

BINARY STARS: A CHEAT SHEET

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Abstract. This talk presented a brief summary of three different types of binary star – astrometric, spectroscopic and eclipsing – and tabulated the properties of these systems that can be determined directly from observations. Eclipsing binary stars are the most valuable, as they are our main source of direct mass and radius measurements for normal stars. In good cases, masses and radii can be obtained to better than 1% precision and accuracy using only photometry, spectroscopy and geometry. These measurements constitute vital empirical data against which theoretical models of stars can be verified and improved. It gave examples of the use of these systems for constraining stellar theory and the distance scale, and concluded with a presentation of preliminary results for the solar-type eclipsing binary 1SWASP J034114.25+201253.5.

Keywords: Stars: binaries: visual, spectroscopic, eclipsing, fundamental parameters

1 Introduction

Binary stars are one of the classical subject areas of *astronomy*, as they represent the only way of determining the masses and radii of normal stars to high precision and accuracy. This makes them *astrophysically* vital: the properties of stars in binary systems are used to calibrate theoretical stellar models, determine the distances to nearby galaxies, and support asteroseismology studies.

This review summarised the various types of binary star, the history of their study, and what physical properties can be obtained from them. It then presented some new work in progress on the eclipsing binary 1SWASP J034114.25+201253.5, detected using SuperWASP data and observed using the NASA K2 mission.

2 Binary stars

Table 1 summarises the properties directly measurable for different types of binary star. Both their symbols and names are given. The organisation of the table follows the observational techniques used—a convenient way of considering these systems.

2.1 Visual binaries

The first type of binary star to be observed was the *spatially-resolved* binary. These are also called visual binaries (because they can be identified by eye) and astrometric binaries (because it is possible to determine their orbits from measurements of the relative positions of the two stars).

Visual binaries were shown to be real, rather than chance alignments of stars at different distances, on statistical grounds, by the Revd. John Michell (1767). William Herschel (1802) introduced the term *binary star* and spent many years proving that some visual doubles show orbital motion. Herschel (1803) found that the double star Castor, with a separation of 3.9", is a binary system with a period of 342 yr, relatively close to the “modern” value of 420 yr (Rabe 1958). Visual binaries are usually close to Earth and have a large orbital separation (and thus a long orbital period) in order for the stars to be far enough apart on the sky to be resolved.

The equations of an astrometric orbit were established by Félix Savary in 1827. From observations of the motion of one of the stars relative to the other, it is possible to determine several of the properties of the orbit (P , e , ω , a). The semimajor axis a that can be found is the *angular size*, not the true length (Table 1). The properties ω , Ω and i can also be measured; they give the orientation of the orbit relative to the observer, so are not intrinsic properties of the system.

If we know the distance to the system, usually via a parallax from the *Hipparcos* or *Gaia*, we can convert *a* from angular units to length units. As this is the semimajor axis of the relative orbit, it can be used to find the sum of the masses (via Kepler’s Third Law) but not the individual masses.

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Table 1. Symbols and names of quantities measurable from various types of binary star.

Name	Symbol	Astrometric binary			Spectroscopic binary		Eclipsing binary		
		alone	with distance	with RVs	SB1	SB2	alone	with RVs (SB1)	with RVs (SB2)
<i>Orbital parameters</i>									
Orbital period	P	*	*	*	*	*	*	*	*
Orbital eccentricity	e	*	*	*	*	*	*	*	*
Argument of periastron	ω	*	*	*	*	*	*	*	*
Longitude of ascending node	Ω	*	*	*					
Projected semimajor axis	$a \sin i$		*	*		*			*
True semimajor axis	a (au)		*	*					*
Orbital inclination	i	*	*	*			*	*	*
Distance	d			*					*
<i>Spectroscopic parameters</i>									
Velocity amplitude of star 1	K_1			*	*	*		*	*
Velocity amplitude of star 2	K_2			*		*			*
Systemic velocity	V_γ			*	*	*		*	*
Mass function	$f(M)$				*	*		*	*
Mass ratio	$q = M_2/M_1$			*		*			*
Mass sum	$M_1 + M_2$		*	*		*			*
Minimum masses	$M_{1,2} \sin^3 i$			*		*			*
Mass of primary star	M_1			*					*
Mass of secondary star	M_2			*					*
<i>Size parameters</i>									
Fractional radii	r_1 and r_2						*	*	*
Radius of primary star	R_1								*
Radius of secondary star	R_2								*
Surface gravity of primary	$\log g_1$								*
Surface gravity of secondary	$\log g_2$							*	*
Density of primary star	ρ_1								*
Density of secondary star	ρ_2								*
<i>Radiative parameters</i>									
Temperature of primary star	$T_{\text{eff},1}$	*	*	*	*	*		*	*
Temperature of secondary star	$T_{\text{eff},2}$	*	*	*		*			*
Luminosity of primary star	L_1			*					*
Luminosity of secondary star	L_2			*					*

If we obtain radial-velocity (RV) observations of the two stars and fit a spectroscopic orbit (see below), we can convert a from angular units to length units, measure the distance to the system (without needing its parallax) and also calculate the individual masses of the stars. We can therefore use these systems as distance indicators (e.g. Torres et al. 1997) and to constrain the predictions of theoretical models of stellar evolution (e.g. Torres et al. 2009).

2.2 Spectroscopic binaries

Spectroscopic binaries are those found from changes in the RVs of the stars due to orbital motion. The first known spectroscopic binary was Algol (β Persei), which was already known to be an eclipsing binary (see below). Vogel (1890) observed the brighter of the two components to be moving away from Earth before primary eclipse and moving towards Earth after primary eclipse. Rudolf Lehmann-Filhés established the equations of a spectroscopic orbit in their current form in 1894.

For a “single-lined” spectroscopic binary system (SB1) – one where we can measure RVs for only one component – we can measure the quantities P , e , ω , K_1 and V_γ (Table 1). From these we can in turn calculate the mass function $f(M)$ and a lower limit to the semimajor axis of the barycentric orbit of the star, $a_1 \sin i$.

For a “double-lined” spectroscopic binary system (SB2) – where RVs are measurable for both components – we can obtain all the quantities for an SB1 system, plus the minimum masses ($M_1 \sin^3 i$ and $M_2 \sin^3 i$) and a

lower limit to the semimajor axis ($a \sin i$). As the orbital inclination i is not known, the true values of these three quantities are not accessible.

Spectroscopic binaries are easier to study if they have short orbital periods, large masses and a high i , in order to maximise the amplitude of the RV variation. On the other hand, it is more difficult to measure RVs for massive stars because they have few spectral lines and high rotational velocities (see Southworth & Clausen 2007). Spectroscopic binaries are useful for measuring the multiplicity fraction of stars (e.g. Duquennoy & Mayor 1991), which varies as a function of mass (Duchêne & Kraus 2013), age (Jaehnig et al. 2017) and chemical composition (Badenes et al. 2018), and can be used to probe the star-formation process (e.g. Bate 2009).

2.3 Eclipsing binaries

These are the most useful kind of binary star because of the huge number of physical properties that can be measured to high precision and accuracy (Table 1). Goodricke (1783) is generally credited as the first to advance the hypothesis of eclipses in order to explain the dimming of the “demon star” Algol every 2.87 days. These eclipses are even recorded in the Ancient Egyptian Calendar dating from around 1100 B.C. (Jetsu & Porceddu 2015). The first eclipsing binary system to be characterised properly was β Aurigae, by Stebbins (1911); the values found for its masses and radii agree reasonably well with modern values (Southworth et al. 2007a). The mathematical framework for eclipse calculations was laid out by Russell (1912a,b); see also Kopal (1959).

The light-curve of an eclipsing binary depends on the fractional radii of the two stars ($r_1 = \frac{R_1}{a}$ and $\frac{R_2}{a}$), so cannot be used in isolation to obtain the full physical properties of the system (Table 1). The orbital eccentricity e and argument of periastron ω can also be found, because $e \cos \omega$ depends on the time of secondary eclipse relative to primary eclipse, and $e \sin \omega$ depends on the relative durations of the eclipses.

If we add RVs of one star to the light-curve of an EB we can measure the mass function, $f(M)$, but not the actual masses of the stars. The surface gravity of the secondary star (the one for which we have no RVs) can also be calculated (Southworth et al. 2004b), something that can also be applied to transiting planets (Southworth et al. 2007b).

If RVs are available for both stars, then the full physical properties of the system can be obtained: the true masses and radii of both stars, which in turn give the surface gravities and densities. In the best cases these quantities can be measured to a precision of 0.5% or better (e.g. Helminiak et al. 2019). It is usually the case that the temperatures of the stars are known, from spectroscopy plus the surface brightness ratio measured from the light-curve. In this case the luminosities of the stars can be calculated from $L = 4\pi R^2 \sigma T_{\text{eff}}^4$, and the distance to the system can be obtained. EB-based distance estimates have been published for nearby galaxies: the LMC (Pietrzyński et al. 2019), SMC (North et al. 2010) and M33 (Bonanos et al. 2006).

For the case of SB2 EBs we can measure the masses, radii and luminosities of two stars of the same age and chemical composition. Such information is valuable for calibrating theoretical stellar models (e.g. Andersen et al. 1990; Pols et al. 1997; Claret 2007), if the two stars have not undergone mass transfer and thus have evolved as single stars. Claret & Torres (2016, 2018) have used a large sample of SB2 EBs to calibrate the strength of convective core overshooting as a function of mass, finding a ramp-up from $1.2 M_{\odot}$ to $2.0 M_{\odot}$. However, this approach has been challenged by Valle et al. (2016) on account of possible biases in the method, and by Constantino & Baraffe (2018) about the reproducibility of the results.

It is possible to add interferometry of SB2 EBs and obtain highly precise distances and physical properties of individual systems (Gallenne et al. 2016, 2019). The study of EBs with pulsating components is a very promising possibility for constraining the interior and atmospheric structure of stars (Debosscher et al. 2013; Aerts 2013; Tkachenko et al. 2014; Themeßl et al. 2018). Catalogues of EBs from photometric surveys are useful for determining the multiplicity fraction of stars, as have been achieved for M-dwarfs by Shan et al. (2015) and for solar-type stars by Moe et al. (2019).

2.3.1 DEBCat: the Detached Eclipsing Binary Catalogue

DEBCat* (Southworth 2015) is a catalogue of EBs suitable for comparison with predictions from theoretical stellar models. It includes all EBs for which there is no evidence for current or past mass transfer, and with mass and radius measurements to precisions of 2% or better. The full physical properties of the systems are collected (mass, radius, surface gravity, temperature, luminosity, orbital period, metallicity) and can be downloaded in ASCII format. At the time of writing (2019/12/01) DEBCat contains 239 systems.

*<https://www.astro.keele.ac.uk/jkt/debcats/>

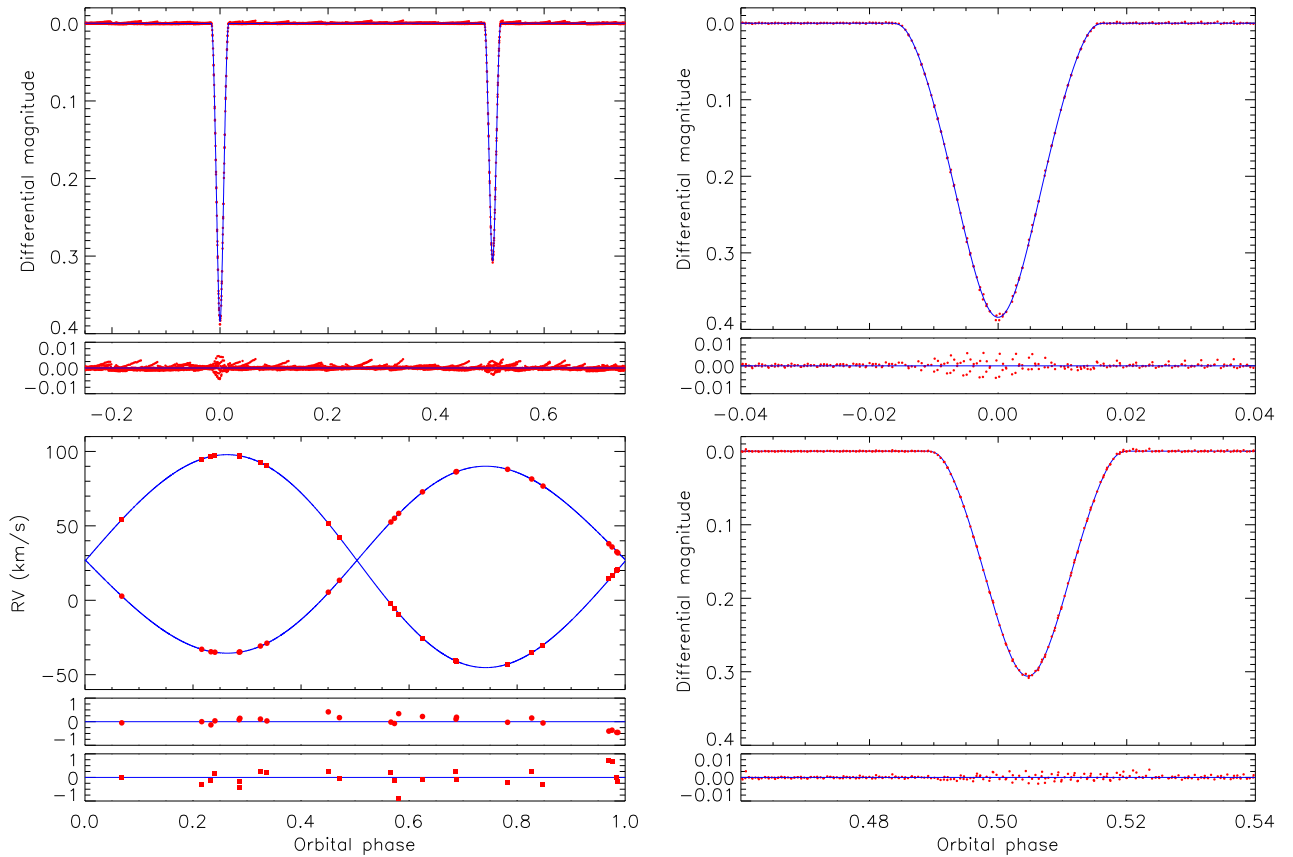


Fig. 1. Observational data for WASP J0341. Top left: *K2* light-curve versus orbital phase. Top right: primary eclipse. Bottom right: secondary eclipse. Bottom left: RV curves.

3 1SWASP J034114.25+201253.5

We now present some preliminary work on the solar-type eclipsing binary system 1SWASP J034114.25+201253.5 (hereafter WASP J0341), performed in collaboration with P. Maxted (Keele), G. Torres (CfA) and K. Pavlovski (Zagreb). WASP J0341 was identified as an eclipsing binary in the vicinity of the Pleiades open cluster by using photometric data from the SuperWASP database (Pollacco et al. 2006). Because of the scientific importance of eclipsing binaries that are cluster members (e.g. Southworth et al. 2004a; Brogaard et al. 2011) we proceeded to obtain several sets of additional observations. We were awarded observations of WASP J0341 from the *K2* mission, which observed it in long cadence for 71 days during Campaign 4. The PDC light-curve from the Kepler data reduction pipeline is shown in Fig. 1; more sophisticated reductions of these data exist and will be used for future analyses.

We also obtained a set of 30 high-resolution spectra of this system from the 1.5 m TRES spectrograph on the 1.5 m Tillinghast telescope at FLWO. RVs from these data are shown in Fig. 1. A joint fit of these RVs and the *K2* light-curve using the JKTEBOP code (Southworth 2013) yielded masses of 1.08 and 0.95 M_{\odot} and radii of 1.21 and 0.93 R_{\odot} , all measured to a precision of 0.5% or better. We have also obtained a set of six high-S/N spectra from VLT/UVES that will be used to measure the chemical composition of the two stars, yielding an even more precise test of theoretical models.

WASP J0341 has an 8-day orbital period and a small eccentricity, so tidal effects in this system are small. Its properties will therefore serve as an excellent test of theoretical models of solar-type stars. A careful inspection of Fig. 1 shows that the scatter of the data increases during both eclipses. This is an indication of temporal and spatial changes in the flux from the stellar surfaces, which is likely due to starspots. Our UVES data do show weak emission in the centres of the Ca H & K lines, a good indicator of magnetic activity in the stars (Baliunas et al. 1995).

4 Conclusions

Binary systems are a common result of the star formation process and offer the only known way of determining the masses of stars directly. Their frequency of occurrence offers a way to probe the star formation process. Eclipsing binary systems are those which can be characterised in detail; precise measurements of their properties enable an exacting assessment of the predictive power of theoretical stellar models, and provide one of the lower rungs of the cosmological distance ladder.

The advent of space satellites has revolutionised the study of eclipsing binaries, with the discovery of several thousand examples in data from CoRoT (Deleuil et al. 2018) and Kepler (Kirk et al. 2016). TESS is currently providing new data for the great majority of known eclipsing binaries, and in future PLATO will provide unparalleled photometric observations of many known and new examples.

I thank the organisers of the conference for their invitation to give this review talk, my many collaborators on eclipsing binaries, and the students of the PHY-30024 module who are gradually and uncomplainingly weeding out all the typographical and mathematical errors in my lecture notes on binary stars and extrasolar planets.

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