Topology-Transparent Distributed Scheduling in Multi-hop Wireless Networks*

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Abstract — **Transmission scheduling is a key design problem in wireless multi-hop networks and many scheduling algorithms have been proposed to maximize the spatial reuse and minimize the time-division multiple-access (TDMA) frame length. Most of scheduling algorithms are graph-based, dependent on the exact network topology information and cannot adapt to the dynamic wireless environment. Some topology-independent TDMA scheduling algorithms have been proposed, and do not need accurate topology information. Our proposed algorithm follows a similar approach but with a different design strategy. Instead of minimizing the TDMA frame length, we maximize the minimum expected throughput, and we consider multicasting and broadcasting. The simulation result shows that the performance of our algorithm is better than the conventional TDMA and other existing algorithms in most cases.**

Keywords — Distributed scheduling, multi-hop wireless networks, TDMA, topology-transparent

I. INTRODUCTION

Scheduling transmissions without collision is one of the key issues in implementing Time Division Multiple Access (TDMA) and Code Division Multiple Access (CDMA) networks. In conventional TDMA network, each node is assigned a unique time slot to transmit. This method works very well when the connectivity relationship among nodes is known and the number of nodes is not large [4]. However, conventional TDMA does not work well in multi-hop environments. The network topology is not fixed because of node movements and limited battery power. Furthermore, conventional TDMA only allows one node to transmit in each slot, and does not facilitate resource sharing. Although spatial reuse can be employed to improve system performance [7], it must still address the contention problem. Hence, allocation techniques have been derived to ensure good system performance [3], [10]. A proper design not only guarantees transmission success but also maximizes the throughput of each node.

In this paper, we design a distributed scheduling algorithm to maximize the minimum expected throughput of each node in TDMA networks. Two common transmission paradigms, namely, multicasting and broadcasting are investigated. The connectivity model used in this paper can be deterministic or probabilistic and both scenarios are investigated. A network is connected probabilistically when any two nodes in the network can communicate with each other with a certain probability.

The major contributions of our work are as follows. First, the algorithm is topology-transparent and can be used under different network conditions with the presence of channel contention. Second, it addresses two popular transmission methods, namely, multicasting and broadcasting. Third, the simulation results show that our proposed algorithm is better than other existing algorithms in most cases.

The rest of paper is organized as follows. Related work is presented in Section II. Section III presents our analytical model. In Section IV, we describe our novel distributed scheduling algorithm and analyze it in Section V. We evaluate our algorithm through simulations in Section VI. Finally, we conclude our work in Section VII.

II. RELATED WORK

Previous studies on transmission scheduling can be divided into two categories: link activation [8] and node activation [3]. Most of these studies are topology-dependent or graph-based. Some of these graph-based scheduling algorithms focus on finding fair conflict-free algorithms which maximize the system throughput [8]. Most algorithms are centralized, while the algorithms proposed in [2] are distributed. Since these algorithms are based on a fixed network topology, their performance and robustness deteriorate substantially in a highly dynamic environment [1], where it may be very difficult to obtain accurate network connectivity information. Furthermore, they may require a large number of information exchanges among nodes, thus consuming valuable network bandwidth.

In order to overcome the above deficiencies, a number of topology-transparent (code-based) scheduling methods have been proposed [3], [9]. However, the algorithm in [3] only guarantees that each node has at least one successful transmission in each frame, and may sometimes perform worse than conventional TDMA [9]. Although the algorithm proposed in [9] can maximize the minimum throughput, it only considers point-to-point communications.

Since multicasting and broadcasting are important in wireless networks, they deserve further study. Our proposed algorithm can maximize the minimum expected throughput for

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each node under multicasting and broadcasting. Furthermore, we investigate these paradigms in both deterministically and probabilistically connected networks.

III. System Model

In this paper, we assume that a reception failure is only due to packet collisions. A collision occurs when 1) two or more nodes transmit simultaneously to the same destination node, and 2) a receiving node is also within the interference range of another transmission not intended for it. We also assume that nodes cannot transmit and receive simultaneously, and all network nodes are assumed to be homogeneous. All notations used in this paper are shown in Table I. Based on the total number of nodes (N) and the maximum degree (D_{max}), we design a scheduling algorithm so that each node in a network gets a predetermined minimum expected throughput no matter how the network topology changes. We assume that D_{max} will remain constant [9].

A wireless multi-hop network can be modeled as a bidirectional graph $G_v(n) = (V, E)$, where V is the set of nodes and E is the set of edges to indicate which pair of nodes is connected. The degree of a node v is defined as the number of its neighbors, which is always less than or equal to the maximum node degree D_{max}. Under this model, the network is called a deterministic network.

A probabilistically connected wireless network can be represented by a random graph $G(N, p)$. In this model, nodes are connected by edges at random. A popular model is the Erdős-Rényi model [6], in which an edge between any two nodes is present with probability p. The probability p_z that the degree of a node is exactly z follows the binomial distribution:

$$p_z = \binom{N-1}{z} p^z (1-p)^{N-1-z} \tag{1}$$

IV. Distributed Scheduling Algorithm

We use coding theory to design a topology-transparent distributed scheduling algorithm for multicasting and broadcasting, so as to maximize the network throughput. When a node transmits a message, all its neighbors can hear the signal, but only some of them, denoted "intended receivers," want to receive the message. A multicasted transmission is defined to be successful whenever all intended receivers receive the transmitted message successfully. A broadcasted transmission is defined to be successful whenever all neighbors of a sender receive the transmitted message successfully.

Consider a polynomial $f(x) = \sum_{i=0}^{k} a_i x^i \pmod q$, where $a_i \in \{0, 1, 2, \ldots, q-1\}$, q is a prime number, and k is the degree of $f(x)$ [11]. By [5], the equation $f(x) = 0 \pmod q$ will have at most k distinct roots, which are integers between 0 and $q-1$, inclusive.

In our algorithm, each node selects a time slot per subframe based on the following rules, as discussed in [10].

Rule 1: For a given network, each node v chooses a unique time slot allocation function (TSAF) $f_v(x) = \sum_{i=0}^{k} a_i x^i \pmod q,\ where\ v \in V$. The function is used to calculate the position of a transmission slot selected in a frame for node v.

TABLE I
NOTATIONS AND DEFINITIONS.

D_{max}	Max. node degree	p_z	Prob. that a node has exactly z neighbors
q	No. of subframes/slots	G_{min_exp}	Min. exp. throughput
k	Most no. of collisions	DT_{exp_max}	Max. exp. delay
p	Connectivity probability	DT_{exp_min}	Min. exp. delay

Rule 2: Let a standard row vector S be (0, 1, 2,…, q-1). $f_v(S) = (f_v(0), f_v(1), \ldots, f_v(q-1))$ is known as the time slot location vector (TSLV) for node v.

A TSLV indicates which time slots are selected by each node per frame. In Fig. 1, a frame is divided into q subframes. Each subframe consists of q time slots. For each node v in subframe i, the selected transmission time slot is given by $f_v(i)$ mod q. For example, if $f_v(0) = 2\ and\ f_v(1) = 1$, node v chooses the third time slot in subframe 0 and the second time slot in subframe 1.

Property 1: For a set of TSAFs $f_v(x)$ with degree k, two TSAFs have the same time slot selection for at most k times. This indicates that the number of collisions for any two nodes per frame is at most k [10].

V. Analysis of the Algorithm

Consider a single channel TDMA network with N mobile nodes and with maximum node degree D_{max}. Let node M be a sender. Denote the one-hop neighbors of M as X_i, where $i \in \{1, 2, \ldots, D_{\max}\}$. There are at most $Y_i \le D_{\max} - 1$ neighbors, excluding M, for X_i. A TDMA frame consists of q subframes, each of which has q slots. Each node selects one transmission slot in each subframe according to its unique TSAF $f_v(S)$. All TSLVs must be unique so that each node can have a certain minimum throughput.

The total number of TSLVs should be at least equal to the total number of nodes in the network. By [10], $q \ge N^{\frac{1}{k+1}}$. By Property 1, the maximum number of possible collisions in each frame for any two nodes is k, where k is the highest degree among all TASFs. Moreover, because each node selects q transmission slots in each frame, we can derive the minimum expected number of successful transmissions per frame as follows.

A. Deterministically Connected Networks

1) **Multicasting**: A node M multicasts a message to its R intended receivers. The probability an intended receiver X_j will receive M's multicasted message successfully is

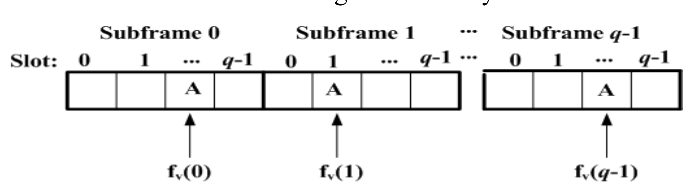

Fig. 1. The frame structure.

P (X_j receives M's multicast successfully)

$$= P(X_j \ does\ not\ transmit) \times \prod_{i=1}^{Y_i} P(i^{th}\ neighbor\ of\ X_j\ does\ not\ send) \quad (2)$$

Since X_j has at most $Y_i \le D_{\max} - 1$ neighbors and two nodes have at most k collisions in a frame, we can get,

P (X_j receives M's multicasted message successfully)

$$\ge (1-\frac{k}{q}) \cdot (1-\frac{k}{q})^{D_{\max}-1} = (1-\frac{k}{q})^{D_{\max}} \quad (3)$$

The probability that M's multicasted message is delivered successfully can be calculated as follows:

P (M's multicasting is successful)

$$= \prod_{j=1}^{R} P(j^{th}\ intended\ receiver\ receives\ successfully)$$

$$\ge [(1-\frac{k}{q})^{D_{\max}}]^R = (1-\frac{k}{q})^{R \cdot D_{\max}} \quad (4)$$

Since a frame has q subframes and M has one transmission slot per subframe, the minimum expected number of successful multicastings by M per frame, T_m, is:

$$T_m = q \cdot (1-\frac{k}{q})^{R \cdot D_{\max}} \quad (5)$$

The throughput is defined as the ratio of the number of guaranteed successful transmissions in each frame to the frame length L, where $L = q^2$. Let $G_{\min\_exp}(m)$ be the minimum expected throughput for multicasting, thus,

$$G_{\min\_exp}(m) = \frac{T_m}{L} = \frac{(1-\frac{k}{q})^{R \cdot D_{\max}}}{q} \quad (6)$$

Theorem 1: Given the value of k, the maximum value of $G_{\min\_exp}(m)$ is given by:

$$\max(G_{\min\_exp}(m)) = \begin{cases} \dfrac{\left\{1-\dfrac{1}{R \cdot D_{\max}+1}\right\}^{R \cdot D_{\max}}}{(R \cdot D_{\max}+1) \cdot k}, & if\ N^{\frac{1}{k+1}} \le (R \cdot D_{\max}+1) \cdot k \\ \dfrac{\left\{1-\dfrac{k}{N^{\frac{1}{k+1}}}\right\}^{R \cdot D_{\max}}}{N^{\frac{1}{k+1}}}, & otherwise \end{cases} \quad (7)$$

Detailed proof is omitted due to page limitations. Please refer to [10] for a similar proof.

The Max./Min. expected transmission delay is the ratio of the frame length to the Min./Max. number of successful transmissions in a frame. Let $DT_{\exp\_max}(m)$ and $DT_{\exp\_min}(m)$ be the Max. and Min. delay, respectively. Thus,

$$DT_{\exp\_max}(m) = 1/G_{\min\_exp}(m),\ DT_{exp\_min}(m) = q \quad (8)$$

2) **Broadcasting**: It is a special case of multicasting when all neighbors of M are the intended receivers. Thus, we have:

Theorem 2: Given the value of k, the maximum value of the minimum expected throughput $G_{\min\_exp}(b)$ is given by:

$$\max(G_{\min\_exp}(b)) = \begin{cases} \dfrac{\left\{1-\dfrac{1}{D_{\max}^2+1}\right\}^{D_{\max}^2}}{(D_{\max}^2+1) \cdot k}, & if\ N^{\frac{1}{k+1}} \le (D_{\max}^2+1) \cdot k \\ \dfrac{\left\{1-\dfrac{k}{N^{\frac{1}{k+1}}}\right\}^{D_{\max}^2}}{N^{\frac{1}{k+1}}}, & otherwise \end{cases} \quad (9)$$

The Max. and Min. expected transmission delay $DT_{\exp\_max}(b)$ and $DT_{\exp\_min}(b)$ are as follows,

$$DT_{\exp\_max}(b) = 1/G_{\min\_exp}(b),\ DT_{\exp\_min}(b) = q \quad (10)$$

B. Probabilistically Connected Networks

1) Multicasting: A node M multicasts to its R intended receivers. Since the probability that any one intended receiver X_j has degree z is p_z, the maximum degree of a node is $N-1$ and two nodes have at most k collisions in a frame, the probability an intended receiver X_j will receive M's multicasting successfully is

P (X_j receives M's multicasted message successfully)

$$= P(X_j\ does\ not\ transmit) \times \prod_{i=1}^{Y_i} P(i^{th}\ neighbor\ of\ X_j\ does\ not\ send)$$

$$\ge \sum_{z=0}^{N-1} p_z \cdot (1-\frac{k}{q}) \cdot (1-\frac{k}{q})^{z-1}$$

$$= \sum_{z=0}^{N-1} \binom{N-1}{z} p^z (1-p)^{N-1-z} \cdot (1-\frac{k}{q})^z \quad (11)$$

$$= [p \cdot (1-\frac{k}{q}) + (1-p)]^{N-1} = (1-\frac{pk}{q})^{N-1}$$

P (M's multicasting is successful)

$$= \prod_{j=1}^{R} P(j^{th}\ intended\ receiver\ receives\ successfully)$$

$$= [(1-\frac{pk}{q})^{N-1}]^R = (1-\frac{pk}{q})^{R \cdot (N-1)} \quad (12)$$

Hence, the average expected number of successful multicasting by M per frame is

$$\overline{T_m} = q \cdot (1-\frac{pk}{q})^{R \cdot (N-1)} \quad (13)$$

Let $\overline{G_{\exp}(m)}$ be the minimum expected throughput for multicasting, thus,

$$\overline{G_{\exp}(m)} = \frac{\overline{T_m}}{L} = (\frac{1}{q}) \cdot (1 - \frac{pk}{q})^{R \cdot (N-1)} \tag{14}$$

Theorem 3: Given the value of *k*, the maximum value of $\overline{G_{\exp}(m)}$ is as follows:

$$\max(\overline{G_{\exp}(m)}) = \begin{cases} \dfrac{[R \cdot (N-1)]^{R \cdot (N-1)}}{p \cdot k \cdot [R \cdot (N-1)+1]^{R \cdot (N-1)+1}}, & \text{if } N^{\frac{1}{k+1}} \leq [R \cdot (N-1)+1] \cdot p \cdot k \\ \dfrac{\left\{1 - \dfrac{p \cdot k}{N^{\frac{1}{k+1}}}\right\}^{R \cdot (N-1)}}{N^{\frac{1}{k+1}}}, & \text{otherwise} \end{cases} \tag{15}$$

The Max. and Min. expected transmission delay $DT_{\max}(m)$ $DT_{\min}(m)$ are as follows,

$$DT_{\max}(m) = 1 / \overline{G_{\exp}(m)},\ DT_{\min}(m) = q \tag{16}$$

2) **Broadcasting:** It is a special case of multicasting when all neighbors of M are intended receivers. Let $\overline{G_{\exp}(b)}$ be the minimum expected throughput. Thus, we have,

Theorem 4: Given the value of *k*, the maximum value of $\overline{G_{\exp}(b)}$ is as follows:

$$\max(\overline{G_{\exp}(b)}) = \begin{cases} \dfrac{[(N-1)^2]^{(N-1)^2}}{p \cdot k \cdot [(N-1)^2+1]^{(N-1)^2+1}}, & \text{if } N^{\frac{1}{k+1}} \leq [(N-1)^2+1] \cdot p \cdot k \\ \dfrac{\left\{1 - \dfrac{p \cdot k}{N^{\frac{1}{k+1}}}\right\}^{(N-1)^2}}{N^{\frac{1}{k+1}}}, & \text{otherwise} \end{cases} \tag{17}$$

The Max. and Min. expected transmission delay $DT_{max}(b)$ $DT_{min}(b)$ are as follows,

$$DT_{\max}(b) = 1 / \overline{G_{\exp}(b)},\ DT_{\min}(b) = q \tag{18}$$

To maximize the minimum expected throughput of the network, we have to choose the optimal values of *k* and *q*. In our analysis, we have found that the optimal throughput occurs when *k* is less than one. This means that the maximum number of collisions between any two nodes in a frame is either zero or one. Conventional TDMA corresponds to the case when k=0. Its minimum expected throughput is equal to 1/*N*.

Based on the above discussion, we can summarize our algorithm as follows: 1) According to Theorems 1 to 4, find the values of *k* and *q* for the given N and degree distribution such that the minimum expected number of successful transmissions for each node is maximized. 2) Each node is randomly assigned a unique TSAF (with degree $\leq k$). 3) Each node calculates its TSLV according to TSAF. 4) Each node transmits its data packets at its assigned slots.

VI. Performance Evaluation

We compare the expected throughput and transmission delay of our algorithm with conventional TDMA. Through the numerical results based on the analytical expressions obtained in Section V, we can study the performance impact of *N, R*, D_{max}, and *p* on our algorithm.

For conventional TDMA, there is exactly one successful transmission per node in each frame because of its unique allocation scheme. Thus, the TDMA throughput per node is 1/*N*. Furthermore, when a new connection comes into the TDMA system during a random time slot, then the expected transmission delay for a new connection should be *N*/2. Thus, the average transmission delay of a node in TDMA is *N*/2.

A. Performance of Deterministically Connected Networks

Case 1: Effect of N on performance

Given the number of intended receivers *R* is 3 and D_{max} is 5, we consider six different cases with number of nodes *N* equal to 32, 64, 128, 256, 512, and 1024, respectively. The effect of the number of nodes *N* on performance is shown in Fig 2. Fig. 2(a) shows that when *N* is more than 32, our throughput is better than conventional TDMA. The throughput of our algorithm decreases very slowly with increasing *N*. In Fig. 2(b), the expected transmission delay of our algorithm is lower than conventional TDMA.

Case 2: Effect of R on performance

Given *N*=512 and D_{max}=10, we consider values of *R* from 1 to 10, to investigate the impact of the number of intended receivers on performance. As shown in Fig. 3, our algorithm achieves higher throughput and lower transmission delay than conventional TDMA. Moreover, with increasing *R*, the throughput decreases while the transmission delay increases. A large value of *R* is more likely to have concurrent transmissions among the sender and the receivers, and so the number of successful transmissions decreases. The sender needs to wait for more time for the next successful transmission. Similarly, when *R*=10=D_{max}, it becomes broadcasting, which is a special case of multicasting. From Fig. 3, the performance of our algorithm for broadcasting is still better than TDMA.

Case 3: Effect of D_{max} on performance

Given *N*=1024, 10 different values of D_{max}, from 4 to 13 and with R equal to half of D_{max} are considered. Bigger D_{max} means higher node density in a certain area. From Fig. 4, we find our algorithm can achieve higher throughput and lower transmission delay than conventional TDMA when *R* is less than 24. With increasing *R* and D_{max}, our algorithm is not better than conventional TDMA. The reason is that larger *R* and D_{max} increase the likelihood of concurrent transmissions among the sender and the receivers. Comparing case 3 with case 1, we know *R* and D_{max} have more impact on the throughputs and the transmission delay than *N*. For broadcasting, our algorithm also outperforms TDMA when *R*=D_{max}.

Based on the evaluation and analysis in the above subsections, our algorithm can achieve higher throughput and lower transmission delay in most cases, especially in large

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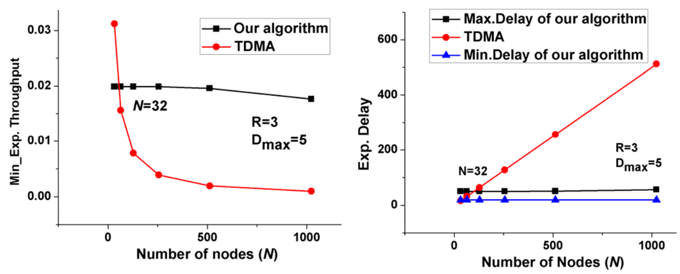
networks. Moreover, from the simulation results, we find that the performance of our algorithm is mainly affected by the number of intended receivers R and D_{max}. Thus, our algorithm is not very sensitive to the number of nodes N.

B. *Performance of Probabilitically Connected Networks*

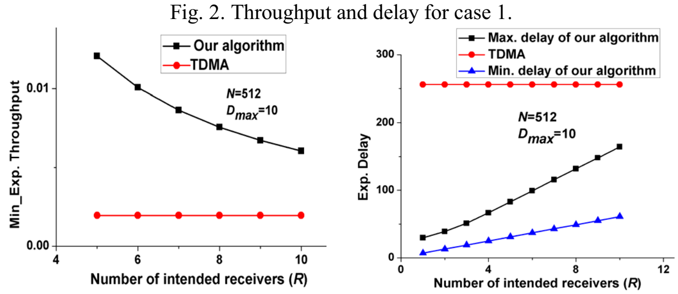
Case 4: Impact of p on performance

We consider two different values of N, which are 128 and 1024, R=10, and p between 0 and 1. We compare the performance of our algorithm with conventional TDMA and observe the impact of p on the performance. From (1), we know that the connectivity probability p mainly determines the average node degree. If the connectivity probability p increases, the maximum expected throughput will decrease and the expected transmission delay will increase, as shown in Fig. 5. The reason is that a lower connectivity probability results in fewer neighbors of a node. Therefore collisions are less likely to occur. Hence, the number of successful transmissions becomes larger. Therefore, the connectivity probability p does have a pivotal effect on the performance.



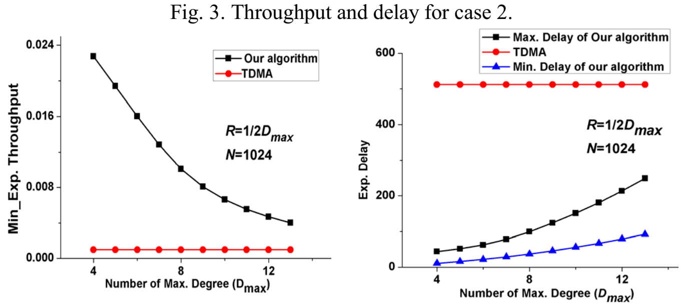
(a) Expected Throughput. (b) Transmission Delay.

Fig. 2. Throughput and delay for case 1.



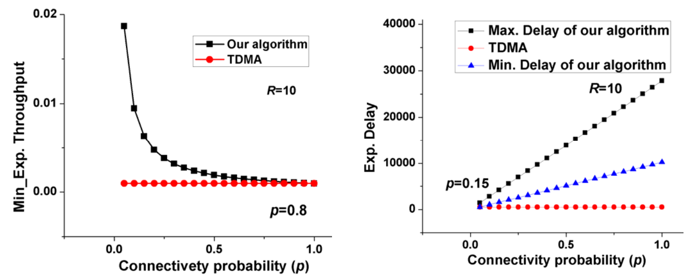
(a) Expected Throughput. (b) Transmission Delay.

Fig. 3. Throughput and delay for case 2.



(a) Expected Throughput. (b) Transmission Delay.

Fig. 4. Throughput and delay for case 3.



(a) Expected Throughput N = 128. (b) Transmission Delay N = 1024.

Fig. 5. Throughput and delay for case 4.

VII. Conclusion

In this paper, we have proposed a distributed topology-transparent scheduling algorithm for multi-hop wireless networks. Our algorithm maximizes the minimum expected throughput for multicasting and broadcasting, and can be applied to an arbitrarily connected multi-hop wireless network. The proposed algorithm is simple and suitable for distributed implementation because it only requires the number of nodes and the maximum node degree in the network. The simulation results show that our algorithm outperforms existing algorithms in almost all cases and its performance is insensitive to the number of nodes in the network.

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