

Multiscale Granger causality analysis by *à trous* wavelet transform

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Abstract—Since interactions in neural systems occur across multiple temporal scales, it is likely that information flow will exhibit a multiscale structure, thus requiring a multiscale generalization of classical temporal precedence causality analysis like Granger’s approach. However, the computation of multiscale measures of information dynamics is complicated by theoretical and practical issues such as filtering and undersampling: to overcome these problems, we propose a wavelet-based approach for multiscale Granger causality (GC) analysis, which is characterized by the following properties: (i) only the candidate driver variable is wavelet transformed (ii) the decomposition is performed using the *à trous* wavelet transform with cubic B-spline filter. We measure GC, at a given scale, by including the wavelet coefficients of the driver times series, at that scale, in the regression model of the target. To validate our method, we apply it to publicly available scalp EEG signals, and we find that the condition of closed eyes, at rest, is characterized by an enhanced GC among channels at slow scales w.r.t. eye open condition, whilst the standard Granger causality is not significantly different in the two conditions.

Index Terms—Granger causality, multiscale analysis, Wavelet transform, scalp EEG

I. INTRODUCTION

Great attention has been devoted in the last years to the identification of information flows in human brains. Wiener [1] and Granger [2] formalized the notion that, if the prediction of one time series could be improved by incorporating the knowledge of past values of a second one, then the latter is said to have a *causal* influence on the former. Initially developed for econometric applications, Granger causality (GC) has gained popularity also among engineers and physicists (see, e.g., [3], [4]). GC is connected to the information flow between variables [5]. A kernel method for GC, introduced in [6], deals with the nonlinear case by embedding data into a Hilbert space, and searching for linear relations in that space. Geweke [7] has generalized GC to a multivariate fashion in order to identify conditional GC; as described in [8], multivariate GC may be used to infer the structure of dynamical networks [9] from data. Another important notion in information theory is the redundancy in a group of variables, formalized in [10] as a generalization of the mutual information. A formalism to recognize redundant and synergetic variables in dynamical networks has been proposed in [11]; the information theoretic treatment of groups of correlated degrees of freedom can

reveal their functional roles in complex systems.

The manuscript is organized as follows: in the next section we describe the *à trous* wavelet transform, whilst in Section 3 we briefly recall the notion of GC and introduce the new method. In Section 4 we describe the application of the proposed approach to scalp EEG signals corresponding to resting conditions with closed eyes and with open eyes [17]. Section 5 summarizes our conclusions.

II. WAVELET TRANSFORM

In the present section we give a brief account of discrete wavelet mathematical aspects that are relevant to our objectives. The wavelet transform is a signal processing technique that represents a transient or non-stationary signal in terms of time and scale distribution, and it is an excellent tool for on-line data compression, analysis and reducing, etc. [18]. The most striking difference between Fourier and wavelet decomposition is that the last allows for a projection on modes simultaneously localized in both time and frequency space, up to the limit of classical uncertainty relations. Unlike the Fourier bases, which are delocalized by definition, the wavelet bases have compact spatial support, therefore being particularly suitable for the study of signals which are known only inside a limited temporal window. We like to stress that wavelet transform is not intended to replace the Fourier transform, which remains very appropriate in the study of all topics where there is no need for local information. Quantitatively, given the integer scale parameter m , the discrete wavelet transform (DWT) of a signal $f(t)$ is defined as

$$W_f(n, m) = \sum_t f(t) \psi_{m,n}^*(t), \quad (1)$$

where

$$\psi_{m,n}(t) = 2^{-m} \psi(2^m t - n)$$

is the dilated and translated version of the *mother* wavelet $\psi(t)$, n constituting a time index running on a scale dependent grid whose spacing is chosen according to the uncertainty relations; this implies that discrete wavelet analysis provides good time resolution and poor frequency resolution at high frequencies and good frequency resolution and poor time resolution at low frequencies. Many mother wavelets have been

proposed (Haar, Daubechies, Coiflets, Symlet, Biorthogonal and etc.): the selection of ψ should be made carefully to better approximate and capture the transient dynamics of the original signal.

The *à trous* wavelet transform (also called stationary wavelet transform SWT) is a wavelet transform algorithm designed to overcome the lack of translation-invariance of the discrete wavelet transform. Translation-invariance is achieved by removing the downsamplers and upsamplers in the DWT. The SWT is an inherently redundant scheme as the output of each level of SWT contains the same number of samples as the input so for a decomposition of N levels there is a redundancy of N in the wavelet coefficients. The input data is decomposed into a set of band-pass filtered components, the wavelet coefficients, plus a low-pass filtered version of the data, the continuum (or background or residual). Given a signal $x(t)$, the *à trous* wavelet transform decomposes it as a sum of a smooth version of the signal and several *detail* signals which take into account the features of the signal at the various scale. Let J be the maximal scale, then x is written as follows:

$$x(t) = c_J(t) + \sum_{j=1}^J w_j(t)$$

where

$$c_j(t) = \sum_{n=1}^5 h(n) c_{j-1}(t - 2^{j-1}(n-1)),$$

$$w_j(t) = c_{j-1}(t) - c_j(t)$$

and $h = \frac{1}{16}[1 \ 4 \ 6 \ 4 \ 1]$ is the B-spline filter. The signals c_j are coarse or smooth version of the original signal, whilst the wavelet coefficients w_j represents the *details* of x at scale 2^{-j} . The indexing is such that $j = 1$ corresponds to the finest scale (high frequencies). The maximum scale J is considered as an input. Unlike widely used non-redundant wavelet transforms, it retains the same computational requirement (linear, as a function of the number of input values). Redundancy (i.e. each scale having the same number of samples as the original signal) is helpful for detecting fine features in the detail signals since no aliasing biases arise through decimation. However this algorithm is still simple to implement and the computational requirement is $O(N)$ per scale.

III. MULTISCALE GRANGER CAUSALITY

Granger causality is a powerful and widespread data-driven approach to determine whether and how two time series exert direct dynamical influences on each other [5]. Quantitatively, let us consider n time series $\{x_\alpha(t)\}_{\alpha=1,\dots,n}$; the lagged state vectors are denoted

$$X_\alpha(t) = (x_\alpha(t-m), \dots, x_\alpha(t-1)), \quad (2)$$

m being the order of the model (window length). Let $\epsilon(x_\alpha|\mathbf{X})$ be the mean squared error prediction of x_α on the basis of all the vectors $\mathbf{X} = \{X_\beta\}_{\beta=1}^n$. The multivariate Granger causality index $\delta_{mv}(\beta \rightarrow \alpha)$ is defined as follows: consider

the prediction of x_α on the basis of all the variables but X_β and the prediction of x_α using all the variables, then the GC is the (normalized) variation of the error in the two conditions, i.e.

$$\delta_{mv}(\beta \rightarrow \alpha) = \log \frac{\epsilon(x_\alpha|\mathbf{X} \setminus X_\beta)}{\epsilon(x_\alpha|\mathbf{X})}; \quad (3)$$

The pairwise GC is given by:

$$\delta_{bv}(\beta \rightarrow \alpha) = \log \frac{\epsilon(x_\alpha|X_\alpha)}{\epsilon(x_\alpha|X_\alpha, X_\beta)}. \quad (4)$$

Here we propose to measure the causality $\beta \rightarrow \alpha$, at scale j , as

$$\Delta_j(\beta \rightarrow \alpha) = \log \frac{\epsilon(x_\alpha|X_\alpha)}{\epsilon(x_\alpha|X_\alpha, W_j)}, \quad (5)$$

where

$$W_j(t) = (w_j(t-m), \dots, w_j(t-1)),$$

is the vector of detail coefficients of the candidate driver time series x_β at scale j . In other words, we substitute the single test where the driver time series is considered as a whole, with multiple testing where a single scale is candidate driver, while obviously correcting for multiple comparison with Bonferroni correction as in [6]. As an example, we tested the approach on the simulated two time series

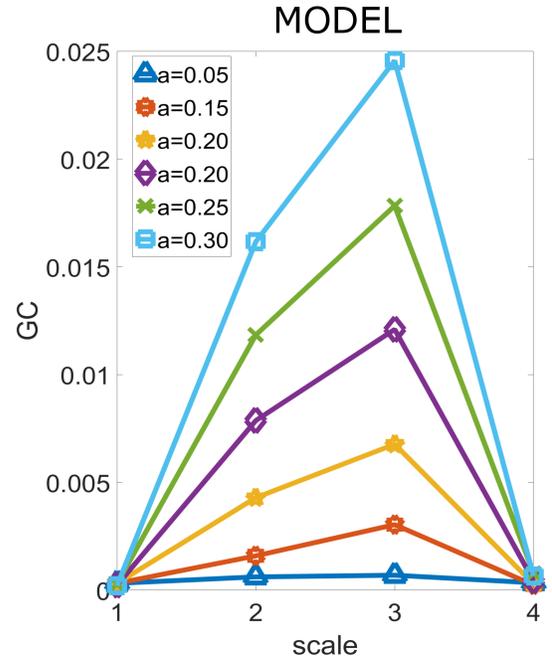


Fig. 1. We depict the multiscale GC evaluated by the proposed approach on a simulated two dimensional linear system, unidirectionally coupled with lag equal to 8 and strength a (see the text): it increases with the strength and peaks in correspondence of the lag.

$$\begin{aligned} x(t) &= 0.3 x(t-1) + 0.5 \eta_1(t) \\ y(t) &= 0.1 y(t-1) + a x(t-8) + 0.5 \eta_2(t) \end{aligned} \quad (6)$$

where η_1 and η_2 are i.i.d. unit variance noise terms, and a is the coupling $x \rightarrow y$. Setting the maximal scale $J = 4$, we plot

in fig.1 the multiscale GC Δ_j as evaluated by the proposed approach: it shows a peak in correspondence of the lag, as depicted in figure 1.

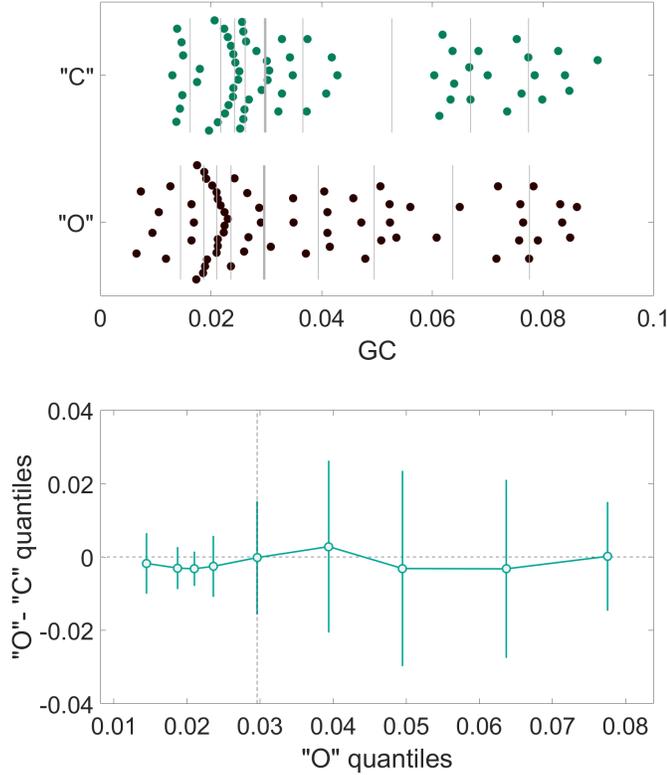


Fig. 2. The standard GC, averaged over pairs of channels, is depicted for the 64 subjects in the two conditions, eye open (O) and eye closed (C). In the bottom, the shift function [20] is depicted.

IV. DATA SET AND RESULTS

We apply the proposed algorithm to scalp EEG signals gathered from the public database PhysioNet BCI [17]. The database consists of healthy subjects recorded in two different baseline conditions, i.e. eyes open (EO) resting state and eyes closed (EC) resting state. In each condition, subjects were comfortably seated on a reclining chair in a dimly lit room. During EO they were asked to avoid ocular blinks in order to reduce signal contamination. The EEG data were recorded with a 64-channel system with an original sampling rate of 160 Hz. All the EEG signals are here referenced to the mean signal gathered from electrodes on the ear lobes. Same data were analyzed in [19]. From 64 subjects and two conditions (EO, EC) the EEG signals epochs of 10 seconds are considered. These epochs are considered as different observations of the same mental state and they are used to assess the differences in directed dynamical connectivity in the two conditions. We evaluate the multiscale GC here proposed in all the EEG segments and average it over all pairs of channels in the two

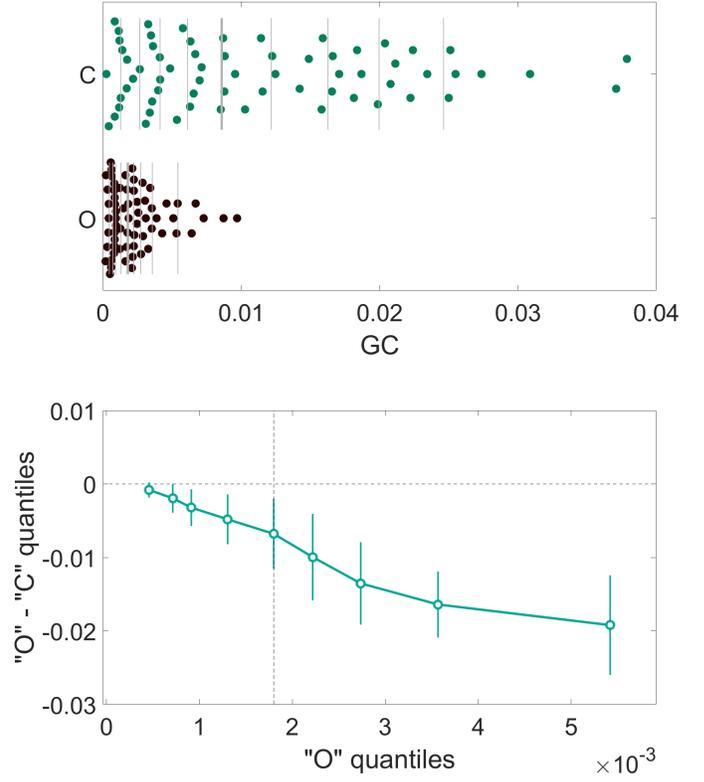


Fig. 3. The multiscale GC at scale $j = 4$, averaged over pairs of channels, is depicted for the 64 subjects in the two conditions, eye open (O) and eye closed (C). In the bottom, the shift function [20] is depicted, showing a clear separation in the two conditions for slow scales.

conditions for all scales $j = 1, \dots, 4$; we also evaluate the classical GC for all EEG epochs.

We find that the global amount of GC among signals is significantly decreasing as the scale j is increased, in both conditions. Furthermore, comparing signals corresponding to resting conditions with closed eyes and with open eyes, we find that at large scales the directed dynamical connectivity, in terms of the proposed measure, is significantly increased when eyes are closed w.r.t. eyes open, whilst using the standard GC no differences between the two conditions are found. Standard GC values for eyes open and closed are depicted in figure 2; the multiscale GC from wavelet coefficients (scale 4) is depicted in figure 3. The latter results in a clear separation of the two conditions at all the quantiles.

V. CONCLUSIONS

A great need exists of effective approaches to measure scale-dependent directed dynamical connectivity, especially in applications where interactions coexist at several scales. Here we have proposed a novel method based on wavelet transform. As Granger causality examines how much the predictability of the target from its past improves when the driver variables past values are included in the regression, we measure it at a

given scale by including the wavelet coefficients of the driver time series in the regression model of the target. Comparing scalp EEG signals in resting subjects we have shown that the wavelet-based multiscale GC at slow scales significantly increases when eyes are closed (w.r.t. open eye condition); this phenomenon is not detected by the classical GC estimation.

REFERENCES

- [1] N. Wiener, *The theory of prediction*. In E.F. Beckenbach, Ed., *Modern mathematics for Engineers*. (McGraw-Hill, New York, 1956).
- [2] C.W.J. Granger, "Investigating causal relations by econometric models and cross-spectral methods", *Econometrica* **37**, 424 (1969).
- [3] M. Wibral, R. Vicente and J.T. Lizier (Eds.) *Directed Information Measures in Neuroscience* (Springer, Berlin, 2014);
- [4] K. Sameshima, L.A. Baccala (Eds.) *Methods in Brain Connectivity Inference through Multivariate Time Series Analysis* (CRC press, 2014);
- [5] K. Hlavackova-Schindler, M. Palus, M. Vejmelka, J. Bhattacharya, "Causality detection based on information-theoretic approaches in time series analysis", *Physics Reports* **441**, 1-46 (2007).
- [6] D. Marinazzo, M. Pellicoro, S. Stramaglia, "Kernel method for nonlinear Granger causality", *Phys. Rev. Lett.* **100**, 144103 (2008).
- [7] J. Geweke, "Measures of conditional linear dependence and feedback between time series", *J. Am. Stat. Assoc.* **79**, 907 (1984).
- [8] D. Marinazzo, M. Pellicoro and S. Stramaglia, "Kernel Granger causality and the analysis of dynamical networks", *Phys. Rev. E* **77**, 056215 (2008).
- [9] S. Boccaletti, V. Latora, Y. Moreno, M. Chavez and D.U. Hwang, "Complex networks: structure and dynamics", *Physics Reports* **424**, 175-308 (2006).
- [10] M. Palus, V. Albrecht, I Dvorak, "Information theoretic test for nonlinearity in time series", *Phys. Lett. A* **175**, 203 (1993).
- [11] S. Stramaglia, L. Angelini, G.R. Wu, J.M. Cortes, L. Faes, D. Marinazzo, "Synergetic and redundant information flow detected by unnormalized Granger causality: application to resting state fMRI", *IEEE Trans. Biomed. Eng.* Vol. 63, Pages 2518-2524 (2016).
- [12] A. Pitti, M. Lungarella, Y. Kuniyoshi, "Information transfer at multiple scales", *Physical Review E* 2007, 76 (5), pp.056117-1, 056117-10.
- [13] J.R. Sato, E.A. Junior, D.Y. Takahashi DY, F.M. de Maria, M.J. Brammer, P.A. Morettin PA, "A Method to produce evolving functional connectivity maps during the course of an fMRI experiment using Wavelet-based time-varying Granger causality", *Neuroimage* 31 2006 18796
- [14] L. Faes, A. Montalto, S. Stramaglia, G. Nollo, D. Marinazzo, "Multi-scale analysis of information dynamics for linear multivariate processes", *Conf Proc IEEE Eng Med Biol Soc.* 2016 Aug;2016:5489-5492. doi: 10.1109/EMBC.2016.7591969.
- [15] L. Barnett, A.K. Seth, "Detectability of Granger causality for subsampled continuous-time neurophysiological processes", *Journal of Neuroscience Methods* 275, pp. 93121 (2017).
- [16] Renaud O, Starck, J-L, Murtagh, F, "Wavelet-based combined signal filtering and prediction", *IEEE Transactions on Systems, Man, and Cybernetics, Part B: Cybernetics* 35, no. 6, p. 1241-1251 (2005).
- [17] <http://www.physionet.org/pn4/eegmmidb>
- [18] J.L. Starck, F. Murtagh, A. Bijaoui, *Image processing and data analysis: the multiscale approach*, Cambridge University Press New York, NY, USA 1998
- [19] D. La Rocca, P. Campisi, B. Vegso, F. De Vico Fallani, "Human brain distinctiveness based on EEG spectral coherence connectivity", *IEEE transactions on Biomedical Engineering* 61(9) April 2014
- [20] G.A. Rousselet, C.R. Pernet, R.R. Wilcox, "Beyond differences in means: robust graphical methods to compare two groups in neuroscience", *European Journal of Neuroscience*; doi: 10.1111/ejn.13610 (2017)