Time-based transmission power policies for energy-efficient wireless control of nonlinear systems

Vineeth S. Varma, Romain Postoyan, Daniel E. Quevedo and Irinel-Constantin Morărescu

Abstract— **We present a controller and transmission policy design procedure for nonlinear wireless networked control systems. Our objective is to ensure the stability of the closed-loop system, in a stochastic sense, together with given control performance, while minimizing the average power used to generate the communication instants. The controller is designed by emulation, i.e. ignoring the network, and the transmission instants are generated by a so-called time-based threshold policy. The latter consists in waiting a given amount of time since the last successful transmission instant before using a constant power to transmit. We explain how to select the waiting time and the power to minimize the induced average communication power while ensuring the desired control objectives.**

I. Introduction

This work aims at minimizing the energy consumption of wireless networks, which are being increasingly being deployed in control systems [1]. Since 2011, about 2–6% of the energy consumption worldwide arises from the communications and information industry, and a significant portion of this is contributed by the wireless and mobile communications companies [2]. Improving the efficiency of this technology has therefore gained a rising amount of interest in recent years [3]. For mobile devices such as cellular phones, laptops, and mobile robots, smart and careful management of the energy utilized is essential due to the limited supply of energy available. For the case of fixed infrastructure connected to wireless networks, energy consumption has become a critical issue due to environmental and economic factors and has led to a large amount of research and publications [4], [5].

In the wireless communication literature, various studies have investigated the design of energy-efficient communication systems, i.e, maximizing the ratio of data rate to the energy consumed or minimizing energy while maintaining a certain quality of service parameter, see [3] for an extensive survey. One of the most relevant techniques to improve energy efficiency is that of *transmission power control*. In works like [5] and [6], transmission power is optimized so that the ratio between the number of packets transmitted successfully to the power consumed is maximized. While these works are fully relevant in the context of regular communication systems, they are a priori not well-suited for wireless networked control systems (WNCS), which have different, specific requirements other than maximizing some kind of data rate.

A few researchers have recently published results, which consider the above mentioned problem. See [7] for example, which mostly concentrates on estimation, or [8] that looks at numerical heuristics. The survey paper [9] provides a list of works that implement energy-efficient communication design in the context of WNCS. Power control has also been studied in the control literature, like [10] and [11] for state estimation. On the other hand, [12] and [13] studied the problem of minimization of a cost defined as the sum of the control cost and the wireless power using state-feedback event-triggered controllers for linear systems. Recently, [14] looks at event-based transmission power policies for linear systems, which reduce the control cost for the same average transmission power as a constant power policy. In [15], the energy minimization problem is studied for time-triggered control when communication is assumed to be always successful but with varying costs. In [16], a framework for energy-efficient time-triggered control was proposed for discrete-time linear systems, in which the authors minimize the average transmission power while ensuring the desired control performance. Despite significant progress, a lot of open issues remain to be solved for other types of transmission policies, plant and controller models and set-ups. In particular, although recent works like [17] explore power control for interference management in nonlinear WNCS, results for nonlinear systems are crucially lacking.

In this context, we propose transmission power policies for nonlinear discrete-time systems controlled over a wireless network. For this purpose, we develop *threshold-based* transmission policies, which have been shown to perform optimally in control systems for state estimation [18]. The proposed policy is such that transmissions are not attempted until a certain threshold is passed on the time since the last successful communication. Once this condition is met, transmissions are attempted with a certain power level until the packet is received. The control law, on the other hand, is based on emulation, i.e. it is designed disregarding the presence of the network. Our main contributions are the following.

- We formulate a framework for the threshold-based transmission policies of nonlinear discrete-time systems, in contrast to several works that focus on energy-efficient transmission policies for linear systems like [13] and [16].
- We provide a set characterizing the length of the time interval before any transmission is attempted after a

V.S. Varma, R. Postoyan and I-C. Morărescu are with Université de Lorraine, CNRS, CRAN, F-54000 Nancy, France, vineeth.satheeskumar-varma@univ-lorraine.fr.
D.E. Quevedo is with the School of Electrical Engineering and Robotics, Queensland University of Technology, Brisbane, Australia, dquevedo@ieee.org.

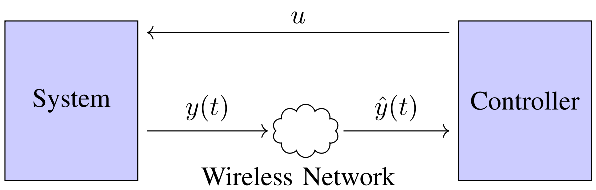

Fig. 1: Schematic of the networked control system.

successful communication, as well as the transmission power which has be used, in order to guarantee stability and performance in a stochastic sense.
- We provide the optimal transmission power to use in order to minimize the average transmission power consumed for the policies belonging to the provided set.

The rest of the paper is organized as follows. In Section II, we present the control and wireless communication models, and the main objectives. In Section III, we propose time-based threshold policies which satisfy the desired objectives. The Lyapunov assumption we make in Section II is discussed in more details in Section IV. In Section V, we provide numerical illustrations of our method. Concluding remarks in Section VI end the paper. All the major proofs are provided in the appendix.

Notation. Let $\mathbb{R} := (-\infty, \infty)$, $\mathbb{R}_{\geq 0} := [0, \infty)$, $\mathbb{Z}_{>0} := \{1, 2, \dots\}$ and $\mathbb{Z}_{\geq 0} := \{0, 1, 2, \dots\}$. We use $\Pr(\cdot)$ for the probability and $\mathbb{E}[\cdot]$ for the expectation taken over the relevant stochastic variables. We say a function $\alpha : \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ is of class $\mathcal{K}_\infty$ ($\alpha \in \mathcal{K}_\infty$) if it is continuous, strictly increasing, $\alpha(0) = 0$ and $\lim_{a\to\infty} \alpha(a) = \infty$. For any $x_1 \in \mathbb{R}^{n_1}$ and $x_2 \in \mathbb{R}^{n_2}$, (x_1, x_2) stands for $(x_1^\top, x_2^\top)^\top$.

II. Problem statement

A. Plant and controller model

We consider the discrete-time system given by

$$\begin{aligned} x_p(t+1) &= f_p(x_p(t), u(t)) \\ y(t) &= g_p(x_p(t)), \end{aligned} \tag{1}$$

where $t \in \mathbb{Z}_{\geq 0}$ is the time, $x_p(t) \in \mathbb{R}^{s_p}$ is the plant state, $u(t) \in \mathbb{R}^{s_u}$ is the control input, $y(t) \in \mathbb{R}^{s_y}$ is the measured output used for control and $s_p, s_u, s_y \in \mathbb{Z}_{>0}$ are their respective dimensions.

We proceed by emulation, we thus assume that we know a stabilizing output-feedback controller for system (1) of the form

$$\begin{aligned} x_c(t+1) &= f_c(x_c(t), y(t)) \\ u(t) &= g_c(x_c(t), y(t)), \end{aligned} \tag{2}$$

where $x_c(t) \in \mathbb{R}^{s_c}$ is the controller state. When the controller is static, we simply have $u(t) = g_c(y(t))$ in (2). At this stage, any controller design techniques can be employed to construct (2), like backstepping, feedback linearization etc. The assumption we make on the closed-loop system (1)-(2) is formalized in the sequel.

We are interested in the scenario where plant (1) and controller (2) communicate over a wireless channel as illustrated in Figure 1. As a result, the feedback loop is no longer closed at every time instant $t \in \mathbb{Z}_{\geq 0}$, but only at the instants $t_k \in \mathcal{T} \subseteq \mathbb{Z}_{\geq 0}, k \in \mathbb{Z}_{>0}$ when communication is successful. In the absence of communication, the controller uses a networked version of the output measurement denoted by $\hat{y}$. Controller (2) becomes in this context

$$\begin{pmatrix} x_c(t+1) \\ u(t) \end{pmatrix} = \begin{cases} \begin{pmatrix} f_c(x_c(t), y(t)) \\ g_c(x_c(t), y(t)) \end{pmatrix} & \text{for } t \in \mathcal{T} \\ \begin{pmatrix} f_c(x_c(t), \hat{y}(t)) \\ g_c(x_c(t), \hat{y}(t)) \end{pmatrix} & \text{for } t \in \mathbb{Z}_{\geq 0} \setminus \mathcal{T}. \end{cases} \tag{3}$$

The networked version of the output $\hat{y}$ generated at the controller evolves according to the following dynamics

$$\hat{y}(t+1) = \begin{cases} \hat{f}(g_p(x_p(t))) & \text{if } t \in \mathcal{T} \\ \hat{f}(\hat{y}(t)) & \text{if } t \in \mathbb{Z}_{\geq 0} \setminus \mathcal{T}, \end{cases} \tag{4}$$

where $\hat{f}$ is the holding function applied, which can take various forms including the zero-order-hold strategy, i.e., $\hat{f}(\hat{y}) = \hat{y}$, or the zeroing policy, i.e., $\hat{f}(\hat{y}) = 0$ for any $\hat{y} \in \mathbb{R}^{s_y}$. Note that $\hat{y}$ is never reset to the actual value of y in (4). This is fine in view of the way we model the closed-loop system below.

We introduce the concatenated state $\chi := (x_p, x_c, \hat{y}) \in \mathbb{R}^{s_\chi}$ with $s_\chi := s_p + s_c + s_y$, and we write the closed-loop dynamics of the WNCS as

$$\chi(t+1) = \begin{cases} f_S(\chi(t)) & \text{for } t \in \mathcal{T} \\ f_U(\chi(t)) & \text{for } t \in \mathbb{Z}_{\geq 0} \setminus \mathcal{T}, \end{cases} \tag{5}$$

where f_S, f_U are defined as, for $\chi \in \mathbb{R}^{s_\chi}$,

$$f_S(\chi) := \begin{pmatrix} f_p(x_p, g_c(x_c, g_p(x_p))) \\ f_c(x_c, g_p(x_p)) \\ \hat{f}(g_p(x_p)) \end{pmatrix}, \tag{6}$$

and

$$f_U(\chi) := \begin{pmatrix} f_p(x_p, g_c(x_c, \hat{y})) \\ f_c(x_c, \hat{y}) \\ \hat{f}(\hat{y}) \end{pmatrix}. \tag{7}$$

The assumptions we make on system (5) are stated next.

Assumption 1: There exist $\overline{\alpha}, \underline{\alpha} \in \mathcal{K}_\infty$, $a_1 \in (0,1)$, $a_0 > a_1$ and $V : \mathbb{R}^{s_\chi} \to \mathbb{R}_{\geq 0}$ such that, for any $\chi \in \mathbb{R}^{s_\chi}$,

$$\underline{\alpha}(|\chi|) \leq V(\chi) \leq \overline{\alpha}(|\chi|) \tag{8a}$$

$$V(f_S(\chi)) \leq a_1 V(\chi), \tag{8b}$$

$$V(f_U(\chi)) \leq a_0 V(\chi). \tag{8c}$$

□

Properties (8a) and (8b) imply that the origin of system $\chi(t+1) = f_S(\chi(t))$ is uniformly globally asymptotically stable (UGAS). This is typically the case when controller (2) has been designed to ensure that the origin of system (1)-(2) is UGAS, see Section IV. The fact that the bound in (8b) is linear in V comes with no loss of generality. If we know a Lyapunov function which does not admit a linear bound as in (8b), we can always modify it to satisfy (8b) and (8a), under mild regularity assumptions, see Theorem 2 in [19]. On the other hand, (8c) in Assumption 1 imposes a condition on the growth rate of V along solutions to (5) when a transmission fails. Typically a_0 is strictly larger than

1, and we assume $a_1 < a_0$ implying that communications improve the convergence speed of the Lyapunov function V to 0. Conditions ensuring the satisfaction of Assumption 1 are discussed in more details in Section IV.

To conclude the description of the closed-loop system (5), we need to explain when a communication attempt is successful or not. Before that, we define for the sake of convenience the function F with the following recursion, for any $\chi \in \mathbb{R}^{s_\chi}$ and $\ell \in \mathbb{Z}_{\geq 0}$

$$F(\chi, \ell) := \begin{cases} f_S(\chi) & \text{for } \ell = 1 \\ f_U(F(\chi, \ell - 1)) & \text{otherwise.} \end{cases} \tag{9}$$

This allows us to write the dynamics between successful communication instants as $\chi(t_k + \ell) = F(\chi(t_k), \ell)$ for all $t_k \in \mathcal{T}$ and $\ell \in \{1, \ldots, t_{k+1} - t_k\}$.

Remark 1: The results presented in this paper apply mutatis mutandis when the network is between the controller and the actuator, and not between the sensors and the controller as in Figure 1, by changing the network variable to be $\hat{u}$ instead of $\hat{y}$. When the network is used in both directions, the analysis becomes quite convoluted, especially if communication events occur independently; the study of this case is left for the future. □

B. Communication setup

We describe in this section, the sequence of successful communication instants $t_k \in \mathcal{T}$. In wireless communication, the signal-to-noise ratio (SNR) determines the probability of successful communication and it depends on the transmission power $P(t) \in [0, P_{\max}]$. Here, $P_{\max} > 0$ is the maximum transmission power allowed by the transmitter at any time. In particular, we make the next assumption.

Assumption 2: The following holds.

(i) The packet error rate, i.e. the probability of failing to communicate, is given by $e(P(t)) \in [0, 1]$ at all time $t \in \mathbb{Z}_{>0}$ and the mapping $P \mapsto e(P)$ is: (i-a) differentiable, (i-b) strictly decreasing on $[0, P_{\max}]$, (i-c) initially concave and then convex, (i-d) $e(0) = 1$ and $\lim_{P \to \infty} e(P) = 0$.

(ii) When a packet sent at time $t \in \mathbb{Z}_{>0}$ is received, the transmitter obtains an acknowledgement (ACK) without any error before $t + 1$. □

Item (i) of Assumption 2 models the packet error rate as a smooth time-invariant function of the transmission power, as is common in wireless communication literature [2], [5]. The additional properties considered are quite standard in wireless literature, see [6], [20] for example. On the other hand, most practical communication setups like Wifi, 4G, etc. use some sort of ACK protocol so that item (ii) of Assumption 2 is reasonable. The ACK packets have a size of the order of a few bits and are typically much smaller than the control/output information packets, and are assumed to be received without any loss [5].

C. Time-based threshold policies

We focus on a class of communication policies, which we call time-based threshold (TT) policies. These are such that communication is attempted only when a certain number of time instants have elapsed since the last successful communication, which is known by the transmitter in view of item (ii) in Assumption 2. To model the latter, we introduce the clock $\tau(t) \in \mathbb{Z}_{>0}$ for all $t \in \mathbb{Z}_{>0}$, which counts the number of time instants elapsed since the last successful communication as follows

$$\tau(t+1) = \begin{cases} 1 & \text{for } t \in \mathcal{T} \\ \tau(t) + 1 & \text{for } t \in \mathbb{Z}_{\geq 0} \setminus \mathcal{T}. \end{cases} \tag{10}$$

We assume that the initial time is a successful communication instant, i.e, we set $t_1 = 0$ resulting in $0 \in \mathcal{T}$ and $\tau(1) = 1$.

For any $t \in \mathbb{Z}_{>0}$, we set the transmission power as

$$P(t) = q(t)p \tag{11}$$

where $q(t) \in \{0, 1\}$ is updated as follows

$$q(t) = \begin{cases} 1 \text{ if } \tau(t) \geq n + 1 \\ 0 \text{ otherwise.} \end{cases} \tag{12}$$

In other words, communication is not attempted before the number of time instants since the last successful one is greater than a given constant $n \in \mathbb{Z}_{\geq 0}$[1]. Afterwards, we use a fixed power $p \in [0, P_{\max}]$ to transmit. This technique is similar in spirit to time-triggered control as commonly done in the sampled-data control literature. Under policy (11), the closed-loop system is

$$\begin{pmatrix} \chi(t+1) \\ \tau(t+1) \end{pmatrix} = \begin{cases} \begin{pmatrix} f_S(\chi(t)) \\ 1 \end{pmatrix} & \text{if } \tau(t) \geq n+1, \text{ with probability } 1 - e(p), \\ \begin{pmatrix} f_U(\chi(t)) \\ \tau(t) + 1 \end{pmatrix} & \text{otherwise.} \end{cases} \tag{13}$$

D. Objectives

The first objective of this work is to preserve the stability of the WNCS. Due to the stochastic nature of communication success, we can no longer ensure the original UGAS property ensured in Assumption 1. Instead, we need to rely on a stochastic notion of stability, as defined next and which is inspired by [21].

Definition 1: We say that the set $\{(\chi, \tau) : \chi = 0\}$ is *stochastically stable* for system (13), if there exists $\alpha \in \mathcal{K}_\infty$, such that for any solution (χ, τ),

$$\sum_{t=0}^{\infty} \mathbb{E}[\alpha(|\chi(t)|)] < \infty. \tag{14}$$

□

Definition 1 implies that we are merely interested in the stability of the origin for χ, and not τ. In addition to the partial stability property described above, we also want to ensure that the Lyapunov function V in Assumption 1 converges in expectation to 0, with a certain given rate $\mu \in (a_1, \min\{1, a_0\})$, along solutions to (13), i.e.,

$$\mathbb{E}[V(\chi(t))] \leq \mu^t V(\chi(0)) \tag{15}$$

[1] In general, one could design $P(t)$ as a function of $\tau(t)$ as is done in [16] for linear systems. We focus on the simpler threshold policies in this paper, which are easier to design and implement.

for all $t \in \mathbb{Z}_{>0}$ and for any solution (χ, τ) to (13). Property (15) serves as a measure of the control performance of system (13). Note that we always pick $\mu < a_0$ as otherwise, never communicating will achieve the objective (15).

An intuitive way to ensure the two above properties is to set $P(t) = P_{\max}$ for all $t \geq 0$ by picking $n = 0$ and $p = P_{\max}$. This would result in frequent successful communications in view of item (i) of Assumption 2, but also, and importantly, in a high power consumption [2]. We want to avoid this issue by reducing the average power consumed while preserving the stability of the origin for (13) according to Definition 1 and satisfying the convergence property (15). The average communication power over an infinite horizon is defined as

$$J(\mathbf{P}) := \lim_{T\to\infty} \mathbb{E}\left[\frac{\sum_{t=1}^{T} P(t)}{T}\right], \tag{16}$$

where $\mathbf{P} = (P(1), P(2), \dots)$ is the sequence of transmission powers applied. Our objective is to find the optimal p and n, which minimizes (16) under the TT policy defined by (11) and (12), while ensuring certain conditions, which we will identify to be sufficient for the satisfaction of the control objective.

Remark 2: Minimizing $J(\mathbf{P})$ over all possible $\mathbf{P}$ is hard to solve due to the nonlinear inter-dependence of $P(t)$ and the stability constraints, which is why we focus on TT policies in this paper. We also feel that TT policies are appealing from an implementation point of view. □

III. Main results

Given a convergence rate $\mu \in (a_1, \min\{1, a_0\})$ for the expected value of V, we first identify a set of n and p ensuring (15) and stochastic stability as in Definition 1. Later, we will minimize (16) over this set.

We define $\gamma(i) := a_1 a_0^i$ for all $i \in \mathbb{Z}_{\geq 0}$, and for any $p \in [0, P_{\max}]$ and $n \in \mathbb{Z}_{\geq 0}$,

$$\mu_n(p) := \exp\left(\frac{(1-e(p))\log(\gamma(n)) + \log(a_0)e(p)}{1 + n(1-e(p))}\right). \tag{17}$$

We prove in Theorem 1 stated below (recall that all proofs are provide in the Appendix) that $\mu_n(p)$ is a possible convergence rate for the expected value of V along solutions of (13) as in (15) while implementing the power control policy defined by (11) and (12). This theorem is one of the main results of this paper, which provides conditions for stochastic stability.

Theorem 1: For given $\mu \in (a_1, \min\{1, a_0\})$, $n \in \mathbb{Z}_{\geq 0}$ and $p \in [0, P_{\max}]$, if $\mu_n(p) \leq \mu$, then the WNCS (13) is stochastically stable and

$$\mathbb{E}[V(\chi(t))] \leq \mu_n(p)^t V(\chi(0)) \leq \mu^t V(\chi(0)) \tag{18}$$

for all $t \in \mathbb{Z}_{\geq 0}$ and any solution (χ, τ) to (13). □

Theorem 1, subject to some conservatism, implies that as long as n and p are chosen such that $\mu_n(p) \leq \mu$, the desired stability and convergence properties are ensured.

Remark 3: The convergence rate $\mu_n(p)$ used in Theorem 1 by setting $\gamma(i) = a_1 a_0^i$ may be conservative in general. When additional properties on F and V are known, less conservative bounds on the growth of V may be used to obtain $\gamma(i)$. □

Next, we provide conditions on p such that $\mu_n(p) \leq \mu$ for given $n \in \mathbb{Z}_{\geq 0}$ and $\mu \in (a_1, \min\{1, a_0\})$ as required in Theorem 1.

Proposition 1: For given $\mu \in (a_1, \min\{1, a_0\})$ and $n \in \mathbb{Z}_{\geq 0}$, if $\mu_n(P_{\max}) \leq \mu$ then $\mu_n(p) \leq \mu$ for any $p \in [\underline{p}_n, P_{\max}]$, where $\underline{p}_n \in [0, P_{\max}]$ is the unique solution to $\mu_n(\underline{p}_n) = \mu$. In addition, the function $\mu_n(\cdot)$ is monotonically decreasing for any $n \in \mathbb{Z}_{\geq 0}$. □

Proposition 1 gives an easy-to-check condition to know whether $\mu_n(p) \leq \mu$ for any $p \in [0, P_{\max}]$ is feasible for given $n \in \mathbb{Z}_{\geq 0}$ and $\mu \in (a_1, 1)$, namely to see if $\mu_n(P_{\max}) \leq \mu$. The minimum admissible power value corresponds to $\underline{p}_n$ in Proposition 1. Using $\underline{p}_n$ does not necessarily imply that the cost (16) is minimized for a fixed $n \in \mathbb{Z}_{\geq 0}$. Indeed, it might be more efficient to use a higher power in (11) because we assume that transmissions are attempted until a packet goes through, and using a smaller power would imply a larger number of attempted transmissions, thereby potentially increasing the net energy consumed [5], [6], see Section V for an illustration.

According to Theorem 1 and Proposition 1, the set

$$\mathcal{S} := \left\{(p, n) : n \in \mathbb{Z}_{\geq 0},\, p \in [\underline{p}_n, P_{\max}] :\, \mu_n(P_{\max}) \leq \mu\right\},$$

contains the values of p and n ensuring the condition of Theorem 1, under which the control goals are satisfied, in view of Proposition 1. In the next proposition, we characterize the associated average communication cost. We use the notation $J_{\mathrm{TT}}(p, n)$ to denote the cost in (16) while implementing (11) and (12).

Proposition 2: Using a transmission policy based on (11) and (12), the cost in (16) for all $p \in [0, P_{\max}]$ and $n \in \mathbb{Z}_{\geq 0}$ is given by

$$J_{\mathrm{TT}}(p, n) = \frac{p}{(1-e(p))n + 1}. \tag{19}$$

The mapping $(p, n) \mapsto J_{\mathrm{TT}}(p, n)$ is

1) strictly increasing in p for small n,
2) "N shaped" for larger values of n, i.e., it is initially increasing upto a local maximum, then decreasing to a local minimum and then finally increasing again in p. □

We can exploit Proposition 2 to characterize the optimal power p minimizing (16) for a given n, such that $(p, n) \in \mathcal{S}$.

Theorem 2: For any $n \in \mathbb{Z}_{\geq 0}$ with $\mu_n(P_{\max}) \leq \mu$, let p_n^* be the optimal power, i.e., $p_n^* \in \arg\min_{p\,|\,(p,n)\in S} J_{\mathrm{TT}}(p, n)$. If a local minimum $p_n^o \in \mathbb{R}_{>0}$ exists such that $\frac{\partial J_{\mathrm{TT}}}{\partial p}(p_n^o, n) = 0$ and $\frac{\partial^2 J_{\mathrm{TT}}}{\partial p^2}(p_n^o, n) > 0$, then $p_n^* \in \left\{\underline{p}_n, p_n^o, P_{\max}\right\}$. Otherwise, $p_n^* = \underline{p}_n$. □

Proof: Proposition 2 implies J_{TT} for a given n is either strictly increasing or N shaped. In the first case, p_n^o does not exist and so, selecting $\underline{p}_n$ is optimal.

In the second case, J_{TT} is N shaped in p, and it has a single local minimum and is concave for small p and then convex. Since we look at $J_{\mathrm{TT}}(p, n)$ for $p \in [\underline{p}_n, P_{\max}]$, a closed and compact set, the global optimum is either the local minimum or one of the boundary points. When $p_n^o > P_{\max}$, J_{TT} may be decreasing or concave in the interval $[\underline{p}_n, P_{\max}]$, which implies that the global minimum is at $\underline{p}_n$ or $P_{\max}$. Otherwise, the optimal power is either $\underline{p}_n$ or the local minimum p_n^o. ■

Theorem 2 characterizes the optimal power to use for a given $n \in \mathbb{Z}_{\geq 0}$ and this power can be found once the local minimum p_n^o for J_{TT} is found, if it exists. In practice, the existence of the local minimum $p_n^o \in [\underline{p}_n, P_{\max}]$ for J_{TT} with a given n can be easily checked by applying a gradient descent initialized at $P_{\max}$. If the gradient descent converges to a point in the interval $[\underline{p}_n, P_{\max}]$, then this point is p_n^o.

Our objective is to find the best pair $(p, n) \in \mathcal{S}$ minimizing J_{TT}. We can show that the set of feasible n such that $\mu_n(P_{\max}) \leq \mu$ is finite, allowing us to do an exhaustive search over all $J_{TT}(p_n^*, n)$ to find the best pair $(p, n) \in \mathcal{S}$. This is done by noticing that even while using infinite transmission power, we have in view of (17),

$$\lim_{p\to\infty} \mu_n(p) = (a_1 a_0^n)^{\frac{1}{n+1}}, \tag{20}$$

by applying the property that $\lim_{p\to\infty} e(p) = 0$ in item (i) of Assumption 2. Therefore, if $(a_1 a_0^n)^{\frac{1}{n+1}} > \mu$ for some $n \in \mathbb{Z}_{\geq 0}$, then $\mu_n(p) > \mu$ for any p applying the decreasing property shown in Proposition 1. Note that since $a_0 > a_1$, $(a_1 a_0^n)^{\frac{1}{n+1}}$ is increasing in n and as $a_0 > \mu$ we define

$$N := \max\Big\{n \in \mathbb{Z}_{\geq 0} | (a_1 a_0^n)^{\frac{1}{n+1}} \leq \mu\Big\}, \tag{21}$$

which is finite and in $\mathbb{Z}_{\geq 0}$. Then for any $n > N$ and any $p \in [0, P_{\max}]$, we have $\mu_n(p, n) > \mu$ implying that such (p, n) does not belong to $\mathcal{S}$, and we thus only consider n in the finite set $\{0, \ldots, N\}$.

IV. About Assumption 1

We present conditions ensuring the satisfaction of Assumption 1 when the strategy used to generate $\hat{y}$ is based on zeroing and zero-order-hold respectively.

A. Zeroing strategy

We suppose that controller (2) has been designed such the next properties hold.

Assumption 3: There exist $W : \mathbb{R}^{s_p+s_c} \to \mathbb{R}$ continuous, $\underline{\alpha}_W, \overline{\alpha}_W \in \mathcal{K}_\infty$, $a_{W,1} \in (0,1)$ and $a_{W,0} > 0$ such that, for any $(x_p, x_c) \in \mathbb{R}^{s_p+s_c}$:

(i) $\underline{\alpha}_W(|(x_p, x_c)|) \leq W(x_p, x_c) \leq \overline{\alpha}_W(|(x_p, x_c)|)$;

(ii) $W\left(f_p(x_p, g_c(x_c, g_p(x_p)), f_c(x_c, g_p(x_p))\right) \leq a_{W,1} W(x)$;

(iii) $W\left(f_p(x_p, g_c(x_c, 0), f_c(x_c, 0)\right) \leq a_{W,0} W(x)$. □

Items (i)-(ii) of Assumption 3 are equivalent to the fact that the origin of (1)-(2) is UGAS when f_p, f_c, g_p and g_c are continuous, see [22]. Item (iii), on the other hand, is an exponential growth condition on W when a transmission fails and $\hat{f}$ is the zero function. The next proposition ensures the satisfaction of Assumption 1.

Proposition 3: Suppose Assumption 3 holds, then Assumption 1 is verified with $V : \chi \mapsto W(x_p, x_c) + |\hat{y}|$, $a_1 = a_{W,1}$, $a_0 = a_{W,0}$, $\underline{\alpha}(s) = \min\{\underline{\alpha}_W(s/2), s/2\}$ and $\overline{\alpha}(s) = \overline{\alpha}_W(s) + s$ for any $s \geq 0$. □

Proof: Let $\chi \in \mathbb{R}^{s_\chi}$, $V(\chi) \leq \overline{\alpha}_W(|(x_p, x_c)|) + |\hat{y}|$ in view of item (i) of Assumption 3, from which we derive that $V(\chi) \leq \overline{\alpha}_W(|\chi|)$ with $\overline{\alpha}_W$ is given in Proposition 3. We proceed similarly to prove the lower-bound on V by invoking [23, Remark 2.3]. On the other hand, in view of item (ii) of Assumption 3, $V(f_S(\chi)) = W\left(f_p(x_p, g_c(x_c, g_p(x_p)), f_c(x_c, g_p(x_p))\right) \leq a_1 W(x) \leq a_{W,1} V(\chi)$. We similarly derive from item (iii) of Assumption 3 that $V(f_S(\chi)) \leq a_{W,0} V(\chi)$, which concludes the proof. ■

B. Zero-order-hold strategy

When zero-order-hold devices are used to generate $\hat{y}$, we need to modify Assumption 4 to conclude about the satisfaction of Assumption 1.

Assumption 4: There exist $W : \mathbb{R}^{s_p+s_c} \to \mathbb{R}$ continuous, $\underline{a}_W, \overline{a}_W > 0$, $a_{W,1} \in (0,1)$ and $a_{W,0}, b_0 \geq 0$ such that, for any $\chi \in \mathbb{R}^{s_\chi}$:

(i) $\underline{a}_W |(x_p, x_c)|^2 \leq W(x_p, x_c) \leq \overline{a}_W |(x_p, x_c)|^2$;

(ii) $W\left(f_p(x_p, g_c(x_c, g_p(x_p)), f_c(x_c, g_p(x_p))\right) \leq a_{W,1} W(x)$;

(iii) $W\left(f_p(x_p, g_c(x_c, \hat{y}), f_c(x_c, \hat{y})\right) \leq a_{W,0} W(x) + b_0 |\hat{y}|^2$. □

Items (i)-(ii) of Assumption 4 are equivalent to the fact that the origin of (1)-(2) is uniformly globally exponentially stable under conditions as mentioned after Assumption 3. Item (iii) is an exponential growth condition on W when a transmission fails, which involves $\hat{y}$ this time because of the use of a zero-order-hold strategy.

We also require the output map to be linearly bounded.

Assumption 5: There exist $c \geq 0$ such that $|g_p(x_p)| \leq c|x_p|$ for any $x_p \in \mathbb{R}^{s_p}$. □

Assumption 5 is verified when $y = C_p x_p$ with C_p a real matrix for instance in which case $c = |C_p|$. The next proposition ensures the satisfaction of Assumption 4.

Proposition 4: Suppose Assumptions 4-5 hold, then Assumption 1 is verified with $V : \chi \mapsto W(x_p, x_c) + \nu|\hat{y}|^2$ with $\nu \in \left(0, (1 - a_{W,1})\frac{\underline{a}_W}{c^2}\right)$, $a_1 = a_{W,1} + \nu c^2/\underline{a}_W$, $a_0 = \max\{a_{W,0}, b_0/\nu + 1\}$ given in Assumption 3, $\underline{\alpha}(s) = \min\{\underline{\alpha}_U(s/2), \nu(s/2)^2\}$ and $\overline{\alpha}(s) = \overline{\alpha}_U(s) + \nu s^2$ for any $s \geq 0$. □

Proof: The proof of (8a) follows similar lines as in the proof of Proposition 3. Let $\chi \in \mathbb{R}^{s_\chi}$. In view of item (ii) of Assumption 4, $V(f_S(\chi)) = W\left(f_p(x_p, g_c(x_c, g_p(x_p)), f_c(x_c, g_p(x_p))\right) + \nu|g_p(x_p)|^2 \leq a_{W,1} W(x) + \nu|g_p(x_p)|^2$. According to Assumption 5, $|g_p(x_p)|^2 \leq c^2|x_p|^2$, and, in view of item (i) of Assumption 4, $|g_p(x_p)|^2 \leq c^2/\underline{a}_W W(x_p, x_c) \leq c^2/\underline{a}_W V(\chi)$. Consequently, $V(f_S(\chi)) \leq a_{W,1} W(x) + \nu c^2/\underline{a}_W V(\chi) \leq \left(a_{W,1} + \nu c^2/\underline{a}_W\right) V(\chi) = a_1 V(\chi)$ and $a_1 \in (0,1)$ in view of the definition of ν in Proposition 4.

On the other hand, $V(f_U(\chi)) \leq a_{W,0} W(x) + b_0|\hat{y}|^2 + \nu|\hat{y}|^2$ in view of (7) and item (iii) of Assumption 4. Hence, $V(f_U(\chi)) \leq \max\{a_{W,0}, b_0/\nu + 1\}(W(x) + \nu|\hat{y}|^2) =$

$\max\{a_{W,0}, b_0/\nu + 1\}V(\chi) = a_0 V(\chi)$. We have proved that the conditions in Assumption 1 are verified. ∎

V. Numerical example

We illustrate the results of Section III on a single link robot arm, whose model is obtained by discretizing the continuous-time system using Euler method with sampling period of 0.001 seconds. System (1) with plant state $x_p = (x_1, x_2) \in \mathbb{R}^2$ is given by

$$\begin{pmatrix} x_1(t+1) \\ x_2(t+1) \end{pmatrix} = \begin{pmatrix} x_1(t) + 0.001x_2(t) \\ x_2(t) + 0.001(\sin(x_1(t)) + u(t)) \end{pmatrix}. \quad (22)$$

The controller (2) is given by $u = -\sin(x_1) - 25x_1 - 10x_2$ and we use zero-order-holds to implement it.

Assumption 1 is verified with $V(\chi) \mapsto \chi^T P \chi$, $a_1 = 0.98$ and $a_0 = 1.0009$ where

$$P = \begin{pmatrix} 0.0384 & -0.0019 & -0.0336 & 0.0031 \\ -0.0019 & 0.0015 & 0.0033 & -0.0008 \\ -0.0336 & 0.0033 & 0.0341 & -0.0032 \\ 0.0031 & -0.0008 & -0.0032 & 0.0009 \end{pmatrix}.$$

For the communication channel model, we consider a Rayleigh slow-fading channel with parameters such that $e(p) \mapsto 1 - \exp(-1/p)$, which verifies Assumption 2, see [24] for details, with $P_{\max} = 10$. We fix $\mu = 0.9999$ and pick $n \in \{1, \ldots, N\}$ with $N = 20$ and $\underline{p}_{20} = 9.5$ using (17) and Theorem 1.

In Figure 2, we plot the optimal power p_n^* minimizing $J_{\mathrm{TT}}(p, n)$ for $n \in \{0, \ldots, 19\}$ and compare it with the required power $\underline{p}_n$ to ensure the convergence property (15). We note that $\underline{p}_n$ is not always the optimal power as mentioned after Proposition 1. In Figure 3, we plot the average power consumed $J_{\mathrm{TT}}(p_n^*, n)$ with respect to feasible values of n for given values of μ, when using the optimal power p_n^* as defined in Theorem 2. We note that using the largest values of feasible n results in a higher communication cost because while the frequency of communications decreases, the power required to stabilize the system also increases with n. The optimal n for $\mu \in \{0.995, 0.999, 0.9999\}$ can be observed to be $1, 8, 18$ respectively. We observe that a smaller μ demands more frequent communication, leading to a higher communication cost.

VI. Conclusions

We have proposed a framework to design a class of energy-efficient transmission power policies for nonlinear WNCS. The main objective of this work is to minimize the average transmission power while maintaining stability of the WCNS is a stochastic sense. We provide expressions to compute the optimal transmission power for the proposed performance criteria under the proposed policy based on time thresholds. Future work will design threshold policies based on the wireless channel state and potentially, the state of the plant as in event-triggered control.

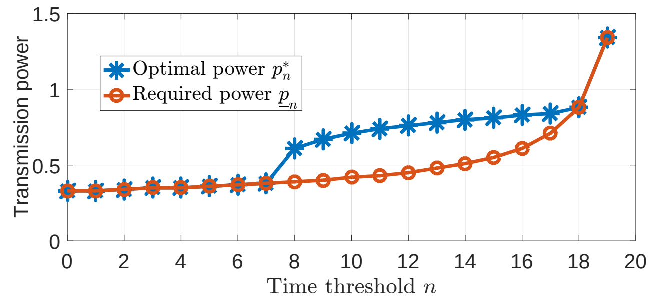

Fig. 2: We plot the optimal power p_n^* minimizing $J_{\mathrm{TT}}(p, n)$, and the required power $\underline{p}_n$ for given values of n.

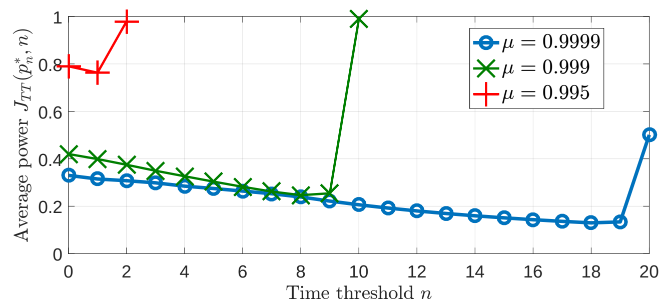

Fig. 3: Average power consumed $J_{\mathrm{TT}}(p_n^*, n)$ using the optimal transmission power p_n^* for the given n and μ.

Appendix

A. A technical lemma

We first provide a lemma on the evolution of $\tau(t)$ along solutions to (13) for the proposed TT policy.

Lemma 1: Under policy (12), the clock state $\tau(t)$ is a Markov process, with steady state probabilities given by

$$\Pr(\tau(t) = j) = \frac{1 - e(p)}{n(1 - e(p)) + 1}, \quad (23)$$

for all $j \in \{1, 2, \ldots, n+1\}$ and

$$\Pr(\tau(t) > n+1) = \frac{e(p)}{n(1 - e(p)) + 1}. \quad (24)$$

□

Proof: Using (11), (12) and Assumption 2 give us $\Pr(\tau(t+1)|\tau(t))$. Since $P(t) = 0$ when $\tau(t) \leq n$, $e(P(t)) = 1$ and this implies

$$\Pr\Big(\tau(t+1) = \tau(t) + 1 | \tau(t) \leq n\Big) = 1. \quad (25)$$

For all $\ell \in \mathbb{Z}_{\geq 0}$ we have

$$\begin{aligned} \Pr\Big(\tau(t+1) = n + \ell + 2 | \tau(t) = n + 1 + \ell\Big) &= e(p) \\ \Pr\Big(\tau(t+1) = 1 | \tau(t) = n + 1 + \ell\Big) &= 1 - e(p) \end{aligned} \quad (26)$$

in view of Assumption 2 and the fact that $P(t) = p$ when $\tau(t) \geq n + 1$ according to (11) and (12). This allows us to evaluate

$$\Pr(\tau(t)=1)=\frac{1-e(p)}{n(1-e(p))+1}, \tag{27}$$

when the Markov chain is in steady state, which will also be the steady state probabilities for $\tau(t)=i$ for any $i\in\{1,\ldots,n+1\}$. Additionally,

$$\Pr(\tau(t)>n+1)=\frac{e(p)}{n(1-e(p))+1}. \tag{28}$$

■

B. Proof of Theorem 1

We first note that, in view of Assumption 1 and (13), we have that

$$V(F(\chi,i+1))\leq a_1 a_0^i V(\chi) \tag{29}$$

for all $\chi\in\mathbb{R}^{s_\chi}$ and $i\in\mathbb{Z}_{\geq 0}$.

Let $\chi_0\in\mathbb{R}^{s_\chi}$ and consider $\chi(t)$ the solution to (13) initialized at χ_0. Recall that due to the structure of (12), once a transmission is successful, the next transmission is attempted only after n steps. Therefore, we define for all $t\in\mathbb{Z}_{\geq 0}$

$$\mathcal{T}_U(t):=\Big\{i\in\{1,2,\ldots,t-1\}\,|\,\tau(i+1)\geq 2+n\Big\}, \tag{30}$$

which denotes the set of all time instances where transmissions were attempted, but communication failed before t. This implies that for any $t\in\mathbb{Z}_{>0}$

$$\forall i\in\mathcal{T}_U(t);\ V(\chi(i+1))\leq a_0 V(\chi(i)) \tag{31}$$

in view of Assumption 1.

On the other hand, we define the set of all time instances where transmission was successful before t for all $t\in\mathbb{Z}_{\geq 0}$ as

$$\mathcal{T}_S(t):=\Big\{i\in\{1,2,\ldots,t-1\}\,|\,\tau(i+1)=1\Big\}, \tag{32}$$

because whenever a communication occurs at some time, we have $\tau(t+1)=1$ according to (10). However, note that $a_1\leq\gamma(n)$ for any $n\in\mathbb{Z}_{\geq 0}$ by definition. This allows us, in view of Assumption 1 to write for any $t\in\mathbb{Z}_{>0}$,

$$\forall i\in\mathcal{T}_S(t);\ V(\chi(i+\ell))\leq\gamma(n)V(\chi(i)). \tag{33}$$

for all $\ell\in\{1,\ldots,n+1\}$. Combining (31) and (33), we can write

$$V(\chi(t))\leq\gamma(n)\prod_{i=0}^{t-1}G(i)V(\chi_0) \tag{34}$$

where $G(i):=a_0$ if $i\in\mathcal{T}_U(t)$, $G(i):=\gamma(n)$ if $i\in\mathcal{T}_S(t)$ and $G(i):=1$ otherwise. This can be done because we have $a_0\leq\gamma(1)\leq\cdots\leq\gamma(n)$. Taking the logarithm on both sides, we have for any $t\in\mathbb{Z}_{>0}$,

$$\log(V(\chi(t)))\leq\log(V(\chi(0)))+\sum_{i=1}^{t}\log(G(i)). \tag{35}$$

Note that under policy (11) and (12), the transmission power $P(t)$ can be seen as a Markov process which depends on the clock state $\tau(t)$, with steady state distribution as stated in Lemma 1. Recall that we initialize $\tau(1)=1$. This allows us to see $G(i)$ as a random variable and we can calculate the distribution of $G(i)$ as

$$\begin{array}{lll}\Pr(G(i)=\gamma(n)) & \leq & \Pr(\tau(i+1)=1)\\ \Pr(G(i)=a_0) & \geq & \Pr(\tau(i+1)=n+1)\end{array} \tag{36}$$

for all $i\in\{0,\ldots,t-1\}$ for any $t\in\mathbb{Z}_{>0}$.

The results of Lemma 1 provides $\Pr(\tau(i+1)=1)$ and we have

$$\begin{aligned}\mathbb{E}[\log(V(\chi(t)))]\leq\ & \log(V(\chi_0))\\ & +t\Big(\Pr(\tau(t)>n+1)\log(a_0))\\ & +\Pr(\tau(t)=n+1)\log(\gamma(n))\Big)\\ \leq\ & \log(V(\chi_0))+t\mu_n(p)\end{aligned} \tag{37}$$

Taking the exponential on both sides, we get the convergence rate

$$\mathbb{E}[V(\chi(t))]\leq\mu_n(p)^t V(\chi_0) \tag{38}$$

Since $\mu_n(p)<\mu$, property (38) automatically implies that

$$\begin{aligned}\textstyle\sum_{t=0}^{\infty}\mathbb{E}[\underline{\alpha}(|\chi(t)|)] & \leq\textstyle\sum_{t=0}^{\infty}\mathbb{E}[V(\chi(t))]\\ & \leq\tfrac{1}{1-\mu}V(\chi_0)<\infty\end{aligned} \tag{39}$$

satisfying condition (14) in Definition 1 as $\mu<1$ and concluding our proof.

C. Proof of Proposition 1

Recall that we consider $a_0>a_1$ in Assumption 1. Due to the property of logarithms, if $\log(\mu_n(p))$ for any $n\in\mathbb{Z}_{\geq 0}$, is monotonically decreasing in p, then so is $\mu_n(p)$.

$$\log(\mu_n(p))=\frac{(\log(\gamma(n))+(\log(a_0)-\log(\gamma(n))e(p)}{1+n(1-e(p))}, \tag{40}$$

which is strictly decreasing in p if $\frac{e(p)}{1+n(1-e(p))}$ is strictly decreasing in p. We can evaluate its derivative as

$$\frac{e'(p)}{1+n(1-e(p))}+\frac{ne(p)e'(p)}{(1+n(1-e(p)))^2}, \tag{41}$$

which has all terms positive except for $e'(p)$, and we have $e'(p)\leq 0$ from item (i) of Assumption 2 concluding the proof of monotonicity.

Next, observe that we have $\mu_n(0)=a_0$ as $e(0)=1$ according to Assumption 2. Since, we consider $\mu<a_0$, if $\mu_n(P_{\max})<\mu$, we have $\mu_n(P_{\max})\leq\mu\leq\mu_n(0)$. Since $e(\cdot)$ is a continuous function, we have $\mu_n(\cdot)$ also continuous implying that there exists at least one $\underline{p}_n\in[0,P_{\max}]$ such that $\mu_n(\underline{p}_n)=\mu$. Finally, due to $\mu_n(\cdot)$ being monotonous, $\underline{p}_n$ is unique and $\mu_n(P_{\max})>\mu$ implies no such $\underline{p}_n$ exists.

If $\underline{p}_n\in[0,P_{\max}]$ exists, since $\mu_n(\cdot)$ is a monotonically decreasing function of p, $p\geq\underline{p}_n$ implies that $\mu_n(p)\leq\mu$.

D. Proof of Proposition 2

Since we know that $P(t)$ is a stochastic variable under policy (11)-(12), we can rewrite the cost (16) as

$$J_{\mathrm{TT}}(p,n)=\mathbb{E}[P(t)]=p\Pr(\tau(t)\geq n+1). \tag{42}$$

Applying Lemma 1, we substitute for $\Pr(\tau(t)\geq n+1)$ which gives us (19).

For $n = 0$, we trivially have that the function $J_{\mathrm{TT}}(p, 0) = p$, which is strictly increasing in p. For all other cases, we will have $\underline{p}_n > 0$ and in order to study the properties of $J_{\mathrm{TT}}(p, n)$ w.r.t p, we look at the properties of the inverse cost which is never zero for $p > 0$ defined as

$$\xi_n(p) = \frac{1}{J_{\mathrm{TT}}(p, n)} = \frac{1}{p} + n\frac{1 - e(p)}{p} \tag{43}$$

Due to the stability requirement, we only look at $\xi_n(p)$ for all $p \in [\underline{p}_n, P_{\max}]$, $n \geq 1$. Note that due to item (i) of Assumption 2, we have $1 - e(p)$ to be a sigmoidal function of p. We can therefore apply Theorem 1 in [20], to conclude that the term $\frac{1-e(p)}{p}$ is quasi-concave and takes the value 0 at the limits when $p \to 0$ and $p \to \infty$. The term $\frac{1-e(p)}{p}$ therefore has a unique maximum at say p^u and is strictly increasing in the interval $(0, p^u)$ and is decreasing in the interval (p^u, ∞).

Now, we can consider the two cases.

- There is no local extremum for $\xi_n(p)$ for $p > 0$.
- There exists at least one p^* which is a local extremum satisfying

$$\frac{\partial \xi(p^*)}{\partial p} = \frac{-ne'(p^*)}{p^*} - \frac{1 + n(1 - e(p^*))}{p^{*2}} = 0. \tag{44}$$

In the first case, since $\xi_n(\cdot)$ is differentiable and has no local extremum, $\frac{\partial \xi(p)}{\partial p}$ is never 0 for $p > 0$. Note that the function ξ_n is decreasing in the interval (p^u, ∞) for any n, and so $p \mapsto \xi_n(p)$ must be decreasing for all $p > 0$. Since $\xi_n(p)$ is differentiable and $\frac{\partial \xi(p^*)}{\partial p}$ is never 0, $\xi_n(p)$ is always decreasing, which implies that $J_{\mathrm{TT}}(p, n)$ is always increasing.

For the second case, there exists at least one p^* satisfying (44). Then, we evaluate

$$\frac{\partial^2 \xi(p)}{\partial p^2} = \frac{-ne''(p)}{p} + \frac{2}{p^2}\left(\frac{1+n(1-e(p))}{p} + ne'(p)\right) \tag{45}$$

However, note that at a local extremum, the above expression will have the second term vanishing due to (44), implying that

$$\frac{\partial^2 \xi(p^*)}{\partial p^2} = \frac{-ne''(p^*)}{p^*} \tag{46}$$

which is positive when e is concave and negative when e is convex. From item (ii) of assumption 2, we know that $(1 - e)$ is initially convex and then concave after a point which means that ξ has only local minima initially (when $1 - e$ is convex), and then only local maxima. Since $\xi(p)$ is continuous and differentiable, this is only possible if the local minimum and maximum are unique.

References

[1] A. Ahlen, J. Akerberg, M. Eriksson, A. J. Isaksson, T. Iwaki, K. H. Johansson, S. Knorn, T. Lindh, and H. Sandberg. Toward wireless control in industrial process automation: A case study at a paper mill. *IEEE Control Systems Magazine*, 39(5):36–57, 2019.

[2] J. Wu, S. Rangan, and H. Zhang. *Green communications: theoretical fundamentals, algorithms, and applications*. CRC Press, 2016.

[3] T. Rault, A. Bouabdallah, and Y. Challal. Energy efficiency in wireless sensor networks: A top-down survey. *Computer Networks*, 67:104–122, 2014.

[4] E. Hossain, V.K. Bhargava, and G.P. Fettweis. *Green radio communication networks*. Cambridge University Press, 2012.

[5] V.S. Varma, S. Lasaulce, M. Debbah, and S.E. Elayoubi. An energy-efficient framework for the analysis of MIMO slow fading channels. *IEEE Transactions on Signal Processing*, 61(10):2647–2659, 2013.

[6] D. Goodman and N. Mandayam. Power control for wireless data. *IEEE Personal Communications*, 7(2):48–54, 2000.

[7] M. Rabi and K.H. Johansson. Event-triggered strategies for industrial control over wireless networks. In *Proceedings of the 4th annual international conference on wireless internet*, page 34. ICST (Institute for Computer Sciences, Social-Informatics and Telecommunications Engineering), 2008.

[8] X. Liu and A. Goldsmith. Wireless medium access control in networked control systems. In *American Control Conference, 2004. Proceedings of the 2004*, volume 4, pages 3605–3610. IEEE, 2004.

[9] N.C. De Castro, C.C. De Wit, and K.H. Johansson. On energy-aware communication and control co-design in wireless networked control systems. In *2nd IFAC Workshop on Distributed Estimation and Control in Networked Systems, Annecy, France*, pages 49–54, 2010.

[10] D. E. Quevedo, J. Østergaard, and A. Ahlen. Power control and coding formulation for state estimation with wireless sensors. *IEEE Transactions on Control Systems Technology*, 22(2):413–427, 2013.

[11] Y. Li, D. E. Quevedo, V. Lau, and L. Shi. Optimal periodic transmission power schedules for remote estimation of ARMA processes. *IEEE Transactions on Signal Processing*, 61(24):6164–6174, 2013.

[12] A. Molin and S. Hirche. On LQG joint optimal scheduling and control under communication constraints. In *IEEE Conference on Decision and Control, held jointly with Chinese Control Conference*, pages 5832–5838, 2009.

[13] K. Gatsis, A. Ribeiro, and G.J. Pappas. Optimal power management in wireless control systems. *IEEE Transactions on Automatic Control*, 59(6):1495–1510, 2014.

[14] M. Balaghiinaloo, D.J. Antunes, V.S. Varma, R. Postoyan, and W.P.M.H. Heemels. LQ-power consistent control: Leveraging transmission power selection in control systems. In *IFAC Europen Control Conference 2020*.

[15] V.S. Varma and R. Postoyan. Energy efficient time-triggered control over wireless sensor/actuator networks. In *IEEE Conference on Decision and Control*, pages 2727–2732, 2016.

[16] V. S. Varma, A. M. de Oliveira, R. Postoyan, I-C. Morarescu, and J. Daafouz. Energy-efficient time-triggered communication policies for wireless networked control systems. *IEEE Transactions on Automatic Control*, 2019.

[17] A.I. Maass, D. Nešić, V.S. Varma, R. Postoyan, and S. Lasaulce. Stabilisation and power control for nonlinear feedback loops communicating over lossy wireless networks. *submitted to IEEE Conference on Decision and Control*, 2020.

[18] A. S. Leong, D. E. Quevedo, D. Dolz, and S. Dey. Transmission scheduling for remote state estimation over packet dropping links in the presence of an eavesdropper. *IEEE Transactions on Automatic Control*, 64(9):3732–3739, Sept. 2019.

[19] J. P. Hespanha, D. Liberzon, and A. R Teel. Lyapunov conditions for input-to-state stability of impulsive systems. *Automatica*, 44(11):2735–2744, 2008.

[20] V. Rodriguez. An analytical foundation for resource management in wireless communication. In *IEEE Global Telecommunications Conference*, volume 2, pages 898–902, 2003.

[21] D. E. Quevedo, V. Gupta, W-J. Ma, and S. Yüksel. Stochastic stability of event-triggered anytime control. *IEEE Transactions on Automatic Control*, 59(12):3373–3379, 2014.

[22] Z.-P. Jiang and Y. Wang. A converse Lyapunov theorem for discrete-time systems with disturbances. *Systems & Control Letters*, 45(1):49–58, 2002.

[23] D.S. Laila and D. Nešić. Lyapunov based small-gain theorem for parameterized discrete-time interconnected ISS systems. In *IEEE Conference on Decision and Control, Las Vegas, U.S.A.*, pages 2292–2297, 2002.

[24] L.H. Ozarow, S. Shamai, and A.D. Wyner. Information theoretic considerations for cellular mobile radio. *IEEE transactions on Vehicular Technology*, 43(2):359–378, 1994.