

Bootstrapped Oblivious Transfer and Secure Two-Party Function Computation¹

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Abstract—We propose an information theoretic framework for the secure two-party function computation (SFC) problem and introduce the notion of SFC capacity. We study and extend string oblivious transfer (OT) to *sample-wise* OT. We propose an efficient, *perfectly private* OT protocol utilizing the binary erasure channel or source. We also propose the *bootstrap* string OT protocol which provides disjoint (weakened) privacy while achieving a multiplicative increase in rate, thus trading off security for rate. Finally, leveraging our OT protocol, we construct a protocol for SFC and establish a general lower bound on SFC capacity of the binary erasure channel and source.

I. INTRODUCTION

Motivated by applications ranging from confidential database access to oblivious contract negotiation [1], we study the problem of *secure two-party function computation* (SFC). In this problem, Alice and Bob each have private data, and they wish to compute functions of both of their data. The objective is to design a protocol that ensures *correctness* of the computed functions while maintaining individual *privacy*, in the sense that neither party gains any information about the other's data other than what can be inferred from the result of their function computation. An important special case of this problem is *string oblivious transfer* (OT) from [2], wherein Alice has two strings \tilde{A}_0 and \tilde{A}_1 and Bob has a single bit B . An OT protocol should reveal \tilde{A}_B to Bob, while Alice remains ignorant of B and Bob of $\tilde{A}_{(1-B)}$.

In this work, we propose an information theoretic framework for SFC and introduce the notion of SFC rates and capacity, in terms of the ratio of samples of computation to samples of *correlated randomness* needed. Correlated randomness is a noisy resource in the form of a noisy communication channel or distributed random source available between the parties. We cast the string OT problem as a special case within our framework and also introduce the *sample-wise* OT problem. We address the string and sample-wise OT problems with an efficient *perfectly private* protocol utilizing the binary erasure channel or source. For the string OT problem, we also propose the *bootstrap* protocol which provides disjoint (weakened) privacy while achieving a multiplicative increase in rate, thus trading off security for rate. Finally, leveraging

our OT protocol, we construct a protocol for SFC and establish a general lower bound on SFC capacity binary erasure channel and source. Due to space limitations, detailed proofs are omitted, but will appear in an extended version of this work.

Our objective is information theoretic (unconditional) security, where even computationally unbounded adversaries must not be able to break the privacy. We work with the assumption of *semi-honest* (or passive) parties, where the parties honestly follow the protocol. It is well-known that in this setting both OT and SFC cannot be realized “from scratch” [3], [4], that is with protocols using only noise-free communication channels and local randomness. It has been observed that OT becomes possible given correlated randomness [5], [6], [7], [8], and that SFC also becomes possible based on OT [3]. Thus, correlated randomness is a valuable resource as an enabling factor for OT and SFC. Recently the concept of *OT capacity* of a channel or source, measuring the fundamental limit of how efficiently the resource can be used toward OT, has been introduced in [9], [7] and further characterized by [8], [10].

II. PROBLEM FORMULATION

In this section, we first formulate the SFC problem within a novel information theoretic framework. This framework utilizes conditional mutual information based privacy measures, and defines achievable function computation rate and capacity. We then discuss OT which is encompassed by this SFC framework as a special case. In the last subsection we present the problem of string OT with the notion of *disjoint privacy*.

A. Secure Two-Party Function Computation

Two parties, Alice and Bob, each have k samples of a jointly distributed source on the finite alphabets $\mathcal{A} \times \mathcal{B}$, where Alice possesses $A^k \triangleq \{A_1, \dots, A_k\} \in \mathcal{A}^k$ and Bob possesses $B^k \triangleq \{B_1, \dots, B_k\} \in \mathcal{B}^k$, with $(A^k, B^k) \sim P_{A^k, B^k}$. For a given function $f: \mathcal{A} \times \mathcal{B} \rightarrow \mathcal{R}_f$, Alice wishes to compute samples of a function of the sources $F^k \triangleq \{f(A_1, B_1), \dots, f(A_k, B_k)\}$. Similarly, Bob wishes to compute $G^k \triangleq \{g(A_1, B_1), \dots, g(A_k, B_k)\}$ where $g: \mathcal{A} \times \mathcal{B} \rightarrow \mathcal{R}_g$. Alice and Bob cooperatively compute these functions via an interactive protocol that may exchange messages over an error-free discussion channel and also utilize n samples of *correlated randomness*. The correlated randomness is a precious resource which comes in two possible forms:

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- *Source-model*: For $i = 1, \dots, n$, $(X_i, Y_i) \stackrel{\text{iid}}{\sim} P_{X,Y}$. $X^n \triangleq (X_1, \dots, X_n)$ is available to Alice and $Y^n \triangleq (Y_1, \dots, Y_n)$ to Bob.
- *Channel-model*: X^n and Y^n are respectively the sequence of inputs and outputs of a discrete memoryless channel (DMC) with conditional distribution $P_{Y|X}$, with X^n selected by Alice and Y^n received by Bob.

An acceptable (n, k) -protocol for *source-model* correlated randomness is defined as follows. First, Alice and Bob receive (A^k, X^n) and (B^k, Y^n) , and generate local random variables Z_A and Z_B respectively, where $Z_A, Z_B, (A^k, B^k)$, and (X^n, Y^n) are mutually independent. Then, over r stages, Alice and Bob exchange messages M_1, \dots, M_r over the error-free discussion channel, where in an odd numbered stage i Alice produces message M_i as a function of everything available to her, namely (A^k, X^n, Z_A, M^{i-1}) , and in an even numbered stage j Bob produces the messages M_j as a function of everything available to him, namely (B^k, Y^n, Z_B, M^{i-1}) . At the end of the protocol, Alice and Bob produce function estimates \hat{F}^k and \hat{G}^k as functions of (A^k, X^n, Z_A, M^r) and (B^k, Y^n, Z_B, M^r) respectively.

An acceptable (n, k) -protocol for *channel-model* correlated randomness is similar to the source-model protocol, but (X^n, Y^n) are not given at the beginning of the protocol. Instead, the samples X^n are generated by Alice, transmitted into the DMC, and outputs Y^n are received by Bob. The DMC transmissions may be arbitrarily interspersed with discussion stages (including happening entirely before or after the discussion messages are exchanged). At each stage or transmission, the discussion message or channel input symbol is a function of everything available to the sending party. A source-model protocol can be realized as a special case of the channel-model if Alice randomizes the inputs, for $i = 1, \dots, n$, $X_i \stackrel{\text{iid}}{\sim} P_X$, and transmits before any discussion messages are sent.

For both models, $R > 0$ is called an *achievable SFC rate* for the particular sources, functions, and correlated randomness if for every $\epsilon > 0$, and all sufficiently large n , there exists an acceptable (n, k) -protocol with $(k/n) > R - \epsilon$ satisfying the following

- (*Correctness*) $\Pr[\hat{F}^k \neq F^k] < \epsilon$ and $\Pr[\hat{G}^k \neq G^k] < \epsilon$,
- (*Privacy for Alice*)

$$I(A^k; Z_B, Y^n, M^r | B^k, G^k) < \epsilon, \quad (1)$$

- (*Privacy for Bob*)

$$I(B^k; Z_A, X^n, M^r | A^k, F^k) < \epsilon. \quad (2)$$

A protocol is said to be *perfectly private* if the privacy constraints of (1) and (2) are exactly zero. The *SFC capacity* C for the particular sources, functions, and correlated randomness is defined as the largest achievable function computation rate, and 0 if no rate $R > 0$ is achievable.

B. Sample-wise Oblivious Transfer

The 1-out-of- m sample-wise OT problem is a special case of the SFC problem, wherein Alice's source alphabet is $\mathcal{A} =$

$\{0, 1\}^m$, Bob's source alphabet is $\mathcal{B} = \{1, \dots, m\}$, Alice's function is constant $f = 0$, and Bob's function is given by $g((a_1, \dots, a_m), b) = a_b$. For clarity of exposition, let \mathbf{A} be the $k \times m$ binary matrix formed by vertically stacking Alice's m -bit samples A_1, \dots, A_k as the rows. Bob wishes to receive the k bits G_1, \dots, G_k , where $G_i = \mathbf{A}_{i, B_i}$. Alice's privacy condition (1) means that Bob obtains no information about the other $k(m-1)$ bits of \mathbf{A} that he did not select. Bob's privacy condition (2) means that Alice obtains no information about Bob's selection B^k . When dealing with the above scenario, we speak of achievable sample-wise OT rate $R_{OT,m}$, and the sample-wise OT capacity $C_{OT,m}$.

C. String Oblivious Transfer with Disjoint Privacy

The 1-out-of- m string OT problem is a special case of the 1-out-of- m sample-wise OT problem, wherein the source distribution is specified as

$$P_{A^k, B^k}(a^k, b^k) = \begin{cases} \frac{1}{m2^{km}}, & \text{if } b_1 = \dots = b_k \\ 0, & \text{otherwise,} \end{cases}$$

that is, Alice's source samples A^k consist of km iid Bernoulli- $(1/2)$ bits and is independent of Bob's source B^k which always consists of identical samples uniformly distributed over $\mathcal{B} = \{1, \dots, m\}$. Interpreting this scenario, Alice has m, k -bit strings $\tilde{A}_1, \dots, \tilde{A}_m$, which are aligned as the *columns* of \mathbf{A} , and Bob has the selection $B \triangleq B_1$ and wishes to receive the k -bit string \tilde{A}_B .

Alice's privacy condition (1) reduces to

$$I(\{\tilde{A}_i\}_{i=1, i \neq B}^k; Z_B, Y^n, M^r | B, \tilde{A}_B) < \epsilon,$$

which implies that Bob is unable to reconstruct any string that he did not select or any non-trivial *joint function* of the strings that he did not select without non-negligible probability of error. The interesting alternative notion of *disjoint privacy* replaces Alice's privacy condition (1) with

$$\text{for } i \in \{1, \dots, m\}, \quad I(\tilde{A}_i; Z_B, Y^n, M^r | B, \tilde{A}_B) < \epsilon, \quad (3)$$

which implies that Bob is unable to reconstruct any non-trivial function of any individual string (including the string itself) that he did not select without non-negligible probability of error. A protocol that satisfies (1) will also satisfy this disjoint privacy condition (3), however the converse is not true. For example, a protocol that reveals to Bob \tilde{A}_B and also the binary exclusive-or (XOR) all of the strings $\tilde{A}_1 \oplus \dots \oplus \tilde{A}_m$ will satisfy the disjoint privacy constraint, but will not satisfy (1). A protocol obtains *perfect disjoint privacy* if the privacy constraints of (3) are exactly zero.

The motivation for considering the weakened sense of disjoint privacy is to explore protocols (see Section IV) that tradeoff privacy in order to achieve higher rates than the sample-wise OT protocol of Section III-A. When dealing with the above scenario with the disjoint privacy condition (3) replacing Alice's standard privacy condition (1), we speak of achievable string OT rate with disjoint privacy $\tilde{R}_{OT,m}$, and string OT capacity with disjoint privacy $\tilde{C}_{OT,m}$.

III. SAMPLE-WISE OBLIVIOUS TRANSFER

In this section, we present the sample-wise oblivious transfer (**SWOT**) protocol. Later on, by leveraging the **SWOT** protocol, we construct protocols for string OT with disjoint privacy (see Section IV-A) and for SFC (see Section V-A). The **SWOT** protocol utilizes correlated randomness in form of the binary erasure channel (BEC) and the binary erasure source (BES). The $\text{BEC}(p)$ has the input and output alphabets $\mathcal{X} = \{0, 1\}$ and $\mathcal{Y} = \{0, 1, e\}$, with the conditional distribution $P_{Y|X}(y|x) = p\mathbf{1}_{(y=e)} + (1-p)\mathbf{1}_{(y=x)}$, where p is the probability of erasure. The $\text{BES}(p)$ has the joint distribution $P_{X,Y}(x,y) = (1/2)P_{Y|X}(y|x)$. The protocol will be described as a source-model protocol for the $\text{BES}(p)$, which is easily adapted into a channel-model protocol for the $\text{BEC}(p)$ by adding the initial step of Alice transmitting n iid Bernoulli- $(1/2)$ bits into the BEC in order to simulate n samples of a BES. Because of the interchangeability of the BEC and BES, we will write $\text{BES/BEC}(p)$ to denote that the correlated randomness is either the $\text{BEC}(p)$ or $\text{BES}(p)$.

A. Sample-wise Oblivious Transfer Protocol

This protocol is inspired by the protocols for the binary erasure channel given by [8], [9]. The novel aspects of our protocol are the treatment of *sample-wise* as opposed to *string* oblivious transfer and the mechanism of failing into an error case when privacy cannot be provided. This yields a perfectly private protocol with roughly the same negligible probability of error, and also simplifies analysis of both privacy and error. The perfect privacy of this protocol is important since it enables it to be leveraged in a secure black-box manner to construct other protocols without complicating the analysis of privacy. The basic idea of this protocol is to use the erasures of the BES/BEC to conceal the $k(m-1)$ bits at the locations in \mathbf{A} that Bob must remain ignorant of, while using the non-erasures to reveal the k bits at the locations in \mathbf{A} that Bob has selected.

- Bob partitions $\{1, \dots, n\}$ into the set of locations of erasures S_e and locations of non-erasures S in Y^n , that is, $Y_i = e$ if and only if $i \in S_e$, and $S = \{1, \dots, n\} \setminus S_e$.
- If there is not enough erasures or non-erasures, $k > |S|$ or $k(m-1) > |S_e|$, then the protocol aborts and Bob sets his function estimate to $\hat{G}^k = 0^k$.
- Otherwise, the protocol continues and Bob creates an $k \times m$ matrix \mathbf{U} , where at each of the k positions specified by $\{(i, B_i) : i = 1, \dots, k\}$, a random, uniform selection, without replacement, from S is placed. Similarly, at the other $k(m-1)$ positions, a random selection from S_e is placed. Bob sends the matrix \mathbf{U} to Alice via the discussion channel.
- The $k \times m$ matrix \mathbf{U} , whose elements belong to $\{1, \dots, n\}$ specifies how Alice should arrange km of the n bits X^n into the $k \times m$ binary matrix $\mathbf{X}_{\mathbf{U}}$, via $\mathbf{X}_{\mathbf{U}}(i, j) = X_{\mathbf{U}(i, j)}$. Alice computes $\mathbf{C} = \mathbf{A} \oplus \mathbf{X}_{\mathbf{U}}$, where \oplus denotes element-wise binary exclusive-or (XOR), and sends \mathbf{C} to Bob over the discussion channel.

- Bob is able create his function estimate \hat{G}^k by reversing the XOR since $\mathbf{Y}_{\mathbf{U}}$ is equal to $\mathbf{X}_{\mathbf{U}}$ at locations corresponding to the locations of \mathbf{A} that he has selected.

B. Analysis and Achievable Rates

The **SWOT** protocol is perfectly private for Bob since any \mathbf{U} is uniformly possible given any realization of Bob's samples B^k because erasures uniformly and independently occur in Y^n . The protocol is perfectly private for Alice since \mathbf{C} is only sent to Bob if the protocol does not abort and there have been enough erased bits, acting as a Bernoulli- $(1/2)$ one-time pad, to mask the $k(m-1)$ bits that should be concealed. The protocol is correct if it does not abort, thus the probability of error is bounded by the probability of aborting, which becomes negligible for n sufficiently large if $k < n(1-p) = \mathbb{E}|S|$ and $k(m-1) < np = \mathbb{E}|S_e|$, by the law of large numbers. Thus, the rate $R_{OT,m} = \min((1-p), p/(m-1))$ is achievable by this protocol. This protocol is distribution-free since the above arguments hold not only for any distribution, but also for any realization of the sources (A^k, B^k) . These results are summarized in the following theorem

Theorem 3.1: For any arbitrary source distribution P_{A_k, B_k} , the **SWOT** protocol, utilizing correlated randomness in the form a $\text{BES/BEC}(p)$, obtains perfect privacy and achieves the 1-out-of- m sample-wise OT rate

$$R_{OT,m} = \min((1-p), p/(m-1)).$$

Hence, the 1-out-of- m sample-wise OT capacity for a $\text{BES/BEC}(p)$ and for arbitrary source distributions is bounded below by

$$C_{OT,m} \geq R_{OT,m} = \min((1-p), p/(m-1)).$$

An upper bound to the sample-wise OT capacity for general correlated randomness is established by the following theorem.

Theorem 3.2: For the uniform source distribution and general source-model correlated randomness, we have

$$C_{OT,m} \leq \min(I(X; Y), H(X|Y)/(m-1)).$$

For channel-model correlated randomness, the right side of the above expression is maximized over P_X .

The proof of this theorem is omitted due to space limitations. It follows from the methods and results used in [8, Theorem 1] and [9, Lemma 7].

For the $\text{BES}(p)$, $H(X|Y) = p$ and $I(X; Y) = 1 - p$. For the $\text{BEC}(p)$, $\max_{P_X} \min(I(X; Y), H(X|Y)/(m-1)) = \min((1-p), p/(m-1))$ with the maximum achieved by $P_X = (1/2)$. Thus, the capacity upper bound of Theorem 3.2 matches the capacity lower bound of Theorem 3.1, implying the following corollary.

Corollary 3.1: The 1-out-of- m sample-wise OT capacity for the uniform source distribution and correlated randomness in the form of a $\text{BES/BEC}(p)$ is given by

$$C_{OT,m} = \min((1-p), p/(m-1)).$$

The **SWOT** protocol achieves capacity.

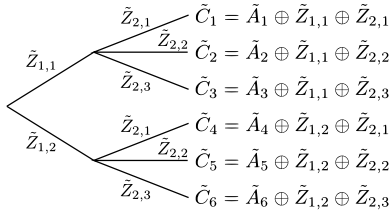


Fig. 1. The encoding tree structure for the **BOOT** protocol for 1-out-of-6 string OT, with parameters $u = 2$, $s_1 = 2$ and $s_2 = 3$.

Note that this capacity is maximized at $C = (1/m)$ for the erasure probability $p^* = (m-1)/m$, where the ratio of erasures to non-erasures matches the ratio of bits concealed to bits revealed. The **SWOT** protocol achieves capacity since it efficiently utilizes the erasures and non-erasures in revealing and concealing the appropriate bits.

IV. STRING OT WITH DISJOINT PRIVACY

The bootstrap OT (**BOOT**) protocol addresses the problem of string OT with the disjoint privacy condition (3). The **SWOT** protocol could also be applied to this problem, yielding the achievable rate given in Theorem 3.1 with the stronger sense of joint privacy (1). However, the **BOOT** protocol achieves rates that are better by a factor up to $((m-1)/\lceil \log_2 m \rceil)$ (when the probability of erasure $p \leq 1/2$) since it provides only disjoint privacy.

A. Bootstrap String Oblivious Transfer Protocol

The **BOOT** protocol for 1-out-of- m string OT is parameterized by a finite sequence of u integers, $s_1, \dots, s_u \in \{2, \dots, m\}$, such that $\prod_{i=1}^u s_i \geq m$. The **BOOT** protocol leverages u uses of the **SWOT** protocol, where the i -th usage is for 1-out-of- s_i OT. For $i = 1, \dots, u$, Alice generates s_i independent k -bit Bernoulli- $(1/2)$ masking strings $\{\tilde{Z}_{i,j}\}_{j=1}^{s_i}$. The basic idea is to encode each one of Alice's strings with the XOR of a different combination of u of these masking strings, taking one from each set $\{\tilde{Z}_{i,j}\}_{j=1}^{s_i}$ for $i = 1, \dots, u$. Alice first sends these encodings, denoted by $\tilde{C}_1, \dots, \tilde{C}_m$, to Bob over the discussion channel. Then, for Bob to decode a particular string of Alice's, Alice and Bob perform u oblivious transfers where in the i -th OT Bob chooses from $\{\tilde{Z}_{i,j}\}_{j=1}^{s_i}$ the masking string that is part of the combination masking the string of Alice's that he wants. The method in which each string of Alice is assigned a unique combination of masking strings can be visualized by a tree structure.

The encoding tree structure for the example of 1-out-of-6 string OT via the **BOOT** protocol with parameters $u = 2$, $s_1 = 2$ and $s_2 = 3$ is illustrated in Figure 1. In this example, if Bob wishes to obtain \tilde{A}_3 , he would select $\tilde{Z}_{1,1}$ in first round of OT and then select $\tilde{Z}_{2,3}$ in second round of OT, allowing him to reconstruct \tilde{A}_3 via $\tilde{C}_3 \oplus \tilde{Z}_{1,1} \oplus \tilde{Z}_{2,3}$.

B. Analysis and Achievable Rates

The perfect privacy of the **SWOT** protocol guarantees that Bob only learns the particular combination of masking strings $\tilde{Z}_{i,j}$ that he selected, however, by the structure of the encoding,

its possible for Bob to learn some information about Alice's strings beyond just the knowledge of \tilde{A}_B . However, Bob will not be able to determine the specific value of any particular string \tilde{A}_i for $i \neq B$. Consider the example of 1-out-of-6 string OT illustrated in Figure 1 for $B = 3$. Since Bob learns $\tilde{Z}_{1,1}$ and $\tilde{Z}_{2,3}$, he can also determine certain joint functions of Alice's strings such as $\tilde{A}_1 \oplus \tilde{A}_4 \oplus \tilde{A}_6$, which can be found from $\tilde{C}_1 \oplus \tilde{C}_4 \oplus \tilde{C}_6 \oplus \tilde{Z}_{1,1} \oplus \tilde{Z}_{2,3}$. Note that however, Bob cannot reduce any of the equations further to determine the value of any \tilde{A}_i for $i \neq 3$. The proof for the general situation is omitted due to space limitations.

The correctness of the protocol follows if each of the u usages of the **SWOT** protocol is correct, which happens if there is enough erasures and non-erasures in BES/BEC samples in each usage. Note that instead, all of the BES/BEC samples can be taken at the beginning, with the erasures and non-erasures being allocated to multiple usages if they are sufficient, which will happen with high probability for n sufficiently large provided that k/n is slightly less than the achievable rate determined by the following rate analysis.

For each round $i = 1, \dots, u$, the **SWOT** protocol for 1-out-of- s_i OT of k -bit strings requires asymptotically $n_i = k/R_{OT,s_i}$ samples of the BES/BEC. The total number of samples of BES/BEC needed is $n = \sum_{i=1}^u n_i = \sum_{i=1}^u k/R_{OT,s_i}$. Thus, the asymptotic rate achieved by this protocol is given by the following theorem.

Theorem 4.1: Let $s_1, \dots, s_u \in \{2, \dots, m\}$ be a finite sequence of integers such that $\prod_{i=1}^u s_i \geq m$. Then, the **BOOT** protocol with parameters (s_1, \dots, s_u) for a BES/BEC(p) obtains perfect disjoint privacy and achieves the string OT rate

$$\tilde{R}_{OT,m} = \left(\sum_{i=1}^u \frac{1}{R_{OT,s_i}} \right)^{-1},$$

where $R_{OT,s} = \min((1-p), p/(s-1))$ is the achievable sample-wise OT rate for the BES/BEC(p) from Theorem 3.1. Hence, the string OT capacity (with disjoint privacy) is bounded below by

$$\tilde{C}_{OT,m} \geq \max_{u, s_1, \dots, s_u} \left(\sum_{i=1}^u \frac{1}{R_{OT,s_i}} \right)^{-1},$$

where the maximization is taken over the set of finite sequences of integers $s_1, \dots, s_u \in \{2, \dots, m\}$ such that $\prod_{i=1}^u s_i \geq m$.

The **SWOT** protocol is most efficient for erasure probability $p^* = (m-1)/m$, since it needs a large proportion of erasures to fully conceal the $k(m-1)$ bits that Bob did not select. The best erasure probability for the **BOOT** protocol, however, is variable and depends on the choice of parameters. For example, setting $u = \lceil \log_2 m \rceil$ and $s_i = 2$ for all i , yields the achievable rate

$$\tilde{R}_{OT,m} = \frac{R_{OT,2}}{\lceil \log_2 m \rceil} = \frac{\min(p, (1-p))}{\lceil \log_2 m \rceil},$$

which is maximized at $\tilde{R}_{OT,m} = 1/(2\lceil \log_2 m \rceil)$ for $p = (1/2)$. Comparing this to the achievable sample-wise OT

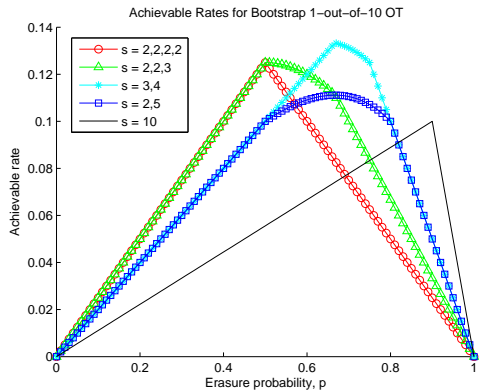


Fig. 2. Achievable rates of the **BOOT** protocol for 1-out-of-10 string OT as a function of erasure probability p of the BES/BEC(p). Each curve represents the achievable rates for a different set of parameters of the **BOOT** protocol.

rate for $p = (1/2)$, $R_{OT,m} = 1/(2(m-1))$ reveals an improvement in rate by a factor of $((m-1)/(\lceil \log_2 m \rceil))$. The **BOOT** protocol achieves higher rates since it effectively recycles the erasures to conceal more bits. Some information is leaked since the erasures are being recycled, however, only joint functions of the strings (specifically the exclusive-or of multiple strings) are revealed while maintaining the disjoint privacy. Note that for the parameters $u = 1$ and $s_1 = m$, the **BOOT** protocol achieves the same rate as the **SWOT** protocol. Thus, the **BOOT** protocol can achieve any rate achieved by the **SWOT** protocol.

Figure 2 illustrates the achievable rates of the **BOOT** protocol, with $m = 10$, as a function of erasure probability p of the BES/BEC(p), for various sets of parameters. Note that in different ranges, different sets of parameters are best. The **BOOT** protocol for parameters $\{s_1 = 10\}$ (giving the performance of the **SWOT** protocol) is best only in the range of erasure probability close to $p = (m-1)/m$ and above.

V. SECURE FUNCTION COMPUTATION

The general secure function computation (**GSFC**) protocol leverages the **SWOT** protocol. It uses two oblivious transfers, where the first is from Alice to Bob and the second is from Bob to Alice, reversing the roles. Since the **SWOT** protocol uses a BES/BEC in the direction of the transfer, the **GSFC** protocol uses a BES/BEC available in both directions. The rate is determined as the ratio of function samples k to the total number of BES/BEC samples used in both directions.

A. General SFC Protocol

This protocol is applicable to any general sources and functions. Without loss of generality, let the finite source alphabets be given by $\mathcal{A} = \{1, \dots, m_A\}$ and $\mathcal{B} = \{1, \dots, m_B\}$, and the ranges of the functions f and g be $\mathcal{R}_f = \{0, 1\}^{h_A}$ and $\mathcal{R}_g = \{0, 1\}^{h_B}$ respectively.

We outline the **GSFC** protocol with the following steps. For Bob to compute G^k , Alice generates m_B , kh_B -bit strings, for $i = 1, \dots, m_B$, $\tilde{A}'_i = (g(A_1, i), \dots, g(A_k, i))$. Bob expands his k source samples B^k to a vector of length kh_B , where each

element of B^k is repeated h_B times to produce the samples B^{kh_B} . Alice and Bob then use the **SWOT** protocol to perform 1-out-of- m_B OT for kh_B -bit strings with $\{\tilde{A}'_i\}_{i=1}^{m_B}$ as Alice's strings, and Bob's selections vector as B^{kh_B} . The result of this OT gives Bob $(g(A_1, B_1), \dots, g(A_k, B_k))$. Similarly, for Alice to compute F^k , Alice and Bob reverse roles and perform 1-out-of- m_A OT for kh_B -bit strings from Bob to Alice.

B. Analysis and Achievable Rates

The perfect privacy, negligible probability of error, and distribution-free properties of **SWOT** protocol imply the same properties in this secure function computation protocol. The 1-out-of- m_B OT for kh_B -bit strings via the **SWOT** protocol asymptotically requires $n_1 = kh_B/R_{OT,m_B}$ samples of a BES/BEC from Alice to Bob, and likewise the other OT requires $n_2 = kh_A/R_{OT,m_A}$ samples of a BES/BEC from Bob to Alice, yielding the following theorem.

Theorem 5.1: Let $m_A = |\mathcal{A}|$, $m_B = |\mathcal{B}|$, $h_A = \lceil \log_2 |\mathcal{R}_f| \rceil$, and $h_B = \lceil \log_2 |\mathcal{R}_g| \rceil$. Then, the **GSFC** protocol, utilizing correlated randomness in the form a BES/BEC(p) available in both directions, is perfectly-private, distribution-free and achieves the function computation rate

$$R = \left(\frac{h_B}{R_{OT,m_A}} + \frac{h_A}{R_{OT,m_B}} \right)^{-1},$$

where $R_{OT,m} = \min((1-p), p/(m-1))$ is the achievable sample-wise OT rate from Theorem 3.1. The function computation capacity is bounded below by $C \geq R$.

The **GSFC** protocol is general, but not optimal since it does not exploit any source correlation or functional structure. Note that only one usage of the **SWOT** is necessary if one of the functions f, g is a function of (or the same as) the other function.

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