Capacity of Wireless Ad-hoc Networks under Ultra Wide Band with Power Constraint

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***Abstract*— In this paper, we study how the achievable throughput scales in a wireless network with randomly located nodes as the number of nodes increases, under a communication model where (i) each node has a maximum transmission power W_0 and is capable of utilizing B Hz of bandwidth and (ii) each link can obtain a channel throughput according to the Shannon capacity. Under the limiting case that B tends to infinity, we show that each node can obtain a throughput of $\Theta(n^{(\alpha-1)/2})$ where n is the density of the nodes and α is the path loss exponent. Both the upper bound and lower bound are derived through percolation theory. In order to derive the capacity bounds, we have also derived an important result on random geometric graphs: if the distance between two points in a Poisson point process with density n is non-diminishing, the minimum power route requires power rate at least $\Omega(n^{(1-\alpha)/2})$. Our results show that the most promising approach to improving the capacity bound in wireless ad hoc networks is to employ unlimited bandwidth resources, such as UWB.**

keywords– **Stochastic processes/Queuing theory, Graph theory, Combinatorics, Information theory**

I. Introduction

A wireless ad hoc network consists of nodes that communicate with each other over a shared wireless channel. Without the need for centralized infrastructure support, wireless ad hoc networks have many salient features such as ease of deployment, low cost and low maintenance. However, wireless radio signal attenuation and interference on the shared wireless medium impose new challenges in building large scale wireless ad hoc networks. A natural question is how the throughput scales with the number n of nodes in this type of networks. In their seminar work, Gupta and Kumar [17] show that assuming each node can transmit with constant rate, the per-node throughput capacity of a random wireless ad hoc network with n static nodes decreases with n as $O(1/\sqrt{n})$ (in the physical model, the definition of which is given in Section VI). They also show that a per-node throughput of rate $\Omega(1/\sqrt{n \log n})$ can be achieved in the same setting.

Gupta and Kumar's result indicates that the network capacity "vanishes" as the number of nodes increases. Due to this pessimistic result, many researchers have investigated different ways of increasing the capacity. The first type of methods uses mobility to increase the capacity. Specifically, in a network where nodes move randomly on a unit-area disk such that their steady state distribution is uniform, Grossglauser and Tse [16] show each source-destination pair can achieve a constant throughput, which is independent of the number of nodes. Several subsequent works [3], [23], [13] study the delay introduced due to mobility, and show that in order to achieve constant per-node throughput, the delay has to be proportional to the inverse of the velocity of node movement [3], [13]. In general, since node movement is much slower than the radio propagation, the delay introduced using node mobility is non-negligible, and to some extent, intolerable for most applications. In addition, the buffer required for mobile nodes to carry data packets to their destinations is another issue.

The second type of methods use static infrastructures to increase the capacity of wireless networks. In this case, m base stations interconnected with wired lines are placed within the ad hoc network with n nodes to help transport packets. Liu *et al.* [20] consider the case where m base stations form a regular hexagonal pattern. They show that the number m of base stations has to grow at a rate faster than $\sqrt{n}$, in order to effectively improve the throughput capacity. Kozat and Tassiulas [18] show that assuming the base stations are also randomly deployed, the per-node throughput capacity can only be $\Theta(1/\log n)$, even if the number m of base stations grows at a rate proportional to n. Therefore, the use of infrastructure support requires a large number of base stations interconnected through wired line.

The third type of methods use directional antennas to increase the capacity of wireless networks. Yi *et al.* [28] show the capacity in a random wireless network can be improved by a constant (w.r.t. the number of wireless nodes) factor that is inverse proportional to the beam width of the antennas. Peraki and Servetto [2] show that even if the transmitter can generate arbitrarily narrow beams (which essentially removes all wireless interference) and the transmission ranges are set as minimal as possible to maintain connectivity, the capacity can only improve by an order of $\Theta(\log^2(n))$. Thus the capacity improvement using directional antenna is very limited.

The fourth type of method to improve the capacity bound is to leverage the unlimited spectrum resources, in particular, the Ultra Wideband (UWB). Negi and Rajeswaran [21] show that under the limiting case when the bandwidth $B \to \infty$ and each node has a power constraint W_0, the per node capacity is upper bounded by $O((n \log n)^{(\alpha-1)/2})$ and lower bounded by $\Omega(\frac{n^{(\alpha-1)/2}}{(\log n)^{(\alpha+1)/2}})$, where α is the path-loss exponent. Note there is a gap of $(\log n)^{\alpha}$ between the derived upper bound and lower bound of the capacity. Dana and Hassibi [5], [6] consider a different scenario in which there are n relay nodes and $r \leq \sqrt{n}$ source-destination pairs. Assuming unlimited bandwidth, they show that given the total rate scales like $\Theta(f(n))$, the minimum power required by each node scales like $\Theta(f(n)/\sqrt{n})$. In addition to the difference in the scenario, these bounds are derived based on a simple "listen and transmit" protocol, which may not be optimal in terms of the capacity-power tradeoff.

It seems that the most promising approach to improving the capacity bound is to use the unlimited bandwidth (spectrum) resources, i.e., the UWB. Since the approval of commercial use by the United States Federal Communications Commission (FCC) in 2002, the UWB technology has received a great deal of attention in the wireless community [10], [9], [1]. UWB is defined as any radio technology using a spectrum that occupies a bandwidth greater than 20 percent of the center frequency, or a bandwidth of at least 500MHz. UWB technology is most appropriate for short range communications ($\leq$ 10 meters). By Shannon's capacity theory, UWB transmitters are capable of transmitting, with very lower power, at a rate ranging from 100Mbps to 500 Mbps. The characteristics of UWB make it well suited for wireless sensor networks (in addition to wireless personal area networks, such as smart home environments). In particular, wireless sensor networks are expected to be deployed with high densities (up to 20 nodes/m^2[24]) and sensors have very limited power supplies.

In this paper we study the same problem as that in [21], i.e., deriving the capacity bound in the limiting case when (i) the bandwidth $B \to \infty$, (ii) each node's power is constrained by W_0, and (iii) each link achieves its Shannon capacity. We tighten both the lower and upper bounds of the network capacity to $\Theta(n^{(\alpha-1)/2})$, and close the gap between the two bounds that exist in [21]. Although we investigate the same problem in [21], we have used a dramatically different proof technique. Our derivation is based on the theory of percolation. We expect the same proof technique can be used to establish tighter bounds in several other related works.

In order to derive the capacity bound in the aforementioned limiting case, we derive an important result on random geometric graphs. Given a Poisson point process with density n in a unit square (in which each link of length ℓ between two points is given a weight ℓ^{α} where $\alpha > 0$), we show that if two points have non-diminishing distance, the minimum weight path between them (which can be obtained using any shortest path algorithm) has weight at least in the order of $n^{(1-\alpha)/2}$. The upper bound is derived based on the above result. The lower bound is derived leveraging the technique used in [11], [12] (in a different context). The derivation is made by constructing a backbone network which consists of many disjoint horizontal paths and disjoint vertical paths.

The rest of the paper is organized as follows. In Section II, we discuss the background that pertains to the problem considered in the paper to facilitate derivation. Following that we derive the upper bound in Section III and the lower bound in Section IV. In Section V, we discuss how the capacity scales as the area increases and density is kept constant. We give a comprehensive overview of related work on deriving the wireless capacity in Section VI and conclude the paper in Section VII.

II. Background

The objective of this paper is to derive bounds on the capacity of the ad hoc network that employs UWB. The derivation is made based on some of the results in [21]. For completeness of the paper, we highlight results in [21] that pertains to our derivation. We start with the assumptions on the system model and the performance metrics that define the feasible rate.

A. Assumptions on the System Model

We consider a square of unit area, in which nodes are distributed according to a Poisson point process of intensity n. Each node has a power constraint W_0. The underlying communication system has an arbitrarily large bandwidth B (i.e., $B \to \infty$). An ambient Gaussian noise model with the power spectral density of N_0 and the signal noise power loss of $1/d^{\alpha}$ is assumed, where d is the distance and $\alpha > 1$ is the distance loss exponent. Capacity-achieving Gaussian channel codes are assumed for each link. Thus each link can support a data rate as determined by the Shannon capacity of that link [4], i.e., $r = B \log(1 + SINR)$. Similar to [21], we use $\log(\cdot)$ to denote $\log_e(\cdot)$ and the capacity is expressed in units of nats [4].

Specifically, let X_i denote node i's position and W_{ij} the transmission power of node i to node j. The power constraint on each node W_0 implies that $W_k \triangleq \sum_j W_{kj} \leq W_0$. Let g_{ij} denote the power loss between node i and j and $g_{ij} = |X_i - X_j|^{-\alpha}$. The SINR of the transmission from node i to node j can be computed as

$$SINR = \frac{W_{ij} g_{ij}}{BN_0 + \sum_{k \in I} W_k g_{kj}}, \tag{1}$$

where I is the set of nodes that are simultaneously transmitting.

B. Performance Metric

All nodes send traffic at a rate of $r(n)$ nats per second to their corresponding destinations. We pick uniformly and randomly source-destination pairs, so that each node is exactly the destination of one source. A uniform throughput $r(n)$ is feasible if there exists a routing and scheduling scheme that can satisfy the throughput requirement of $r(n)$ nats per second for each source-destination pair. The maximum feasible uniform throughput is the uniform throughput capacity, and is the performance metric studied in this paper.

The objective is to bound the uniform throughput capacity by a function of n. Since the underlying network is random, so is the capacity. The capacity bounds are often derived to be certain functions with high probability (*w.h.p.*), i.e., with probability approaching 1 as the density $n \to \infty$. Specifically, we say that the uniform throughput capacity $r(n)$ is of order $\Theta(f(n))$ if there exist deterministic constants $c_1 > c_0 > 0$ (w.r.t. n) such that

$$\lim_{n\to\infty} \text{Prob}(r(n) = c_0 f(n)) \text{ is feasible }) \;=\; 1, \tag{2}$$

$$\lim_{n\to\infty} \text{Prob}(r(n) = c_1 f(n)) \text{ is feasible }) \;<\; 1. \tag{3}$$

If only Eq. (2) is satisfied, we say that the uniform throughput capacity $r(n)$ is of order (or lower bounded by) $\Omega(f(n))$. If only Eq. (3) is satisfied, we say that the uniform throughput capacity $r(n)$ is of order (or upper bounded by) $O(f(n))$.

C. Bandwidth Scaling

It has been shown in [21] that with high probability no pair of nodes has distance less than $\frac{1}{n\sqrt{\log n}}$, and if the bandwidth scales as fast as $\Theta(n(n^2 \log n)^{\alpha/2})$, the interference is negligible with respect to ambient noise. Since the bandwidth is arbitrarily large, each link's Shannon capacity r_{ij} is proportional to the received power, i.e.,

$$r_{ij} = \lim_{B\to\infty} B \log(1 + \frac{W_{ij} g_{ij}}{N_0 B}) = \frac{W_{ij} g_{ij}}{N_0}. \tag{4}$$

The bandwidth requirement for Eq. (4) to hold is later reduced to $\Theta(n^{(\alpha+1)/2})$ in [21].

D. Optimality of CDMA MAC

It is shown in [21] that CDMA performs as well as any other optimal scheduling scheme under the assumptions of UWB and bounded power. This means the optimal capacity can be achieved by all nodes simultaneously transmitting without applying TDMA or FDMA schemes.

III. An Upper Bound on throughput capacity

The upper bound is derived under a relaxed assumption that the average power constraint of all the nodes is W_0, instead of that each node a power constraint W_0. This is sufficient because the upper bound under a more relaxed assumption is clearly an upper bound under a more restricted assumption.

As has been proved in [21], the optimal route that maximizes the throughput capacity under this relaxed assumption is the minimum power route for each source-destination pair, because minimizing the power consumption of a route for each source destination pair is equivalent to minimizing the average power consumption of all nodes. Let R_i denote the minimum power route for a given source-destination pair i, i.e., $R_i = [X_{i_1}, X_{i_2}, \cdots X_{i_K}]$. Let r_i denote the achieved throughput on the route R_i. The minimum power on this route is (according to Eq. (4))

$$W(R_i) = r_i(n) \cdot N_0 \sum_{k=1}^{K-1} |X_{i_k} - X_{i_{k+1}}|^\alpha. \tag{5}$$

Intuitively, since the average power consumption on each route is bounded, the achievable rate r_i is determined by the bounds on the *power rate* defined as

$$Q_i \triangleq \sum_{k=1}^{K-1} |X_{i_k} - X_{i_{k+1}}|^\alpha. \tag{6}$$

Let d_i denote the distance between the source X_{i_1} and its destination X_{i_K}, i.e.,

$$d_i = |X_{i_1} - X_{i_K}|. \tag{7}$$

In what follows, we establish a bound on Q_i and consequently on the throughput capacity. The key to the derivation is that, if it is possible for R_i to be composed of mostly short hops, then potentially the minimum power rate (Q_i) of a route can be very small. Thus, our major task is to show that there are a sufficiently large number of long hops. The proof is based on the site percolation model.

A. Construction of the Site Percolation Model

We divide the area into grids of edge length $c_0/\sqrt{n}$ as depicted in Fig. 1. By adjusting the constant c_0, we can adjust the probability that a grid contains at least one node:

$$P(\text{a grid contains at least one node}) = 1 - e^{-c_0^2} \triangleq p. \tag{8}$$

A grid is said to be *open* if it contains at least one node, and *closed* otherwise. Two grids are said to be *adjacent* if they share an edge or a vertex. Any grid is thus adjacent to 8 other grids. For notational convenience, we use (i) a *path* to refer to a list of grids such that any two neighboring grids in the list are adjacent; and (ii) a *route* to refer to a list of wireless nodes that are actually used to transport packets from the source to the destination. By convention in graph theory, we assume a path does not include any grid twice, except that its first grid may be the same as the last grid. A path is said to be *open* (*closed*) if all the grids on the path are open (closed).

As a first step, we observe that if there is an open path in the percolation model from the grid where the source is

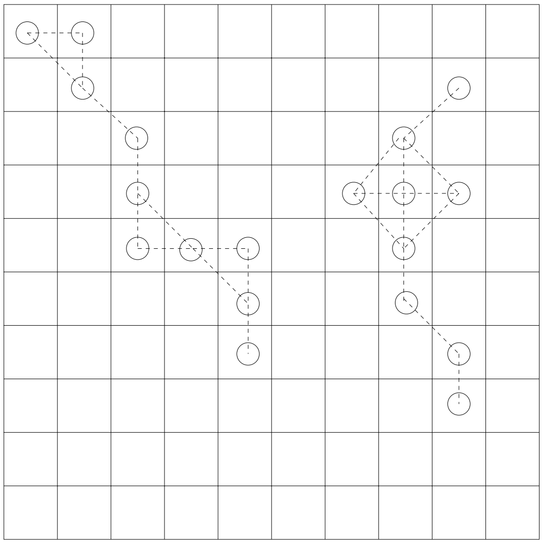


Fig. 1. Construction of the site percolation model. We divide the area into grids of edge length $c_0/\sqrt{n}$. A grid is said to be open if there is at least one Poisson point inside it; and closed otherwise. Two grids are said to be adjacent if two grids share an edge or a vertex, i.e., grid (i,i) is adjacent to $(i-1,i-1),(i-1,i),(i-1,i+1),(i,i-1),(i,i+1),(i+1,i-1),(i+1,i),(i+1,i+1)$. An open grid is denoted with a circle inside it. The dashed lines show all the possible open links.

located to the grid where the destination is located, then we can form a route from the source to the destination by picking one node from each grid on the path. Every hop on this route is bounded from above by $2\sqrt{2}c_0/\sqrt{n}$. On the other hand, if there is no such an open path in the percolation model, then in any route (including the minimum power route) from the source to the destination, at least one hop is of length at least $c_0/\sqrt{n}$. Indeed, if c_0 and consequently p are sufficiently small, and the distance, d_i, between the source and the destination is sufficiently large, there exists no open path between them in the percolation model *w.h.p.*.

Important Properties of the Site Percolation Model: We formally state and prove the above property in the lemma below.

Lemma 1 *Let p be the probability that a grid is open in the site percolation model we have defined (Eq. (8)). Then the probability that there exists an open path of length m starting from a source is upper bounded by*

$$P(N(m) \geq 1) \leq \frac{8}{7}(7p)^m, \tag{9}$$

where $N(m)$ is the number of open paths of length m starting from a given source.

Proof. The total number of paths of length m are upper bounded by $8 \cdot 7^{m-1}$, because in the first hop there are at most 8 choices, and in each subsequent hop there are at most 7 choices. Each path is open with a probability of p^m. Thus, the expected number of open paths of length m starting from a given source is $E[N(m)] = 8 \cdot 7^{m-1} \cdot p^m$. It then follows by the Markov inequality that

$$P(N(m) \geq 1) \leq E[N(m)] = \frac{8}{7}(7p)^m. \tag{10}$$

$\square$

If we choose $p < 1/7$ and the distance (in terms of grids) between the source and the destination goes to infinity, then *w.h.p.* there is no open path between them.

The next result is patterned on the results derived in [15] (Eq. (2.49)) in which the bond percolation model is used. Since we consider the site percolation model, we give the proof. Let P_p be denoted as the probability measure with the site-open probability (the probability that a grid is open) p.

Lemma 2 *Let A be the event that there exists an open path of length m starting from a given source and F_A the minimum number of grids that need to be turned open from closed in order for the event A to take place. Then we have*

$$P_p(A) \geq \left(\frac{p-p'}{1-p'}\right)^r P_{p'}(F_A \leq r) \tag{11}$$

for any $0 < p' < p < 1$.

Proof. See Appendix I.

B. Derivation of Upper Bound of Network Capacity

We are now ready to prove the following result. Note that the results can be applied to other fields such as random geometric graphs.

Theorem 1 *Assume that nodes are distributed in a unit square area according to a Poisson point process with density n. If the distance between a source-destination pair is $d_i \geq \epsilon > 0$, the power rate Q_i (Eq. (6)) of the minimum power route between them is at least $c_1 n^{(1-\alpha)/2}$* w.h.p. *for some constant $c_1 > 0$. Specifically,*

$$P(Q_i > c_1 n^{(1-\alpha)/2}) \geq 1 - \frac{8}{7} \cdot \exp(-c_2\sqrt{n}), \tag{12}$$

as $n \to \infty$, for some constant $c_1, c_2 > 0$.

Proof. For any route between the source and the destination, we can construct a walk (which may include some grids more than once) in the site percolation model by including all the grids that intersect with the route. The walk can be further trimmed into a path which contains the minimum number of closed grids by removing unnecessary grids (see an illustration in Fig. 2). We denote T^* as an optimally trimmed path that contains the minimum number of closed grids. In what follows, we bound the probability that the optimally trimmed path T^* contains at most $c_3\sqrt{n}$ closed grids, where c_3 is a constant yet to be determined.

Note that the distance between the source-destination pair in terms of grids is at least $m \triangleq d_i/(\sqrt{2}c_0/\sqrt{n}) =$

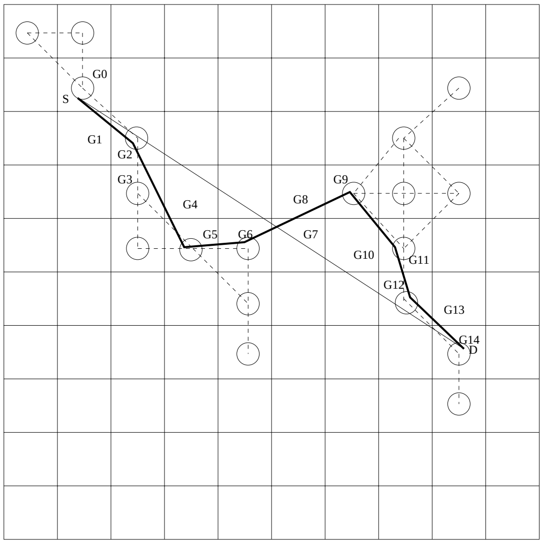


Fig. 2. The bold lines show a route from source S to destination D. We can construct a walk (which is also a path) that is composed of grids that intersect with the route: $[G_0, G_1, G_2, G_3, G_4, G_5, G_6, G_7, G_8, G_9, G_{10}, G_{11}, G_{12}, G_{13}, G_{14}]$. Some of the grids can be removed from the path. For example, G_1 can be removed because G_0 and G_2 are connected (in our percolation model). Similarly, G_4, G_8, G_{10}, G_{13} can all be removed. There are multiple ways of trimming the path. For example, we can also remove G_3, G_5 but keep G_4. Among all the trimmed paths, we pick as T^* the one that contains the minimum number of closed grids. Ties are broken arbitrarily. In the above example, the path $[G_0, G_2, G_3, G_5, G_6, G_7, G_9, G_{11}, G_{12}, G_{14}]$ contains minimum number (which is 1 in this case) of closed grids.

$d_i\sqrt{n}/(\sqrt{2}c_0)$. This implies the path length of T^* is at least m.

If T^* contains at most $c_3\sqrt{n}$ closed grids, then we can construct an open path from the source to the destination by turning at most $c_3\sqrt{n}$ closed grids into open grids. This further indicates that by turning at most $c_3\sqrt{n}$ closed grids into open ones, we can obtain an open path of length at least m starting from the source. Now we can apply Lemma 2. Let A denote the event that there is an open path of length m starting from the source, and F_A the minimum number of closed grids that need to be turned into open in order for event A to take place. We conclude that $F_A \le r = c_3\sqrt{n}$ if the trimmed path T^* contains at most $c_3\sqrt{n}$ closed grids. By Lemma 2,

$$P_{p'}(F_A \le c_3\sqrt{n}) \quad \le \quad P_p(A)\left(\frac{p-p'}{1-p'}\right)^{-c_3\sqrt{n}}. \tag{13}$$

By Lemma 1,

$$P_p(A) = \frac{8}{7}\cdot(7p)^m = \frac{8}{7}\cdot(7p)^{d_i\sqrt{n}/(\sqrt{2}c_0)}. \tag{14}$$

We can choose c_0 such that $p = 1 - e^{-c_0^2} < 1/7$. After fixing c_0 and p, we can choose $k > 1/p$ and $p' = \frac{kp-1}{k-1} < p$. Now plugging the equation of p' and Eq. (14) into Eq. (13), we have

$$\begin{aligned} P_{p'}(F_A \le c_3\sqrt{n}) & \\ \le\ & \frac{8}{7}\cdot(7p)^{d_i\sqrt{n}/(\sqrt{2}c_0)}\cdot k^{c_3\sqrt{n}} \\ =\ & \frac{8}{7}\cdot\exp\left(\sqrt{n}\left(\frac{d_i\log(7p)}{\sqrt{2}c_0} + c_3\log k\right)\right). \end{aligned} \tag{15}$$

If we choose $0 < c_3 < -\frac{\epsilon\log(7p)}{\sqrt{2}c_0\log k} < -\frac{d_i\log(7p)}{\sqrt{2}c_0\log k}$, we obtain

$$P_{p'}(F_A \le c_3\sqrt{n}) \le \frac{8}{7}\cdot\exp(-c_2\sqrt{n}) \to 0 \tag{16}$$

as $n \to \infty$, where

$$c_2 = -\frac{\epsilon\log(7p)}{\sqrt{2}c_0} - c_3\log k > 0. \tag{17}$$

Hence, the optimally trimmed path T^* contains more than $c_3\sqrt{n}$ closed grids with probability at least $p_1 \triangleq 1 - \frac{8}{7}\cdot\exp(-c_2\sqrt{n})$ if we choose the grid size $c_0'/\sqrt{n}$ such that $1 - e^{-c_0'^2} = p'$.

It is not difficult to see that for each closed grid on T^*, there is exclusively one line segment completely contained in a link on the minimum power route with length at least $c_0'/\sqrt{n}$ (An illustration is given in Fig. 3). In addition, if a link on the route intersects with j closed grids on T^*, the link has length at least $jc_0'/\sqrt{n}$. To derivate the lower bound of the power rate of the route, we can assume each link only intersects at most one grid in T^*, because if a link intersects with j grids in T^*, its power rate will be greater than the power rate of j links each with length $c_0'/\sqrt{n}$. Thus the route contains at least $c_3\sqrt{n}$ links each with length at least $c_0'/\sqrt{n}$ with probability at least p_1. Hence the total power rate of the route is at least $c_3\sqrt{n}\cdot\left(\frac{c_0'}{\sqrt{n}}\right)^\alpha = c_3c_0'^\alpha n^{(1-\alpha)/2}$ with probability at least p_1. Let $c_1 = c_3c_0'^\alpha$. We obtain

$$P(Q_i > c_1n^{(1-\alpha)/2}) \ge p_1 = 1 - \frac{8}{7}\cdot\exp(-c_2\sqrt{n}). \tag{18}$$

□

The following results are intuitively true and a tedious but rigorous proof is given in Appendix II.

Lemma 3 *(i)* w.h.p., *the number of nodes in the field is between $n/2$ and $2n$.*
(ii) With our ways of choosing source-destination pairs, there exist $\epsilon > 0$ such that the number of pairs with distance at least ϵ is at least $n/8$ w.h.p..

We now prove the main result in this section.

Theorem 2 *With the assumptions we have made in Section II-A, the network capacity is upper bounded by $O(n^{(\alpha-1)/2})$* w.h.p. *as $n \to \infty$.*

Proof. Combining Eqs. (5) and (6), we obtain

$$W(R_i) = r_i(n)N_0Q_i. \tag{19}$$

Let I_1 denote the set of routes with the distance between the source-destination pair at least ϵ. By Lemma 3, we have

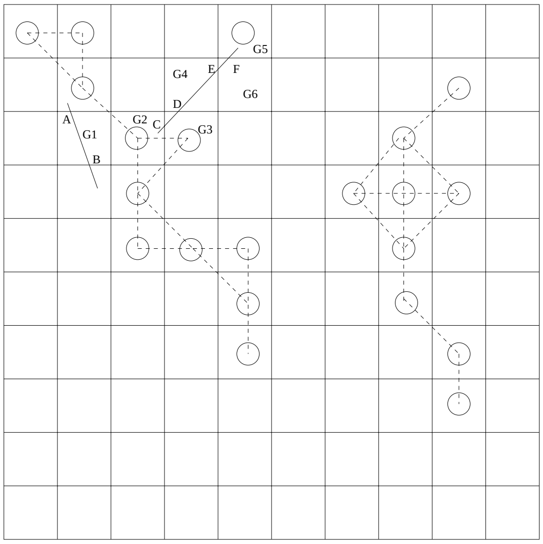


Fig. 3. Illustration of the relation between a closed grid on T^* and a line segment of length at least $c_0'/\sqrt{n}$ on a link of the minimum power route. If a link crosses a closed grid at the two opposite edges (as in the grid G_1), the line segment (AB) on the link that is contained by the grid have length at least $c_0'/\sqrt{n}$. Hence without loss of generality, we can assume a link enters a closed grid from its bottom and exits from its right (such as grid G_4). If grid G_3 is on the path T^*, then G_6 is either not on the path T^* or open, because otherwise G_4 can be removed from the path. In this case, the line segment DF has length at least $c_0'\sqrt{n}$. Similarly if G_2 but not G_3 is on the path T^*, the line segment CE has length at least $c_0'\sqrt{n}$. If a link intersects more than one grid on the path T^*, similar analysis can be performed.

$n/8 \leq |I_1| \leq 2n$ *w.h.p.*. Summing over all the routes in I_1, we have

$$\sum_{i \in I_1} W(R_i) = \sum_{i \in I_1} r_i(n) N_0 Q_i. \tag{20}$$

Since we are interested in the uniform capacity bound $r(n)$ achieved by all routes, we have

$$\sum_{i \in I_1} W(R_i) \geq r(n) \sum_{i \in I_1} N_0 Q_i. \tag{21}$$

In Theorem 1, we have shown that there exists $c_1, c_2 > 0$ such that

$$P(Q_i \leq c_1 n^{(1-\alpha)/2}) \leq \frac{8}{7} \exp(-c_2\sqrt{n}) \tag{22}$$

for a given source destination pair i with distance at least ϵ. Without loss of generality, we can assume $|I_1| \leq 2n$ because otherwise we may only keep the first $2n$ routes in I_1. Thus

$$P(\exists i \in I_1, \text{ s.t. } Q_i \leq c_1 n^{(1-\alpha)/2}) \leq 2n \cdot \frac{8}{7} \exp(-c_2\sqrt{n}). \tag{23}$$

The right equation in the above converges to 0 as $n \to \infty$. Thus, *w.h.p.* we have at least $n/8$ routes, each with at least power $c_1 n^{(1-\alpha)/2}$. In addition, *w.h.p.* the total power of all routes in I_1 is at most $2nW_0$ by Lemma 3(i). Plugging these results into Eq. (21), we obtain that *w.h.p.*

$$r(n) \leq \frac{2W_0}{N_0 c_1 n^{(1-\alpha)/2}/8} = c_4 n^{(\alpha-1)/2}. \tag{24}$$

This completes our proof. □

IV. A Lower Bound on Throughput Capacity

In order to derive a lower bound on the throughput capacity, we leverage a routing scheme used in [11], [12][1]. We show that the routing scheme can achieve a capacity bound that is of the same order of the upper bound we have derived in Section III. For completeness of the paper, we summarize the routing scheme first.

A. Construction of the Backbone Network

The routing scheme lays a wireless backbone network that carries packets across the network at the desired rate. The backbone network is composed of short hops (and hence is able to transmit at high rates), and is obtained through the percolation theory.

To construct the backbone network, we divide the area into square grids of edge length $c_5/(2\sqrt{n})$. The new grid system is depicted in Fig. 4 (a). Note that the grid system is constructed differently from that in Section III. As depicted in Fig. 4 (b), we draw a horizontal edge across half of the grids and a vertical edge across the others. An edge is said to be open if there exists at least one node (from the Poisson point process) in the grid that contains the edge and closed otherwise. In this way we obtain a bond percolation model. The probability that an edge is open is independent of all other edges, and can be expressed as

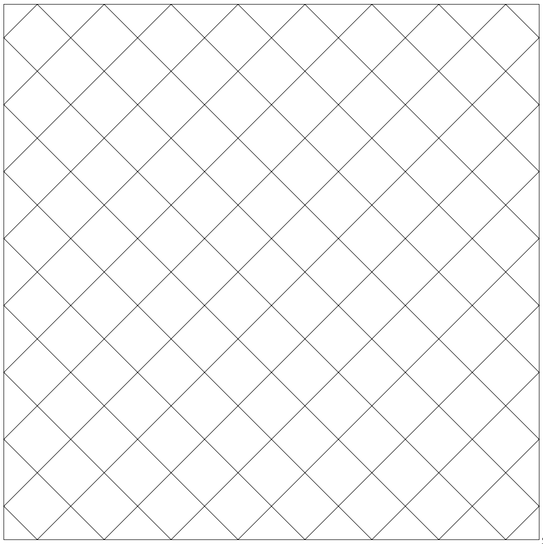
$$p = 1 - e^{-c_5^2/4}. \tag{25}$$

Next we divide the network area into horizontal rectangles, $\bar{R}_n$, of size $1 \times \frac{c_5}{\sqrt{2n}} \log \frac{\sqrt{2n}}{c_5}$. Each of the rectangles thus has $m \times \log m$ grids in the bond percolation model, with $m = \sqrt{2n}/c_5$ (as the edges have length $\frac{1}{m}$). As proved in [11] (Theorem 1), there exist many open paths from left to right inside each such rectangle $\bar{R}_n$.

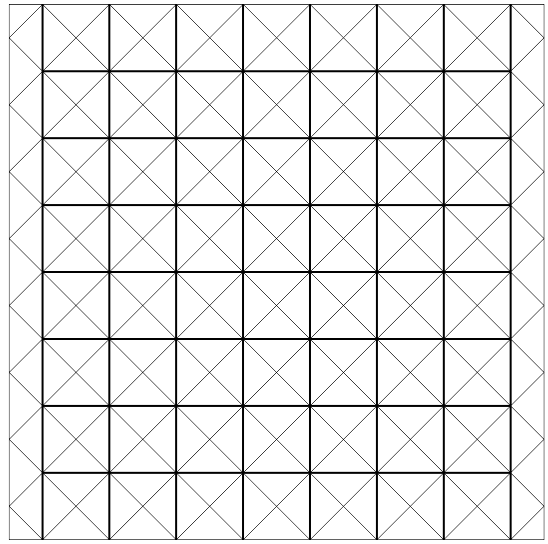
Lemma 4 *(Theorem 1 in [11]) If c_5 is sufficiently large, there exists a constant $\beta = \beta(c_5) > 0$ such that* w.h.p. *there are $\beta \log m = \beta \log \frac{\sqrt{2n}}{c_5}$ disjoint open paths that cross each rectangle $\bar{R}_n$ from left to right.*

With all the rectangles, we obtain βm open paths from left to right. We can also divide the area into vertical rectangles and reach similar results for paths that cross the area from bottom to top. With the use of a simple union bound argument, we conclude that there exist βm horizontal disjoint paths and βm vertical disjoint paths simultaneously *w.h.p.*. These paths constitute the backbone network.

[1]We make minor revision on the choice of the exit point to facilitate our derivation.



(a) New grid system



(b) Edges for the bond percolation model

Fig. 4. Construction of the bond percolation model. We divide the unit square area into square grids of side length $c_5/(2\sqrt{n})$. A grid is said to be *open* if it contains at least one point in the Poisson point process and *closed* otherwise. The edge that crosses an open (closed) grid is said to be *open* (*closed*).

B. Routing in the Backbone Network

Packets are transported from sources to destinations in the above backbone network via three phases: *draining phase*, *backbone phase*, and *delivery phrase*. In the first (draining) phase, the source sends packets directly to a node on a horizontal path of the backbone network. In the second (backbone) phase, packets are transported along the horizontal path and reach a vertical path. In the third (delivery) phase, a node in the vertical path sends packets directly to the destination. In what follows we discuss the detailed operations in each phase.

1) Draining phase: In the draining phase, packets are carried from the source to the backbone network. We first evenly divide the square area into βm horizontal slabs of width $\frac{1}{\beta m}$. Now since there are exactly as many slabs as horizontal paths, we can enforce that nodes in the ith slab send their packets using the ith horizontal path. More precisely, an entry point in the ith horizontal path can be assigned to each source in the ith slab. As shown in Fig. 5, the entry point is chosen to be the node on the ith horizontal path that is closest to the vertical line drawn from the source point. By Lemma 4, the distance between a source and its corresponding entry point is never larger than $(c_5/\sqrt{2n})\log(\sqrt{2n}/c_5)+c_5/\sqrt{2n}$ (Since the source and the entry point are in the same rectangle $\bar{R}_n$ their vertical distance is at most $(c_5/\sqrt{2n})\log(\sqrt{2n}/c_5)$, and their horizontal distance is at most $c_5/\sqrt{2n}$ by the choice of the entry point).

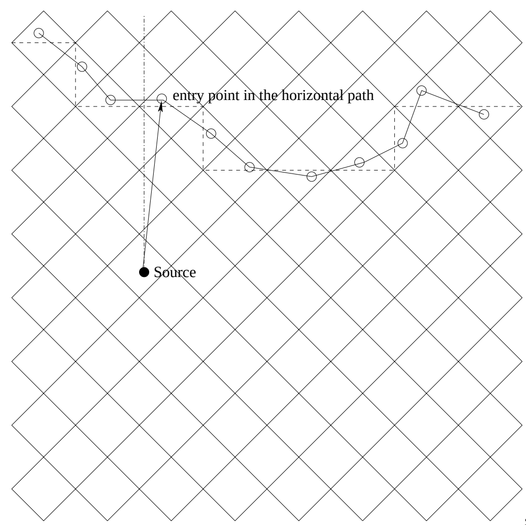


Fig. 5. A source transmits packets directly to the entry point on a horizontal path.

2) Backbone phase: Similarly we can divide the square area into βm vertical slabs. Once a packet is transmitted to the entry point, it is carried along the corresponding horizontal path until it reaches the crossing point with the target vertical path. The target vertical path is determined by the vertical slab that contains the destination node, i.e, if the destination is in the ith vertical slab, the target vertical path is the ith vertical path. The following result is proved in [11]:

Lemma 5 *The probability that each slab contains less than $c_5\sqrt{2n}/\beta$ nodes tends to one when $n \to \infty$.*

3) Delivery phase: In the delivery phase, packets are transported from the exit point of the vertical path to the destination directly. The *exit point* for a given destination is defined as a node in the grid on the vertical path whose center (i.e., the center of the grid) is closest to the horizontal line drawn from the destination. Again, the destination from the exit point to the destination is at most $(c_5/\sqrt{2n})\log(\sqrt{2n}/c_5)+c_5/\sqrt{2n}$.

C. Achievable Throughput

We now show that the achievable throughput using the routing scheme presented in Section IV-B is at least $c_6 n^{\frac{\alpha-1}{2}}$ where $c_6>0$ is to be determined later. Clearly it is sufficient to show this is true in each phase of the routing scheme.

1) Draining phase: Since the distance from each source X_i to the entry point X_{i_1} is never larger than $\frac{c_5}{\sqrt{2n}}(\log\frac{\sqrt{2n}}{c_5}+1)$, the achievable rate from each source X_i to the entry point is

$$\begin{aligned} r_i &= \frac{W_0}{N_0|X_i-X_{i_1}|^\alpha} \\ &\geq \frac{W_0}{N_0}\left(\frac{\sqrt{2n}}{c_5(1+\log\sqrt{2n}/c_5)}\right)^\alpha \\ &\geq c_6 n^{(\alpha-1)/2}. \end{aligned} \tag{26}$$

Clearly the last inequality holds if n is sufficiently large and c_6 is sufficiently small (but independent of n). Therefore, the rate $c_6 n^{(\alpha-1)/2}$ is achievable as long as there are $\beta\log(\sqrt{2n}/c_5)$ horizontal paths in every rectangle $\bar{R}_n$. Since the latter takes place *w.h.p.*, the rate $c_6 n^{(\alpha-1)/2}$ is achievable *w.h.p.*.

2) Backbone phase: By Lemma 5, every slab has less than $c_5\sqrt{2n}/\beta$ nodes *w.h.p.*. Thus *w.h.p.*, every node in the backbone (on the horizontal path, the vertical path, or both), will need to relay traffic at a rate $r_i \leq 2\cdot(c_5\sqrt{2n}/\beta)\cdot c_6 n^{(\alpha-1)/2} = 2\sqrt{2}c_5c_6 n^{\alpha/2}/\beta$. In the backbone phase, a node only need to transmit packets to its next hop node and the transmission distance is at most $c_5\sqrt{2/n}$. Thus the power consumption on each node W_i is

$$\begin{aligned} W_i &\leq r_i N_0 (c_5\sqrt{2/n})^\alpha \\ &\leq (2\sqrt{2}c_5c_6 n^{\alpha/2}/\beta)\cdot N_0(c_5\sqrt{2/n})^\alpha \\ &= 2c_6(c_5\sqrt{2})^{\alpha+1}N_0/\beta. \end{aligned} \tag{27}$$

If we choose $c_6 \leq \frac{W_0\beta}{2N_0(c_5\sqrt{2})^{\alpha+1}}$, we have $W_i\leq W_0$. Thus the backbone can support a rate of $c_6 n^{(\alpha-1)/2}$ for each source *w.h.p.*.

3) Delivery phase: In the delivery phase, an exit point on the vertical path sends packets to the destination node directly. The following Lemma bounds the number of destination nodes each exit point needs to handle.

Lemma 6 *The probability that there are less than* $(c_5^2/(2\beta))\log n$ *destination nodes for each exit point approaches one as* $n\to\infty$.

Proof. See appendix III.

Again the maximum distance between an exit point and the corresponding destination node is $(c_5/\sqrt{2n})(\log(\sqrt{2n}/c_5)+1)$. Using Lemma 6 and Eq. (5), we can conclude that, *w.h.p.*, the power consumption of every exit point is less than

$$\begin{aligned} & N_0 c_6 n^{\frac{\alpha-1}{2}}\cdot\frac{c_5^2}{2\beta}\log n\cdot\left(\frac{c_5}{\sqrt{2n}}(\log\frac{\sqrt{2n}}{c_5}+1)\right)^\alpha \\ =\ & \frac{N_0c_6c_5^2\log n}{2\beta\sqrt{n}}\cdot\left(\frac{c_5}{\sqrt{2}}(\log\frac{\sqrt{2n}}{c_5}+1)\right)^\alpha \end{aligned} \tag{28}$$

Clearly, when n is sufficiently large, c_6 can be chosen sufficiently small (but independent of n) to satisfy the power consumption constraint ($\leq W_0$) for each exit point.

In summary, we have proved the lower bound of the network capacity as follows.

Theorem 3 *With the assumptions we have made in Section II-A, the network capacity is lower bounded by* $\Omega(n^{(\alpha-1)/2})$ w.h.p..

Remark: A node may play multiple roles. For example, it can be a source node, a transit node on a horizontal path or a vertical path, and/or an exit point. In such cases, we can evenly distribute its power for each role. The achievable rate is still $\Omega(n^{(\alpha-1)/2})$.

V. Discussions

Area Rescaling Since the assumption of unit area is an abstraction of the real world with larger area, we consider the rescaled network where the side length of the square is L and the node density in the rescaled network is n_0. Thus if we envision the side length L as 1 unit, the network density in the unit area is $n=n_0L^2$. In the rescaled network, we keep the density n_0 fixed and let $L\to\infty$. Since the edge length in the rescaled network is multiplied by L, the power rate function is multiplied by $L^{\alpha/2}$. For the upper bound, Theorem 1 should be revised to that if a source and a destination have distance at least ϵL, the total power rate Q_i of the minimum power route between them is at least $\Omega(n^{(1-\alpha)/2}L^\alpha)=\Omega(n^{1/2}n_0^{-\alpha/2})$. Thus by Eq. (19), the upper bound of the per node capacity is of order $n^{-1/2}$. For the lower bound, since the transmission distance in the backbone network is upper bounded by a constant, the transmission rate in the backbone is lower bounded by a constant. Since each node in the backbone is responsible to relay traffics for $\Theta(\sqrt{n})$ source-destination pairs, the achievable rate is at least $\Omega(1/\sqrt{n})$. It is not hard to verify that the draining phase and the delivery phase can also achieve this rate since they are not the bottleneck. This is not surprising because the assumption of bounded power in the unit-area network is equivalent to that the power of each node is of order $\Theta(L^\alpha)=\Theta(n^{\alpha/2})$ in the rescaled (large-area) network. By Eq. (4), the rate of each link is proportional to the transmission power. So if in the rescaled network each node has transmission power $\Theta(n^{\alpha/2})$, the (per node) network capacity is still $\Theta(n^{(\alpha-1)/2})$.

VI. Related Work

In their ground breaking work [17][2], Gupta and Kumar first derive the transport capacity of wireless ad hoc network. Specifically, they assume that n nodes are independently and uniformly randomly distributed, either on the surface of a three-dimensional sphere of unit area, or on a disk of unit area in the plane, that the destination is independently chosen as the node that is closest to a randomly located point (according to the uniform distribution), and that all nodes employ the same transmission range or power. They further assume two transmission models: protocol model and physical model. In the protocol model, a transmission from node i to j is successful if and only if (i) $|X_i - X_j| \le r$ and (ii) $|X_k - X_j| \ge (1+\Delta)r$ for every other simultaneously transmission, where X_i is the location of node i. In the physical model, all nodes choose a common power P for their transmissions. A transmission from node i to node j is successful if and only if

$$\frac{\frac{P}{|X_i - X_j|^\alpha}}{N + \sum_{k\in\Gamma, k\neq i}\frac{P}{|X_k - X_j|^\alpha}} \ge \beta, \tag{29}$$

where Γ is the set of simultaneously transmitting nodes, N is the ambient noise power level. In addition, they assume the transmission rate is constant if the transmission is successful.

The authors show that (i) under the protocol model, the per-node capacity of the wireless network is both upper bounded and lower bounded by $\Theta(1/\sqrt{n\log n})$, and (ii) under the physical model, the per-node capacity is upper bounded by $O(1/\sqrt{n})$ and lower bounded by $\Omega(1/\sqrt{n\log n})$.

Since then, many research efforts have been made to investigate the wireless network capacity. Some of them aim to improve the capacity bound in different ways, while others attempt to derive the capacity bound under under different (usually more realistic) assumptions or different traffic patterns. We roughly classify existing work into those that improve the capacity bound (Section VI-A) and those that derive the bound under different assumptions (Section VI-B). In the former category, we further group existing methods for improving the network capacity bound into four types.

A. Work that Improves the Capacity Bounds

Improving the network capacity bound by mobility: The first type of methods employs mobility to improve the capacity bound. Under the assumption that nodes are mobile and the position of each node is ergodic with stationary uniform distribution on an open disk, Grossglauser and Tse [16] show that the average long-term throughput per source-destination pair can be kept constant *w.h.p.* as the number n of nodes in each unit area goes to infinity. Diggavi *et al.* [7] further show that even if nodes are only allowed to move in one dimension (each node are constrained to move on a single-dimensional great circle on the unit sphere), each node can still obtain constant capacity as the number of nodes in the unit area increases. Their derivation is based on the physical model.

Following that, several researchers study the delay incurred using mobility to improve the capacity. Bansal and Liu [3] study the achievable rate together with the maximum delay incurred. Specifically, under the assumptions that n static nodes and m mobile nodes (that move according to the random mobility model given in [16]) are randomly distributed, and that n sender-receiver pairs are chosen randomly among the static nodes according to a uniform distribution, they show that the achievable capacity is at least $\Theta(\frac{\min(m,n)}{n\log^3 n})$ and the maximum delay incurred by packets is at most $2d/v$, where d is the diameter of the network and v is the velocity of the mobile nodes.

Perevalov and Blum [23] obtain an expression for the capacity as a function of the maximum allowable delay in an all mobile network. They show that there exists a critical value of the delay such that for delays below the critical value, the capacity does not benefit from the motion significantly. For delays d above the critical value, the capacity increases approximately as $d^{2/3}$. In addition, they show that the value of the critical delay increases approximately as the order of $n^{1/14}$ with the number n of nodes. They assume the physical model as in [16].

Gamal *et al.* [13] characterize the optimal throughput-delay tradeoff for both the static network model and the mobile network model. For the static network model, the optimal throughput-delay tradeoff is $D(n) = \Theta(nT(n))$ where $T(n)$ and $D(n)$ are the throughput and delay respectively. For the mobile network model, they show that delay scales as $\Theta(n^{1/2}/v(n))$ if the per node capacity scales at $\Theta(1)$. Their derivation is based on a relaxed protocol model where a transmission from node i to j is successful if for any other node k that transmits simultaneously, $d(k,j) \ge (1+\Delta)d(i,j)$ for some fixed $\Delta > 0$, where $d(i,j)$ is the distance between nodes i and j.

Improving the network capacity bound by infrastructure support: The second type of methods use the infrastructure support to improve the capacity bound, where a number of wired base stations are deployed in the network to help transport packets. (Networks of this type are called *hybrid* networks.) Liu *et al.* [20] consider the case where m base stations are placed in a regular hexagonal pattern within the ad hoc network with n nodes. Under a deterministic routing strategy, they show that if m grows asymptoti-

[2]Some of the works reported here also contain results on the capacity of an arbitrary network where node positions, traffic patterns and transmission ranges are optimally chosen. For the brevity of the paper, we only cite the results on random networks, which is the main focus of this paper.

cally slower than $\sqrt{n}$, the maximum throughput capacity is $\Theta(\sqrt{n/\log\frac{n}{m^2}})$; and if m grows faster than $\sqrt{n}$, the maximum capacity is $\Theta(m)$. Under a probabilistic routing strategy, they show that if m grows slower than $\sqrt{\frac{n}{\log n}}$, the maximum throughput capacity has the same asymptotic behavior as a pure ad hoc network; and if m grows faster than $\sqrt{\frac{n}{\log n}}$, the maximum throughput capacity scales as $\Theta(m)$. Kozat and Tassiulas [18] consider the case where both the wireless nodes and base stations are deployed randomly. They show that the per source node capacity of $\Theta(1/\log(n))$ is achievable, if the ratio of the number n of ad hoc nodes to the number m of the base stations are bounded from above.

Improving the network capacity bound via directional antennas: The third type of methods employs directional antennas to improve the capacity bound. Yi *et al.* [28] show that in a random wireless network, use of directional antennas with beamwidth α for the transmitters can increase the capacity by a factor of $2\pi/\alpha$ and use of directional antennas with beamwidth β for the receivers can increase the capacity by a factor of $2\pi/\beta$. In addition, if both the transmitter and the receiver employ directional antenna, the capacity can be improved by a factor of $4\pi^2/\alpha\beta$. Peraki and Servetto [2] shows that even if transmitter can generate arbitrarily narrow beams (which essentially removes all wireless interference) and the transmission ranges are set as minimal as possible to maintain connectivity, the capacity can only improve by an order of $\Theta(\log^2(n))$.

Improving the network capacity bound with the use of UWB: The fourth type of methods to improve capacity leverage unlimited bandwidth resources to improve the network capacity bound. Negi and Rajeswaran [21] show that under the limiting case when bandwidth $B \to \infty$ and that each node has a power constraint W_0, the per node capacity is upper bounded by $O((n\log n)^{(\alpha-1)/2})$ and lower bounded by $\Omega(\frac{n^{(\alpha-1)/2}}{(\log n)^{(\alpha+1)/2}})$.

Dana and Hassibi [5], [6] consider a different scenario in which there are n relay nodes and $r \leq \sqrt{n}$ source-destination pairs. Assuming unlimited bandwidth, they show that given the total rate scales like $\Theta(f(n))$, the minimum power required by each node scales like $\Theta(f(n)/\sqrt{n})$. The required bandwidth for achieving the minimum power is $\Theta(f(n))$. In addition to the difference in the scenario, these bounds are based on a simple "listen and transmit" protocol, which may not be optimal in terms of the capacity-power-bandwidth tradeoff.

B. Work that Derive the Capacity Bound Under Different Assumptions

Some other researchers study capacity bounds under different (usually more realistic) assumptions. Dousse and Thiran [8] show the available rate per node decreases like $1/n$ under the assumption that the attenuation function is uniformly bounded at the origin. Their derivation is based on the physical model. Toumpis and Goldsmith [26] study the network capacity under a general fading channel model. They show that in a static network, each node can send data to its destination with a rate of $\Theta(n^{-1/2}(\log n)^{-3/2})$. In a mobile network each of the n mobile nodes can achieve the same order of magnitude throughput with a fixed maximum delay constraint that does not depend on n. If each node is willing to tolerate packet delay $\Theta(n^d)$ where $0 < d < 1$, they show that each mobile node can send data to its destination with rate $\Theta(n^{(d-1)/2}(\log n)^{-5/2})$.

Xie and Kumar [27] study the capacity bound in a setting where nodes can employ sophisticated cooperative strategies to achieve interference cancellation. They show that the aggregate capacity of an arbitrary network is upper bounded by $O(\sqrt{n})$ (in a large-area network), assuming some natural signal attenuation law, and the upper bound is sharp for regular planar networks where the nodes reside at integer lattice sites in a square.

Some other researchers develop capacity bounds under different traffic patterns. Gastpar and Vetterli [14] consider the same physical model as in [17], but a different traffic pattern, namely the *relay traffic pattern*. There exists only one (randomly chosen) source-destination pair and all other nodes serve as relay nodes. They show that if arbitrarily complex network coding is allowed, the upper bound and lower bound of the capacity of a wireless network with n nodes under the relay traffic pattern meet asymptotically at $\Theta(\log n)$ as the number n of nodes in the network goes to infinity. Marco *et al.* study the network capacity under the *many-to-one* scenario where there is only one destination and every node needs to transmit packets to the destination. They show that per node capacity scales as $\Theta(1/n)$ as the number n of nodes increases. This is due to the bottleneck at the single destination.

In [25], Toumpis studies the capacity bounds of three classes of wireless networks under fading channels. The first class is asymmetric networks where there are n source nodes and around n^d destination nodes, and each source picks a destination at random. The author show that if $1/2 < d < 1$, an aggregate throughput of $\Omega(n^{1/2}(\log n)^{-3/2})$ is achievable; and if $0 < d < 1/2$, an aggregate throughput of $\Omega(n^d/\log n)$ is achievable. In both cases, the aggregate throughput is upper bounded by $O(n^d \log n)$. The second class is cluster networks where there are n client nodes and around n^d cluster heads. Each client communicates with one of the cluster heads, but the particular choice of the cluster head is not important. They show in this setting, the maximum aggregate throughput is lower bounded by $\Omega(n^d(\log n)^{-2})$ and upper bounded by $O(n^d \log n)$. The third class is hybrid networks where there are n wireless nodes and n^d base stations, and the base stations are connected through wired lines and only used to support the operation of wireless nodes. They show that if $1/2 < d < 1$, the maximum aggregate throughput is lower bounded by $\Omega(n^d(\log n)^{-2})$ and if $0 < d < 1/2$, there is no significant gain of employing the infrastructure. We note the last result is similar to that in [20].

Li *et al.* [19] study the capacity of small ad hoc networks through extensive simulations, which verifies the capacity bound of order $\Theta(1/\sqrt{n})$ to some extent. Finally in a very recent work, Franceschetti *et al.* [11], [12] close the gap between the capacity upper bound and lower bound in Gupta and Kumar's original results [17] under the physical model. They use percolation theory to devise a routing strategy which achieves a per node capacity bound of $\Theta(1/\sqrt{n})$.

VII. Conclusion

In this paper, we have derived lower and upper bounds of the uniform capacity of a power constrained wireless ad hoc network with an arbitrarily large bandwidth. The problem was first introduced and studied in [21]. We close the gap between the lower and upper bounds that exist in [21] and show that both the bounds scale at $\Theta(n^{(\alpha-1)/2})$. Contrary to the results in [17], we demonstrate an increasing per-node throughput capacity as the number n of wireless nodes increases. This is because the bandwidth (spectrum) is assumed to scale with the density of nodes, and the throughput of each link is determined by the Shannon capacity instead of being a constant as in [17].

In order to derive the aforementioned capacity bounds, we have also derived an important result on random geometric graphs: if the distance between two points in a Poisson point process with density n is non-diminishing, the minimum power route requires power rate at least $\Omega(n^{(1-\alpha)/2})$. The upper bound is obtained with this geometric result. The lower bound is obtained by constructing a backbone network that is composed of a sufficient number of disjoint horizontal and vertical paths.

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Appendix I
Proof of Lemma 2

We prove a generalized version of Lemma 2 in the context of the site percolation model. Let $\Omega = \Pi_{s\in\mathbb{Z}^d}\{0,1\}$ be the sample space in the underline probability space, where $\mathbb{Z} = \{\cdots,-1,0,1,\cdots\}$. Points in Ω are represented as $\omega = (\omega(s) : s \in \mathbb{Z}^d)$ and called *configurations*. The value $\omega(s) = 0$ corresponds to the site (grid) s being closed and $\omega(s) = 1$ corresponds to the site s being open. An event A is called *increasing* if $I_A(\omega) \le I_A(\omega')$ whenever $\omega \le \omega'$, where I_A is the indicator function of the event A. (Interested readers should refer to [15] for more details of the definitions.) Let

A be an increasing event. For $\omega \in \Omega$, let $F_A(\omega)$ denote the "distance" of ω from A, i.e.,

$$F_A(\omega) = \inf\left\{\sum_s (\omega'(s) - \omega(s)) : \omega' \geq \omega, \omega' \in A\right\}. \quad (30)$$

Note that $F_A(\omega) = 0$ if $\omega \in A$. The generalized version of Lemma 2 is

$$P_{p_2}(A) \geq \left(\frac{p_2 - p_1}{1 - p_1}\right)^r P_{p_1}(F_A \leq r) \quad (31)$$

for any $0 < p_1 < p_2 < 1$.

Proof. Suppose that $X(s) : s \in \mathbb{Z}^d$ is a family of independent random variables indexed by the grid (site) set $\mathbb{Z}^d$, where each $X(s)$ is uniformly distributed on [0,1]. We may couple together all the site percolation processes on $\mathbb{Z}^d$ in the following way. Let $0 \leq p \leq 1$ and define $\eta_p \in \Omega$ by

$$\eta_p(s) = \begin{cases} 1 & \text{if } X(s) \leq p, \\ 0 & \text{otherwise.} \end{cases} \quad (32)$$

We may think of η_p as the random outcome of the site percolation process on $\mathbb{Z}^d$ with the site-open probability p. It is clear that $\eta_{p_1} \leq \eta_{p_2}$ whenever $p_1 < p_2$. Thus we may couple two percolation processes with site-open probability p_1 and p_2 in such a way that the set of open sites of the first process is a subset of the set of the open sites of the second.

Suppose that $0 \leq p_1 \leq p_2 \leq 1$ and A is an increasing event. Denote $I_r(A) = \{\omega : F_A(\omega) \leq r\}$. If $\eta_{p_1} \in I_r(A)$, there exists a (random) collection $C = C(\eta_{p_1})$ of sites such that

(a) $|C| \leq r$;
(b) $\eta_{p_1}(s) = 0$ for all $s \in C$; and
(c) the configuration η obtained from η_{p_1} by declaring all edges in C to be open, satisfies $\eta \in A$.

Suppose now that every s in the set C satisfies $p_1 \leq X(s) \leq p_2$. It follows from (c) above that $\eta_{p_2} \in A$. Conditioning on (b) above, the probability of $p_1 \leq X(s) \leq p_2$ is $((p_2 - p_1)/(1 - p_1))^{|C|}$. Therefore,

$$P(\eta_{p_2} \in A | \eta_{p_1} \in I_r(A)) \geq \left(\frac{p_2 - p_1}{1 - p_1}\right)^r, \quad (33)$$

since $|C| \leq r$. Eq. (31) follows easily.

Appendix II
Proof of Lemma 3

(i) follows directly from Lemma 1.2 in [22]. Alternatively, this can be proved using Chernoff bound.

(ii) Let N be the number of nodes in the field. By (i) *w.h.p.*, $N \geq n/2$. Now conditioning on $N \geq n/2$, all nodes' locations are uniformly independently distributed on the unit square area. Let d_i be the distance between the ith source-destination pairs and d'_i be the distance between the ith source-destination pairs under Torus convention (for a definition, see [29]). Clearly $d_i \geq d'_i$. Let $I(\cdot)$ denote an indicator function. For any $0 < \epsilon < 1/2$,

$$I(d_i \geq \epsilon) \geq I(d'_i \geq \epsilon) = 1 - \pi\epsilon^2. \quad (34)$$

Among the N source-destination pairs, we can pick $N' = N/3$ pairs such that any two of them do not share a node. Since nodes' locations are independently uniformly distributed, if two source-destination pairs i, j do not share nodes, their distance d_i, d_j (and d'_i, d'_j, respectively in the Torus convention) is independent. Without loss of generality, we can assume the first N' pairs do not share nodes. Thus $I_i \triangleq I(d'_i \geq \epsilon)$ is ***i.i.d.*** Bernoulli random variable with parameter $1 - \pi\epsilon^2$. Let $S_{N'} = \sum_{i \leq N'} I_i$. By Chernoff inequality, for any $\theta < 0, a > 0$,

$$\begin{aligned} & P(S_{N'} \leq aN') \\ \leq\ & E[\exp(\theta(S_{N'} - aN'))] \\ =\ & \exp(N'(\ln E[e^{\theta I_i}] - \theta a)) \\ =\ & \exp(N'(\ln(\pi\epsilon^2 + (1 - \pi\epsilon^2)e^\theta) - \theta a) \end{aligned} \quad (35)$$

Let $\theta = -1, a = 3/4$ and ϵ sufficiently small, we have $\delta \triangleq \ln(\pi\epsilon^2 + (1 - \pi\epsilon^2)e^\theta) - \theta a < 0$ and

$$P(S_{N'} \leq 3N'/4) \leq \exp(N'\delta) \to 0 \text{ as } N' \to \infty. \quad (36)$$

Thus *w.h.p.*, the number of pairs with distance at least ϵ is at least $3N'/4 = N/4 \geq n/8$. □

Appendix III
Proof of Lemma 6

By the choice of the exit points, a given exit point i will only be responsible to the destination nodes in a rectangle of size not larger than $(1/(\beta\sqrt{2n}/c_5)) \times (c_5/\sqrt{2n})$. The number M_i of nodes inside this area has Poisson distribution with parameter $c_7 = c_5^2/(2\beta)$. By Chernoff inequality, for any $\theta \geq 0$, we have

$$\begin{aligned} & P(M_i \geq c_7 \log n) \\ \leq\ & E[\exp(\theta M_i - \theta c_7 \log n)] \\ =\ & \exp(c_7(e^\theta - 1) - \theta c_7 \log n) \end{aligned} \quad (37)$$

Let $\theta = 2/c_7$, we have

$$P(M_i \geq c_7 \log n) \leq \exp(c_7(e^{2/c_7} - 1))\ n^{-2} \quad (38)$$

Again *w.h.p.*, there are less than $2n$ exit points. Conditioning on this,

$$P(\exists i, s.t.\ M_i \geq c_7 \log n) \leq 2n \cdot \exp(c_7(e^{2/c_7} - 1))\ n^{-2}. \quad (39)$$

The right equation tends to 0 as $n \to \infty$. Therefore, *w.h.p.*, every exit point needs to deliver packets to at most $c_7 \log n$ destinations.