

Privacy and optimality of distributed schemes for secondary frequency regulation in power networks

Kanwal Khan, Andreas Kasis, Marios M. Polycarpou and Stelios Timotheou

Abstract—The increasing participation of local generation and controllable demand units within the power network motivates the use of distributed schemes for their control. Simultaneously, it raises two issues; achieving an optimal power allocation among these units, and securing the privacy of the generation/demand profiles. This study considers the problem of designing distributed optimality schemes that preserve the privacy of the generation and controllable demand units within the secondary frequency control timeframe. We propose a consensus scheme that includes the generation/demand profiles within its dynamics, providing guarantees that those cannot be inferred from the communicated signals, even when eavesdroppers possess knowledge of the underlying system dynamics. For the proposed scheme, we provide analytic stability and optimality guarantees and show that the secondary frequency control objectives are satisfied. The presented scheme is distributed, locally verifiable and applicable to arbitrary network topologies. Our analytic results are verified with simulations on a 9-bus system, where we demonstrate that the proposed scheme enables an optimal power allocation and preserves the privacy of the generation/demand and the stability of the power network.

I. INTRODUCTION

Motivation and literature survey: The increasing penetration of renewable sources of generation is expected to cause more frequent generation-demand imbalances within the power network, which may harm power quality and even cause blackouts [1]. Controllable demand is considered to be a means to address this issue, since loads may provide a fast response to counterbalance intermittent generation [2]. However, the increasing number of such active units makes traditionally implemented centralized control schemes expensive and inefficient, motivating the adoption of distributed schemes. Such schemes offer many advantages, such as scalability, reduced expenses associated with the necessary communication infrastructure and enhanced reliability due to the absence of a single point of failure.

The introduction of controllable loads and local renewable generation raises an issue of economic optimality in the power allocation. In addition, the introduction of smart meters for the monitoring of generation and demand units poses a privacy threat for the citizens, since readings may be used to expose customers daily life and habits, by inferring the users energy consumption patterns and types of appliances [3]. For example, this issue led the Dutch Parliament to prohibit the deployment of smart meters until the privacy concerns are resolved [4]. These concerns motivate the design of distributed schemes that will simultaneously achieve an optimal power allocation and preserve the privacy of local prosumption profiles.

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In recent years, various studies considered the use of decentralized/distributed control schemes for generation and controllable demand with applications to both primary [5], [6], [7] and secondary [8], [9] frequency regulation, where the objectives are to ensure generation-demand balance and that the frequency attains its nominal value at steady state respectively. In addition, the problem of obtaining an optimal power allocation within the secondary frequency control timeframe has received broad attention in the literature [10], [11]. These studies considered suitably constructed optimization problems and designed the system equilibria to coincide with the solutions to these problems. In many studies, the control dynamics were inspired from the dual of the considered optimization problems [12], [13]. Such schemes, usually referred to in the literature as *Primal-Dual schemes*, yield an optimal power allocation and at the same time allow operational constraints to be satisfied. Alternative distributed schemes, which ensure that frequency attains its nominal value at steady state by using the generation outputs, have also been proposed [14]. However, the use of real-time knowledge of the prosumption in the proposed schemes may result in privacy issues.

The topic of preserving the privacy of generation and demand units has recently attracted wide attention in the literature. Different types of privacy concerns, resulting from the integration of information and communication technologies in the smart grid, are mentioned in [15]. In addition, [16] analyzes various smart grid privacy issues and discusses recently proposed solutions for enhanced privacy, while [17] proposes a privacy-preserving power request scheme. Moreover, [18] uses the differential privacy framework to provide privacy guarantees and [19] studies the effect of differential privacy on smart metering data. A privacy-preserving aggregation scheme is proposed in [20] which considers various security threats. The use of energy storage units to preserve the privacy of user consumption has been considered in [21] and [22]. Furthermore, [23] and [24] aim to simultaneously preserve the privacy of individual agents and enable an optimal power allocation using homomorphic encryption and differential privacy respectively. Both approaches result in suboptimal allocations, which suggests a trade-off between optimality and privacy. Several existing techniques that aim at preventing disclosure of private data are also discussed in [25].

Although the problems of preserving the privacy of power prosumption and obtaining an optimal power allocation in power networks have been independently studied, the problem of simultaneously achieving these goals has not been adequately investigated. In addition, to the authors best knowledge, no study has considered the impact of such schemes on the stability and dynamic performance of the power grid. This study aims to jointly consider these objectives within the secondary frequency control timeframe.

Contribution: This paper studies the problem of providing optimal frequency regulation within the secondary frequency control timeframe while preserving the privacy of generation and controllable demand profiles. We first propose an optimization problem that ensures that secondary

frequency regulation objectives, i.e. achieving generation-demand balance and frequency attaining its nominal value at steady state, are satisfied. In addition, to facilitate the interpretation of our privacy results, we define the notion of *intelligent eavesdroppers* that utilize knowledge of the underlying system dynamics and the communicated information to infer the prosumption profiles.

We consider a distributed scheme that has been extensively studied in the literature, usually referred to as the *Primal-Dual scheme*, that enables an optimal power allocation and the satisfaction of system constraints, and explain why it causes privacy issues. Inspired by the *Primal-Dual scheme*, we propose the *Privacy-Preserving scheme* that incorporates a distributed controller at each privacy-seeking unit of the power grid. The latter replaces the transmission of the prosumption profiles with a consensus signal. In addition, it incorporates two important, privacy-enhancing features. In particular, the proposed scheme periodically alters the speed of response of each controller, making model based inference inaccurate. Moreover, it adds bounded noise to the prosumption information within each controller, with a maximum magnitude proportional to the local frequency deviation. The latter yields changes in all controllers when a disturbance occurs, making it hard to detect the origin of the disturbance. These properties enable privacy guarantees against intelligent eavesdroppers when the *Privacy-Preserving scheme* is implemented. For the proposed scheme, we provide analytic stability guarantees and show that an optimal power allocation is achieved at steady state. In addition, the proposed *Privacy-Preserving scheme* is distributed, locally verifiable and applicable to arbitrary network topologies.

Our analytic results are illustrated with numerical simulations on the Western System Coordinating Council (WSCC) 9-bus system which validate that the proposed scheme enables an optimal power allocation and satisfy the secondary frequency regulation objectives. In addition, we demonstrate how the *Privacy-Preserving scheme* offers privacy of the prosumption profiles against intelligent eavesdroppers.

Paper structure: In Section II we present the dynamics of the power network, the considered optimization problem and the problem statement. In Sections III and IV we present the proposed *Privacy-Preserving scheme* and provide our main analytic results respectively. In Section V we validate our main results through numerical simulations on the WSCC 9-bus system. Finally, conclusions are drawn in Section VI. The proofs of the main results are omitted due to space restrictions and are provided in [26].

Notation: Real numbers and the set of n -dimensional vectors with real entries are denoted by \mathbb{R} and \mathbb{R}^n respectively. The p -norm of a vector $x \in \mathbb{R}^n$ is given by $\|x\|_p = (|x_1|^p + \dots + |x_n|^p)^{1/p}$, $1 \leq p < \infty$. A function $f : \mathbb{R}^n \rightarrow \mathbb{R}^m$ is said to be locally Lipschitz continuous at x if there exists some neighbourhood X of x and some constant L such that $\|f(x) - f(y)\| \leq L \|x - y\|$ for all $y \in X$, where $\|\cdot\|$ denotes any p -norm. A matrix $A \in \mathbb{R}^{n \times n}$ is called diagonal if $A_{ij} = 0$ for all $i \neq j$. The image of a vector x is denoted by $\text{Im}(x)$. The cardinality of a discrete set \mathcal{S} is denoted by $|\mathcal{S}|$. For a graph with sets of nodes and edges denoted by \mathcal{A} and \mathcal{B} respectively, we define the incidence matrix $H \in \mathbb{R}^{|\mathcal{A}| \times |\mathcal{B}|}$ as follows

$$H_{ij} = \begin{cases} +1, & \text{if } i \text{ is the positive end of edge } j \in \mathcal{B}, \\ -1, & \text{if } i \text{ is the negative end of edge } j \in \mathcal{B}, \\ 0, & \text{otherwise.} \end{cases}$$

We use $\mathbf{0}_n$ and $\mathbf{1}_n$ to denote the n -dimensional vectors with all elements equal to 0 and 1 respectively. Finally, for a state $x \in \mathbb{R}^n$, we let x^* denote its equilibrium value.

II. PROBLEM FORMULATION

We describe the power network by a connected graph $(\mathcal{N}, \mathcal{E})$ where $\mathcal{N} = \{1, 2, \dots, |\mathcal{N}|\}$ is the set of buses and $\mathcal{E} \subseteq \mathcal{N} \times \mathcal{N}$ the set of transmission lines connecting the buses. The term (i, j) denotes the link connecting buses i and j . The graph $(\mathcal{N}, \mathcal{E})$ is assumed to be directed with an arbitrary direction, so that if $(i, j) \in \mathcal{E}$ then $(j, i) \notin \mathcal{E}$. For each $j \in \mathcal{N}$, we define the sets of predecessor and successor buses by $\mathcal{N}_j^p = \{k : (k, j) \in \mathcal{E}\}$ and $\mathcal{N}_j^s = \{k : (j, k) \in \mathcal{E}\}$ respectively. It should be noted that the form of the considered dynamics is unaffected by changes in the graph ordering and the results presented in this paper are independent of the choice of direction. The following assumptions are made for the network:

- 1) Bus voltage magnitudes are $|V_j| = 1$ per unit, $j \in \mathcal{N}$.
- 2) Lines $(i, j) \in \mathcal{E}$ are lossless and characterized by the magnitudes of their susceptances $B_{ij} = B_{ji} > 0$.
- 3) Reactive power flows do not affect bus voltage phase angles and frequencies.
- 4) The relative phase angles are sufficiently small such that the approximation $\sin \eta_{ij} = \eta_{ij}$ is valid.

The first three assumptions have been frequently used in the literature in frequency regulation studies [8], [6], [5], [12]. They are valid in medium to high voltage transmission systems since transmission lines are dominantly inductive and voltage variations are small. In addition, they are valid in distribution networks with tight voltage control. The fourth assumption is valid when the network operates in nominal conditions, where relative phase angles are small. It should be noted that the theoretical results presented in this paper are validated with numerical simulations in Section V, on a comprehensive power network model.

We use the swing equations to describe the rate of change of frequency at buses [27]. In particular, at each bus we consider a set of generation and controllable and uncontrollable demand units. This motivates the following system dynamics:

$$\dot{\eta}_{ij} = \omega_i - \omega_j, (i, j) \in \mathcal{E}, \quad (1a)$$

$$\begin{aligned} M_j \dot{\omega}_j &= \sum_{k \in \mathcal{N}_j^G} p_{k,j}^M - \sum_{k \in \mathcal{N}_j^L} d_{k,j}^c - \sum_{k \in \mathcal{N}_j} p_{k,j}^L - D_j \omega_j \\ &\quad - \sum_{i \in \mathcal{N}_j^s} p_{ji} + \sum_{i \in \mathcal{N}_j^p} p_{ij}, j \in \mathcal{N}, \end{aligned} \quad (1b)$$

$$p_{ij} = B_{ij} \eta_{ij}, (i, j) \in \mathcal{E}. \quad (1c)$$

In system (1), variable ω_j represents the deviation of the frequency at bus j from its nominal value, namely 50 Hz (or 60 Hz). Variable $p_{k,j}^M$ represents the mechanical power injection associated with the k th generation unit at bus j . Moreover, $d_{k,j}^c$ denotes the demand associated with the k th controllable load at bus j . \mathcal{N}_j^G and \mathcal{N}_j^L represent the sets of generation units and controllable loads, which are jointly referred to as active elements or active units, at bus j respectively. Each of these units are associated with a privacy-seeking user or entity. The set of active units at bus j is given by $\mathcal{N}_j = \mathcal{N}_j^G \cup \mathcal{N}_j^L$. The variable $p_{k,j}^L$ represents the uncontrollable demand associated with the k th active unit at bus j . Furthermore, the time-dependent variables η_{ij} and p_{ij} represent, respectively, the power angle difference and the power transmitted from bus i to bus j . The quantities B_{ij} represent the line susceptances between buses i and j . Finally, the positive constants D_j and M_j represent the generation damping and inertia respectively. The generation and consumption will be jointly referred to as prosumption.

We will study the behavior of the power system under the

following dynamics for generation and controllable loads,

$$\tau_{k,j} \dot{x}_{k,j} = -x_{k,j} + m_{k,j}(u_{k,j} - \omega_j), k \in \mathcal{N}_j^G, j \in \mathcal{N}, \quad (2a)$$

$$p_{k,j}^M = x_{k,j} + h_{k,j}(u_{k,j} - \omega_j), k \in \mathcal{N}_j^G, j \in \mathcal{N}, \quad (2b)$$

$$d_{k,j}^c = -h_{k,j}(u_{k,j} - \omega_j), k \in \mathcal{N}_j^L, j \in \mathcal{N}, \quad (2c)$$

where $x_{k,j} \in \mathbb{R}$ represents the internal state, and $\tau_{k,j} > 0$ and $m_{k,j} > 0$ the time and droop constants associated with generation unit k at bus j respectively. The positive constant $h_{k,j}$ represents the damping associated with active unit k (generation or controllable load) at bus j . In addition, $u_{k,j}$ represents the control input to the k th active unit at bus j , the dynamics of which are discussed in the following sections.

We consider first-order generation dynamics and static controllable demand for simplicity and to keep the focus of the paper on developing a privacy-preserving scheme. More involved generation and demand dynamics could be considered by applying existing results (e.g. [5], [8]).

For convenience, we define the vectors $p_j^M = [p_{k,j}^M]_{k \in \mathcal{N}_j^G}$, $d_j^c = [d_{k,j}^c]_{k \in \mathcal{N}_j^L}$, $p_j^L = [p_{k,j}^L]_{k \in \mathcal{N}_j}$, $p^M = [p_j^M]_{j \in \mathcal{N}}$, $d^c = [d_j^c]_{j \in \mathcal{N}}$ and $p^L = [p_j^L]_{j \in \mathcal{N}}$.

A. Prosumption cost minimization problem

In this section we form an optimization problem that aims to minimize the costs associated with generation and controllable demand and simultaneously achieve generation-demand balance. The considered optimization problem is described below.

A cost $\frac{1}{2}q_{k,j}(p_{k,j}^M)^2$ is incurred when the generation unit k at bus j produces a power output of $p_{k,j}^M$. In addition, a cost $\frac{1}{2}q_{k,j}(d_{k,j}^c)^2$ is incurred when controllable load k at bus j adjusts its demand to $d_{k,j}^c$. The optimization problem is to obtain the vectors p^M and d^c that minimize the cost associated with the aggregate generation and controllable demand and simultaneously achieve power balance. The considered optimization problem is presented below.

$$\begin{aligned} \min_{p^M, d^c} \quad & \sum_{j \in \mathcal{N}} \left(\sum_{k \in \mathcal{N}_j^G} \frac{1}{2} q_{k,j} (p_{k,j}^M)^2 + \sum_{k \in \mathcal{N}_j^L} \frac{1}{2} q_{k,j} (d_{k,j}^c)^2 \right) \\ \text{subject to} \quad & \sum_{j \in \mathcal{N}} \left(\sum_{k \in \mathcal{N}_j^G} p_{k,j}^M - \sum_{k \in \mathcal{N}_j^L} d_{k,j}^c - \sum_{k \in \mathcal{N}_j} p_{k,j}^L \right) = 0. \end{aligned} \quad (3)$$

The equality constraint in (3) requires all the uncontrollable loads to be matched by the generation and controllable demand, such that generation-demand balance is achieved. The equality constraint also guarantees that the frequency attains its nominal value at equilibrium, which is a main objective of secondary frequency control. The latter follows by summing (1b) at steady state over all buses, which yields $\sum_{j \in \mathcal{N}} D_j \omega_j = 0$, and noting that frequency synchronizes at equilibrium from (1a).

B. Eavesdropper and privacy definitions

We provide a definition of intelligent eavesdroppers to facilitate the interpretation and intuition of our results.

Definition 1: An intelligent eavesdropper aims to extract private information by using knowledge of:

- (K1) All signals communicated to and from a given unit, for which it aims to obtain private information.
- (K2) The underlying control dynamics of the system.

Definition 1 presents the knowledge that intelligent eavesdroppers may possess. Informed eavesdroppers analyze the intercepted signals using knowledge of the underlying system dynamics. It is intuitive to note that privacy against intelligent eavesdroppers implies privacy against eavesdroppers with knowledge of K1 or K2 only.

Below we provide the definition of private prosumption profiles, used throughout the rest of the manuscript.

Definition 2: A prosumption profile is called private against an eavesdropper type if the knowledge available to the eavesdropper does not allow the estimation of its trajectory and steady state values, i.e. of $s(t), t \geq 0$.

C. Problem Statement

This paper aims to design control schemes that enable stability and optimality guarantees and simultaneously preserve the privacy of all active units. The problem is stated below.

Problem 1: Design a control scheme that:

- (i) Provides privacy of the prosumption profiles against intelligent eavesdroppers.
- (ii) Enables asymptotic stability guarantees.
- (iii) Uses local information and locally verifiable conditions.
- (iv) Yields an optimal steady-state power allocation.
- (v) Applies to arbitrary connected network configurations.

Problem 1 aims to design a control scheme that enables stability guarantees, ensures an optimal power allocation at steady state, and provides privacy of the generation/demand profiles against intelligent eavesdroppers, following Definitions 1 and 2. In addition, we aim to design a scheme that relies on locally available information and locally verifiable conditions, to enable scalable designs. Finally, it is desired that the proposed scheme is applicable to general network topologies.

III. PRIVACY-PRESERVING SCHEME

In this section, we propose a scheme that aims to solve Problem 1. We first present a distributed scheme that has been widely studied in the literature [8], [12], usually referred to as the *Primal-Dual scheme*, that enables an optimal power allocation, and discuss its resulting privacy issues. To resolve these issues, we propose the *Privacy-Preserving scheme*, which enables the privacy of the prosumption profiles.

A. Primal-Dual scheme

To describe the *Primal-Dual scheme*, we consider a connected communication graph $(\mathcal{N}, \mathcal{E})$, where \mathcal{E} represents the set of communication lines among the buses, i.e. $(i, j) \in \mathcal{E}$ if buses i and j communicate. In addition, we let \hat{H} be the incidence matrix of $(\mathcal{N}, \mathcal{E})$ and define the variable $\zeta_j = \mathbf{1}_{|\mathcal{N}_j|}^T p_j^L + \mathbf{1}_{|\mathcal{N}_j^L|}^T d_j^c - \mathbf{1}_{|\mathcal{N}_j^G|}^T p_j^M$ for all $j \in \mathcal{N}$. The prosumption input dynamics are given by

$$\hat{\Gamma} \dot{\psi} = \hat{H}^T p^c, \quad (4a)$$

$$\bar{\Gamma} \dot{p}^c = \zeta - \hat{H} \psi, \quad (4b)$$

$$u_{k,j} = p_j^c, k \in \mathcal{N}_j, j \in \mathcal{N}, \quad (4c)$$

where the diagonal matrices $\hat{\Gamma} \in \mathbb{R}^{|\mathcal{E}| \times |\mathcal{E}|}$ and $\bar{\Gamma} \in \mathbb{R}^{|\mathcal{N}| \times |\mathcal{N}|}$ contain the time constants associated with (4a) and (4b) respectively and p_j^c is a power command variable associated with bus j and shared with communicating buses. In addition, variable ψ is a state of the *Primal-Dual scheme* that integrates the difference in power command variables between communicating buses. The input for all active elements at bus j is given by the local power command value p_j^c .

The dynamics in (4a) enable the synchronization of the power command variables at steady state. This property is useful to provide an optimality interpretation of the system's equilibria. In addition, (4b) ensures that the secondary frequency control objectives, i.e. ensuring generation/demand balance and the frequency attaining its nominal value, are satisfied at steady state. The latter follows by summing (1b) and (4b) at steady state over all $j \in \mathcal{N}$, which yields $\sum_{j \in \mathcal{N}} D_j \omega_j^* = 0$, which in turn implies that $\omega^* = \mathbf{0}_{|\mathcal{N}|}$ from the synchronization of frequency at equilibrium, as follows from (1a). It should be noted that the stability and

optimality of the *Primal-Dual scheme* (4) for a wide class of generation/demand dynamics, including those in (2), have been analytically shown in the literature (e.g. [8]).

Remark 1: A shortcoming of the *Primal-Dual scheme* (4) is the requirement for real-time knowledge of the generation and demand from all active units in the network. In practice, this requires the transmission of this information to a central controller at each bus, exposing the local generation/demand profiles to an eavesdropper who intercepts these signals, even when knowledge of the underlying dynamics (K2) is not present. The latter compromises the presumption privacy.

B. Privacy-Preserving scheme

In this section we present a scheme that aims to improve the privacy properties of the generation/demand profiles. In contrast to (4), which includes a controller at each bus, the proposed scheme employs a controller at each privacy-seeking unit. We show that the proposed scheme offers privacy against intelligent eavesdroppers and simultaneously enables an optimal power allocation.

To describe the new scheme, we consider a communication network characterized by a connected graph $(\tilde{\mathcal{N}}, \tilde{\mathcal{E}})$, where $\tilde{\mathcal{N}} = \cup_{j \in \mathcal{N}} \mathcal{N}_j$ represents the set of active units within the power network and $\tilde{\mathcal{E}} \subseteq \tilde{\mathcal{N}} \times \tilde{\mathcal{N}}$ the set of connections. Moreover, we let $H \in \mathbb{R}^{|\tilde{\mathcal{N}}| \times |\tilde{\mathcal{E}}|}$ be the incidence matrix of $(\tilde{\mathcal{N}}, \tilde{\mathcal{E}})$. In addition, the following variables are defined for compactness in presentation,

$$s_j^T = [(-p_j^M)^T, (d_j^c)^T], j \in \mathcal{N}, \quad (5a)$$

$$\tilde{s}_j = s_j + p_j^L, j \in \mathcal{N}, \quad (5b)$$

where $\tilde{s} \in \mathbb{R}^{|\tilde{\mathcal{N}}|}$ is a vector that includes all generation and controllable and uncontrollable demand.

The proposed *Privacy-Preserving scheme*, is presented below

$$\tilde{\Gamma} \dot{\psi} = H^T p^c, \quad (6a)$$

$$\Gamma p^c = \tilde{s} - H\psi + n, \quad (6b)$$

$$u = p^c, \quad (6c)$$

where $\tilde{\Gamma} \in \mathbb{R}^{|\tilde{\mathcal{E}}| \times |\tilde{\mathcal{E}}|}$ and $\Gamma \in \mathbb{R}^{|\tilde{\mathcal{N}}| \times |\tilde{\mathcal{N}}|}$ are diagonal matrices containing the positive time constants associated with (6a) and (6b) respectively, and $p_{k,j}^c$ corresponds to the power command variable associated with active unit k at bus j , that is also used as the input to (2) following (6c). In (6) above, the locally Lipschitz, privacy-enhancing signal $n = [n_i]_{i \in \mathcal{N}}$, where $n_i = [n_{k,i}]_{k \in \mathcal{N}_i}$, adapts the derivative of the power command variables to enable enhanced privacy properties. Its design approach is explained below.

Following the *Privacy-Preserving scheme*, privacy-seeking users share power command signals instead of their generation and demand values. Hence, the presumption profiles are not communicated among local controllers.

The design of the signal n is crucial in providing enhanced privacy properties and simultaneously enabling stability and optimality guarantees for the *Privacy-Preserving scheme* (6). Some desired properties of the privacy-enhancing signal n are: (i) to permit the existence of equilibria, by taking a constant value when the states of the system are at equilibrium, and (ii) to allow an optimality interpretation of the resulting equilibria. Both objectives can be achieved if n is zero at steady state.

The following design condition is imposed on the privacy-enhancing signal n . As demonstrated below, this condition ensures the privacy of the presumption profiles against intelligent eavesdroppers and allows stability and optimality to be deduced. It should be noted that the trajectories of n are in general non-unique.

Design Condition 1: The privacy-enhancing signals satisfy $n = n^d + n^f$ where:

- (i) $n_{k,j}^d = -\delta_{k,j} \dot{p}_{k,j}^c$, $k \in \mathcal{N}_j, j \in \mathcal{N}$, where $\delta_{k,j} \geq 0$ is periodically randomly assigned, at periods significantly slower than the secondary frequency control timeframe,
- (ii) $|n_{k,j}^f(t)| < \beta_{k,j} |\omega_j(t)|$, $k \in \mathcal{N}_j, j \in \mathcal{N}$ for all $t \geq 0$, where $\beta_{k,j} = 2\sqrt{(h_{k,j} + D_j/|\mathcal{N}_j|)(h_{k,j})} - 2h_{k,j}$.

Design Condition 1 splits the privacy-enhancing signal n to two other signals, n^d and n^f , that serve different purposes. The signal $n_{k,j}^d$ is proportional to the power command derivative $\dot{p}_{k,j}^c$ with a non-negative gain $\delta_{k,j}$. The latter adjusts the rate at which the power command variables respond to external signals and makes any prior estimates of the power command model inaccurate. Hence, a potential eavesdropper utilizing model-based observations will produce inaccurate results. The component n^f introduces a noise signal that is mixed with the generation/demand values. The latter offers improved privacy properties since: (i) the generation/demand profile information in the controller is distorted, (ii) it perturbs the communicated signals of all controllers when a disturbance occurs, making it harder to detect the origin of the disturbance from a change in the transmitted signal. Design Condition 1(ii) restricts the magnitude of n^f in relation with the magnitude of the local frequency. The values of $\beta_{k,j}$ are selected to satisfy Design Condition 1(ii) such that convergence is guaranteed, as demonstrated in Theorem 1 later on. These properties enable the privacy of presumption against intelligent eavesdroppers since the same power command trajectories result from a (wide) class of presumption profiles due to different potential trajectories of the privacy-enhancing signal n . The latter is analytically shown in Section IV below. In addition, note that since all communicated power command signals synchronize at steady state, their equilibrium values do not convey any information about local generation/demand.

IV. MAIN RESULTS

In this section, we present the main analytic results of the paper. To facilitate their presentation, we provide a definition of an equilibrium point to the interconnected dynamical system (1), (2), (5), (6).

Definition 3: The point $\alpha^* = (\eta^*, \psi^*, \omega^*, x^*, p^{c,*})$ defines an equilibrium of the system (1), (2), (5), (6) if all time derivatives of (1), (2), (5), (6) are equal to zero at this point.

The following proposition demonstrates that the proposed *Privacy-Preserving scheme* preserves the privacy of the presumption profiles. It should be clarified that for an intelligent eavesdropper, K1 and K2 imply knowledge of all power command signals communicated to and from a considered unit and of the *Privacy-Preserving scheme* dynamics (6) respectively.

Proposition 1: Consider any supply unit k, j implementing the *Privacy-Preserving scheme* (6). Then its presumption profile $\tilde{s}_{k,j}$ is private against intelligent eavesdroppers with knowledge of K1 and K2.

Proposition 1 provides privacy guarantees for the presumption profiles when the *Privacy-Preserving scheme* is implemented. The latter demonstrates that the proposed scheme satisfies objective (i) within Problem 1.

The following proposition provides necessary and sufficient conditions that ensure that the equilibrium values of p^M and d^c globally minimize the optimization problem (3).

Proposition 2: Let Design Condition 1, $q_{k,j}(m_{k,j} + h_{k,j}) = 1, k \in \mathcal{N}_j^G, j \in \mathcal{N}$ and $q_{k,j}h_{k,j} = 1, k \in \mathcal{N}_j^L, j \in \mathcal{N}$ hold. Then, the equilibrium values $p^{M,*}$ and $d^{c,*}$ of system (1), (2), (5), (6), globally minimize problem (3).

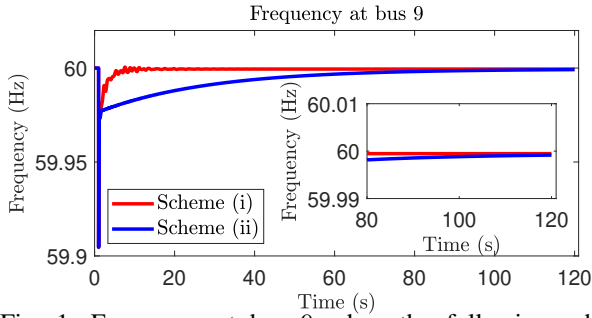


Fig. 1: Frequency at bus 9 when the following schemes are implemented: (i) Primal-Dual scheme, and (ii) Privacy-Preserving scheme.

Proposition 2 demonstrates how the controller gains in prosumption units should be designed such that an optimal power allocation is ensured, when Design Condition 1 holds.

The following theorem demonstrates that when Design Condition 1 holds, then the set of equilibria of (1), (2), (5), (6), is attracting. The latter shows that the proposed *Privacy-Preserving scheme* does not compromise the stability of the power network.

Theorem 1: Let Design Condition 1 hold. Then, the solutions of (1), (2), (5), (6), globally asymptotically converge to the set of its equilibria, where $\omega^* = \mathbf{0}_{|\mathcal{N}|}$.

Theorem 1 guarantees the convergence of solutions to (1), (2), (5), (6), to the set of its equilibria. In addition, the dynamics of (1), (2), (5), (6), are distributed, applicable to arbitrary network configurations and locally verifiable. Moreover, as demonstrated in Proposition 1, the *Privacy-Preserving scheme* enables the privacy of prosumption profiles against intelligent eavesdroppers. Finally, as demonstrated in Proposition 2, the presented scheme allows an optimal power allocation among generation and controllable demand. Hence, all objectives of Problem 1 are satisfied.

V. SIMULATION ON THE WSCC 9-BUS SYSTEM

In this section, we illustrate our analytic results with simulations using the Power System Toolbox [28] on Matlab. For our simulations, we use the Western System Coordinating Council (WSCC) 9-bus interconnection system. This model is more detailed and realistic than the considered analytical model, including voltage dynamics, line resistances, and a transient reactance generator model.

The test system consists of three generation buses and six load buses and has a total real power of 248 MW. Controllable demand was considered at three load buses, where at each bus the number of controllable loads was randomly selected from an integer uniform distribution with range [90, 180]. In addition, quadratic cost functions were considered for controllable demand following the description in (3). The values for $q_{k,j}$, $k \in \mathcal{N}_j$, $j \in \mathcal{N}$ were selected from a uniform distribution with range [50, 250]. For the simulation, a step change in demand of magnitude 0.05 per unit (100 MW) at 10 randomly selected loads at bus 9 was considered at $t = 1$ second. The time step for the simulations was set at 10 ms.

The system was tested under the following schemes:

- (i) The *Primal-Dual scheme*, described by (4).
- (ii) The *Privacy-Preserving scheme* that we proposed, described by (6) and Design Condition 1. The values of $\delta_{k,j}$, associated with Design Condition 1(i), were randomly assigned from the uniform distribution [0, 1] seconds. In addition, the values of $n_{k,j}^f$ were selected at each time step from the uniform distribution $[-0.9\beta_{k,j}|\omega_j|, 0.9\beta_{k,j}|\omega_j|]$ where $\beta_{k,j}$ followed from Design Condition 1(ii) and the factor 0.9 was

included to allow a 10% margin from the obtained safety bound.

In schemes (i), (ii), the dynamics of the implemented prosumption units followed from (2) and the controller gains were selected such that the optimality conditions presented in Proposition 2 were satisfied. The communication network associated with scheme (i) had the same structure as the power network. A random connected communication network was generated when scheme (ii) was implemented.

The frequency response at a randomly selected bus (bus 9) is depicted in Fig. 1. From Fig. 1, it follows that the frequency converges to its nominal value at both simulated cases. The latter suggests that the proposed *Privacy-Preserving scheme* yields a stable response. Note also that the frequency returns to within 0.005 Hz from its nominal value in less than one minute, which is well within the secondary frequency control timeframe. Nevertheless, the *Privacy-Preserving scheme* results in slower convergence of frequency to its nominal value. This is due to a larger number of controllers that need to synchronize for convergence.

To demonstrate the optimality of the proposed analysis, we consider the marginal costs of each active unit, defined as the absolute value of the cost derivative of the local cost functions. The marginal costs for all controllable loads and local generators are depicted in Fig. 2 (left). From Fig. 2, it follows that for both schemes (i) and (ii), the marginal costs for all units converge to the same value. The latter suggests that an optimal power allocation is attained at steady state and validates the presented optimality analysis.

To validate the enhanced privacy properties associated with the *Privacy-Preserving scheme*, compared with the *Primal-Dual scheme*, we considered the communicated signals from three randomly selected loads (loads 9, 18 and 27 at bus 9). The results are shown on Fig. 2 (middle, right), which demonstrate that when the *Primal-Dual scheme* is applied, the privacy of the controllable demand units is compromised since the demand values are communicated. By contrast, when the *Privacy-Preserving scheme* is implemented, power command signals are communicated and hence the privacy of the controllable load profiles cannot be directly compromised.

To demonstrate that the *Privacy-Preserving scheme* preserves the privacy of the prosumption profiles even when the underlying control dynamics (K2) are known, we considered an observer scheme that aims to infer the controllable demand using a model of the power command dynamics and knowledge of the power command signals. In particular, by evaluating the power command derivative and the value of ψ , an eavesdropper may attempt to observe the generation and controllable demand profiles by reversing (6b), i.e. using $\tilde{s} = \Gamma \dot{p}^c + H\psi$. Figure 3 demonstrates the result from such observer scheme for the same three loads considered in Fig. 2, when the *Privacy-Preserving scheme* is implemented. From Fig. 3, it follows that the application of the *Privacy-Preserving scheme* ensures that the demand is private against intelligent eavesdroppers, since the retrieved information is distorted by the signal $n_{k,j}$.

VI. CONCLUSION

We have considered the problem of simultaneously enabling an optimal power allocation and preserving the privacy of generation and controllable demand profiles within the secondary frequency control timeframe. To enhance the intuition on our results, we defined the notion of intelligent eavesdroppers who intercept all communicated information and simultaneously possess knowledge of the internal system dynamics. We proposed the *Privacy-Preserving scheme*, which implements a controller at each privacy-seeking unit in the power grid to provide improved privacy properties.

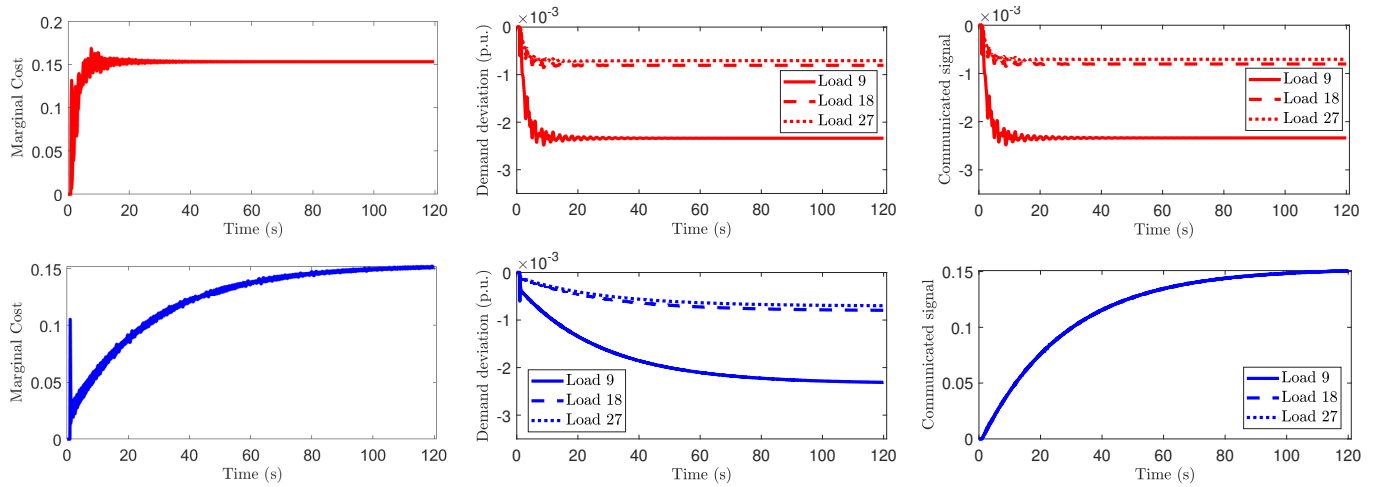


Fig. 2: Marginal costs for all loads (left), controllable demand (middle) and communicated signals (right) for loads 9, 18 and 27 at bus 9 under the Primal-Dual scheme (top) and the Privacy-Preserving scheme (bottom).

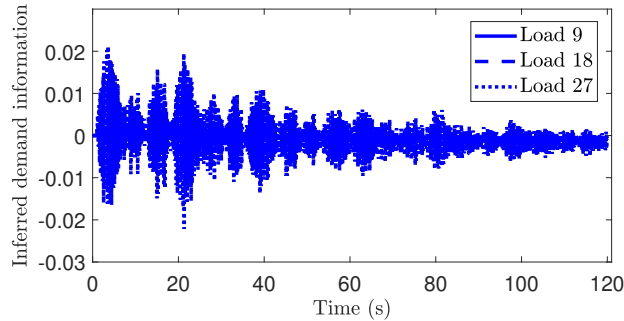


Fig. 3: Inferred demand information on loads 9, 18 and 27 at bus 9 using the power command trajectories under the Privacy-Preserving scheme.

In addition, the proposed scheme introduces a privacy-enhancing signal at each controller, which adjusts their response speed, making model based observations inaccurate, and disturbs the generation/demand profile information, to enable privacy against intelligent eavesdroppers. For the proposed scheme, we provide analytic stability, optimality and privacy guarantees. Our presented results are distributed, locally verifiable and applicable to general network configurations. The applicability of the *Privacy-Preserving scheme* is demonstrated with simulations on the WSCC 9-bus system where we show that stability and privacy are preserved and an optimal power allocation is attained.

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