caps, the first effect is necessarily small (since the range of demand scenarios for which the price cap is binding is small), such that the second effect always dominates. Note that it depends on the exact distribution of θ whether the investment–maximizing price cap lies in $\left[\overline{MR}, \bar{\rho}^{\infty}\right)$, or below. It is furthermore worthwhile to notice that the investment maximizing price cap constitutes an upper bound on the welfare maximizing price cap that is, overall welfare is maximized for price caps which always lie below $\bar{p}^*$ as characterized in theorem 5. The reason is that

[24] Recall that in theorem 3 and 4, we could not make statements about monotonicity without further assumptions on the distribution.

[25] In order to prove the theorem it is sufficient to assume $P_{q\theta} = 0$, such that demand can be decomposed such that $P(Q,\theta) = v(\theta) + \widetilde{P}(Q)$. Then, the hazard rate of the transformed random variable $v(\theta)$ would have to be increasing. In the theorem we present a slightly less general statement for easier exposition.

[26] Note that marginal revenue in scenario θ without binding price cap is $P_q(X^*, \theta)x_i + P(X^*, \theta)$ and thus, a binding price cap $\bar{p} = P(X^*, \tilde{\theta}^{\bar{p}})$ increases marginal revenue in scenario $\tilde{\theta}^{\bar{p}}$ by $-P_q(X^*, \tilde{\theta}^{\bar{p}})x_i > 0$.

decreasing the price cap below $\bar{p}^*$ on the one hand decreases investment but, on the other hand, eliminates the incentive to withhold in additional demand scenarios such that the average quantity *sold* may increase and thus, also total and consumer welfare. Price caps higher than $\bar{p}^*$ lead to lower investment without having the desirable effect on the firms' withholding decisions.

All those findings are illustrated in the empirical part in section 4, where we analyze the impact of price–caps on capacity choice in the German electricity market.

4 Empirical Analysis: Price caps and Capacity Choice in the German Electricity Market

In this section we demonstrate how our theoretical insights can be used to assess (long run) capacity and welfare effects of price caps. In the following we analyze the impact of a price cap on the German wholesale electricity market on firms investment decisions and overall welfare. We quantify the capacity and welfare effects of the introduction of price caps at the spot market for different degrees of market concentration.

Our aim is to fit the theoretical model as closely as possible to the data of the German Electricity market for the year 2006 and to compute resulting investment in gas turbine generation capacity for the scenarios with and without price cap. Note that this approach yields total industry capacity under the assumption that each firm's marginal generating unit is always a gas turbine. Since investment in the last unit of capacity (which, of course, determines total capacity) is always a marginal decision, we do not need to specify the inframarginal technology mix for the empirical analysis. Note however, that we need to assume that firms are symmetric in size (but not necessarily with respect to their inframarginal technology mix). Since mark-ups in the Cournot model generally increase if firms become asymmetric, our results yield a lower bound for the extent of market power for a given number of firms.

In order to use our theoretical model for the analysis we chose to make the following specifications. We assume linear fluctuating demand $P(Q) = \theta - bQ$

We furthermore determine average production cost and investment cost and can model firms capacity choice for different degrees of market power and different price-caps by making use of the solutions obtained in theorems 1 and 2.

In order to assess the robustness of our results we do not perform the analysis for single parameter values, but rather for plausible ranges of parameter distributions. This concerns the following parameters of the model: The demand elasticity (determined by the slope of the demand function, b), marginal cost of production, c, and marginal investment cost, k. From the possible ranges of those parameters, our algorithm selects one random combination in each iteration. The resulting distributions of capacities and welfare differences give an impression of the sensitivity of our results to changes in the parameters. In the following we provide some details on the relevant ranges of our cost and demand parameters.

Market demand: To construct fluctuating market demand, we depart from hourly market prices (from the European Energy Exchange (EEX)[27]) and hourly quantities consumed (from the Union for the Co-ordination of Transmission of Electricity (UCTE)[28]) for the year 2006. We chose the value of b in line with other studies on energy markets. Most studies

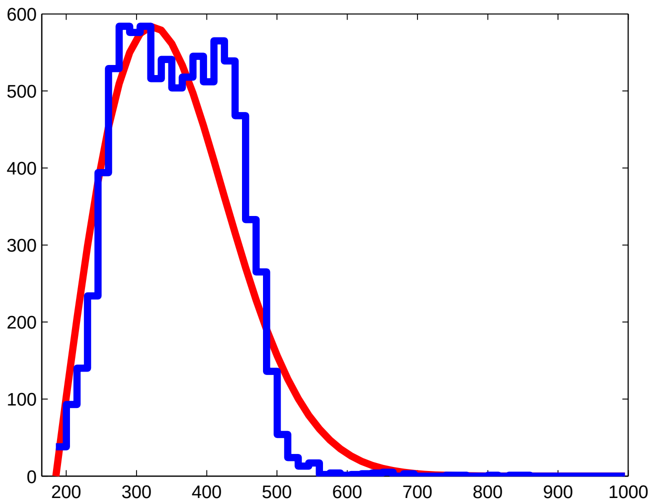

Figure 1: Fitting a smooth distribution to observed frequencies (for b=0.0055). MLE's of a shifted Weibull distribution are given by $\widehat{\alpha} = 2.004$ and $\widehat{\beta} = 193.434$.

that estimate demand for electricity[29] find short run elasticities between 0.1 and 0.5 and long run elasticities between 0.3 and 0.7.[30] The relevant range of prices is around $P = 100$ €/MWh and corresponding consumption is approximately $Q = 50$ GW. In our simulations

[27]See www.EEX.com

[28]See www.UCTE.org

[29]See, for example, Lijsen (2006) for an overview of recent contributions on that issue.

[30]E.g. Beenstock et al. (1999), Bjorner and Jensen (2002), Filippini Pachuari (2002), Booinekamp (2007), and many others.

we thus use a uniform distribution of b on the interval $[0.004, 0.007]$, which corresponds to elasticities between 0.5 and 0.29.

In order to illustrate our results concerning investment maximizing price caps as analyzed in section 3.3 we fit a shifted Weibull distribution over the observed frequencies. In figure 4 this is illustrated for the parameter $b = 0.0055$, which yield the following maximum likelihood estimates for the parameters: .

Production cost: The major components of variable production cost are gas prices[31] and prices for CO_2 emission allowances.[32] The average TTF gas price in 2006 was 20 €/MWh and CO_2 permissions traded on average for 9.30 €/MWh.[33] The efficiency of gas turbines currently ranges at around $37,5\%$.[34] The resulting daily production cost for the year 2006 was on average 66.30 €/MWh. In order to assess robustness of our results, however we consider uniformly distributed errors ($\pm 10\%$) and thus consider average marginal cost to be in the interval [59.67, 72.93] €/MWh.

Investment Cost: Since we analyze investment incentives based solely on one year, we break down investment cost to annuities.[35] In order to take construction time of gas turbine plants into account we consider investment cost on the basis of data from the year 2000. We assume perfect foresight, i.e. all cost components have been predicted accurately by the firms at the time of their investment decision. We base investment cost on the following two studies: First, a study on the German energy market commissioned by the German Parliament (2002), with scenarios for investment decisions summarized in Weber and Swider (2004) [in the following GP/WS]. Second, Energiereport III, a study conducted by the Institute of Energy Economics (EWI) in Cologne and Prognos (2000) for the German Ministry of Economics [in the following EWI/P].

The relevant annuity is determined as follows: Total investment cost ranges between 279 €/KW (GP/WS) and 300 €/KW (EWI/P). Annual fixed cost of running a gas turbine

[31] Daily values from the Dutch Hub TTF, corrected for transportation cost.

[32] Daily data taken from the EEX. The emission-coefficient for natural gas is set by the German ministry of environment at 56t CO_2/TJ which corresponds to 0.2016t CO_2/MWh. Compare Umweltbundesamt (2004).

[33] Recall that we do not use the averages but the daily values in our simulation.

[34] See 2006 GTW Handbook or EWI and Prognos (2005).

[35] The results will thus only yield a benchmark for current profitability of investment. Provided, however, that yearly demand is increasing over time (and that strategic timing of investment is not an issue) our procedure should yield accurate predictions, even though once installed capacities cannot be removed the subsequent year.

is already included in GP/WS, and is given by 8 €/KWa in EWI/P. This value is corrected by the average availability of gas turbines, which, in Germany, is given by 94%.[36] Based on a financial horizon of 20 years and an interest rate of 10 % this yields annuities of 34863 €/MWa (GP/WS) and 45998 €/MWa (EWI/P). Finally, the free allotment of CO_2 allowances granted to new power plants results in a de facto reduction of the annuity by the net value of the allocated allowances. Calculating their value on the basis of the average market price in 2006 yields 6305.3 €/MWa. The range of relevant annuities which we use in our simulation is consequently given by [28558, 39692] €/MWa.

Results: We determine industry capacity choice for different number of firms, and for different price caps. The left graph of figure 4 shows mean values (solid black lines) and 95% confidence intervals for industry capacity choice as a function of price caps (for different number of firms).

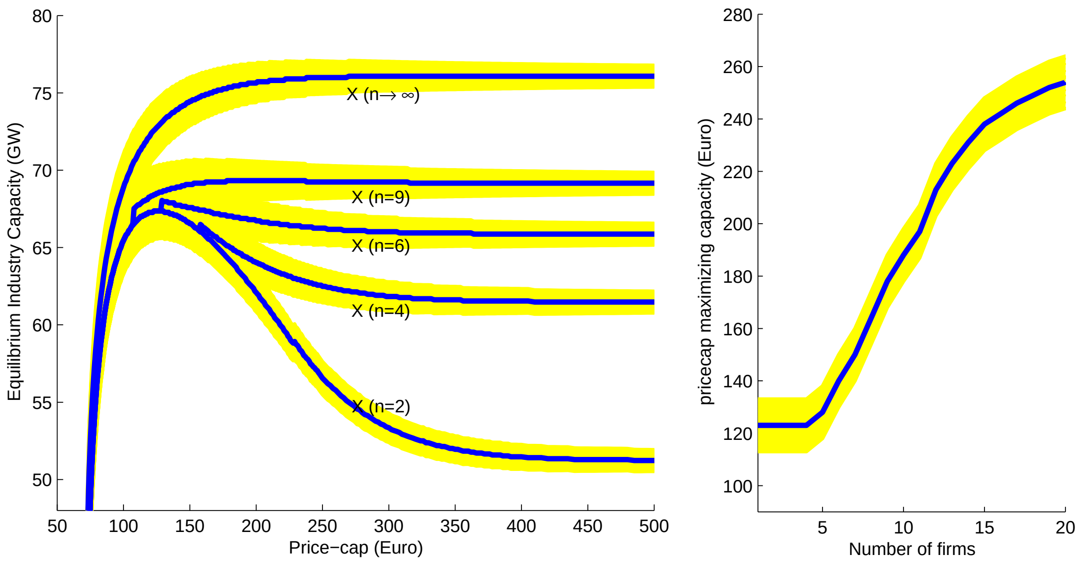

Figure 2: Left: Industry capacity choice over spot market price caps for different degrees of concentration. Right: Price caps maximizing investment over market concentration. Means and 95%-confidence intervals.

We observe that especially for concentrated markets surprisingly low caps lead to a dramatic increase of industry capacities (in the case of duopoly industry capacity is increased by up to 30%). Observe furthermore that, in line with the findings of theorem 2 (and the existing literature, e.g. see Joskow (2007)), for the case of perfect competition any binding

[36] Compare VGB Powertech (2006).

price cap leads to reduced industry capacity. The right graph of figure 4 gives investment maximizing price caps as a function of the number of firms in the industry. We observe that price caps below 260 €/MWh increase industry capacity for any market structure based on up to 20 firms. For concentrated markets we find surprisingly low investment maximizing price caps at around 130 €/MWh for the case of up to 4 firms in the industry. Finally, confirming the findings of theorem 5 The locus of those price caps is increasing in the number of firms.

We then analyze the impact of price caps on overall welfare.

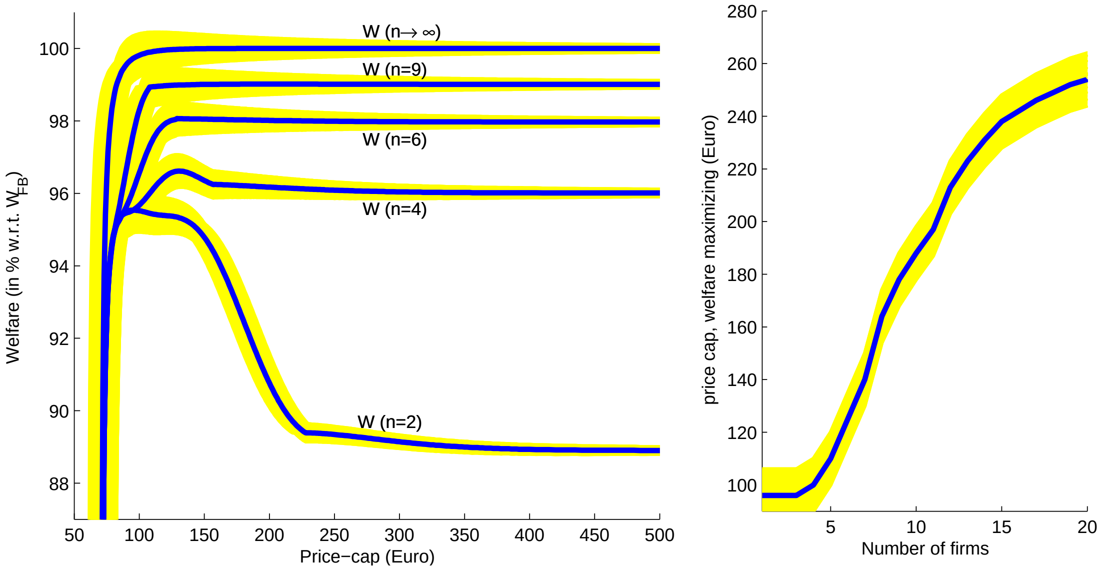

Figure 3: Left: Welfare over spot market price caps for different degrees of concentration. Right: Price caps maximizing welfare over market concentration. Means and 95%-confidence intervals.

The left graph of figure 4 shows mean values (solid black lines) and 95% confidence intervals for Welfare[37] as a function of price caps (for different number of firms). The right graph of figure 4 gives welfare maximizing price caps as a function of the number of firms in the industry. We observe that price caps below 250 €/MWh increase industry capacity for any market structure based on up to 20 firms. For concentrated markets we find surprisingly low welfare maximizing price caps at around 110 €/MWh for the case of up to 4 firms in the industry.

The empirical results thus clearly confirm our theoretical findings. Under imperfect

[37]Notice that all values are normalized with respect to welfare generated in the first best scenario, which is given by roughly 100 bn €.

competition, price caps at the spot market, if appropriately chosen have a positive impact on industry investments and overall welfare. Our empirical results for the case of the German Market document, that it is a very large range of price caps having those beneficial properties. Those price caps not only do increase welfare in the short run for fixed investment decisions but also have a positive impact on welfare in the long run, when investment decisions are considered to be chosen endogenously by strategic firms.

5 Conclusion

In this article we have analyzed the impact of price caps at electricity spot markets on investment decisions of strategic firms. In the short run, when firms investment decisions can be assumed to remain fixed, there is undisputed consensus regarding the impact of such price caps. They reduce firms incentives to withhold production and are thus considered to be a powerful tool to discipline market power. In the long run, however, in liberalized electricity markets, firms can decide on their investment in generation facilities. Interventions at the spot markets designed to mitigate wholesale market power, should take this into account, as an increasing number of experts in the field point out.[38]

It was the aim of the present article to thoroughly model the impact of a uniform price cap on the capacity choice of strategic firms in such markets, contributing to this ongoing debate. We analyzed a two stage market game where strategic firms choose their capacities prior to the spot market. Demand at the spot market is assumed to be either fluctuating or uncertain.

In section 3 we first characterized the market equilibrium for a given price caps. We then show that appropriately chosen price caps not only increase welfare at the spot market but do also enhance investment choice in equilibrium, whenever firms have market power. Finally we have characterized price caps which maximize equilibrium capacity for a given market structure. Those caps are found to be increasing in the number of firms and vanish in the limit, when approaching the case of perfect competition.

In section 4 we have empirically analyzed overall capacity choice in the German electricity market and determined the impact of introducing a price cap at the wholesale market. We find a surprisingly large interval of price caps leading to an increase of equilibrium capacity choice for almost any market structure[39].

[38] e.g. compare Hogan (2005), Joskow (2007).

[39] That is for any number of firms with $n \leq 20$ and all price caps $\bar{p} \geq 130/MWh$ lead to an increase of equilibrium capacity.

Our insights crucially enrich the ongoing discussion regarding price caps in electricity spot markets. In the current perception, price caps distort the price signals of the market and always lead to reduced investment incentives (compare Joskow (2007))[40]. A policy maker, when considering the introduction of such price caps would thus always face the tradeoff between increasing welfare at the spot market against undesirable reduction of investment in the long run. Our results reveal, however, that this is not necessarily true, when explicitly modeling firms investment incentives in such a market. Both our theoretical and our empirical results suggest, that for appropriately chosen price caps, welfare can be increased both from a short run and a long run perspective without tradeoff to be solved.

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Appendix

Proof of Theorem 1

Existence and Uniqueness In a compagnion paper, Grimm Zoettl (2008), we show that the market game in absence of a price cap has a unique equilibrium which is symmetric. Now consider the following modified demand function that captures the situation that firms face for a given price cap.

$$\bar{P}(Q,\theta) = \begin{cases} \bar{p} & \text{if} \quad 0 \le Q \le Q^{\bar{p}}(\theta) \\ P(Q,\theta) & \text{if} \quad Q > Q^{\bar{p}}(\theta), \end{cases} \tag{4}$$

where $Q^{\bar{p}}(\theta) = \{Q : P(Q,\theta) = \bar{p}\}$.

Note that $\bar{P}(Q,\theta)$ does not satisfy assumption 1 of Grimm Zoettl (2008) and thus, the proof does apply to the case of a price caps at the spot market only after slight extensions. We have to deal with two issues:

(i) The fact that $\bar{P}_q(Q,\theta) = 0$ for $Q \in [0, Q^{\bar{p}}(\theta)]$. This implies that, given θ, the equilibrium at the second stage is not necessarily unique, which leads potentially to multiple equilibria of the overall game. Our assumption that for a binding price cap, production is as equal as possible (see section 2) overcomes this problem. It can be interpreted either as (i) a special equilibrium selection[41], or (ii) as analyzing only a special class of equilibria of the overall game, focusing on symmetric outcomes at the second stage.

(ii) Furthermore demand $\bar{P}(Q,\theta)$ is kinked at $Q^{\bar{p}}(\theta)$. This implies that firms' stage two profits are not twice continuously differentiable. It straight forward that this kink does not pose any problems when carrying through the proof along the lines of Grimm Zoettl (2008), due to concavity of the kink. Since computations are mainly redundant to the proof conducted there, (the only difference being the concave kink) they are not reported in detail again.

We conclude that the market game has a unique equilibrium which is symmetric.

Characterization of Equilibrium As we have shown, no asymmetric equilibria exist in our framework. We can now focus on the symmetric case and characterize equilibrium capacity choice.

Let us first consider profits for each realization of θ, given capacity choices x and $\bar{p}$. Recall that firms at the spot markets can produce at most their overall capacity choice made in stage one. For ease of exposition denote by $\hat{Q}(\theta)$ the unrestricted Cournot equilibrium quantity in scenario θ, which may exceed capacity X of the firms. For the moment without taking into account price caps, denote by $Q^*(x,\theta)$ equilibrium production in scenario θ, i. e. $Q^*(x,\theta) = \min\{\hat{Q}(\theta), X\}$.

Depending on the level of the price cap $\bar{p}$ Profits in scenario θ are given as follows:

$$\pi_i(x,\bar{p},\theta) = \begin{cases} (P(Q^*(x,\theta),\theta) - c)\ q_i^*(x,\theta) & \text{if} \quad \bar{p} \geq P(Q^*(x,\theta),\theta) \\ (\bar{p} - c)\,\frac{Q^{\bar{p}}(x,\theta)}{n} & \text{if} \quad \bar{p} < P(Q^*(x,\theta),\theta) \end{cases}, \tag{5}$$

where $Q^{\bar{p}}(x,\theta) = \min(X, \{Q : \bar{p} = P(Q,\theta)\})$ Note that if capacity is binding we obtain $Q^*(x,\theta) = X$, otherwise $Q^*(x,\theta) < X$. If capacity is binding, two cases are possible: (i) the firms do not have an incentive to withhold, but the price cap is not yet met; (b) the price

[41]Note that this would be the unique equilibrium at the second stage if even for a binding price cap, inverse demand would have an (infinitely small) slope.

cap is effective and the firms produce at the capacity bound[42]. Let us denote by $\tilde{\theta}^X(x,\bar{p})$ the demand scenario from which on the firms are capacity constrained and by $\tilde{\theta}^{\bar{p}}(x,\bar{p})$ the demand realization where firms are capacity constrained *and* the price cap is met at the spot market. $\tilde{\theta}^{\bar{p}}(x,\bar{p})$ and $\tilde{\theta}^X(x,\bar{p})$ are implicitly defined as follows:

$$\begin{aligned}\tilde{\theta}^{\bar{p}}(x,\bar{p}) &= \{\theta : \bar{p} = P(X,\theta)\}\\ \tilde{\theta}^X(x,\bar{p}) &= \min\Big(\tilde{\theta}^{\bar{p}}(x,\bar{p}), \{\theta : P_q(X,\theta)q + P(X,\theta) = 0\}\Big)\end{aligned}$$

Since in $\theta \in [\tilde{\theta}^X(x,\bar{p}), \overline{\theta}]$ the price is monotonically increasing in θ, profits in this interval are given by

$$\pi_i(x,\bar{p},\theta) = \begin{cases} (P(X,\theta)-c)\ x_i & \text{for} \quad \theta \in [\tilde{\theta}^X(x,\bar{p}), \tilde{\theta}^{\bar{p}}(x,\bar{p})] \\ (\bar{p}-c)\, x_i & \text{for} \quad \theta \in [\tilde{\theta}^{\bar{p}}(x,\bar{p}), \overline{\theta}] \end{cases}, \tag{6}$$

Denote spot market profits whenever capacity is not binding by $\pi_i^0(\bar{p},\theta) = \pi_i(x,\bar{p},\theta)|_{\theta<\tilde{\theta}^X(x,\bar{p})}$.

Now we can finally write down a firm's expected profit at stage one,

$$\pi_i(x,\bar{p}) = \int_{\underline{\theta}}^{\tilde{\theta}^X(x,\bar{p})} \pi_i^0(\bar{p},\theta)dF(\theta) + \int_{\tilde{\theta}^X(x,\bar{p})}^{\tilde{\theta}^{\bar{p}}(x,\bar{p})} (P(X,\theta)-c)\, x_i dF(\theta) + \int_{\tilde{\theta}^{\bar{p}}(x,\bar{p})}^{\overline{\theta}} (\bar{p}-c)\, x_i dF(\theta) - K(x_i).$$

Differentiation yields the first order condition (note that $\pi_i^0(\bar{p},\theta)$ does not depend on x_i and that the derivatives of the integration limits cancel out due to Leibnitz' rule)

$$\frac{d\pi_i(x,\bar{p})}{dx_i} = \int_{\tilde{\theta}^X(x,\bar{p})}^{\tilde{\theta}^{\bar{p}}(x,\bar{p})} [P(X,\theta) + P_q(X,\theta)x_i - c]\, dF(\theta) + \int_{\tilde{\theta}^{\bar{p}}(x,\bar{p})}^{\overline{\theta}} (\bar{p}-c)\, dF(\theta) - K_x(x_i) = 0. \tag{7}$$

From (7) the characterization of the unique and symmetric equilibrium of Theorem 1 can be easily derived. If the price cap does not bind in any demand scenario, we obtain $\tilde{\theta}^{\bar{p}}(x,\bar{p}) = \overline{\theta}$ and the second integral on the LHS of the first order condition cancels out. If $\tilde{\theta}^{\bar{p}}(x,\bar{p}) < \overline{\theta}$, the price cap is binding in some demand scenarios. Two cases are possible: (1) capacity is binding before the price cap is met ($\tilde{\theta}^X(x,\bar{p}) < \tilde{\theta}^{\bar{p}}(x,\bar{p})$), or capacity and price cap become binding in the same demand scenario ($\tilde{\theta}^X(x,\bar{p}) = \tilde{\theta}^{\bar{p}}(x,\bar{p})$).

[42]Note that if at a certain θ the price cap is binding **and** firms are capacity constrained, then the price cap is also binding for all higher demand scenarios. This is not necessarily true if only the price cap is binding but firms produce below their capacity bound. Assumption 1, part (iii), guarantees that the Cournot quantity $\hat{Q}(\theta)$ is increasing in θ. It does not guarantee, however, that equilibrium *prices* are increasing in θ. Consequently, as θ increases, the price cap might be binding for low realizations and non–binding for higher ones.

Proof of theorem 2

The proof of existence and uniqueness for the case of perfect competition follows the same lines as the proof in case of oligopoly. Note that under perfect competition firms do not strategically choose production for the spot markets but always produce at marginal cost given by c whenever capacity is not binding, and rise only whenever capacity is binding. Let us denote by $\hat{\theta}^X(x) = \{\max\theta : P(X,\theta) = c\}$ the demand scenario from which on capacity is binding and by $\hat{\theta}^{\bar{p}}(x,\bar{p}) = \{\theta : \bar{p} = P(X,\theta)\}$ the demand realization where the price cap is met. Obviously, under perfect competition we always obtain $\hat{\theta}^X(x) < \hat{\theta}^{\bar{p}}(x,\bar{p})$, since the price cap can only be met if the prices rise above marginal cost of production (i. e. capacity is binding). The profit function under perfect competition is given by[43]

$$\pi_i(x,\bar{p}) = \int\limits_{\hat{\theta}^X(x,\bar{p})}^{\hat{\theta}^{\bar{p}}(x,\bar{p})} (P(X,\theta) - c)\, x_i dF(\theta) + \int\limits_{\hat{\theta}^{\bar{p}}(x,\bar{p})}^{\bar{\theta}} (\bar{p} - c)\, x_i dF(\theta) - K(x_i).$$

Thus, the first order condition of a perfectly competitive firm (that cannot affect the market price by changing its quantity) is given by

$$\int\limits_{\hat{\theta}^X(x,\bar{p})}^{\hat{\theta}^{\bar{p}}(x,\bar{p})} (P(X,\theta) - c)\, dF(\theta) + \int\limits_{\hat{\theta}^{\bar{p}}(x,\bar{p})}^{\bar{\theta}} (\bar{p} - c)\, dF(\theta) = K_x\left(\frac{X}{n}\right). \tag{8}$$

From (8) the characterization of industry capacity $\widehat{X}(\bar{p})$ in theorem 2 follows immediately. In order to prove the last part of the theorem, we derive the welfare optimum and show that it coincides with the solution under perfect competition in the absence of a price cap. Welfare given x and $\bar{p}$ is

$$W(x,\bar{p}) = \int\limits_{\underline{\theta}}^{\hat{\theta}^X(x,\bar{p})} \int\limits_0^Q (P(s,\theta) - c)\, ds dF(\theta) + \int\limits_{\hat{\theta}^X(x,\bar{p})}^{\bar{\theta}} \int\limits_0^X (P(s,\theta) - c)\, ds dF(\theta) - nK\left(\frac{X}{n}\right)$$

Thus, welfare is maximized at

$$X^{FB} = \left\{ X : \int\limits_{\hat{\theta}^X(x,\bar{p})}^{\bar{\theta}} (P(X,\theta) - c)\, dF(\theta) = K_x\left(\frac{X}{n}\right) \right\}. \tag{9}$$

[43] Note that for $\theta < \hat{\theta}^X(x,\bar{p})$ profits are equal to zero.

Note that this coincides with the solution under perfect competition for $\hat{\theta}^{\bar{p}}(x, \bar{p}) = \overline{\theta}$, i. e. for a non–binding price cap. Pointwise comparison of expressions (8) and (9) directly reveals that for any fixed X the LHS of (8) is smaller than the LHS of (9) for all binding price caps. This yields lower industry capacity choice under perfect competition for any biding price cap than in the first best solution.

Proof of Theorem 3

1) Capacity: First we show $X^*(\bar{p}) > X^{*\infty}$ for all $\bar{p} \in [\overline{MR}, \bar{\rho}^\infty]$. Denote equilibrium production in absence of a price cap by $X^{*\infty} := X^*(\bar{\rho}^\infty)$, which solves the following first order condition:

$$\int_{\theta^X(x^{*\infty})}^{\bar{\theta}} \left[P\left(X^{*\infty}, \theta\right) + P_q\left(X^{*\infty}, \theta\right) \frac{X^{*\infty}}{n} - c \right] dF\left(\theta\right) - K_x\left(\frac{X^{*\infty}}{n}\right) = 0 \tag{10}$$

Remember $\overline{MR}$ is defined to be given by $\overline{MR} = \left[P\left(X^{*\infty}, \theta\right) + P_q\left(X^{*\infty}, \theta\right) \frac{X^{*\infty}}{n} - c\right]_{\theta = \bar{\theta}}$, which is the highest value the integrand of expression (10)can take. In order to prove that a price cap $\bar{p} \in [\overline{MR}, \bar{\rho}^\infty]$ increases equilibrium capacity choice, we compare pointwise the first order condition without price cap (given above by (10)) with the first order condition with binding price cap as given by expression (2):

$$\int_{\theta^{X^*}(x^*)}^{\theta^{\bar{p}}(x^*, \bar{p})} \left[P\left(X^*, \theta\right) + P_q\left(X^*, \theta\right) \frac{X^*}{n} - c \right] dF\left(\theta\right) + \int_{\tilde{\theta}^{\bar{p}}(x^*, \bar{p})}^{\bar{\theta}} \left(\bar{p} - c\right) dF(\theta) - K_x\left(\frac{X^*}{n}\right) = 0 \tag{11}$$

By construction of $\overline{MR}$, for any fixed value X^0, the LHS of (11) is strictly bigger than the LHS of (10). Since profits of firms along the symmetry line are concave and cost of investment weakly convex, both expressions (11) and (10) are strictly decreasing in X. This implies however that condition (11) can only be satisfied for strictly bigger investment levels than (11), which proofs $X^*(\bar{p}) > X^{*\infty}$ for all $\bar{p} \in [\overline{MR}, \bar{\rho}^\infty]$.

2) Welfare: In order to show that total welfare always increases if a price cap price cap $\bar{p} \in [\overline{MR}, \bar{\rho}^\infty]$ is chosen (as compared to no price cap) just need to show that from a welfare perspective capacities chosen by strategic firms for a given price-cap are always below the welfare maximizing level and thus any increase in total capacity increases welfare.

Welfare maximizing capacity choice solves expression (9) and equilibrium investment of strategic firms for any binding price cap solves (11). As above we observe that for any fixed investment level X^0, the LHS of (9) is strictly bigger than the LHS of (11), since the LHS of both expressions are decreasing in capacity choice X, first best capacity choice is strictly bigger than capacity choice of strategic firms for any price cap.

3) Consumer Surplus: Since production is always increasing for any price cap $\bar{p} \in [\overline{MR}, \bar{\rho}^{\infty}]$ (as compared to no price cap), consumer surplus also increases.

4) Average Prices: We show that average prices decrease in the relevant range. For a given $X^*(\overline{p})$, average prices are uniquely given by

$$\begin{aligned} \mathbf{E}\Big[P\Big] \quad &= \quad \int_{\underline{\theta}}^{\theta^X(x^*(\bar{p}))} 0 dF(\theta) \\ &+ \int_{\tilde{\theta}^X(x^*(\bar{p}))}^{\theta^{\bar{p}}(x^*(\bar{p}),\bar{p})} P(X^*(\overline{p}),\theta) dF(\theta) + \int_{\theta^{\bar{p}}(x^*(\bar{p}),\bar{p})}^{\overline{\theta}} \overline{p} dF(\theta). \end{aligned} \tag{12}$$

If we lower price caps from $\bar{\rho}^{\infty}$ to some $\overline{p}^0 \in [\overline{MR}, \bar{\rho}^{\infty}]$, average prices are affected as follows. For the limits of integration in (12) we obtain $\theta^X(x^*(\bar{p}))$ and $\theta^p(x^*(\bar{p}))$ increase. For the integrands we obtain $P(X^{*\infty},\theta) > P(X^*(\bar{p}),\theta)$ and $\bar{\rho}^{\infty} > \bar{p}^0$. Thus, average prices decrease.

Proof of Theorem 4

We proof the theorem by showing that $\widehat{\bar{p}^{\infty}} = \overline{MR}$ under the assumptions of the theorem. This allows then to apply theorem 3. In other words we show that in absence of a price cap, the perfectly competitive equilibrium price just equals marginal equilibrium revenue:

$$P(X^*_{PC},\theta) = P(X^{*\infty},\theta) + P_q(X^{*\infty},\theta)\frac{X^{*\infty}}{n} \quad \forall \quad \theta \in [\tilde{\theta}^X(x^{*\infty}) = \theta^X(x^{*\infty}), \overline{\theta}],$$

Thus, setting the highest price observed in the perfectly competitive benchmark is equivalent to setting the lowest price from the interval of welfare increasing price caps identified in theorem 3.

In order to do so we show that in absence of a price cap the equilibria of the market game under perfect competition and the Cournot market game can be characterized by the same condition. To this end, define $T(X^{FB}) := \widetilde{P}(X^{FB})$ and $T(X^{*\infty}) := \widetilde{P}(X^{*\infty}) + \widetilde{P}_q(X^{*\infty})\frac{X^{*\infty}}{n}$ (recall X^{FB} denotes first best equilibrium quantity, while $X^{*\infty}$ denotes the market outcome in the absence of a price cap). Observe that the unique equilibrium production in both cases is characterized by the following equation,

$$\int_{\tilde{\theta}^T}^{\bar{\theta}} \left(a(\theta) + b(\theta)T - c\right) dF(\theta) = K_x, \tag{13}$$

where $\tilde{\theta}^T = \{\theta : a(\theta) + b(\theta)T = c\}$.

Thus both, the solution under perfect competition and under imperfect competition involve an identical value T^* solving (13). For all $\theta \in [\theta^{T^*}, \overline{\theta}]$ we obtain

$$a(\theta) + b(\theta)T^* = P(X^*_{PC}, \theta) = P(X^{*\infty}, \theta) + P_q(X^{*\infty}, \theta)\frac{X^{*\infty}}{n}.$$

Proof of Theorem 5

We prove theorem 5 by setting $\frac{dX^*(\bar{p})}{d\bar{p}}$ equal to zero. We then show that for an increasing hazard rate and $P_{x\theta} = 0$, $X^*(\bar{p})$ is quasiconcave by proving $\left.\frac{d^2X^*(\bar{p})}{d\bar{p}^2}\right|_{\frac{dX^*(\bar{p})}{d\bar{p}}=0} < 0$.

Preliminaries As a first step we derive properties and slopes of $\tilde{\theta}^{\bar{p}}(x, \bar{p})$. Notice that at the demand realization where the price cap is met in equilibrium we obtain

$$P(X, \tilde{\theta}^{\bar{p}}(x, \bar{p})) \equiv \bar{p}$$

We first calculate the partial derivative by differentiation with respect to $\bar{p}$ given some fixed X,[44]

$$\frac{\partial \tilde{\theta}^{\bar{p}}(x, \bar{p})}{\partial \bar{p}} = \frac{1}{P_\theta(X, \tilde{\theta}^{\bar{p}})} = 1.$$

Now we derive the total derivative at $X^*(\bar{p})$,

$$P_q(\cdot)\frac{dX^*(\bar{p})}{d\bar{p}} + P_\theta(\cdot)\frac{d\tilde{\theta}^{\bar{p}}(x^*(\bar{p}), \bar{p})}{d\bar{p}} = 1 \quad \Leftrightarrow \quad \frac{d\tilde{\theta}^{\bar{p}}(x^*(\bar{p}), \bar{p})}{d\bar{p}} = 1 - P_q(X^*, \theta)\frac{dX^*(\bar{p})}{d\bar{p}}. \tag{14}$$

Characterization of $\bar{p}^*$ Since expected profits are strictly concave for all $\bar{p}$, we can apply the Implicit Function Theorem in order to derive $\frac{dX^*(\bar{p})}{\bar{p}}$. Differentiation of the first order condition

$$\int_{\tilde{\theta}^X(x^*)}^{\tilde{\theta}^{\bar{p}}(x^*, \bar{p})} \left[P\left(X^*, \theta\right) + P_q\left(X^*, \theta\right)\frac{1}{n}X^* - c\right] dF\left(\theta\right) + \int_{\tilde{\theta}^{\bar{p}}(x^*, \bar{p})}^{\bar{\theta}} (\bar{p} - c)dF\left(\theta\right) - K_x\left(\frac{X^*}{n}\right) = 0.$$

[44]Note that $P_\theta = 1$ under our assumptions.

with respect to $\bar{p}$ yields

$$\frac{dX^*(\bar{p})}{d\bar{p}} = -\frac{\partial^2 \pi_i(x,\bar{p})}{\partial x_i \partial \bar{p}} \Big/ \frac{\partial^2 \pi_i(x,\bar{p})}{\partial x_i \partial X}$$
$$= \left[\frac{P_q(X^*,\tilde{\theta}^{\bar{p}})}{P_\theta(X^*,\tilde{\theta}^{\bar{p}})} \frac{X^*(p)}{n} f(\tilde{\theta}^{\bar{p}}) + 1 - F(\tilde{\theta}^{\bar{p}})\right] \Big/ -\frac{\partial^2 \pi_i(x,\bar{p})}{\partial x_i \partial X}, \tag{15}$$

Notice that it may happen that $\tilde{\theta}^{\bar{p}}(x^*,\bar{p}) = \tilde{\theta}^X(x^*,\bar{p})$, in which case

$$-P_q(X^*,\tilde{\theta}^{\bar{p}})\frac{X^*}{n} = -P_q(X^*,\tilde{\theta}^X)\frac{X^*}{n} = P(X^*,\tilde{\theta}^X) - c = P(X^*,\tilde{\theta}^{\bar{p}}) - c = \bar{p} - c$$

(we denote this type of equilibrium by EX^{II}, and the other type by EX^I in the following). Rewriting equation (15) using the hazard rate yields

$$EX^I: \quad \frac{dX^*(\bar{p})}{d\bar{p}} = \left[P_q(\cdot)\frac{X^*(p)}{n} + \frac{1}{h(\tilde{\theta}^{\bar{p}})}\right] \Big/ \left[-\frac{\partial \pi_i(x,q^*)}{\partial x_i \partial X}\frac{1}{f(\tilde{\theta}^{\bar{p}})}\right] \tag{16}$$
$$EX^{II}: \quad \frac{dX^*(\bar{p})}{d\bar{p}} = \left[-(\bar{p}-c) + \frac{1}{h(\tilde{\theta}^{\bar{p}})}\right] \Big/ \left[-\frac{\partial \pi_i(x,q^*)}{\partial x_i \partial X}\frac{1}{f(\tilde{\theta}^{\bar{p}})}\right] \tag{17}$$

Define $\xi(\bar{p}) = \left[-\frac{\partial \pi_i(x,q^*)}{\partial x_i \partial X}\frac{1}{f(\tilde{\theta}^{\bar{p}})}\right]$. Note that $\xi(\bar{p}) > 0$ since profits (along the symmetry line) are concave. Thus, $\frac{dX^*(\bar{p})}{d\bar{p}} = 0$ gives exactly the characterization of the price caps that maximize industry capacity choice as stated in the theorem.

In order to show existence and uniqueness of these maximizers, however, we also need to show that second order conditions are satisfied, namely $\left.\frac{d^2X^*(\bar{p})}{d\bar{p}^2}\right|_{\frac{dX^*(\bar{p})}{d\bar{p}}=0} < 0$. Differentiation of (16) and (17) with respect to $\bar{p}$ yields[45]

$$EX^I: \quad \frac{d^2X^*(\bar{p})}{d\bar{p}^2} = \frac{1}{\xi(\bar{p})^2}\left[\left(\frac{dX^*(\bar{p})}{d\bar{p}}\left(P_{qq}\frac{X^*(\bar{p})}{n} + P_q\right) - \frac{h_\theta(\tilde{\theta}^{\bar{p}})}{h^2(\tilde{\theta}^{\bar{p}})}\frac{d\tilde{\theta}^{\bar{p}}}{d\bar{p}}\right)\xi(\bar{p}) - \frac{dX^*(\bar{p})}{d\bar{p}}\xi(\bar{p})\xi_{\bar{p}}(\bar{p})\right]$$
$$EX^{II}: \quad \frac{d^2X^*(\bar{p})}{d\bar{p}^2} = \frac{1}{\xi(\bar{p})^2}\left[-\left(1 + \frac{h_\theta(\tilde{\theta}^{\bar{p}})}{h^2(\tilde{\theta}^{\bar{p}})}\frac{d\tilde{\theta}^{\bar{p}}}{d\bar{p}}\right)\xi(\bar{p}) - \frac{dX^*(\bar{p})}{d\bar{p}}\xi(\bar{p})\xi_{\bar{p}}(\bar{p})\right]$$

We get

$$EX^I: \quad \left.\frac{d^2X^*(\bar{p})}{d\bar{p}^2}\right|_{\frac{dX^*(\bar{p})}{d\bar{p}}=0} = -\frac{1}{\xi(\bar{p})}\frac{h_\theta(\tilde{\theta}^{\bar{p}})}{h^2(\tilde{\theta}^{\bar{p}})} < 0$$
$$EX^{II}: \quad \left.\frac{d^2X^*(\bar{p})}{d\bar{p}^2}\right|_{\frac{dX^*(\bar{p})}{d\bar{p}}=0} = -\frac{1}{\xi(\bar{p})}\left(1 + \frac{h_\theta(\tilde{\theta}^{\bar{p}})}{h^2(\tilde{\theta}^{\bar{p}})}\right) < 0$$

Thus, under our assumptions all (local) extrema are necessarily maxima. Since we have local minima at $p = \bar{\rho}^0$ and $p = \bar{\rho}^\infty$, we conclude that the above condition characterizes the unique capacity maximizing price cap.

[45] Notice that $P_{x\theta} = P_{\theta\theta} = 0$ and that $\frac{d\tilde{\theta}^{\bar{p}}}{d\bar{p}} = 1$ at $\frac{dX^*(\bar{p})}{d\bar{p}} = 0$. Moreover, $\frac{dX^*(\bar{p})}{d\bar{p}}\xi_i(\bar{p})$ is just the enumerator of expressions (16) and (17).

Finally show that the optimal price cap $\bar{p}^*$ is weakly increasing in the number of firms. In order to do so we depart from the unique characterization of the optimal price cap as derived above, given by expressions (16) and (17) applying the envelope theorem:

$$
\begin{aligned}
EX^I: \quad \frac{d\bar{p}^*(n)}{dn} &= -\frac{\partial}{\partial n}\left[P_q(X^*,\tilde{\theta}^{\bar{p}})\frac{X^*}{n}+\frac{1}{h(\tilde{\theta}^{\bar{p}})}\right] \Big/ \frac{\partial}{\partial \bar{p}^*}\left[P_q(X^*,\tilde{\theta}^{\bar{p}})\frac{X^*(p)}{n}+\frac{1}{h(\tilde{\theta}^{\bar{p}})}\right] \\
EX^{II}: \quad \frac{d\bar{p}^*(n)}{dn} &= -\frac{\partial}{\partial n}\left[-(\bar{p}-c)+\frac{1}{h(\tilde{\theta}^{\bar{p}})}\right] \Big/ \frac{\partial}{\partial \bar{p}^*}\left[-(\bar{p}-c)+\frac{1}{h(\tilde{\theta}^{\bar{p}})}\right]
\end{aligned}
$$

Since at $\bar{p}=\bar{p}^*$ we obtain $\frac{dX^*(\bar{p})}{d\bar{p}}=0$ this simplifies to give:

$$
\begin{aligned}
EX^I: \quad \frac{d\bar{p}^*(n)}{dn} &= -P_q X^* \frac{-1}{n^2} \Big/ \frac{-h_\theta(\tilde{\theta}^{\bar{p}})}{h(\tilde{\theta}^{\bar{p}})^2}\frac{d\tilde{\theta}^{\bar{p}}}{d\bar{p}^*} > 0 \\
EX^{II}: \quad \frac{d\bar{p}^*(n)}{dn} &= 0
\end{aligned}
$$

Whenever we have an equilibrium of type I, i.e. first capacity is binding then the price cap, then the optimal price cap is strictly increasing in the number of firms. Notice that the above expression is positive since hazard rate is assumed to be increasing $h_\theta > 0$ and for higher price caps the price cap starts to be binding for higher demand realizations, i.e. $\frac{d\tilde{\theta}^{\bar{p}}}{d\bar{p}^*} > 0$. Whenever we have an equilibrium of type II, i.e. the price cap is already binding when capacity starts to become binding then neither equilibrium capacity choice nor the optimal price cap are a function of the number of firms. Notice however for high enough number of firms we will always have an equilibrium of type I.

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