Manifold dimension reduction based clustering for multi-objective evolutionary algorithm

Yanan Sun
College of Computer Science
Sichuan University
Chengdu, China 610065
sunkevin1214@gmail.com

Gary G. Yen
School of Electrical and Computer Engineering
Oklahoma State University
Stillwater, USA 74075
gyen@okstate.edu

Hua Mao and Zhang Yi
College of Computer Science
Sichuan University
Chengdu, China 610065
{huamao, zhangyi}@scu.edu.cn

***Abstract*—Real world optimization problems always possess multiple objectives which are conflict in nature. Multi-objective evolutionary algorithms (MOEAs), which provide a group of solutions in region of Pareto front, increasingly draw researchers attention for their excellent performance. In this regard, solutions with a wide diversity would be more favored as they give decision makers more choices to evaluate upon their problems. Based on the insight of investigating the evolution, the Pareto front often lies in a manifold space, not Euclidian space. However, most MOEAs utilize Euclidian distance as a sole mechanism to keep a wide range of diversity for solutions, which is not suitable somewhat from this aspect. To this end, manifold dimension reduction algorithm which has the ability to map solutions in the same front of objective space into Euclidian space is adapted in further. And then, general clustering algorithm are utilized. At the end, we use this technology to replace the crowding distance technology in NSGA-II to choose individuals when there is not enough slots in mating selection process. Based on a range of experiments over benchmark problems against state-of-the-art, it is fully expected benefit of performance improvement will be more significant when applied in many objectives optimization problems. This will be pursuit in our future study.**

I. INTRODUCTION

Multiobjective Optimization Problems (MOPs) are mainly concerned with a number of simultaneously optimized objectives over the decision variables space. These optimization problems, such as traction for DC railway system [1], containership loading design [2], gas turbine engine configuration [3], medical image reconstruction [4], supersonic wing design [5], and cancer chemotherapy [6], are very common under real-world complications. Generally, these objectives are conflict in nature and practitioners cannot obtain one perfect solution that can outperform others over every objective. If the exact trade-off among objective solutions is known, a preference based classical method might be good enough to search for the corresponding solution. However a user is usually not sure of the exact trade-off relationship among objectives. Naturally, a set of Pareto optimal solutions could be made available at first based on the Pareto-optimal principle and then a solution is chosen from the set by introducing some higher level decision-making. Algorithms for this class of problems begin with preference-based approaches if a prior knowledge about the importance of each objective is available. Subsequently, population-based algorithms are developed to find multiple Pareto-optimal solutions without any assumed knowledge, which are often referred to as multi-objective evolutionary algorithms (MOEAs), and have been utilized in a wide variety of applications. These MOEAs, e.g. elitist non-dominated sorting genetic algorithm (NSGA-II) [7], advanced version of strength Pareto evolutionary algorithm (SPEA2) [8], and multi-objective evolutionary algorithm based on decomposition (MOEA/D) [9], not only provide the convergence needed, but also the diversity of solutions are kept, so as to give decision makers more alternatives to select for their preferences.

Multiple objective genetic algorithms (MOGAs) of Murata et al. [10] improved the diversity of solutions by niche count in which distance is exploited to estimate how far individual i is away from individual j. In non-dominated sorting genetic algorithms (NSGAs) [11], diversity is maintained based on the number of neighboring solutions sharing function and the distance is measured between solutions i and j. Diversity preservation of NSGA-II is named crowding distance (CD) which is composed of two steps. First, CDs of the boundary solutions in objective space are ranked as infinite magnitude. Afterward the values of objectives are sorted in descending order for each dimension, and CD of solution i is computed through its neighbors. Strength Pareto evolutionary algorithm (SPEA) [12] and SPEA2 employ clustering and k-th nearest neighbor for maintaining diversity, respectively. Coincidentally, the distance metric for strengthening diversity mentioned above is based on Euclidean distance, which is more suitable for data lied in Euclidean space. Recently, based on the insight of investigating the evolution and the geometric regularity of Pareto front in MOEAs, it reveals that the Pareto front lies in manifold under mild conditions [13][1].

Manifold is one type of topological spaces whose local geometric regularity is homeomorphic to Euclidean space. Data lies in manifold are ubiquitous in many real-world applications, especially for high-dimensional data. The essential preprocessing step for a large number of further data analysis processes is understanding the intrinsic low-dimensional pattern of these high-dimensional data [14]–[17]. Recently, man-

[1]Strictly specking, Euclidean space is s special instance of manifold. We distinct these two concept in this context for better following the weakness of Euclidean distance metrics introduced in manifold for MOEAs.

ifold related technologies introduced in MOEAs are mainly concerned with how effectively the MOEAs would interpolate new individuals depending on the basis that the Pareto set is a piece-wise continuous manifold with dimension $m-1$ where m denotes the number of objectives [18], [19]. Manifold dimension reduction (MDR) is one approach for non-linear dimensionality reduction technologies which is comparable with principal component analysis. MDR tries to find the intrinsic representation of the raw data which lies in manifold and maps the data from manifold to Euclidean space. Increasing number of algorithms are proposed for this aspect of data analysis, such as Laplacian eigenmaps (LE) [20], locally linear embedding (LLE) [21], isomap algorithm (ISOMAP) [22], principal curve [23], semi-definite embedding [24], and self-organizing map [25]. In this paper, we focus on making use of MDR technology to map Pareto front lying in manifold into Euclidean space in which traditional clustering technology is then employed for enhancing diversity. As a case study, MDR based clustering NSGA-II is implemented for evaluating the performance of manifold dimension reduction based clustering (MC) based MOEAs.

In the rest of this paper, we first provide the problem formulation in Euclidean distance metric commonly adopted in MOEAs, especially the clustering technology utilized in SPEA2 and NSGA-II in Section II. Then the mathematical description of MDR is presented, and MC based NSGA-II (in short, MC-MSGA-II) for improving the diversity is proposed in Section III. For the purpose of examining the promising performance of MC-NSGA-II, quantitative and qualitative experiments are compared over benchmark problems against a few chosen state-of-the-art MOEAs in Section IV. Finally, Section V concludes this paper and provides directions to some future works.

II. Related Works

Convergence and diversity are two critical ingredients in the design of MOEAs. Convergence prefers a better non-dominated set being closer to the true Pareto front of conflicting objectives, while diversity favors diversified alternatives for decision makers preference. Most, if not all, existing MOEAs apply Euclidian distance as their basic distance metric for improving the efficacy of diversity. On the contrary geodesic distance is more suitable since the Pareto front lies in a manifold. In this paper, our focus will not be emphasized on how the geodesic distance metric can be implemented for the Pareto front, but on more reasonable to take advantage of traditional clustering technology. Consequently, only clustering technology utilized in SPEA and Clustering NSGA-II will be mainly discussed in details. It should be noted that the same treatment can be extended to other MOEA designs.

Crowding distance strategy is replaced by clustering in environmental selection of NSGA-II when there is not enough available slots, which gives birth to the Clustering NSGA-II algorithm for enhancing diversity of solutions. For convenience, $F_i(i=1,2,3,\cdots,n)$ denotes n different Pareto fronts in the current generation of NSGA-II, and $|F_i|$ denotes the number of individuals in F_i; N denotes the population size; $\sum_{k=1}^{j-1}|F_k| < N$ and $\sum_{k=1}^{j}|F_k| - N = Q$. Consequently, clustering technology will be adapted in F_j to select representative individuals with the number of Q, which are organized in details as follows:

1) Each individual i is clustered into one group g_i. $G=\{g_1,g_2,g_3,\cdots,g_N\}$, and $|\cdot|$ is a countable operator.
2) d_{ij} and $d_{g_ig_j}$ denotes the Euclidean distance between individuals i and j and groups i and j in objective space, respectively. Step 3 will be done until $|G|=Q$, and then go to Step 4.
3) $g_i = g_i \cup g_j$ and g_j is removed from G, where $d_{g_ig_j}$ is the smallest distance between each two groups in G and $d_{g_ig_j} = (\sum_{p=1}^{|g_i|}\sum_{q=1}^{|g_j|} d_{pq})/(|g_i||g_j|)$.
4) One representative individual i is picked from each group $g_i \in G$ with the condition that the distance between i and the centroid of g_i is the smallest against that of other members in g_i and the centroid.

The same clustering technology described above is also employed by SPEA.

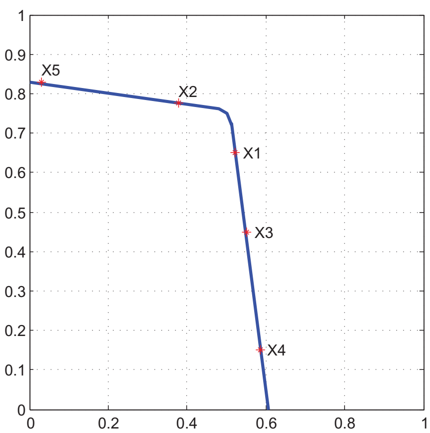


Fig. 1. An example of five solutions in the Pareto front.

It is obvious that this clustering algorithm is not reasonable as its distance metrics. For a better understand of this drawback, an example is designed to demonstrate it. In Figure 1 the solid line denotes the Pareto front, and there are five solutions (i.e., x_1, x_2, x_3, x_4, and x_5) while only four slots are available. With the clustering technology the first step is to measure the distances between each two individuals in the objective space. x_1 and x_2 are clustered into the same group as their Euclidean distance is the smallest. However, it is more reasonable for enhancing the diversity that x_1 and x_3 should be in the same cluster as the geodesic distance between x_1 and x_3 is less than others.

MDR technology is utilized in mapping data that lies in manifold to Euclidean space and preserve the neighboring relationship simultaneously by reducing dimension assuming that all of the data are continuous in manifold[2]. Figure 2 explains the effectiveness of MDR algorithms utilized over the example of Figure 1[3]. ISOMAP, LLE, and LE are conventional MDR algorithms reported in literature.

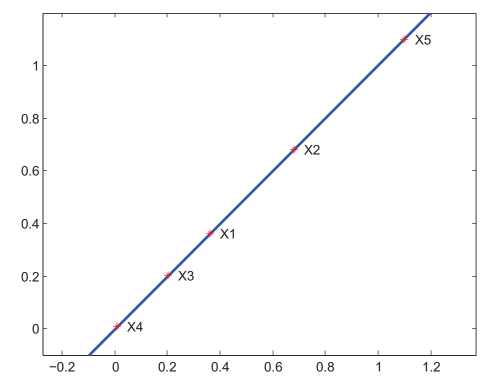


Fig. 2. The effectiveness of manifold dimension reduction algorithm utilized over the example in Figure 1.

LE makes use of spectral technologies to perform MDR relying on the theory that Laplacian operator in the graph converges to that in manifold [26]–[28] and eigenvectors of Laplacian matrix converges to Laplacian function [29], [30]. LE presumes that the data lies in a low dimensional manifold which is embedded in a high dimensional space. Given m data points $x_1, x_2, \cdots, x_m$ in $n-$dimensional space, LE begins with formulating a weighted adjacency graph $G(V, E)$ where V denotes the vertices and E denotes the weighted edges in graph G which can be introduced by detecting the k nearest neighbors or by appointing all the points within some fixed radius ϵ (i.e., $\|x_i - x_j\|^2 < \epsilon$ where $\| \cdot \|$ is the Euclidean norm in R^n). There are two variations for weighting the edges: 1) each edge is weighted by $W_{ij} = e^{-\|x_i - x_j\|^2/d}$, where d is a parameter for controlling the width of neighbors and usually chosen with a priori knowledge; 2) W_{ij} is set to 0 if vertices i and j are disconnected, otherwise 1. Consequently, the embedding map is given by optimizing Function 1

$$F(Y) = min \sum_{i=1}^{m} \sum_{j=1}^{m} (y_i - y_j)^2 W_{ij}, \tag{1}$$

where $Y = \{y_1, y_2, \cdots, y_m\}$ denotes the projected points in low dimensional Euclidean space. Based on the theory proposed in [29], [30] Function 1 can be reformulated as figuring out eigenvalues and eigenvectors for the generalized eigenvector problem:

$$Lf = \lambda Df, \tag{2}$$

[2]The concept *continuous* is highlighted for theoretically distinguishing manifold clustering technology which is frequently introduced for data which exists in manifold in the form of few different continuous geometry. In practice, MSR is also suitable for simple discrete manifold data which is favored by ZTD3 benchmark problem in our experiments

[3]Examples in Figure 1 and Figure 2 are mainly introduced for comparing method the different behavior of general clustering in Euclidean space and manifold, which will affect the mechanism of improving diversity in real problems.

where $L = R - W$ and R is a diagonal matrix whose diagonal element $R_{ii} = \sum_{j=1}^{m} W_{ij}$. L is named the Laplacian function and is a symmetrical, positive semi-definite matrix. Let $f_0, f_1, \cdots, f_{m-1}$ be the solutions of Function 2 and sorted according to their corresponding eigenvalues in a descending order. Finally, eigenvector f_0 corresponding to the smallest eigenvalue with 0 is dropped and the next k eigenvectors for embedding in k-dimensional Euclidean space:

$$y_i = (f_1(i), f_2(i), \cdots, f_k(i)).$$

Practically, the similar performance achieved over ISOMAP, LLE, and LE in this context. Because of the higher complexity of ISOMAP and LLE algorithms, LE is employed as a preliminary processing for clustering the solutions lying in the Pareto front of manifold in our proposed MC-NSGA-II.

III. Proposed Algorithm

In this section, we first propose the general framework of manifold dimension reduction based clustering for MOEAs (MC-NSGA-II). In addition, the architecture and pseudo codes of MC-NSGA-II are given.

Before we introduce the clustering algorithm, one fact need to be clarified. The most problem suffered by MDR is the number of dimensions that we should preserve in the projected Euclidean space. This dilemma is also faced by other clustering approaches, because sometimes we do not know exactly the intrinsic dimension of this manifold. Generally speaking, manifold with dimension k is viewed as being embedded in Euclidean space with dimension d under the condition $k < d$. In this paper, we utilize MDR in multi-objective evolutionary algorithm with objectives no more than three. Subsequently, it is known that the dimension of the projected data in Euclidean space into which we will map the manifold data is two (for three objectives problems) or one (for two objectives problems). Manifold data with one-dimension in Euclidean space is a line with slope equaling to zero, and with two-dimension in Euclidean space is the line with the slope not equaling to zero.

A. General framework of MDR based clustering for MOEAs

In Algorithm 1, n denotes the number of individuals needed to be clustered, d denotes the number of variables. Moreover, P only includes all the individuals who lies in the same Pareto front. $| \cdot |$ is a countable operator. The dimension of mapped data we kept is two because this clustering algorithm is designed for MOEAs with two or three objectives.

For conveniently measuring the complexity of Algorithm 1, we roughly divide it into three steps: constructing adjacency matrix; performing eigen factorization; and selecting representative individuals. The time complexity of each step is $O(2n + n * (n-1))$, $O(1/3 * n^3)$, and $O((n-k) * n^2)$, respectively. For this reason the time complexity of the whole Algorithm 1 is $O(n^3)$, and n denotes the number of clustered individuals.

The difference between MC-NSGA-II and NSGA-II occurs in the environment selection. NSGA-II selects individuals

Algorithm 1 Manifold dimension Reduction based Clustering Algorithm for MOEA

Input: 1)the number of clusters k; 2)individuals $P \in R^{n\times d}$; 3)all the objective functions $F = \{f_1, f_2, \cdots, f_m\}$

Output: k representative individuals

1: Evaluate the fitness of P over F(denoted as $Y = [y_1, y_2, \cdots, y_n]^T$);
2: Construct adjacency matrix $W \in R^{n\times n}$, where $W_{ij} = exp(-\frac{\|y_i - y_j\|^2}{d})$;
3: Compute $D = L - W$, where L is a Diagonal matrix with $L_{ii} = \sum_{j=1}^{n} W_{ij}$;
4: Compute the eigenvalues of D, sort the eigenvalues in ascending order, and compute the corresponding eigenvectors $V = [v_1, v_2, \cdots, v_n]^T$;
5: Set $B = [v_2, v_3]$, compute the mapped data $X = B^T Y$, where $X = [x_1, x_2, \cdots, x_n]^T$;
6: Partition X into group $G = g_1, g_2, \cdots, g_n$, where $g_i = \{x_i\}$;
7: Find the smallest distance d_{ij} between g_i and g_j by $d_{ij} = (\sum_{p=1}^{|g_i|} \sum_{q=1}^{|g_j|} d_{g_p g_q})/(|g_i||g_j|)$, set $g_i = g_i \cup g_j$ and remove g_j from G;
8: if $|G| > k$, repeat Step 7, otherwise go to next;
9: Set $Q = \oslash$, for each group g_i in G, find the closest individual p to the centroid of g_i, and set $Q = Q \cup p$;
10: **return** Q

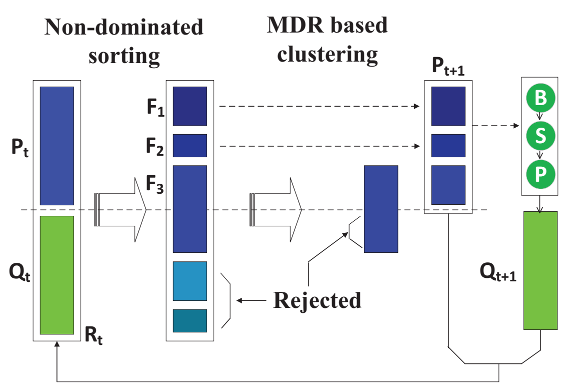


Fig. 3. Architecture of MC-NSGA-II algorithm. Circle with alphabet B, S, and P denote binary tournament selection, simulated crossover operator, and polynomial mutation operator, respectively.

with proper number based on crowding distance metric, while MC-NSGA-II chooses individuals by MDR based clustering algorithm. The architecture of MC-NSGA-II is illustrated in Figure 3, and the pseudo codes are listed in Algorithm 2.

Algorithm 2 Manifold Dimension Reduction based Clustering NSGA-II

1: Merge populations of parent and offspring to build $P_t = R_t \cup Q_t$. Assign a non-dominated sorting on P_t and specify various fronts: $F_i, i = 1, 2, \cdots, etc$;
2: Set new population $R_{t+1} = \oslash$. Set a counter $j = 1$. Until $|R_{t+1}| + |F_t| < N$, perform $R_{t+1} = R_{t+1} \cup F_t$, and $j = j + 1$;
3: Perform Algorithm 1 over individuals in F_{t+1} and include the most representative solutions with number $N - |R_{t+1}|$;
4: Establish offspring population Q_{t+1} from R_{t+1} by making use of the crowded tournament selection, crossover and mutation operators.

IV. Experiments

In order to evaluate the performance of MC-NSGA-II, ZTD [31] and DTLZ [32] benchmark problems are employed to simulate experiments upon the proposed algorithm against chosen state-of-the-art peer competitors. In this section, qualitative experiments are first made over DTLZ test suits by plotting the distribution of solutions generated by MC-NSGA-II to show the obtained non-dominated landscape. Furthermore, two performance metrics for quantitative comparison are also applied over ZTD and DTLZ test suits: 1) spacing metric [33]–[35] is utilized to measure how evenly the solutions of the final non-dominated front are distributed; 2) inverted generational distance (IGD) [36] performance metric is employed for investigating both the diversity and convergence of our proposed algorithm.

A. Test problems

ZDT test suites contain six extensively employed two objectives test problems. Because ZTD5 is a Boolean function which needs binary encoding, it is omitted so as to ZTD1-4 and ZTD6 test problems are included in our study. In addition, DTLZ1-7 test problems with two and three objectives from DTLZ test suites are also utilized. The dimensions, features, and sample size of each test problem in the true Pareto front are described in Table I.

B. Peer Algorithms

For the purpose of justifying the performance of MC based MOEA, MC-NSGA-II is employed to perform comparisons over NSGA-II[4], MOEA/D[5], and SPEA2[6]. Furthermore, the existing MOEA most similar to MC-NSGA-II, clustering NSGA-II, is also included into the list of peer algorithms for comparisons. All the algorithms are performed in Matlab platform except NSGA-II (given the original code of NSGA-II by the authours is implemented by C language), In order to accelerate the speed of clustering, an common extension version of clustering algorithm for Matlab based on C language is implemented[7].

[4]The code of NSGA-II is downloaded from: http://www.iitk.ac.in/kangal/codes.shtml

[5]The code of MOEA/D is downloaded from: http://dces.essex.ac.uk/staff/zhang/webofmoead.htm

[6]The code of SPAE2 is downloaded from: http://delta.cs.cinvestav.mx/~ccoello/EMOO/EMOOsoftware.html

[7]The code is downloaded from: http://legacy.machineilab.org/users/sunyanan/cluster.zip

TABLE I
THE DIMENSIONS, FEATURES, AND SAMPLE SIZE OF EACH TEST PROBLEM IN THE TRUE PARETO FRONT.

Benchmark	Dimensions of		Feature of PF	Sample size in PF
	Variables	Objectives		
ZTD1	30	2	ii	200
ZTD2	30	2	i	200
ZTD3	30	2	iii,v	200
ZTD4	30	2	ii,v	200
ZTD6	30	2	ii,iii,vii	200
DTLZ1	6	2	iv,v	200
	7	3		2000
DTLZ2	11	2	i	200
	12	3		2000
DTLZ3	11	2	i,v	200
	12	3		2000
DTLZ4	11	2	i,vi	200
	12	3		2000
DTLZ5	11	2	vi	200
	12	3		2000
DTLZ6	11	2	vi	200
	12	3		2000
DTLZ7	21	2	iii,v	200
	22	3		2000

The symbols i, ii, iii, iv, v, and vi denote concave, convex, disconnected, linear, multi-modal, and nonuniform regularity of the PF, respspectively.

C. Simulation settings

The size of population for each algorithm is set to 500. The number of function evaluations is fixed at 500. The performance on each test problem is obtained from 50 independent runs since the test algorithms are stochastic. Besides, simulated crossover (SBX) rate is fixed at 0.9, and polynomial mutation rate is set to $1/n$ where n denotes the number of decision variables. Both Distribution index for polynomial mutation and SBX are formulated to 20. Controlling width of neighbors in Algorithm 1 is heuristically set to 0.2. All the parameters in MOEA/D are utilized by its default settings given in the original implementation.

D. Qualitative experiments

For obtaining an intuitive perception about the performance of our proposed algorithm, the obtained approximate Pareto fronts by MC-NSGA-II with 500 generations over DTLZ1-7 test problems with two and three objectives are plotted in Figure 4. Solutions briefly give a landscape of the true Pareto front, which can be viewed as the effectiveness of our proposed algorithm. It is noticed that the landscape of DTLZ6 problem with two and three objectives simulated by MC-NSGA-II is not as good as expected because of the base function g in DTLZ6. This deficiency can be viewed as the ineffectiveness of NSGA-II reported in [32] not due to MDR based clustering.

E. Quantitative experiments

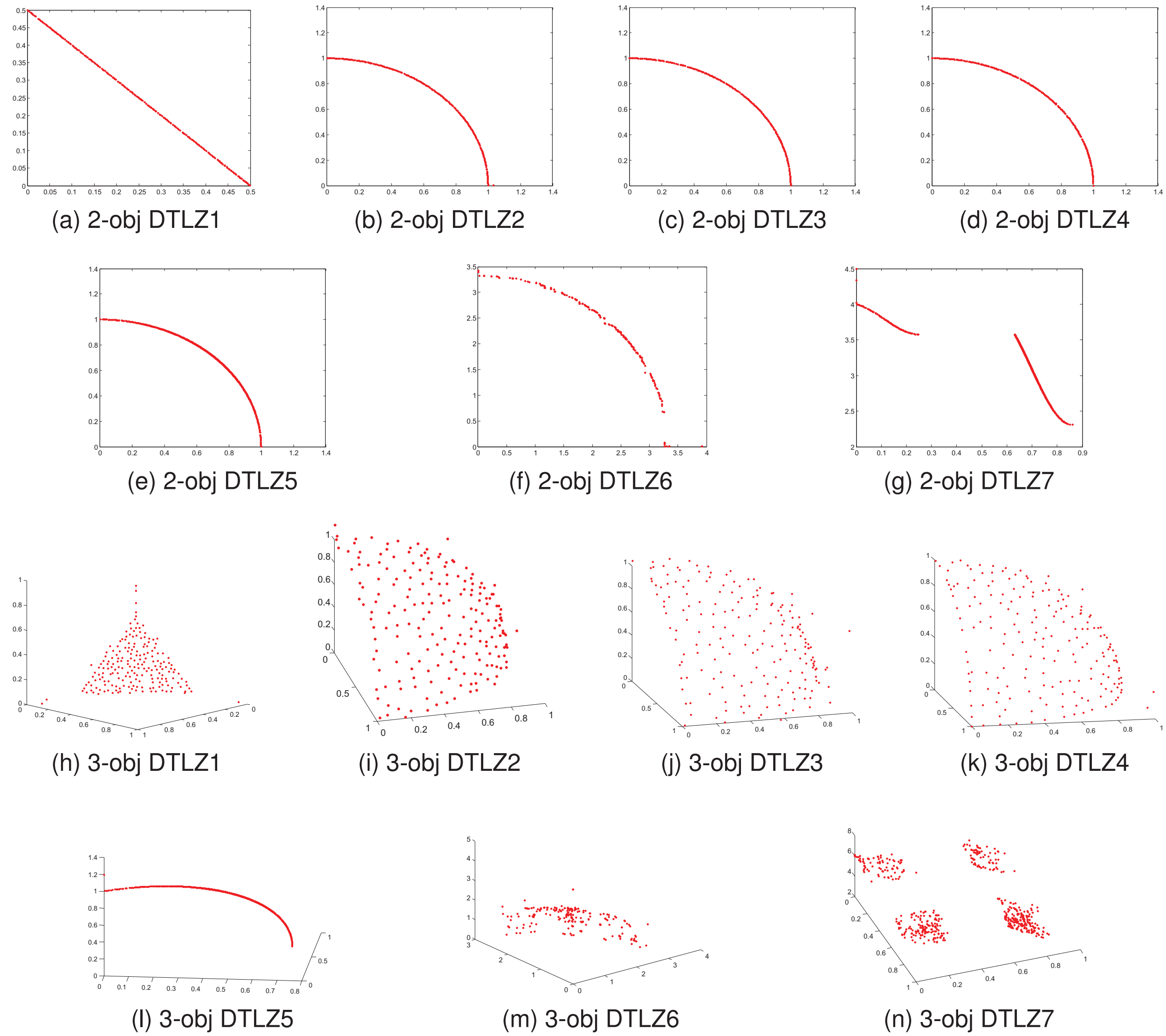
Two comprehensive performance metrics, spacing and IGD, are employed to measure the quality of obtained approximate Pareto fronts generated by algorithms mentioned above. S-pacing is a metric estimating how evenly the non-dominated solutions are distributed in the approximation front, the less the better. IGD metric quantifies both convergence and diversity of the approximate Pareto front at the same time. The dimensions, features, and sample size of the true Pareto fronts, which is required for IGD metric to evaluate the performance of given algorithms, are described in Table I. Spacing metric is used by MC-NSGA-II against clustering NSGA-II and NSGA-II while IGD is employed by MC-NSGA-II against clustering NSGA-II, NSGA-II, MOEA/D, and SPEA2.

Experiments results of spacing metric, generated by MC-NSGA-II, clustering NSGA-II, and NSGA-II over 13 test problems with two and three objectives, are plotted in Figure 5. Square markers denote the experimental results of MC-NSGA-II, and it is clear that the result of MC-NSGA-II is superior than others in most benchmark problems. In observing Figure 5, we believe that MC algorithm improves the diversity of solutions in MOEA.

IGD results are descripbed in Table II, and the best results are highlighted in bold face. Besides, Man-Whitney-Wilcoxon rank-sum test [37], a non-parametric statistical hypothesis testing, is also employed for comparing the mean IGD of MC-NSGA-II with that of the other algorithms for highlighting the significance of the findings. Moreover, MC-NSGA-II performs better than other algorithm in ZTD1, DTLZ1 with three objectives, DTLZ2 with two and three objectives, DTLZ3 with three objectives, DTLZ4 with two and three objectives, DTLZ5 with two and three objectives, and DTLZ7 with three objectives over the mean value. MC-NSGA-II and MEOA/D have the similar performance over DTLZ7 with two objectives, and MEOA/D has a better performance over DTLZ6 problem. In summary, MC-NSGA-II wins over most problems of DTLZ. It is highly expected that MC-NSGA-II would perform even better in more complicated problems when Pareto front are most likely in manifold. Moreover, it is also justified that convergence and diversity cannot be treated separately. From the results of IGD, it can be seen that improvement in diversity also enhances the convergence performance. At last, it is concluded from Table II that the proposed algorithm is superior to those chosen MOEAs in terms of IGD.

V. CONCLUSION

Convergence and diversity are two critical ingredients in the design of MOEAs. Based on extended investigation in enhancing diversity employed in most MOEAs and the geometrical regularity of Pareto front, conventional distance metrics, such as Euclidean distance, is not suitable for manifold in which Pareto front often lies. In order to provide a reasonable metric in improving diversity during the evolutionary process, manifold dimension reduction based clustering algorithm for MOEAs is designed in this paper. We first find the intrinsic dimension of this manifold



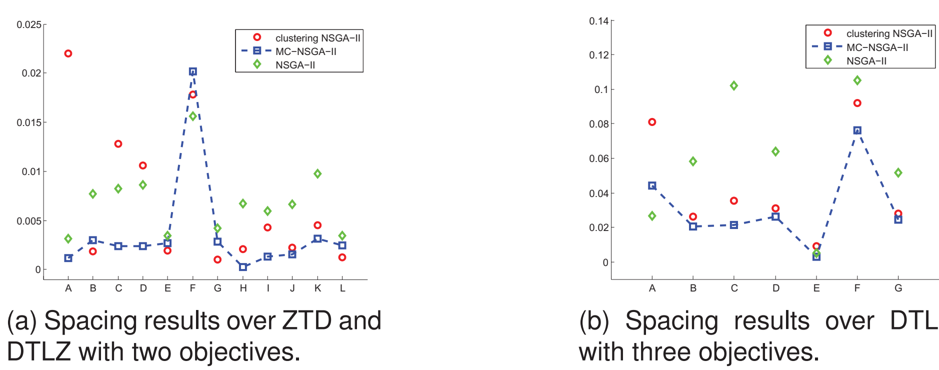
(a) 2-obj DTLZ1 (b) 2-obj DTLZ2 (c) 2-obj DTLZ3 (d) 2-obj DTLZ4

(e) 2-obj DTLZ5 (f) 2-obj DTLZ6 (g) 2-obj DTLZ7

(h) 3-obj DTLZ1 (i) 3-obj DTLZ2 (j) 3-obj DTLZ3 (k) 3-obj DTLZ4

(l) 3-obj DTLZ5 (m) 3-obj DTLZ6 (n) 3-obj DTLZ7

Fig. 4. Solutions of MC-NSGA-II over DTLZ1-7 problems with two and three objectives. Figure 4a-4g are DTLZ1-7 benchmark problems with two objectives, while Figure 4h-4n are DTLZ1-7 benchmark problems with three objectives.



(a) Spacing results over ZTD and DTLZ with two objectives. (b) Spacing results over DTLZ with three objectives.

Fig. 5. Experimental results of spacing metrics of MC-NSGA-II against clustering NSGA-II, and NSGA-II over ZDT and DTLZ test suits. In Figure 5a, the alphabet A-G, H-K, and L in x axis denote DTLZ1-DTLZ7, ZTD1-ZTD4, and ZTD5 test instances with two objectives, respectively. In Figure 5b, the alphabet A-G in x axis denote DTLZ1-DTLZ7 test instances with three objectives, respectively.

TABLE II

IGD RESULTS OF MC-NSGA-II AGAINST MOEA/D, CLUSTERING NSGA-II, NSGA-II, AND SPEA2 OVER DTLZ1-7, ZTD1-4 AND ZTD6 BENCHMARK PROBLEMS, RESPECTIVELY. BEST PERFORMANCE IS HIGHLIGHTED IN BOLD FACE.

Benchmark	Objectives		MC-NSGA-II	MOEA/D	clustering NSGA-II	NSGA-II	SPEA2
ZTD1	2	Mean	**1.5496E-03**	1.9224E-03	5.3206E-03	3.3490E-02	3.6269E-01
		Std.	**2.9672E-04**	3.4131E-06	1.1593E-03	8.8962E-03	1.1173E-01
		u-test		+	+	+	+
ZTD2	2	Mean	2.3793E-03	**1.8866E-03**	8.9021E-03	6.0540E-02	4.6582E-01
		Std.	9.7218E-04	**2.0664E-06**	2.4019E-03	1.4910E-02	8.5019E-02
		u-test		-	+	+	+
ZTD3	2	Mean	**1.4288E-03**	5.1194E-03	4.3060E-03	1.9376E-02	3.5379E-01
		Std.	**1.1256E-04**	1.6278E-05	7.4736E-04	4.0491E-03	7.9018E-02
		u-test		+	+	+	+
ZTD4	2	Mean	1.3067E-01	**1.1040E-02**	1.4162E-01	5.4035E-01	6.9050E-01
		Std.	7.8498E-02	**1.5836E-02**	6.0549E-02	9.0410E-03	4.4653E-02
		u-test		-	=	+	+
ZTD6	2	Mean	6.3339E-01	**9.5683E-04**	7.2226E-01	7.9739E-01	8.5169E-01
		Std.	2.5419E-02	**1.8975E-06**	1.4324E-02	5.2593E-03	2.5183E-02
		u-test		-	+	+	+
DTLZ1	2	Mean	8.8152E-04	**1.3681E-01**	1.2483E-03	3.8455E-03	2.9524E-01
		Std.	4.2279E-04	**3.4483E-04**	6.9020E-04	1.6959E-03	8.0477E-02
		u-test		-	=	+	+
DTLZ2	2	Mean	**1.4514E-03**	2.3500E-01	2.2760E-03	5.4032E-03	2.2125E-01
		Std.	**8.5411E-05**	1.4779E-04	7.1092E-05	2.7585E-04	7.6501E-02
		u-test		+	+	+	+
DTLZ3	2	Mean	4.7554E-02	3.7115E-01	**3.4310E-03**	7.1675E-02	1.4559E+00
		Std.	3.5240E-02	1.4759E-02	**2.5639E-03**	2.2915E-02	5.0823E-01
		u-test		+	=	=	+
DTLZ4	2	Mean	**1.7958E-03**	7.1570E-01	2.7618E-03	7.9018E-03	1.9369E-01
		Std.	**3.1225E-04**	1.1430E-01	7.8122E-04	3.2677E-03	1.4719E-01
		u-test		+	+	+	+
DTLZ5	2	Mean	**1.2752E-03**	1.8761E-03	2.2639E-03	2.4661E-03	1.9372E-01
		Std.	**7.3931E-05**	4.3957E-06	9.1368E-05	9.6746E-05	5.9462E-02
		u-test		+	+	+	+
DTLZ6	2	Mean	2.7186E+00	**1.8762E-03**	2.7921E+00	2.6934E+00	4.1019E-01
		Std.	2.5243E-01	**2.1149E-06**	2.8109E-01	2.6648E-01	3.0402E-02
		u-test		-	=	=	-
DTLZ7	2	Mean	4.3792E-01	**4.3715E-01**	4.4370E-01	4.4363E-01	8.9965E-01
		Std.	1.6292E-03	**6.2006E-05**	3.1866E-03	2.7260E-03	1.1058E-01
		u-test		-	+	+	+
DTLZ1	3	Mean	**1.3112E-02**	5.6007E-02	1.7813E-02	4.2618E-02	3.2565E-01
		Std.	**5.0085E-04**	5.8864E-03	1.1353E-03	5.3748E-03	7.6096E-02
		u-test		+	+	+	+
DTLZ2	3	Mean	**3.4364E-02**	1.4361E-01	4.0800E-02	7.2915E-02	2.6791E-01
		Std.	**1.2471E-03**	1.6022E-03	7.4677E-04	2.6104E-03	6.0448E-02
		u-test		+	+	+	+
DTLZ3	3	Mean	**4.0476E-02**	2.3383E-01	8.3955E-02	2.7614E-01	1.9278E+00
		Std.	**4.5799E-03**	5.4913E-02	1.0816E-02	5.3895E-02	5.1641E-01
		u-test		+	+	+	+
DTLZ4	3	Mean	**3.9959E-02**	1.9362E-01	4.1402E-02	7.2781E-02	5.6307E-01
		Std.	**2.8136E-03**	1.1608E-01	6.4221E-04	3.7324E-03	2.2253E-02
		u-test		+	+	+	+
DTLZ5	3	Mean	**1.2792E-03**	3.4857E-03	2.2620E-03	2.6532E-03	1.5815E-01
		Std.	4.3469E-05	**7.2483E-06**	3.9646E-05	8.6946E-05	5.0846E-02
		u-test		+	+	+	+
DTLZ6	3	Mean	2.0901E+00	**3.3236E-03**	1.9609E+00	2.0666E+00	5.1351E-01
		Std.	8.5626E-02	**9.0184E-06**	1.1723E-01	1.6752E-01	1.2888E-01
		u-test		-	+	=	+
DTLZ7	3	Mean	**3.8611E-02**	1.0511E-01	4.3625E-02	5.4819E-02	9.5132E-01
		Std.	4.9645E-03	5.8120E-03	2.4488E-03	**2.1796E-03**	1.9104E-01
		u-test		+	+	+	+
Better(+)				12	15	16	18
Same(=)				0	4	3	0
Worse(-)				7	0	0	1
Score				5	15	16	17

The value of u-test is tested based on its corresponding p-value that is generated by a two-sided Wilcoxon rank sum test upon the results of IGD of independent 50 runs. The marker "-", "=", and "+"denotes the performance on the benchmark problem is worse, same, and better than that of others with a significance level of 5%.

Pareto front embedd in Euclidean space, and then employ Laplacian eigenmaps technology to map the manifold into the corresponding Euclidean space in which conventional distance metric is properly utilized for clustering solutions within closer neighborhood. For evaluating the performance of our proposed clustering algorithm, manifold dimension reduction based clustering for NSGA-II is implemented by replacing the crowding distance metric in environment selection for preserving diversity. At last, qualitative and quantitative experiments are performed over ZTD and DTLZ test suits against clustering NSGA-II, NSGA-II, MOEA/D, and SPEA2. Results of experiments clearly justify that our proposed algorithm not only improving the diversity, but retaining its convergence performance.

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