

# Bearing-only Landmark Initialization with Unknown Data Association

Al Costa  
Department of Mechanical Engineering  
Carnegie Mellon University  
Email: [acosta@andrew.cmu.edu](mailto:acosta@andrew.cmu.edu)

George Kantor  
Robotics Institute  
Carnegie Mellon University  
Email: [kantor@ri.cmu.edu](mailto:kantor@ri.cmu.edu)

Howie Choset  
Department of Mechanical Engineering  
Carnegie Mellon University  
Email: [choset@cs.cmu.edu](mailto:choset@cs.cmu.edu)

*Abstract*—It is essential in many applications that mobile robots localize themselves with respect to an unknown environment. This means that the robot must build a map of its environment and then localize using the map. This process is called simultaneous localization and mapping (SLAM). This paper presents an iterative solution to the landmark initialization problem inherent in a bearing-only implementation of SLAM. No prior knowledge of the environment is required, and furthermore, there are no requirements about having the data association problem solved. Once landmarks are initialized, they are inserted into an extended Kalman Filter (EKF) to solve the SLAM problem. Both indoor and outdoor experiments are presented to validate the method.

## I. INTRODUCTION

Here we discuss two major problems that must be addressed in any implementation of simultaneous localization and mapping (SLAM): data association and landmark initialization. The data association problem deals with matching measurements to the appropriate landmark, or similarly, matching measurements from the current step to measurements from prior steps. If the landmark locations are known, then the data association problem is fairly straightforward to solve [2]. The landmark initialization problem deals with using the measurements to determine a spatial estimate for the landmark. These problems are particularly important in the case where available sensors provide only bearing information. While a single range and bearing measurement is usually enough to initialize a landmark relative to the robot, two or more measurements are required if only bearing information is available [1].

This paper presents a new method of dealing with the landmark initialization problem in bearing-only SLAM. Bearing-only SLAM allows for SLAM algorithms to be run on mobile platforms with inexpensive sensors such as low resolution monocular cameras. Furthermore, this aids in SLAM implementations in which a camera is used to detect natural landmarks such as edges.

The approach presented in this paper is an iterative method for initializing landmarks given only bearing measurements. The iterative nature of the algorithm allows it to be incorporated into an extended Kalman filter (EKF) in a straightforward manner. This method makes no assumptions about having the data association problem solved or that disambiguating between any subset of the landmarks is possible.

## II. BACKGROUND

Leonard and Durrant-Whyte [2] first coined the term simultaneous localization and mapping or SLAM. This field has received considerable attention in the past five years. Conventional SLAM involves fusing observations of landmarks with dead-reckoning information in order to track the location of the robot and build a map of the landmark locations. Implementations of SLAM are usually done using Kalman Filters [3] [4] [5] [6] or particle filters [7] [8]. The EKF [9] [3] [4] uses a linear approximation of the system in order to maintain a state vector that contains the locations of the robot and landmarks. In addition to this, it maintains a covariance matrix that contains an approximation of correlated uncertainty.

The problem of conducting SLAM using only bearing measurements has also been discussed in the recent literature. The nonlinear nature of the bearing-only SLAM problem has led to some difficulties in estimating the location of the landmarks given a series of robot poses and measurements from those poses. Techniques have been developed to determine when the problem is well-conditioned for landmark initialization [10] [1]. These methods ignore the data association problem, i.e., they assume it is known as to which measurements correspond to which landmarks. This is important in that it allows for merging measurements from the current pose with the appropriate measurements from prior poses. A fast, accurate, and robust method for running SLAM algorithms on a bearing-only problem was also considered [11]. This method employs a batch technique in order to initialize the landmarks.

Monocular vision cameras are an increasingly popular sensor for SLAM in light of the developments in bearing-only SLAM. As this is the case, various methods from the vision community have been applied to SLAM, such as color-based feature tracking [12] and structure from motion (SFM) [13] [14]. Feature tracking-based approaches, as well as the SFM methods, have solved the data association problem; landmarks are tracked from one frame to another so the measurements are automatically associated. Additionally, batch optimization techniques do not directly require that the landmarks be initialized; however, they are sensitive to the initial conditions used in the optimization process. The work presented here goes beyond these results by presenting a method of landmark

initialization that does not rely on feature tracking or batch optimization.

### III. LANDMARK INITIALIZATION

The bearing-only SLAM landmark initialization methods mentioned in Section II all assume that the data association is known *a priori*; however, if a system is to be used in an unknown environment with landmarks that are not easily distinguishable from each other, then it is unlikely that such an assumption about the data association would be reasonable. Similarly, if a robot needs to reach its goal in a short period of time, it becomes harder to obtain frames that are close together causing feature tracking to fail. In this section, a method is presented for initializing landmarks without any *a priori* data association or requirements about the distance the robot can travel between frames.

#### A. Determining Spatial Estimates for the Landmark Locations

The locations of the robot and landmarks are maintained in a state vector,  $\mathbf{X}$ , with an associated covariance matrix,  $\mathbf{P}$ ,

$$\mathbf{X} = [x_r, y_r, \theta_r, x_{l1}, y_{l1}, \dots, x_{ln}, y_{ln}]^T, \quad (1)$$

where  $[x_r, y_r, \theta_r]$  are the position and orientation of the robot, and  $[x_{li}, y_{li}]$  are the location of the  $i$ th landmark.

The bearing measurements are maintained in a vector  $\mathbf{Z}$ ,

$$\mathbf{Z} = [\alpha_1, \dots, \alpha_m]^T, \quad (2)$$

where  $\alpha_j$  is the  $j$ th measurement from the current robot pose.

Consider an ideal system with perfect positioning and no sensing uncertainty. Note that a single measurement corresponds to a ray emanating from the robot position at which the measurement was taken. For measurements taken from a sequence of robot positions, a landmark that has been seen from multiple locations will lie at the point where rays emanating from those locations intersect. Without a method for determining the data association, it is unknown which rays to intersect (Figure 1). So the only option is to consider every possible intersection. Only some of these calculated intersections correspond to actual landmark locations. The remaining intersection points are due to intersecting measurements that do not correspond to the same landmark (i.e. a misassociation). Some intersections can be eliminated by ensuring that the point lies in the detectable range of the robot at the poses from which the measurements originated. Storing the number of times that a given point has been calculated as an intersection will yield some measure of *persistence* of that landmark pose estimate as a possibility. As additional measurements are taken from new robot locations, the *persistence* of an intersection can be tracked. As the intersection's *persistence* increases, the probability of that intersection corresponding to a landmark increases<sup>1</sup>. Once the probability reaches an acceptable range, the landmark can be initialized.

<sup>1</sup>This is only qualitative. We have not yet derived a formal relationship between persistence and probability.

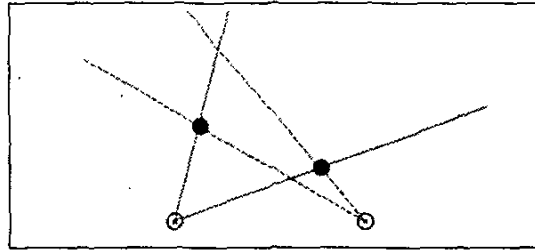


Fig. 1. Two measurements are obtained from each of the two poses. These two measurements can be modeled as lines and intersected resulting in four intersection points. Of these four intersection points, only two correspond to the actual landmark locations (denoted by the solid black circles).

In the presence of positioning and sensing uncertainties, this process is not as straightforward, but can still serve as a guideline for landmark initialization. A set of possible landmark locations must still be generated by intersecting measurements, and this set can be further reduced by taking into consideration the sensing range limitations of the robot. The difficulty due to sensing and positioning error arises in determining the *persistence* of the landmarks. Some measure of the uncertainty of the intersection point due to the robot's positioning and sensing errors must be determined; therefore, as each intersection is determined, so is a corresponding covariance matrix. The choice of Gaussian representations for this uncertainty follows naturally from the Kalman filtering-based solution to the SLAM problem since the robot's positioning and sensing uncertainties are already represented as Gaussian distributions and stored in covariance matrices.

#### B. Creating Gaussian Representations

In order to represent the uncertainty of the calculated intersection point of two measurements, several pieces of information need to be fused. We opt to perform this fusion in the plane, which means that we must project measurements from sensor space onto the plane. To accomplish this, we present a technique of approximating the projection of a measurement as a bivariate Gaussian distribution in the plane. In order to create the Gaussian then, three pieces of information are needed: a major and minor axis,  $a_1$  and  $a_2$  respectively, for the ellipse and an angle of rotation,  $\phi = (\theta_r + \alpha)$  where  $\theta_r$  is the orientation of the robot and  $\alpha$  is the bearing measurement to a landmark. Since the measurements are relative bearings to the landmarks, bearing is a natural selection for one of the axes of the ellipse. While the uncertainty of the bearing measurement is constant, the spatial uncertainty of the estimate increases as the distance to the estimate increases. Therefore, this axis of the ellipse can be given by

$$a_2 = D \sin \sigma_\alpha, \quad (3)$$

where  $\sigma_\alpha$  is the standard deviation of the measurement and  $D$  is the distance from the robot's pose to the intersection point (Figure 2). If the standard deviation of the measurement is suf-

ficiently small, then the following small angle approximation can be made,

$$a_2 = D\sigma_\alpha. \quad (4)$$

To create the other axis, of the ellipse, some notion of uncertainty in the direction perpendicular to the bearing is necessary. Since the sensor does not return any range information, the best that can be done is to create a very large uncertainty in this direction (e.g. some constant times the distance to the intersection). The ellipse is now given by

$$\mathbf{E} = \begin{bmatrix} a_1^2 & 0 \\ 0 & a_2^2 \end{bmatrix}, \quad (5)$$

where  $a_1$  is some large uncertainty and  $a_2$  is given by Equation 4. All that remains is to rotate the ellipse such that the range axis is normal to the robot and the bearing axis lies in a tangential direction (Figure 2):

$$\mathbf{C}_2 = \mathbf{R}\mathbf{E}\mathbf{R}^T, \quad (6)$$

where  $\mathbf{R}$  is given by

$$\mathbf{R} = \begin{bmatrix} \cos(\theta_r + \alpha) & -\sin(\theta_r + \alpha) \\ \sin(\theta_r + \alpha) & \cos(\theta_r + \alpha) \end{bmatrix}. \quad (7)$$

This covariance matrix,  $\mathbf{C}_2$ , represents the uncertainty of the landmark estimate given only sensor noise; it does not take into consideration the uncertainty of the robot's pose,  $\mathbf{C}_1$ . In order to combine the newly calculated sensor uncertainty with the robot's position uncertainty, *compounding* from [15] is used. Compounding is a method of determining the uncertainty of an estimate due to a measurement from another uncertain location. This method requires functions,  $\mathbf{f}$  and  $\mathbf{g}$ , that describe the transformation from the robot frame to the intersection frame, and also the Jacobian,  $\mathbf{J}$ , of these functions (for more details see [15]). Specifically, the new covariance matrix is:

$$\mathbf{C}_3 = \mathbf{T}\mathbf{C}_1\mathbf{T}^T + \mathbf{C}_2, \quad (8)$$

where the transformation matrix,  $\mathbf{T}$ , that translates the robot's pose uncertainty,  $\mathbf{C}_1$ , to the intersection point,  $\mathbf{X}_3$ , is given by:

$$\mathbf{T} = \begin{bmatrix} 1 & 0 & -(y_l - y_r) \\ 0 & 1 & (x_l - x_r) \end{bmatrix}, \quad (9)$$

where  $[x_l, y_l]$  is the location of the landmark estimate and  $[x_r, y_r]$  is the robot's location.  $\mathbf{C}_3$  contains the uncertainty of the robot's pose at the time of the measurement,  $\mathbf{C}_1$ , and the uncertainty of the pose due to measurement noise,  $\mathbf{C}_2$ .

The landmark pose estimate,  $\mathbf{X}_3$ , and its covariance matrix,  $\mathbf{C}_3$ , represent the landmark estimate due to a single measurement, but two measurements formed the intersection. This means that a second estimate,  $\mathbf{X}_4$  and  $\mathbf{C}_4$ , must be created in a similar fashion for the other measurement. At the intersection point, there are now two estimates that need to be combined to yield a single estimate for the intersection. In order to combine

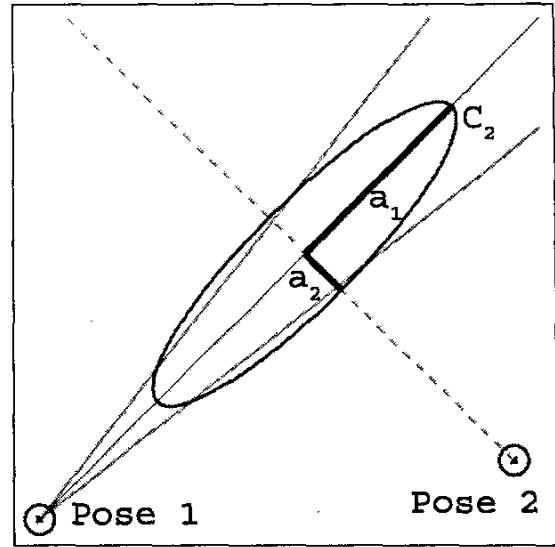


Fig. 2. Two rays corresponding to measurements from two distinct poses intersect at a point.  $a_2$  is determined using the standard deviation of the sensor's uncertainty.  $a_1$  is chosen to be large, and the ellipse created using  $a_1$  and  $a_2$  is rotated to its proper orientation.

the estimates, the *merging* method discussed in [15] will be used. The merging equations are

$$\mathbf{K} = \mathbf{C}_3 [\mathbf{C}_3 + \mathbf{C}_4]^{-1} \quad (10)$$

$$\mathbf{C}_{int} = \mathbf{C}_3 - \mathbf{K}\mathbf{C}_3 \quad (11)$$

$$\mathbf{X}_{int} = \mathbf{X}_3 + \mathbf{K}(\mathbf{X}_4 - \mathbf{X}_3), \quad (12)$$

where  $\mathbf{K}$  is the Kalman gain factor,  $\mathbf{C}_{int}$  is the covariance matrix of the intersection, and  $\mathbf{X}_{int}$  is the resulting landmark pose estimate (Figure 4).

Now that an estimate of each intersection as well as a representation of its uncertainty have been obtained, the persistence of each estimate can be determined.

### C. Clustering and Combining Intersections

The next step in the process is to group the intersections into clusters. In order to do this a notion of closeness is needed. Therefore, the distance between intersections must be determined. The representation of an intersection was chosen to be a mean value and corresponding Gaussian distribution, and numerous distance metrics between such distributions can be found in Statistics literature. One such distance metric is the Bhattacharyya distance metric [16]. The Bhattacharyya distance between two multivariate Gaussian distributions is given by

$$B = \frac{1}{8} (\mathbf{m}_1 - \mathbf{m}_2)^T \left( \frac{\boldsymbol{\Sigma}_1 + \boldsymbol{\Sigma}_2}{2} \right)^{-1} (\mathbf{m}_1 - \mathbf{m}_2) + \frac{1}{2} \ln \left( \frac{\det(\boldsymbol{\Sigma}_1 + \boldsymbol{\Sigma}_2) / 2}{\sqrt{\det(\boldsymbol{\Sigma}_1)} \sqrt{\det(\boldsymbol{\Sigma}_2)}} \right), \quad (13)$$

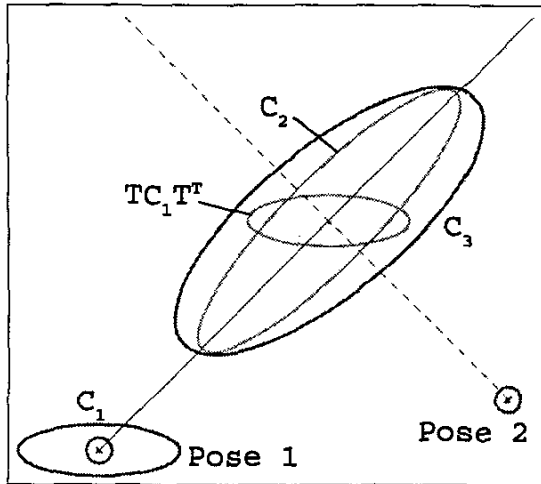


Fig. 3. An intersection is found from a pose with some uncertainty,  $C_1$ . The uncertainty of the robot is translated to the intersection point. The uncertainty due to sensing noise,  $C_2$ , is determined. The two sources of uncertainty are compounded.

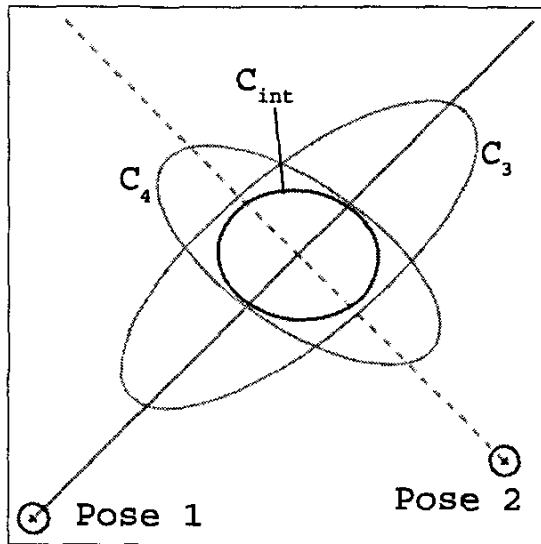


Fig. 4. Merging  $C_3$  and  $C_4$  yields  $C_{int}$ .

where  $m_1$  and  $m_2$  are the means of the estimates and similarly  $\Sigma_1$  and  $\Sigma_2$  are the corresponding covariance matrices. If the distance between two estimates is close enough for the given system, then the two estimates should be combined and the persistence count should be increased. This is done for subsequent estimates as well. This method doesn't explicitly cluster, but it does use estimates from several steps in order to determine a better estimate. In order to combine the estimates, the merging method mentioned above (Equations 10 - 12) is used.

As the persistence count increases, the certainty of the estimate corresponding to a landmark also increases. Once this

certainty is sufficiently large, the landmark can be initialized with the intersection's mean vector being appended to the system's state vector, and the covariance matrix being appended to the covariance matrix of the system.

#### IV. BEARING-ONLY SLAM

Section III discussed initializing a landmark given only the robot's pose and bearing measurements to the landmark. This section discusses how this initialization can be included into an EKF implementation of SLAM. Section IV-A summarizes the EKF and how it is used in SLAM. Section IV-B discusses data association for previously initialized landmarks. And Section IV-C discusses how the aforementioned landmark initialization method can be included in the EKF.

##### A. EKF

The locations of the robot and the landmarks are stored in a state vector,  $\mathbf{X}$ , as given in Equation 1 with a corresponding covariance matrix,  $\mathbf{P}$ . Linearizing about the current robot pose estimate in a fashion similar to a first order Taylor series approximation allows the EKF to compute estimates despite the systems nonlinearities. The EKF consists of two major steps, the time update or propagation step and the measurement update step. During the time update, the odometry information obtained from the robot is treated as an input,  $u_k$ , to generate an estimate of the robot's new position and uncertainty

$$\hat{\mathbf{x}}_k^- = \mathbf{f}(\hat{\mathbf{x}}_{k-1}, u_k) \quad (14)$$

$$\mathbf{P}_k^- = \mathbf{A}_k \mathbf{P}_{k-1} \mathbf{A}_k^T + \mathbf{W}_k \mathbf{Q}_{k-1} \mathbf{W}_k^T, \quad (15)$$

where  $\mathbf{f}$  is a nonlinear function that describes how the state of the system propagates as a function of the previous state and the current inputs,  $\mathbf{Q}$  is the process noise covariance matrix, and  $\mathbf{A}$  and  $\mathbf{W}$  are the Jacobians of the state propagation function,  $\mathbf{f}$ , with respect to the state and process noise respectively, i.e.

$$\mathbf{A} = \left. \frac{\partial \mathbf{f}}{\partial \mathbf{x}} \right|_{(\mathbf{x}=\hat{\mathbf{x}}_{k-1}, u=u_k)} \quad (16)$$

$$\mathbf{W} = \left. \frac{\partial \mathbf{f}}{\partial \mathbf{w}} \right|_{(\mathbf{x}=\hat{\mathbf{x}}_{k-1}, u=u_k)}. \quad (17)$$

If any landmarks are detected, the SLAM algorithm tries to associate the new measurement with landmarks contained in the state vector (see Section IV-B). If any measurements are associated, then a measurement update is run. The measurement update calculates what it would expect the measurements to be given the current estimate of the robot's and landmarks' locations

$$\hat{\mathbf{z}}_k = \mathbf{h}(\hat{\mathbf{x}}_k^-). \quad (18)$$

The Kalman gain,  $\mathbf{K}$ , and posterior estimates of the state and covariance matrix can be determined by

$$\mathbf{K}_k = \mathbf{P}_k^- \mathbf{H}_k^T (\mathbf{H}_k \mathbf{P}_k^- \mathbf{H}_k^T + \mathbf{V}_k \mathbf{R}_k \mathbf{V}_k^T)^{-1} \quad (19)$$

$$\hat{\mathbf{x}}_k = \hat{\mathbf{x}}_k^- + \mathbf{K}_k (\mathbf{z}_k - \hat{\mathbf{z}}_k) \quad (20)$$

$$\mathbf{P}_k = (\mathbf{I} - \mathbf{K}_k \mathbf{H}_k) \mathbf{P}_k^-, \quad (21)$$

where  $\mathbf{R}$  is the sensor noise covariance matrix, and  $\mathbf{H}$  and  $\mathbf{V}$  are the Jacobians of the sensor model,  $\mathbf{h}$ , with respect to the state and sensor noise respectively, i.e.

$$\mathbf{H} = \left. \frac{\partial \mathbf{h}}{\partial \mathbf{x}} \right|_{(\mathbf{x}=\hat{\mathbf{x}}_k^-)} \quad (22)$$

$$\mathbf{V} = \left. \frac{\partial \mathbf{h}}{\partial \mathbf{v}} \right|_{(\mathbf{x}=\hat{\mathbf{x}}_k^-)}. \quad (23)$$

If no landmarks are associated, then

$$\hat{\mathbf{x}}_k = \hat{\mathbf{x}}_k^- \quad (24)$$

$$\mathbf{P}_k = \mathbf{P}_k^-. \quad (25)$$

This means that in the absence of any measurements, the best estimate that can be determined comes simply from propagating the odometry.

### B. Data association

Data association between a measurement and a previously initialized landmark is typically done using a Chi-squared test. The Chi-squared test is given by

$$\chi^2 \geq \nu^T \mathbf{S}^{-1} \nu, \quad (26)$$

where  $\nu$  is the innovation,  $\nu = [\alpha_{observed} - \alpha_{estimate}]$ , and  $\mathbf{S}$  is the innovation covariance given by

$$\mathbf{S}_k = \mathbf{H}_k \mathbf{P}_k^- \mathbf{H}_k^T + \mathbf{V}_k \mathbf{R}_k \mathbf{V}_k^T. \quad (27)$$

The value of  $\chi$  can be obtained from a Chi-squared table. The observation has 1-degree-of-freedom, and if the system was to associate measurements to landmarks with a confidence of 95%, then the value of  $\chi$  would be 3.84.

If a given measurement passes the Chi-squared test, it is added as a possible match for the landmark. If multiple measurements are associated to the same landmark, or multiple landmarks are associated with the same measurement, several options are available. The measurement could simply be disregarded to avoid a possible misassociation, the measurement-landmark combination with the lowest chi-squared value could be associated since it will have the largest probability of being correct, or as done in [12], the set of measurement-landmark combinations that yields the highest number of jointly-compatible matches can be selected.

### C. Landmark initialization

In order for the bearing-only SLAM algorithm to be complete, it must be able to initialize landmarks and add them to the state of the system. In Section III, a method for initializing landmarks with only bearing information was presented. For this method to be used in conjunction with the EKF presented here, it must be added after the data association step. This means that the robot takes a step and propagates its odometry and error covariance matrix. Then a measurement is taken, and data association is attempted. If there are unassociated measurements, they are used in the landmark initialization algorithm along with the current estimate of the robot's position,  $\hat{\mathbf{x}}_k^-$ . If any new landmarks are initialized as a result of these new measurements, the state vector and covariance matrix need to have the new information added:

$$\hat{\mathbf{x}}_{new}^- = \begin{bmatrix} (\hat{\mathbf{x}}_{old}^-)^T & x_l & y_l \end{bmatrix}^T \quad (28)$$

$$\mathbf{P}_{new}^- = \begin{bmatrix} \mathbf{P}_{old}^- & \mathbf{0} \\ \mathbf{0} & \mathbf{P}_{landmark} \end{bmatrix}. \quad (29)$$

In this manner, each new landmark can be added as it is initialized. And the EKF can be run with no other modifications.

## V. EXPERIMENTS

Current tests of the EKF and landmark initialization method consist of simple outdoor SLAM experiments where the robot was driven for fifty feet, Section V-A, and also an indoor mapping experiment where the robot was driven for over one hundred feet, Section V-B.

### A. Outdoor SLAM

The outdoor experiments were conducted on a differential drive robot built at the Sensor Based Planning Lab, Carnegie Mellon University. The robot was equipped with encoders attached to the two motors used for determining the robot's odometry and a monocular camera used to measure bearings to pink golf balls that were used as landmarks. A picture of the robot during a run can be seen in Figure 5. The robot drove for about fifty feet (fifteen meters) while gathering bearing measurements to several landmarks during three separate experiments. In the first experiment, there were eight landmarks spread out in order to allow the robot to see at least one landmark at all times. The second experiment was set up such that there were fewer landmarks (only five landmarks), but they were still set up such that the robot could always obtain measurements to at least one. The third experiment was designed such that there were periods in which the robot could not see any landmarks and must therefore rely totally on its odometry at those points. The results can be seen in Figures 6a, 6b, and 6c. The final position for all three experiments was designed to be fifty feet away from where the robot started and the robot arrived within five inches of this point at the conclusion of each experiment. At the conclusion of the first experiment, the robot had the least uncertainty of

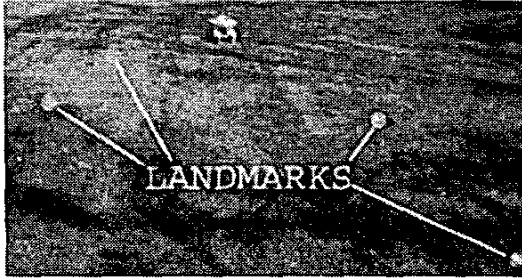


Fig. 5. A picture of the robot, 50 feet away, during an outdoor run. The landmarks are pink golfballs tossed in the grass. Several golf balls are highlighted in the picture.

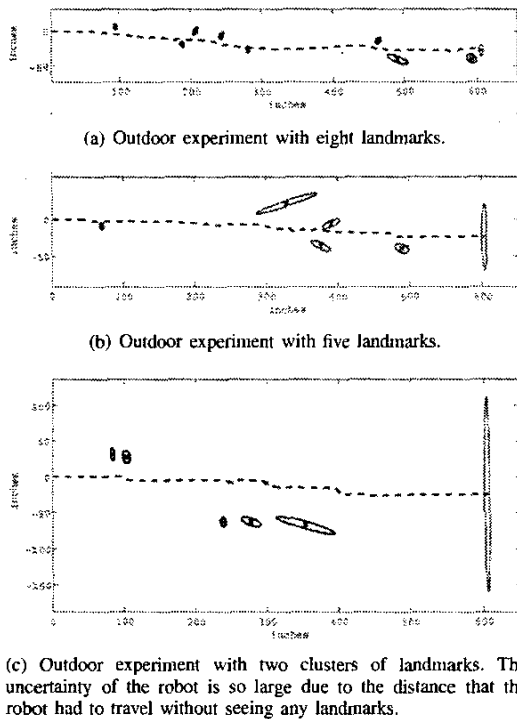


Fig. 6. A series of outdoor experiments. The black ellipses represent the uncertainty of the landmark estimates. The slightly lighter ellipse at the right side of each figure represents the uncertainty of the robot at the end of the experiment.

all the trials, the Frobenius norm of the robot's covariance matrix was  $7.05 \text{ in}^2$  ( $45.48 \text{ cm}^2$ ). The covariance matrix at the conclusion of the second experiment had a Frobenius norm of  $153.2 \text{ in}^2$  ( $988.4 \text{ cm}^2$ ), and the covariance matrix at the end of the third experiment was  $206.5 \text{ in}^2$  ( $1332.3 \text{ cm}^2$ ). This result is expected. As the robot had to rely on its odometry outside more frequently, the robot's certainty at the end of the experiment decreased.

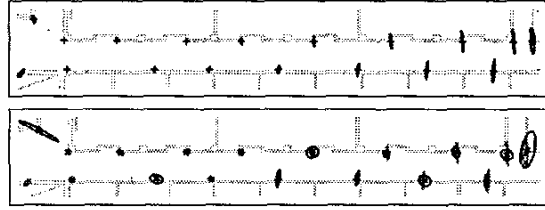


Fig. 7. One corridor of Wean Hall at Carnegie Mellon University. The top figure shows results from an earlier experiment using both bearing and range information to the landmarks. The bottom figure contains results from the bearing-only SLAM implementation discussed in this paper (processed offline) in black superimposed on the results from the top figure in gray.

### B. Indoor Mapping

The indoor experiment was conducted on a Nomadic Scout robot equipped with a firewire camera and overhead omnidirectional mirror. Again encoders were used for odometric reasons and the camera-mirror combination was used to obtain bearing information to landmarks. The indoor experiment was conducted in Wean Hall at Carnegie Mellon University. The bearing-only SLAM results presented in Figure 7 were processed offline. The only reason for this was that the data had been gathered previously for a different experiment.

In Figure 7, the ellipses and crosses in the top figure correspond to the landmark estimates and their corresponding  $2\sigma$  uncertainty ellipses. These results are from a SLAM implementation that uses both bearing and range measurements to the landmarks. Similarly, the black ellipses and crosses in the bottom figure correspond to a different set of landmark estimates and their corresponding uncertainty ellipses. These are the result of the method for landmark initialization and SLAM discussed in this paper. The bearing-range implementation yields a set of landmarks with less uncertainty than the bearing-only method. This is to be expected; the bearing-range implementation has access to more information regarding each landmark at a given step. Using the bearing-range SLAM algorithm, the robot also arrives at the final destination with less uncertainty than it did with the bearing-only SLAM algorithm, in fact the Frobenius norm of the robot's covariance matrix due to the bearing-range information was  $14.9 \text{ in}^2$  ( $96.1 \text{ cm}^2$ ) while the norm due to the bearing-only information was  $45.2 \text{ in}^2$  ( $291.3 \text{ cm}^2$ ).

## VI. CONCLUSION

This paper presented an iterative approach to solving the landmark initialization problem found in bearing-only SLAM. The method does not require a solution for data association or require any sort of batch process to be run.

However, as the method is presented, it does have difficulties initializing landmarks in a few circumstances. Since the landmarks are being initialized with noisy measurements, if the angle between the intersecting measurements is small, the error in calculating the intersection point gets large. Consider the case of nearly parallel measurements; calculating an accurate intersection point for them is nearly impossible in

the presence of the measurement noise as well as numerical errors in the calculation. This can be solved by simply ignoring measurements that do not intersect at a large enough angle. While this approach would potentially take longer to initialize the landmarks, it would yield a more robust approach to the initialization process.

This also brings up the issue that taking a straight line path may yield a situation in which one or more landmarks can never be initialized. So if the goal of the application is to generate a complete (or near complete) map, the navigational algorithm or planner must generate paths that take this conclusion into consideration.

It is worth noting that bearing-only SLAM is generally no better than a bearing-range implementation. The bearing-only SLAM problem is faced with singularities during the landmark initialization process that the a bearing-range implementation does not have. Furthermore, only one measurement per step is taken to each landmark. This means that less information can be gathered per step. Despite this, bearing-only SLAM has its uses. If a sensor is incapable of gathering range information or if the range information is extremely unreliable, then using a bearing-only landmark initialization method may be desirable if not essential.

Our experimental results, both indoors and outdoors, show that this iterative approach to landmark initialization is certainly a practical and computationally efficient method if there is no means of disambiguating landmarks and having the data association problem solved or if only a low frame rate is obtainable from a vision sensor. This method will be useful for scenarios in which bearing-only SLAM is desirable, but may prove essential when batch estimation becomes intractable or initial conditions are hard to determine as well as when feature tracking is not possible due to low frame rates or other such restrictions.

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