

SOCIAL NETWORK ANALYSIS: METHODS AND APPLICATIONS

STANLEY WASSERMAN

University of Illinois

KATHERINE FAUST

University of South Carolina



CAMBRIDGE
UNIVERSITY PRESS

Contents

<i>List of Tables</i>	page xxi
<i>List of Illustrations</i>	xxiv
<i>Preface</i>	xxix

Part I: Networks, Relations, and Structure	1
1 Social Network Analysis in the Social and Behavioral Sciences	3
1.1 The Social Networks Perspective	4
1.2 Historical and Theoretical Foundations	10
1.2.1 Empirical Motivations	11
1.2.2 Theoretical Motivations	13
1.2.3 Mathematical Motivations	15
1.2.4 In Summary	16
1.3 Fundamental Concepts in Network Analysis	17
1.4 Distinctive Features	21
1.5 Organization of the Book and How to Read It	22
1.5.1 Complexity	23
1.5.2 Descriptive and Statistical Methods	23
1.5.3 Theory Driven Methods	24
1.5.4 Chronology	24
1.5.5 Levels of Analysis	25
1.5.6 Chapter Prerequisites	26
1.6 Summary	27
2 Social Network Data	28
2.1 Introduction: What Are Network Data?	28
2.1.1 Structural and Composition Variables	29

2.1.2	Modes	29
2.1.3	Affiliation Variables	30
2.2	Boundary Specification and Sampling	30
2.2.1	What Is Your Population?	31
2.2.2	Sampling	33
2.3	Types of Networks	35
2.3.1	One-Mode Networks	36
2.3.2	Two-Mode Networks	39
2.3.3	Ego-centered and Special Dyadic Networks	41
2.4	Network Data, Measurement and Collection	43
2.4.1	Measurement	43
2.4.2	Collection	45
2.4.3	Longitudinal Data Collection	55
2.4.4	Measurement Validity, Reliability, Accuracy, Error	56
2.5	Data Sets Found in These Pages	59
2.5.1	Krackhardt's High-tech Managers	60
2.5.2	Padgett's Florentine Families	61
2.5.3	Freeman's <i>EIES</i> Network	62
2.5.4	Countries Trade Data	64
2.5.5	Galaskiewicz's CEOs and Clubs Network	65
2.5.6	Other Data	66
Part II: Mathematical Representations of Social Networks		67
3	Notation for Social Network Data	69
3.1	Graph Theoretic Notation	71
3.1.1	A Single Relation	71
3.1.2	○Multiple Relations	73
3.1.3	Summary	75
3.2	Sociometric Notation	77
3.2.1	Single Relation	79
3.2.2	Multiple Relations	81
3.2.3	Summary	83
3.3	○Algebraic Notation	84
3.4	○Two Sets of Actors	85
3.4.1	⊗Different Types of Pairs	86
3.4.2	○Sociometric Notation	87
3.5	Putting It All Together	89

4 Graphs and Matrices	92
4.1 Why Graphs?	93
4.2 Graphs	94
4.2.1 Subgraphs, Dyads, and Triads	97
4.2.2 Nodal Degree	100
4.2.3 Density of Graphs and Subgraphs	101
4.2.4 Example: Padgett's Florentine Families	103
4.2.5 Walks, Trails, and Paths	105
4.2.6 Connected Graphs and Components	109
4.2.7 Geodesics, Distance, and Diameter	110
4.2.8 Connectivity of Graphs	112
4.2.9 Isomorphic Graphs and Subgraphs	117
4.2.10 \bigcirc Special Kinds of Graphs	119
4.3 Directed Graphs	121
4.3.1 Subgraphs – Dyads	124
4.3.2 Nodal Indegree and Outdegree	125
4.3.3 Density of a Directed Graph	129
4.3.4 An Example	129
4.3.5 Directed Walks, Paths, Semipaths	129
4.3.6 Reachability and Connectivity in Digraphs	132
4.3.7 Geodesics, Distance and Diameter	134
4.3.8 \bigcirc Special Kinds of Directed Graphs	134
4.3.9 Summary	136
4.4 Signed Graphs and Signed Directed Graphs	136
4.4.1 Signed Graph	137
4.4.2 Signed Directed Graphs	138
4.5 Valued Graphs and Valued Directed Graphs	140
4.5.1 Nodes and Dyads	142
4.5.2 Density in a Valued Graph	143
4.5.3 \bigcirc Paths in Valued Graphs	143
4.6 Multigraphs	145
4.7 \otimes Hypergraphs	146
4.8 Relations	148
4.8.1 Definition	148
4.8.2 Properties of Relations	149
4.9 Matrices	150
4.9.1 Matrices for Graphs	150
4.9.2 Matrices for Digraphs	152
4.9.3 Matrices for Valued Graphs	153
4.9.4 Matrices for Two-Mode Networks	154

4.9.5	○Matrices for Hypergraphs	154
4.9.6	Basic Matrix Operations	154
4.9.7	Computing Simple Network Properties	159
4.9.8	Summary	164
4.10	Properties	164
4.10.1	Reflexivity	164
4.10.2	Symmetry	165
4.10.3	Transitivity	165
4.11	Summary	165
Part III: Structural and Locational Properties		167
5	Centrality and Prestige	169
5.1	Prominence: Centrality and Prestige	172
5.1.1	Actor Centrality	173
5.1.2	Actor Prestige	174
5.1.3	Group Centralization and Group Prestige	175
5.2	Nondirectional Relations	177
5.2.1	Degree Centrality	178
5.2.2	Closeness Centrality	183
5.2.3	Betweenness Centrality	188
5.2.4	⊗Information Centrality	192
5.3	Directional Relations	198
5.3.1	Centrality	199
5.3.2	Prestige	202
5.3.3	A Different Example	210
5.4	Comparisons and Extensions	215
6	Structural Balance and Transitivity	220
6.1	Structural Balance	222
6.1.1	Signed Nondirectional Relations	223
6.1.2	Signed Directional Relations	228
6.1.3	○Checking for Balance	230
6.1.4	An Index for Balance	232
6.1.5	Summary	232
6.2	Clusterability	233
6.2.1	The Clustering Theorems	235
6.2.2	Summary	238
6.3	Generalizations of Clusterability	239

6.3.1 Empirical Evidence	239
6.3.2 \circ Ranked Clusterability	240
6.3.3 Summary	242
6.4 Transitivity	243
6.5 Conclusion	247
7 Cohesive Subgroups	249
7.1 Background	250
7.1.1 Social Group and Subgroup	250
7.1.2 Notation	252
7.2 Subgroups Based on Complete Mutuality	253
7.2.1 Definition of a Clique	254
7.2.2 An Example	254
7.2.3 Considerations	256
7.3 Reachability and Diameter	257
7.3.1 n -cliques	258
7.3.2 An Example	259
7.3.3 Considerations	260
7.3.4 n -clans and n -clubs	260
7.3.5 Summary	262
7.4 Subgroups Based on Nodal Degree	263
7.4.1 k -plexes	265
7.4.2 k -cores	266
7.5 Comparing Within to Outside Subgroup Ties	267
7.5.1 LS Sets	268
7.5.2 Lambda Sets	269
7.6 Measures of Subgroup Cohesion	270
7.7 Directional Relations	273
7.7.1 Cliques Based on Reciprocated Ties	273
7.7.2 Connectivity in Directional Relations	274
7.7.3 n -cliques in Directional Relations	275
7.8 Valued Relations	277
7.8.1 Cliques, n -cliques, and k -plexes	278
7.8.2 Other Approaches for Valued Relations	282
7.9 Interpretation of Cohesive Subgroups	283
7.10 Other Approaches	284
7.10.1 Matrix Permutation Approaches	284
7.10.2 Multidimensional Scaling	287
7.10.3 \circ Factor Analysis	290
7.11 Summary	290

8 Affiliations and Overlapping Subgroups	291
8.1 Affiliation Networks	291
8.2 Background	292
8.2.1 Theory	292
8.2.2 Concepts	294
8.2.3 Applications and Rationale	295
8.3 Representing Affiliation Networks	298
8.3.1 The Affiliation Network Matrix	298
8.3.2 Bipartite Graph	299
8.3.3 Hypergraph	303
8.3.4 \circ Simplices and Simplicial Complexes	306
8.3.5 Summary	306
8.3.6 An example: Galaskiewicz's CEOs and Clubs	307
8.4 One-mode Networks	307
8.4.1 Definition	307
8.4.2 Examples	309
8.5 Properties of Affiliation Networks	312
8.5.1 Properties of Actors and Events	312
8.5.2 Properties of One-mode Networks	314
8.5.3 Taking Account of Subgroup Size	322
8.5.4 Interpretation	324
8.6 \otimes Analysis of Actors and Events	326
8.6.1 \otimes Galois Lattices	326
8.6.2 \otimes Correspondence Analysis	334
8.7 Summary	342
 Part IV: Roles and Positions	 345
9 Structural Equivalence	347
9.1 Background	348
9.1.1 Social Roles and Positions	348
9.1.2 An Overview of Positional and Role Analysis	351
9.1.3 A Brief History	354
9.2 Definition of Structural Equivalence	356
9.2.1 Definition	356
9.2.2 An Example	357
9.2.3 Some Issues in Defining Structural Equivalence	359
9.3 Positional Analysis	361
9.3.1 Simplification of Multirelational Networks	361

9.3.2	Tasks in a Positional Analysis	363
9.4	Measuring Structural Equivalence	366
9.4.1	Euclidean Distance as a Measure of Structural Equivalence	367
9.4.2	Correlation as a Measure of Structural Equivalence	368
9.4.3	Some Considerations in Measuring Structural Equivalence	370
9.5	Representation of Network Positions	375
9.5.1	Partitioning Actors	375
9.5.2	Spatial Representations of Actor Equivalences	385
9.5.3	Ties Between and Within Positions	388
9.6	Summary	391
10	Blockmodels	394
10.1	Definition	395
10.2	Building Blocks	397
10.2.1	Perfect Fit (Fat Fit)	398
10.2.2	Zeroblock (Lean Fit) Criterion	399
10.2.3	Oneblock Criterion	400
10.2.4	α Density Criterion	400
10.2.5	Comparison of Criteria	401
10.2.6	Examples	401
10.2.7	Valued Relations	406
10.3	Interpretation	408
10.3.1	Actor Attributes	408
10.3.2	Describing Individual Positions	411
10.3.3	Image Matrices	417
10.4	Summary	423
11	Relational Algebras	425
11.1	Background	426
11.2	Notation and Algebraic Operations	428
11.2.1	Composition and Compound Relations	429
11.2.2	Properties of Composition and Compound Relations	432
11.3	Multiplication Tables for Relations	433
11.3.1	Multiplication Tables and Relational Structures	435
11.3.2	An Example	439
11.4	Simplification of Role Tables	442
11.4.1	Simplification by Comparing Images	443

11.4.2	⊗ Homomorphic Reduction	445
11.5	⊗ Comparing Role Structures	449
11.5.1	Joint Homomorphic Reduction	451
11.5.2	The Common Structure Semigroup	452
11.5.3	An Example	453
11.5.4	Measuring the Similarity of Role Structures	457
11.6	Summary	460
12	Network Positions and Roles	461
12.1	Background	462
12.1.1	Theoretical Definitions of Roles and Positions	462
12.1.2	Levels of Role Analysis in Social Networks	464
12.1.3	Equivalences in Networks	466
12.2	Structural Equivalence, Revisited	468
12.3	Automorphic and Isomorphic Equivalence	469
12.3.1	Definition	470
12.3.2	Example	471
12.3.3	Measuring Automorphic Equivalence	472
12.4	Regular Equivalence	473
12.4.1	Definition of Regular Equivalence	474
12.4.2	Regular Equivalence for Nondirectional Relations	475
12.4.3	Regular Equivalence Blockmodels	476
12.4.4	○ A Measure of Regular Equivalence	479
12.4.5	An Example	481
12.5	“Types” of Ties	483
12.5.1	An Example	485
12.6	Local Role Equivalence	487
12.6.1	Measuring Local Role Dissimilarity	488
12.6.2	Examples	491
12.7	⊗ Ego Algebras	494
12.7.1	Definition of Ego Algebras	496
12.7.2	Equivalence of Ego Algebras	497
12.7.3	Measuring Ego Algebra Similarity	497
12.7.4	Examples	499
12.8	Discussion	502

Part V: Dyadic and Triadic Methods	503
13 Dyads	505
13.1 An Overview	506
13.2 An Example and Some Definitions	508
13.3 Dyads	510
13.3.1 The Dyad Census	512
13.3.2 The Example and Its Dyad Census	513
13.3.3 An Index for Mutuality	514
13.3.4 \otimes A Second Index for Mutuality	518
13.3.5 \circ Subgraph Analysis, in General	520
13.4 Simple Distributions	522
13.4.1 The Uniform Distribution – A Review	524
13.4.2 Simple Distributions on Digraphs	526
13.5 Statistical Analysis of the Number of Arcs	528
13.5.1 Testing	529
13.5.2 Estimation	533
13.6 \otimes Conditional Uniform Distributions	535
13.6.1 Uniform Distribution, Conditional on the Number of Arcs	536
13.6.2 Uniform Distribution, Conditional on the Outdegrees	537
13.7 Statistical Analysis of the Number of Mutuals	539
13.7.1 Estimation	540
13.7.2 Testing	542
13.7.3 Examples	543
13.8 \otimes Other Conditional Uniform Distributions	544
13.8.1 Uniform Distribution, Conditional on the Indegrees	545
13.8.2 The $U MAN$ Distribution	547
13.8.3 More Complex Distributions	550
13.9 Other Research	552
13.10 Conclusion	555
14 Triads	556
14.1 Random Models and Substantive Hypotheses	558
14.2 Triads	559
14.2.1 The Triad Census	564
14.2.2 The Example and Its Triad Census	574
14.3 Distribution of a Triad Census	575
14.3.1 \otimes Mean and Variance of a k -subgraph Census	576

14.3.2	Mean and Variance of a Triad Census	579
14.3.3	Return to the Example	581
14.3.4	Mean and Variance of Linear Combinations of a Triad Census	582
14.3.5	A Brief Review	584
14.4	Testing Structural Hypotheses	585
14.4.1	Configurations	585
14.4.2	From Configurations to Weighting Vectors	590
14.4.3	From Weighting Vectors to Test Statistics	592
14.4.4	An Example	595
14.4.5	Another Example — Testing for Transitivity	596
14.5	Generalizations and Conclusions	598
14.6	Summary	601
 Part VI: Statistical Dyadic Interaction Models		 603
15	Statistical Analysis of Single Relational Networks	605
15.1	Single Directional Relations	607
15.1.1	The Y -array	608
15.1.2	Modeling the Y -array	612
15.1.3	Parameters	619
15.1.4	⊗Is p_1 a Random Directed Graph Distribution?	633
15.1.5	Summary	634
15.2	Attribute Variables	635
15.2.1	Introduction	636
15.2.2	The W -array	637
15.2.3	The Basic Model with Attribute Variables	640
15.2.4	Examples: Using Attribute Variables	646
15.3	Related Models for Further Aggregated Data	649
15.3.1	Strict Relational Analysis — The V -array	651
15.3.2	Ordinal Relational Data	654
15.4	○Nondirectional Relations	656
15.4.1	A Model	656
15.4.2	An Example	657
15.5	⊗Recent Generalizations of p_1	658
15.6	⊗Single Relations and Two Sets of Actors	662
15.6.1	Introduction	662
15.6.2	The Basic Model	663
15.6.3	Aggregating Dyads for Two-mode Networks	664

<i>Contents</i>	xix
15.7 Computing for Log-linear Models	665
15.7.1 Computing Packages	666
15.7.2 From Printouts to Parameters	671
15.8 Summary	673
16 Stochastic Blockmodels and Goodness-of-Fit Indices	675
16.1 Evaluating Blockmodels	678
16.1.1 Goodness-of-Fit Statistics for Blockmodels	679
16.1.2 Structurally Based Blockmodels and Permutation Tests	688
16.1.3 An Example	689
16.2 Stochastic Blockmodels	692
16.2.1 Definition of a Stochastic Blockmodel	694
16.2.2 Definition of Stochastic Equivalence	696
16.2.3 Application to Special Probability Functions	697
16.2.4 Goodness-of-Fit Indices for Stochastic Blockmodels	703
16.2.5 \odot Stochastic a posteriori Blockmodels	706
16.2.6 Measures of Stochastic Equivalence	708
16.2.7 Stochastic Blockmodel Representations	709
16.2.8 The Example Continued	712
16.3 Summary: Generalizations and Extensions	719
16.3.1 Statistical Analysis of Multiple Relational Networks	719
16.3.2 Statistical Analysis of Longitudinal Relations	721
Part VII: Epilogue	725
17 Future Directions	727
17.1 Statistical Models	727
17.2 Generalizing to New Kinds of Data	729
17.2.1 Multiple Relations	730
17.2.2 Dynamic and Longitudinal Network Models	730
17.2.3 Ego-centered Networks	731
17.3 Data Collection	731
17.4 Sampling	732
17.5 General Propositions about Structure	732
17.6 Computer Technology	733
17.7 Networks and Standard Social and Behavioral Science	733

