Auxiliary file to the paper titled

On the reconstruction of the center of a projection by distances and incidence relations

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by the authors

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unified_irreducibility

Andras

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```
var('u,a,b,c,x,y,A,B,C,D,E,F')
(u, a, b, c, x, y, A, B, C, D, E, F)
```

```
d_4=(a^4 - 2*a^2*b^2 + b^4 - 2*a^2*c^2 - 2*b^2*c^2 + c^4)
d_3=4*(u*a^4 + (1-u)*a^2*b^2 - u*a^2*c^2 - b^4 + b^2*c^2)
d_2x=2*(3*u^2*a^4 + (-u^2 + 4*u - 1)*a^2*b^2 - u^2*a^2*c^2 + 3*b^4 -\
    b^2*c^2)
d_2y=2*(u^2*a^4 - (u^2+1)*a^2*b^2 - u^2*a^2*c^2 + b^4 - b^2*c^2)
d_1=4*(u^3*a^4 + (u^2-u)*a^2*b^2 - b^4)
d_0=(u^4*a^4 - 2*u^2*a^2*b^2 + b^4)
```

```
################################
################################
# Case (1)
################################
################################
```

```
var('u,a,b,c,x,y,A,C')

poly_fact_1=expand(
(x^2+2*i*x*y-y^2+ A*x + i*A*y +C)*(x^2-2*i*x*y-y^2+ A*x -i*A*y +C)
)

poly_fact_1.maxima_methods().collectterms(x,y)
(u, a, b, c, x, y, A, C)
2*A*x^3 + x^4 + 2*A*x*y^2 + 2*x^2*y^2 + y^4 + 2*A*C*x + (A^2 + 2*C)*x^2 + (A^2 -
2*C)*y^2
+ C^2
```

```
# Clearly 2*(d_2x + d_2y)*d_0 - d_1^2 = 0.
```

```
# We verify that the left hand side is  -8*(u^2*a^2-b^2)^2*((u^2*a^2+\
    b^2)*c^2+(u+1)^2*a^2*b^2), which is zero iff b=u*a.
# However, that is impossible: in that case, C=0, and then d_2x-d_2y\
    =0, a contradiction.

expr_1=expand(
(2*(d_2x + d_2y)*d_0 - d_1^2) +
8*(u^2*a^2-b^2)^2*((u^2*a^2+b^2)*c^2+(u+1)^2*a^2*b^2)
)

expr_1.maxima_methods().collectterms(a,b)
0
```

```
##################################
##################################
# Case (2)
##################################
##################################

var('u,a,b,c,x,y,A,C,D,F')

poly_fact_2=expand(
(x^2+y^2+ A*x + C)*(x^2+y^2+ D*x + F)
)

poly_fact_2.maxima_methods().collectterms(x,y)
(u, a, b, c, x, y, A, C, D, F)
(A + D)*x^3 + x^4 + (A + D)*x*y^2 + 2*x^2*y^2 + y^4 + (A*D + C + F)*x^2 + (C + F)*y^2 +
C*F + (C*D + A*F)*x
```

```
# From two quadratic equations, we obtain the following necessary \
    condition (see the paper):
# d_1^2*d_4-d_1*d_3*d_2y+(d_2x-d_2y)*d_2y^2+d_0*d_3^2-4*d_0*(d_2x-\
    d_2y)*d_4.
# We verify that the left hand side is 64*(u^4+2*u^3+u^2)*a^4*b^4*c\
    ^4, which is negative, a contradiction.

expr_2=expand(
(d_1^2*d_4-d_1*d_3*d_2y+(d_2x-d_2y)*d_2y^2+d_0*d_3^2-4*d_0*(d_2x-\
    d_2y)*d_4) -
64*(u^4+2*u^3+u^2)*a^4*b^4*c^4
)

expr_2.maxima_methods().collectterms(a,b)
0
```

```
var('u,a,b,c,x,y,A,B,C')

poly_fact_3=expand(
(x^2+y^2+ A*x + B*y + C)*(x^2+y^2+ A*x - B*y + C)
)

poly_fact_3.maxima_methods().collectterms(x,y)
```
(u, a, b, c, x, y, A, B, C)
2*A*x^3 + x^4 + 2*A*x*y^2 + 2*x^2*y^2 + y^4 + 2*A*C*x + (A^2 + 2*C)*x^2 - (B^2 - 2*C)*y^2
+ C^2


```
# Clearly, d_1^2*d_4 - d_3^2*d_0 = 0.
# We verify that the above expression is 64*a^2*b^2*c^2*(u^2*a^2 - b\
    ^2)^2*(u*c^2-((u^2+u)*a^2 + (u+1)*b^2)).
# Thus b=u*a, or c^2=(u+1)*(a^2 + (1/u)*b^2).

expr_3_1=expand(
(d_1^2*d_4 - d_3^2*d_0) -
64*a^2*b^2*c^2*(u^2*a^2 - b^2)^2*(u*c^2-((u^2+u)*a^2 + (u+1)*b^2))
)
expr_3_1.maxima_methods().collectterms(a,b,c)
```
0

```
# Our next goal is to conclude b=u*a from the other possibility c\
    ^2=(u+1)*(a^2 + (1/u)*b^2) as well.
# We have (4*d_2x*d_4 - d_3^2)^2 - 64*d_0*d_4^3 = 0.
# The left hand side is computed below.

expr_3_2=expand(
((4*d_2x*d_4 - d_3^2)^2 - 64*d_0*d_4^3)
)
(expr_3_2).maxima_methods().collectterms(c)
```
256*a^2*b^2*c^12*u^2 + 256*(a^4*b^2*u^4 - 4*a^4*b^2*u^3 - 5*a^4*b^2*u^2 - 5*a^2*b^4*u^2 -

4*a^2*b^4*u + a^2*b^4)*c^10 - 512*(2*a^4*b^4*u^4 - 8*a^6*b^2*u^3 - 4*a^4*b^4*u^3 -
5*a^6*b^2*u^2 - 18*a^4*b^4*u^2 - 5*a^2*b^6*u^2 - 4*a^4*b^4*u - 8*a^2*b^6*u +
2*a^4*b^4)*c^8 - 512*(3*a^8*b^2*u^4 - 3*a^4*b^6*u^4 + 12*a^8*b^2*u^3 + 20*a^6*b^4*u^3 +
5*a^8*b^2*u^2 + 27*a^6*b^4*u^2 + 27*a^4*b^6*u^2 + 5*a^2*b^8*u^2 + 20*a^4*b^6*u +

12*a^2*b^8*u - 3*a^6*b^4 + 3*a^2*b^8)*c^6 + 256*(8*a^10*b^2*u^4 + 12*a^8*b^4*u^4 - 4*a^4*b^8*u^4 + 16*a^10*b^2*u^3 + 24*a^8*b^4*u^3 + 32*a^6*b^6*u^3 - 8*a^4*b^8*u^3 + 5*a^10*b^2*u^2 + 20*a^8*b^4*u^2 + 46*a^6*b^6*u^2 + 20*a^4*b^8*u^2 + 5*a^2*b^10*u^2 -

8*a^8*b^4*u + 32*a^6*b^6*u + 24*a^4*b^8*u + 16*a^2*b^10*u - 4*a^8*b^4 + 12*a^4*b^8 + 8*a^2*b^10)*c^4 - 256*(3*a^12*b^2*u^4 - 8*a^10*b^4*u^4 + 6*a^8*b^6*u^4 - a^4*b^10*u^4 + 4*a^12*b^2*u^3 - 8*a^10*b^4*u^3 + 8*a^6*b^8*u^3 - 4*a^4*b^10*u^3 + a^12*b^2*u^2 - 3*a^10*b^4*u^2 + 2*a^8*b^6*u^2 + 2*a^6*b^8*u^2 - 3*a^4*b^10*u^2 + a^2*b^12*u^2 - 4*a^10*b^4*u + 8*a^8*b^6*u - 8*a^4*b^10*u + 4*a^2*b^12*u - a^10*b^4 + 6*a^6*b^8 - 8*a^4*b^10 + 3*a^2*b^12)*c^2

```
# Now we replace c^2 in expr_3_2 by (u+1)*(a^2 + (1/u)*b^2) \
    everywhere, and verify that the expression obtained is 256*a^2*b\
    ^2*((u+1)*(a^2 + (1/u)*b^2))*(u^2*a^2-b^2)^4*((1 + 1/u)*a^2 + (1/\
    u^3 + 1/u^4)*b^2). Hence, b=u*a indeed.

expr_3_3=expand(
(256*a^2*b^2*((u+1)*(a^2 + (1/u)*b^2))^6*u^2 +
256*(a^4*b^2*u^4 - 4*a^4*b^2*u^3 - 5*a^4*b^2*u^2 - 5*a^2*b^4*u^2 - \
    4*a^2*b^4*u + a^2*b^4)*((u+1)*(a^2 + (1/u)*b^2))^5 -
512*(2*a^4*b^4*u^4 - 8*a^6*b^2*u^3 - 4*a^4*b^4*u^3 - 5*a^6*b^2*u^2 -\
    18*a^4*b^4*u^2 - 5*a^2*b^6*u^2 - 4*a^4*b^4*u - 8*a^2*b^6*u + 2*a\
    ^4*b^4)*((u+1)*(a^2 + (1/u)*b^2))^4 -
512*(3*a^8*b^2*u^4 - 3*a^4*b^6*u^4 + 12*a^8*b^2*u^3 + 20*a^6*b^4*u^3\
    + 5*a^8*b^2*u^2 + 27*a^6*b^4*u^2 + 27*a^4*b^6*u^2 + 5*a^2*b^8*u\
    ^2 + 20*a^4*b^6*u + 12*a^2*b^8*u - 3*a^6*b^4 + 3*a^2*b^8)*((u+1)\
    *(a^2 + (1/u)*b^2))^3 +
256*(8*a^10*b^2*u^4 + 12*a^8*b^4*u^4 - 4*a^4*b^8*u^4 + 16*a^10*b^2*u\
    ^3 + 24*a^8*b^4*u^3 + 32*a^6*b^6*u^3 - 8*a^4*b^8*u^3 + 5*a^10*b\
    ^2*u^2 + 20*a^8*b^4*u^2 + 46*a^6*b^6*u^2 + 20*a^4*b^8*u^2 + 5*a\
    ^2*b^10*u^2 - 8*a^8*b^4*u + 32*a^6*b^6*u + 24*a^4*b^8*u + 16*a^2*\
    b^10*u - 4*a^8*b^4 + 12*a^4*b^8 + 8*a^2*b^10)*((u+1)*(a^2 + (1/u)\
    *b^2))^2 -
256*(3*a^12*b^2*u^4 - 8*a^10*b^4*u^4 + 6*a^8*b^6*u^4 - a^4*b^10*u^4 \
    + 4*a^12*b^2*u^3 - 8*a^10*b^4*u^3 + 8*a^6*b^8*u^3 - 4*a^4*b^10*u\
    ^3 + a^12*b^2*u^2 - 3*a^10*b^4*u^2 + 2*a^8*b^6*u^2 + 2*a^6*b^8*u\
    ^2 - 3*a^4*b^10*u^2 + a^2*b^12*u^2 - 4*a^10*b^4*u + 8*a^8*b^6*u -\
    8*a^4*b^10*u + 4*a^2*b^12*u - a^10*b^4 + 6*a^6*b^8 - 8*a^4*b^10 \
    + 3*a^2*b^12)*((u+1)*(a^2 + (1/u)*b^2))) -
256*a^2*b^2*((u+1)*(a^2 + (1/u)*b^2))*(u^2*a^2-b^2)^4*((1 + 1/u)*a^2\
    + (1/u^3 + 1/u^4)*b^2)
)
(expr_3_3).maxima_methods().collectterms(a,b)
0
```

```
# Thus d_0=0, and consequently, C=0, which implies d_3^2-4*d_2x*d_4\
    =0. See the computation of the left hand side below.

expr_3_4=expand(
(d_3^2-4*d_2x*d_4)
)
(expr_3_4).maxima_methods().collectterms(a,c)
-8*a^8*u^2 + 24*a^6*c^2*u^2 - 24*a^4*c^4*u^2 + 8*a^2*c^6*u^2 - 8*b^8 + 24*b^6*c^2 -
24*b^4*c^4 + 8*b^2*c^6 + 8*(3*b^2*u^2 + b^2)*a^6 + 8*(6*b^2*u^2 + 8*b^2*u - b^2)*a^4*c^2 -

8*(b^2*u^2 + 8*b^2*u + b^2)*a^2*c^4 - 24*(b^4*u^2 + b^4)*a^4 - 8*(b^4*u^2 - 8*b^4*u -
6*b^4)*a^2*c^2 + 8*(b^6*u^2 + 3*b^6)*a^2

#  We verify that if b=u*a, then the left hand side is 16*u^2*a^2*c\
    ^2*(c^2-(u+1)^2*a^2)^2. Thus c=(u+1)*a, and in particular c=a+b, \
    a contradiction.

b=u*a

expr_3_5=expand(
(-8*a^8*u^2 + 24*a^6*c^2*u^2 - 24*a^4*c^4*u^2 + 8*a^2*c^6*u^2 - 8*b\
    ^8 + 24*b^6*c^2 - 24*b^4*c^4 + 8*b^2*c^6 + 8*(3*b^2*u^2 + b^2)*a\
    ^6 + 8*(6*b^2*u^2 + 8*b^2*u - b^2)*a^4*c^2 - 8*(b^2*u^2 + 8*b^2*u\
    + b^2)*a^2*c^4 - 24*(b^4*u^2 + b^4)*a^4 - 8*(b^4*u^2 - 8*b^4*u -\
    6*b^4)*a^2*c^2 + 8*(b^6*u^2 + 3*b^6)*a^2) -
16*u^2*a^2*c^2*(c^2-(u+1)^2*a^2)^2
)
(expr_3_5).maxima_methods().collectterms(a,c)
0


#######################################
#######################################
# Case (4)
#######################################
#######################################

var('u,a,b,c,x,y,A,C,D')

poly_fact_4=expand(
(x^3 - i*x^2*y + x*y^2 - i*y^3 + A*x^2 + i*(C-A)*x*y + C*y^2 + D*x +\
    i*(A*C-C^2-D)*y + C*(C^2-A*C+D))*(x + i*y + C)
)

poly_fact_4.maxima_methods().collectterms(x,y)
(u, a, b, c, x, y, A, C, D)
-A*C^3 + C^4 + (A + C)*x^3 + x^4 + (A + C)*x*y^2 + 2*x^2*y^2 + y^4 + C^2*D + (A*C + D)*x^2
```

- (A*C - 2*C^2 - D)*y^2 - (A*C^2 - C^3 - 2*C*D)*x

#(2*A*C - 2*C^2)*C^2 - 2*(A*C^2 - C^3 - 2*C*D)*C + 4*(-A*C^3 + C^4 +\
    C^2*D)=0, so C is a root of the quadratic polynomial (d_2x-d_2y)\
    *z^2 - 2*d_1*z+4*d_0 (after multiplication by d_4). The \
    discriminant of this polynomial is zero, shown by the following \
    caculation. (The leading coefficient is not zero, thus the \
    polynomial is really quadratic.)

expr_4_1=expand(
4*d_1^2-4*(d_2x-d_2y)*4*d_0
)
expr_4_1.maxima_methods().collectterms(a,b,c)
0

Hence, C = d_1/(d_{2,x}-d{2,y}). We find another quadratic \
    equation for C, and substitute this in it.
That other equation is obtained by A= d_3/d_4-C and 2*C(A-C)=(d_2x\
    -d_2y)/d_4. Hence, we have 4*C^2 - 2*d_3/d_4*C + (d_2x-d_2y)/d_4 \
    =0, that is, 4*d_4*C^2 - 2*d_3*C + (d_2x-d_2y) =0. Putting C = \
    d_1/(d_2x-d_2y) yields 4*d_4*d_1^2 - 2*d_3*d_1*(d_2x-d_2y) + (\
    d_2x-d_2y)^3 =0. See the left hand side below.

expr_4_2=expand(
4*d_4*d_1^2 - 2*d_3*d_1*(d_2x-d_2y) + (d_2x-d_2y)^3
)
expr_4_2.maxima_methods().collectterms(c)
64*a^8*b^4*u^6 + 256*a^8*b^4*u^5 + 128*a^6*b^6*u^5 + 384*a^8*b^4*u^4 + 512*a^6*b^6*u^4 +
64*a^4*b^8*u^4 + 256*a^8*b^4*u^3 + 768*a^6*b^6*u^3 + 256*a^4*b^8*u^3 + 64*a^8*b^4*u^2 +
512*a^6*b^6*u^2 + 384*a^4*b^8*u^2 + 128*a^6*b^6*u + 256*a^4*b^8*u + 64*a^4*b^8 +
64*(a^8*u^6 + 2*a^6*b^2*u^5 - 2*a^6*b^2*u^4 + a^4*b^4*u^4 - 4*a^4*b^4*u^3 + a^4*b^4*u^2 -

2*a^2*b^6*u^2 + 2*a^2*b^6*u + b^8)*c^4 - 128*(a^8*b^2*u^6 + 2*a^6*b^4*u^5 - a^8*b^2*u^4 -

a^6*b^4*u^4 + a^4*b^6*u^4 - 2*a^6*b^4*u^3 - 2*a^4*b^6*u^3 + a^6*b^4*u^2 - a^4*b^6*u^2 -

a^2*b^8*u^2 + 2*a^4*b^6*u + a^2*b^8)*c^2

The expression expr_4_2 is quadratic in c^2, hence its \
    discriminant is non-negative. See the discriminant below.

expr_4_3=expand(
(128*(a^8*b^2*u^6 + 2*a^6*b^4*u^5 - a^8*b^2*u^4 - a^6*b^4*u^4 + a^4*\
    b^6*u^4 - 2*a^6*b^4*u^3 - 2*a^4*b^6*u^3 + a^6*b^4*u^2 - a^4*b^6*u\
    ^2 - a^2*b^8*u^2 + 2*a^4*b^6*u + a^2*b^8))^2 -

```
4*64*(a^8*u^6 + 2*a^6*b^2*u^5 - 2*a^6*b^2*u^4 + a^4*b^4*u^4 - 4*a^4*\
    b^4*u^3 + a^4*b^4*u^2 - 2*a^2*b^6*u^2 + 2*a^2*b^6*u + b^8)*(64*a\
    ^8*b^4*u^6 + 256*a^8*b^4*u^5 + 128*a^6*b^6*u^5 + 384*a^8*b^4*u^4 \
    + 512*a^6*b^6*u^4 + 64*a^4*b^8*u^4 + 256*a^8*b^4*u^3 + 768*a^6*b\
    ^6*u^3 + 256*a^4*b^8*u^3 + 64*a^8*b^4*u^2 + 512*a^6*b^6*u^2 + \
    384*a^4*b^8*u^2 + 128*a^6*b^6*u + 256*a^4*b^8*u + 64*a^4*b^8)
)
expr_4_3.maxima_methods().collectterms(a,b)
-65536*(u^11 + 2*u^10 + u^9)*a^16*b^4 - 131072*(2*u^10 + 3*u^9 - u^7)*a^14*b^6 -
65536*(6*u^9 + 4*u^8 - 9*u^7 - 6*u^6 + u^5)*a^12*b^8 - 262144*(u^8 - u^7 - 4*u^6 - u^5 +
u^4)*a^10*b^10 - 65536*(u^7 - 6*u^6 - 9*u^5 + 4*u^4 + 6*u^3)*a^8*b^12 + 131072*(u^5 -
3*u^3 - 2*u^2)*a^6*b^14 - 65536*(u^3 + 2*u^2 + u)*a^4*b^16


# See the verification below that expr_4_3 coincides with 65536*a^4*\
    b^4*(b^2-u^2*a^2)^2*(-(u^3 + 2*u^2 + u)*b^8 + (-4*u^4-8*u^3-4*u\
    ^2)*a^2*b^6 + (-6*u^5-12*u^4-6*u^3)*a^4*b^4 + (-4*u^6-8*u^5-4*u\
    ^4)*a^6*b^2 + (-u^7-2*u^6-u^5)*a^8), which is clearly negative \
    unless b=u*a.
# Hence, b=u*a, and then d_0=d_1=0. Note that C cannot be zero, as \
    otherwise d_2x-d_2y=0. Thus A*C^2-C^3-CD=0, and then CD=0, which \
    yields D=0. Then by d_0=0 we have A=C, and thus d_2x-d_2y=0, a \
    contradiction.

expr_4_4=expand(
(-65536*(u^11 + 2*u^10 + u^9)*a^16*b^4 - 131072*(2*u^10 + 3*u^9 - u\
    ^7)*a^14*b^6 - 65536*(6*u^9 + 4*u^8 - 9*u^7 - 6*u^6 + u^5)*a^12*b\
    ^8 - 262144*(u^8 - u^7 - 4*u^6 - u^5 + u^4)*a^10*b^10 - 65536*(u\
    ^7 - 6*u^6 - 9*u^5 + 4*u^4 + 6*u^3)*a^8*b^12 + 131072*(u^5 - 3*u\
    ^3 - 2*u^2)*a^6*b^14 - 65536*(u^3 + 2*u^2 + u)*a^4*b^16) -
65536*a^4*b^4*(b^2-u^2*a^2)^2*(-(u^3 + 2*u^2 + u)*b^8 + (-4*u^4-8*u\
    ^3-4*u^2)*a^2*b^6 + (-6*u^5-12*u^4-6*u^3)*a^4*b^4 + (-4*u^6-8*u\
    ^5-4*u^4)*a^6*b^2 + (-u^7-2*u^6-u^5)*a^8)
)
expr_4_4.maxima_methods().collectterms(a,b)
0


################################
################################
# Case (5)
################################
################################

var('u,a,b,c,x,y,A,C,D')

poly_fact_5=expand(
```

```
(x^2+y^2+ A*x +C)*(x + D)
)


poly_fact_5.maxima_methods().collectterms(x,y)
(u, a, b, c, x, y, A, C, D)
(A + D)*x^2 + x^3 + D*y^2 + x*y^2 + C*D + (A*D + C)*x


# D=d_2y/d_3, A=(d_2x-d_2y)/d_3, and then C=d_1/d_3-A*D= (d_1*d_3-\
    d_2y*(d_2x-d_2y))/d_3^2.
# This yields C*D=(d_1*d_3-d_2y*(d_2x-d_2y))*d_2y/d_3^3, but we also\
     have C*D=d_0/d_3.
# Hence, (d_1*d_3-d_2y*(d_2x-d_2y))*d_2y - d_0*d_3^2 =0. See the \
    left hand side below: it is clearly negative, a contradiction.


var('a,b,c,u')
expr=expand(
(d_1*d_3-d_2y*(d_2x-d_2y))*d_2y - d_0*d_3^2
)

expr.maxima_methods().collectterms(a,b,c)
(a, b, c, u)
-64*(u^4 + 2*u^3 + u^2)*a^4*b^4*c^4

#################################
#################################
# Case (6)
#################################
#################################

var('u,a,b,c,x,y,A,B')

poly_fact_6=expand(
(x^2+i*x*y+ A*x + B*y +B*(B-i*A))*(x -i*y + (A + i*B))
)

poly_fact_6.maxima_methods().collectterms(x,y)
(u, a, b, c, x, y, A, B)
-I*A^2*B + 2*A*B^2 + I*B^3 + (2*A + I*B)*x^2 + x^3 - I*B*y^2 + x*y^2 + (A^2 + B^2)*x


# x^3 + x*y^2 + (2*A + I*B)*x^2 + (-I*B)*y^2 + (A^2 + B^2)*x + (- I*\
    A^2*B + 2*A*B^2 + I*B^3)
#
# d_2x/d_3 = (2*A + I*B)
# d_2y/d_3 = (-I*B)
# d_1/d_3      = (A^2 + B^2)
```

32*a^12*u^6 + 64*a^11*b*u^6 - 32*a^10*b^2*u^6 - 128*a^9*b^3*u^6 - 64*a^8*b^4*u^6 -
128*a^8*b^4*u^5 - 256*a^7*b^5*u^5 - 128*a^6*b^6*u^5 - 32*a^10*b^2*u^4 - 64*a^9*b^3*u^4 -

416*a^8*b^4*u^4 - 1280*a^7*b^5*u^4 - 1728*a^6*b^6*u^4 - 1152*a^5*b^7*u^4 - 320*a^4*b^8*u^4
- 640*a^8*b^4*u^3 - 2304*a^7*b^5*u^3 - 3328*a^6*b^6*u^3 - 2304*a^5*b^7*u^3 -
640*a^4*b^8*u^3 - 320*a^8*b^4*u^2 - 1152*a^7*b^5*u^2 - 1728*a^6*b^6*u^2 - 1280*a^5*b^7*u^2
- 416*a^4*b^8*u^2 - 64*a^3*b^9*u^2 - 32*a^2*b^10*u^2 - 128*a^6*b^6*u - 256*a^5*b^7*u -
128*a^4*b^8*u - 64*a^4*b^8 - 128*a^3*b^9 - 32*a^2*b^10 + 64*a*b^11 + 32*b^12 + 64*(a^8*u^6
- 2*a^6*b^2*u^5 - 2*a^6*b^2*u^4 + a^4*b^4*u^4 + 4*a^4*b^4*u^3 + a^4*b^4*u^2 -
2*a^2*b^6*u^2 - 2*a^2*b^6*u + b^8)*c^4 - 32*(3*a^10*u^6 + 4*a^9*b*u^6 - 4*a^8*b^2*u^5 -
8*a^7*b^3*u^5 - 8*a^6*b^4*u^5 - 5*a^8*b^2*u^4 - 8*a^7*b^3*u^4 - 14*a^6*b^4*u^4 -
12*a^5*b^5*u^4 - 8*a^4*b^6*u^4 - 12*a^6*b^4*u^3 - 16*a^5*b^5*u^3 - 12*a^4*b^6*u^3 -
8*a^6*b^4*u^2 - 12*a^5*b^5*u^2 - 14*a^4*b^6*u^2 - 8*a^3*b^7*u^2 - 5*a^2*b^8*u^2 -

8*a^4*b^6*u - 8*a^3*b^7*u - 4*a^2*b^8*u + 4*a*b^9 + 3*b^10)*c^2

c=a+b

expr_6_4=expand(
32*a^12*u^6 + 64*a^11*b*u^6 - 32*a^10*b^2*u^6 - 128*a^9*b^3*u^6 - \
    64*a^8*b^4*u^6 - 96*a^10*c^2*u^6 - 128*a^9*b*c^2*u^6 + 64*a^8*c\
    ^4*u^6 - 128*a^8*b^4*u^5 - 256*a^7*b^5*u^5 - 128*a^6*b^6*u^5 + \
    128*a^8*b^2*c^2*u^5 + 256*a^7*b^3*c^2*u^5 + 256*a^6*b^4*c^2*u^5 -\
     128*a^6*b^2*c^4*u^5 - 32*a^10*b^2*u^4 - 64*a^9*b^3*u^4 - 416*a\
    ^8*b^4*u^4 - 1280*a^7*b^5*u^4 - 1728*a^6*b^6*u^4 - 1152*a^5*b^7*u\
    ^4 - 320*a^4*b^8*u^4 + 160*a^8*b^2*c^2*u^4 + 256*a^7*b^3*c^2*u^4 \
    + 448*a^6*b^4*c^2*u^4 + 384*a^5*b^5*c^2*u^4 + 256*a^4*b^6*c^2*u^4\
     - 128*a^6*b^2*c^4*u^4 + 64*a^4*b^4*c^4*u^4 - 640*a^8*b^4*u^3 - \
    2304*a^7*b^5*u^3 - 3328*a^6*b^6*u^3 - 2304*a^5*b^7*u^3 - 640*a^4*\
    b^8*u^3 + 384*a^6*b^4*c^2*u^3 + 512*a^5*b^5*c^2*u^3 + 384*a^4*b\
    ^6*c^2*u^3 + 256*a^4*b^4*c^4*u^3 - 320*a^8*b^4*u^2 - 1152*a^7*b\
    ^5*u^2 - 1728*a^6*b^6*u^2 - 1280*a^5*b^7*u^2 - 416*a^4*b^8*u^2 - \
    64*a^3*b^9*u^2 - 32*a^2*b^10*u^2 + 256*a^6*b^4*c^2*u^2 + 384*a^5*\
    b^5*c^2*u^2 + 448*a^4*b^6*c^2*u^2 + 256*a^3*b^7*c^2*u^2 + 160*a\
    ^2*b^8*c^2*u^2 + 64*a^4*b^4*c^4*u^2 - 128*a^2*b^6*c^4*u^2 - 128*a\
    ^6*b^6*u - 256*a^5*b^7*u - 128*a^4*b^8*u + 256*a^4*b^6*c^2*u + \
    256*a^3*b^7*c^2*u + 128*a^2*b^8*c^2*u - 128*a^2*b^6*c^4*u - 64*a\
    ^4*b^8 - 128*a^3*b^9 - 32*a^2*b^10 + 64*a*b^11 + 32*b^12 - 128*a*\
    b^9*c^2 - 96*b^10*c^2 + 64*b^8*c^4
)
(expr_6_4).maxima_methods().collectterms(a,b)
0

Finally, we verify below that if c=|a-b|, then expr_6_2+4*a*b*(u\
    ^2*a^2-(u^2+1)*a*b+b^2)*expr_6_1 = 256*u^2*(u + 1)^2*a^4*b^4*(a-b\
)^4. As c=\pm(a-b) is not zero, this is a contradiction.
var('u,a,b,c,x,y,A,B')

expr_6_6=expand(
expr_6_2+4*a*b*(u^2*a^2-(u^2+1)*a*b+b^2)*expr_6_1 -
256*u^2*(u + 1)^2*a^4*b^4*(a-b)^4
)
(expr_6_6).maxima_methods().collectterms(c)
(u, a, b, c, x, y, A, B)
32*a^12*u^6 - 64*a^11*b*u^6 - 32*a^10*b^2*u^6 + 128*a^9*b^3*u^6 - 64*a^8*b^4*u^6 -
128*a^8*b^4*u^5 + 256*a^7*b^5*u^5 - 128*a^6*b^6*u^5 - 32*a^10*b^2*u^4 + 64*a^9*b^3*u^4 -

416*a^8*b^4*u^4 + 1280*a^7*b^5*u^4 - 1728*a^6*b^6*u^4 + 1152*a^5*b^7*u^4 - 320*a^4*b^8*u^4
- 640*a^8*b^4*u^3 + 2304*a^7*b^5*u^3 - 3328*a^6*b^6*u^3 + 2304*a^5*b^7*u^3 -

640*a^4*b^8*u^3 - 320*a^8*b^4*u^2 + 1152*a^7*b^5*u^2 - 1728*a^6*b^6*u^2 + 1280*a^5*b^7*u^2 - 416*a^4*b^8*u^2 + 64*a^3*b^9*u^2 - 32*a^2*b^10*u^2 - 128*a^6*b^6*u + 256*a^5*b^7*u - 128*a^4*b^8*u - 64*a^4*b^8 + 128*a^3*b^9 - 32*a^2*b^10 - 64*a*b^11 + 32*b^12 + 64*(a^8*u^6 - 2*a^6*b^2*u^5 - 2*a^6*b^2*u^4 + a^4*b^4*u^4 + 4*a^4*b^4*u^3 + a^4*b^4*u^2 - 2*a^2*b^6*u^2 - 2*a^2*b^6*u + b^8)*c^4 - 32*(3*a^10*u^6 - 4*a^9*b*u^6 - 4*a^8*b^2*u^5 + 8*a^7*b^3*u^5 - 8*a^6*b^4*u^5 - 5*a^8*b^2*u^4 + 8*a^7*b^3*u^4 - 14*a^6*b^4*u^4 + 12*a^5*b^5*u^4 - 8*a^4*b^6*u^4 - 12*a^6*b^4*u^3 + 16*a^5*b^5*u^3 - 12*a^4*b^6*u^3 - 8*a^6*b^4*u^2 + 12*a^5*b^5*u^2 - 14*a^4*b^6*u^2 + 8*a^3*b^7*u^2 - 5*a^2*b^8*u^2 - 8*a^4*b^6*u + 8*a^3*b^7*u - 4*a^2*b^8*u - 4*a*b^9 + 3*b^10)*c^2

Note that all powers of c that occur in the above expression have \
    even exponent, thus we may put c=a-b for simplicity.

c=a-b

expr_6_7=expand(
32*a^12*u^6 - 64*a^11*b*u^6 - 32*a^10*b^2*u^6 + 128*a^9*b^3*u^6 - \
    64*a^8*b^4*u^6 - 128*a^8*b^4*u^5 + 256*a^7*b^5*u^5 - 128*a^6*b^6*\
    u^5 - 32*a^10*b^2*u^4 + 64*a^9*b^3*u^4 - 416*a^8*b^4*u^4 + 1280*a\
    ^7*b^5*u^4 - 1728*a^6*b^6*u^4 + 1152*a^5*b^7*u^4 - 320*a^4*b^8*u\
    ^4 - 640*a^8*b^4*u^3 + 2304*a^7*b^5*u^3 - 3328*a^6*b^6*u^3 + \
    2304*a^5*b^7*u^3 - 640*a^4*b^8*u^3 - 320*a^8*b^4*u^2 + 1152*a^7*b\
    ^5*u^2 - 1728*a^6*b^6*u^2 + 1280*a^5*b^7*u^2 - 416*a^4*b^8*u^2 + \
    64*a^3*b^9*u^2 - 32*a^2*b^10*u^2 - 128*a^6*b^6*u + 256*a^5*b^7*u \
    - 128*a^4*b^8*u - 64*a^4*b^8 + 128*a^3*b^9 - 32*a^2*b^10 - 64*a*b\
    ^11 + 32*b^12 + 64*(a^8*u^6 - 2*a^6*b^2*u^5 - 2*a^6*b^2*u^4 + a\
    ^4*b^4*u^4 + 4*a^4*b^4*u^3 + a^4*b^4*u^2 - 2*a^2*b^6*u^2 - 2*a^2*\
    b^6*u + b^8)*c^4 - 32*(3*a^10*u^6 - 4*a^9*b*u^6 - 4*a^8*b^2*u^5 +\
     8*a^7*b^3*u^5 - 8*a^6*b^4*u^5 - 5*a^8*b^2*u^4 + 8*a^7*b^3*u^4 - \
    14*a^6*b^4*u^4 + 12*a^5*b^5*u^4 - 8*a^4*b^6*u^4 - 12*a^6*b^4*u^3 \
    + 16*a^5*b^5*u^3 - 12*a^4*b^6*u^3 - 8*a^6*b^4*u^2 + 12*a^5*b^5*u\
    ^2 - 14*a^4*b^6*u^2 + 8*a^3*b^7*u^2 - 5*a^2*b^8*u^2 - 8*a^4*b^6*u\
     + 8*a^3*b^7*u - 4*a^2*b^8*u - 4*a*b^9 + 3*b^10)*c^2
)
(expr_6_7).maxima_methods().collectterms(a,b)
0