

Approximate Nash Region of the Gaussian Interference Channel with Noisy Output Feedback

Victor Quintero, Samir M. Perlaza, Jean-Marie Gorce, and H. Vincent Poor

Abstract—In this paper, an achievable η -Nash equilibrium (η -NE) region for the two-user Gaussian interference channel with noisy channel-output feedback is presented for all $\eta \geq 1$. This result is obtained in the scenario in which each transmitter-receiver pair chooses its own transmit-receive configuration in order to maximize its own individual information transmission rate. At an η -NE, any unilateral deviation by either of the pairs does not increase the corresponding individual rate by more than η bits per channel use.

Index Terms—Gaussian Interference Channel, Noisy channel-output feedback, η -Nash equilibrium region.

I. INTRODUCTION

The interference channel (IC) is one of the simplest yet insightful multi-user channels in network information theory. An important class of ICs is the two-user Gaussian interference channel (GIC) in which there exist two point-to-point links subject to mutual interference and independent Gaussian noise sources. In this model, each output signal is a noisy version of the sum of the two transmitted signals affected by the corresponding channel gains. The analysis of this channel can be made considering two general scenarios: (1) a centralized scenario in which the entire network is controlled by a central entity that configures both transmitter-receiver pairs; and (2) a decentralized scenario in which each transmitter-receiver pair autonomously configures its transmission-reception parameters. In the former, the fundamental limits are characterized by the capacity region, which is approximated to within a fixed number of bits in [1] for the case without feedback; in [2] for the case with perfect channel-output feedback; and in [3] and [4] for the case with noisy channel-output feedback. In the latter, the fundamental limits are characterized by the η -Nash equilibrium (η -NE) region. The η -NE of the GIC is approximated in the cases without feedback and with perfect channel-output feedback in [5] and [6], respectively.

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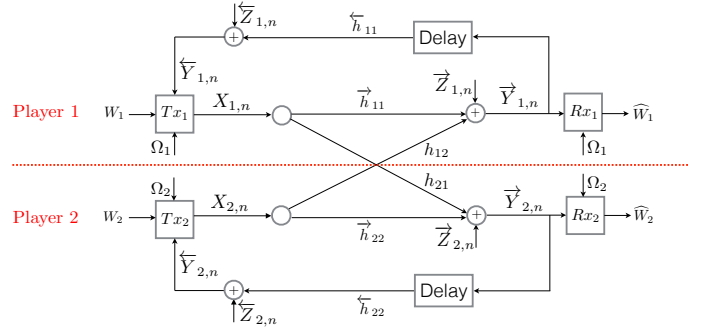


Fig. 1. Two-User Decentralized Gaussian interference channel with noisy channel-output feedback at channel use n .

In this paper the η -NE region of the GIC is studied assuming that there exists a noisy feedback link from each receiver to its corresponding transmitter. The η -NE region is approximated by two regions for all $\eta \geq 1$: a region for which an equilibrium transmit-receive configuration is presented for each of the information rate pairs (an achievable region); and a region for which any information rate pair that is outside of this region cannot be an η -NE (impossibility region). The focus of this paper is on the achievable region.

The results presented in this paper are a generalization of the results presented in [5] and [6], and they are obtained thanks to the analysis of linear deterministic approximations in [7] and [8].

II. DECENTRALIZED GAUSSIAN INTERFERENCE CHANNELS WITH NOISY CHANNEL-OUTPUT FEEDBACK

Consider the two-user decentralized Gaussian interference channel with noisy channel-output feedback (D-GIC-NOF) depicted in Figure 1. Transmitter i , with $i \in \{1, 2\}$, communicates with receiver i subject to the interference produced by transmitter j , with $j \in \{1, 2\} \setminus \{i\}$. There are two independent and uniformly distributed messages, $W_i \in \mathcal{W}_i$, with $\mathcal{W}_i = \{1, 2, \dots, \lfloor 2^{N_i R_i} \rfloor\}$, where N_i denotes the fixed block-length in channel uses and R_i the information transmission rate in bits per channel use. At each block, transmitter i sends the codeword $\mathbf{X}_i = (X_{i,1}, X_{i,2}, \dots, X_{i,N_i})^T \in \mathcal{C}_i \subseteq \mathcal{R}^{N_i}$, where \mathcal{C}_i is the codebook of transmitter i . The channel coefficient from transmitter j to receiver i is denoted by h_{ij} ; the channel coefficient from transmitter i to receiver i is denoted by \overrightarrow{h}_{ii} ; and the channel coefficient from channel-output i to transmitter i is denoted by \overleftarrow{h}_{ii} . All channel coefficients are assumed to be non-negative real numbers. At a given channel

use $n \in \{1, 2, \dots, N\}$, with

$$N = \max(N_1, N_2), \quad (1)$$

the channel output at receiver i is denoted by $\vec{Y}_{i,n}$. During channel use n , the input-output relation of the channel model is given by

$$\vec{Y}_{i,n} = \vec{h}_{ii} X_{i,n} + h_{ij} X_{j,n} + \vec{Z}_{i,n}, \quad (2)$$

where $X_{i,n} = 0$ for all n such that $N \geq n > N_i$ and $\vec{Z}_{i,n}$ is a real Gaussian random variable with zero mean and unit variance that represents the noise at the input of receiver i . Let $d > 0$ be the finite feedback delay measured in channel uses. At the end of channel use n , transmitter i observes $\overleftarrow{Y}_{i,n}$, which consists of a scaled and noisy version of $\vec{Y}_{i,n-d}$. More specifically,

$$\overleftarrow{Y}_{i,n} = \begin{cases} \overleftarrow{Z}_{i,n} & \text{for } n \in \{1, 2, \dots, d\} \\ \overleftarrow{h}_{ii} \vec{Y}_{i,n-d} + \overleftarrow{Z}_{i,n} & \text{for } n \in \{d+1, d+2, \dots, N\}, \end{cases} \quad (3)$$

where $\overleftarrow{Z}_{i,n}$ is a real Gaussian random variable with zero mean and unit variance that represents the noise in the feedback link of transmitter-receiver pair i . The random variables $\vec{Z}_{i,n}$ and $\overleftarrow{Z}_{i,n}$ are assumed to be independent. In the following, without loss of generality, the feedback delay is assumed to be one channel use, i.e., $d = 1$. The encoder of transmitter i is defined by a set of deterministic functions $f_{i,1}^{(N)}, f_{i,2}^{(N)}, \dots, f_{i,N_i}^{(N)}$, with $f_{i,1}^{(N)} : \mathcal{W}_i \times \mathbb{N} \rightarrow \mathcal{X}_i$ and for all $n \in \{2, 3, \dots, N_i\}$, $f_{i,n}^{(N)} : \mathcal{W}_i \times \mathbb{N} \times \mathbb{R}^{n-1} \rightarrow \mathcal{X}_i$, such that

$$X_{i,1} = f_{i,1}^{(N)}(W_i, \Omega_i), \quad \text{and} \quad (4a)$$

$$X_{i,n} = f_{i,n}^{(N)}(W_i, \Omega_i, \overleftarrow{Y}_{i,1}, \overleftarrow{Y}_{i,2}, \dots, \overleftarrow{Y}_{i,n-1}), \quad (4b)$$

where Ω_i is an additional index randomly generated. The index Ω_i is assumed to be known by both transmitter i and receiver i , while unknown by transmitter j and receiver j .

The components of the input vector \mathbf{X}_i are real numbers subject to an average power constraint

$$\frac{1}{N_i} \sum_{n=1}^{N_i} \mathbb{E}_{X_{i,n}} [X_{i,n}^2] \leq 1. \quad (5)$$

The decoder of receiver i is defined by a deterministic function $\psi_i^{(N)} : \mathbb{N} \times \mathbb{R}^N \rightarrow \mathcal{W}_i$. At the end of the communication, receiver i uses the vector $(\vec{Y}_{i,1}, \vec{Y}_{i,2}, \dots, \vec{Y}_{i,N})$ and the index Ω_i to obtain an estimate

$$\widehat{W}_i = \psi_i^{(N)}(\Omega_i, \vec{Y}_{i,1}, \vec{Y}_{i,2}, \dots, \vec{Y}_{i,N}), \quad (6)$$

A *transmit-receive configuration* for transmitter-receiver pair i , denoted by s_i , can be described in terms of the block-length N_i , the rate R_i , the codebook \mathcal{C}_i , the encoding functions $f_{i,1}^{(N)}, f_{i,2}^{(N)}, \dots, f_{i,N_i}^{(N)}$, and the decoding function $\psi_i^{(N)}$, etc. The average error probability at decoder i given the configurations s_1 and s_2 , denoted by $p_i(s_1, s_2)$, is given by

$$p_i(s_1, s_2) = \Pr [W_i \neq \widehat{W}_i]. \quad (7)$$

Within this context, a rate pair $(R_1, R_2) \in \mathbb{R}_+^2$ is said to be achievable if it complies with the following definition.

Definition 1 (Achievable Rate Pairs): A rate pair $(R_1, R_2) \in \mathbb{R}_+^2$ is achievable if there exists at least one pair of configurations (s_1, s_2) such that the decoding bit error probabilities $p_1(s_1, s_2)$ and $p_2(s_1, s_2)$ can be made arbitrarily small by letting the block-lengths N_1 and N_2 grow to infinity.

The aim of transmitter i is to autonomously choose its transmit-receive configuration s_i in order to maximize its achievable rate R_i . Note that the rate achieved by transmitter-receiver i depends on both configurations s_1 and s_2 due to mutual interference. This reveals the competitive interaction between both links in the decentralized interference channel. The fundamental limits of the two-user D-GIC-NOF in Figure 1 can be described by six parameters: $\overrightarrow{\text{SNR}}_i$, $\overleftarrow{\text{SNR}}_i$, and INR_{ij} , with $i \in \{1, 2\}$ and $j \in \{1, 2\} \setminus \{i\}$, which are defined as follows:

$$\overrightarrow{\text{SNR}}_i \triangleq \vec{h}_{ii}^2, \quad (8)$$

$$\text{INR}_{ij} \triangleq h_{ij}^2 \quad \text{and} \quad (9)$$

$$\overleftarrow{\text{SNR}}_i \triangleq \overleftarrow{h}_{ii}^2 (\vec{h}_{ii}^2 + 2\vec{h}_{ii}h_{ij} + h_{ij}^2 + 1). \quad (10)$$

The analysis presented in this paper focuses exclusively on the case in which $\text{INR}_{ij} > 1$ for all $i \in \{1, 2\}$ and $j \in \{1, 2\} \setminus \{i\}$. The reason for exclusively considering this case follows from the fact that when $\text{INR}_{ij} \leq 1$, the transmitter-receiver pair i is impaired mainly by noise instead of interference. In this case, feedback does not bring a significant rate improvement. Denote by \mathcal{C} the capacity region of the two-user GIC-NOF with fixed parameters $\overrightarrow{\text{SNR}}_1, \overrightarrow{\text{SNR}}_2, \text{INR}_{12}, \text{INR}_{21}, \overleftarrow{\text{SNR}}_1$, and $\overleftarrow{\text{SNR}}_2$. The achievable region \mathcal{C} in [4, Theorem 2] and the converse region $\overline{\mathcal{C}}$ in [4, Theorem 3] approximate the capacity region \mathcal{C} to within 4.4 bits [4].

III. GAME FORMULATION

The competitive interaction between the two transmitter-receiver pairs in the interference channel can be modeled by the following game in normal-form:

$$\mathcal{G} = (\mathcal{K}, \{\mathcal{A}_k\}_{k \in \mathcal{K}}, \{u_k\}_{k \in \mathcal{K}}). \quad (11)$$

The set $\mathcal{K} = \{1, 2\}$ is the set of players, that is, the set of transmitter-receiver pairs. The sets \mathcal{A}_1 and \mathcal{A}_2 are the sets of actions of players 1 and 2, respectively. An action of a player $i \in \mathcal{K}$, which is denoted by $s_i \in \mathcal{A}_i$, is basically its transmit-receive configuration as described above. The utility function of player i is $u_i : \mathcal{A}_1 \times \mathcal{A}_2 \rightarrow \mathbb{R}_+$ and it is defined as the information rate of transmitter i ,

$$u_i(s_1, s_2) = \begin{cases} R_i, & \text{if } p_i(s_1, s_2) < \epsilon \\ 0, & \text{otherwise,} \end{cases} \quad (12)$$

where $\epsilon > 0$ is an arbitrarily small number. This game formulation for the case without feedback was first proposed in [9] and [10].

A class of transmit-receive configurations that are particularly important in the analysis of this game is referred to as the set of η -Nash equilibria (η -NE), with $\eta > 0$. This type of configurations satisfy the following definition.

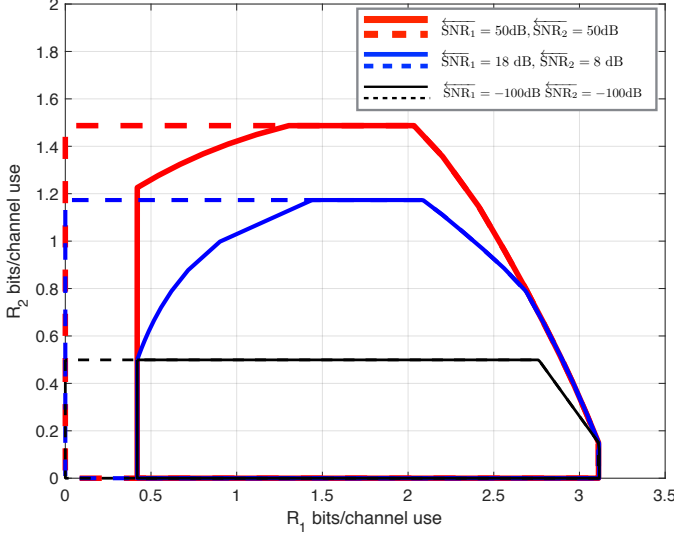


Fig. 2. Achievable capacity regions \mathcal{C} (dashed-lines) in [4, Theorem 2] and achievable η -NE regions \mathcal{N}_η (solid lines) in Theorem 1 of the two-user GIC-NOF and two-user D-GIC-NOF with parameters $\overline{\text{SNR}}_1 = 24$ dB, $\overline{\text{SNR}}_2 = 3$ dB, $\text{INR}_{12} = 16$ dB, $\text{INR}_{21} = 9$ dB, $\overline{\text{SNR}}_1 \in \{-100, 18, 50\}$ dB, $\overline{\text{SNR}}_2 \in \{-100, 8, 50\}$ dB and $\eta = 1$.

Definition 2 (η -Nash equilibrium): Given a positive real η , an action profile (s_1^*, s_2^*) is an η -Nash equilibrium (NE) in the game $\mathcal{G} = (\mathcal{K}, \{\mathcal{A}_k\}_{k \in \mathcal{K}}, \{u_k\}_{k \in \mathcal{K}})$, if for all $i \in \mathcal{K}$ and for all $s_i \in \mathcal{A}_i$, it follows that

$$u_i(s_i, s_j^*) \leq u_i(s_i^*, s_j^*) + \eta. \quad (13)$$

Let (s_1^*, s_2^*) be an η -Nash equilibrium action profile. Then, none of the transmitters can increase its own transmission rate more than η bits per channel use by changing its own transmit-receive configuration and keeping the average bit error probability arbitrarily close to zero. Note that for η sufficiently large, from Definition 2, any pair of configurations can be an η -NE. Alternatively, for $\eta = 0$, the definition of Nash equilibrium is obtained [11]. In this case, if a pair of configurations is a Nash equilibrium ($\eta = 0$), then each individual configuration is optimal with respect to each other. Hence, the interest is to describe the set of all possible η -NE rate pairs (R_1, R_2) of the game in (11) with the smallest η for which there exists at least one equilibrium configuration pair.

The set of rate pairs that can be achieved at an η -NE is known as the η -Nash equilibrium (η -NE) region.

Definition 3 (η -NE Region): Let $\eta > 0$ be fixed. An achievable rate pair (R_1, R_2) is said to be in the η -NE region of the game $\mathcal{G} = (\mathcal{K}, \{\mathcal{A}_k\}_{k \in \mathcal{K}}, \{u_k\}_{k \in \mathcal{K}})$ if there exists a pair $(s_1^*, s_2^*) \in \mathcal{A}_1 \times \mathcal{A}_2$ that is an η -NE and the following holds:

$$u_1(s_1^*, s_2^*) = R_1 \quad \text{and} \quad u_2(s_1^*, s_2^*) = R_2. \quad (14)$$

The η -NE regions of the two-user GIC with and without perfect channel-output feedback have been approximated to within a constant number of bits in [5] and [6], respectively. The next section introduces a generalization of these results.

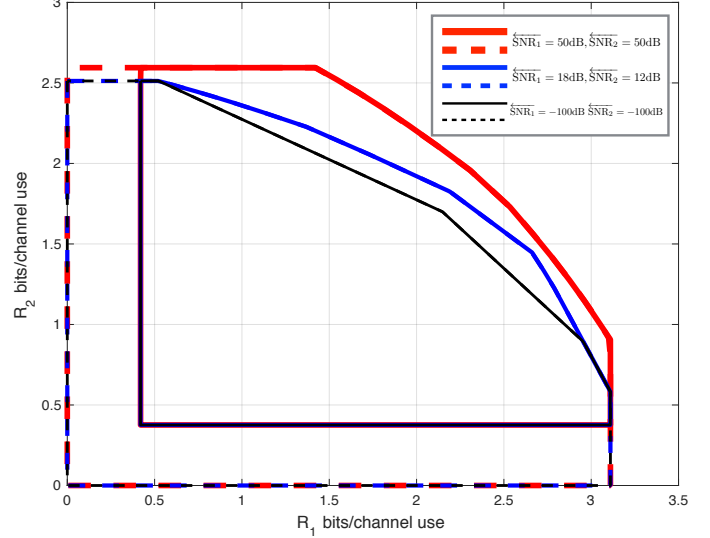


Fig. 3. Achievable capacity regions \mathcal{C} (dashed-lines) in [4, Theorem 2] and achievable η -NE regions \mathcal{N}_η (solid lines) in Theorem 1 of the two-user GIC-NOF and two-user D-GIC-NOF with parameters $\overline{\text{SNR}}_1 = 24$ dB, $\overline{\text{SNR}}_2 = 18$ dB, $\text{INR}_{12} = 16$ dB, $\text{INR}_{21} = 10$ dB, $\overline{\text{SNR}}_1 \in \{-100, 18, 50\}$ dB, $\overline{\text{SNR}}_2 \in \{-100, 12, 50\}$ dB and $\eta = 1$.

IV. MAIN RESULTS

A. Achievable η -Nash Equilibrium Region

Let the η -NE region (Definition 3) of the D-GIC-NOF be denoted by \mathcal{N}_η . This section introduces a region $\mathcal{N}_\eta \subseteq \mathcal{N}_\eta$ that is achievable using a coding scheme that combines rate splitting [12], common randomness [5], [6], block Markov superposition coding [13] and backward decoding [14]. In the following, this coding scheme is referred to as randomized Han-Kobayashi scheme with noisy channel-output feedback (RHK-NOF). This coding scheme is presented in [8] and uses the same techniques of the schemes in [5] and [6]. Therefore, the focus of this section is on the results rather than the description of the scheme. A motivated reader is referred to [15]. The RHK-NOF is proved to be an η -NE action profile with $\eta \geq 1$. That is, any unilateral deviation from the RHK-NOF by any of the transmitter-receiver pairs might lead to an individual rate improvement which is at most one bit per channel use. The description of the achievable η -Nash region \mathcal{N}_η is presented using the constants $a_{1,i}$; the functions $a_{2,i} : [0, 1] \rightarrow \mathbb{R}_+$, $a_{1,i} : [0, 1]^2 \rightarrow \mathbb{R}_+$, with $l \in \{3, \dots, 6\}$; and $a_{7,i} : [0, 1]^3 \rightarrow \mathbb{R}_+$, which are defined as follows, for all $i \in \{1, 2\}$, with $j \in \{1, 2\} \setminus \{i\}$:

$$a_{1,i} = \frac{1}{2} \log \left(2 + \frac{\overline{\text{SNR}}_i}{\text{INR}_{ji}} \right) - \frac{1}{2}, \quad (15a)$$

$$a_{2,i}(\rho) = \frac{1}{2} \log \left(b_{1,i}(\rho) + 1 \right) - \frac{1}{2}, \quad (15b)$$

$$a_{3,i}(\rho, \mu) = \frac{1}{2} \log \left(\frac{\overline{\text{SNR}}_i (b_{2,i}(\rho) + 2) + b_{1,i}(1) + 1}{\overline{\text{SNR}}_i ((1-\mu)b_{2,i}(\rho) + 2) + b_{1,i}(1) + 1} \right), \quad (15c)$$

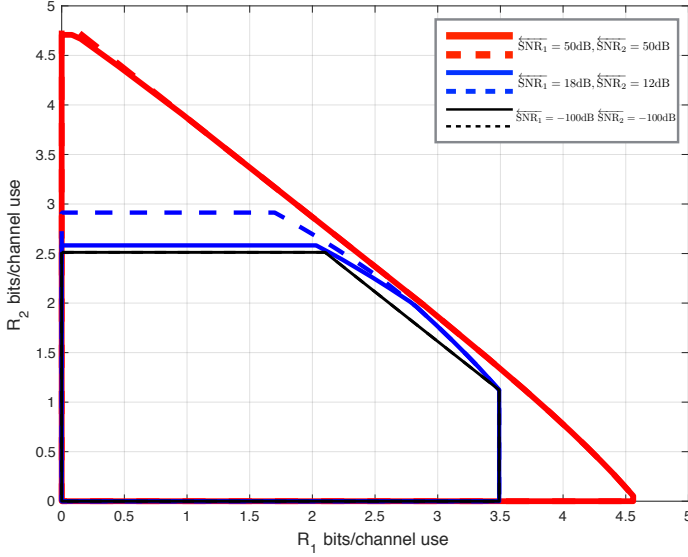


Fig. 4. Achievable capacity regions \mathcal{C} (dashed-lines) in [4, Theorem 2] and achievable η -NE regions \mathcal{N}_η (solid lines) in Theorem 1 of the two-user GIC-NOF and two-user D-GIC-NOF with parameters $\overrightarrow{\text{SNR}}_1 = 24$ dB, $\overrightarrow{\text{SNR}}_2 = 18$ dB, $\text{INR}_{12} = 48$ dB, $\text{INR}_{21} = 30$ dB, $\overrightarrow{\text{SNR}}_1 \in \{-100, 18, 50\}$ dB, $\overrightarrow{\text{SNR}}_2 \in \{-100, 12, 50\}$ dB and $\eta = 1$.

$$a_{4,i}(\rho, \mu) = \frac{1}{2} \log \left((1 - \mu) b_{2,i}(\rho) + 2 \right) - \frac{1}{2}, \quad (15d)$$

$$a_{5,i}(\rho, \mu) = \frac{1}{2} \log \left(2 + \frac{\overrightarrow{\text{SNR}}_i}{\text{INR}_{ji}} + (1 - \mu) b_{2,i}(\rho) \right) - \frac{1}{2}, \quad (15e)$$

$$a_{6,i}(\rho, \mu) = \frac{1}{2} \log \left(\frac{\overrightarrow{\text{SNR}}_i}{\text{INR}_{ji}} \left((1 - \mu) b_{2,j}(\rho) + 1 \right) + 2 \right) - \frac{1}{2},$$

$$a_{7,i}(\rho, \mu_1, \mu_2) = \frac{1}{2} \log \left(\frac{\overrightarrow{\text{SNR}}_i}{\text{INR}_{ji}} \left((1 - \mu_i) b_{2,j}(\rho) + 1 \right) + (1 - \mu_j) b_{2,i}(\rho) + 2 \right) - \frac{1}{2}, \quad (15f)$$

where the functions $b_{l,i} : [0, 1] \rightarrow \mathbb{R}_+$, with $l \in \{1, 2\}$ are defined as follows:

$$b_{1,i}(\rho) = \overrightarrow{\text{SNR}}_i + 2\rho \sqrt{\overrightarrow{\text{SNR}}_i \text{INR}_{ij}} + \text{INR}_{ij} \quad \text{and} \quad (16a)$$

$$b_{2,i}(\rho) = (1 - \rho) \text{INR}_{ij} - 1. \quad (16b)$$

Note that the functions in (15) and (16) depend on $\overrightarrow{\text{SNR}}_1$, $\overrightarrow{\text{SNR}}_2$, INR_{12} , INR_{21} , $\overrightarrow{\text{SNR}}_1$, and $\overrightarrow{\text{SNR}}_2$, however as these parameters are fixed in this analysis, this dependence is not emphasized in the definition of these functions. Finally, using this notation, the achievable η -NE region is presented by Theorem 1 on the next page. The proof of Theorem 1 is presented in [8]. The inequalities in (17) are additional conditions to those defining the region \mathcal{C} in [4, Theorem 2]. More specifically, the η -NE region is described by the intersection of the achievable region \mathcal{C} and the set of rate pairs (R_1, R_2) satisfying (17).

Figure 2 shows the achievable region \mathcal{C} in [4, Theorem 2] of a two-user centralized GIC-NOF and the achievable η -NE region \mathcal{N}_η in Theorem 1 of a two-user D-GIC-NOF with

parameters $\overrightarrow{\text{SNR}}_1 = 24$ dB, $\overrightarrow{\text{SNR}}_2 = 3$ dB, $\text{INR}_{12} = 16$ dB, $\text{INR}_{21} = 9$ dB, $\overrightarrow{\text{SNR}}_1 \in \{-100, 18, 50\}$ dB, $\overrightarrow{\text{SNR}}_2 \in \{-100, 8, 50\}$ dB and $\eta = 1$. Note that in this case, the feedback parameter $\overrightarrow{\text{SNR}}_2$ does not have an effect on the achievable η -NE region \mathcal{N}_η and the achievable capacity region \mathcal{C} ([4, Theorem 2]). This is due to the fact that when one transmitter-receiver pair is in low interference regime (LIR) and the other transmitter-receiver pair is in high interference regime (HIR), feedback is useless on the transmitter-receiver pair in HIR [15], [16].

Figure 3 shows the achievable region \mathcal{C} in [4, Theorem 2] of a two-user centralized GIC-NOF and the achievable η -NE region \mathcal{N}_η in Theorem 1 of a two-user D-GIC-NOF with parameters $\overrightarrow{\text{SNR}}_1 = 24$ dB, $\overrightarrow{\text{SNR}}_2 = 18$ dB, $\text{INR}_{12} = 16$ dB, $\text{INR}_{21} = 10$ dB, $\overrightarrow{\text{SNR}}_1 \in \{-100, 18, 50\}$ dB, $\overrightarrow{\text{SNR}}_2 \in \{-100, 12, 50\}$ dB and $\eta = 1$. Figure 4 shows the achievable region \mathcal{C} in [4, Theorem 2] of a two-user centralized GIC-NOF and the achievable η -NE region \mathcal{N}_η in Theorem 1 of a two-user D-GIC-NOF with parameters $\overrightarrow{\text{SNR}}_1 = 24$ dB, $\overrightarrow{\text{SNR}}_2 = 18$ dB, $\text{INR}_{12} = 48$ dB, $\text{INR}_{21} = 30$ dB, $\overrightarrow{\text{SNR}}_1 \in \{-100, 18, 50\}$ dB, $\overrightarrow{\text{SNR}}_2 \in \{-100, 12, 50\}$ dB and $\eta = 1$. In this case, the achievable η -NE region \mathcal{N}_η in Theorem 1 and achievable region \mathcal{C} on the capacity region [4, Theorem 2] are almost identical, which implies that in the cases in which $\overrightarrow{\text{SNR}}_i < \text{INR}_{ij}$, for both $i \in \{1, 2\}$, with $j \in \{1, 2\} \setminus \{i\}$, the achievable η -NE region is almost the same as the achievable capacity region in the centralized case studied in [4]. At low values of $\overrightarrow{\text{SNR}}_1$ and $\overrightarrow{\text{SNR}}_2$, the achievable η -NE region approaches the rectangular region reported in [5] for the case of the two-user decentralized GIC (D-GIC). Alternatively, for high values of $\overrightarrow{\text{SNR}}_1$ and $\overrightarrow{\text{SNR}}_2$, the achievable η -NE region approaches the region reported in [6] for the case of the two-user decentralized GIC with perfect channel-output feedback (D-GIC-POF). These observations are formalized by the following corollaries.

Denote by $\mathcal{N}_{\eta\text{PF}}$ the achievable η -NE region of the two-user D-GIC-POF presented in [6]. The region $\mathcal{N}_{\eta\text{PF}}$ can be obtained as a special case of Theorem 1 as shown by the following corollary.

Corollary 1 (η -NE Region with Perfect Output Feedback): Let $\mathcal{N}_{\eta\text{PF}}$ denote the achievable η -NE region of the two-user D-GIC-POF with fixed parameters $\overrightarrow{\text{SNR}}_i$ and INR_{ij} , with $i \in \{1, 2\}$ and $j \in \{1, 2\} \setminus \{i\}$. Then, the following holds:

$$\mathcal{N}_{\eta\text{PF}} = \lim_{\substack{\overrightarrow{\text{SNR}}_1 \rightarrow \infty \\ \overrightarrow{\text{SNR}}_2 \rightarrow \infty}} \mathcal{N}_\eta. \quad (18)$$

Denote by $\mathcal{N}_{\eta\text{WF}}$ the achievable η -NE region of the two-user D-GIC presented in [5]. The region $\mathcal{N}_{\eta\text{WF}}$ can be obtained as a special case of Theorem 1 as shown by the following corollary.

Corollary 2 (η -NE Region without Output Feedback): Let $\mathcal{N}_{\eta\text{WF}}$ denote the achievable η -NE region of the two-user D-GIC, with fixed parameters $\overrightarrow{\text{SNR}}_i$ and INR_{ij} , with $i \in \{1, 2\}$

Theorem 1: Let $\eta \geq 1$ be fixed. The achievable η -NE region \mathcal{N}_η is given by the closure of all possible achievable rate pairs $(R_1, R_2) \in \underline{\mathcal{C}}$ in [4, Theorem 2] that satisfy, for all $i \in \{1, 2\}$ and $j \in \{1, 2\} \setminus \{i\}$, the following conditions:

$$R_i \geq \left(a_{2,i}(\rho) - a_{3,i}(\rho, \mu_j) - a_{4,i}(\rho, \mu_j) - \eta \right)^+, \quad (17a)$$

$$R_i \leq \min \left(a_{2,i}(\rho) + a_{3,j}(\rho, \mu_i) + a_{5,j}(\rho, \mu_i) - a_{2,j}(\rho) + \eta, \quad (17b)$$

$$a_{3,i}(\rho, \mu_j) + a_{7,i}(\rho, \mu_1, \mu_2) + 2a_{3,j}(\rho, \mu_i) + a_{5,j}(\rho, \mu_i) - a_{2,j}(\rho) + \eta, \\ a_{2,i}(\rho) + a_{3,i}(\rho, \mu_j) + 2a_{3,j}(\rho, \mu_i) + a_{5,j}(\rho, \mu_i) + a_{7,j}(\rho, \mu_1, \mu_2) - 2a_{2,j}(\rho) + 2\eta \right),$$

$$R_1 + R_2 \leq a_{1,i} + a_{3,i}(\rho, \mu_j) + a_{7,i}(\rho, \mu_1, \mu_2) + a_{2,j}(\rho) + a_{3,j}(\rho, \mu_1) - a_{2,i}(\rho) + \eta, \quad (17c)$$

for all $(\rho, \mu_1, \mu_2) \in \left[0, \left(1 - \max \left(\frac{1}{\text{INR}_{12}}, \frac{1}{\text{INR}_{21}} \right) \right)^+ \right] \times [0, 1] \times [0, 1]$.

and $j \in \{1, 2\} \setminus \{i\}$. Then, the following holds:

$$\mathcal{N}_{\eta\text{WF}} = \lim_{\substack{\frac{\text{SNR}_1}{\text{SNR}_2} \rightarrow 0 \\ \rho = 0}} \mathcal{N}_\eta. \quad (19)$$

B. Impossibility Region

This section introduces an impossibility region, denoted by $\overline{\mathcal{N}}_\eta$. That is, $\overline{\mathcal{N}}_\eta \supseteq \mathcal{N}_\eta$. More specifically, any rate pair $(R_1, R_2) \in \overline{\mathcal{N}}_\eta^c$ is not an η -NE. This region is described in terms of the convex region $\overline{\mathcal{B}}_{\text{G-IC-NOF}}$. Here, for the case of the two-user D-GIC-NOF, the region $\overline{\mathcal{B}}_{\text{G-IC-NOF}}$ is given by the closure of the rate pairs $(R_1, R_2) \in \mathbb{R}_+^2$ that satisfy for all $i \in \{1, 2\}$, with $j \in \{1, 2\} \setminus \{i\}$:

$$\overline{\mathcal{B}}_{\text{G-IC-NOF}} = \left\{ (R_1, R_2) \in \mathbb{R}_+^2 : R_i \geq L_i, \right. \\ \left. \text{for all } i \in \mathcal{K} = \{1, 2\} \right\}, \quad (20)$$

where,

$$L_i \triangleq \left(\frac{1}{2} \log \left(1 + \frac{\overline{\text{SNR}}_i}{1 + \text{INR}_{ij}} \right) - \eta \right)^+. \quad (21)$$

Note that L_i is the rate achieved by the transmitter-receiver pair i when it saturates the power constraint in (5) and treats interference as noise. Following this notation, the impossibility region of the two-user GIC-NOF, i.e., $\overline{\mathcal{N}}_\eta$, can be described as follows.

Theorem 2: Let $\eta \geq 1$ be fixed. The impossibility region $\overline{\mathcal{N}}_\eta$ of the two-user D-GIC-NOF is given by the closure of all possible non-negative rate pairs $(R_1, R_2) \in \overline{\mathcal{C}} \cap \overline{\mathcal{B}}_{\text{G-IC-NOF}}$ for all $\rho \in [0, 1]$.

The impossibility region in Theorem 2 has been first presented in [6] and it is very loose in this case. A better impossibility region is presented in [15].

V. CONCLUSIONS

In this paper, an achievable η -Nash equilibrium (η -NE) region for the two-user Gaussian interference channel with noisy channel-output feedback has been presented for all $\eta \geq 1$. This result generalizes the existing achievable regions of the η -NE for the the cases without feedback and with perfect channel-output feedback.

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