Match-Fixing under Competitive Odds

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Abstract

Two bookmakers compete in Bertrand fashion while setting odds on the outcomes of a sporting contest where an influential punter (or betting syndicate) may bribe some player(s) to fix the contest. Zero profit and bribe prevention may not always hold together. When the influential punter is quite powerful, the bookies may coordinate on prices and earn positive profits for fear of letting the 'lemons' (i.e., the influential punter) in. On the other hand, sometimes the bookies make zero profits but also admit match-fixing.

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1 Introduction

Match-fixing and gambling related corruption often grab news headlines. Almost any sport – horse races, tennis, soccer, cricket, to name a few – is susceptible to negative external influences.[1] Someone involved in betting on a specific sporting event may have access to player(s) and induce under-performance through bribery. In high visibility sports many unsuspecting punters, bookmakers and the general viewing public may therefore be defrauded in the process.[2]

We adapt the horse-race betting models due to Shin (1991; 1992) to analyze match-fixing. In Shin (1991) a monopolist bookmaker sets odds on each one of two horses winning a race, whereas in Shin (1992) two bookmakers simultaneously set odds, as in Bertrand competition, in an n-horse race game ($n > 2$).[3] In both models, there is an *insider* who knows precisely which horse would win the race, while the remaining are noise punters with their different exogenous beliefs about the horses' winning probabilities that are uncorrelated with the true probabilities. The bookmaker(s) know only the true winning probabilities.[4]

Rather than assuming an insider who knows before betting the identity of the winner (as in Shin's models), we consider the prospect of a gambler *influencing* the contestants' winning odds through bribery and match-fixing. Ex ante (before bribing), this gambler, to be called the 'influential punter', is no better informed than the bookmakers and is less privileged than Shin's insider. However, different from Shin's framework, the influential punter may

[1]See "Race-fixing probed in Fallon trial" and similar reports at http://www.channel4.-com/news/articles/sports/racefixing+probed+in+fallon+trial+/894147. See also a BBC panorama on this subject (http://news.bbc.co.uk/1/hi/programmes/panorama/2290356.stm).

For tennis, see reports such as "Tennis chiefs battle match-fixers" and "ITF working with ATP, WTA and Grand Slam Committee to halt match-fixing in tennis" (http://news.bbc.co.uk/sport1/hi/tennis/7035003.stm ; http://www.signonsandiego.com/sports/20071009-0552-ten-tennis-gambling.html).

In March 2009, Uefa president Michel Platini publicly issues the following warning: "There is a grave danger in the world of football and that is match-fixing." Uefa general secretary says, "We are setting up this betting fraud detection system across Europe to include 27,000 matches in the first and second division in each national association." See http://news.bbc.co.uk/sport2/hi/football/europe/7964790.stm.

For cricket, see http://news.bbc.co.uk/sport1/low/cricket/719743.stm. For basketball, Wolfers (2006) estimated that nearly 1 percent of all games in NCAA Division one basketball (about 500 games between 1989 and 2005) involved gambling related corruption. A striking account of match-rigging in Sumo wrestling in Japan appears in Duggan and Levitt (2002).

[2]Corruption in sports has been only occasionally highlighted by economists without the consideration of its causal relationship to betting – see Duggan and Levitt (2002), and Preston and Szymanski (2003). Wolfers (2006), Winter and Kukuk (2008), and Strumpf (2003) are few exceptions.

[3]Actually, Shin's (1992) price setting game is slightly different from one-shot Bertrand: the bookmakers first submit bids specifying a maximum combined price for bets on all the horses, the low bidder wins and then sets prices for individual bets so that the total for all bets combined does not exceed the winning bid.

[4]In an empirical framework, Shin (1993) provides estimates for the incidence of insider trading in UK betting markets.

become better informed than the bookmakers through his secret dealings with one of the contestants, if chance presents it and bookmakers' odds make it worthwhile. Thus, we shift the focus from the use of insider *information* (i.e. pure adverse selection) to manipulative *action* that generates inside information for the influential gambler. As actions are choices, our bookmakers can control these by appropriately setting their odds – a possibility absent in models of pure adverse selection such as Shin's.

Moreover, there is an issue of legality. While betting on the basis of inside information may not be illegal, match-fixing through bribery clearly is. However, in the existing literature on betting (including Shin) a key objective has been to explain the favorite-longshot bias in race-track betting.[5] But we raise concern also about the unfairness of the contest not only for the sake of unsuspecting bettors, but for the viewing public and the media that rely on the public's interest in sports. How the threat of bribery and manipulation influences betting odds and the eventual occurrence of match-fixing is the main subject of our interest.

We focus on the bookmakers' odds setting behavior in the shadow of match-fixing under Bertrand competition. We ask: when is match-fixing a serious threat, and when it is a threat how bookmakers combat or even perversely trigger match-fixing. We are going to argue that competition does not necessarily yield zero profits, nor does it guarantee fair play.

Our model involves two bookmakers (or bookies) and two types of punters (ordinary/naive and influential). The bookies set odds and the punters place bets on the outcome of a sporting contest between two teams (or contestants).[6] The influential punter (or equivalently a large betting syndicate), who shares the same beliefs as the bookies (and the players) about the teams' winning chances, may be able to gain access to some members in one of the teams and bribe them to sabotage the team.[7] When bribing a team, the influential punter would place a bet on the other team. The anti-corruption authority may investigate the losing team and punish the match-fixing punter and the corrupt player(s) whenever it catches them.[8]

With the threat of match-fixing looming, in selecting odds the bookies take into account both the benefit and the danger of undercutting each other. When the influential punter

[5]In a parimutuel market setting Winter and Kukuk (2008) allow a participant jockey to underperform and bet on a likely favorite. But they do not show when underperforming will be optimal and what betting strategy to follow. Their main objective is to study the favorite-longshot bias in an empirical context.

[6]Fixed-odds betting, as opposed to parimutuel betting, is a more relevant format for analysis of match-fixing where the bookie plays a significant role without direct involvement in the act of bribery and/or placement of surrogate bets. For a contrast between how odds are set (or determined) in these two betting markets (but without the issues of match-fixing), see Ottaviani and Sorensen (2005; 2008).

[7]In contests involving rival firms or lobbies, sabotage is a well-studied theme; see, for instance, Konrad (2000). Our sports contest model is much simpler than the 'effort contest' games (such as the one analyzed by Konrad) in that we assume exogenous winning probabilities of the contestants due to their inherent skills (or characteristics), and sabotage is a deliberate under-performance relative to one's own skills.

[8]The law enforcement is one of *investigation* rather than *monitoring* (Mookherjee and Png, 1992).

cannot place too large a bet, the following results occur. If, *ex ante*, teams are relatively more even, competition yields zero expected profits for the bookies without attracting the risk of bribery and match-fixing. With the influential punter kept at bay, prices correspond to fair odds, which is different from Shin's result;[9] ordinary punters will gain. But if (*ex ante*) teams are more uneven, Bertrand competition cannot guarantee elimination of bribery; the bookies make zero profits and the influential punter earns rent. Match-fixing will occur with positive probability, and it can be attributed to *opportunism*. If undercutting triggers match-fixing, its adverse impact (loss) is shared by both bookies, but if match-fixing is not triggered then the gain is exclusive (positive profit).

On the other hand, when the influential punter can place a significantly large bet, the adverse impact of match-fixing could be so severe that undercutting becomes very risky. In particular, in contests that are *ex ante* nearly even, the bookies will coordinate on prices strictly above fair odds and sustain, non-cooperatively, positive profits and prevent bribery. Positive profits seem to go against common wisdoms of competition. Here, the *fear* of triggering (the 'lemons' of) match-fixing forces the bookies to coordinate on prices. Ironically, without the corrupting influential punter the bookies would compete away profits.

Before we proceed to detailed analysis, we would like to note that in practice bookmakers are well aware of the potential risks of the influential punter's involvement and as a precaution they may limit the size of trades at posted prices.[10] Even more, the bookmakers may set new odds seeing the increasing volume of bets being placed on a particular outcome so that the influential punter may face a quantity-price trade-off. Further, odds revisions may generate and disseminate new information even among the ordinary punters leading to an erosion of the value of insider information, similar to the market micro structure literature (Glosten and Milgrom, 1985; Kyle, 1985). While our model does not incorporate these features employed in models of financial economics, we do not see the basic insights of our analysis changing qualitatively even if a more sophisticated and much more complex model were formulated. We also assume exogenous investigation probabilities and fines by the prosecution authorities, to keep the analysis tractable. Nor do we model the role of sports bodies that may regulate the betting market in large to prevent cheating. These considerations are important no doubt, but beyond the scope of the present work.

In section 2 we present the model, followed by an analysis of the betting and bribing decisions in section 3. In section 4 we analyze the Bertrand duopoly competition. Section 5 concludes. The formal proofs appear in an appendix, and a separate supplementary material

[9]In Shin (1991; 1992), prices exaggerate the true odds.

[10]This may be difficult to implement, however, as any corrupt betting syndicate may have multiple punters on its team.

reports an extra derivation.

2 The Model

There are two bookmakers, called the bookies, who set the odds on each of two teams winning a competitive sports match (equivalently, sets the prices of two tickets); the match being drawn is not a possibility. Ticket i with price π_i yields a dollar whenever team i wins the contest and yields nothing if team i loses.

There are a continuum of naive punters, to be described as *punters* or sometimes *ordinary punters*, parameterized by individual belief (i.e., the probability) q that team 1 will win ($1-q$ is the probability that team 2 will win); q is distributed 'uniformly' over (0, 1). Ordinary punters stubbornly stick to their beliefs.

There is also a knowledgeable and potentially corrupt/influential punter, to be referred as punter I, who may influence a team's winning chances by bribing its corruptible players to under-perform. Punter I gains access to team i with probability $0 \leq \mu_i \leq 1$; with probability $1 - \mu_1 - \mu_2$, he fails to gain any access. At best, punter I can access only one team. The bookies and the prosecution authority know only (μ_1, μ_2).

The distribution of ordinary punters' wealth is 'uniform' over [0, 1], with a collective wealth of y dollars; the wealth of punter I is $z = 1 - y$ dollars.

In the absence of any external influence, the probability that team 1 will win is $0 < p_1 < 1$ and the corresponding probability for team 2 is $p_2 = 1 - p_1$. The bookies, punter I, and the players – all initially observe the draw p_1.[11] The prosecution need not observe p_1, or even when the prosecution observes p_1 it does not employ sophisticated game-theoretic inferences whether match-fixing has occurred or not based on the betting odds and p_1.

The prosecution authority investigates team i only when team i loses the contest. Assume that the probability of investigation of team i, $0 < \alpha_i < 1$, is known to all, and the investigation detects bribery, if any, with probability one. The investigation probability may differ across teams.[12] Given our focus on the bookies' pricing strategies, we take the prosecution to be non-strategic rule-book follower.

On conviction, the corrupt player (or players) will be imposed a total fine $0 < f \leq \bar{f}$ and punter I is imposed a fine $0 < f_I \leq \bar{f}$.[13]

When punter I gets access to team i, by making a bribe promise of b_i *conditional on team i losing* he can lower the probability of team i winning from the *true* probability p_i

[11]Levitt (2004) recognizes that bookmakers are usually more skilled at predicting match outcomes than ordinary punters. In any case, without such confidence in abilities the bookies won't be in the business.

[12]This difference could be due to the teams' different susceptibility to corruption.

[13]The finding of bribery is assumed to reveal the identity of punter I.

to $\lambda_i p_i$, where $0 \leq \lambda_i < 1$, provided the corrupt players of team i cooperate with punter I in undermining the team performance. λ_i depends on the susceptibility to corruption and bribery of team i's members, i.e., whether a small or a significant section of the team takes part in undermining the team cause. Also, the particular player (or players) to whom punter I is likely to have an access may be of varied importance to the team's overall performance. We take λ_i to be exogenous and common knowledge.

The bookies, two types of punters and the corruptible team members – all are assumed to be risk-neutral and maximize their respective expected profits/payoffs. Define the 'betting and bribery' game, Γ, as follows:

Stage 1. Nature draws p_1 and reveals it to the bookies, punter I and the players; the ordinary punters draw their respective private signals q. Then the bookies simultaneously set the prices (π_1, π_2) for the tickets on respective teams' win, where $0 \leq \pi_1, \pi_2 \leq 1$; to keep the notations simple, the bookie indices will be omitted from the prices.

Stage 2. Punter I finds out if he could get access to team 1 or team 2 or neither,[14] and decides whether to bribe the team (in the event of gaining access) to influence the contest outcome.

Stage 3. The ordinary punters as well as punter I place bets according to their 'eventual' beliefs. When the bookies charge the same price for a given ticket, market is evenly shared, and when they charge unequal prices, lower price captures the whole market. The match is played out according to the teams' winning probabilities $(p_1, 1-p_1)$ or $(\lambda p_1, 1-\lambda p_1)$ (where team 1 is bribed), or $(1-\lambda p_2, \lambda p_2)$ (where team 2 is bribed) and the outcome of the match is determined.

Stage 4. Finally, the prosecution follows its anti-corruption investigation policy, (α_1, α_2). On successful investigation, fines are imposed on the corrupt player(s) and punter I. ||

Note that in stage 3 only punter I privately forms his beliefs about the teams' eventual winning prospects, while the bookies will already have set the betting odds and the ordinary punters choose their bets based on initial, exogenous signals. Thus, the game is one of imperfect information and we will solve for the subgame perfect equilibrium (*SPE*). To remember the extensive form game specified above, the following time line may be helpful.

p_1 drawn — punter I access/not — I bribe/not-bribe — match played — ACU exams.

● —— ● —— ● —— ● —— ● —— ●

bookies post odds — bets settled — penalties apply

Time line

[14]The timing of the influential punter's access to teams (after or before the odds are posted) is immaterial. What is important is that the bookie does not know whether the influential punter got the access or not.

3 Betting and Bribing Decisions

Ordinary Punters' Betting Decision

The ordinary punters adopt the following betting rule:[15]
If $q \geq \pi_1$ but $1 - q < \pi_2$, bet on team 1;
If $1 - q \geq \pi_2$ but $q < \pi_1$, bet on team 2;
If $q \geq \pi_1$ and $1 - q \geq \pi_2$, then bet on team 1 if $\frac{q}{\pi_1} \geq \frac{1-q}{\pi_2}$ and bet on team 2 if $\frac{q}{\pi_1} \leq \frac{1-q}{\pi_2}$;
If $q < \pi_1$ and $1 - q < \pi_2$, do not bet on either team.

Player Incentives for Bribe-taking and Sabotage

Given the prosecution's investigation strategy, let us consider the incentives of players to accept bribes. In a team context, the incentives concern the corruptible member(s) of a team. We assume a single corruptible member in each team; the analysis applies equally to a consortium of corruptible members. Suppose the corruptible player of team i (with whom punter I establishes contact) gets the reward w in the event team i wins, and receives nothing if team i loses.[16] Given any belief p_i, a bribe b_i is accepted and honored by the corruptible player by under-performing

$$\text{if and only if} \quad (\lambda_i p_i)w + (1 - \lambda_i p_i)(b_i - \alpha_i f) \geq p_i w + (1 - p_i)(b_i - \alpha_i f)$$

$$\text{i.e., if and only if} \quad b_i \geq w + \alpha_i f. \tag{1}$$

A player can renege on his promise to under-perform even after entering into an agreement with punter I. The right-hand side of (1) recognizes this possibility. A player will be penalized for taking bribes, even if he might not have deliberately under-performed.[17]

The minimum bribe required to induce the corruptible player to accept the bait is $\underline{b}_i = w + \alpha_i f$. That is, the *reservation bribe* covers the loss of the prize w and the expected penalty. We assume that punter I holds all the bargaining power so that $b_i = \underline{b}_i$.[18]

Influential Punter's Betting and Bribing Incentives

First consider the betting incentives. Having learnt the true probabilities p_i and observed

[15] This is same as the betting rule by the *Outsiders* in Shin (1991).
[16] The prize w includes both direct and indirect rewards, with the latter in the form of lucrative endorsement opportunities for commercials. The player may additionally receive unconditional retainer wage/appearance fee that does not affect the player's bribe-taking incentives.
[17] To prove that a player has deliberately under-performed is very difficult. On the other hand, bribery can be established based on hard evidence.
[18] Our analysis can be easily extended to bargaining over bribe.

the prices π_i $(i = 1, 2)$, if punter I fails to contact either team or decides not to bribe,

$$\begin{aligned} \text{he will bet } z \text{ on team } i \text{ if} \quad & \frac{p_i}{\pi_i} \;\geq\; max\{1, \frac{p_j}{\pi_j}\}, \;\; i \neq j^{19} \\ \text{and will bet on neither if} \quad & \frac{p_i}{\pi_i} \;<\; 1, \quad i = 1, 2. \end{aligned}$$

The expected profit to punter I from betting exclusively on team i is $E\Pi_{0i}^I = p_i \frac{z}{\pi_i} - z, \quad i = 1, 2$, and zero when he bets on neither.[20] His expected profit from not bribing is $\max\{E\Pi_{0i}^I, E\Pi_{0j}^I, 0\}$.

If, however, punter I contacts a corruptible member of team i, offers him a bribe and places a bet on team j, his expected profit equals: $E\Pi^I(b_i) = (1 - \lambda_i p_i)\left[\frac{z}{\pi_j} - \underline{b}_i - \alpha f_I\right] - z$. Substituting $\underline{b}_i = w + \alpha_i f$,

$$E\Pi^I(b_i) = (1 - \lambda_i p_i) z \left[\frac{1}{\pi_j} - \Omega_i\right] - z = (1 - \lambda_i p_i) z \left[\frac{1}{\pi_j} - \frac{1}{\phi_i}\right],$$

$$\text{where} \qquad \Omega_i = \frac{w + \alpha_i (f + f_I)}{z}, \qquad \text{and} \qquad \phi_i = \frac{1 - \lambda_i p_i}{1 + (1 - \lambda_i p_i)\Omega_i}.$$

Clearly, if $E\Pi^I(b_i) > 0$ and greater than the profit from 'not bribing', he will bribe (upon access). But there are several situations of indifference, for which we impose two tie-breaking rules:

Assumption 1. (***Tie-breaking rule-***(i)**.**) *If 'bribing and betting' and 'betting without bribing' yield identical and positive expected profits for the influential punter, then he will choose bribing and betting.*

(***Tie-breaking rule-***(ii)**.**) *If 'bribing and betting' and 'betting without bribing' yield zero expected profits for the influential punter, then he will not bet at all.*

The first rule would bring to bear the full impact of the (negative) influence. Tie-breaking rule-(ii) is to ensure that the bribe prevention prices are well-defined.

Now we specify three scenarios of punter I's decision making that will be relevant for our analysis. Suppose $\pi_i \geq p_i, \forall i$ (such that $\max\{E\Pi_{0i}^I, E\Pi_{0j}^I, 0\} = 0$). Then punter I *does not* bribe team i $(i = 1, 2)$, if $E\Pi^I(b_i) \leq 0$, i.e.,

$$\pi_j \geq \frac{1 - \lambda_i p_i}{1 + (1 - \lambda_i p_i)\Omega_i} \equiv \phi_i. \tag{2}$$

(Tie-breaking rule (ii) applies when $E\Pi^I(b_i) = 0$.)

[20] Betting on both teams yield the same profit as exclusive betting, given the betting rule specified above.

Alternatively, suppose $\pi_i < p_i$ (and $\pi_j > p_j$) such that $\max\{E\Pi^I_{0i}, E\Pi^I_{0j}, 0\} = z(\frac{p_i}{\pi_i} - 1) > 0$. Then punter I *bribes* team i and bets on team j (as opposed to betting on team i), if $E\Pi^I(b_i) \geq z(\frac{p_i}{\pi_i} - 1)$, i.e.,

$$\pi_j \leq \frac{(1 - \lambda_i p_i)\pi_i}{p_i + (1 - \lambda_i p_i)\Omega_i \pi_i} \equiv \psi_i(\pi_i). \tag{3}$$

(Tie-breaking rule (i) applies when $E\Pi^I(b_i) = z(\frac{p_i}{\pi_i} - 1)$.)

Continuing with the assumption that $\pi_i < p_i$ (and $\pi_j > p_j$) such that $\max\{E\Pi^I_{0i}, E\Pi^I_{0j}, 0\} = z(\frac{p_i}{\pi_i} - 1) > 0$, punter I *bribes* team j and bets on team i if $E\Pi^I(b_j) \geq z(\frac{p_i}{\pi_i} - 1)$, i.e.,

$$\pi_i \leq \frac{(1 - \lambda_j)p_j}{(1 - \lambda_j p_j)\Omega_j} \equiv h_j. \tag{4}$$

Condition (2), which is our bribe prevention constraint, says that by setting the price of ticket j high enough, team i can be protected from match-fixing, and by doing so for both tickets punter I can be altogether kept out of the market. Indeed, that will be the outcome in any equilibrium featuring bribe prevention. If punter I does not bribe, but bets on team i, he must earn strictly positive profit. This is possible if and only if $\pi_i < p_i$, which is clearly loss-making for the bookies. Competition will eliminate such loss-making situations. Thus, if bribery is prevented, punter I will not participate at all.

Condition (3) is the bribe inducement constraint for team i, when team i is otherwise attractive to bet on. Essentially by reducing the price of ticket j below a threshold level, the betting incentive of punter I can be reversed. The threshold level will evidently depend on the price of ticket i. In particular, $\psi_i < \phi_i$ for $\pi_i < p_i$; if $\pi_i = p_i$ then $\phi_i = \psi_i$.

Finally, condition (4) is the bribe inducement constraint for team j. Here by reducing the price ticket of i below a threshold level (so that bribery can by financed from the potential gains), the incentive to bet on team i is strengthened.

4 Bertrand Competition in Bookmaking

Throughout we impose the following restriction known as the *Dutch-book restriction*:

Assumption 2. *The bookies must always choose prices* $0 \leq \pi_1, \pi_2 \leq 1$ *such that* $\pi_1 + \pi_2 \geq 1$.

Assumption 2 can be defended as follows. If instead $\pi_1 + \pi_2 < 1$, it gives rise to the "money pump" scenario implying someone who otherwise might not have bet on the sporting event (for reasons of risk aversion and the likes) can make free money by spending less than a dollar to earn a dollar for sure. This would drive the bookies out of business.

Our principal observations will be on two important issues concerning the effects of competition. First, a basic fact of (Bertrand) competition is that firms earn zero profits. Second, an often held perception of competition is that it generally discourages rent-seeking and corruption (Rose-Ackerman, 1978). In our case, while under certain conditions Bertrand competition ensures zero profits (to the bookies) *and* prevention of bribery and match-fixing (Proposition 1), either of these two expected results may fail to obtain in isolation (Propositions 2 and 3) under complementary conditions, that is, bribery/match-fixing may be triggered with positive probability or firms may make positive expected profits. Moreover, it is never the case that positive expected profits and bribery/match-fixing will occur at the same time. In the remainder of this section, we analyze these possibilities.

Bribe Prevention with Zero Profit

Let us first determine the prices at which expected profit is zero and bribery is prevented. Focusing on identical prices (and therefore suppressing bookie indices for notational simplicity) consider the posting of (π_1, π_2). Given the Dutch-book restriction, ordinary punters whose beliefs, q, are in the interval $[0, 1-\pi_2]$ will buy ticket 2 and those with beliefs in $[\pi_1, 1]$ will buy ticket 1; punters with beliefs in $(1-\pi_2, \pi_1)$ will buy neither. This allows the bookie's objective function to be written as:

$$E\Pi_{BP}^d = \frac{y}{2}\left[\int_{\pi_1}^{1}(1-\frac{p_1}{\pi_1})\,\mathrm{d}q\right] + \frac{y}{2}\left[\int_{0}^{1-\pi_2}(1-\frac{p_2}{\pi_2})\,\mathrm{d}q\right] = y\left[3-\pi_1-\pi_2-\frac{p_1}{\pi_1}-\frac{p_2}{\pi_2}\right].$$

The bribe prevention constraints are: $\pi_1 \geq \max\{p_1, \phi_2(p_1)\}$, $\pi_2 \geq \max\{p_2, \phi_1(p_1)\}$.

From the objective function one might expect that competition in each market should induce $\pi_1 = p_1$ and $\pi_2 = p_2$ (i.e., prices equal the true probabilities of winning) leading to $E\Pi_{BP}^d = 0$. But to ensure such an outcome, the prices must also prevent bribery. To analyze the possibility of such an equilibrium, let us introduce two critical probabilities:

Definition 1. *Let $\tilde{p}_1$ be the unique p_1 such that $\phi_2(p_1) = p_1$, and $\hat{p}_1$ be the unique p_1 such that $\phi_1(p_1) = p_2$.*

It can be readily checked that $\phi_2'(p_1) > 0$, $\phi_2''(p_1) < 0$ with $\phi_2(0) > 0$ and $\phi_2(1) < 1$, as shown in Fig. 1. Therefore, a unique $\tilde{p}_1$ must exist and is in $(0, 1)$. It then follows that at all $p_1 < \tilde{p}_1$, $\phi_2(p_1) > p_1$, and at all $p_1 > \tilde{p}_1$, $\phi_2(p_1) < p_1$. Similarly, $\phi_1'(p_1) < 0, \phi''(p_1) < 0$ with $\phi_1(0) > 0$, $\phi_1(1) < 1$, as shown in Fig. 1. Therefore, $\hat{p}_1$ also exists and it is unique. Further, at all $p_1 > \hat{p}_1$, $\phi_1(p_1) > p_2$, and at all $p_1 < \hat{p}_1$, $\phi_1(p_1) < p_2$.

If Ω_i is large enough, which requires z to be small relative to $w + \alpha_i(f + f_I)$, $\tilde{p}_1$ will be smaller than $\hat{p}_1$. In other words, the influential punter should not be 'too powerful'. Until specified otherwise, we will assume:

Assumption 3. $\tilde{p}_1 < \frac{1}{2} < \hat{p}_1$.

Fig. 1 is drawn on the basis of this assumption. Clearly, over the interval $[\tilde{p}_1, \hat{p}_1]$ the zero-profit prices $\pi_i = p_i$ $(i = 1, 2)$ prevent bribery, and these can be sustained as Bertrand equilibrium by applying the usual logic: unilateral price increase(s) by a bookie do not improve profits, and any price reduction(s) inflict losses (in addition to violating the Dutch-book constraint).

But outside $[\tilde{p}_1, \hat{p}_1]$ we cannot have bribe prevention (along with zero profits) in equilibrium – a direct implication of the constructed interval $[\tilde{p}_1, \hat{p}_1]$. Here can bribery be prevented with certainty while profit remaining positive? The answer is 'no'. The reason is that due to the natural interlinkage of the two market gains from undercutting in one market must be evaluated in light of the possible bribery implication in the other market.

If bribe prevention with positive profit were to be an equilibrium, then we must have one of the following three possibilities: (i) both tickets are generating profit; (ii) only one ticket is generating profit, while the other ticket is generating loss, but the overall profit is positive; and (iii) only one ticket yields positive profit, while the other ticket yields zero profit. In all three cases, given Assumption 3 and also evident from Fig. 1, only one ticket needs to be protected from the betting of the influential punter, and it is this ticket which will yield positive profit. For instance in the region $[0, \tilde{p}_1)$ ticket 1 needs to be protected. On the other ticket (namely ticket 2 when $p_1 < \tilde{p}_1$) competition will wither away profit. Therefore, possibility (i) is ruled out. Possibility (ii) is also ruled out, because one can raise the price of the loss making ticket and lose the market altogether.

So, we are left with only possibility (iii). For the sake of concreteness consider $p_1 < \tilde{p}_1$. Here as argued above, the profit generating ticket must be ticket 1, due to the bribe prevention constraint $\pi_1 \geq \phi_2$ (recall (2)). But it can be easily seen that competition will force the constraint to bind. Thus, we will have $\pi_1 = \phi_2 > p_1$. For ticket 2 we have $\pi_2 = p_2$. Now, from this proposed equilibrium, we argue, *both* tickets can be undercut without increasing the prospect of bribery and profit will improve. To see that suppose, one of the bookies can reduces π_2 slightly below p_2 and takes a small loss on ticket 2. But simultaneously he makes the bribe prevention constraint $\pi_1 = \phi_2$ irrelevant. As punter I now can gainfully bet on ticket 2 without committing bribery, the new bribe prevention constraint should be $\pi_1 > \psi_2$ (recall (3) and tie-breaking rule (i)). As can be checked from (2) and (3), $\psi_2 < \phi_2$ as long as $\pi_2 < p_2$. Therefore π_1 can be suitably reduced to π_1' (in accordance with the reduction in π_2 below p_2) such that $\psi_2 < \pi_1' < \phi_2$. Thus, bribery is still prevented and the bookie fully captures both markets. As long as price reductions are of small order, loss in ticket 2 will be compensated by gains from ticket 1. Hence, possibility

(iii) is also ruled out.

Proposition 1. (***Bribe prevention***) *Suppose Assumption 3 holds.*

(i) For $p_1 \in [\tilde{p}_1, \hat{p}_1]$, the unique and symmetric equilibrium under Bertrand competition is $\pi_1 = p_1$ and $\pi_2 = p_2$, such that bribery is prevented surely and each bookie earns zero expected profit.

(ii) For p_1 outside the interval $[\tilde{p}_1, \hat{p}_1]$, there is no pure strategy equilibrium under Bertrand competition in which bribery is prevented with probability one.

Thus, it is possible that the influential punter will not bribe (so that match-fixing is not a threat) and Bertrand competition leads to zero profits with prices of bets equalling fair odds. In contrast, in Shin (1992; Proposition 1) with competition in bookmaking, the likely presence of an insider meant distortion in the prices of bets away from fair odds. So there may not be any inside information to worry the bookmakers, if such information were to be generated endogenously (as assumed in our model), and this means the betting odds follow the usual prediction of competition.

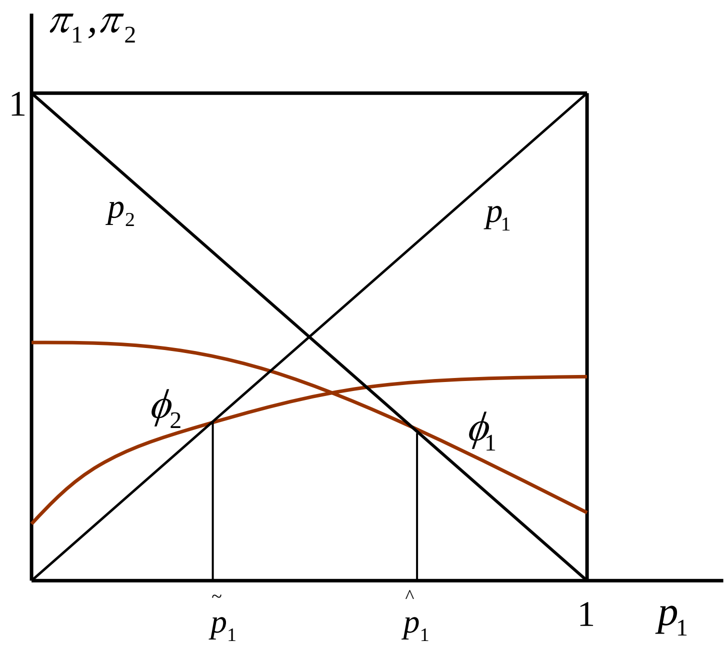

Figure 1: Bribe prevention zero profit prices

Bribe Inducement with Zero Profit

Outside $[\tilde{p}_1, \hat{p}_1]$ we now look for a (pure strategy pricing) equilibrium in which bribery occurs with positive probability. Consider $p_1 \in [0, \tilde{p}_1)$; a symmetric argument will apply to $p_1 \in (\hat{p}_1, 1]$. Below we detail an equilibrium and outline the conditions that will lead up to the equilibrium.

Definition 2. *Let π_{20} be the price of ticket 2 and π_{10} be the price of ticket 1, such that the bookies' expected profit from each ticket is zero, when upon access team 2 is expected to be bribed.*

In our conjectured equilibrium, on access (only) team 2 will be bribed; when team 2 is bribed, punter I will bet on team 1, and otherwise he will bet on team 2. When team 2 is bribed (which occurs with probability μ_2), its winning probability falls from p_2 to $\lambda_2 p_2$ and therefore its ex-ante winning probability falls from p_2 to $\mu_2\lambda_2 p_2 + (1-\mu_2)p_2$. Setting $\pi_2 = p_2$ would imply zero profit from the bets placed by punter I (which occurs with probability $(1-\mu_2)$), but strictly positive profit from naive punters (as $\pi_2 > \lambda_2 p_2$). Alternatively, setting $\pi_2 = \mu_2\lambda_2 p_2 + (1-\mu_2)p_2$ would imply zero (ex-ante, expected) profit from naive punters, but negative profit from punter I. So the zero-profit price π_{20} must lie within these two bounds, i.e., $\pi_{20} \in \Big(\mu_2\lambda_2 p_2 + (1-\mu_2)p_2,\ p_2\Big)$.

Simultaneously, in the event of bribery team 1's winning probability rises to $(1-\lambda_2 p_2)$; with probability $(1-\mu_2)$ of course its winning probability remains unchanged at p_1. The ex-ante probability of team 1 winning rises from p_1 to $\mu_2(1-\lambda_2 p_2) + (1-\mu_2)p_1$. If π_1 is set equal to $(1-\lambda_2 p_2)$, profit from punter I will be zero, but still the price is high enough to give strictly positive profit from naive punters (because with some probability team 2 will not be accessed in which case team 1's winning chance is $1-p_2 < 1-\lambda_2 p_2$. Similarly, if $\pi_1 = \mu_2(1-\lambda_2 p_2) + (1-\mu_2)p_1$ profit from naive punters will be zero, but there will be bookies' losses to punter I (which is profit for punter I). So the zero-profit price π_{10} must lie within these two bounds, i.e., $\pi_{10} \in \Big(\mu_2(1-\lambda_2 p_2) + (1-\mu_2)p_1,\ (1-\lambda_2 p_2)\Big)$.

In addition, π_{10} and π_{20} must satisfy the bribe inducement constraint for team 2 (condition (3)), i.e. $\pi_{10} \leq \psi_2(\pi_{20})$, and violate the bribe inducement constraint for team 1 (condition (4)), i.e. $\pi_{20} > h_1$. If (3) is satisfied and profits are positive, then the bookies, by undercutting each other, can only lead to the zero-profit prices (π_{10}, π_{20}).

From the zero-profit prices, neither bookie would raise the price(s) (and lose one or both markets), nor would they lower the price(s) and face losses (while the bribing incentives are unchanged). Condition (4) ensures that if team 1 is accessed it will not be bribed. Further, one must ensure against two possible deviations from the conjectured equilibrium (that may alter the bribing incentives): both π_1 and π_2 are reduced to π_1' and π_2' such that either (i) $\pi_2' \leq h_1$ and $\pi_1' > \psi_2(\pi_2')$, or (ii) $\pi_2' \leq h_1$ and $\pi_1' \leq \psi_2(\pi_2')$. The first deviation leads to bribing of team 1 (instead of team 2). The second deviation leads to bribing of either team (upon access). It can be shown that in a zero-profit bribe inducement equilibrium it is never the case that only the underdog is bribed (Lemma 1). Therefore, the first deviation is ruled out.

The second deviation requires some consideration. By opening up to the possibility of team 1 being bribed, the deviating bookie will find ticket 1 a bit more profitable (or less loss-making), but at the same time due to the reduction in π_2 below π_{20} ticket 2 becomes clearly loss-making. If this loss is more than compensated by any gains from ticket 1, this deviation is worthwhile. In Proposition 2 we specify two alternative conditions to prevent such deviation. Condition (5) says that the minimum undercutting needed to induce bribery of either team (π_2 to be lowered to h_1 and π_1 to be lowered to $\psi_2(h_1)$) does not satisfy the Dutch-book constraint. Condition (6) will ensure that profit from ticket 1 at these deviation prices will be non-positive. If the corrupted probability of team 1's winning (which is given in the left hand side of (6)) exceeds a critical level, then inducing bribery of team 1 will be unprofitable. In Example 1 we show for several parametric configurations that condition (5) is easily satisfied.

Lemma 1. (***No bribery of the underdog***) *In a zero-profit bribe inducement equilibrium it is never the case that only the underdog is bribed* (*upon access*)*, when the influential punter can profitably bet on the favorite without bribing.*

Proposition 2. (***Bribe inducement***) *Suppose* $p_1 < \tilde{p}_1$*, and at* $\pi_1 = \pi_{10}$*,* $\pi_2 = \pi_{20}$ (*as defined above*) *the bribe inducement constraint* (3) *is satisfied for* π_{10} *and the bribe inducement constraint* (4) *is violated for* π_{20}*. Then* (π_{10}, π_{20}) *constitute a competitive equilibrium, if*

$$h_1 + \psi_2(h_1) < 1; \tag{5}$$

or, if

$$(1-\mu_1-\mu_2)p_1+\mu_1\lambda_1 p_1+\mu_2(1-\lambda_2 p_2) \geq \frac{p_1(1-\lambda_1)(1-\lambda_2 p_2)}{p_1(1-\lambda_1)(1-\lambda_2 p_2)\Omega_2 + p_2(1-\lambda_1 p_1)\Omega_1}. \tag{6}$$

At these prices,

(*i*) *punter I will bribe team 2 whenever he gets an access to team 2,*

(*ii*) *each bookie will earn zero expected profit, and*

(*iii*) *punter I will earn strictly positive expected profit.*

While the zero-profit outcome conforms to Bertrand competition, it also involves match-fixing, and so in some instances competition can *induce* corruption. This runs counter to a commonly accepted notion that competition should reduce corruption (Rose-Ackerman, 1978). But this principle usually applies to the context where corruption is of rent-seeking type and rent is created by government regulation. When corruption occurs through market activities (as in our model), competition is no guarantee for a corruption-free outcome.

Having identified the possibility of a zero-profit bribe inducement equilibrium, we should stress that if the bribe inducement constraint (3) is not satisfied at the proposed equilibrium prices, there will be no pure strategy equilibrium. As punter I will not bribe at these prices, rather he will bet on one of the teams and that would yield losses for the bookies. So the bookies must then raise the price of the loss-making bet. But raising prices will also not ensure bribe prevention with zero or positive profit (for $p_1 < \tilde{p}_1$ and $p_1 > \hat{p}_1$). So the usual Bertrand instability will resurface.

Example 1. To illustrate the existence of such an equilibrium, we consider some numerical examples in Table 1.

Table 1: Zero profit, bribe inducement (of team 2) prices

Parameters		**Parameters**		**Parameters**	
Ω_1=Ω_2=2, z=0.3, μ_1=μ_2=0.3, λ_1=λ_2=0.5 → $\tilde{p}_1$ =0.28, $\hat{p}_1$ =0.72		Ω_1=Ω_2=2, z=0.3, μ_1=μ_2=0.15, λ_1=λ_2=0.5 → $\tilde{p}_1$ =0.28, $\hat{p}_1$ =0.72		Ω_1=Ω_2=2, z=0.3, μ_1=μ_2=0.15, λ_1=λ_2=0.3 → $\tilde{p}_1$ =0.3, $\hat{p}_1$ =0.7	
Bribe inducement range of p_1: p_1 < 0.28		Bribe inducement range of p_1: p_1 < 0.28		Bribe inducement range of p_1: p_1 < 0.3	
Prob.	**Zero profit prices**	**Prob.**	**Zero profit prices**	**Prob.**	**Zero profit prices**
p_1=0.03, p_2=0.97	π_{10}=0.224, π_{20}=0.95; $\Psi_2(\pi_{20})$=0.251, h_1=0.008, $\Psi_2(h_1)$=0.004	p_1=0.05, p_2=0.95	π_{10}=0.15, π_{20}=0.94; $\Psi_2(\pi_{20})$=0.255, h_1=0.013, $\Psi_2(h_1)$=0.007	p_1=0.05, p_2=0.95	π_{10}=0.192, π_{20}=0.935; $\Psi_2(\pi_{20})$=0.292, h_1=0.018, $\Psi_2(h_1)$=0.013
p_1=0.05, p_2=0.95	π_{10}=0.24, π_{20}=0.925; $\Psi_2(\pi_{20})$=0.253, h_1=0.013, $\Psi_2(h_1)$=0.007	p_1=0.1, p_2=0.9	π_{10}=0.2, π_{20}=0.885; $\Psi_2(\pi_{20})$=0.260, h_1=0.026, $\Psi_2(h_1)$=0.015	P_1=0.1, p_2=0.9	π_{10}=0.236, π_{20}=0.875; $\Psi_2(\pi_{20})$=0.293, h_1=0.036, $\Psi_2(h_1)$=0.028
p_1=0.06, p_2=0.94	π_{10}=0.25, π_{20}=0.91; $\Psi_2(\pi_{20})$=0.253, h_1=0.015, $\Psi_2(h_1)$=0.008	p_1=0.16, p_2=0.84	π_{10}=0.251, π_{20}=0.81; $\Psi_2(\pi_{20})$=0.264, h_1=0.043, $\Psi_2(h_1)$=0.028	P_1=0.16, p_2=0.84	π_{10}=0.29, π_{20}=0.81; $\Psi_2(\pi_{20})$=0.295, h_1=0.059, $\Psi_2(h_1)$=0.048
Comment: Above $p_1 = 0.06$ the bribery incentive constraint $\pi_{10} \leq \Psi_2(\pi_{20})$ is violated.		**Comment**: Above $p_1 = 0.16$ the constraint $\pi_{10} \leq \Psi_2(\pi_{20})$ is violated.		**Comment**: Above $p_1 = 0.16$ the constraint $\pi_{10} \leq \Psi_2(\pi_{20})$ is violated.	

Suppose conditions are symmetric for both teams, specifically $\Omega_1 = \Omega_2 = 2$, $\mu_1 = \mu_2 = 0.3$, $\lambda_1 = \lambda_2 = 0.5$, and $z = 0.3$; see the first column. In the second column μ_1, μ_2 are reduced to 0.15, while in column 3 λ_1, λ_2 are reduced to 0.3.

For the parameter specification in column 1, $\tilde{p}_1 = 0.28$ and $\hat{p}_1 = 0.72$. Now consider $p_1 = 0.03 < \tilde{p}_1$. At this probability the zero-profit prices of ticket 1 and ticket 2 are 0.224 and 0.95 respectively, obtained by solving equations (9) and (8) in the Appendix. π_{20} is strictly less than $p_2 = 0.97$ and it gives rise to the highest bribe inducement price of π_1, $\psi_2 = 0.251$; ψ_2 is defined in (3). That ψ_2 is strictly greater than $\pi_1 = 0.224$ implies that if team 2 is accessed it will be bribed. Further, that team 1 will not be bribed is evident from the fact that $\pi_{20} > h_1 = 0.008$. Notably $h_1 < \phi_1 = 0.332$. Further, condition (5) as specified in Proposition 2 ensures that undercutting on both tickets and inducing bribery of either team, are not possible. Minimum prices to do so ($\pi_2 = h_1 = 0.008$ and $\psi_2(h_1) = 0.004$) do not satisfy the Dutch-book restriction.

Also note that the zero-profit prices lie within the intervals specified earlier: $\pi_{20} > \mu_2\lambda_2 p_2 + (1-\mu_2)p_2 = 0.82$ and $\mu_2(1-\lambda_2 p_2) + (1-\mu_2)p_1 = 0.175 < \pi_{10} < (1-\lambda_2 p_2) = 0.515$.

At higher values of p_1 the gap between the two prices gets narrower, as is evident from the two successive rows in column 1. Also with an increase in p_1, π_{10} increases and π_{20} decreases; but in all cases $\pi_{20} < p_2$ and π_{10} remains strictly less than $\psi_2(\pi_{20})$, confirming the inducement of bribery of team 2.

Column 2 shows the effects of a decrease in the probability of accessing team 2. With a lower chance of corruption, a bookie's loss to punter I on ticket 1 decreases; so to offset this smaller loss (and ensure zero profit) expected gains from naive punters on ticket 1 must fall; so π_{10} must decrease. At $p_1 = 0.05$, π_{10} falls from 0.24 to 0.15. Conversely a bookie's loss to punter I on ticket 2 (which he buys if he fails to access team 2) rises, and hence expected gains from naive punters on ticket 2 must also rise; hence π_{20} rises from 0.925 to 0.94. All other relevant constraints are satisfied including $\pi_{10} < \psi_2(\pi_{20}) = 0.255$ and $h_1 + \psi_2(h_1) < 1$. Thus, bribe inducement of only team 2 is maintained, and deviations are rendered unprofitable.

Finally, if λ_i falls corruption becomes more costly. This is shown in column 3. By applying a similar reasoning one can see why now (at $p_1 = 0.05$) π_{10} rises from 0.15 to 0.192, and π_{20} falls from 0.94 to 0.935.

Positive Profit, Bribe Prevention Equilibrium

Suppose Assumption 3 is violated and we have $\hat{p}_1 < \frac{1}{2} < \tilde{p}_1$. This will be true if Ω_i is relatively small, i.e., z is large relative to $(w + \alpha_i(f + f_I))$ so that the influential punter is quite powerful in terms of wealth. Fig. 2 presents this case.[21] We will restrict attention to $p_1 \in (\hat{p}_1, \tilde{p}_1)$. Here, we propose a bribe prevention equilibrium, $(\pi_1 = \phi_2, \pi_2 = \phi_1)$, which generates strictly positive profit since $\phi_2 > p_1, \phi_1 > p_2$. Each bookie then earns at the proposed equilibrium:

$$E\Pi^d_{BP} = \frac{y}{2}\left[3 - \phi_2 - \phi_1 - \frac{p_1}{\phi_2} - \frac{p_2}{\phi_1}\right] \equiv k.$$

k can be large around $p_1 = 1/2$.[22] We need to show that this equilibrium is immune to all possible undercutting, which are undercutting on both tickets and undercutting on each ticket separately. In what follows we provide an informal argument by suggesting conditions that will ensure immunity against all undercuttings. The formal proof and precise conditions are provided in the Appendix.

Undercutting on both tickets: First consider undercutting on both tickets. In Fig. 2, let us select p_1 to be m, at which $\phi_2 = b$ and $\phi_1 = a$. Suppose one bookie deviates from the equilibrium by undercutting π_2 slightly below a, and π_1 slightly below b. As long as the reduced π_i is strictly greater than p_i, by the violation of bribe prevention constraint (2) punter I will be strictly better off by bribing. Therefore, team 1 will be bribed with probability μ_1 and team 2 with probability μ_2. With both markets being monopolized by the undercutting bookie, he will face significant losses with probability $\mu_1 + \mu_2$, against significant gains with probability $(1 - \mu_1 - \mu_2)$. Intuitively, it then seems that his expected overall profit is likely to be smaller than the non-deviation duopoly profit if $\mu_1 + \mu_2$ is sufficiently high. Let $\bar{\mu}$ be such that the deviation profit $E\Pi_{BI}$ is just equal to the duopoly profit k, and for $\mu_1 + \mu_2 > \bar{\mu}$, $E\Pi_{BI} < k$. Thus, a lower bound on the total probability of access seems in order. In the Appendix we formally specify $\bar{\mu}$.

Undercutting on a single ticket: Suppose the deviating bookie lowers π_1 slightly from b while maintaining $\pi_2 = \phi_1 = a$. As long as $\pi_1 > p_1$ (which is indeed possible at $p_1 = m$ in Fig. 2), any slight reduction in π_1 from ϕ_2 will trigger bribery of team 2 with probability μ_2 (but team 1 will not be bribed). In the event of bribery capturing market 1 becomes a curse, and therefore, if μ_2 is sufficiently high the bookie will be deterred from such undercutting. Let the critical value of μ_2 be denoted as μ_2^*, such that at all $\mu_2 \geq \mu_2^*$, $E\Pi_{BI} \leq k$. Symmetrically, let μ_1^* be the critical value of μ_1 such that at all $\mu_1 \geq \mu_1^*$ slight undercutting on ticket 2 is deterred.

Thus, we need to have lower bounds on individual μ_is as well as their sum $(\mu_1 + \mu_2)$ to

[21] Bribe prevention then requires ticket prices to be set high, with ϕ_1, ϕ_2 shifting upwards; contrast Fig. 2 with Fig. 1, especially the reversal of positions of $\tilde{p}_1$ and $\hat{p}_1$.

[22] Note that even if the volume of bets on the two tickets may be fairly even if the ticket prices, for $p = 1/2$, are symmetric (or fairly close), large prices of bets means the bookies' profits may be substantial.

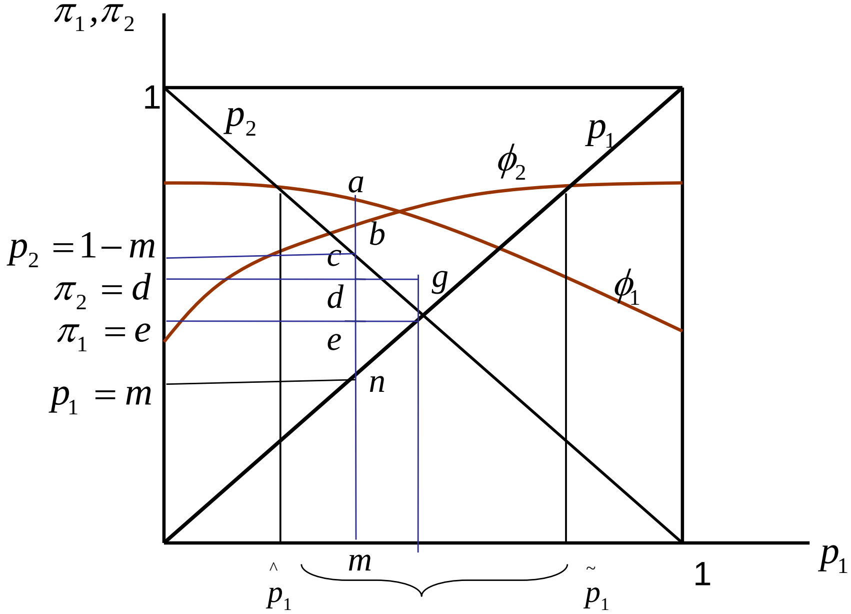

Figure 2: Positive profit, bribe prevention prices

support the proposed equilibrium. In addition to slight undercutting, we need to consider large-scale undercutting as well. For example, in Fig. 2 π_1 can be reduced from b to any point between b and n, while π_2 can be reduced to any point between a and c. Since both markets are captured, the deviating bookie will earn monopoly profit with the prospect of match-fixing for either team.[23] Identifying conditions under which this deviation (monopoly) profit falls short of the duopoly profit k proves to be difficult under the general case. But we can say that if the deviation-profit is found to be increasing at ϕ_2 and ϕ_1 (and not decreasing at $\pi_1 < \phi_2$ and $\pi_2 < \phi_1$), then restricting attention to small-scale undercutting is sufficient. In the following proposition we provide a sufficient condition to ensure that indeed that is the case. Then with this monotonicity condition and lower bound restrictions on μ_1 and μ_2 we can support the bribe prevention, positive profit equilibrium.

Proposition 3. (***Bribe prevention and positive profit***) *Suppose* $\hat{p}_1 < \frac{1}{2} < \tilde{p}_1$, *and* $p_1 \in (\hat{p}_1, \tilde{p}_1)$. *If (i)* μ_1 *and* μ_2 *exceed some threshold levels (to be precisely determined in the Appendix), and*

$$(ii) \qquad \sqrt{\rho + \frac{z\mu_2(1-\lambda_2 p_2)}{y}} \geq \phi_2, \quad \textit{and} \quad \sqrt{(1-\rho) + \frac{z\mu_1(1-\lambda_1 p_1)}{y}} \geq \phi_1, \qquad (7)$$

where $\rho = \mu_1\lambda_1 p_1 + \mu_2(1-\lambda_2 p_2) + (1-\mu_1-\mu_2)p_1$ *is the ex-ante probability of team 1 winning,*

[23] One can also see that undercutting of π_1 and π_2 need not end at p_1 and p_2. For example, one may consider $\pi_1 = e$ and $\pi_2 = d$, so that π_2 is reduced below p_2 but π_1 is raised above p_1. Mapping $\pi_1 = e$ along the horizontal axis and adding $\pi_2 = d$ to it vertically results in a point like g, which is above the 45° line, thus the price combination satisfies Dutch-book and can induce bribery.

then $\pi_1 = \phi_2$ and $\pi_2 = \phi_1$ is a bribe prevention equilibrium in which each bookie makes a positive expected profit.

The above is a possibility result which, to our knowledge, is new. One would normally expect competition to drive down bookmakers' profits to zero (as was the result in Shin's (1992) exogenous insider information model, for instance). Our intuition is that high chances of corruption make undercutting a dangerous proposition as it may create a 'lemon' (Akerlof, 1970) and give rise to an ***adverse selection*** problem similar to the credit rationing story of Stiglitz and Weiss (1981). Potential entry by the influential punter who can fix the match works as a disciplining influence deterring deviation by the bookies from the implicit 'collusive' equilibrium.

Example 2. Suppose $\lambda_1 = \lambda_2 = 0$, $\Omega_1 = \Omega_2 = \Omega$. Then $\phi_2 = \phi_1 = \frac{1}{1+\Omega}$, and $\tilde{p}_1 = \frac{1}{1+\Omega}$ and $\hat{p}_1 = \frac{\Omega}{1+\Omega}$. If $\Omega < 1$, then $\hat{p}_1 < \tilde{p}_1$. As in Proposition 3, we consider $p_1 \in (\hat{p}_1, \tilde{p}_1)$.

Next, it can be shown that at the proposed bribe prevention equilibrium $\pi_1 = \pi_2 = \frac{1}{1+\Omega}$, each bookie's expected profit $E\Pi_{BP} = \frac{y}{2}\left[\frac{\Omega(1-\Omega)}{1+\Omega}\right]$. Now consider a unilateral deviation from the proposed equilibrium by slight undercutting (on both tickets). This gives an expected profit approximately, $E\Pi_{BI} = y\left[\frac{\Omega(1-\Omega)}{1+\Omega}\right] - z(\mu_1 + \mu_2)\Omega$. Such deviation is unprofitable, if $\frac{y}{2}\left[\frac{\Omega(1-\Omega)}{1+\Omega}\right] < z(\mu_1 + \mu_2)\Omega$. Now substituting $y = (1-z)$, $\Omega = \frac{w+\alpha(f+f_I)}{z} < 1$ in the above inequality we can show that there are many values of (μ_1, μ_2) such that the above condition is satisfied. Further, to show that no other deviation is to be considered, condition (7) also has to be satisfied. Table 2 provides a numerical example to this effect.

For the numerical example, we assume symmetry with $\lambda_1 = \lambda_2 = 0$, and set $w + \alpha(f + f_I) = 0.2$ and $z = 0.25$. This specification is reported in column 1. Here, $\Omega = 0.8$, and $\hat{p}_1 = \frac{\Omega}{1+\Omega} = 0.44$, $\tilde{p}_1 = \frac{1}{1+\Omega} = 0.56$. Moreover, $\phi_2 = \phi_1 = \frac{1}{1+\Omega} = 0.56$, which is equal to our proposed equilibrium prices. We then consider several values of p_1 from the interval $(0.44, 0.56)$ and show that at each of these p_1 there is a non-empty set of μ_1 and μ_2 such that no deviation from $\pi_1 = \pi_2 = 0.56$ is profitable. Since these prices exceed p_1 and p_2, expected profit for each bookie under bribe prevention is strictly positive. Under this parameter specification one constraint on (μ_1, μ_2) that would commonly occur at all $p_1 \in (0.44, 0.56)$ is $\mu_1 + \mu_2 \geq 0.167$; this ensures that slight undercutting on *both* tickets is not profitable. Then there are four additional constraints to examine, which will vary depending on p_1.

Consider $p_1 = 0.456$. There are individual restrictions on μ_2 and μ_1 to rule out undercutting on a single ticket, as given in (12) and (13) in the Appendix. The other two constraints are given by (7), which rules out large-scale undercutting. The same constraints are then reproduced at higher values of p_1 in the interval $(\hat{p}_1, \tilde{p}_1)$. As can be seen, in each cases, the feasible set of (μ_1, μ_2) is non-empty.

Table 2: Feasible (μ_1,μ_2) for positive profit, bribe prevention

Parameter specification – Case 1: $\lambda_1=\lambda_2=0$, $z=0.25$, $\Omega=0.8 \rightarrow$ $\hat{p}_1=\frac{\Omega}{1+\Omega}=0.44, \tilde{p}_1=\frac{1}{1+\Omega}=0.56$		**Parameter specification – Case 2:** $\lambda_1=\lambda_2=0$, $z=0.4$, $\Omega=0.5 \rightarrow$ $\hat{p}_1=\frac{\Omega}{1+\Omega}=0.33, \tilde{p}_1=\frac{1}{1+\Omega}=0.67$	
	$\mu_1+\mu_2 \geq 0.167$, and		$\mu_1+\mu_2 \geq 0.25$, and
p_1=0.456	$\mu_2 \geq 0.083$, $\mu_1 \geq 0.01$ $\mu_2 \geq -0.167+0.519\mu_1$, $\mu_2 \leq 0.236+1.449\mu_1$	p_1=0.367	$\mu_2 \geq 0.15$, $\mu_1 \geq 0.02$ $\mu_2 \geq 0.06+0.28\mu_1$ $\mu_2 \leq 0.19+1.63\mu_1$
p_1=0.47	$\mu_2 \geq 0.06$, $\mu_1 \geq 0.03$ $\mu_2 \geq -0.2+0.56\mu_1$ $\mu_2 \leq 0.21+1.55\mu_1$	p_1=0.43	$\mu_2 \geq 0.12$, $\mu_1 \geq 0.06$ $\mu_2 \geq 0.009+0.35\mu_1$ $\mu_2 \leq 0.122+1.94\mu_1$
p_1=0.5	$\mu_2 \geq 0.05$, $\mu_1 \geq 0.05$ $\mu_2 \geq -0.23+0.6\mu_1$ $\mu_2 \leq 0.19+1.67\mu_1$	p_1=0.5	$\mu_2 \geq 0.09$, $\mu_1 \geq 0.09$ $\mu_2 \geq -0.48+0.43\mu_1$ $\mu_2 \leq 0.06+2.33\mu_1$
p_1=0.52	$\mu_2 \geq 0.03$, $\mu_1 \geq 0.065$ $\mu_2 \geq -0.26+0.64\mu_1$ $\mu_2 \leq 0.17+1.79\mu_1$	p_1=0.56	$\mu_2 \geq 0.05$, $\mu_1 \geq 0.12$ $\mu_2 \geq -0.11+0.51\mu_1$ $\mu_2 \leq -0.011+2.84\mu_1$
p_1=0.54	$\mu_2 \geq 0.01$, $\mu_1 \geq 0.08$ $\mu_2 \geq -0.3+0.69\mu_1$ $\mu_2 \leq 0.14+1.93\mu_1$	p_1=0.63	$\mu_2 \geq 0.02$, $\mu_1 \geq 0.15$ $\mu_2 \geq -0.18+0.61\mu_1$ $\mu_2 \leq -0.08+3.54\mu_1$

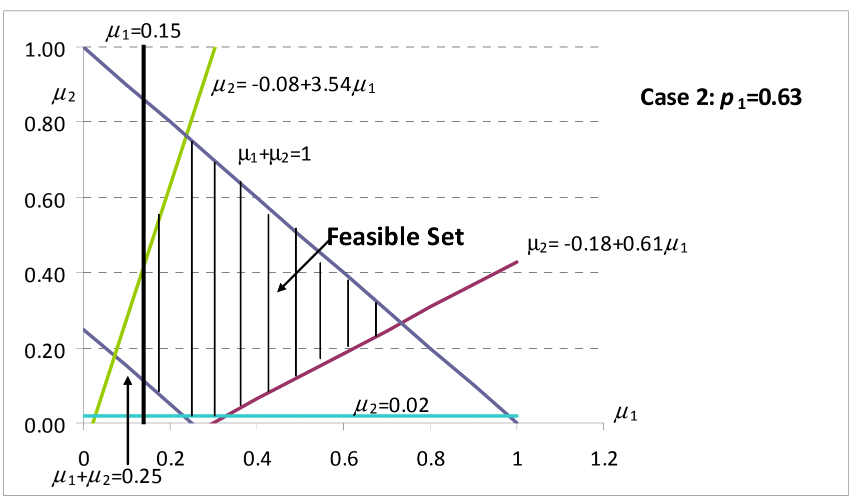

Figure 3: Feasible access probabilities

Next, in column 2 we set $z = 0.4$ leaving everything else unchanged. As punter I's wealth increases, Ω falls (to 0.5) leading to an expansion of the interval $(\hat{p}_1, \tilde{p}_1)$ to $(0.33, 0.67)$. Here too, we illustrate by taking five points inside the interval that there are many (μ_1, μ_2) that would support the proposed bribe prevention equilibrium. Fig. 3 depicts the case of $(z = 0.4, \Omega = 0.5)$ and $p_1 = 0.63$ with μ_1 on the horizontal axis and μ_2 on the vertical axis.

5 Conclusion

Match-fixing in a number of sports and its implications for betting have attracted a great deal of media attention in recent times. Building on Shin's (1991; 1992) horse-race betting model with fixed odds, we analyze the match-fixing and bribing incentives of a potentially corrupt gambler and how competition in bookmaking affects match-fixing, taking the anti-corruption authority's investigation strategy as exogenous. At the set prices, the bookies are obliged to honor the bets using deep pockets. The bookies' pricing decisions determine whether the corrupt influence comes into play or kept out. We show that the competitive equilibrium may not always ensure zero profit, nor does it always prevent bribery.

We did not comment on the favorite-longshot bias. In the bribe prevention equilibrium of Proposition 1, the bias clearly disappears. But in other cases (Propositions 2 and 3), the favorite-longshot bias reappears. We also think that studying the monopoly case should be interesting. The monopolist can control the influential punter's incentive without having to worry about losing the market to a rival. It is conceivable that sometimes the monopolist may even want to engineer match-fixing. Characterization of the monopolist's optimal strategy, i.e. whether to prevent or induce match-fixing, depending on the type of contest (close or uneven) is the subject matter of our related work, Bag and Saha (2009).

Appendix

Proof of Proposition 1. First note that $\tilde{p}_1$ and $\hat{p}_1$ can be verified to be as follows:

$$\begin{aligned}
\tilde{p}_1 &= \frac{1}{2}\Big\{\frac{(1-\lambda_2)}{\lambda_2}\frac{1+\Omega_2}{\Omega_2}\Big\}\Big[\sqrt{1+\frac{\lambda_2}{1-\lambda_2}\frac{4\Omega_2}{(1+\Omega_2)^2}}-1\Big],\\
\hat{p}_1 &= 1-\frac{1}{2}\Big\{\frac{(1-\lambda_1)}{\lambda_1}\frac{1+\Omega_1}{\Omega_1}\Big\}\Big[\sqrt{1+\frac{\lambda_1}{1-\lambda_1}\frac{4\Omega_1}{(1+\Omega_1)^2}}-1\Big].
\end{aligned}$$

Part (i): By the definition of $\tilde{p}_1$ and $\hat{p}_1$, if $\pi_1 \geq \phi_2$ ticket 1 is immune to match-fixing and if $\pi_2 \geq \phi_1$ ticket 2 is immune to match-fixing. The proof of part (i) then follows the standard Bertrand argument and the Dutch-book restriction.

Part (ii): Consider $p_1 < \tilde{p}_1$, where the focus is restricted to bribing of team 2 only; symmetric argument will apply to $p_1 > \hat{p}_1$. If there was a pure strategy bribe prevention equilibrium, it must be from one of the following two price configurations:

[1] $\pi_1 = \phi_2(p_1)$ and $\pi_2 = p_2$. (We can rule out $\pi_1 > \phi_2$ and $\pi_2 > p_2$ due to Bertrand competition in each of ticket 1 and 2 respectively.)

[2] $\pi_1 < \phi_2$ and $\pi_2 < p_2$; but π_1 is such that punter I finds bribing team 2 (when accessed) less profitable than betting on team 2. That is, $E\Pi^I(b_2) = (1-\lambda_2 p_2)z\big[\frac{1}{\pi_1} - \frac{1}{\phi_2}\big] < E\Pi^I_{02} = z\big[\frac{p_2}{\pi_2} - 1\big]$, or $\pi_1 > \psi_2$ where ψ_2 is defined in (3).

From (2) and (3) check that $\psi_2 < \phi_2$ if $\pi_2 < p_2$, and $\psi_2 = \phi_2$ if $\pi_2 = p_2$. Thus, the second set of price configuration is: $\pi_1 \in (\psi_2(p_1), \phi_2(p_1))$, $\pi_2 < p_2$.

Of these configurations, [2] cannot be equilibrium: starting from $\pi_2 < p_2$, a bookie can raise only π_2 and avoid the loss on ticket 2.

For configuration [1], starting from $(\pi_1 = \phi_2(p_1), \pi_2 = p_2)$ we show that a deviation in the form of slight undercutting on both tickets (by either bookie) will be profitable. In the posited equilibrium each bookie earns the profit $E\Pi_C = \frac{y}{2}(1-\phi_2)\left[1 - \frac{p_1}{\phi_2}\right] > 0$. Now consider the following deviation: $\pi_2 = p_2 - \epsilon$ and $\pi_1 = \phi_2 - \delta$, such that $\psi_2(p_2 - \epsilon) < \pi_1 = \phi_2 - \delta < \phi_2$. As long as $\epsilon > 0$, $\psi_2(p_2 - \epsilon) < \phi_2$ since ψ_2 is increasing in π_2 (easy to check), and then some $\delta > 0$ can be chosen to satisfy the above inequality. Further, as ϵ becomes small, the permissible δ will also become small.

With this deviation bribery of team 2 is not induced; but it brings monopoly in both markets. For ϵ and δ arbitrarily small, the deviation profit must be greater than $E\Pi_C$. This is a contradiction. Thus, at no prices bribery is prevented with certainty. **Q.E.D.**

Proof of Lemma 1. Without loss of generality assume $p_1 \leq \tilde{p}_1 < 1/2$ (i.e. team 2 is favorite), and consider π_{10} and π_{20} as zero-profit prices that induce bribery of some team. Further assume that $\pi_{20} < p_2$ (and $\pi_{10} > p_1$ by the Dutch-book restriction). So the influential punter can bet on team 2 without bribing and expect a positive return. Now contrary to the claim of the lemma suppose only team 1 (which is the longshot here) is bribed (upon access). Then the influential punter will not bet on team 1.

From the zero-profit condition on ticket 1,

$$E\Pi_1 = \frac{y}{2}(1-\pi_{10})\left[1 - \frac{\mu_1\lambda_1 p_1 + (1-\mu_1)p_1}{\pi_{10}}\right] = 0,$$

we obtain $\pi_{10} = [\mu_1\lambda_1 + (1-\mu_1)]p_1 < p_1$, which is a contradiction to our assumption that $\pi_{10} > p_1$. **Q.E.D.**

Proof of Proposition 2 (**Bribe inducement equilibrium**). Below we provide conditions that would guarantee the particular type of equilibrium. Example 1 in the text shows that the conditions are not vacuous.

Formally, the equilibrium (π_{10}, π_{20}) must satisfy the following conditions:

[1] $\pi_{20} < p_2, \quad \pi_{10} > p_1;$

[2] (Zero profit from ticket 2)

$$E\Pi_2 = \frac{y}{2}(1-\pi_{20})\big[1 - \frac{(1-\mu_2)p_2 + \mu_2\lambda_2 p_2}{\pi_{20}}\big] + \frac{z}{2}(1-\mu_2)\big[1 - \frac{p_2}{\pi_{20}}\big] = 0; \tag{8}$$

[3] (Zero profit from ticket 1)

$$E\Pi_1 = \frac{y}{2}(1-\pi_{10})\big[1 - \frac{(1-\mu_2)p_1 + \mu_2(1-\lambda_2 p_2)}{\pi_{10}}\big] + \frac{z}{2}\mu_2\big[1 - \frac{1-\lambda_2 p_2}{\pi_{10}}\big] = 0; \tag{9}$$

[4] (If team 1 is accessed, it will not be bribed): $\pi_{20} > h_1$ because of the violation of the bribe inducement constraint (4).

[5] (If team 2 is accessed, it will be bribed, and a bet will be placed on team 1 as per the bribe inducement constraint (3)):

$$E\Pi^I(b_2) = (1-\lambda_2 p_2)z\big[\frac{1}{\pi_{10}} - \frac{1}{\phi_2}\big] \geq E\Pi^I_{02} = z\big[\frac{p_2}{\pi_{20}} - 1\big] \qquad \text{or,} \qquad \pi_{10} \leq \psi_2 \ \ (\leq \phi_2).$$

From the two zero-profit conditions it is evident that for $\mu_2 > 0$,

$$\begin{aligned} \lambda_2 p_2 &\leq \pi_{20} \in [\mu_2\lambda_2 p_2 + (1-\mu_2)p_2, p_2), \\ \text{and} \qquad p_1 &< \pi_{10} \in [\mu_2(1-\lambda_2 p_2) + (1-\mu_2)p_1, (1-\lambda_2 p_2)]. \end{aligned}$$

In the price ranges identified above, the Dutch-book constraint will be satisfied:

$$\pi_{20} + \pi_{10} > p_2[1 - \mu_2(1-\lambda_2)] + \mu_2(1-\lambda_2 p_2) + (1-\mu_2)p_1 = 1.$$

From these prices (satisfying (8)–(9) and (3)–(4)) any undercutting that leaves the bribe incentive of punter I unchanged (i.e. only team 2 will be bribed) will only inflict losses. But if undercutting leads to a change in the bribe incentive of punter I, we need to ensure that the undercutting is unprofitable or infeasible. There are two such deviations we need to guard against. Both π_1 and π_2 can be reduced to π_1' and π_2' such that either (i) $\pi_2' \leq h_1$ and $\pi_1' > \psi_2(\pi_2')$ (in which case team 1 will be bribed instead of team 2 for satisfying condition (4) and violating condition (3)), or (ii) $\pi_2' \leq h_1$ and $\pi_1' \leq \psi_2(\pi_2')$ (in this case either team can be bribed). Deviation (i) is ruled out by Lemma 1; that is, bribery of team 1 alone cannot occur in a zero-profit equilibrium. For deviation (ii) consider the highest values of π_1 and π_2 (i.e., $\pi_2' = h_1$ and $\pi_1' = \psi_2(\pi_2')$) that induce bribery of either team, which also correspond to the minimum undercutting necessary. If condition (5) holds, the Dutch-book restriction is violated at these deviation prices, and thus such deviations will be infeasible.

Alternatively, we can specify a condition to render such deviations unprofitable. If such undercutting were to be profitable, profit from the sale of ticket 1 to the naive punters must be strictly positive, because sale on all other counts will be loss-making (at the deviation prices). The expected profit from the sale of ticket 1 to the naive punters is non-positive if

$$y(1-\psi_2(h_1))\left[1-\frac{(1-\mu_1-\mu_2)p_1+\mu_1\lambda_1 p_1+\mu_2(1-\lambda_2 p_2)}{\psi_2(h_1)}\right]\leq 0.$$

Substituting appropriate expressions for h_1 and $\psi_2(h_1)$ we then get condition (6).

The incentive conditions for bribery [(2) and (3)] are likely to hold under appropriate parameter specifications (as demonstrated in Example 1-Table 1). In summary, zero profit, bribe inducement equilibrium obtains under the conditions (8) – (9) and (3) – (4). Finally, strictly positive profit to punter I follows from the fact that $\pi_{20} < p_2$. **Q.E.D.**

Proof of Proposition 3. Below we derive conditions that would guarantee the particular type of positive-profit, price coordination equilibrium in which bribery is prevented. Example 2 in the text shows that the conditions are not vacuous.

Slight undercutting on both tickets: By undercutting on both tickets, $\pi_1' \in (p_1, \phi_2), \pi_2' \in (p_2, \phi_1)$, bookie 1 earns the following profit:

$$\begin{aligned} E\Pi_{BI} \;=\;& \mu_1 \underbrace{y[\int_{\pi_1'}^{1}(1-\frac{\lambda_1 p_1}{\pi_1'})\,\mathrm{d}q+\int_0^{1-\pi_2'}(1-\frac{(1-\lambda_1 p_1)}{\pi_2'})\,\mathrm{d}q]}_{\equiv k_1} \\ &+\mu_2 \underbrace{y[\int_{\pi_1'}^{1}(1-\frac{(1-\lambda_2 p_2)}{\pi_1'})\,\mathrm{d}q+\int_0^{1-\pi_2'}(1-\frac{\lambda_2 p_2}{\pi_2'})\,\mathrm{d}q]}_{\equiv k_2} \\ &+(1-\mu_1-\mu_2)y[3-\pi_1'-\pi_2'-\frac{p_1}{\pi_1'}-\frac{p_2}{\pi_2'}] \\ &+z\big[\mu_1\big\{1-\frac{(1-\lambda_1 p_1)}{\pi_2'}\big\}+\mu_2\big\{1-\frac{(1-\lambda_2 p_2)}{\pi_1'}\big\}\big]. \end{aligned} \tag{10}$$

Let $\pi_1' = \phi_2 - \epsilon_1$ and $\pi_2' = \phi_1 - \epsilon_2$, ϵ_1 and ϵ_2 both arbitrarily small, and rewrite (10) as

$$E\Pi_{BI} \approx \mu_1 k_1+\mu_2 k_2+(1-\mu_1-\mu_2)2k-z\big[\mu_1 \underbrace{\frac{(1-\lambda_1 p_1)}{\phi_1-\epsilon_2}}_{>1;\,\phi_1<1-\lambda_1 p_1} +\mu_2 \underbrace{\frac{(1-\lambda_2 p_2)}{\phi_2-\epsilon_1}}_{>1;\,\phi_2<1-\lambda_2 p_2} -(\mu_1+\mu_2)\big]. \tag{11}$$

The first (and second) term(s) indicate expected profit from naive punters when team 1 (team 2) is bribed. The third term captures the no-bribery profit; this is twice the bribe prevention duopoly profit due to monopolization of both markets. The fourth term is the expected net payout to punter I which is positive-valued. The overall value of $E\Pi_{BI}$ varies inversely with $\mu_1 + \mu_2$, if one changes μ_1 and μ_2 in the same proportion. If $\mu_1 + \mu_2$ is sufficiently large (say, $\mu_1 + \mu_2 \rightarrow 1$) the magnitude of $E\Pi_{BI}$ will crucially depend on the magnitude of $\mu_1 k_1 + \mu_2 k_2$. If $\max\{k_1, k_2\}$ is not too large relative to k (or is smaller than k), then clearly $E\Pi_{BI} < k = E\Pi^d_{BP}$. On the other hand, by letting $\mu_1 + \mu_2 \rightarrow 0$ we will get $E\Pi_{BI} = 2k > k$. Therefore, by the intermediate value theorem there exists $\bar{\mu} = \mu_1 + \mu_2$ such that $E\Pi_{BI}(\bar{\mu}) = k$. Thus, slight under cutting on both tickets are ruled out, if $\mu_1 + \mu_2 \geq \bar{\mu}$.

Slight undercutting on ticket 1 alone: Now consider the possibility that the price of ticket 1 is reduced below ϕ_2, while the price of ticket 2 is held at ϕ_1. The market for ticket 1 is captured, but then team 2 will be bribed with probability μ_2 in which case punter I will bet on ticket 1. Formally we can set $\pi_2' = \phi_1$, $\mu_1 = 0$, $\epsilon_2 = 0$ and adjust for sharing of market 2 in equation (10) and $\epsilon_1 > 0$ but arbitrarily close to zero in equation (11) and reproduce the first bookie's deviation payoff as

$$\begin{aligned} E\Pi_{BI} \;\approx\;& \mu_2 k_2 + (1-\mu_2)k - \mu_2 z[\frac{(1-\lambda_2 p_2)}{\phi_2} - 1] \\ & -\mu_2 \frac{y}{2}\{(1-\phi_1)(1-\frac{\lambda_2 p_2}{\phi_1})\} + (1-\mu_2)\frac{y}{2}\{(1-\phi_2)(1-\frac{p_1}{\phi_2})\}. \end{aligned}$$

The first and fourth terms together capture the bribery profit; here since market 2 is not captured, the profit is less than k_2. The second and fifth terms together give the profit in the event of no-bribery. Here there is a gain over the duopoly bribe prevention profit, k, due to undercutting in market 1. The third term indicates the net loss to punter I. Therefore, so long as the sum of the last three terms is negative, we will have $E\Pi_{BI} < k$ provided μ_2 satisfies the following condition:

$$\mu_2 \geq \frac{\frac{y}{2}\big\{(1-\phi_2)(1-\frac{p_1}{\phi_2})\big\}}{\frac{y}{2}\big\{(1-\phi_2)(1-\frac{p_1}{\phi_2})\big\} + \frac{y}{2}\big\{(1-\phi_1)(1-\frac{\lambda_2 p_2}{\phi_1})\big\} + z\big\{\frac{(1-\lambda_2 p_2)}{\phi_2} - 1\big\}} \equiv \mu_2^*. \tag{12}$$

Slight undercutting on ticket 2 alone: The analysis is similar to the previous case. Now ticket 2 price is lowered slightly below ϕ_1, while $\pi_1' = \phi_2$. Bookie 1's deviation profit can be calculated as

$$E\Pi_{BI} \approx \mu_1 k_1 + (1-\mu_1)k - \mu_1 z\big[\frac{(1-\lambda_1 p_1)}{\phi_1} - 1\big]$$
$$-\mu_1 \frac{y}{2}\big\{(1-\phi_2)(1-\frac{\lambda_1 p_1}{\phi_2})\big\} + (1-\mu_1)\frac{y}{2}\big\{(1-\phi_1)(1-\frac{p_2}{\phi_1})\big\}.$$

The deviation can be ruled out if

$$\mu_1 \geq \frac{\frac{y}{2}\big\{(1-\phi_1)(1-\frac{p_2}{\phi_1})\big\}}{\frac{y}{2}\big\{(1-\phi_1)(1-\frac{p_2}{\phi_1})\big\} + \frac{y}{2}\big\{(1-\phi_2)(1-\frac{\lambda_1 p_1}{\phi_2})\big\} + z\big\{\frac{(1-\lambda_1 p_1)}{\phi_2} - 1\big\}} \equiv \mu_1^*. \quad (13)$$

Large-scale undercutting on both tickets: However, the above conditions do not apply to large-scale deviations. What if the prices are significantly reduced and profit rises? Let ρ denote the probability of team 1 winning (from the bookie's point of view) when either team may be bribed, where $\rho = \mu_1\lambda_1 p_1 + \mu_2(1-\lambda_2 p_2) + (1-\mu_1-\mu_2)p_1$.

The deviating bookie's bribe inducement problem is to maximize

$$E\Pi_{BI} = y\big[3 - \pi_1 - \pi_2 - \frac{\rho}{\pi_1} - \frac{(1-\rho)}{\pi_2}\big] - z\big[\mu_1\frac{(1-\lambda_1 p_1)}{\pi_2} + \mu_2\frac{(1-\lambda_2 p_2)}{\pi_1} - (\mu_1+\mu_2)\big], \quad (14)$$

subject to $p_1 \leq \pi_1 < \phi_2$ and $p_2 \leq \pi_2 < \phi_1$.

The unconstrained solutions (ignoring the two constraints) are:

$$\pi_1^* = \sqrt{\rho + \frac{z\mu_2(1-\lambda_2 p_2)}{y}}, \qquad \pi_2^* = \sqrt{(1-\rho) + \frac{z\mu_1(1-\lambda_1 p_1)}{y}}.$$

If $\pi_1^* \geq \phi_2$ and $\pi_2^* \geq \phi_1$ as in (7), then $E\Pi_{BI}$ must be non-decreasing at $\pi_1 \leq \phi_2$ and $\pi_2 \leq \phi_1$. Therefore, the deviating bookie would like to capture both markets only by undercutting slightly.

There are two other possible deviations – large-scale undercutting on ticket 1 only, and large-scale undercutting on ticket 2 only. For the specific equilibrium in this proposition, it can be shown that such deviations cannot yield higher profits for the bookies given the two conditions in (7) (see Supplementary material). **Q.E.D.**

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