Additional Materials C: More on Assortativity

This appendix contains further investigations on the results displayed in Sect. "A network with mixed assortative/disassortative behaviour", where we showed how low degree nodes interact with high degree nodes. We captured a strong disassortative signal both in the knnk plot in Fig.8, where we showed how the mean degree of each k-degree node's neighbors is very high for low degree nodes and the other way around, and in the density heatmap in Fig. 10, that we decomposed in Fig.11(b) and Fig.11(c), isolating those parts of the network mostly made of disassortative links.

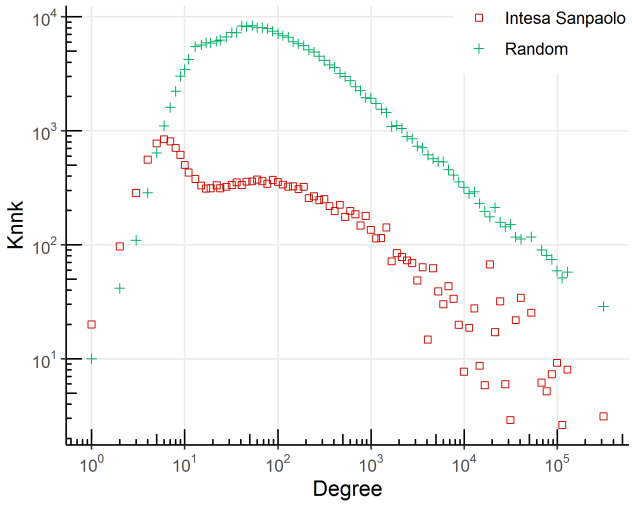

Figure C1: A variant of knnk: median degree of all the neighbors of k-degree nodes. It is more visible here the local assortative tendency between small nodes. For high degree nodes we observe a disassortative behaviour similar to the regular knnk we showed in Fig. 8 of the paper.

When computing the knnk in, we took the mean degree of the neighbors of each k-degree node. However, the mean could be easily carried away by a single outlier. We computed the same measure but replacing the mean with the median, which is less affected by single nodes with a very high degree. Results are shown in Fig. C1. It emerges here the assortative tendency for small degree ($k \in (10, 100)$) that we observed in the degree correlation heatmaps in Fig.10

and Fig.11. On the other hand, for higher degrees we have a behaviour largely similar to knnk that confirms all our observations in the paper.

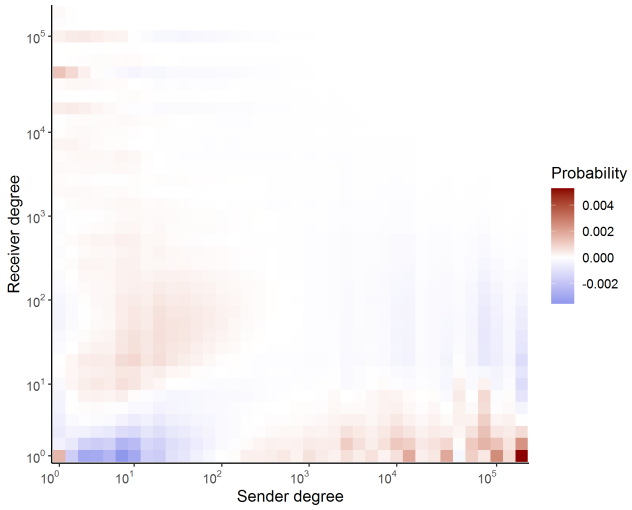

Figure C2: Comparison between ISP network and a null model: each tile $< x, y >$ shows the difference between the probability of a wire transfer from x-degree nodes to y-degree nodes in ISP network and in the null model. Red zones are those in which ISP network is denser, blue zones are where ISP network is less dense than the null model.

We then computed a degree correlation heatmap similar to the one in Fig. 10, but where we compared Intesa Sanpaolo's network with an in-degree and out-degree preserving randomization, in order to evaluate if there is a disassortative trend beyond what we would expect from the degree distribution, as in [1]. Precisely, in each tile $< x, y >$ we show the difference between the probabilities $P_{ISP}(x, y)$, defined as the probability of a wire transfer from an x-degree node to a y-degree node in Intesa Sanpaolo's network, and $P_{RAND}(x, y)$, defined as the probability of a wire transfer from an x-degree node to a y-degree node in the randomization. Results are displayed in Fig. C2. We colored in red those tiles in which Intesa Sanpaolo's network is denser, meaning that there is a higher probability of wire transfers, and in blue the tiles in which Intesa Sanpaolo's network is less dense than the random one. From the plot is clear that Intesa Sanpaolo's is denser on disassortative zones, i.e. the zones where low-degree nodes pay high-degree nodes (mostly taxes and subscriptions) and where high-degree nodes pay low-degree nodes (mostly salaries and pensions). On the other hand, the null model is denser on the bottom-left zone, with same-degree nodes paying each other.

This analyses confirm the general disassortative trend of Intesa Sanpaolo's wire transfers network, also hinting to the low-degree to low-degree patterns we

discussed in the paper.

If we look at the amount of money shared instead of the number of wire transfers, we can capture the same strong signal arising from salaries and pensions. Fig. C3 shows a correlation heatmap, in which each tile $< x, y >$ represents the cumulative amount of money sent by x-degree nodes to y-degree nodes. Unlike the heatmaps in Fig.10 and Fig. C2, in Fig. C3 we could not see any specific pattern we could link to low-degree to high-degree payments, yet we observe a dense horizontal band involving receivers whose cumulative money received is approx in [20k€, 75k€], that get money from all ranges of senders: many of the related payments are probably salaries from companies to natural persons.

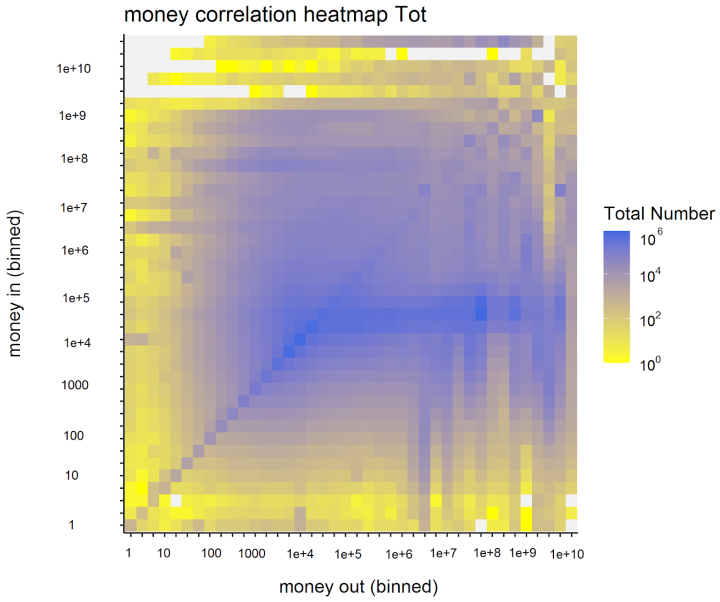

Figure C3: Money density heatmap: each tile $< x, y >$ shows the cumulative amount of money sent by x-degree nodes to y-degree nodes. We can observe a dense horizontal band involving receivers whose cumulative money received is approx in [20k€, 75k€], that get money from all ranges of senders, probably an effect of salaries and pensions.

References

[1] Maslov S, Sneppen K. Specificity and Stability in Topology of Protein Networks. Science. 2002;296(5569):910–913. Available from: `https://science.sciencemag.org/content/296/5569/910`.