Supplementary material to:

# An analysis about the accuracy of Probabilistic Geographic Profiling in relation to the number of observations

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Fig. 1S. Descriptive Plot.

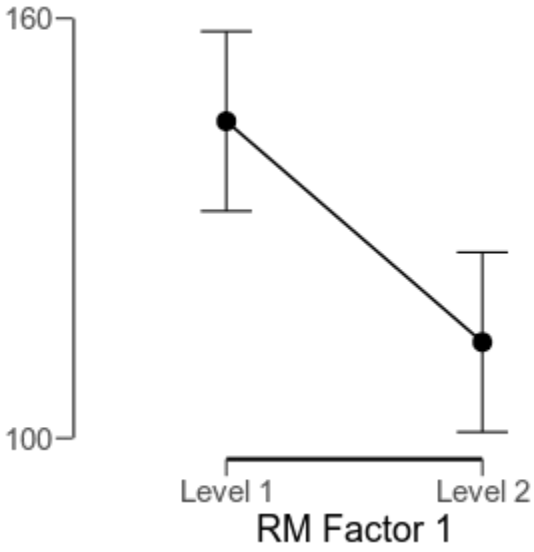

Fig. 2S. Distance between the curves with delta = 10%.

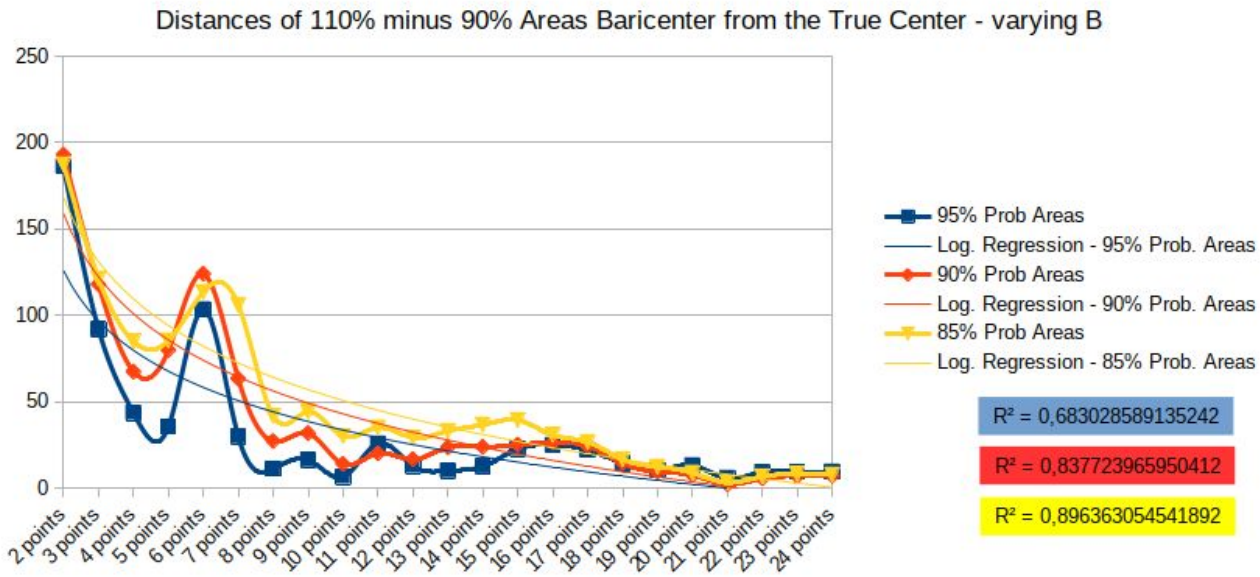


Fig. 3S. Distances of 120% minus 80% areas baricenter from the true center by variation of the B value.

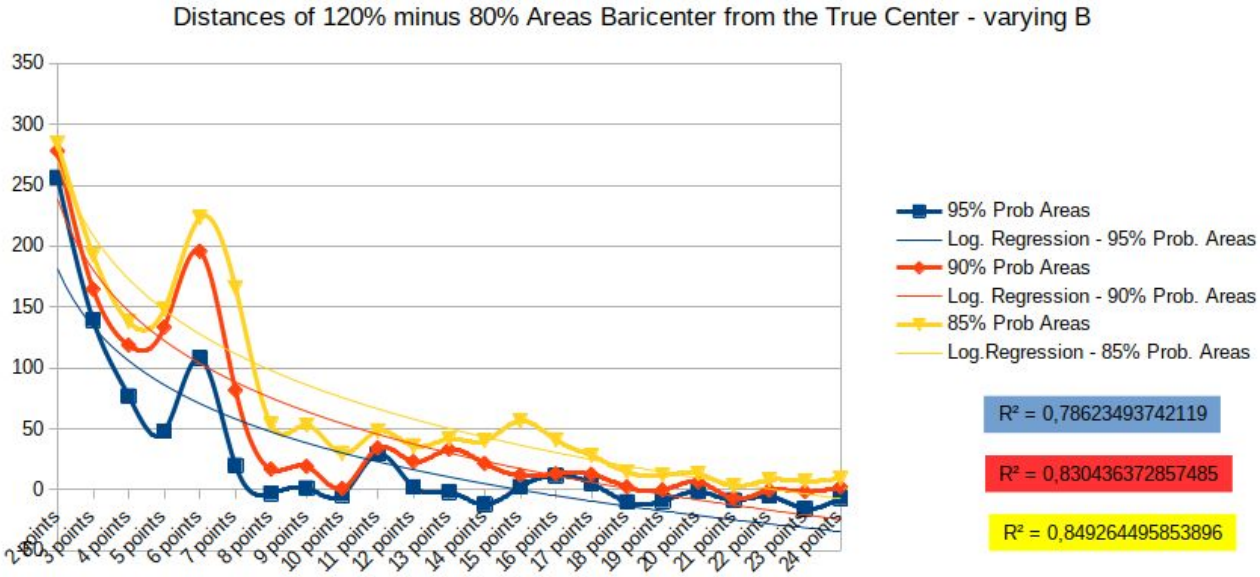

Table 1S. List of the cases related to Boston strangler case.

| N. | Name | Surname | Age | Date | Address | | City |
|---|---|---|---|---|---|---|---|
| 1 | Anna | Slesers | 56 | June 14 1962 | 77, Gainsborough St. | Back Bay | Boston |
| 2 | Mary | Mullen | 85 | June 28 1962 | 1435, Commonwealth Ave. | | Boston |
| 3 | Nina | Nichols | 68 | June 30 1962 | 1940, Commonwealth Ave. | | Boston |
| 4 | Helen | Blake | 65 | June 30 1962 | 73, Newhall St. | | Lynn |
| 5 | Ida | Irga | 75 | August 19 1962 | 7, Grove St. | Beacon Hill | Boston |
| 6 | Jane | Sullivan | 67 | August 21 1962 | 435, Columbia Rd. | Dorchester South | Boston |
| 7 | Sophie | Clark | 20 | December 5 1962 | 315, Huntington Ave. | Back Bay | Boston |
| 8 | Patricia | Bissette | 23 | December 31 1962 | 515, Park Drive | Back Bay | Boston |
| 9 | Mary | Brown | 69 | March 6 1963 | 319, Park Ave. | | Lawrence |
| 10 | Beverly | Samans | 23 | May 6 1963 | 4, University Rd. | | Cambridge |
| 11 | Evelyn | Corben | 58 | September 6 1963 | 224, Lafayette St. | | Salem |
| 12 | Joann | Graff | 23 | November 23 1963 | 54, Essex St. | | Lawrence |
| 13 | Mary | Sullivan | 19 | January 4 1964 | 44-A, Charles St. | | Boston |

Table 2S. Table of values conversion to the pixel position on the map.

| **Case ID** | **Real Distance from the offender's home (km)** | **X Coordinate** | **Y Coordinate** | **Distance from the offender's home (pixels)** | | **B Values (Km)** | **B Values (pixels)** |
|---|---|---|---|---|---|---|---|
| Home | 0,00 | 493 | 633 | 0 | | | |
| 1 | 9,63 | 476 | 761 | 129,12 | | 5,7544 | 75 |
| 2 | 10,41 | 421 | 748 | 135,68 | | 6,5216 | 85 |
| 3 | 12,08 | 404 | 762 | 156,72 | | 7,2889 | 95 |
| 4 | 11,50 | 635 | 587 | 149,26 | | 8,0561 | 105 |
| 5 | 7,40 | 498 | 729 | 96,13 | | 8,8233 | 115 |
| 6 | 12,85 | 498 | 800 | 134,73 | | 9,5906 | 125 |
| 7 | 9,80 | 479 | 767 | 121,84 | | 10,3578 | 135 |
| 8 | 9,32 | 459 | 750 | 431,73 | | 11,1251 | 145 |
| 9 | 33,00 | 385 | 215 | 98,18 | | 11,8923 | 155 |
| 10 | 7,36 | 439 | 715 | 227,79 | | 12,6596 | 165 |
| 11 | 17,41 | 686 | 512 | 412,28 | | 13,4268 | 175 |
| 12 | 31,79 | 406 | 230 | 412,28 | | 14,1941 | 185 |
| 13 | 7,85 | 495 | 736 | 103,02 | | 14,9613 | 195 |
| | | | | | | | |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| Mean | 13,88 km | 200,68 pix | | | | | |
| Standard Deviation | 8,65 km | 128,82 pix | | | | | |
| Median | 10,41 km | 135,68 pix | | | | Pixel Linear Dimension. | 0,0767 Km |

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How the Pareto algorithm works (see Python script My_Pareto_1.py)

***Pseudo-algorithm**:*

It generates a sequence of n_max/2 points with a Pareto distribution as follows:

1. Generates a sequence of n_max/2 points with a Pareto distribution
2. Unites the two sequences one after the other and randomly mixes the data (radiuses vector)
3. Generates a sequence of n_max points with uniform distribution (angles vector)
4. Multiplies the radiuses vector so that the maximum is 95% of min (image_width/2, image_heigth/2) (normalization on the map).
5. Converts the two sequences in a two dimensional vector
6. Converts the vector from polar coordinates to cartesian coordinates
7. Translates the resulting vector so the centre coincides with that of the image (image_width/2, image_heigth/2)
8. Visualizes the radial and spatial distribution
9. Writes data and reconstruction parameters

***Execution of reconstructions***

*Pseudo-algorithm:*

*For  n = 1 to n_max*

1. *extracts the first n points from the data set*
2. *executes the reconstruction of the PGF after the parameters provided by the previous procedure (it will generate 2 images)*
3. *discards the redundant images*

***Execution of counts***

One critical aspect of PGF for the reconstruction of the areas with highest probability is the use of a B parameter that can be as close as possible to the real B value ($B_{exact}$).

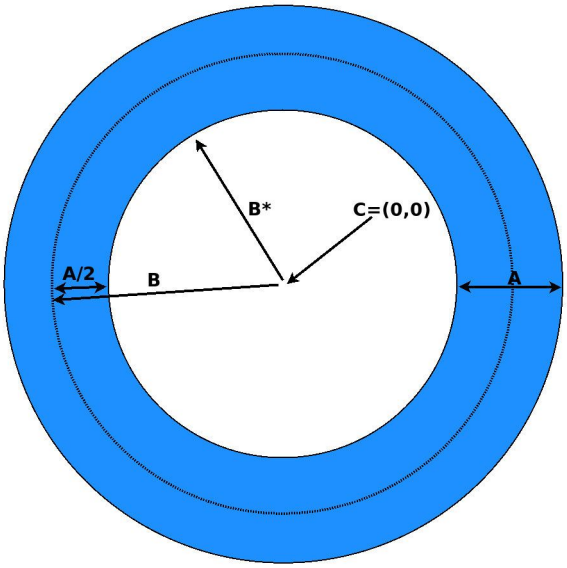
In our case we know $B_{exact}$, since we provided it during the procedure of generation of the points (on the map). For this reason, we executed a series of reconstructions based on the same data set, with B values corresponding to 90%, 80% and 60% of $B_{exact}$ in order to verify how the use of incorrect values of B affect the identification of the spread centre and in which amount. For this purpose it is needed to investigate the shape and dimension of the red zone.

The choice of the above mentioned B values allows testing of B values that are not too far from the real value, since no objective criterion is currently known to define a range of "feasible" B values only on the base of the events distribution, while in the real world, the B value should be evaluated case by case by analysing the type of event and cause (for instance, the biology and propagation mode of an invading organism). For too high B values the buffer zone could enlarge so much to create a "border effect", and then some of the points/events may localize outside the map and then be truncated. In this last case we would have an alteration of the results based on counting.

Currently the choice of B values is justified either on the basis of a priori knowledge of the spreading mechanism, the events and the cause (spreading centre) or on the basis of previous analogous studies. For this reason the choice of B appears to be crucial and can affect the feasibility of the PGF reconstruction. Further investigations will be necessary to provide insights on this last topics.

GPF v3.0.2 and the other programs here used were written by the authors (Santosuosso and Papini 2018) and are released under GPL license and available on Bitbucket (https://bitbucket.org/ugosnt/al_and_ugo/).*

**About the probability distribution around the spreading center**

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$F(x)=\alpha\frac{H}{x^{\alpha+1}}$ where 0 < H ≤ x < ∞ describes the envelope of the probability distribution towards the centre of the buffer zone and with α >0 the same but outside the buffer zone (Fig. 1).

**My pareto Python script**

```
#!/usr/bin/python

#
# Ugo Santosuosso - actual version: 0.1.2
#
# Version History - vers 0.1.2 - need code optimization and a better output
(now only on terminal)
#                 - vers 0.1.1 - need any debug
#                 - vers 0.1.0 - translated comments and variables to
english
#                 - vers 0.0.3 - transformed the most frequent codes into
functions
#                 - vers 0.0.2 - added matplotlib graphics, utf-8
conversion and timer base random seed
```

```
#                 - vers 0.0.1 - initial working codes
#
# Instruction for bypass UTF-8 Exceptions
# encoding=utf8
import sys
reload(sys)
#
import random as rnd
import numpy as np
import time
import argparse
import math
import matplotlib.pyplot as plt

def Random_Angles(n_max,factor):
    #
    # function that generates a sequence of random angles
    #
    Angles=[]
    #
    # scale_factor=factor # unused here...
    #
    RAND=int(time.time())
    rnd.seed(RAND)
    i=0
    base=0.0
    while i<n_max :
        Angle=base+((rnd.uniform(0,2*math.pi)))
        Angles.append(Angle)
        base += Angle
        i += 1
    return Angles

def my_pareto(alpha,n_max, Max_Value):
    #
    # function that generates a vector of given length containing the
values of the pareto distribution
    #
    i=0
    Values=[]
    while i<n_max:
        valore=(int(rnd.paretovariate(alpha)))
        if valore < Max_Value:
            Values.append(valore)
            i += 1
    return Values

def polar_2_rect_coords(Radius,Angle):
    #
    # function that transforms 2 values that express polar coordinates
    # of one point into two other values that express
    # the same point in Cartesian coordinates
    #
    x=Radius*math.sin(Angle)
```

```
    y=Radius*math.cos(Angle)
    return x,y

def vect_pol_2_rect(radiuses,Angles):
    #
    # function that transforms 2 vectors that
    # express polar coordinates of one point into two other
    # vectors that express the same point in Cartesian coordinates
    #
    coord_X=[]
    coord_Y=[]
    for i in range (len (radiuses)):
        temp1,temp2=polar_2_rect_coords(radiuses[i],Angles[i])
        coord_X.append(temp1)
        coord_Y.append(temp2)
    return coord_X,coord_Y
#
# ****** main ******
#
# initially it generates data in polar coordinates and then it will convert
them to cartesian coordinates
# hence the radial distribution will be that of the Rossmo formula, while
the radial one
# will be uniform, so to reach the maximal number of necessary hypotheses
for the correct working
# of the method
#
# B is equal to Rossmo formula "B"
# alpha is Rossmo formula "f" & "g" ( assuming f and g are equals )
#
B = 100
alfa=0.4
n_max=12
#
# random variation amplitude
#
x0=28
Values_a=[]
Values_b=[]
Angles=[]
#
# init of random numbers generation
#
RAND=int(time.time())
rnd.seed(RAND)
rng=rnd.getstate()
rnd.randrange(0, x0)
Max_Value=45
#
#
# generate a pareto distribution for Values variing  from zero to Max_Value
#
Values_a=my_pareto(alfa,n_max, Max_Value)
#
```

```
# generate a second pareto distribution for Values variing  from zero to
Max_Value
#
Values_b=my_pareto(alfa,n_max, Max_Value)
#
# flip the distribution around simmetry axe x=1
#
for i in range(len(Values_b)):
    Values_b[i]=1-(Values_b[i])
#
# concatenate the lists Values_a and Values_b, so we have a two tiles
pareto distribution with max in x=1
#
Values=Values_a+Values_b
#
# Multiply for B the radius otherwise the maximum is in 1 ( normalized
distribution )
# to simulate the dead zone in polar coordinates

for i in range (len(Values)):
    Values[i]=Values[i]*B
#
# shuffle the vector values
#
np.random.shuffle(Values)
#
# create the vector containing the angles
# they are generated with a uniform distribution
#
scale_factor=3.0/47.0 #(ratio between 2 prime numbers, just to make sure
itâ€™s irrational)
Angles=Random_Angles((2*n_max),(math.pi*scale_factor))
#
# Polar to rectagnular coords conversion
#
Coord_X,Coord_Y=vect_pol_2_rect(Values,Angles)
#
# generate and plot distribution histogram
#
hist1,_=np.histogram(Values,2*n_max)
plt.figure()
#
# Instructions to construct the graph of the generated data
#
Range=np.linspace(hist1.min(), hist1.max(), 20)
Range_A=np.array(Range)
plt.subplot(2,1,1)
plt.title('Distribution of Points Distances From Center - Histogram ')
plt.hist(Values,bins=n_max)
plt.subplot(2,1,2)
plt.title('Spatial Distribution of Generated Points')
#plot raw data
plt.scatter(Coord_X,Coord_Y,marker=".",color="red")
#plot distribution center
```

```
plt.scatter(np.zeros(1),np.zeros(1),marker="*",color="black")
#print data labes
for i in range(len(Coord_X)):
    plt.text(Coord_X[i],Coord_Y[i], i,fontsize=9)
plt.text(0.3, 0.3, "Diffusion Center", fontsize=9)
plt.xlabel('X Coord')
plt.ylabel('Y Coord')
plt.show()
#
# I print the data to video, then I have to save data to csv
#
for i in range(len(Coord_X)):
    print i,";", int(Coord_X[i]),";",int(Coord_Y[i]), ";100"
print
#
# Saving data to csv - with semicolon as separator of values
#
#
Output_file_name="out_file.csv"
#
# surely not the most elegant way to do it, but I will improve it later
#
my_File = open(Output_file_name,â€Žwâ€Ž)
for i in range(len(Coord_X)):
    my_string=str(i)+";"+str(int(Coord_X[i])+";"+str(int(Coord_Y[i])) +
";100"
    my_File.write=(my_string)
print
my_File.close()
#
print "End"
#
```

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**Linear correlation between the curves obtained with the different B values**

The linear correlation between the curves obtained with the different B values (correlation of data obtained with the simulation with the theoretical data obtained with the authors' provided parameters) produce the results shown in Table 2 (ANOVA with $p<10^{-15}$):

Table 3S

**Model Summary**

| Model | R | $R^2$ | Adjusted $R^2$ | RMSE |
|---|---|---|---|---|
| 1 | 0.9816 | 0.9635 | 0.9618 | 24.8706 |

**ANOVA**

| Model | | Sum of Squares | df | Mean Square | F | p |
|---|---|---|---|---|---|---|
| 1 | Regression | 343174.3334 | 1 | 343174.3334 | 554.8074 | 1.3913e -16 |
| | Residual | 12989.4823 | 21 | 618.5468 | | |
| | Total | 356163.8157 | 22 | | | |

**Coefficients**

| Model | | Unstandardized | Standard Error | Standardized | t | p |
|---|---|---|---|---|---|---|
| 1 | (Intercept) | -11.4123 | 8.4353 | | -1.3529 | 0.1905 |
| | Media_95_B110 | 1.3780 | 0.0585 | 0.9816 | 23.5544 | 1.391e -16 |

**Table 3S. Ex table 2. Linear Regression between curves obtained with the different B values**

**Table 4S. Ex table 3. Bayesian Repeated Measures ANOVA**

**Model Comparison**

| Models | P(M) | P(M\|data) | $BF_M$ | $BF_{10}$ | error % |
|---|---|---|---|---|---|
| RM Factor 1 | 0.5000 | 0.9496 | 18.8511 | 1.0000 | |
| Null model (incl. subject) | 0.5000 | 0.0504 | 0.0530 | 0.0530 | 1.0701 |

*Note.* All models include subject.

**Descriptives**

**Descriptives**

| RM Factor 1 | Mean | SD | N |
|---|---|---|---|
| Level 1 | 145.2928 | 127.2370 | 23.0000 |
| Level 2 | 113.7168 | 90.6332 | 23.0000 |