Clustering algorithm based on minimal path loss ratio for vehicular communication

Y Harikrishnan, *Samsung, India*[1]
Jianhua He, *School of Engineering and Applied Science, Aston University, UK*

***Abstract*—Emerging vehicular comfort applications pose a host of completely new set of requirements such as maintaining end-to-end connectivity, packet routing, and reliable communication for internet access while on the move. One of the biggest challenges is to provide good quality of service (QoS) such as low packet delay while coping with the fast topological changes. In this paper, we propose a clustering algorithm based on minimal path loss ratio (MPLR) which should help in spectrum efficiency and reduce data congestion in the network. The vehicular nodes which experience minimal path loss are selected as the cluster heads. The performance of the MPLR clustering algorithm is calculated by rate of change of cluster heads, average number of clusters and average cluster size. Vehicular traffic models derived from the Traffic Wales data are fed as input to the motorway simulator. A mathematical analysis for the rate of change of cluster head is derived which validates the MPLR algorithm and is compared with the simulated results. The mathematical and simulated results are in good agreement indicating the stability of the algorithm and the accuracy of the simulator. The MPLR system is also compared with V2R system with MPLR system performing better.**

I. INTRODUCTION

The rapid growth in technologies and the need for ubiquitous connection have steered the Intelligent Transport Systems (ITS) in the direction of comfort applications such as data transfer and downloading, file sharing, and infotainment services apart from road safety applications [1]. There are organisations such as Car 2 Car Consortium [2] aiming to standardise the optimal communication between vehicles and projects such as COMeSafety2 – "Communication for e-Safety" [3] , DriveC2X – "Drive Car-2-X" [4], shows some of the current research activity undertaken in vehicular communication.

One of the biggest challenges in vehicular networks is maintaining the robustness of the links while handling the topology changes. In a highly mobile network, the communication links become inconsistent not suiting the standard protocol requirements [5]. Due to challenging conditions in VANET environment, a good clustering algorithm is required to reduce the fast reconfigurable condition of the dynamic network ensuring a better performance of the MAC protocols [6, 7]. A hierarchical clustering architecture is used in this work, as it is an efficient method to optimise communication and improve spectrum efficiency. A cluster is formed by a group of nodes which are in communication range of each other and a cluster head is selected from this group based on different constraints.

The remainder of the paper is organised as follows. Section 2 discusses the related work. Section 3 gives description of the proposed system setup. Section 4 explains the proposed clustering algorithm to reduce the cluster head changes based on minimal path loss ratio (MPLR) and the performance metrics for the algorithm. A concise summary of mathematical modelling of the clustering algorithm is discussed in Section 5. Section 6 describes the vehicular arrivals modelling for motorway simulator. The simulator is used to simulate the vehicular movements. Section 7 compares the mathematical analysis of the proposed clustering algorithm with the simulated results for rate of change of cluster head metric. Section 8 compares the performance of the MPLR system with a pure V2R system and finally Section 9 gives the conclusion.

II. RELATED WORK

This section introduces about various clustering techniques used currently. There are two popular clustering algorithms - lowest id cluster algorithm (LIC) and highest connectivity cluster algorithm.

In LIC [8], the network is arranged into a group of nodes called cluster where each node belongs to at least one cluster. Each node in the cluster is assigned a distinct ID and the node with the lowest ID is selected as the cluster head (CH). There are two main drawbacks of the LIC algorithm. One is the node ids are arbitrarily assigned numbers without considering any other qualifications of a node for election as a cluster head. And the second drawback is nodes are susceptible to power drain due to serving of cluster head for longer periods of time.

The highest connectivity (degree) algorithm [7] is based on the number of neighbour nodes with which it can communicate and the node with the maximum number of neighbours (connectivity) becomes the CH. A node is made a CH if it has the most number of neighbour nodes who doesn't have a CH. In case if there are two nodes with the highest number of connectivity, the lowest ID prevails. The disadvantages with this algorithm are creation of unstable clusters and no upper bound restriction on the number of nodes in the cluster thereby resulting in low throughput.

III. SYSTEM SETUP

Our proposed system scenario is based on a motorway with 3 lanes unidirectional. The lanes typically have three different speeds 60mph, 70mph and 80mph. The arrival rates of the vehicles are obtained from the Traffic Wales data of Swansea-London M4 motorway. Base stations are installed every 4 km typically in the motorway. The communication architecture

[1] This work was done at Swansea University as part of the author's PhD candidature.

considered for this system setup is a hybrid setup and has two tiers of communication network.

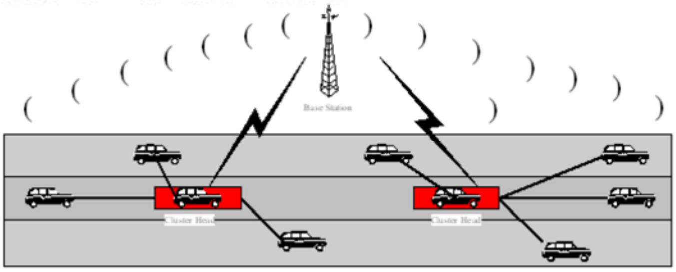

Fig.1. Proposed system setup

The lower tier is vehicle to vehicle (V2V) [9] communication and the upper tier is vehicle to base station/roadside (V2R) [10] communication. The vehicles within the transmission range of each other form a network group called cluster and select a cluster head based on the path loss parameter between the vehicle and the base station. The vehicles in the cluster communicate with each other using the 802.11p [11] and any communication to the base station is routed via the cluster head. The cluster head changes are dependent on the speed of the vehicles in the motorway.

The upper tier is used for the communication between cluster head and the base station. It is assumed that the base stations employ LTE [20] technology which can provide large service area coverage and provide higher data rates. Data is communicated to the cluster head/base station using a transceiver present in each vehicle.

IV. Clustering Algorithm Based on Minimal Path Loss Ratio (MPLR)

A new clustering algorithm is proposed based on the physical constraints - a) path loss which is characterised by the distance between the vehicle and base station and b) interference between vehicles due to overlapping coverage. A good cluster head should have strong signal to the base station, minimal path loss and should take in consideration the interference from the surrounding vehicles. The path loss is defined by the distance between the vehicle and the roadside infrastructure. Hence a larger distance of separation between the vehicle and roadside infrastructure has a higher path loss which implies a drop in the signal strength. A vehicle with minimal path loss is a good candidate for the cluster head. The other physical constraints will be part of the future work.

The clustering algorithm has the following procedures:-

1. Node entering the motorway looks for any existing cluster heads within its transmission range. If there is a cluster head already present, the new node checks its distance from the base station and compares it with the distance between the existing cluster head and base station. Based on the shortest distance to the station the cluster head is selected. This is a standard procedure for selecting a cluster head.
2. When cluster head moves out of range of cluster, it performs the standard procedure for selecting the cluster head. If there are no other cluster heads nearby, it becomes it own cluster head.
3. When a cluster member is moving out of range of the cluster, it tries to look for a cluster head within its transmission range and if unable to find a cluster head nearby, it becomes its own cluster head.
4. When two clusters come in the proximity range of each other, a new cluster head is selected based on shortest distance between the node and the base station.
5. Single cluster formation is influenced in 3 cases; first case, when the vehicle is isolated from the rest of the vehicles with no vehicles within the transmission range, second case, the nearest cluster will not be able to service more requests and in the third case, the next to nearest cluster doesn't have good signal quality.

The performance of the algorithm is measured by a main metric – rate of change of cluster head. The rate of change of cluster head metric indicates the reconfiguration speed of the network. This will assess the robustness of the algorithm. A mathematical analysis is derived for the rate of change of cluster heads. Additionally there are two other minor metrics – a) average cluster size, which is useful in determining the average number of requests that can emerge from a cluster and b) average number of clusters, which will help in the efficient bandwidth allocation and communication relay of both cluster head –base station and intra cluster communication

V. Mathematical Analysis for Rate of Change of Cluster Heads for MPLR Algorithm

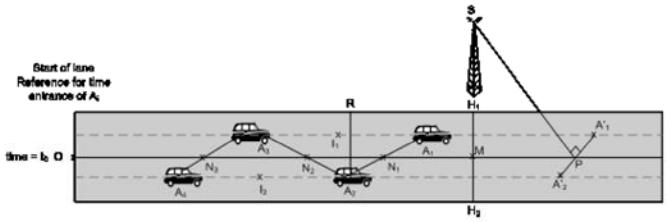

Fig.2. Depiction of change of cluster heads

Fig.2 demonstrates a setup where all the vehicles A_1, A_2, A_3 and A_4 in a cluster , are travelling in two lanes. A_1 and A_3 travel in one lane with A_3 following A_1. A_2 and A_4 travel in the second lane with A_4 following A_2. The base station or the roadside infrastructure (SH) is positioned by the side of the lane. The point O denotes of the start of the motorway lanes which is used as a reference for the time of entrance for the vehicles. The vehicle closest to the base station becomes the cluster head. In the first instance, A_1 is the cluster head.

The table 1 gives the notations used in the fig.2 and in the following derivations.

A_1	Cluster head
A_2, A_3	Immediate cluster members
I_1	Middle point of the segment A_1A_3 (imaginary point and travelling at the same speed as A_1& A_3)
SH_1	Distance of the base station to the lane
H_1H_2	Lane width (1.75m)
N_i	Mid point of A_{2i+1} and $A_{2(i+1)}$ and is moving.
P_i	Imaginary static point
M	Mid point of H_1H_2

Table 1 Notations for change of cluster head for motorway

For representation, all odd numbered vehicles are denoted by A_{2i+1} and even numbered vehicles $A_{2(i+1)}$.

The distance $A_{2i+1}A_{2(i+1)}$ will vary when the speeds vary which implies I_i speed and N_i speed is variable.
N_i is the mid-point formed between the vehicles in different lanes (e.g. N_1 is the imaginary mid point between A_1 and A_2). There are 2 different scenarios for vehicular placements – a) vehicles are moving at the same speed in both lanes; b) vehicles are moving at different speeds with respect to each other while maintaining a constant average speed.

Scenario A – Vehicles are moving at the same speed

The rule for cluster head change from A_i to A_{i+1} is when I_i crosses H. The Δt_1 and Δt_2 are given as:

$$\Delta t_1 = \frac{I_1 I_2}{v};\ \Delta t_2 = \frac{I_2 I_3}{v}$$

Where v is the speed of the vehicle and
$I_i I_{i+1}$ is the distance between the two middle points.
Therefore average time period for cluster change can be generalised as:

$$\langle \Delta t \rangle = \frac{\sum_{i=1}^{n} \frac{I_i I_{i+1}}{v}}{n-1} \quad (1)$$

Where n is the number of participating vehicles.

The average rate of change of cluster head (ΔCH) = $\frac{1}{\langle \Delta t \rangle}$

$$\Delta CH = \frac{n-1}{\sum_{i=1}^{n} \frac{I_i I_{i+1}}{v}} \quad (2)$$

Scenario B – Vehicles moving at different speeds with respect to each other but moving at a constant average speed.

The cluster head will change from A_{2i+1} to $A_{2(i+1)}$ after the point N_i coincides with point P_i where $P_iA'_{2i+1} = P_iA'_{2(i+1)}$. SP_i is orthogonal to $A'_{2i+1}A'_{2(i+1)}$, where A'_{2i+1} and $A'_{2(i+1)}$ are the expected positions of A_{2i+1} and $A_{2(i+1)}$ at Δt_0. SP_i intersects MN_i.

The distance MP_i is given as: $MP_i = \frac{SM}{\tan\left(a\cos\left(\frac{H_1H_2}{A_{2n+1}A_{2(n+1)}}\right)\right)}$ (3)

At time t_1, the cluster head changes from A_1 to A_2 when N_1 crosses the static point P_1 and at time $t_2 = t_1 + \Delta t_1$, the cluster head changes from A_2 to A_3. The generalisation of the cluster head changes with respect to time could be represented as:

$$n \in N : \Delta t_{2n} = \frac{N_{2n+1}P_{2n+1}(t_{2n})}{v(N_{2n+1})} \quad (4)$$

$$n \in N : \Delta t_{2n+1} = \frac{N_{2(n+1)}P_{2(n+1)}(t_{2n+1})}{v(N_{2(n+1)})} \quad (5)$$

Where N_iP_i gives the distance between the mid-point N_i and the imaginary static point P_i
$v(N_i)$ gives the speed of the N_i and is the average of speed of vehicles A_{2i+1} and $A_{2(i+1)}$

In lane 1, A_i (odd numbered vehicle) is entering the lane, i.e., A_{2n+1} entering at time T_{2n+1} for all $n \in N$. In lane 2, A_i (even numbered vehicle) is entering the lane, i.e., $A_{2(n+1)}$ entering at time $T_{2(n+1)}$ for all $n \in N$.

- ON_{2n+1} with N_{2n+1} middle of $[A_{2n+1}A_{2(n+1)}]$ is formed at time $T_{2(n+1)}$ i.e., when vehicle $A_{2(n+1)}$ enters the motorway.
- $ON_{2(n+1)}$ with $N_{2(n+1)}$ middle of $[A_{2(n+1)} A_{2(n+1)+1}]$ is formed at time $T_{2(n+1)+1}$ i.e., when vehicle $A_{2(n+1)+1}$ enters the motorway

The distance between the start of the motorway O to the mid point N_i at the time when a new vehicle enters the motorway is given by:

For vehicles in lane 1 (odd numbered vehicles),

$$ON_{2n+1}(T_{2(n+1)}) = \frac{OA_{2n+1}(T_{2(n+1)})}{2} = \frac{v(A_{2n+1})*(T_{2(n+1)} - T_{2n+1})}{2} \quad (6)$$

For vehicles in lane 2 (even numbered vehicles),

$$ON_{2(n+1)}(T_{2(n+1)+1}) = \frac{OA_{2(n+1)}(T_{2(n+1)+1})}{2} = \frac{v(A_{2(n+1)})*(T_{2(n+1)+1} - T_{2(n+1)})}{2} \quad (7)$$

A change in cluster head will take place at time t_{2n} as given by equation (4) and the distance $N_{2n+1}P_{2n+1}$ at time t_{2n} is given by:

$$N_{2n+1}P_{2n+1}(t_{2n}) = OP_{2n+1} - ON_{2n+1} = OM - ON_{2n+1}(t_{2n}) + MP_{2n+1} \quad (8)$$

Where $ON_{2n+1}(t_{2n}) = v(N_{2n+1})*(t_{2n} - T_{2(n+1)}) + ON_{2n+1}(T_{2(n+1)})$

$$MP_{2n+1} = \frac{SM}{\tan\left[a\cos\left(\frac{H_1H_2}{A_{2n+1}A_{2(n+1)}}\right)\right]}$$

$$A_{2n+1}A_{2(n+1)} = \sqrt{(OA_{2n+1}(t_i) - OA_{2(n+1)}(t_i))^2 + (H_1H_2)^2}$$

$$OA_i(t_i) = v(A_i) * t_i$$

A change in cluster head will take place at time t_{2n+1} as given by equation (5) and the distance $N_{2(n+1)}P_{2(n+1)}$ at time t_{2n+1} is:

$$N_{2(n+1)}P_{2(n+1)}(t_{2n+} = OM - ON_{2(n+1)}(t_{2n+1}) + MP_{2(n+1)} \quad (9)$$

Where $ON_{2(n+1)}(t_{2n+1}) = v(N_{2(n+1)})*(t_{2n+1} - T_{2(n+1)+1}) + ON_{2(n+1)}(T_{2(n+1)+1})$,

$$MP_{2(n+1)} = \frac{SM}{\tan\left[a\cos\left(\frac{H_1H_2}{A_{2(n+1)}A_{2(n+1)+1}}\right)\right]}$$

$$A_{2(n+1)}A_{2(n+1)+1} = \sqrt{(OA_{2(n+1)}(t_i) - OA_{2(n+1)+1}(t_i))^2 + (H_1H_2)^2}$$

$$OA_i(t_i) = v(A_i) * t_i$$

Speed of mid point N_i is given as

$$v(N_i) = \frac{v(A_{2i+1}) + v(A_{2i})}{2} \quad (10)$$

Hence $v(N_{2n+1})$ and $v(N_{2(n+1)})$ is given by

$$\therefore\ v(N_{2n+1}) = \frac{v(A_{2n+1}) + v(A_{2(n+1)})}{2},\ v(N_{2(n+1)}) = \frac{v(A_{2(n+1)}) + v(A_{2(n+1)+1})}{2}$$

Average rate of change of cluster head at time t_{2n} is given by:

$$\Delta CH = \frac{v(A_{2n+1}) + v(A_{2(n+1)})}{2(OM - v(N_{2n+1})*(t_{2n} - T_{2(n+1)}) + ON_{2n+1}(T_{2(n+1)}) + MP_{2n+1})} \quad (11)$$

Average rate of change of cluster head at time t_{2n+1}:

$$\Delta CH = \frac{v(A_{2(n+1)}) + v(A_{2(n+1)+1})}{2(OM - v(N_{2(n+1)})*(t_{2n} + 1 - T_{2(n+1)} + 1) + ON_{2(n+1)}(T_{2(n+1)+1}) + MP_{2(n+1)})} \quad (12)$$

The above expressions for cluster head changes can be used for different vehicular placement scenarios based on speed. For different speeds, there is change in the cluster head

- On I_i crossing H_i
- N_i coincides with P_i
- The speed N_i and I_i governs the rate of change of cluster heads.

The above equations could be used for rate of change of cluster heads for both one and three lanes scenarios.

VI. Vehicular Arrival Model and Motorway Simulator

The vehicular arrival model is derived from the Traffic Wales data and is fed into the simulator. The motorway traffic simulator uses this model as an input and simulates vehicular movements based on time-discrete microscopic traffic model [13]. The clustering algorithm is implemented on top of this simulator. Traffic Wales, responsible for traffic management in Wales UK have provided traffic profiles on the M4 motorway for the past 5 years to help in research for implementing an accurate vehicular traffic model. The data set comprises of data obtained from the inductive loops in M4 between Swansea and London. There are 585 inductive loops in the stretch each spaced 500m apart. A statistical analysis was done on the M4 data to derive an accurate vehicular arrival model.

Fig.3 shows the average vehicular arrival rate for every 5 minute interval in the motorway obtained from the M4 data. A 2 peak Gaussian distribution for multiple peaks caused by the peak hours in the morning and evening is used to fit the data. The chi square test (X^2) yields a value of 0.064759. A low value of X^2 indicates the level of independence. This is used as an input to the vehicular simulator for simulating realistic vehicular traffic. There appears to be a good agreement between the real traffic flow data and the fitted data.

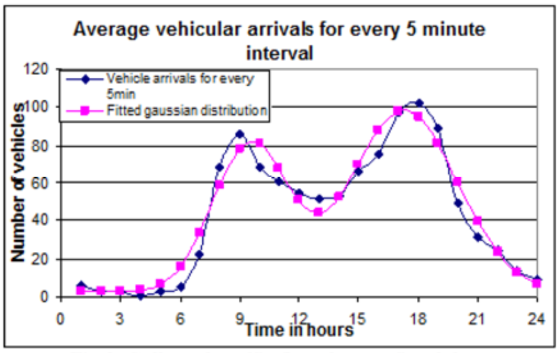

Fig.3. Collected traffic flow data vs fitted data

For design of the motorway simulator, the time-discrete microscopic model is used [14]. In the simulation, each lane in the motorway is divided into boxes of 2m size as shown in fig.4. According to official Highway Code, an average length of car is 4m and a box size 2 is an easier representation of all vehicles and help in the discrete movement of the vehicles based on the speed.

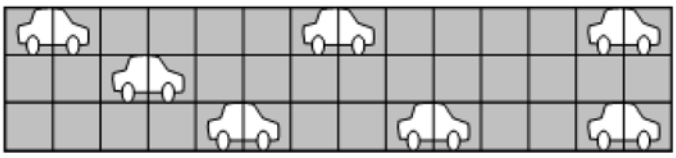

Fig.4. Depiction of lanes and vehicle placements

VII. Performance Metrics for the Proposed MPLR Clustering Algorithm

The rate of change of cluster head metric is compared to the analytical results derived from the mathematical model. Simulated results for the other two performance metrics - average number of clusters and average cluster size are also given in the following sections

A. Comparison of simulated and analytical results for rate of change of cluster heads

The rate of change of cluster heads are measured for each of 24 consecutive hours using the motorway simulator for all the 3 lanes and is compared with the mathematical analysis for the rate of change of cluster heads.

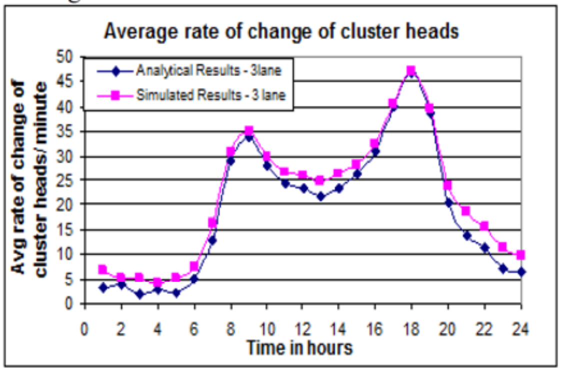

Fig.5. Comparison of simulated and analytical results for rate of change of cluster heads

Fig. 5 shows that there is a good agreement existing between the analytical and simulated model, thereby validating the soundness of the analytical model. As can be seen from the figure, the cluster head changes are higher during the peak hours due to the increased number of vehicles during those hours.

B. Average cluster size & Average number of clusters

The average cluster size helps in determining the average number of service requests that can emerge from the cluster. Average number of clusters will help in efficient bandwidth allocation and anticipate the relay of communication. Fig. 6 represents both the metrics for different hours of the day. It could be seen in both the metrics for the peak hours, number of clusters and cluster size reach the maximum and during the off peak hours, it gradually reduces.

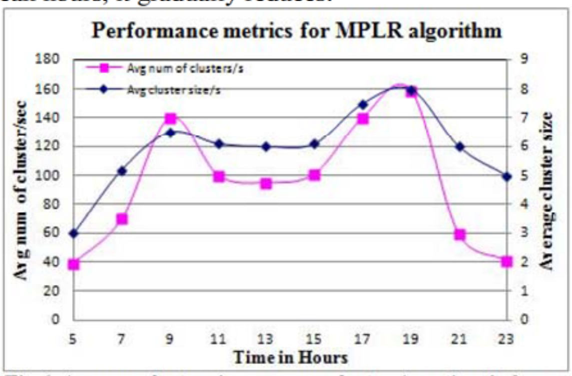

Fig.6. Average cluster size, average clusters/s vs time in hours

VIII. Comparison of the Proposed MPLR System with a Vehicle to Roadside (V2R) Communication System

In a V2R system, each vehicle can communicate to the base station with its long-range radio.In these kinds of networks, each vehicle can communicate to the base station directly. There is no hierarchy in the networks for coordinating on sending messages to the base stations. In comparison, by clustering, the cluster head acts as a controller for vehicles within its cluster and forward the messages to and from the base station to the vehicles in the cluster. Clustering can be an efficient method to reduce bandwidth consumption and network congestion for large scale networks at no extra cost for infrastructure. It is important that the limited radio spectrum resources should be used as economically as possible. The simulation for comparison of MPLR and V2R systems considers a fixed size data file is transmitted from base station to all the vehicles in the motorway. This file could contain information ranging from safety messages to infotainment messages. The data is sent out to all the vehicles and resources are allocated based on the round robin scheduling in the downlink scheduler in the base station. The spectrum efficiency of MPLR network is compared to the pure V2R system in terms of the end to end delay and packet dropping probability. The packet arrival rate follows a Poisson distribution

Parameters	Values
Channel bit rate	50 Mbps
Data bit rate	500 Kbps
Packet Size	4800 bits

Table 2 Simulation Parameters

Average Access Delay is the delay that is experienced by a packet while waiting in the queue at the base station. Considered to be one of the most important parameters in determining the performance of the system, PDP (%) is a percentage ratio of the number of packets dropped to the total number of packets.

On observing the results in Fig. 7 it can be seen that there is little variation in the access delay for lower number of vehicles, around 2 ms for both V2R and the MPLR systems. As the number of vehicles increases the delay for a V2R system reaches around the margin for 55ms compared to 18ms delay for the MPLR system. Also for a MPLR system, when the number of vehicles is less than 20, the delay is in the insensitive region and the delay is quite minimal and any variation in the number of vehicles does not affect the delay significantly. To guarantee good quality of service (QoS), PDP should be less than 10-15%. The packets are sensitive to delay and once delay reaches a maximum limit D_{max} (=150ms), the packets are dropped to maintain the quality of the service. It can be seen that the PDP is a lot higher for V2R system than the MPLR system. The number of participating vehicles in the clustering mechanism is reduced and this improves the packet dropping probability at least by 20%.

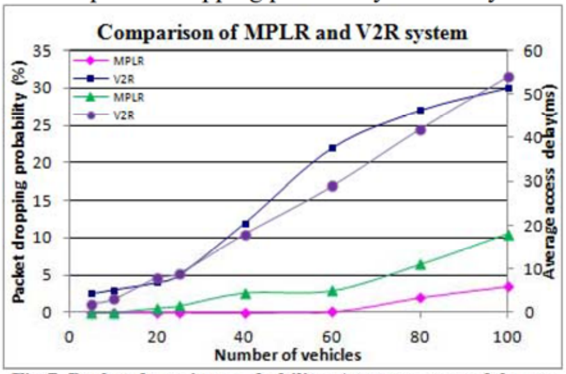

Fig.7. Packet dropping probability, Average access delay vs number of vehicles

IX. CONCLUSION

This paper proposed a novel clustering algorithm based on minimal path loss for vehicular communication. A good clustering algorithm should optimise the communication and improve network stability between nodes by avoiding flooding of data in the network. Hence there is a need for a new clustering algorithm which takes into consideration the physical channel impairments such as path loss and interference in selecting a cluster head. The cluster head acts as a central relay entity between vehicles in a cluster and also between vehicles and the roadside base station. The vehicle clustering is periodically updated to reflect the topological changes and vehicle movements. The clustering should feature low cluster head changes for the stability of the network. An accurate vehicular traffic model is deduced from the real time data of Traffic Wales and used as an input to a motorway simulator. The performance of the clustering algorithm are evaluated in terms of rate of change of cluster heads and average cluster size over different hours of the day. A mathematical analysis is presented for rate of change of cluster heads and is compared with the simulated results. The results are in good agreement with each other. The MPLR system is compared with a V2R system and it is found that the MPLR system performs better than a V2R in terms average access delay and packet dropping probability.

REFERENCES

[1] R Nagel, S Eichler and J Eberspächer , "Intelligent Wireless Communication for future autonomous and cognitive automobiles", *Proceedings of the 2007 IEEE Intelligent Vehicles Symposium*, Istanbul, Turkey, June 13-15 2007, pp:716-721.

[2] C2C-CC: CAR 2 CAR Communication Consortium.

[3] COMeSafety2 – "Communication for e-Safety"

[4] DriveC2X – "Drive Car-2-X", *iCarSupport – iMobility Forum Plenary Meeting Presentations*, October 2011.

[5] G Abdalla, M A AbuRgheff and S M Senouci, "Current Trends in Vehicular Adhoc Networks," *Ubiquitous Computing and Communication Journal (UbiCC)* – Special issue of UbiRoads 2007C. J. Kaufman, Rocky Mountain Research Lab., Boulder, CO, private communication, May 1995.

[6] Hans-J. Reumerman. M. Roggero, M. Ruffini, "The application-based clustering concept and requirements for intervehicle networks", *IEEE Communications Magazine*, April 2005,pp.108-113.

[7] M. Gerla and J. T. C. Tsai, 'Multicluster mobile multimedia radio networks", *ACM-Baltzer Journal of Wireless Networks*, 1995, pp: 255-265.

[8] S Chinara and S K Rath, "A Survey on One-hop clustering algorithms in mobile ad hoc networks", *Journal of Network and Systems Management*, Vol.17, No.1-2, June 2009, pp.183-207

[9] H. Wu, "Analysis and Design of Vehicular Networks," *PhD Thesis*, Department of Computer Science, Georgia Institute of Technology, December 2005.

[10] J Miller, "Vehicle-to-Vehicle-to-Infrastructure (V2V2I) Intelligent Transportation System Architecture", *IEEE Intelligent Vehicles Symposium*, June 2008, pp.715-720

[11] D Jiang and L Delgrossi, "IEEE 802.11p:Towards an international standard for wireless access invehicular environments", *Vehicular Technology Conference* 2008, p.p. 2036-2040

[12] E Dahlman et al, "The 3G Long Term Evolution – Radio Interface Concepts, Performance and Evaluation" , *IEEE 63rd Vehicular Technology Conference*, May 2006, pp.137-141

[13] T Bellemans, B D Schutter and B D Moor, "Models for traffic control," *Journal A*, Vol. 43, No. 3–4, 2002, pp. 13–22.

[14] M Brackstone and M McDonald, "Modelling of Motorway Operations," *Transportation Research Record*, Issue.1485, 1995.