The Impact of Resale on Entry in Second Price Auctions

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Abstract

This paper investigates the effect of resale allowance on entry strategies in a second price auction with two bidders whose entries are sequential and costly. We first characterize the perfect Bayesian equilibrium in cutoff strategies. We then show that there exists a unique threshold such that if the reseller's bargaining power is greater (less) than the threshold, resale allowance causes the leading bidder (the following bidder) to have a higher (lower) incentive on entry; i.e., the cutoff of entry becomes lower (higher). We also discuss asymmetric bidders and the original seller's expected revenue.

Keywords: Second price auctions; costly participation; sequential entry; resale
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1 Introduction

In this paper, we study the impact of resale opportunities on bidders' entry behavior in a sealed bid second price auction with sequential and costly participation. Under the independent private value environment, each bidder sequentially arrives at the auction and knows her private value before making the entry decision. The bidder upon arrival can observe the entry history in the auction, and then decide whether or not to enter. If she does so, an unavoidable participation cost needs to be incurred. In this circumstance, it is well known that a *deterrence effect* exists in the auction, which means that a bidder who has observed the information of the previous participation will have less incentive to incur the cost to enter. Due to this effect, the auction outcome would mis-assign the object to the low-valuation bidder, creating a possibility for resale after the auction.

In reality, resale can be conducted in different formats, such as bargaining, re-auction, and posted price. In this paper, we follow Gupta and Lebrun (1999) and Pagnozzi (2007) to assume that after the initial auction stage, bidders' valuations become commonly known and the resale stage is conducted in a standard Nash bargaining game.[1] Under this model setting, the resale opportunity creates a tradeoff for bidders: on the one hand, bidders would prefer to directly attend the resale market to avoid the participation cost in the auction; on the other hand, resale allowance would also encourage bidders to enter the auction, because the possibility of reselling the object to the other bidders may generate a higher expected payoff. Our interest is to examine how the resale allowance will affect entry strategies of bidders in such an auction.

In this paper, after introducing a resale stage in the auction game, we first characterize the perfect Bayesian equilibrium in cutoff strategies, where each bidder enters and bids truthfully if and only if her private valuation is no less than a cutoff. Second, by comparing the absence and the presence of resale in the auction, there exists a unique threshold such that if the reseller's bargaining power is greater than the threshold, the expected payoff from resale dominates the benefit of avoiding the entry cost for the leading bidder, causing her to become more aggressive on participating in the auction; i.e., her equilibrium cutoff of entry becomes lower. However, if this bargaining power is less than the threshold, our comparison shows that the cutoff of entry for the following bidder becomes higher, implying that she has a lower incentive to enter after resale is allowed.[2]

[1]See Zheng (2002), Haile (2003), and Hafalir and Krishna (2008) for resale with incomplete information.

[2]This result corresponds only to the cutoffs when entry is possible for both bidders. However, if the leading bidder's private value is lower than the cutoff of entry, we will show, in Proposition 2, that resale allowance leads the following bidder to become more aggressive on entry, because there exists an opportunity for the following bidder to resell the object to the leading bidder.

We will also provide some extended discussion. First, we relax the assumption of symmetric bidders in the auction and show that the impact of resale allowance on the cutoffs of entry is unaffected when the distributions of the bidders' valuations become asymmetric. We then comment on the change in the original seller's expected revenue after resale is allowed.

Resale has been partially studied in the auction theory literature. Haile (2000, 2001, 2003) shows that resale may occur because bidders' private values change after auction; in a two-stage auction model, a bidder may win the item in the initial auction and subsequently resell it to bidders who turn out to have high private values after auction. Even if bidders' valuations do not change, Garratt and Tröger (2006) demonstrate the possibility of resale in auctions when a speculator with zero valuation exists. They show that, to avoid bidding competition, bidders with real valuations choose to drop out of the auction and buy the item from the speculator in the resale market.

Under the setting of asymmetric bidders, Hafalir and Krishna (2008) consider resale in first and second price auctions via monopoly pricing, and conclude that a first price auction is more profitable for the seller than a second price auction. Following a similar manner, an extension of the resale stage to a dynamic bargaining game is investigated by Cheng (2011), who shows that the opposite ranking in a seller's expected revenue will be achieved when this more general type of bargaining is considered.

Additionally, Zheng (2002) and Mylovanov and Tröger (2009) identify the conditions under which the optimal auction, as characterized by Myerson (1981), can be achieved via resale. Lebrun (2012) proves that after resale is permitted, the optimal allocation can still be achieved through an English auction with a special class of asymmetric n bidders.

The rest of the paper is organized as follows. Section 2 characterizes the equilibria in cutoff strategies in a second price auction with and without resale, and examines the effect of resale allowance on bidders' cutoff strategies. In Section 3, we provide some extended discussion as remarks. Section 4 concludes this study.

2 The Setup

Consider a seller selling a single indivisible object by employing a second price auction. There are two risk-neutral potential bidders indexed by $i = 1$ or 2. Each bidder in turn decides whether or not to enter and place a bid. Prior to making the entry decision, each bidder knows her private value v_i, which is randomly drawn over $[0, 1]$ according to an accumulative distribution $F(.)$ and $f(.) \equiv F'(.)$. When entry is taken, a cost is incurred, denoted by $c \in (0,1)$. This entry cost c is the same across the two potential bidders.[3]

We further assume that bidder 1 is the leading bidder and bidder 2 is the following bidder. First, the leading bidder makes a decision regarding whether or not to enter. If she decides to enter and place a bid, she has to pay c; otherwise, she leaves the auction. In the following bidder's turn, she observes the leading bidder's decision, and then decides whether or not to pay c to participate in the auction. If a bidder decides to leave, she cannot revisit the auction. The seller's valuation of the object is zero and a reserve price cannot be set.

Obviously, it is not always a weakly dominant strategy for a bidder to place her true value when there exist a participation cost and sequential arrival of bidders in the auction. However, given other bidders bidding truthfully, conditional on entry, a bidder cannot do better than placing her true value. We therefore restrict our attention to the equilibrium where bidders use cutoff strategies; bidder i enters and bids truthfully if and only if her private valuation is no less than a cutoff $x_i \in (0,1]$. All of our results should be interpreted accordingly. Then, bidder i's strategy, denoted by $b_i(v_i)$, can be expressed as follows:

$$b_i(v_i) = \begin{cases} v_i & if\ v_i \geq x_i, \\ No & otherwise, \end{cases}$$

where "*No*" denotes no participation. In particular, if $x_i = 1$, bidder i never enters the auction, regardless of her private valuation.

2.1 Equilibrium Cutoffs of Entry without Resale

It is well known that when resale is restricted in the auction, the unique equilibrium in cutoff strategies is characterized by[4]

$$x_1' F(x_2') = c, \tag{1}$$

and

$$\frac{1}{1 - F(x_1')} \int_{x_1'}^{x_2'} (F(v) - F(x_1'))dv = c, \tag{2}$$

where x_1' and x_2' are the equilibrium cutoffs of entry for bidders 1 and 2, respectively. In this equilibrium, bidder 1 enters if $v_1 \geq x_1'$. Then, bidder 2, observing that bidder 1 has

[3] See Samuelson (1985), McAfee and McMillan (1987), Levin and Smith (1994), Tan and Yilankaya (2006), and Bulow and Klemperer (2009) for auctions with costly entry.

[4] The entry cost c should be in $(0, c')$, where c' is given by $c' = \frac{1}{1-F(c')} \int_{c'}^{1} (F(v) - F(c'))dv$. Tian and Xiao (2009) provide the technical details on the proof of this equilibrium.

entered, chooses to enter if $v_2 \geq x'_2$. Obviously, $x'_1 < x'_2$ in the equilibrium; the participation information of the leading bidder lowers the possibility of the following bidder's entry. As has been mentioned previously, this is identified as the deterrence effect in the auction. Due to this effect, the following bidder with private value $v_2 \in [x'_1, x'_2)$ cannot enter the auction, even if her private value is greater than the leading bidder's (i.e., $v_2 > v_1$). This therefore yields a possibility of resale between the two bidders.

Of course, along the equilibrium path, if bidder 2 upon arrival observes that no bid has been placed by bidder 1 in the auction, she believes that $v_1 < x'_1$ and then chooses to enter if $v_2 \geq c$; this cutoff of entry for bidder 2 is denoted by $x'_{2N} = c$. In this case, resale would also occur if $v_2 < v_1$. We will study both resale possibilities in the next subsection.

2.2 Equilibrium Cutoffs of Entry with Resale

In this subsection, we characterize the equilibrium when resale is allowed after the auction. We assume that after the auction, each bidder's private valuation becomes common knowledge, and resale (if possible) is conducted in a standard Nash bargaining game.[5] The bargaining power parameters of the reseller and the buyer are λ and $(1-\lambda)$, respectively, where $\lambda \in (0, 1)$.

Given that both bidders submit their true values conditional on entry, the only task that remains is to characterize the equilibrium cutoffs of entry in the auction with resale. Let x_1^*, x_2^*, and x_{2N}^* denote the cutoffs of entry for bidder 1 and bidder 2, where $x_1^*, x_2^*, x_{2N}^* \in (0, 1)$. Furthermore, when x_1^* and x_{2N}^* satisfy $x_{2N}^* + \frac{\lambda}{F(x_1^*)} \int_{x_{2N}^*}^{x_1^*} (F(x_1^*) - F(v))dv = c$, we write this implicit function as $x_{2N}^*(x_1^*)$, and assume that

Assumption 1. *$x_1^* F(x_{2N}^*)$ is non-decreasing in x_1^*.*

This assumption ensures that the derivative of $x_1^* F(x_{2N}^*)$ with respect to x_1^* is non-negative, which is useful to prove the existence of the equilibrium in the following analysis. Moreover, let c_M be implicitly defined by $c_M = \frac{\lambda}{1-F(c_M)} \int_{c_M}^{1} (F(v) - F(c_M))dv$. The following result is then guaranteed:

Proposition 1. *Suppose that $c \in (0, c_M)$ and Assumption 1 holds. If resale is allowed in the auction, there exists a perfect Bayesian equilibrium in cutoff strategies, which can be*

[5]This assumption can be easily justified by the fact that the entry cost in a resale market is relatively smaller than in auctions, because auctions are normally conducted by formal authorities (such as auction houses and the government), which require bidders to pass entrance examinations and qualifications for bidding. However, a resale market would be more decentralized and organized with fewer restrictions; normally, this type of trading only refers to bargaining between two bidders, and hence, a lower participation cost has to be paid. For analytical convenience, we normalize this cost to be zero in the resale market.

characterized by

$$F(x_2^*)x_1^* + \lambda \int_{x_1^*}^{x_2^*} (F(x_2^*) - F(v))dv - (1-\lambda) \int_{x_{2N}^*}^{x_1^*} (F(v) - F(x_{2N}^*))dv = c, \tag{3}$$

$$\frac{\lambda}{1-F(x_1^*)} \int_{x_1^*}^{x_2^*} (F(v) - F(x_1^*))dv = c, \tag{4}$$

and

$$x_{2N}^* + \frac{\lambda}{F(x_1^*)} \int_{x_{2N}^*}^{x_1^*} (F(x_1^*) - F(v))dv = c, \tag{5}$$

In this equilibrium, bidder 1 chooses to enter if $v_1 \geq x_1^*$. Upon arrival, if bidder 2 observes a bid in the auction, she chooses to enter if $v_2 \geq x_2^*$. If, however, there is no bid in the auction, bidder 2 chooses to enter if $v_2 \geq x_{2N}^*$. Resale occurs after the auction stage if $x_1^* \leq v_1 < v_2 \leq x_2^*$ or $x_{2N}^* \leq v_2 < v_1 \leq x_1^*$.

Proof. Before proceeding further, it is worth examining the relationship among the three cutoffs of entry x_1^*, x_2^*, and x_{2N}^*. This will help us construct the equilibrium in the following analysis.

First of all, it is clear to see that $x_{2N}^* < x_2^*$, as bidder 2 has a higher incentive to enter, when observing no bid in the auction. We further see that in any equilibrium, it is impossible to have $x_2^* \leq x_1^*$, as bidder 2 with a private value slightly above x_2^* always has an incentive to deviate (chooses not to enter), if she observes that a bid has been placed by bidder 1, implying that $v_1 \geq x_1^*$. Thus, if any equilibrium exists in the auction, the equilibrium cutoffs should be either $x_{2N}^* < x_1^* < x_2^*$ or $x_1^* \leq x_{2N}^* < x_2^*$. As the first situation gives more analytical interests than the second one, in the main text, we restrict our attention to the situation where $x_{2N}^* < x_1^* < x_2^*$.[6] We solve the game by backward induction.

Case 1.1. Observing that a bid has been placed, bidder 2 knows that bidder 1 has entered the auction, implying that $v_1 \geq x_1^*$. Then, she has two choices: either competing with bidder 1 in the auction by paying c and placing her true value, or bargaining with bidder 1 in the resale market. Now suppose that $v_2 = x_2^*$. If bidder 2 enters the auction, the expected payoff is $\frac{1}{1-F(x_1^*)} \int_{x_1^*}^{x_2^*} (F(v) - F(x_1^*))dv - c$. If, however, bidder 2 chooses to enter the resale market, resale will happen if and only if $v_1 < v_2$; bidder 1 as the reseller receives the surplus $\lambda(v_2 - v_1)$

[6]Readers in the following analysis will see that under the equilibrium where $x_{2N}^* < x_1^* < x_2^*$, resale may occur in two different situations: either bidder 2 is the reseller and bidder 1 is the buyer, or bidder 1 is the reseller and bidder 2 is the buyer. In contrast, in the equilibrium with $x_1^* \leq x_{2N}^* < x_2^*$, resale only occurs when bidder 1 is the reseller and bidder 2 is the buyer. Interested readers can see the full analysis for the second type of equilibrium in the Appendix.

and bidder 2 as the buyer gains $(1-\lambda)(v_2 - v_1)$. Thus, the expected payoff of bidder 2 from the resale market is given by $\int_{x_1^*}^{x_2^*}(1-\lambda)(x_2^* - v)\frac{f(v)}{1-F(x_1^*)}dv$. Obviously, given that the bidders play cutoff strategies, we know that bidder 2 with $v_2 = x_2^*$ should be indifferent between the two choices, namely, $\frac{1}{1-F(x_1^*)}\int_{x_1^*}^{x_2^*}(F(v) - F(x_1^*))dv - c = \int_{x_1^*}^{x_2^*}(1-\lambda)(x_2^* - v)\frac{f(v)}{1-F(x_1^*)}dv$. Simplifying it yields equation (4).

Case 1.2. Observing that the leading bidder has not entered the auction, bidder 2 believes that $v_1 < x_1^*$ and chooses to enter the auction if $v_2 \geq x_{2N}^*$. Of course, resale will also occur if $x_{2N}^* \leq v_2 < v_1 < x_1^*$. Contrary to Case 1.1, bidder 2 will be the reseller and bidder 1 will be the buyer. Clearly, in this case bidder 2 with value x_{2N}^* should be indifferent between entering and not entering, implying that $x_{2N}^* + \frac{\lambda}{F(x_1^*)}\int_{x_{2N}^*}^{x_1^*}(F(x_1^*) - F(v))dv = c$, where the first term is the expected payoff from the auction and the second term is the expected payoff from reselling to bidder 1. This gives equation (5).

Moving backward to bidder 1's entry decision and assuming that $v_1 = x_1^*$, if bidder 1 chooses to enter the auction, the only difference when resale is allowed, in contrast to no resale, is that there exists an opportunity for bidder 1 to sell the object to bidder 2 if she realizes $v_1 < v_2$ after the auction. Therefore, corresponding to Case 1.1, the total payoff of bidder 1 from entry is $x_1^* F(x_2^*) + \lambda \int_{x_1^*}^{x_2^*}(v - x_1^*)dF(v) - c$, where the first term is the expected payoff from winning the auction and the second term is the expected payoff from reselling the object to bidder 2. However, if bidder 1 chooses to enter the resale market directly, which corresponds to Case 1.2, she can only obtain the object via resale and receive the expected payoff $(1-\lambda)\int_{x_{2N}^*}^{x_1^*}(F(v) - F(x_{2N}^*))dv$.[7] When $v_1 = x_1^*$, bidder 1 should be indifferent between entering the auction and entering the resale market, and thus, $x_1^* F(x_2^*) + \lambda \int_{x_1^*}^{x_2^*}(v - x_1^*)dF(v) - c = (1-\lambda)\int_{x_{2N}^*}^{x_1^*}(F(v) - F(x_{2N}^*))dv$. Simplifying it gives equation (3).[8]

We then prove that the cutoffs x_1^*, x_2^*, and x_{2N}^* are unique in equations (3), (4), and (5) by the following three steps.

Step 1.1. Differentiating x_{2N}^* with respect to x_1^* in equation (5) shows $\frac{dx_{2N}^*}{dx_1^*} < 0$.

[7] Note that in this case, bidder 1 is the buyer and bidder 2 is the reseller.

[8] We are grateful to an anonymous referee who pointed this out.

Step 1.2. Differentiating x_2^* with respect to x_1^* in equation (3) yields

$$\begin{aligned}\frac{dx_2^*}{dx_1^*}f(x_2^*)[\lambda x_2^* + (1-\lambda)x_1^*] =& -(1-\lambda)F(x_2^*) - \lambda F(x_1^*) + (1-\lambda)[F(x_1^*) - F(x_{2N}^*)] \\ & -(1-\lambda)f(x_{2N}^*)(x_1^* - x_{2N}^*)\frac{dx_{2N}^*}{dx_1^*} \\ =& -(1-\lambda)[F(x_2^*) - F(x_1^*)] - \lambda F(x_1^*) + (1-\lambda)f(x_{2N}^*)x_{2N}^*\frac{dx_{2N}^*}{dx_1^*} \\ & -(1-\lambda)\left[F(x_{2N}^*) + f(x_{2N}^*)x_1^*\frac{dx_{2N}^*}{dx_1^*}\right].\end{aligned}$$

Since $\frac{dx_{2N}^*}{dx_1^*} < 0$ by Step 1.1, the first three terms are negative in the second equality, and thus we focus on the last term. Given the fact that $dx_1^*F(x_{2N}^*)/dx_1^* = F(x_{2N}^*) + f(x_{2N}^*)x_1^*\frac{dx_{2N}^*}{dx_1^*}$, by Assumption 1, we know that $x_1^*F(x_{2N}^*)$ is non-decreasing in x_1^*, implying $dx_1^*F(x_{2N}^*)/dx_1^* \geq 0$. Therefore, $\frac{dx_2^*}{dx_1^*} < 0$.

Step 1.3. We define $\Omega(x_1^*) \equiv \frac{\lambda}{1-F(x_1^*)}\int_{x_1^*}^{x_2^*}(F(v) - F(x_1^*))dv - c$. Differentiating $\Omega(x_1^*)$ with respect to x_1^* and noting $\frac{dx_2^*}{dx_1^*} < 0$ by Step 1.2, we have

$$\begin{aligned}\frac{d\Omega}{dx_1^*} =& \frac{\lambda}{1-F(x_1^*)}\left[(F(x_2^*) - F(x_1^*))\frac{dx_2^*}{dx_1^*} - f(x_1^*)(x_2^* - x_1^*)\right] \\ & + \frac{\lambda f(x_1^*)}{(1-F(x_1^*))^2}\int_{x_1^*}^{x_2^*}(F(v) - F(x_1^*))dv \\ & < \frac{-\lambda f(x_1^*)\int_{x_1^*}^{x_2^*}(1-F(v))dv}{(1-F(x_1^*))^2} < 0.\end{aligned}$$

If $x_1^* \to x_2^*$, $\lim_{x_1^* \to x_2^*}\Omega(x_1^*) = -c < 0$. If $x_1^* \to c$, $\lim_{x_1^* \to c}\Omega(x_1^*) = \frac{\lambda}{1-F(c)}\int_c^{x_2^*}(F(v) - F(c))dv - c$ and it is easy to check that $\Omega(c) > 0$ if $c \in (0, c_M)$.[9] Therefore, we can conclude that given any $c \in (0, c_M)$, there exists a unique x_1^* satisfying $\Omega(x_1^*) = 0$. This also implies that x_{2N}^* and x_2^* are uniquely determined.

Finally, given the cutoffs x_1^*, x_2^*, and x_{2N}^* characterized by the equations (3)-(5), we consider a strategy profile as follows. Bidder 1 upon arrival chooses to enter and to bid her true value if $v_1 \geq x_1^*$. When bidder 2 arrives, if she observes a bid in the auction, she knows that bidder 1 has already entered the auction, and she chooses to enter and to bid truthfully

[9] Differentiating $\Omega(c)$ with respect to c shows that $\frac{d\Omega}{dc} < 0$, and further, $\lim_{c \to 0}\Omega(c) = \lambda\int_0^{x_2^*}F(v)dv > 0$ and $\lim_{c \to c_M}\Omega(c) = \Omega(c_M) = 0$. Thus, given any $c \in (0, c_M)$, $\Omega(c) > 0$.

if $v_2 \geq x_2^*$. If, however, there is no bid in the auction, bidder 2 chooses to enter and to submit her private value if $v_2 \geq x_{2N}^*$. Resale occurs after the auction in both cases: either $x_1^* \leq v_1 < v_2 \leq x_2^*$ or $x_{2N}^* \leq v_2 < v_1 \leq x_1^*$.

Obviously, bidder 1 does not have an incentive to deviate from the strategy profile, given bidder 2's strategy. Along the equilibrium path, given bidder 1's strategy, it is also optimal for bidder 2 to choose not to deviate. Thus, this strategy profile constitutes a perfect Bayesian equilibrium in the auction.

□

2.3 Comparison of Cutoffs with and without Resale

In this subsection, we compare the cutoffs of entry for bidders 1 and 2 with and without resale, and present the following result:

Proposition 2. *Resale allowance results in $x_{2N}^* < x_{2N}'$, and there exists a unique threshold λ' such that $x_1^* < x_1'$ if $\lambda \geq \lambda'$ and $x_2^* > x_2'$ if $\lambda < \lambda'$.*

Proof. We first compare x_{2N}^* and x_{2N}'. Obviously, following equation (5), we have $x_{2N}^* < c = x_{2N}'$. In this case, resale allowance leads bidder 2 to become more aggressive on entry, conditional on bidder 1 having not entered the auction.

Given x_{2N}^*, we then utilize equations (3) and (4) to examine the changes in the cutoffs of entry x_1^* and x_2^*. For convenience, let $G(x_1^*, x_2^*)$ and $H(x_1^*, x_2^*)$ denote the left hand side of equations (3) and (4), respectively; this allows us to rewrite equations (3) and (4) as $G(x_1^*, x_2^*) = c$ and $H(x_1^*, x_2^*) = c$. Further, differentiating $G(x_1^*, x_2^*)$ with respect to x_1^* and x_2^* yields $\frac{\partial G}{\partial x_1^*} > 0$ and $\frac{\partial G}{\partial x_2^*} > 0$, respectively, indicating that $G(x_1^*, x_2^*)$ is increasing in both arguments (i.e., x_1^* and x_2^*). Additionally, differentiating $H(x_1^*, x_2^*)$ with respect to x_1^* and x_2^* yields $\frac{\partial H}{\partial x_1^*} < 0$ and $\frac{\partial H}{\partial x_2^*} > 0$, respectively, showing that $H(x_1^*, x_2^*)$ is decreasing in the first argument (x_1^*) but increasing in the second (x_2^*). Given $\lambda \in (0, 1)$ and x_{2N}^*, substituting x_1' and x_2' into functions H and G yields $H(x_1', x_2') < c$, and $G(x_1', x_2') \geq (<)c$ if $\lambda \geq (<)\lambda'$, where λ' satisfies that $\lambda' \int_{x_1'}^{x_2'} (F(x_2') - F(v))dv = (1 - \lambda') \int_{x_{2N}^*}^{x_1'} (F(v) - F(x_{2N}^*))dv$.

Given the properties of functions G and H above, we prove the proposition by contradiction.

(I) $\lambda \geq \lambda'$, implying that $G(x_1', x_2') \geq c$ and $H(x_1', x_2') < c$.

Case 2.1. If $x_1^* = x_1'$, given that $G(x_1^*, x_2^*) = c$ and $G(x_1^*, x_2') \geq c$, then x_2^* is no greater than x_2'. However, this case would not support the equilibrium outcomes, as the fact that $H(x_1^*, x_2^*) = c$ and $H(x_1^*, x_2') < c$ requires $x_2^* > x_2'$, creating a contradiction.

Case 2.2. If $x_1^* > x_1'$ and $x_2^* \geq x_2'$, given that $G(x_1', x_2') \geq c$ and that $G(x_1^*, x_2^*)$ is increasing in both x_1^* and x_2^*, then $G(x_1^*, x_2^*) > c$, which contradicts the fact that $G(x_1^*, x_2^*) = c$. If $x_1^* > x_1'$ and $x_2^* < x_2'$, both $G(x_1', x_2') \geq c$ and $G(x_1^*, x_2^*) = c$ can be satisfied. However, given that $H(x_1', x_2') < c$ and that $H(x_1^*, x_2^*)$ is decreasing in x_1^* but increasing in x_2^*, then $H(x_1^*, x_2^*) < c$, contradictory to $H(x_1^*, x_2^*) = c$. Thus, $x_1^* > x_1'$ cannot be supported in the equilibrium outcomes.

Therefore, $x_1^* < x_1'$. Nevertheless, given that the leading bidder becomes more aggressive on entry after resale is allowed, we cannot rule out all of the possibilities for the change in the following bidder's cutoff of entry where $x_2^* \gtreqqless x_2'$.

(II) $\lambda < \lambda'$, implying that $G(x_1', x_2') < c$ and $H(x_1', x_2') < c$.

Case 2.3. If $x_2' = x_2^*$, given that $G(x_1^*, x_2^*) = c$ and $G(x_1', x_2^*) < c$, then x_1^* is greater than x_1'. However, this case would not support the equilibrium outcomes, as the fact that $H(x_1^*, x_2^*) = c$ and $H(x_1', x_2^*) < c$ requires $x_1' > x_1^*$, creating a contradiction.

Case 2.4. If $x_2' > x_2^*$ and $x_1' \geq x_1^*$, given that $G(x_1^*, x_2^*)$ is increasing in both x_1^* and x_2^*, then $G(x_1', x_2') > c$, which contradicts the fact that $G(x_1', x_2') < c$. If $x_2' > x_2^*$ and $x_1' < x_1^*$, both $G(x_1', x_2') < c$ and $G(x_1^*, x_2^*) = c$ can be satisfied. However, given that $H(x_1^*, x_2^*)$ is decreasing in x_1^* but increasing in x_2^*, then $H(x_1', x_2') > c$, contradicting $H(x_1', x_2') < c$. Thus, $x_2' > x_2^*$ cannot be supported in the equilibrium outcomes.

Therefore, $x_2' < x_2^*$. Nevertheless, given that the following bidder has a lower incentive on entry after resale is allowed, we cannot rule out all of the possibilities for the change in the leading bidder's cutoff of entry where $x_1^* \gtreqqless x_1'$.

$\square$

3 Some Remarks

In this section, we provide some remarks as follows.

Asymmetric Bidders. Under the same setting but with asymmetric distributions of the bidders' valuations, i.e., $F_1(.) \neq F_2(.)$, the equilibrium in cutoff strategies can still be easily established. If the auction is without resale, the equilibrium outcome can be rewritten as follows: $x_1' F_2(x_2') = c$ and $\frac{1}{1-F_1(x_1')} \int_{x_1'}^{x_2'} (F_1(v) - F_1(x_1'))dv = c$. In addition, the equilibrium with resale is characterized by $F_2(x_2^*)[\lambda x_2^* + (1-\lambda)x_1^*] - \lambda \int_{x_1^*}^{x_2^*} F_2(v)dv - (1-\lambda) \int_{x_{2N}^*}^{x_1^*} (F_2(v) - F_2(x_{2N}^*))dv = c$, $\frac{\lambda}{1-F_1(x_1^*)} \int_{x_1^*}^{x_2^*} (F_1(v) - F_1(x_1^*))dv = c$, and $x_{2N}^* + \frac{\lambda}{F_1(x_1^*)} \int_{x_{2N}^*}^{x_1^*} (F_1(x_1^*) - F_1(v))dv =$

c. To ensure the existence of the equilibrium, similar to Assumption 1, we assume that $x_1^* F_2(x_{2N}^*)$ is non-decreasing.[10]

Given the above equilibrium outcomes, it is easy to see that the setting of asymmetric bidders does not affect the result in Proposition 2; after the resale stage is introduced, there exists a unique threshold such that the leading bidder becomes more aggressive on entry if the reseller's bargaining power is greater than the threshold, whereas the following bidder's entry incentive becomes lower if this bargaining power is less than the threshold.[11]

Remark 1. *The impact of resale allowance on the cutoffs of entry is unaffected if the bidders' valuations are not identically distributed.*

Expected Revenue of the Original Seller. Let Π_s denote the original seller's expected revenue in the auction. Clearly, Π_s is given by $\Pi_s(k_1, k_2) = (1 - F(k_2)) \int_{k_1}^{k_2} v f(v) dv + 2 \int_{k_2}^{1} v(1 - F(v)) f(v) dv$, where $k_1 = x_1'$ and $k_2 = x_2'$ without a resale stage, and $k_1 = x_1^*$ and $k_2 = x_2^*$ with a resale stage. The first term is the expected payment when v_1 is in $[k_1, k_2]$ and v_2 is in $[k_2, 1]$, whereas the second term is the expected payment when both bidders' valuations are in $[k_2, 1]$. In other cases, the expected payment for the original seller is zero.

Remark 2. *Given the ambiguity of the changes in the bidders' cutoffs of entry, the effect of resale allowance on the original seller's expected revenue remains unclear.*

Note that we can utilize $\Pi_s(k_1, k_2)$ to compute the seller's expected revenues in both types of equilibria with resale.[12] Thus, the unclearness of the effect of resale allowance on the original seller's expected revenue (i.e., Remark 2) applies in both types of equilibria.

Example. Assume that the bidders' private values are uniformly distributed on the unit interval (i.e., $v \in [0, 1]$, $F(v) = v$). If $c = 0.2$ and $\lambda = 0.59$, it is easy to compute that no resale yields $x_1' = 0.251$, $x_2' = 0.798$, and $\Pi_s(x_1', x_2') = 0.093$; resale allowance gives $x_1^* = 0.017$, $x_2^* = 0.833$, and $\Pi_s(x_1^*, x_2^*) = 0.082$. The original seller is worse off with resale. However, if $c = 0.16$ and $\lambda = 0.61$, we obtain $x_1' = 0.222$, $x_2' = 0.721$, and $\Pi_s(x_1', x_2') = 0.129$ without resale; while we have $x_1^* = 0.011$, $x_2^* = 0.731$, and $\Pi_s(x_1^*, x_2^*) = 0.131$ with resale. The original seller's revenue increases after resale is allowed.

[10] We also provide the equilibrium outcome with asymmetric bidders when $x_1^* \leq x_{2N}^* < x_2^*$ in the Appendix.

[11] Similarly, if the leading bidder's private value is lower than the cutoff of entry, resale allowance will still lead the following bidder to become more aggressive on entry.

[12] $\Pi_s(k_1, k_2)$ only depends on the bidders' cutoffs when both bidders enter the auction.

4 Conclusion

In this paper, we study how resale allowance affects the entry strategies of two bidders who sequentially decide to participate in a second price auction. Moreover, this participation is a costly activity for both bidders. We first characterize the perfect Bayesian equilibrium in cutoff strategies. We then demonstrate that resale allowance increases the leading bidder's incentive to enter the auction if the reseller's bargaining power is sufficiently large (i.e., greater than a threshold). Conversely, if this bargaining power is lower than the threshold, the cutoff of entry for the following bidder increases.

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Appendix

The equilibrium where $x_1^* \leq x_{2N}^* < x_2^*$

In the following proposition, we provide the existence of the equilibrium in cutoff strategies, where $x_1^* \leq x_{2N}^* < x_2^*$.

Proposition 3. *If resale is allowed in the auction and $c \in (0, c_M)$, there exists a perfect Bayesian equilibrium in cutoff strategies, which can be characterized by*

$$F(x_2^*)[\lambda x_2^* + (1-\lambda)x_1^*] - \lambda \int_{x_1^*}^{x_2^*} F(v)dv = c, \tag{6}$$

$$\frac{\lambda}{1-F(x_1^*)} \int_{x_1^*}^{x_2^*} (F(v) - F(x_1^*))dv = c, \tag{7}$$

and

$$x_{2N}^* = c. \tag{8}$$

In this equilibrium, bidder 1 chooses to enter if $v_1 \geq x_1^*$. Upon arrival, if bidder 2 observes a bid in the auction, she chooses to enter if $v_2 \geq x_2^*$. If, however, there is no bid in the auction, bidder 2 chooses to enter if $v_2 \geq x_{2N}^* = c$. Resale occurs after auction if $x_1^* \leq v_1 < v_2 \leq x_2^*$.

Proof. After resale is allowed in the auction, there are two cases when bidder 2 arrives.

Case 3.1. Observing that a bid has been placed, bidder 2 knows that bidder 1 has entered the auction and then she has two choices: either competing with bidder 1 in the auction by paying c and placing her true value, or bargaining with bidder 1 in the resale market. Suppose that $v_2 = x_2^*$. If bidder 2 enters the auction, the expected payoff becomes $\frac{1}{1-F(x_1)} \int_{x_1^*}^{x_2^*} (F(v) - F(x_1))dv - c$. If, however, bidder 2 chooses to enter the resale market, resale will happen if and only if $v_1 < v_2$; bidder 1 as the reseller shares the surplus $\lambda(v_2 - v_1)$ and bidder 2 as the buyer gains $(1-\lambda)(v_2 - v_1)$. Thus, the expected payoff of bidder 2 from the resale market is given by $\int_{x_1^*}^{x_2^*} (1-\lambda)(x_2^* - v)\frac{f(v)}{1-F(x_1^*)}dv$. Obviously, given that the bidders play cutoff strategies, we know that bidder 2 should be indifferent to both choices; these two expected payoffs should be the same: $\frac{1}{1-F(x_1^*)} \int_{x_1^*}^{x_2^*} (F(v) - F(x_1^*))dv - c = \int_{x_1^*}^{x_2^*} (1-\lambda)(x_2^* - v)\frac{f(v)}{1-F(x_1^*)}dv$. Simplifying it yields equation (7).

Moving backward to bidder 1's entry decision, following the same logic, when $v_1 = x_1^*$, bidder 1 should be indifferent about entering the auction or not. However, in contrast to no resale, the only difference when resale is allowed is that there exists an opportunity for bidder 1 to sell the object to bidder 2 if she realizes $v_1 < v_2$ after the auction. Therefore, the total payoff of bidder 1 with $v_1 = x_1^*$ from entry becomes $x_1^* F(x_2^*) + \lambda \int_{x_1^*}^{x_2^*} (v - x_1^*) dF(v) = c$, where the first term is the expected payoff from winning the auction and the second term is the expected payoff by reselling the object to bidder 2. Simplifying it gives us equation (6).

Case 3.2. Observing that the leading bidder does not enter the auction, bidder 2 believes that $v_1 < x_1^*$, and hence chooses to enter the auction if $v_2 \geq c$. This implies that $x_{2N}^* = c$. Note that, contrary to Case 3.1, there is no resale in this case, as $x_1^* \leq x_{2N}^*$ in equilibrium.

We further prove that given any $c \in (0, c_M)$, the cutoffs x_1^* and x_2^* are uniquely determined. Fixing any c, differentiating x_2^* with respect to x_1^* in equation (6) shows $\frac{dx_2^*}{dx_1^*} < 0$. We then define $\Omega(x_1^*) \equiv \frac{\lambda}{1-F(x_1^*)} \int_{x_1^*}^{x_2^*} (F(v) - F(x_1^*)) dv - c$. Differentiating $\Omega(x_1^*)$ with respect to x_1^* and noting $\frac{dx_2^*}{dx_1^*} < 0$, we have

$$
\begin{aligned}
\frac{d\Omega}{dx_1^*} &= \frac{\lambda}{1 - F(x_1^*)} \left[(F(x_2^*) - F(x_1^*)) \frac{dx_2^*}{dx_1^*} - f(x_1^*)(x_2^* - x_1^*) \right] \\
&\quad + \frac{\lambda f(x_1^*)}{(1 - F(x_1^*))^2} \int_{x_1^*}^{x_2^*} (F(v) - F(x_1^*)) dv \\
&< \frac{-\lambda f(x_1^*) \int_{x_1^*}^{x_2^*} (1 - F(v)) dv}{(1 - F(x_1^*))^2} < 0.
\end{aligned}
$$

If $x_1^* \to x_2^*$, $\Omega(x_1^*) = -c < 0$. If $x_1^* \to c$, $\lim\limits_{x_1^* \to c} \Omega(x_1^*) = \frac{\lambda}{1-F(c)} \int_c^{x_2^*} (F(v) - F(c)) dv - c$ and it is easy to check that $\Omega(c) > 0$.[13] Therefore, we can conclude that given any $c \in (0, c_M)$, there exists a unique x_1^* satisfying $\Omega(x_1^*) = 0$ and this also implies that x_2^* is uniquely determined.

Finally, given the cutoffs x_1^* and x_2^* characterized by the equation system, we consider a strategy profile such that bidder 1 after arrival chooses to enter if $v_1 \geq x_1^*$ and to leave the auction if $v_1 < x_1^*$. When bidder 2 arrives, if she observes a bid in the auction, she knows that bidder 1 has already entered the auction and chooses to enter if and only if $v_2 \geq x_2^*$. If, however, there is no bid in the auction, bidder 2 chooses to enter if $v_2 \geq c$.

Obviously, bidder 1 does not have an incentive to deviate from the strategy profile, given bidder 2's strategy. Along the equilibrium path, given bidder 1's strategy, it is also optimal for bidder 2 to choose not to deviate. Thus, this strategy profile constitutes a perfect Bayesian equilibrium in the auction. In this equilibrium, resale occurs after the auction when

[13] Differentiating $\Omega(c_M) = 0$ with respect to c shows $\frac{d\Omega}{dc} < 0$, and further, $\lim\limits_{c \to 0} \Omega(c) = \lambda \int_0^{x_2^*} F(v) dv > 0$ and $\lim\limits_{c \to c_M} \Omega(c) = \Omega(c_M) = 0$. Thus, $\Omega(c) > 0$ given any $c \in (0, c_M)$.

$x_1^* \leq v_1 < v_2 \leq x_2^*$. □

Comparison of Cutoffs with and without Resale

We compare the cutoffs of entry for bidders 1 and 2 with and without resale, and present the following result:

Proposition 4. *If resale is allowed, then $x_{2N}^* = x_{2N}'$, $x_1^* < x_1'$ but x_2^* may be greater than, less than, or equal to x_2'.*

Proof. Obviously, $x_{2N}^* = x_{2N}'$; resale allowance does not affect bidder 2's entry decision, conditional on bidder 1 having not entered the auction.

We then utilize equations (6) and (7) to examine the changes in the cutoffs of entry x_1^* and x_2^*. For convenience, let $G(x_1^*, x_2^*)$ and $H(x_1^*, x_2^*)$ denote the left hand sides of equations (6) and (7), respectively. This allows us to rewrite the equilibrium outcome with resale as follows: $G(x_1^*, x_2^*) = c$ and $H(x_1^*, x_2^*) = c$.

Given any $\lambda \in (0, 1)$, it is obvious that substituting x_1' and x_2' into functions G and H yields $G(x_1', x_2') > c$ and $H(x_1', x_2') < c$. Further, differentiating $G(x_1^*, x_2^*)$ with respect to x_2^* and $H(x_1^*, x_2^*)$ with respect to x_1^* yields $\frac{\partial G}{\partial x_2^*} > 0$ and $\frac{\partial H}{\partial x_1^*} < 0$. Thus, we know that $G(x_1^*, x_2^*)$ is increasing in both arguments (i.e., x_1^* and x_2^*), and $H(x_1^*, x_2^*)$ is decreasing in the first argument (x_1^*) but increasing in the second (x_2^*). Given the properties of functions G and H above, we prove this proposition by contradiction. Consider the following two cases:

Case 4.1. If $x_1 = x_1'$, given the fact that $G(x_1^*, x_2^*) = c$ and $G(x_1^*, x_2') > c$, then x_2^* is no greater than x_2'. However, this case would not support the equilibrium, as the facts that $H(x_1^*, x_2^*) = c$ and $H(x_1^*, x_2') < c$ require $x_2^* > x_2'$. This creates a contradiction.

Case 4.2. If $x_1^* > x_1'$ and $x_2^* \geq x_2'$, given that $G(x_1', x_2') > c$ and $G(x_1^*, x_2^*)$ is increasing in both x_1^* and x_2^*, then $G(x_1^*, x_2^*) > c$, obviously showing a contradiction with the fact that $G(x_1^*, x_2^*) = c$. If $x_1^* > x_1'$ and $x_2^* < x_2'$, both $G(x_1', x_2') > c$ and $G(x_1^*, x_2^*) = c$ can be satisfied. However, given that $H(x_1', x_2') < c$ and $H(x_1^*, x_2^*)$ is decreasing in x_1^* but increasing in x_2^*, then $H(x_1^*, x_2^*) < c$, contradicting with $H(x_1^*, x_2^*) = c$. Therefore, $x_1^* > x_1'$ cannot be supported in the equilibrium with resale.

Therefore, we can conclude that $x_1' < x_1^*$. Nevertheless, given that the leading bidder becomes more aggressive on entry after resale is allowed, the comparison of the cutoffs for the following bidder relies on the distribution function and the bargaining power. Thus, we cannot rule out all the possibilities where $x_2^* \gtreqless x_2'$. □

Asymmetric Bidders

If the auction is with resale, the equilibrium can be rewritten as follows:

$$F_2(x_2^*)[\lambda x_2^* + (1-\lambda)x_1^*] - \lambda \int_{x_1^*}^{x_2^*} F_2(v)dv = c,$$

$$\frac{\lambda}{1 - F_1(x_1^*)} \int_{x_1^*}^{x_2^*} (F_1(v) - F_1(x_1^*))dv = c,$$

and

$$x_{2N}^* = c.$$

Given this equilibrium outcome, it is simple to see that the setting of asymmetric bidders does not change the result in Proposition 4. Accordingly, Remark 1 also holds for the equilibrium where $x_1^* \leq x_{2N}^* < x_2^*$.