Low-complexity dominance-based Sphere Decoder for MIMO Systems*

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Abstract

The sphere decoder (SD) is an attractive low-complexity alternative to maximum likelihood (ML) detection in a variety of communication systems. It is also employed in multiple-input multiple-output (MIMO) systems where the computational complexity of the optimum detector grows exponentially with the number of transmit antennas. We propose an enhanced version of the SD based on an additional cost function derived from conditions on worst case interference, that we call dominance conditions. The proposed detector, the king sphere decoder (KSD), has a computational complexity that results to be not larger than the complexity of the sphere decoder and numerical simulations show that the complexity reduction is usually quite significant.

1 Introduction

Currently, system design for wireless communications assumes the presence of multiple antennas at both transmit and receive locations in order to meet the requirements for high data rate transmission [1]. The main reason is found in the equivalent multiple-input multiple-output (MIMO) channel providing diversity and/or capacity gains to the system, where in the last case, compared to single-antenna systems, capacity is increased by a factor equal to the minimum number of transmit and receive antennas.

The problem of (optimal) maximum-likelihood (ML) decoding in MIMO systems is known to be exponentially complex in the number of transmit antennas [2,3]. Various suboptimal algorithms have been developed as low-complexity alternatives to ML decoding, e.g. branch and bound techniques [4], lattice-based approaches [5] and other tree-search algorithms as the A* algorithm [6]. A comprehensive study highlighting the connections among various approaches for low-complexity ML decoding in wireless communications is found in [7].

In the framework of communication and information theory, the term s*phere decoder* (SD) usually refers to a collection of extremely efficient algorithms based on number-theoretic tools, providing optimal or nearly-optimal solutions with

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reduced average computational complexity with respect to the exhaustive search of standard ML decoding. Inspired from the work on vector search in lattices [8, 9], various SD algorithms have been proposed, e.g. for ML sequence estimation in channels with memory [10] and ML decoding for multidimensional modulations in fading channels [11]. SD has been then extended in the context of multiantenna systems, both for uncoded and space-time coded transmissions [12]. Description and performance comparison of different methods for SD-based ML decoding are found in [13,14]: both works conclude that Schnorr–Euchner-based SD (SESD) outperforms other SD variants. Furthermore, the limitation of the algorithm to underloaded scenarios, i.e. with number of transmit antennas not exceeding the number of receive antennas, has been tackled in successive works dealing with optimal decoding in (underdetermined) overloaded systems [15–17]. It is worth noticing that some works showed that the expected complexity of SD is polynomial for a wide range of number of antennas and signal-to-noise ratio (SNR) values [18, 19], however according to a more rigorous definition of expected complexity other works state that SD exhibits reduced (w.r.t. ML) exponential complexity [20]. Other SD algorithms approaching near-ML performance and suitable for implementation with very large scale integration (VLSI) architectures have been proposed in [21].

A different approach for ML decoding, based on dominance conditions, has been studied in [22–24] for systems adopting BPSK or QPSK modulation, and then extended in [25] to arbitrary-size PSK modulation. Such an algorithm, namely *king decoder* (KD), provides the ML solution and thus it is optimal from the point of view of Symbol Error Rate (SER) performance. Two major advantages are: (i) no matrix inversion and/or factorization is needed; (ii) the same algorithm applies to both underloaded and overloaded systems.

The main contribution of this paper is an enhanced version of SD, which is based on an additional cost function derived from dominance conditions, thus exploiting the properties of KD. The new algorithm presents a significantly reduced computational complexity, measured as the average number of visited nodes, w.r.t. the classic SD.

The rest of the paper is organized as follows: in Section 2 we present the mathematical model for the system under investigation; Section 3 describes the SD; dominance conditions, representing the core of the improving innovation, are analytically studied in Section 4; the proposed KSD for MIMO detection is described in Section 5; in Section 6 we show and compare the performance in terms of computational complexity obtained via numerical simulations; finally, concluding remarks are given in Section 7.

Notation - Lower-case bold letters denote vectors, with a_n denoting the nth entry of $\mathbf{a}$; upper-case bold letters denote matrices, with $a_{n,m}$ and $\mathbf{a}_m$ denoting the (n, m)th entry and the mth column of $\mathbf{A}$, respectively; $\mathbb{E}\{\cdot\}$, $(\cdot)^*$, $(\cdot)^T$, $(\cdot)^H$, and $\|\cdot\|_2$, denote expectation, conjugate, transpose, conjugate-transpose and squared Frobenius norm operators, respectively.

2 System Model

We consider a narrowband MIMO system with K transmit antennas and N receive antennas, described by the following vector model

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n}, \tag{1}$$

where $\mathbf{y} \in \mathbb{C}^N$ is the received vector, whose entry y_i represents the signal received by the ith receive antenna; $\mathbf{H} \in \mathbb{C}^{N \times K}$ is the channel matrix, whose entry h_{ij} represents the fading coefficient between the jth transmit antenna and the ith receive antenna; $\mathbf{x} \in \mathbb{C}^K$ is the transmitted vector, whose entry x_j represents the symbol transmitted by the jth transmit antenna; $\mathbf{n} \in \mathbb{C}^N$ is the additive noise vector modeled according to a zero-mean complex Gaussian distribution with variance $\mathbb{E}\left\{\mathbf{n}\mathbf{n}^H\right\} = \eta_0 \mathbf{I}_N$. Transmitted symbols are drawn from a finite set of complex symbols χ which depends on the specific chosen modulation scheme. The channel vector from the kth transmit antenna is $\mathbf{h}_k$, i.e. the kth column of channel matrix. Also, we assume perfect channel state information at the receiver.

The problem of optimal decoding $\mathbf{x}$ from the knowledge of $\mathbf{y}$ is formulated as follows

$$\mathbf{x}_{ML} = \arg \min_{\mathbf{x} \in \chi^K} \|\mathbf{y} - \mathbf{H}\mathbf{x}\|_2 \tag{2}$$

where exhaustive search is apparently prohibitive for sizes of interest, thus the need for low-complexity alternatives. Assuming the constraint that the total average energy to be transmitted over the single symbol period cannot exceed E_x, system performance are evaluated with respect to the SNR per single receive antenna, i.e. SNR $\triangleq E_x/\eta_0$.

It is worth noticing that other kinds of systems for multiuser communications, such as direct-sequence code-division-multiple-access (DS-CDMA) [2] and multi-carrier code-division-multiple-access (MC-CDMA) [26], share the same linear model with additive noise described by (1).

3 Sphere Decoder

The idea of sphere decoding is to restrict the search to transmitted vectors whose received constellation counterparts are included in a hyper-sphere with radius r centered on the received signal $\mathbf{y}$, that is

$$\|\mathbf{y} - \mathbf{H}\mathbf{x}\|^2 < r^2. \tag{3}$$

If the sphere contains no vectors the algorithm either fails or restarts with an increased radius. In the latter case the result of the algorithm is always the optimal ML solution, obtained with reduced computational complexity when the number of vectors in the sphere is small compared to the overall number of possible transmitted vectors, i.e. $|\chi|^K$. The choice of the radius is crucial in order to obtain a computational complexity gain; in the ideal case the sphere should include just one vector.

The test in (3) is efficiently performed by exploiting the **QL** (corresp. **QR**) factorization of the channel matrix $\mathbf{H}$ in terms of a unitary matrix $\mathbf{Q}$ (i.e.

$\mathbf{Q}^H\mathbf{Q} = \mathbf{I}_N$) and a lower-triangular matrix $\mathbf{L}$ (corresp. upper-triangular matrix $\mathbf{R}$). In this case (2) can be equivalently formulated as

$$\mathbf{x}_{ML} = \arg\min_{\mathbf{x}\in\chi^K} \|\tilde{\mathbf{y}} - \mathbf{L}\mathbf{x}\|^2 \tag{4}$$

$$= \arg\min_{\mathbf{x}\in\chi^K} \sum_{i=1}^{K} \left| \tilde{y}_i - \sum_{j=1}^{i} l_{ij}x_j \right|^2, \tag{5}$$

where $\tilde{\mathbf{y}} \triangleq \mathbf{Q}^T\mathbf{y}$. The QR factorization enables the test in (3) to be formulated as a tree search with pruning [7]. In fact the summation in (5) can be performed on a tree with $K+1$ layers where the term

$$\left| \tilde{y}_i - \sum_{j=1}^{i} l_{ij}x_j \right|^2, \tag{6}$$

can be computed at each node of the layer i. The advantage of this formulation is that the partial distance in (6) is always positive; this fact implies that the children nodes have always greater partial distances, i.e. the metric is said to be *cumulative*. Therefore at each node at layer i we can compute the accumulated partial distance

$$\sum_{k=1}^{i} \left| \tilde{y}_i - \sum_{j=1}^{i} l_{ij}x_j \right|^2, \tag{7}$$

and compare it with a threshold, corresponding to r^2. The algorithm selects only the nodes leading to leaves that are within a sphere and at the same time computes the metric that will be used at the end to select the optimal solution. As stated before, if no leaves are contained in the sphere then the radius is increased and the search on the tree is restarted.

There are two possible strategies to perform the tree search: the breadth-first search (BFS) and the depth-first search (DFS) [7]. In the breadth-first search, all surviving nodes of the same level are visited before moving to the next level, until the leaves are reached. In the depth-first, at each level only one node is visited, and following its child in K steps a leaf is reached. At this point the radius is updated and the algorithm proceeds with other nodes starting from upper levels. While in BFS the tree is traversed from top to bottom, in DFS the tree is traversed horizontally. In the latter case the algorithm can be started with an infinite radius as it can be updated as soon as the first leave is reached. The performance of the SD algorithm can be improved by choosing a proper enumeration order. In Fincke-Pohst enumeration [8] branches are enumerated in a natural fashion, while in Schnorr-Euchner enumeration [9, 13] branches are selected in a *zig-zag* fashion for QAM constellations along each dimension [14].

The computational complexity of the sphere decoding algorithm is measured by the average number of visited nodes needed to obtain (4) [20]. That figure is closely related to the time required by the algorithm to provide the solution and clearly related to the throughput that is achievable in currently available digital hardware [27]. The computational cost of **QL** factorization is not considered here, since it is computed once for all and it represents a negligible factor in the overall complexity.

4 Dominance Conditions

In the sphere decoding algorithm at each node the partial distance is checked in order to exclude some branches in the tree. Another condition can be derived from the Euclidean distance that can improve the computational complexity of the sphere decoder. In this section we derive a set of sufficient conditions that can be used to exclude some possible transmitted vectors from the set of candidates in the ML search.

Geometrically the ML solution is given by the vector $\mathbf{x}$ that minimizes the Euclidean distance

$$f(\mathbf{x}) = (\mathbf{y} - \mathbf{H}\mathbf{x})^H (\mathbf{y} - \mathbf{H}\mathbf{x}). \tag{8}$$

We first define the difference of the Euclidean distance between two generic points of χ^K.

Definition 1. Given two generic vectors $\mathbf{x}$ and $\hat{\mathbf{x}}$, with $\{\mathbf{x}, \hat{\mathbf{x}}\} \in \chi^K$, the *discrete difference* is defined as $\Delta f(\mathbf{x}; \hat{\mathbf{x}}) \triangleq f(\mathbf{x}) - f(\hat{\mathbf{x}})$.

Definition 2. The discrete difference related to vectors differing only in the kth component is called *kth discrete difference* along the kth coordinate and denoted $\Delta_k f(\mathbf{x}; \hat{\mathbf{x}})$.

A necessary and sufficient condition for $\mathbf{x}$ to be a global minimum for the cost function $f(\mathbf{x})$ is then that all discrete differences $\Delta f(\mathbf{x}; \hat{\mathbf{x}})$ are non positive for each $\hat{\mathbf{x}} \in \chi^K$. The search of the global minimum just by looking at the differences does not reduce the computational complexity of the ML search alone. The number of differences to compute is still exponential with the number of inputs and the size of the constellation. However, as it will be clearer in the following, we can avoid to look at all differences and still get the optimal solution.

In the special case of the Euclidean distance the discrete difference along the generic kth coordinate takes on a specific expression, as stated by the following proposition.

Proposition 1. *For any pair of vectors $\mathbf{x}$ and $\hat{\mathbf{x}}$ that belong to χ^K and differ only in the kth position*

$$\Delta_k f(\mathbf{x}; \hat{\mathbf{x}}) = -2\Re\left\{ (x_k - \hat{x}_k)^* \left[\mathbf{h}_k^H \mathbf{y} - \sum_{i \neq k} x_i \mathbf{h}_k^H \mathbf{h}_i \right] \right\} + \left(|x_k|^2 - |\hat{x}_k|^2 \right) \mathbf{h}_k^H \mathbf{h}_k. \tag{9}$$

Proof. See appendix 8. □

The kth discrete difference in (9) depends on the observed vector $\mathbf{y}$ and on the symbols of the other elements of the input vector $\mathbf{x}$, i.e. x_i, $i \neq k$. The sign of the discrete difference $\Delta_k f(\mathbf{x}; \hat{\mathbf{x}})$ determines which of the two possible transmit vectors $\mathbf{x}$ and $\hat{\mathbf{x}}$ is closer to the observation $\mathbf{y}$.

The discrete difference expression in (9) can be simplified if a constellation with constant modulus is employed, as the second term on the right hand side of (9) becomes zero. In this case the discrete difference reduces to

$$\Delta_k f(\mathbf{x}; \hat{\mathbf{x}}) = \\ -2\Re\left\{ (x_k - \hat{x}_k)^* \left[\mathbf{h}_k^H \mathbf{y} - \sum_{i \neq k} x_i \mathbf{h}_k^H \mathbf{h}_i \right] \right\}. \quad (10)$$

4.1 Dominance conditions for 4-QAM

Since 4-QAM constellations are separable, we can equivalently consider a real-valued system model, whose dimensions are doubled, with binary signaling, i.e. $\chi = \{-1, +1\}$. In the following, theoretical results will be derived referring to the real-valued system model. In this case, the kth discrete difference is

$$\Delta_k f(\mathbf{x}; \hat{\mathbf{x}}) = -2(x_k - \hat{x}_k) \left[\mathbf{h}_k^T \mathbf{y} - \sum_{i \neq k} x_i \mathbf{h}_k^T \mathbf{h}_i \right]. \quad (11)$$

Eq. (11) can be used to make an optimal decision under the assumption that the contribution due to the other components of vector $\mathbf{x}$ are known. From (11), a necessary condition can be derived for BPSK constellations as follows. The discrete difference is non positive when the two terms on the right-hand side of (11) have the same sign

$$\text{sign}(x_k - \hat{x}_k) = \text{sign} \left[\mathbf{h}_k^T \mathbf{y} - \sum_{i \neq k} x_i \mathbf{h}_k^T \mathbf{h}_i \right]. \quad (12)$$

Note however that in the binary case there exists only one adjacent point, i.e. $\hat{x}_k = -x_k$, and the above equation can be written as

$$\text{sign}(2x_k) = \text{sign} \left[\mathbf{h}_k^T \mathbf{y} - \sum_{i \neq k} x_i \mathbf{h}_k^T \mathbf{h}_i \right], \quad (13)$$

that can be equivalently rewritten as

$$x_k = \text{sign} \left[\mathbf{h}_k^T \mathbf{y} - \sum_{i \neq k} x_i \mathbf{h}_k^T \mathbf{h}_i \right]. \quad (14)$$

From (14) we have that the ML solution must satisfy the set of equations

$$x_k^{ML} = \text{sign} \left[\mathbf{h}_k^T \mathbf{y} - \sum_{i \neq k} x_i^{ML} \mathbf{h}_k^T \mathbf{h}_i \right], \quad k = 1, \ldots, K \quad (15)$$

which provide the set of local minima of the Euclidean distance, thus representing a necessary condition for the ML solution, as for these points all the kth discrete differences are non positive.

It is interesting to note that the same set of equations has been derived in the context of Hopfield neural network (HNN) [28, 29] and applied to ML decoding. In [30, 31], detectors for code division multiple access (CDMA) have been proposed for the first time and then the idea has been further developed in [32–35]. The Eq. (14) represents the discrete-time approximation of the equation of motion of neurons, as the metric of ML optimum detector can be mapped to the energy function of the HNN and the ML solution is the result of the dynamic update of (14) (see for example [35] and references therein). Therefore the search is based on a gradient descent algorithm that may not provide the exact ML solution, but rather only a local minimum. Furthermore when the updates of the discrete-time equations are done in parallel, the solution may also present limit cycles and no convergence to a fixed point [36]. In order to prevent the updating rule to enter a limit cycle and to force the dynamic update through increasing likelihood towards the global minimum, in [37] a modified HNN approach to ML decoding is proposed, leading to a family of likelihood ascent sub-optimal detectors (LAS). All these algorithms are sub-optimal and can approach optimal performances only under specific conditions.

The necessary conditions in (15) suggest to restrict the search for the ML solution to the set of local minima. Unfortunately, no method is known to enumerate all equilibrium points, i.e. points that satisfy (15) with a computational complexity that it is not exponential. However, we can still identify cases where the determination of the sign of the kth discrete difference, i.e. the determination of the kth component of local minima, can be made regardless of the contribution of all other components of $\mathbf{x}$. A sufficient condition for the determination of the sign of the kth discrete difference is given by the following proposition.

Proposition 2. *If the following condition is satisfied*

$$\left|\mathbf{h}_k^T \mathbf{y}\right| > \sum_{i \neq k} \left|\mathbf{h}_k^T \mathbf{h}_i\right| \tag{16}$$

then the sign of the corresponding kth discrete difference for BPSK constellation is determined regardless of the contribution of all other components of $\mathbf{x}$.

Proof. See appendix 9. □

Eq. (16) is a *dominance condition* because, when it holds, the kth component of the projected received vector is so strong that dominates all other components. The dominance condition assumes that in (16) no symbols x_i, $i \neq k$, are known. However, in sequential decoding, partial knowledge may be available. In such cases the sign of the discrete difference depends only on the subset of x_i that are still to be decoded. A dominance condition when only a subset $\mathcal{W}$ of symbols is already available, can be given.

Proposition 3. *Given the set of known symbols $\mathcal{W}$ and a set of unknown symbols $\mathcal{O}$, if the following condition holds*

$$\left|\mathbf{h}_k^T \mathbf{y} - \sum_{m \in \mathcal{W}, m \neq k} x_m \mathbf{h}_k^T \mathbf{h}_m\right| > \sum_{i \in \mathcal{O}, i \neq k} \left|\mathbf{h}_i^T \mathbf{h}_k\right|, \tag{17}$$

then the sign of the corresponding kth discrete difference for BPSK constellation is determined regardless of the contribution of all components of $\mathbf{x}$, x_i, $i \in \mathcal{O}$.

Proof. Analogous to the proof of Prop. 2. □

Eq. (17) generalizes (16): if some antenna i is not dominant over his multiantenna interference, it may happen that it is *conditionally* dominant, as the interference by the already known bits is canceled out.

The sufficient condition in (16) was also derived in [33], where it has been used in a multiuser detection algorithm based on Hopfield Neural Networks. Eqs. (16) and (17) have also been used in [22] for maximum-likelihood sequence detection and then in [23] with a preprocessing algorithm for multiuser detection. In [24] they have been used for a stand-alone tree-search algorithm for low-complexity ML detection in spatial multiplexing MIMO systems, the king decoder.

Eqs. (16) and (17) are satisfied if the off-diagonal terms of the channel correlation matrix are small compared to the terms $\left|\mathbf{h}_k^T \mathbf{y}\right|$, $k = 1, \ldots, K$. Whether the conditions are satisfied or not depends on the received vector $\mathbf{y}$ and on the structure of the channel or of the correlation channel matrix.

4.2 Dominance conditions for M-QAM

In the case of M-QAM constellations, dominance conditions can be expressed in terms of those for 4-QAM, when $M = 2^n$ and n is an even number, e.g. 16-QAM. In fact such QAM constellations can be written as weighted linear combination of $n/2$ 4-QAMs [15]. For example, the 16-QAM transmit vector can be expressed as

$$\mathbf{x} = \mathbf{x}_1 + 2\mathbf{x}_2 \tag{18}$$

where $\mathbf{x}_1, \mathbf{x}_2$ are 4-QAM vectors. Consequently, the system model (1) can be written as

$$\mathbf{y} = \begin{bmatrix} \mathbf{H} & 2\mathbf{H} \end{bmatrix} \begin{pmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \end{pmatrix} + \mathbf{n} \tag{19}$$

which represents the equivalent model for 16-QAM MIMO systems with K transmit and N receive antennas in terms of 4-QAM MIMO system with $2K$ transmit and N receive antennas. Based on this equivalence, we can restrict our attention to 4-QAM MIMO systems without loss of generality.

4.3 Dominance conditions for M-PSK

It is possible to derive analogous dominance conditions in the general case of M-PSK, however in this case the real-valued system model does not hold. Dominance conditions based on the complex-valued system model have been derived and analyzed in [25]. Results are not reported here, as they are not necessary for popular systems supporting QAM.

5 King Sphere Decoder

The main contribution of this paper is the integration of the conditions (16) and (17) in *any* sphere decoding algorithm. The idea that we propose is to use the conditional dominance condition given by (17) at each node of the decoding tree in addition to the partial distance condition of the standard sphere decoding algorithm. The dominance conditions, when satisfied, allow to cut branches off

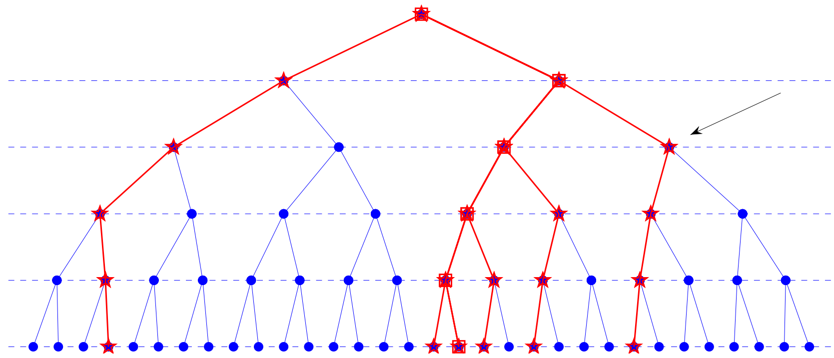

Figure 1: Tree-search algorithm for a system with $N = 5$ and $K = 5$ antennas. The transmitted bit vector is $\mathbf{x} = (1, -1, -1, -1, 1)^T$. Paths with stars are provided by the dominance conditions alone, while the path with square nodes is the ML solution

that cannot correspond to the optimal solution and then reduce the number of the visited nodes, i.e. the computational complexity of the search. The operation of the proposed algorithm is shown with the help of Fig. 1 that shows a decoding tree for a system with $N = 5$ and $M = 5$ antennas. At each node we can check whether (17) is satisfied or not. For example the node pointed by the arrow corresponds to the dominance condition

$$\left|\mathbf{h}_3^T\mathbf{y} - \mathbf{h}_3^T\mathbf{h}_2 x_2 - \mathbf{h}_3^T\mathbf{h}_1 x_1\right|_{x_1=1,x_2=1} > \left|\mathbf{h}_3^T\mathbf{h}_4\right| + \left|\mathbf{h}_3^T\mathbf{h}_5\right|. \quad (20)$$

If the condition is satisfied then a decision on the corresponding bit can be made and only one of the two branches that departs from that node is selected, and half of child nodes can be cut off. In our example such condition is satisfied and a decision on bit 3 can be made: $x_3 = -1$, if $x_1 = 1$ and $x_2 = 1$ or, equivalently, we can exclude all the vectors that have $x_1 = 1$, $x_2 = 1$, $x_3 = 1$. At the end we obtain a set of possible ML solutions, as shown in Fig. 1, where only 6 out of 32 paths survive.

The tree-search algorithm that makes use of the (conditional) dominance conditions alone has already been presented in [24, 25], where it has been called king decoder (KD). In general at the end of the tree-search the selection of the optimal solution is made among the survivors by computing the corresponding metric and then the last step of the search involves the computation of Euclidean distances for all survivors. In KD rather than compute the Euclidean distance at the end of the enumeration process, a different equivalent metric, that is cumulative and re-uses the computations done for dominance conditions, has been introduced [24].

We propose in this paper the inclusion of the dominance conditions as an additional step in a generic tree-search algorithm for ML decoding. For simplicity we restrict our attention to sphere decoding and we show that at the expenses of a marginal increase of computational complexity at each node, a significant reduction of the average number of visited node can be achieved. By integrating the dominance conditions into sphere decoding, we can exclude points that can-

Algorithm 1 Generic tree-search algorithm (adapted from [7])

```
reset_tree() {initialize the tree}
init_search() {ex.:  reset partial distance}
init_ACTIVE() {Create an empty list of active nodes}
cn = root {current node (cn) is root}
//- Main loop
while  cn is not empty  do
  if  cn is not a leaf  then
    if  cn is a valid node  then
      get valid child nodes of cn
      sort valid child nodes
      insert valid child nodes in ACTIVE
      update node counter
    end if
  else
    select best node
    update bounding function
  end if
  get next node in ACTIVE
end while
//
if  best node is empty  then
  restart with a reduced radius
else
  get the ML solution corresponding to the best node
end if
```

not be ML solution before checking if they lie within the sphere. At the end of the tree-search the partial metric computation carried on by the sphere decoder can be used to select the optimal solution. We call this enhanced version of the sphere decoder, *king sphere decoder* (KSD).

We consider the formulation of a generic tree-search algorithm based on the pseudo-code provided by Murugan *et al.* in [7] which describes a generic branch-and-bound algorithm. More generally in a tree-search algorithm at each node a decision is made based on a boolean condition that it is not necessarily expressed as a cost function compared to a bounding function, but as combination of several boolean conditions.

In the algorithm, `ACTIVE` contains an ordered set of nodes to be visited. The data structure used to implement `ACTIVE` determines the traverse strategy in the tree. In case of BFS a queue data structure can be employed, while a stack is suitable for DFS. The algorithm starts with the initialization of the radius, that can also be infinite, as in DFS. The main loop visits each valid node of the tree, starting from the root. At each node, unless a leaf is reached, the (conditional) dominance is checked first. If it is satisfied then one of the two child nodes can be excluded and will not be visited, otherwise no action is taken. Then, for each child nodes that has not been excluded, the partial distance is computed and compared against to the current radius. At this point only nodes that lie within the partial distance will be considered valid nodes. Therefore for each node, in general, a sub-set of child nodes are valid node and will be visited in the loop. Note that dominance conditions are applied to the current node and partial distances are computed on its child nodes only if they are not excluded by the previous check. If valid nodes that are generated from the current node need to be sorted, as for example in Schnorr-Euchner enumeration, a sort function is called before nodes are inserted in `ACTIVE`.

If the current visited node is a leaf then according to the metric, that is cumulatively computed, then the best candidate can be chosen and, depending on the tree traversing strategy, the radius may be updated. If a BFS is employed then it may happen that no leaf nodes are available at the end of the main loop (there is no best node) and a new search must be performed with an increased radius.

Note that the only required modification with respect to the sphere decoding algorithm is contained in the function that generates valid child nodes.

Dominance conditions introduced in the king sphere decoder can be seen as new set of constraints reducing the number of points to be visited, and for which no partial distance needs to be computed, because the new algorithm discards paths that surely cannot be local minima and then cannot be the ML solution.

As for sphere decoder, the advantage of the KSD is the expected large reduction of the number of the visited nodes and then of surviving paths. In the best-case scenario, in every visited node the dominance condition is satisfied, and then the algorithm returns a unique solution that corresponds to the ML solution, and only M nodes are visited, regardless of the choice of radius. In general the number of visited nodes is greater than M because the condition in (17) is not always satisfied. In the worst-case scenario no dominant bit is found, and then no decrease in the number of visited nodes with respect to the original tree-search algorithm is achieved. While the added conditions might increase the computational complexity at each node, the average number of visited nodes can only be decreased. Therefore the algorithm can only perform better in terms of computational complexity measured in terms of the average number of visited nodes at the expenses of increased computation at each node. In practical implementations this represents a good trade-off between speed, and then achievable throughput, and area on VLSI devices.

The efficiency of the algorithm will depend on the structure of the channel, i.e. on the matrix $\mathbf{H}$ and is higher in those cases where the off-diagonals elements of the channel correlation matrix are relatively small. This might be the case of some correlative MIMO channel models that take into account correlation among transmit and receive antennas or keyhole channels [38].

Note that the dominance conditions do not require any matrix inversion or matrix factorization and can be employed unmodified both in underloaded and overloaded systems.

6 Simulation Results

The proposed algorithm has always optimal performances in terms of SER, by construction. Performances are then measured in terms of the average number of visited nodes. We have run Monte-Carlo simulations in order to verify the improvement that can be gained with our proposed algorithm as in the worst case scenario performances are the same as those of SD.

Simulation results are presented with reference to two typologies of wireless channels, with different mathematical structures in their channel matrices: (i) independent fading, where entries of the channel matrix are assumed to be i.i.d. according to a zero-mean complex Gaussian distribution with unit variance; (ii) correlated fading, where a Kronecker model is assumed to take spatial correlation into account [39]. More specifically, in the case of correlated fading we assume that the channel matrix follows the structure [40]

$$\mathbf{H} = \mathbf{R}_R^{1/2}\mathbf{G}\mathbf{R}_T^{1/2}, \tag{21}$$

where $\mathbf{R}_T$ and $\mathbf{R}_R$ describe spatial correlation at transmit and receive locations, respectively, and $\mathbf{G}$ matches the independent fading structure.

In Fig. 2 results from simulations are shown for MIMO systems with different number of transmit and receive antennas. Two MIMO channel models are considered. The first is the standard MIMO channel model where the channel matrix elements are drawn from a complex Gaussian distribution. The second model is the correlative MIMO channel where correlation between transmit antennas and between receive antennas as in (21) where, according the model proposed in [40], we have

$$\mathbf{R}_T = \begin{pmatrix} 1 & \rho_T & \rho_T^4 & \cdots & \rho_T^{(K-1)^2} \\ \rho_T & 1 & \ddots & \ddots & \vdots \\ \rho_T^4 & \rho_T & 1 & \ddots & \rho_T^4 \\ \vdots & \ddots & \ddots & \ddots & \rho_T \\ \rho_T^{(K-1)^2} & \cdots & \rho_T^4 & \rho_T & 1 \end{pmatrix} \tag{22}$$

and

$$\mathbf{R}_R = \begin{pmatrix} 1 & \rho_R & \rho_R^4 & \cdots & \rho_R^{(N-1)^2} \\ \rho_R & 1 & \ddots & \ddots & \vdots \\ \rho_R^4 & \rho_R & 1 & \ddots & \rho_R^4 \\ \vdots & \ddots & \ddots & \ddots & \rho_T \\ \rho_R^{(N-1)^2} & \cdots & \rho_R^4 & \rho_R & 1 \end{pmatrix} \tag{23}$$

with ρ_T and ρ_R transmit and receive correlation indexes, respectively. Results are obtained for $\rho_T = 0.5$ and $\rho_R = 0.5$ and both SD and KSD, with deep first (DF) search strategy, in terms of the number of visited nodes averaged over the channel and noise realizations as well as the possible transmitted vectors [20].

Figs. 2, 3 and 4 show that in practice dominance conditions can effectively reduce the computational complexity of SD in all cases under consideration. The reduction is greater with correlated MIMO systems, suggesting that dominance conditions are more frequently satisfied in this case.

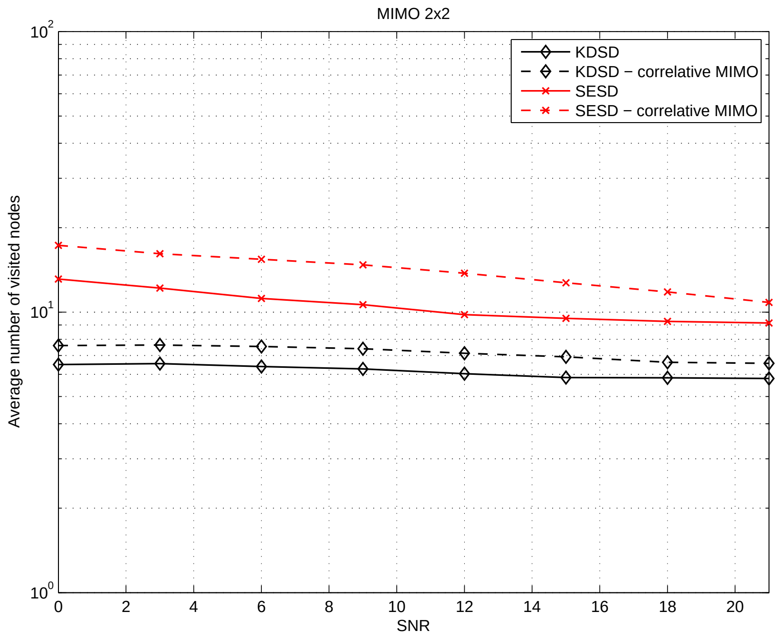

Figure 2: Average number of visited nodes as function of average signal-to-noise ratio. 4-QAM system with $K = 2$, $N = 2$.

7 Conclusions

We have proposed an enhanced version of the SD, namely KSD, that presents a lower computational complexity measured in terms of average number of visited nodes, w.r.t classic SD implementation. The reduction in complexity is possible because an additional cost function is considered in the standard tree-search based SD. The cost function is based on the dominance conditions that allows to take a decision when multiantenna interference is not too strong. Therefore the KSD has all the features of any SD algorithm and has always better performances. Numerical simulations show that for MIMO systems, both with independent and correlated fading statistics, the dominance conditions effectively reduce the computational complexity of the SD.

8 Proof of proposition 1

We explicitly write the discrete difference as:

$$\Delta_k f(\mathbf{x}; \hat{\mathbf{x}}) = \\ -2\Re\left\{(x_k - \hat{x}_k)^* \mathbf{h}_k^H \mathbf{y}\right\} + \mathbf{x}^H\mathbf{H}^H\mathbf{H}\mathbf{x} - \hat{\mathbf{x}}^H\mathbf{H}^H\mathbf{H}\hat{\mathbf{x}} \quad (24)$$

The term $\mathbf{x}^H\mathbf{H}^H\mathbf{H}\mathbf{x} - \hat{\mathbf{x}}^H\mathbf{H}^H\mathbf{H}\hat{\mathbf{x}}$ is a real scalar, so we can apply the conjugate-transpose operator with no change to obtain

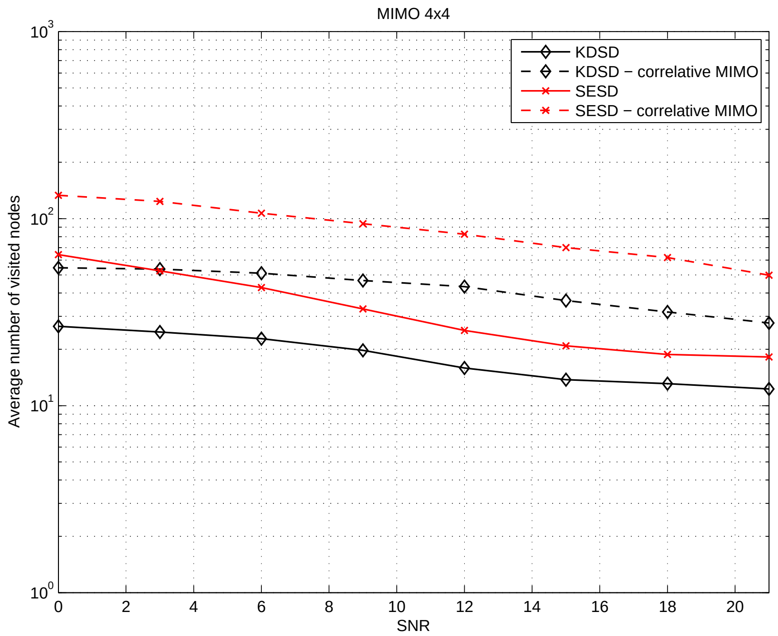

Figure 3: Average number of visited nodes as function of average signal-to-noise ratio. 4-QAM system with $K = 4$, $N = 4$.

$$\mathbf{x}^H\mathbf{H}^H\mathbf{H}\mathbf{x} - \hat{\mathbf{x}}^H\mathbf{H}^H\mathbf{H}\hat{\mathbf{x}} = \Re\left\{\sum_i\sum_j x_i^*\mathbf{h}_i^H\mathbf{h}_j x_j - \sum_m\sum_n \hat{x}_m^*\mathbf{h}_m^H\mathbf{h}_n\hat{x}_n\right\} = 2\Re\left\{(x_k - \hat{x}_k)^*\left[\mathbf{h}_k^H\mathbf{y} - \sum_{i\neq k} x_i\mathbf{h}_k^H\mathbf{h}_i\right]\right\} + \left(|x_k|^2 - |\hat{x}_k|^2\right)\mathbf{h}_k^H\mathbf{h}_k \quad (25)$$

By substituting (25) into (24) we can write the discrete difference as stated by the proposition.

9 Proof of Proposition 2

The sign of the kth discrete difference for $\mathbf{x}$ (w.r.t. its unique adjacent vector $\hat{\mathbf{x}}$ along the kth coordinate) is determined by (14), reported in the following for convenience:

$$x_k = \text{sign}\left[\mathbf{h}_k^T\mathbf{y} - \sum_{i\neq k} x_i\mathbf{h}_k^T\mathbf{h}_i\right]. \quad (26)$$

When the sufficient condition (16) holds, the first term in r.h.s. of (26) is dominant over the sum representing the second term, independently on x_i, $i \neq k$.

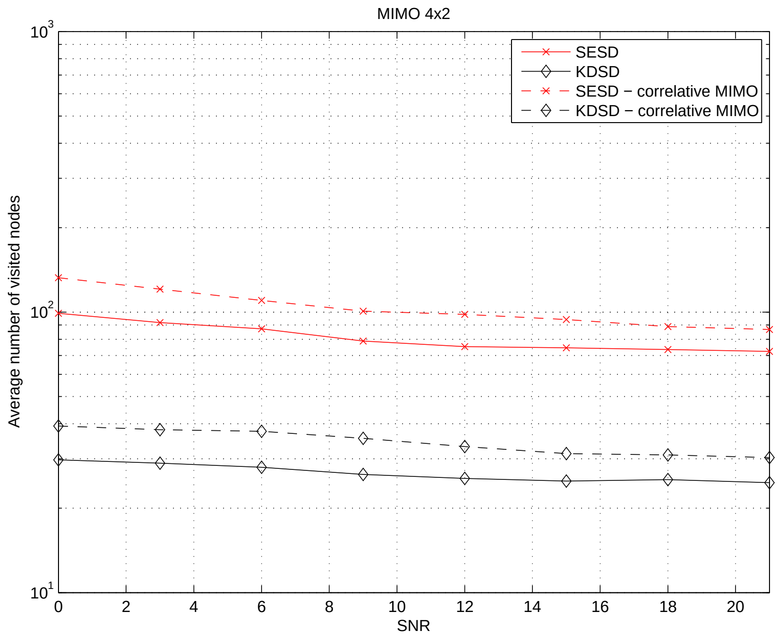

Figure 4: Average number of visited nodes as function of average signal-to-noise ratio. 16-QAM system with $K = 2$, $N = 4$.

In such a case (26) reduces to

$$x_k = \text{sign}\left[\mathbf{h}_k^T \mathbf{y}\right],$$

that is the kth discrete difference depends only on x_k, as stated by the proposition.

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