This full text paper was peer reviewed at the direction of IEEE Communications Society subject matter experts for publication in the IEEE ICC 2011 proceedings

Efficient MIMO Detection Based on Eigenspace Search with Complexity Analysis

Ronald Y. Chang, Wei-Ho Chung* and Cheng-Yu Hung
Research Center for Information Technology Innovation, Academia Sinica, Taiwan

***Abstract*— A low-complexity, effective detection method for multiple-input multiple-output (MIMO) systems based on post-equalization eigenspace search is proposed in this paper. Motivated by the observation that solutions yielded by linear equalization-based detectors are corrupted by color noise, the proposed scheme introduces a new constellation-search procedure to augment linear detectors. Specifically, calibrated search is conducted around the initial solution yielded by linear detectors, in the directions guided by the eigenvectors corresponding to the dominant eigenvalues of the covariance matrix of the color noise to identify improved solutions. Complexity analysis is performed to understand the cost of this search procedure. Simulation results demonstrate that the proposed scheme yields an approximate 5 dB gain over linear equalization-based detectors in terms of symbol error rate (SER), at moderate additional computational cost.**

I. INTRODUCTION

MIMO technology can greatly enhance the capacity of wireless cellular networks and/or the reliability of data transmission through wireless media. It has been adopted in current and next-generation cellular systems such as Long Term Evolution Advanced (LTE-A) [1] and IEEE 802.16m advanced WiMAX [2]. To fully exploit the potential of MIMO, it is crucial to devise and employ a high-fidelity and low-complexity detection scheme at the receiving end. Maximum likelihood (ML) detection, relying on exhaustive search over the transmitted symbol alphabet, is theoretically optimal as it minimizes the error probability given equally probable transmitted symbol vectors, yet at the cost of intense computational demand. In fact, ML detection becomes infeasible for systems with medium to large number of antennas and/or using high-order modulation. Therefore, a practical method with reasonable computational complexity is highly desirable for next-generation cellular systems.

Extensive research has been conducted to remedy the infeasibility of ML detection in real systems. Proposed MIMO detection methods can broadly be classified into two categories. The first category aims at achieving near-ML performance at a reduced complexity cost. This group of work includes sphere decoding (SD) and its variants [3]–[5]. While SD achieves significant complexity reduction, it still requires intensive and varying amount of computations. The second category of work aims at retaining the low-complexity advantage of linear detectors while improving their suboptimal performance. This group of work develops modifications and augmentations on the conventional zero-forcing (ZF) or minimum-mean-square-error (MMSE) detection schemes [6] [7].

Along the line of the second group of work, we propose a new linear detection-based method that incorporates a novel constellation-search procedure. Motivated by the fact that the solutions yielded by linear detectors are corrupted by color noise and the performance of linear detectors largely depends on the "skewness" of the color noise in the constellation hyperspace [8], we develop a constellation-search algorithm that conducts search in specific directions along which the most severe noise disruption may have been imposed on the transmitted signal. This post-linear detection procedure determines the search range and directions based on the channel information available to the receiver. Simulation demonstrates that the proposed scheme offers substantial performance advantage over conventional linear detectors.

The rest of the paper is organized as follows. In Sec. II, the system model is presented and the commonly adopted MIMO detection methods are reviewed. Our proposed methods are described in Sec. III, with complexity analysis conducted in Sec. IV and the performance demonstrated in Sec. V. Finally, concluding remarks are given in Sec. VI.

II. SYSTEM MODEL AND REVIEW OF RELATED DETECTION SCHEMES

We consider a MIMO transmission system with M_T transmit antennas and M_R receive antennas ($M_R \geq M_T$). Then the baseband signal model is given by

$$\mathbf{y} = \mathbf{H}\tilde{\mathbf{x}} + \mathbf{n}, \tag{1}$$

where $\mathbf{y}$ is the $M_R \times 1$ received signal composed of the $M_T \times 1$ transmitted signal $\tilde{\mathbf{x}}$ passed through the $M_R \times M_T$ flat-fading channel $\mathbf{H}$ and the $M_R \times 1$ perturbing noise vector $\mathbf{n}$. The transmitted symbol vector $\tilde{\mathbf{x}}$ contains uncorrelated entries from the countably finite set of modulation constellation points, denoted as $\mathbb{S}$, and has zero mean and covariance matrix $\sigma_x^2\mathbf{I}_{M_T}$, where $\mathbf{I}_{M_T}$ is the $M_T \times M_T$ identity matrix. The complex-valued channel matrix $\mathbf{H}$ has independent and identically distributed (i.i.d.) Gaussian entries with zero mean and covariance matrix $\sigma_H^2\mathbf{I}_{M_R}$, where $\sigma_H^2 = 1$. The channel information is assumed perfectly known to the receiver. The noise $\mathbf{n}$ is additive white Gaussian noise (AWGN) with i.i.d.

*Corresponding author: Wei-Ho Chung. Email: whc@citi.sinica.edu.tw.
This work was supported by National Science Council of Taiwan, under grant number 99-2218-E-001-004.

complex elements and has zero mean and covariance matrix $\sigma_n^2\mathbf{I}_{M_R}$.

In the following, we briefly review three MIMO detection schemes related to this work. The first scheme is optimal ML detection and the other two are suboptimal linear detection with significantly lower computational complexity than ML detection.

A. ML Detection

Given the signal model in (1), ML detection is equivalent to solving a constrained least-square problem, i.e.,

$$\tilde{\mathbf{x}}_{\text{ML}} = \arg\min_{\mathbf{x}\in\mathbb{S}^{M_T}} ||\mathbf{y}-\mathbf{H}\mathbf{x}||^2, \quad (2)$$

where $||\cdot||$ denotes the l_2-norm of a vector. ML detection is optimal in the sense that it finds a solution that minimizes the error probability given equally probable transmitted symbol vectors. The major challenge of ML detection, however, lies in the huge search space $\mathbb{S}^{M_T}$, which renders this method computationally expensive, even impractical, when M_T and/or $|\mathbb{S}|$ is large, where $|\cdot|$ denotes the cardinality of a set.

B. ZF Detection

ZF detection performs linear equalization on the received symbol vector $\mathbf{y}$ followed by entry-wise quantization to the closest constellation point. Assuming $\mathbf{H}$ has full rank and $M_R \geq M_T$, the equalization matrix $\mathbf{G}_{\text{ZF}}$ is given by the Moore-Penrose pseudo-inverse [9] of $\mathbf{H}$, i.e.,

$$\mathbf{G}_{\text{ZF}} = (\mathbf{H}^H\mathbf{H})^{-1}\mathbf{H}^H, \quad (3)$$

where $(\cdot)^{-1}$ and $(\cdot)^H$ denote matrix inverse and Hermitian matrix transpose, respectively. The equalized symbol vector, $\hat{\mathbf{x}}_{\text{ZF}}$, is given by

$$\begin{aligned}\hat{\mathbf{x}}_{\text{ZF}} = \mathbf{G}_{\text{ZF}}\mathbf{y} &= (\mathbf{H}^H\mathbf{H})^{-1}\mathbf{H}^H\mathbf{y} \quad (4)\\ &= \tilde{\mathbf{x}}+\tilde{\mathbf{n}}, \quad (5)\end{aligned}$$

where $\tilde{\mathbf{n}} = (\mathbf{H}^H\mathbf{H})^{-1}\mathbf{H}^H\mathbf{n}$. The ZF detected symbol vector is $\tilde{\mathbf{x}}_{\text{ZF}} = q(\hat{\mathbf{x}}_{\text{ZF}})$, where $q(\cdot)$ denotes entry-wise quantization.

C. MMSE Detection

Similar to ZF detection, MMSE detection performs linear equalization on the received signal $\mathbf{y}$. The equalization matrix, however, is derived by minimizing the mean square error $\text{E}[||\mathbf{G}\mathbf{y}-\tilde{\mathbf{x}}||^2]$ so as to reduce noise enhancement as observed in ZF detection. The equalization matrix is given by [10]

$$\mathbf{G}_{\text{MMSE}} = \left(\mathbf{H}^H\mathbf{H}+\frac{\sigma_n^2}{\sigma_x^2}\mathbf{I}_{M_T}\right)^{-1}\mathbf{H}^H. \quad (6)$$

It follows that the equalized symbol vector is

$$\hat{\mathbf{x}}_{\text{MMSE}} = \mathbf{G}_{\text{MMSE}}\mathbf{y} = \left(\mathbf{H}^H\mathbf{H}+\frac{\sigma_n^2}{\sigma_x^2}\mathbf{I}_{M_T}\right)^{-1}\mathbf{H}^H\mathbf{y}, \quad (7)$$

and the MMSE detected symbol vector is $\tilde{\mathbf{x}}_{\text{MMSE}} = q(\hat{\mathbf{x}}_{\text{MMSE}})$.

III. Proposed Methods

Equalization-based detection methods generally achieve significantly lower computational complexity compared to ML detection at the cost of suboptimal performance. To improve the performance of equalization-based methods while still maintaining the advantage of low complexity, we propose a new method that performs "intelligent" constellation search to augment ZF or MMSE detection without introducing substantial complexity increase. More specifically, it is observed from (5) that even though the crosstalk of $\tilde{\mathbf{x}}$ generated by the channel $\mathbf{H}$ in the received $\mathbf{y}$ is removed, the performance of ZF detection is still greatly compromised due to colored Gaussian noise $\tilde{\mathbf{n}}$. By examining the covariance matrix of this color noise $\tilde{\mathbf{n}}$, which is an ellipsoid in M_T dimensions, the major axes that match eigenvectors corresponding to dominant eigenvalues of the covariance matrix can be identified. These major axes are directions along which the color noise exhibits the largest variance (i.e., reaches farest from the center of the ellipsoid). Therefore, conducting constellation search along these major axes may correct the "most probable" errors incurred in the ZF or MMSE detected symbol due to noise perturbation, and consequently, yield an improved solution. This approach is formally described in the following.

A. Eigenspace Search (ES) Method

The singular value decomposition (SVD) of the channel matrix $\mathbf{H}$ is given by

$$\mathbf{H} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^H, \quad (8)$$

where $\mathbf{U}$ and $\mathbf{V}$ are $M_R\times M_R$ and $M_T\times M_T$ unitary matrices, and $\mathbf{\Sigma}$ is an $M_R\times M_T$ diagonal matrix with positive-valued singular values $\sigma_1,\ldots,\sigma_{M_T}$ on the diagonal (since $M_R \geq M_T$ and $\mathbf{H}$ has full rank). It is assumed that these singular values are ordered such that $\sigma_1 \leq \sigma_2 \leq \cdots \leq \sigma_{M_T}$. By substituting (8) into the covariance matrix of the color noise $\tilde{\mathbf{n}}$, denoted by $\mathbf{R}_{\tilde{n}}$, it can be shown that

$$\begin{aligned}\mathbf{R}_{\tilde{n}} &= \sigma_n^2(\mathbf{H}^H\mathbf{H})^{-1} \quad (9)\\ &= \mathbf{V}\mathbf{\Omega}\mathbf{V}^H, \quad (10)\end{aligned}$$

where $\mathbf{\Omega} = \sigma_n^2(\mathbf{\Sigma}^H\mathbf{\Sigma})^{-1}$ is an $M_T\times M_T$ diagonal matrix with $\sigma_n^2/\sigma_1^2,\ldots,\sigma_n^2/\sigma_{M_T}^2$ on the diagonal. The major axes of $\mathbf{R}_{\tilde{n}}$ are indicated by the columns of $\mathbf{V}$ corresponding to the smallest singular values of $\mathbf{H}$. Let $\mathbf{v}_i$ represent the column of $\mathbf{V}$ that corresponds to singular value σ_i, $i = 1,\ldots,M_T$.

As an example, suppose we are interested in initiating an ES around the ZF equalized symbol vector $\hat{\mathbf{x}}_{\text{ZF}}$ (i.e., the unconstrained least-square solution) for improved estimates. The ES method entails searching the constellation hyperspace with $\hat{\mathbf{x}}_{\text{ZF}}$ at the center, in the direction of the major axes specified by $\mathbf{v}_1,\mathbf{v}_2,\ldots,\mathbf{v}_L$, where L $(1 \leq L \leq M_T)$ is the dimension of the search, and with search steps in the unit of $d/2$, where d is the distance between two nearest constellation points. For a particular axis $\mathbf{v}_i$, we can obtain the search steps for given values of σ_n, d, and $\sigma_1,\ldots,\sigma_L$. More explicitly, the

TABLE I
EIGENSPACE SEARCH ALGORITHM

Algorithm III.1: ZF-ES(L)

Given σ_n and d for some MIMO channel/modulation.
1) Calculate $\hat{\mathbf{x}}_{\text{ZF}}$ and $\{\mathbf{v}_i, \sigma_i\}_{i=1,\ldots,L}$.
2) Obtain $\mathbb{B}_i$, the set of search steps along axis $\mathbf{v}_i$, $i = 1, \ldots, L$.
3) Obtain Ψ, the set of all search points, $\Psi = \{q(\hat{\mathbf{x}}_{\text{ZF}} + \sum_{i=1}^{L} b_i \mathbf{v}_i) | \forall b_i \in \mathbb{B}_i, i = 1, \ldots, L\}$.
4) Output the solution $\tilde{\mathbf{x}}_{\text{ZF-ES}} = \arg\min_{\mathbf{x}\in\Psi} ||\mathbf{y} - \mathbf{H}\mathbf{x}||^2$.

set of search steps along axis $\mathbf{v}_i$ is determined as

$$\mathbb{B}_i = \{0, \pm d/2, \pm d, \ldots, \pm \alpha_i d/2\}, \tag{11}$$

where $\alpha_i = \lceil (3\sigma_n/\sigma_i)/(d/2) \rceil$, $\lceil\cdot\rceil$ being the ceiling function. In other words, search along axis $\mathbf{v}_i$ covers the range $[-3\sigma_n/\sigma_i, 3\sigma_n/\sigma_i]$, where most of the noise power is contained.

The ES method works in conjunction with standard linear equalization-based detectors and its procedure is presented in Table I as Algorithm ZF-ES (i.e., a ZF detector is employed). First, an initial solution is obtained from the linear detector (Step 1). Then, the set of search steps and search points are obtained (Steps 2 and 3, respectively). Lastly, the squared Euclidean distance between these points and $\mathbf{y}$ is evaluated and the one that yields the smallest value is designated as the detected symbol (Step 4). Note that if $\hat{\mathbf{x}}_{\text{ZF}}$ in Algorithm III.1 is replaced by the MMSE equalized symbol vector $\hat{\mathbf{x}}_{\text{MMSE}}$, the algorithm becomes MMSE-ES and the detected symbol is denoted by $\tilde{\mathbf{x}}_{\text{MMSE-ES}}$.

B. Number of Search Points

As shown in Steps 3 and 4 of Algorithm III.1, the complexity of the ES method depends on the total number of search points $|\Psi| = |\mathbb{B}_1| \times |\mathbb{B}_2| \times \cdots \times |\mathbb{B}_L|$, where $|\mathbb{B}_1| \geq |\mathbb{B}_2| \geq \cdots \geq |\mathbb{B}_L|$. To get an idea of the dominant contributor to the complexity, we examine the properties of $|\mathbb{B}_1|$. Note that $|\mathbb{B}_1|$ is a random variable that depends on σ_n, σ_1, and d. Our interest is to know the expected number of search points, $\text{E}[|\mathbb{B}_1|]$, given a particular system setting of MIMO channel, modulation, and signal-to-noise ratio (SNR). The derivation for $\text{E}[|\mathbb{B}_1|]$ will form an upper bound for search complexity in other dimensions (if $L > 1$). We will see in Sec. V, however, that usually $L = 1$ is sufficient for the sake of performance.

First, from (11) and using the property of the ceiling function, we can directly obtain that

$$|\mathbb{B}_1| < 12 \cdot \left(\frac{\sigma_n}{d}\right) \cdot \delta_1 + 3, \tag{12}$$

where $\delta_1 = 1/\sigma_1$. Note that σ_n and d are fully determined by the system setting; δ_1, on the other hand, is a random variable. To analyze δ_1, the following result is found useful.

Theorem 1 [11]: Let $\tilde{\mathbf{W}}(m,m)$ be a Hermitian $m \times m$ random matrix $\mathbf{A}\mathbf{A}^H$, where $\mathbf{A}$ is an $m \times m$ random matrix with i.i.d. elements whose real and imaginary parts are i.i.d. $\mathcal{N}(0,1)$. Then, the probability density function (pdf) of the smallest eigenvalue of $\tilde{\mathbf{W}}(m,m)$, λ_{min}, is $f_{\lambda_{min}}(x) = (m/2)e^{-(m/2)x}$; i.e., it is exponentially distributed with parameter $m/2$.

In our system model, $\mathbf{H}^H\mathbf{H}$ is a $1/(2M_T)\tilde{\mathbf{W}}(M_T, M_T)$ matrix. Thus, σ_1^2, the square of the smallest singular value of $\mathbf{H}$ and the smallest eigenvalue of $\mathbf{H}^H\mathbf{H}$, is exponentially distributed with pdf given by $f_{\sigma_1^2}(x) = M_T^2 e^{-M_T^2 x}, x \geq 0$. It follows that σ_1 is Rayleigh distributed with pdf $f_{\sigma_1}(x) = 2M_T^2 x e^{-M_T^2 x^2}, x \geq 0$. Then, the pdf of $\delta_1 = 1/\sigma_1$ is derived by using the standard change-of-variable procedure, as $f_{\delta_1}(x) = \frac{2M_T^2}{x^3} e^{-\frac{M_T^2}{x^2}}, x > 0$. After some algebraic manipulations it can be shown that $\text{E}[\delta_1] = M_T\sqrt{\pi}$.

To complete the analysis for $\text{E}[|\mathbb{B}_1|]$, we need to determine σ_n/d in (12) for different system settings. First note that the SNR, ϑ, is defined as

$$\begin{aligned} \vartheta &\triangleq \frac{\text{E}[||\mathbf{H}\tilde{\mathbf{x}}||^2]}{\text{E}[||\mathbf{n}||^2]} \\ &= \frac{\text{E}[\tilde{\mathbf{x}}^H\mathbf{H}^H\mathbf{H}\tilde{\mathbf{x}}]}{\text{E}[\mathbf{n}^H\mathbf{n}]} = \frac{M_R\sigma_H^2 M_T \text{E}[\tilde{\mathbf{x}}_i^2]}{M_R\sigma_n^2} = \frac{M_T\sigma_x^2}{\sigma_n^2}, \end{aligned} \tag{13}$$

where $\tilde{\mathbf{x}}_i$ is the ith element of $\tilde{\mathbf{x}}$. By normalizing the energy per bit of the transmitted symbol to one, σ_n/d can be represented in terms of M_T and ϑ for different modulations. Taking expectation on both sides of (12) and substituting $\text{E}[\delta_1] = M_T\sqrt{\pi}$ into it yields

$$\text{E}[|\mathbb{B}_1|] < \sqrt{\beta\frac{\pi M_T^3}{\vartheta}} + 3, \tag{14}$$

where $\beta = 144 \cdot \sigma_x^2/d^2$, which is equal to 360 and 1512 for 16-QAM and 64-QAM modulation, respectively. Figure 1 shows the simulation results for $\text{E}[|\mathbb{B}_1|]$ alongside the analytical upper bound derived in (14) for two system settings. It is seen that the analysis approximates the simulation results well. In the following section, this analytical result will be used to complete the computational complexity analysis for all detection schemes of interest.

IV. COMPLEXITY ANALYSIS FOR MIMO DETECTION SCHEMES

We evaluate the computational complexity in terms of the number of complex operations for each detection method. For the simplicity of presentation, in all the following discussion it is assumed that $M_T = M_R$. First, the computational complexity of calculating $||\mathbf{y} - \mathbf{H}\mathbf{x}||^2$ is determined to be $C_0 = 2M_T^2 + 2M_T - 1$. Then, based on this result, the computational complexity for all detection schemes is evaluated and presented as follows.

ML detection: Since ML detection searches the entire symbol alphabet to find the one that minimizes $||\mathbf{y} - \mathbf{H}\mathbf{x}||^2$, its complexity is $C_{\text{ML}} = |\mathbb{S}|^{M_T} C_0$.

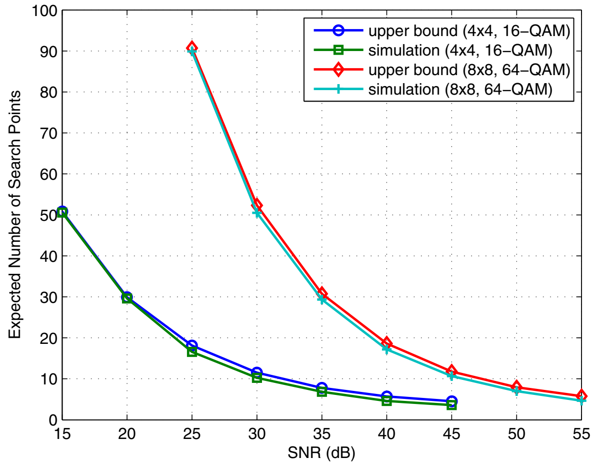

Fig. 1. Expected number of search points along the major axis $\mathbf{v}_1$ vs. SNR for a 4×4 system with 16-QAM modulation and an 8×8 system with 64-QAM modulation.

ZF detection: By using the Gauss-Jordan Elimination algorithm to compute the matrix inversion in (4), the complexity of ZF detection is $C_{\text{ZF}} = (14/3)M_T^3 + 5M_T^2 - (8/3)M_T$.

MMSE detection: Similar to the computation for ZF detection, the complexity of MMSE detection is $C_{\text{MMSE}} = (14/3)M_T^3 + 5M_T^2 - (2/3)M_T + 1$.

ZF-ES detection: In general, the computational complexity of ZF-ES detection is the sum of C_{ZF}, the complexity of calculating the SVD of $\mathbf{H}$, and the complexity involved in the eigenspace search. The complexity of using a popular SVD algorithm, Golub-Reinsch SVD, to get $\mathbf{\Sigma}$ and $\mathbf{V}$ in (8) is $12M_T^3$ [9]. Note that, however, for the particular case of $L = 1$, the power method [12] can be used to compute the dominant (largest) eigenvalue and eigenvector of $(\mathbf{H}^H\mathbf{H})^{-1}$ in (9) with much lower complexity. The complexity of the power method is given by $\mathcal{O}(4kM_T^2 + 3kM_T)$ if k iterations are taken. Therefore, the complexity of ZF-ES detection with $L = 1$ can be approximated by

$$C_{\text{ZF-ES}(L\,=\,1)} = C_{\text{ZF}} + (4M_T^2 + 3M_T) + \text{E}[|\mathbb{B}_1|]C_0,$$

where the last term approximates the complexity involved in the eigenspace search.

For $L = 2$ or above, the SVD of $\mathbf{H}$ needs to be performed, and the constellation search is conducted in L dimensions. By approximating the complexity of searching in the second dimension by the first (i.e., approximating $|\mathbb{B}_1| \times |\mathbb{B}_2|$ by $|\mathbb{B}_1|^2$, which is an upper bound), the approximate complexity of ZF-ES detection with $L = 2$ is given by

$$C_{\text{ZF-ES}(L\,=\,2)} = C_{\text{ZF}} + (12M_T^3) + \text{E}[|\mathbb{B}_1|]^2C_0.$$

MMSE-ES detection: Similar to ZF-ES detection with $L = 1$, the power method can also be applied on $(\mathbf{H}^H\mathbf{H} + (\sigma_n^2/\sigma_x^2)\mathbf{I}_{M_T})^{-1}$ in (7) to obtain the dominant eigenvalue and eigenvector that are needed. Therefore, the complexity of MMSE-ES detection is given by

$$C_{\text{MMSE-ES}(L\,=\,1)} = C_{\text{MMSE}} + (4M_T^2 + 3M_T) + \text{E}[|\mathbb{B}_1|]C_0,$$
$$C_{\text{MMSE-ES}(L\,=\,2)} = C_{\text{MMSE}} + (12M_T^3) + \text{E}[|\mathbb{B}_1|]^2C_0.$$

TABLE II
COMPLEXITY IN COMPLEX OPERATIONS FOR VARIOUS MIMO DETECTION SCHEMES (UNIT: 10^3)

MIMO System Modulation SNR (dB)	4×4 16-QAM 25	4×4 16-QAM 45	8×8 64-QAM 35	8×8 64-QAM 55
ML	2555.9	2555.9	4.03×10^{13}	4.03×10^{13}
ZF	0.368	0.368	2.688	2.688
MMSE	0.377	0.377	2.705	2.705
ZF-ES ($L = 1$)	1.151	0.620	7.363	3.794
ZF-ES ($L = 2$)	13.954	1.930	143.890	13.598
MMSE-ES ($L = 1$)	1.160	0.629	7.380	3.811
MMSE-ES ($L = 2$)	13.963	1.939	143.910	13.615

Table II summarizes the complexity for two different system settings, at two SNR values each. It is seen that the proposed ES method incurs diminishing additional cost as SNR increases. At SNR $= 45$ dB for a 4×4 MIMO system with 16-QAM modulation, both ZF-ES ($L = 1$) and MMSE-ES ($L = 1$) pose a complexity less than two times of that of pure ZF and MMSE. In addition, since the ES method does not involve search that depends on the type of modulation, it presents efficiency in high-order modulation settings compared to other constellation-search methods. The complexity incurred by an additional axis search in ES with $L = 2$ is high at low SNR and moderate at high SNR. The actual complexity, however, may be significantly lower than the figures presented in Table II, as the second-smallest singular value of $\mathbf{H}$ may be much greater than the smallest singular value. Further, as will be seen in Sec. V, ES with $L = 2$ does not pose remarkable performance advantages as compared to $L = 1$, suggesting that ES with $L = 1$ is generally a sufficient and efficient augmentation to ZF and MMSE schemes.

V. SIMULATION RESULTS

In this section, we study the performance of the proposed schemes by computer simulation. Figures 2 and 3 show the SER performance for a 4×4 system with 16-QAM modulation and an 8×8 system with 64-QAM modulation, respectively. Consistent observations can be drawn from these two figures. First, among the schemes that involve the ES procedure, it is seen that the advantage of using $L = 2$ as opposed to $L = 1$ is slight. Secondly, MMSE-ES ($L = 1$) approaches the best performance among ES schemes, presenting a good performance and complexity tradeoff. Specifically, at SER $= 10^{-1}$, MMSE-ES ($L = 1$) poses a 4 dB gain over MMSE in Fig. 2 and a 5 dB gain over MMSE in Fig. 3. Thirdly, all suboptimal schemes exhibit a significant performance gap as compared to the ML scheme. This is because no exhaustive constellation search is conducted but instead only partial constellation search along specific directions determined by the channel information is performed in the ES method. The complexity reduction, being $\mathcal{O}(M_T^3)$ vs. $\mathcal{O}(|\mathbb{S}|^{M_T})$ as discussed previously, is the

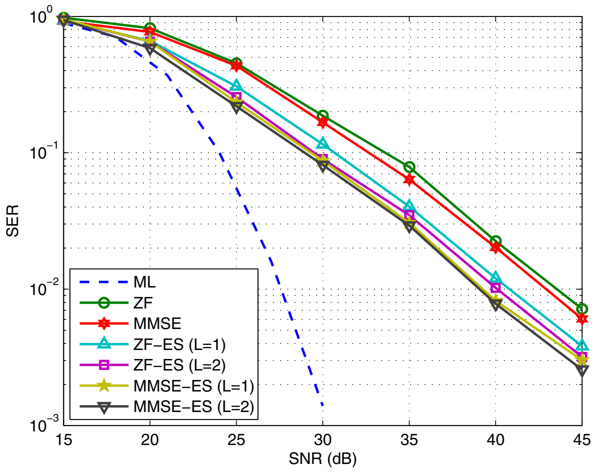

Fig. 2. SER performance of MIMO detection schemes for a 4×4 MIMO system with 16-QAM modulation.

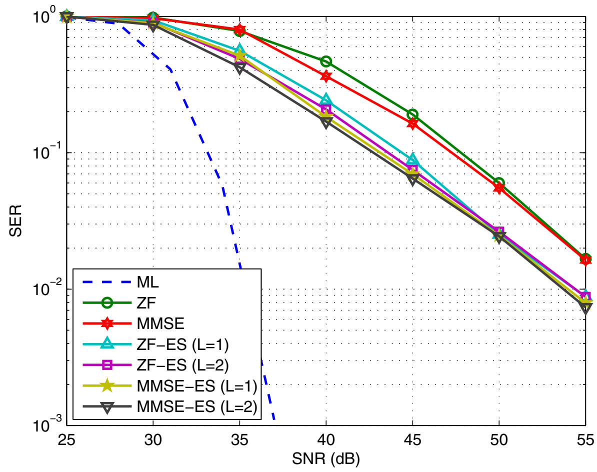

Fig. 3. SER performance of MIMO detection schemes for an 8×8 MIMO system with 64-QAM modulation.

advantage gained by paying the price of such performance degradation.

VI. Conclusion

A new MIMO detection scheme has been proposed in this paper. The proposed method consists of two phases: standard linear equalization-based detection and eigenspace search. An efficient constellation-search procedure has been presented, which is accomplished by an informed search within a small range of the ZF or MMSE equalized symbol and in specific directions along which the most pronounced noise disruption may have been imposed on the transmitted signal. The proposed method can deliver substantial SER performance improvement over conventional linear detectors, as confirmed by computer simulation. Due to its practicality and low complexity, the proposed scheme can be applied in next-generation cellular systems such as LTE-A and IEEE 802.16m advanced WiMAX.

References

[1] Long Term Evolution Advanced (LTE-A). [Online]. Available: http://www.3gpp.org/article/lte-advanced

[2] IEEE 802.16 Task Group m. [Online]. Available: http://ieee802.org/16/tgm/

[3] Z. Guo and P. Nilsson, "Algorithm and implementation of the K-Best sphere decoding for MIMO detection," *IEEE J. Select. Areas Commun.*, vol. 24, pp. 491–503, Mar. 2006.

[4] L. G. Barbero and J. S. Thompson, "Fixing the complexity of the sphere decoder for MIMO detection," *IEEE Trans. Wireless Commun.*, vol. 7, pp. 2131–2142, June 2008.

[5] M. O. Damen, H. El Gamal, and G. Caire, "On maximum-likelihood detection and the search for the closest lattice point," *IEEE Trans. Inform. Theory*, vol. 49, pp. 2389–2402, Oct. 2003.

[6] T.-H. Liu and Y.-L. Y. Liu, "Modified fast recursive algorithm for efficient MMSE-SIC detection of the V-BLAST system," *IEEE Trans. Wireless Commun.*, vol. 7, pp. 3713–3717, Oct. 2008.

[7] R. Böhnke, D. Wübben, V. Kühn, and K.-D. Kammeyer, "Reduced complexity MMSE detection for BLAST architectures," in *Proc. IEEE GLOBECOM '03*, Dec. 2003, pp. 2258–2262.

[8] H. Artés, D. Seethaler, and F. Hlawatsch, "Efficient detection algorithms for MIMO channels: A geometrical approach to approximate ML detection," *IEEE Trans. Signal Processing*, vol. 51, pp. 2808–2820, Nov. 2003.

[9] G. H. Golub and C. F. Van Loan, *Matrix Computations*, 3rd ed. Baltimore: Johns Hopkins University Press, 1996.

[10] S. M. Kay, *Fundamentals of statistical signal processing: estimation theory*. Englewood Cliffs, NJ: Prentice-Hall, 1993.

[11] A. Edelman, "Eigenvalues and condition numbers of random matrices," *Ph.D. Thesis*, Massachusetts Institute of Technology, 1989.

[12] I. Dimov and A. Karaivanova, "A power method with Monte Carlo iterations," in *Recent Advances in Numerical Methods and Applications, World Scientific*, Singapore, 1999, pp. 239–247.