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On the Performance of Full-Duplex Relaying Schemes for Point-to-Point MIMO with Large Antenna Arrays

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***Abstract*—The performance of full-duplex (FD) decode-and-forward (DF) relaying systems for point-to-point multi-antenna transmission is considered. Three different relaying schemes are investigated: co-located, distributed cooperative and distributed non-cooperative. To mitigate the effects of loop interference (LI) caused by FD operation at the relay, a digital cancellation scheme based on pilot-aided channel estimation is used. Asymptotic analysis shows that all considered systems are inter-pair and LI free when the number of antennas at the source and destination grows without bound while the relay has a finite number of antennas. More careful analysis of the achievable rate reveals, however, that the LI has a significant impact on the performance of finite sized systems. The numerical results illustrate that non-cooperative distributed relaying suffers severely from FD operation under realistic scenarios, while cooperation allows for efficient LI cancellation and improved spectral efficiency over half-duplex (HD) systems. The results also demonstrate that the optimal number of antennas used by a FD relay is only 10%–30% of the size of the array used at the source and destination, while HD relaying benefits from fractions of up to 50%.**

I. Introduction

Full-duplex (FD) relaying has been intensively studied in recent years due to its ability to double the spectral efficiency of half-duplex relaying under optimal conditions [1], [2]. The improvement in spectral efficiency is obtained by simultaneously transmitting and receiving signals in the same frequency band [3], so that the precious frequency and time resources do not need to be shared as in half-duplex (HD) systems. However, short distance between transmit and receive antennas in the FD device causes strong self-interference, or loop interference (LI), that can severely degrade the performance of FD systems. Thus, effective loop interference mitigation schemes are needed for the FD systems to operate optimally. Recent studies have shown that a combination of hardware implementations, such as array placement and cross-polarization, along with analog and digital LI cancellation schemes can mitigate LI to a tolerable level [1], [4], [5]. In addition to FD transceivers, the future wireless networks will almost surely rely on multiple-input multiple-output (MIMO) techniques to take advantage of the enormous multiplexing and array gain offered by rich scattering wireless environments. Indeed, combination of these two techniques for relay channels has been an active field of research recently (see, e.g. [1], [6], [7] and references therein).

In this paper, a decode-and-forward (DF) relay system is considered. The relaying is carried out either by a single relay node with multiple co-located antennas, or a set of individual distributed nodes that may operate cooperatively or fully independently. In case of geographically distributed nodes, inter-relay cooperation facilitates the relay stations to exchange information that can be used, for example to cancel LI, at the cost of processing delay and additional bandwidth [8]. On the other hand, single relay node equipped with multiple antennas will suffer from more serious initial LI since the antennas are closely placed, but does not suffer from difficulties in acquiring information needed for LI cancellation.

Given the three scenarios described above, some interesting questions arise: 1) Does the attenuation provided by the propagation environment mitigate LI sufficiently, so that a simple distributed solution with no cooperation works efficiently in FD mode; 2) Are there any scenarios where simple HD relaying is more effective than using FD; 3) How many antennas should the relay node(s) be equipped with, given that the source and destination have (relatively) large antenna arrays. Especially the last question is not trivial since while it is clear that configuring more antennas at the relay will provide higher multiplexing gains, transmitting large numbers of pilots for channel estimation in turn degrades the spectral efficiency and potentially increases the LI in FD mode.

The present work analyzes the above questions under the assumption that the source and destination employ linear processing adhering to the "Massive MIMO" principles [9]–[11]. Similar system, but with a large antenna array at the relay node instead of the source and destination, was studied in [6], where it was shown that while the LI vanishes asymptotically, it can still cause significant problems for FD operation in finite sized systems. Our results show that the "mirror image" scenario studied herein exhibits a similar behavior, that is, the interference vanishes asymptotically for all considered scenarios, but is far from negligible for realistic system sizes. This leads to a conclusion that a system based on non-cooperative distributed relay nodes is a viable solution in practice only when placed very far apart so that LI between the nodes is heavily attenuated. The numerical results also demonstrate that the need for channel estimation based LI cancellation leads to relatively small FD relay antenna numbers of around 10%–30% of the antennas employed at the source and destination.

II. System model

Consider a decode-and-forward relaying system depicted in Fig. 1. where the source (S) and destination (D) are equipped with M antennas each. We assume that the direct link between the source and destination is blocked, so that a relay node (R) is needed in-between to facilitate the transmission. Herein two types of FD relaying strategies are considered: 1) a single relay station with $2K$ antennas (co-located) and 2) a set of K individual relay stations equipped with 2 antennas each (distributed). Under both scenarios, the relay(s) operate in FD mode by using each antenna either for transmitting or receiving but not both at the same time. The co-located relay station in this study can always process signals jointly, while the distributed setup can be either cooperative or non-cooperative. The source operates always in HD mode.

At time instant i, the received signals at the relay and destination are given by

$$\mathbf{y}_R = \sqrt{\rho_S}\mathbf{G}_{SR}^T\mathbf{x}[i] + \sqrt{\rho_R}\mathbf{G}_{RR}(\mathbf{s}[i]+\mathbf{u}_R) + \mathbf{n}'_R, \quad (1)$$
$$\mathbf{y}_D = \sqrt{\rho_R}\mathbf{G}_{RD}(\mathbf{s}[i]+\mathbf{u}_R) + \mathbf{n}'_D, \quad (2)$$

respectively. The transmitted signals $\mathbf{x}[i] \in \mathbb{C}^{M\times 1}$, $\mathbf{s}[i] \in \mathbb{C}^{K\times 1}$ satisfy power constraints $\mathbb{E}\{|\mathbf{x}[i]|^2\} = \mathbb{E}\{|\mathbf{s}[i]|^2\} = 1$ so that ρ_S and ρ_R are the total transmit powers of the source and relay. $\mathbf{G}_{SR}^T \in \mathbb{C}^{K\times M}$ and $\mathbf{G}_{RD} \in \mathbb{C}^{M\times K}$ represent the S$\rightarrow$R and R$\rightarrow$D channel matrices, respectively. We assume TDD operation and reciprocity so that the R$\rightarrow$S channel is $\mathbf{G}_{SR}$. For the S$\rightarrow$R and R$\rightarrow$D links, we let $\mathbf{G}_* = \mathbf{H}_*\mathbf{D}_*^{1/2}$, where the entries of $\mathbf{H}_*$ are i.i.d. standard complex Gaussian random variables (RVs) representing small scale fading. The large scale attenuation is modeled by diagonal matrices $\mathbf{D}_{SR}$ and $\mathbf{D}_{RD}$ whose kth diagonal entries are denoted $\beta_{SR,k}$ and $\beta_{RD,k}$, respectively. The LI matrix $\mathbf{G}_{RR} \in \mathbb{C}^{K\times K}$ has independent circularly symmetric complex Gaussian (CSCG) elements, where the kjth element has variance $\beta_{kj} = \mathbb{E}\{|g_{kj}|^2\}$. Throughout the paper we assume that the variance of diagonal elements g_{kk} of the LI channel are the same for all nodes, i.e. $\beta_{kk} = \beta\ \forall k$ since they are always located in same device and, thus, close to each other. By the same argument, in the co-located case (single relay node) we have $\beta_{kj} = \beta\ \forall k, j$.

Finally, the elements of the receive-side noise vectors $\mathbf{n}'_R$ and $\mathbf{n}'_D$ are i.i.d. standard complex Gaussian RVs. The system model also encompasses transmit-side noise that is modeled by vector $\mathbf{u}_R$ whose entries are i.i.d. CSCG RVs $\mathcal{CN}(0,\sigma_u^2)$ [12]. For simplicity, we denote the combined transmit- and receive-side noise terms at the destination and relay as

$$\mathbf{n}_D = \mathbf{n}'_D + \sqrt{\rho_R}\mathbf{G}_{RD}\mathbf{u}_D, \quad (3)$$
$$\mathbf{n}_R = \mathbf{n}'_R + \sqrt{\rho_R}\mathbf{G}_{RR}\mathbf{u}_R, \quad (4)$$

respectively. Since the receiver knows only the statistics of the channel a priori, we take the worst case scenario for the channel estimation, where the noise is spatially uncorrelated CSCG and independent of the channel. The variance of the kth element in $\mathbf{n}_D$ and $\mathbf{n}_R$ are given by $\sigma_{R,k}^2 = \rho_R\sigma_u^2(\beta + \sum_{j\neq k}\beta_{kj}) + 1$ and $\sigma_{D,k}^2 = \rho_R\sigma_u^2\sum_K \beta_{RD,k} + 1$, respectively.

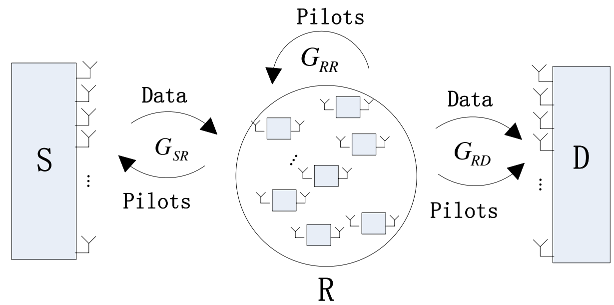

Fig. 1. Full-duplex relaying with distributed relay nodes.

A. Channel Estimation

Block fading channel with coherence time of T symbols is considered. Pilot-aided channel estimation is used to obtain the instantaneous channel state information (CSI) needed for precoding at the source, loop interference cancellation at the relay and detection at the source. If t_p symbols are allocated for pilots, then $t_d = T - t_p$ symbols are available for data transmission. There are two phases in pilot transmission, which take t_{p1}, t_{p2} symbols each, so that $t_p = t_{p1} + t_{p2}$.

1) Channel estimation for precoding at the source: The antennas in the relay station that are used to receive the data transmissions from the source send orthogonal pilot sequences $\mathbf{\Phi}_S \in \mathbb{C}^{K\times t_{p1}}$ to the source. To satisfy the power constraint and guarantee orthogonality, we require $\mathbf{\Phi}_S\mathbf{\Phi}_S^H = \frac{1}{K}\mathbf{I}$ with $t_{p1} \geq K$. The received pilot matrix at the source reads

$$\mathbf{Y}_S = \sqrt{t_{p1}\rho_{p1}}\mathbf{G}_{SR}\mathbf{\Phi}_S + \mathbf{N}_S, \quad (5)$$

where $t_{p1}\rho_{p1}$ is the total energy consumed in this training phase at the relay(s). The source estimates $\mathbf{G}_{SR}$ and uses it for precoding, owing to the reciprocity provided by the TDD operation. Analogous to the R$\rightarrow$D link and (3), the noise matrix $\mathbf{N}_S$ in (5) has independent CSCG entries with the elements in the kth row having variance $\sigma'^2_{S,k} = \frac{t_{p1}}{K}\rho_{p1}\sigma_u^2\sum_K \beta_{SR,k} + 1$.

2) Channel estimation for decoding at the destination and LI cancellation at the relay(s): Just like the source node above, the destination estimates the channel $\mathbf{G}_{RD}$ based on orthogonal pilot sequences $\mathbf{\Phi}_D \in \mathbb{C}^{K\times t_{p2}}$ transmitted by the relay. We let $t_{p2}\rho_{p2}$ to be the total energy consumed in this training phase. At the same time, the relay estimates the LI channel from the same pilots. By (3) and (4), the noise matrices during this training phase are as in (5), i.e., independent CSCG entries with kth row's elements having variances $\sigma'^2_{D,k} = \frac{t_{p2}}{K}\rho_{p2}\sigma_u^2\sum_K \beta_{RD,k} + 1$ for destination and $\sigma'^2_{R,k} = \frac{t_{p2}}{K}\rho_{p2}\sigma_u^2(\beta + \sum_{l\neq j}\beta_{kj}) + 1$ for the relay.

Having described the signal model in the training phase, we now assume that the minimum mean square error (MMSE) channel estimator is used at all nodes to obtain the instantaneous channel estimates $\hat{\mathbf{G}}_{SR}$, $\hat{\mathbf{G}}_{RD}$ and $\hat{\mathbf{G}}_{RR}$. By the properties of the MMSE estimator [13], the error $\tilde{\mathbf{G}}_* = \mathbf{G}_* - \hat{\mathbf{G}}_*$ is uncorrelated with the estimate and they both have independent CSCG entries. The LI error matrix $\tilde{\mathbf{G}}_{RR}$ has then independent CSCG elements with the variance of the kjth element being

$$\tilde{\beta}_{kj} = \frac{\beta_{kj}}{t_{p2}\rho_{p2}\beta_{kj}/K\sigma'^2_{R,k} + 1}. \quad (6)$$

The error matrices $\tilde{\mathbf{G}}_{SR}$ and $\tilde{\mathbf{G}}_{RD}$ have independent CSCG elements, the variance of the entries in the kth column being

$$\tilde{\beta}_{SR,k} = \frac{\beta_{SR,k}}{t_{p1}\rho_{p1}\beta_{SR,k}/K\sigma'^2_{S,k}+1}, \quad (7)$$

$$\tilde{\beta}_{RD,k} = \frac{\beta_{RD,k}}{t_{p2}\rho_{p2}\beta_{RD,k}/K\sigma'^2_{D,k}+1}, \quad (8)$$

respectively. The properties of the MMSE estimator also guarantee that $\hat{\beta}_* = \beta_* - \tilde{\beta}_*$ holds for all channels.

B. Data Transmission

After training, data transmission phase ensues and the source uses $\hat{\mathbf{G}}_{SR}$ to carry out linear precoding [9], [11]

$$\mathbf{x}[i] = \mathbf{A}\mathbf{m}[i], \quad (9)$$

of the information vector $\mathbf{m}[i] \in \mathbb{C}^K$ intended for the destination. Here we focus on matched-filtering (MF) precoding

$$\mathbf{A}_{MF} = \alpha_{MF}\hat{\mathbf{G}}^*_{SR}, \quad (10)$$

$$\alpha_{MF} = \frac{1}{\sqrt{\frac{M}{K}\sum_{k=1}^{K}\hat{\beta}_{SR,k}}}, \quad (11)$$

for simplicity, where α_{MF} guarantees that the long-term transmit power constraint $\mathbb{E}\{|\mathbf{x}[i]|^2\} = 1$ is satisfied.

By (1) and (9), the the kth relay node (or kth antenna element in the co-located setup) receives the signal

$$y_{R,k}[i] = \underbrace{\sqrt{\rho_S}\mathbf{g}^T_{SR,k}\mathbf{a}_k m_k[i]}_{\text{desired signal}} + \underbrace{\sum_{j=1,j\neq k}^{K}\sqrt{\rho_S}\mathbf{g}^T_{SR,k}\mathbf{a}_j m_j[i]}_{\text{inter-pair interference}} + \underbrace{\sqrt{\rho_R}\mathbf{g}^T_{RR,k}\mathbf{s}[i]}_{\text{loop interference}} + \underbrace{n_{R,k}}_{\text{noise}} \quad (12)$$

where $\mathbf{g}_{SR,k}$, $\mathbf{g}_{RR,k}$ and $\mathbf{a}_k$ are the kth column of $\mathbf{G}_{SR}$, $\mathbf{G}_{RR}$ and $\mathbf{A}$, respectively, and $m_k[i]$ is the kth elements of $\mathbf{m}[i]$. For all FD cases, some form of LI cancellation is then applied before detection and decoding. More precisely, we express the cancellation at the kth node / antenna as

$$y'_{R,k}[i] = y_{R,k}[i] - c_k[i], \quad (13)$$

where $c_k[i]$ is a function of $\hat{\mathbf{g}}_{RR,k}$. After LI cancellation, the details of which will be presented in the next subsection, information from $y'_{R,k}[i]$ is decoded[1] to $s_k[i]$ and sent forward (after re-encoding) to the destination. We note that following the common assumption in decode-and-forward relaying there is a processing delay $d \geq 1$ symbols at the relay

$$\mathbf{s}[i] = \mathbf{m}[i-d], \quad (14)$$

so that for any time i, transmit signal signals at relay stations are uncorrelated with receive signals [1].

[1]The relay does not know the instantaneous (pre-coded) channel since it is not estimated at any stage. Indeed, estimating the S→R channel at relay would require another training period that could be prohibitively long if $M \gg 1$. We follow the method proposed in [9], where the relay knows only the average precoded channel coefficient(s) $\mathbb{E}\{\mathbf{g}^T_{SR,k}\mathbf{a}_k\}$. See also Section IV.

After receiving the signals from the relay node(s), the destination uses linear estimation to separate the streams, so that the kth information stream after estimation reads

$$r_{D,k}[i] = \sqrt{\rho_R}\mathbf{w}^H_k\mathbf{g}_{RD,k}s_k[i] + \sum_{j=1,j\neq k}^{K}\sqrt{\rho_S}\mathbf{w}^H_k\mathbf{g}_{RD,j}s_j[i] + \mathbf{w}^H_k\mathbf{n}_D, \quad (15)$$

where $\mathbf{g}_{RD,k}$ represents the kth column of matrix $\mathbf{G}_{RD}$. For simplicity, we concentrate in this study on the MF based estimation so that $\mathbf{w}_k = \hat{\mathbf{g}}_{RD,j}$.

C. Loop Interference Cancellation

Let us now consider the details of the two LI cancellation schemes used in this paper for relaying.

1) Cancellation scheme for cooperative relay stations: If one relay station with multiple antennas is used, it is clear that the device has full knowledge of the received data $\mathbf{y}_R$, the transmitted signal $\mathbf{s}[i]$ given in (14) as well as the estimated CSI $\hat{\mathbf{G}}_{RR}$ discussed in Section II-A. All of the above can be obtained virtually without delay using internal circuitry. In principle, cooperative distributed relays can also obtain the same information if dedicated control channel with sufficient capacity is available for them. In practice, the information needs to be quantized and there might be delays due to transmissions, albeit the latter should not cause problems if the relays operate with buffers and the initial transmissions are "ramped up" appropriately. Here we assume for simplicity that the control channel can be used instantaneously and perfectly to share the transmitted symbols and estimated CSI. However, it should be noted that this is a highly optimistic scenario that provides an upper bound for the performance of a practical system using quantization and finite capacity control channel.

For the cooperative relaying, the LI cancellation factor is

$$c_k[i] = \sqrt{\rho_S}\hat{\mathbf{g}}^T_{RR,k}\mathbf{s}[i], \quad (16)$$

so that by (13), the received signal at the kth node, or antenna stream in colocated case, after cancellation reads

$$y'_{R,k}[i] = \sqrt{\rho_S}\mathbf{g}^T_{SR,k}\mathbf{a}_k m_k[i] + \sum_{j=1,j\neq k}^{K}\sqrt{\rho_S}\mathbf{g}^T_{SR,k}\mathbf{a}_j m_j[i] + \underbrace{\sqrt{\rho_R}\tilde{\mathbf{g}}^T_{RR,k}\mathbf{s}[i]}_{LI_k^{\text{coop}}} + n_{R,k}, \quad (17)$$

where $\tilde{\mathbf{g}}_{RR,k}$, $\hat{\mathbf{g}}_{RR,k}$ are the kth columns of the estimation error matrix $\tilde{\mathbf{G}}_{RR}$ and the channel estimation matrix $\hat{\mathbf{G}}_{RR}$, respectively, both of which have i.i.d. CSCG elements whose variances are as discussed in Section II-A. We denote the remaining LI as $LI_k^{\text{coop}} \triangleq \sqrt{\rho_R}\tilde{\mathbf{g}}^T_{RR,k}\mathbf{s}[i]$ for later use.

2) Cancellation scheme for non-cooperative relay stations: While non-cooperating relay stations can estimate the channels that cause LI, they cannot obtain the transmit signals of other stations. Thus, only loop interference from the kth station itself can be cancelled by using the estimated CSI and the interference from the other stations remains. This means that

$$c_k[i] = \sqrt{\rho_S}\hat{g}_{RR,kk}s_k[i] \quad (18)$$

is used to cancel the loop interference caused by the node's own transmission while the inter-node interference remains unaffected. The received signal at the kth relay node reads

$$y'_{R,k}[i] = \sqrt{\rho_S}\mathbf{g}_{SR,k}^T\mathbf{a}_k m_k[i] + \sum_{j=1,j\neq k}^{K}\sqrt{\rho_S}\mathbf{g}_{SR,k}^T\mathbf{a}_j m_j[i] + \underbrace{\sqrt{\rho_R}\tilde{g}_{RR,kk}s_k[i] + \sum_{l=1,l\neq k}^{K}\sqrt{\rho_R}g_{RR,kl}s_l[i]}_{LI_k^{\text{non}}} + n_{R,k}, \quad (19)$$

where $g_{RR,ij}$ and $\tilde{g}_{RR,ij}$ are the ijth elements of $\mathbf{G}_{RR}$ and $\tilde{\mathbf{G}}_{RR}$, respectively. The transmit symbol of the kth relay station is $s_k[i]$. As before, we write $LI_k^{\text{non}} \triangleq \sqrt{\rho_R}\tilde{g}_{RR,kk}s_k[i] + \sum_{l=1,l\neq k}^{K}\sqrt{\rho_R}g_{RR,kl}s_l[i]$ for the residual LI in the case of non-cooperative relaying. It is clear that $LI_k^{\text{non}} > LI_k^{\text{coop}}$ and the question whether the LI is dominating in non-cooperative setup depends mostly on how strong the channels $g_{RR,kl}$ between the relay nodes are.

III. Asymptotic Interference Analysis

We now investigate the asymptotic region where the number of antennas at the source and destination grow without bound while the number of relay antennas is fixed and finite.

Proposition 1. *Assume that MF processing with estimated channel is used for precoding at the source, and estimation at the destination. Assume further that the LI cancellation schemes described in Section II-C are not used. Then, as $K/M \to 0$ for fixed K,*

$$\frac{y_{R,k}[i]}{\sqrt{M}} \xrightarrow{a.s} \sqrt{\frac{\rho_S}{\frac{1}{K}\sum_{k=1}^{K}\hat{\beta}_{SR,k}}}\hat{\beta}_{SR,k}m_k[i], \quad (20)$$

$$\frac{r_{D,k}[i]}{M} \xrightarrow{a.s} \sqrt{\rho_R}\hat{\beta}_{RD,k}s_k[i], \quad (21)$$

where $\xrightarrow{a.s}$ denotes almost sure convergence and $y_{R,k}[i]$ is the received signal at the kth station before loop interference cancellation for any of the considered relaying schemes.

The proposition implies that regardless of the relaying scheme, if massive antenna arrays are used at the source and destination then the system is free of both LI and inter-stream or inter-pair interference. This is an analogous result to the one obtained in [6] for a system that was a "mirror image" of the one considered in the present paper.

Proof: The received signal $y_{R,k}$ in (12) for MF reads

$$\frac{y_{R,k}[i]}{\sqrt{M}} = \sqrt{\frac{\rho_S}{\frac{1}{K}\sum_{k=1}^{K}\hat{\beta}_{SR,k}}}\frac{\mathbf{g}_{SR,k}^T\hat{\mathbf{g}}_{SR,k}^*}{M}m_k[i] + \sum_{j=1,j\neq k}^{K}\sqrt{\frac{\rho_S}{\frac{1}{K}\sum_{k=1}^{K}\hat{\beta}_{SR,k}}}\frac{\mathbf{g}_{SR,k}^T\hat{\mathbf{g}}_{SR,j}^*}{M}m_j[i] + \frac{\sqrt{\rho_R}\mathbf{g}_{RR,k}^T\mathbf{s}[i]}{\sqrt{M}} + \frac{n_{R,k}}{\sqrt{M}}. \quad (22)$$

Due to the properties of the MMSE estimates and the fact that the channels have CSCG elements, $\hat{\mathbf{g}}_{SR,k}$ and $\tilde{\mathbf{g}}_{SR,k}$ are independent. By applying the strong law of large numbers [14], the first term in (22) becomes

$$\sqrt{\frac{\rho_S}{\frac{1}{K}\sum_{k=1}^{K}\hat{\beta}_{SR,k}}}\frac{\hat{\mathbf{g}}_{SR,k}^T\hat{\mathbf{g}}_{SR,k}^* + \tilde{\mathbf{g}}_{SR,k}^T\hat{\mathbf{g}}_{SR,k}^*}{M}m_k[i] \xrightarrow{a.s} \sqrt{\frac{\rho_S}{\frac{1}{K}\sum_{k=1}^{K}\hat{\beta}_{SR,k}}}\hat{\beta}_{SR,k}m_k[i], \quad \text{as} \quad M\to\infty. \quad (23)$$

Similarly, since $\mathbf{g}_{SR,k}$ and $\hat{\mathbf{g}}_{SR,j}$ are independent for $k\neq j$, the inter-pair interference term in (22) converges almost surely to 0 when $M\to\infty$. Moreover, the third term representing LI also approaches 0 almost surely, when $M\to\infty$, since $\mathbf{g}_{RR,k}^T\mathbf{s}[i]$ is a summation of K random variables that are almost surely finite. Finally, we note that the noise term is almost surely finite and converges to zero when divided by M that grows without bound. Therefore, even without LI cancellation the received signals at the relay are interference and noise free. The proof of (21) is similar and omitted. ■

IV. Achievable Rate Analysis

The asymptotic result in Proposition 1 shows that the considered systems can operate free of noise and interference in the limit $M\to\infty$. However, the question how the performance of finite sized systems behaves still remains. In particular, we are interested in the achievable rate of the relaying schemes when MF processing and the LI cancellation schemes proposed in Section II-C are used.

Since the relay station does not know the instantaneous CSI and instead uses statistical channel gains, the "effective" received signal at relay station k can be expressed as [9]

$$y_{R,k}[i] = \underbrace{\sqrt{\rho_S}\mathbb{E}\{\mathbf{g}_{SR,k}^T\mathbf{a}_k\}m_k[i]}_{\text{desired signal}} + \underbrace{\sqrt{\rho_S}(\mathbf{g}_{SR,k}^T\mathbf{a}_k - \mathbb{E}\{\mathbf{g}_{SR,k}^T\mathbf{a}_k\})m_k[i]}_{\text{noise}} + \underbrace{\sum_{j=1,j\neq k}^{K}\sqrt{\rho_S}\mathbf{g}_{SR,k}^T\mathbf{a}_j m_j[i] + LI'_k + n_{R,k}}_{\text{noise}}, \quad (24)$$

where $\mathbb{E}\{\mathbf{g}_{SR,k}\mathbf{a}_k\}$ is the statistical channel gain and $LI'_k = LI_k^{\text{non}}$ for non-cooperative relaying and $LI'_k = LI_k^{\text{coop}}$ for cooperative relaying. The destination, on the other hand, has an estimate of the instantaneous CSI, i.e. $\hat{\mathbf{g}}_{RD,k}$ that it uses for detection, so that we have the signal at the destination as

$$r_{D,k}[i] = \underbrace{\sqrt{\rho_R}\mathbf{w}_k^H\hat{\mathbf{g}}_{RD,k}s_k[i]}_{\text{desired signal}} + \underbrace{\sqrt{\rho_R}\mathbf{w}_k^H\tilde{\mathbf{g}}_{RD,k}s_k[i]}_{\text{noise}} + \underbrace{\sum_{j=1,j\neq k}^{K}\sqrt{\rho_S}\mathbf{w}_k^H\mathbf{g}_{RD,j}s_j[i] + \mathbf{w}_k^H\mathbf{n}_D}_{\text{noise}}. \quad (25)$$

By using the fact that the worst case noise is when the additive noise and interference terms are independent of data with

CSCG distribution of the same variance [15], the achievable rate for the kth stream of S→R link is lower bounded by (26) shown at the top of the next page, where we denoted $c(x) \triangleq \log_2(1+x)$ and $P_{LI'_k} = \mathbb{E}\{|LI'_k|\}$ for notational convenience. By the same arguments we obtain a lower bound for the achievable rate of the R→D link as given in (27) at the top of the next page. It should be pointed out that these equations are valid also for other precoders / estimators such as zero-forcing (ZF), but they are not considered here due to space constraints.

Since the ergodic achievable rate of the kth stream is limited by the rate of the weaker link rate, the end-to-end rate of the kth stream is given by

$$R_k = \min\{R_{SR,k}, R_{RD,k}\}. \tag{28}$$

While the rates can be obtained through Monte Carlo simulation, it can still be time consuming. For this reason, we provide next a simple approximation for the achievable rate when MF is used both for precoding and detection.

Proposition 2. *Assume that MF processing with estimated channel is used at the source and destination. Assume further that the relay uses the LI cancellation schemes described in Section II-C. The end-to-end achievable rate for the source-to-destination link with DF relaying is then approximated as*

$$\begin{aligned} R_k^{MF} = & c\Bigg(\min\Bigg\{ \frac{\rho_S \hat{\beta}_{SR,k}^2 M}{(\rho_S \beta_{SR,k} + P_{LI'_k} + \sigma_R^2)\sum_K \hat{\beta}_{SR,k}}, \\ & \frac{\rho_R \hat{\beta}_{RD,k}(M+1)}{\rho_R \sum_K \beta_{RD,k} - \rho_R \hat{\beta}_{RD,k} + K} \Bigg\}\Bigg) \end{aligned} \tag{29}$$

where the remaining loop interference power $P_{LI'_k}$ is given for the different relying strategies as

$$P_{LI'_k} = \begin{cases} \rho_R \tilde{\beta}, & \textit{co-located;} \\ \frac{\rho_R}{K}(\tilde{\beta}_k + \sum_{j=1,j\neq k}^{K} \tilde{\beta}_{kj}), & \textit{cooperative;} \\ \frac{\rho_R}{K}(\tilde{\beta}_k + \sum_{j=1,j\neq k}^{K} \beta_{kj}), & \textit{non-cooperative.} \end{cases} \tag{30}$$

Spectral efficiency of the system reads then

$$SE^{MF} = \frac{T - t_{p1} - t_{p2}}{T} \sum_{k=1}^{K} R_k^{MF}. \tag{31}$$

Proof: Omitted due to space constraints. ■

V. Numerical Results

Here the performance of finite-sized systems is investigated via numerical examples. Unless stated otherwise, the normalized transmit-side noise variance is $\sigma_u^2 = 10^{-3}/K$ that corresponds to EVM $= -30$ dB, and the coherence time is set to $T = 200$ symbols. For simplicity, we assume that the large scale fading of all S→R and R→D links are equal, i.e. $\beta_{SR,k} = \beta_{RD,k} = 1\ \forall k$. For co-located relay the self-interference channel strength is constant between all antenna pairs $\beta_{kj} = 20$ dB. In distributed case $\beta_{kk} = 20$ dB and the inter-node interference power $\beta_{kj}\ \forall k \neq j$ is assumed to be identical between all node pairs for simplicity. We set equal average transmit power constraint for source and relay (shared between the distributed nodes) and let the pilots and data have the same average powers so that $\rho = \rho_S = \rho_R = \rho_{p1} = \rho_{p2}$ represents also the signal-to-noise ratio (SNR) of the system.

Fig. 2 shows the spectral efficiency of the system as a function of transmit power ρ (or SNR). The analytical results based on Proposition 2 match well with the Monte Carlo simulations depicted by the markers. The result demonstrates that if the channel estimation based LI cancellation can be done jointly over all antennas, FD has significant advantage over HD at moderate-to-high SNR. For distributed setup where only device's own LI can be cancelled, the remaining inter-pair interference channel strength of $\beta_{kj} = 10$ dB is too strong in this scenario for effective operation in FD mode.

The effect of inter-pair interference channel power β_{kj} on the spectral efficiency is illustrated in Fig. 3. The co-located and HD setups are independent of β_{kj} and distributed coordinated system depends on it only weakly. As expected, the fully distributed setup with independent FD nodes is superior to HD only when β_{kj} is small, here around $5-6$ dB. This implies that independent FD nodes should be spaced very far apart of each other, or blocked by large obstacles. If this is not possible and co-located single device is not an option, then distributed HD relaying is a more design effective option.

The optimal number of antennas at the relay(s) for a given size of the antenna arrays at the source and destination is illustrated in Fig. 4. The results are obtained through exhaustive search using Proposition 2, which has low computational complexity since the rate is given in closed form. The results show that HD relaying benefits from large antenna numbers, up to 50% of the antennas used by the source and destination. As expected, the ratio diminishes as M increases since the coherence time $T = 200$ is fixed. It is worth noting that the FD relaying uses only 10%–30% of antennas even in cooperative case, the reason being training overhead and additional LI caused by larger K. Thus, for relaying between two nodes with large antenna arrays, relatively small sized relay nodes achieve optimal performance in FD case. On the other hand, for HD relaying up to twice the number of antennas are required at the relay to obtain the best performance. Further improvements in the FD case may be achieved by optimizing the power allocation between data and pilots [16] or using spatial LI suppression [1], [17] in case of cooperative relaying.

VI. Conclusion

In this paper, the performance of co-located, distributed cooperative and distributed non-cooperative full-duplex relaying schemes for point-to-point MIMO with large antennas were investigated. Asymptotic analysis showed that all considered schemes are interference and noise free when number of antennas at the source and destination grows without bound, even when no LI cancellation is performed at the relay. Analytical achievable rate analysis and numerical examples for MF showed that given effective LI cancellation, FD operation can provide significant gains over HD relaying with a smaller antenna array at the relay node.

$$R_{SR,k} = c\left(\frac{\frac{\rho_S}{K}|\mathbb{E}\{\mathbf{g}_{SR,k}^T\mathbf{a}_k\}|^2}{\frac{\rho_S}{K}\mathbf{Var}\left(\mathbf{g}_{SR,k}^T\mathbf{a}_k\right)+\sum_{j=1,j\neq k}^{K}\frac{\rho_S}{K}\mathbb{E}\{|\mathbf{g}_{SR,k}^T\mathbf{a}_j|^2\}+P_{LI_k'}+\sigma_{R,k}^2}\right) \tag{26}$$

$$R_{RD,k} = \mathbb{E}\left\{c\left(\frac{\frac{\rho_R}{K}|\mathbf{w}_k^H\hat{\mathbf{g}}_{RD,k}|^2}{\sum_{j=1,j\neq k}^{K}\frac{\rho_R}{K}\mathbb{E}\{|\mathbf{w}_k^H\mathbf{g}_{RD,j}|^2\}+\frac{\rho_R}{K}\mathbb{E}\{|\mathbf{w}_k^H\tilde{\mathbf{g}}_{RD,k}|^2\}+\mathbb{E}\{\mathbf{n}_D^H\mathbf{w}_k\mathbf{w}_k^H\mathbf{n}_D\}}\right)\right\} \tag{27}$$

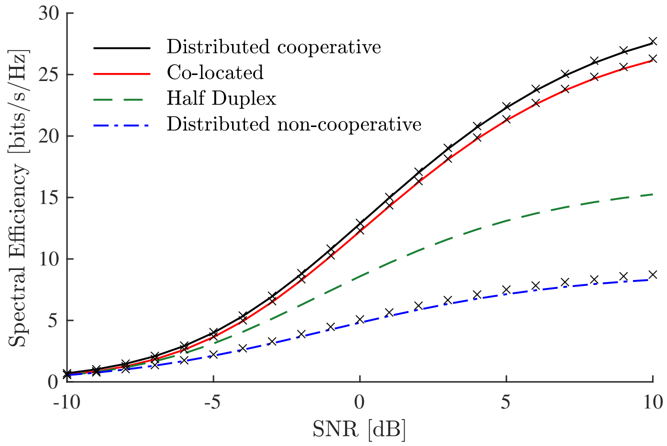

Fig. 2. Spectral efficiency vs. SNR for different relaying schemes. The lines depict analytical results and markers correspond to Monte Carlo simulations. ($\beta_{kk} = 20$ dB, $\beta_{kj} = 10$ dB, $M = 100$ and $K = 10$)

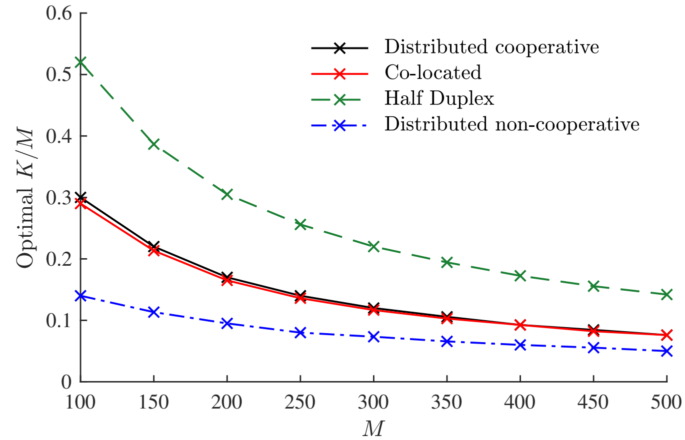

Fig. 4. Optimal K/M vs. M for different relaying schemes. ($\rho = 10$ dB, $\beta_{kk} = 20$ dB and $\beta_{kj} = 10$ dB)

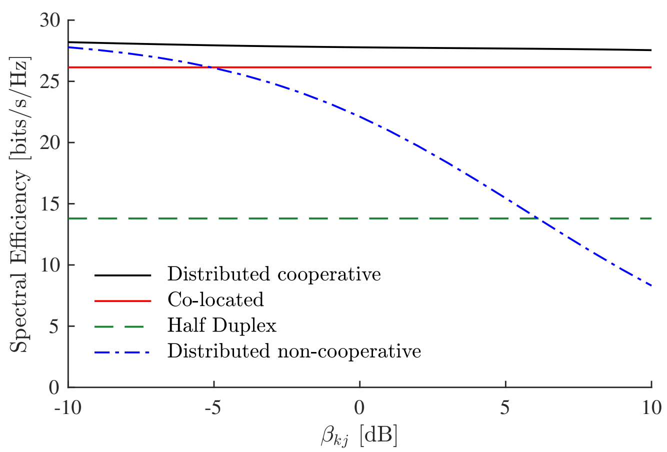

Fig. 3. Spectral efficiency vs. β_{kj} for different relaying schemes. ($\rho = 10$ dB, $\beta_{kk} = 20$ dB, $M = 100$ and $K = 10$)

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